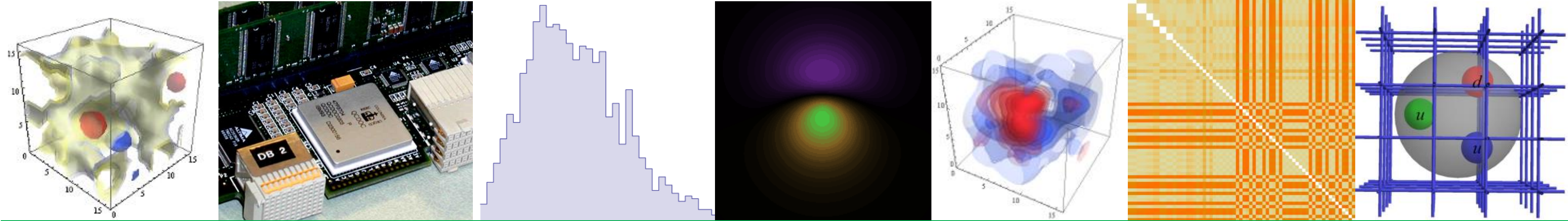


Parton Distributions in Meson and Baryons from Lattice QCD

HUEY-WEN LIN

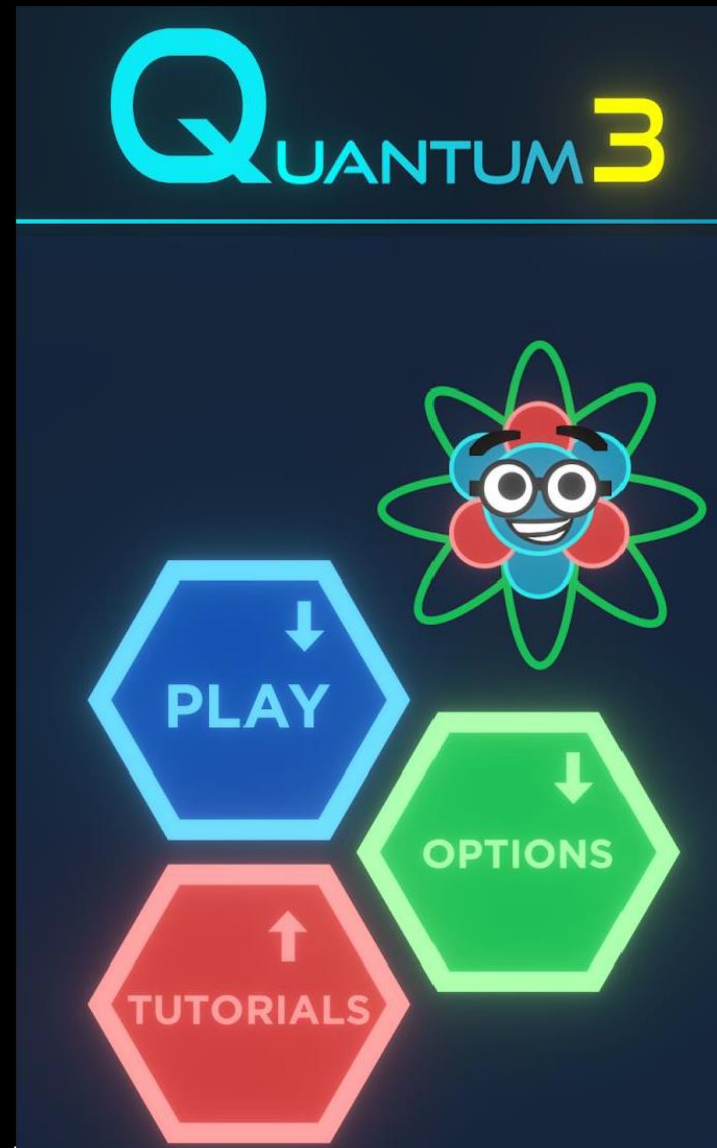
This work is supported by the NSF under grant PHY 1653405
"CAREER: Constraining Parton Distribution Functions for New-Physics Searches"

Everyone Should Learn QCD

§ Learn QCD on your phone

Google Play Store   Apple Appstore

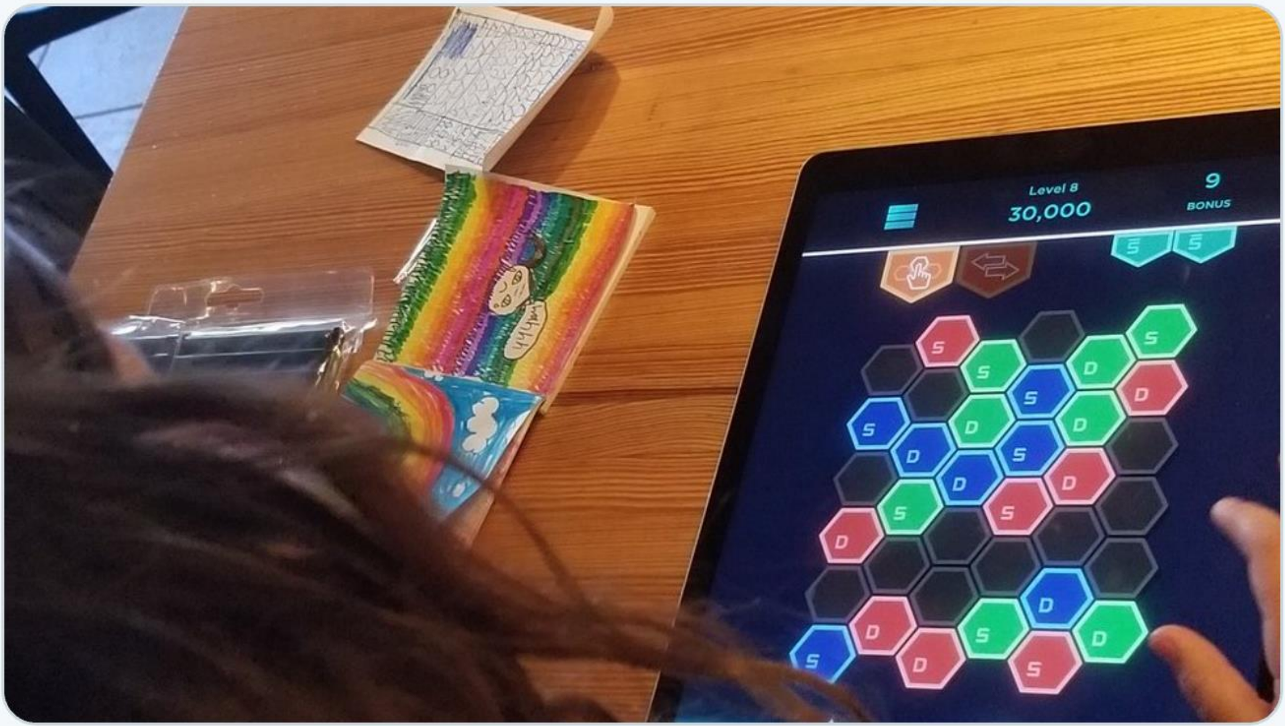
It's only fair that those without physics PhDs get to play with QCD too!



Kids Become the Teachers

- Love to see more tweets like this

Chris Oakley @DrPhysOaks · Mar 21
Replying to @NSF_MPS and @michiganstateu
...and my seven year old is explaining to me how to create Xi - ...



https://twitter.com/NSF_MPS/status/1106577806673264640

Outline

§ LaMET and Nucleon Parton Distributions

§ New Results on Meson Distributions

- Pion GPD, kaon DA/PDF

§ Future prospects

- Systematics study, machine learning, prediction

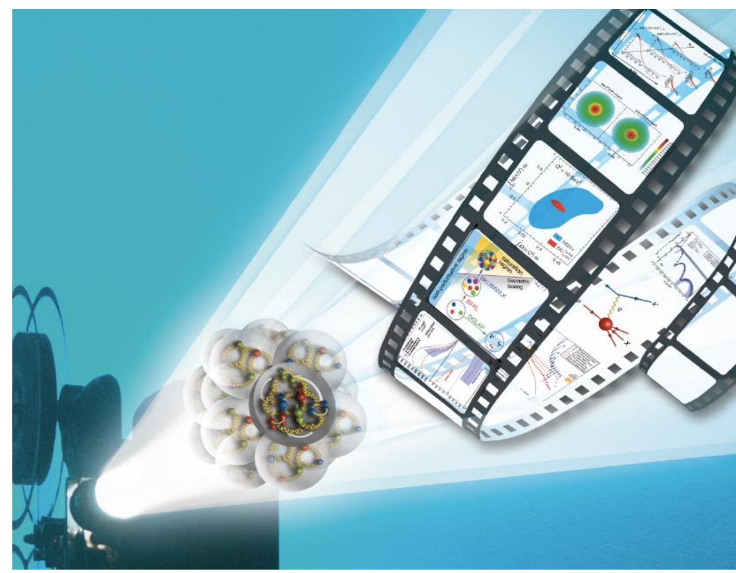
Thanks to MILC collaboration for sharing 2+1+1 HISQ lattices and RBC/UKQCD for sharing 2+1 DWF lattices

Parton Distribution Functions

§ PDFs are universal quark/gluon distributions of nucleon

- Many ongoing/planned experiments
  (BNL, JLab, J-PARC, COMPASS, GSI, EIC, LHeC, …)

Electron Ion Collider:
The Next QCD Frontier



Imaging of the proton

How are the sea quarks and gluons, and their spins, distributed in space and momentum inside the nucleon?

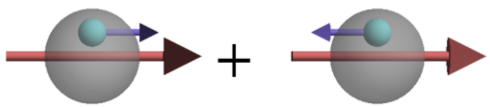
EIC White Paper, 1212.1701

What can we do on the lattice?

PDFs on the Lattice

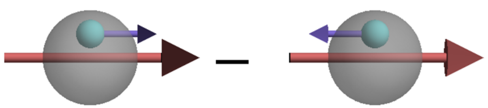
- Lattice calculations rely on operator product expansion, only provide moments

most well known



Quark density/unpolarized

$$\langle x^n \rangle_q = \int_{-1}^{1} dx\, x^n q(x)$$



Helicity

longitudinally polarized

$$\langle x^n \rangle_{\Delta q} = \int_{-1}^{1} dx\, x^n \Delta q(x)$$

Transversity

transversely polarized

$$\langle x^n \rangle_{\delta q} = \int_{-1}^{1} dx\, x^n \delta q(x)$$

very poorly known

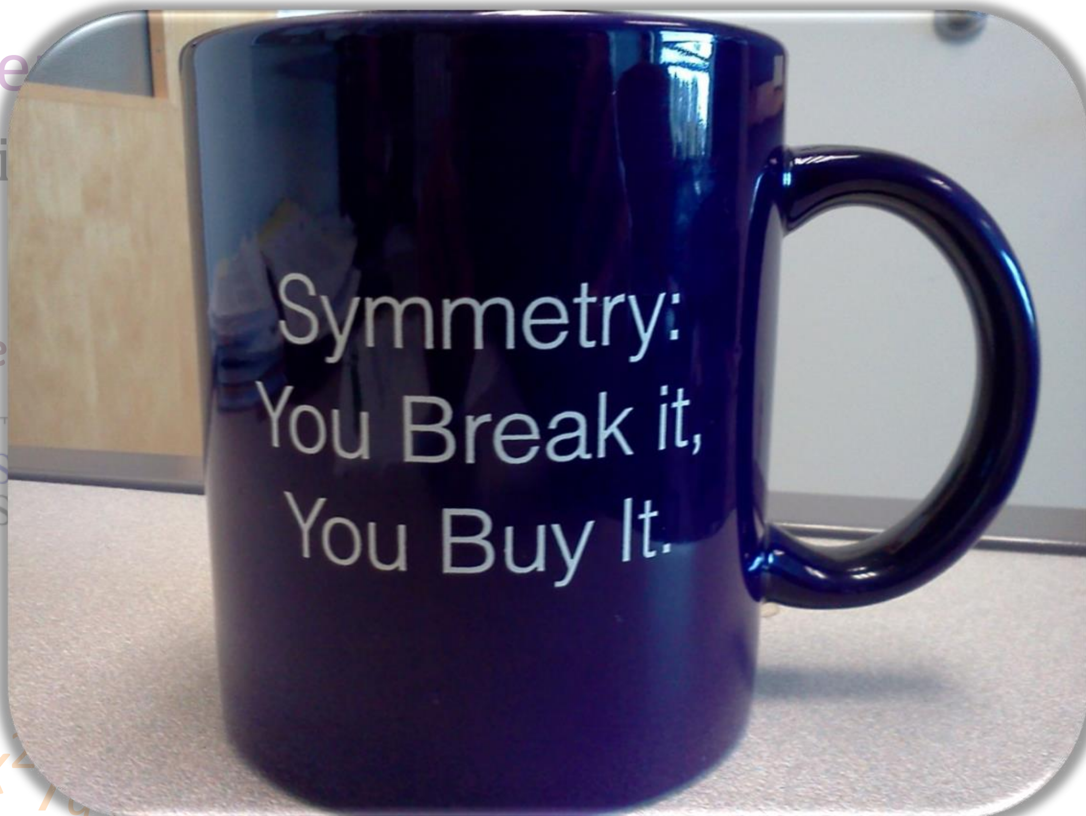
- True distribution can only be recovered with all moments

Problem with Moments

- § For higher moments, ops mix with lower-dimension ops
  - Renormalization is difficult too
- § Relative e
  - Calculati

Dolgov et
Göckeler e

SAM):
clover



A NEW HOPE

It is a period of war and economic uncertainty.

Turmoil has engulfed the galactic republics.

Basic truths at foundation of the human civilization are disputed by the dark forces of the evil empire.

A small group of QCD Knights from United Federation of Physicists has gathered in a remote location on the third planet of a star called Sol on the inner edge of the Orion–Cygnus arm of the galaxy.

The QCD Knights are the only ones who can tame the power of the Strong Force, responsible for holding atomic nuclei together, for giving mass and shape to matter in the Universe.

They carry secret plans to build the most powerful

LaMET

Large-Momentum Effective Theory (LaMET)

X. Ji, PRL. 111, 262002 (2013)

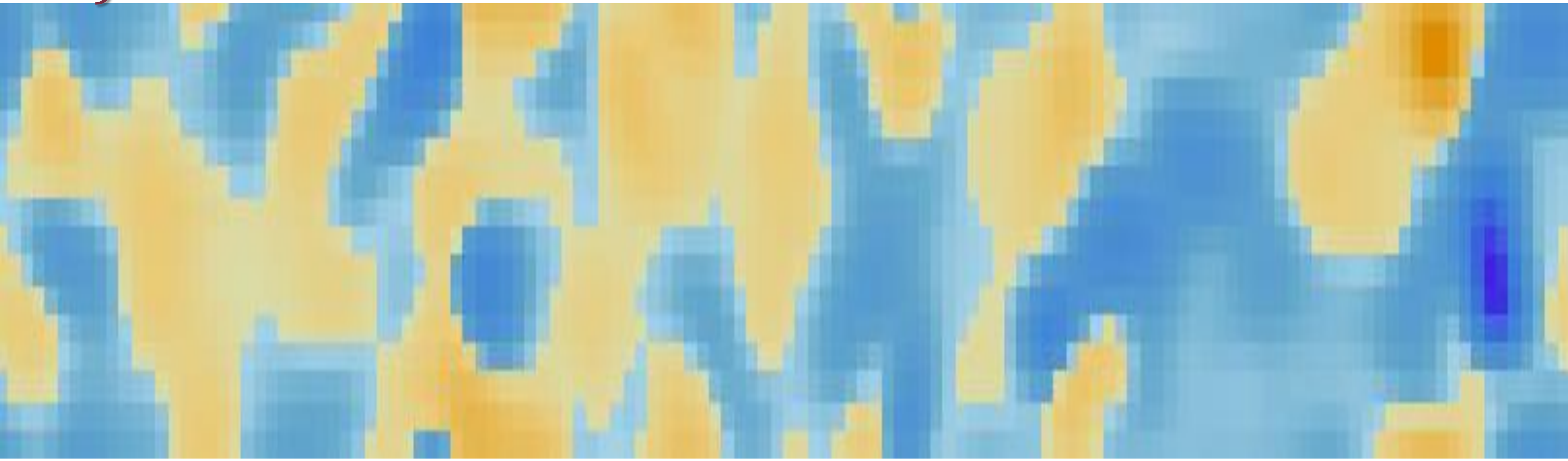
- Calculate the parton distributions through the infinite-momentum frame Feynman, Phys. Rev. Lett. 23, 1415 (1969)

LaMET Recipe

Large-Momentum Effective Theory for PDFs

X. Ji, PRL. 111, 262002 (2013)

1) Calculate nucleon matrix elements on the lattice



Thanks to MILC collaboration for sharing their 2+1+1 HISQ lattices

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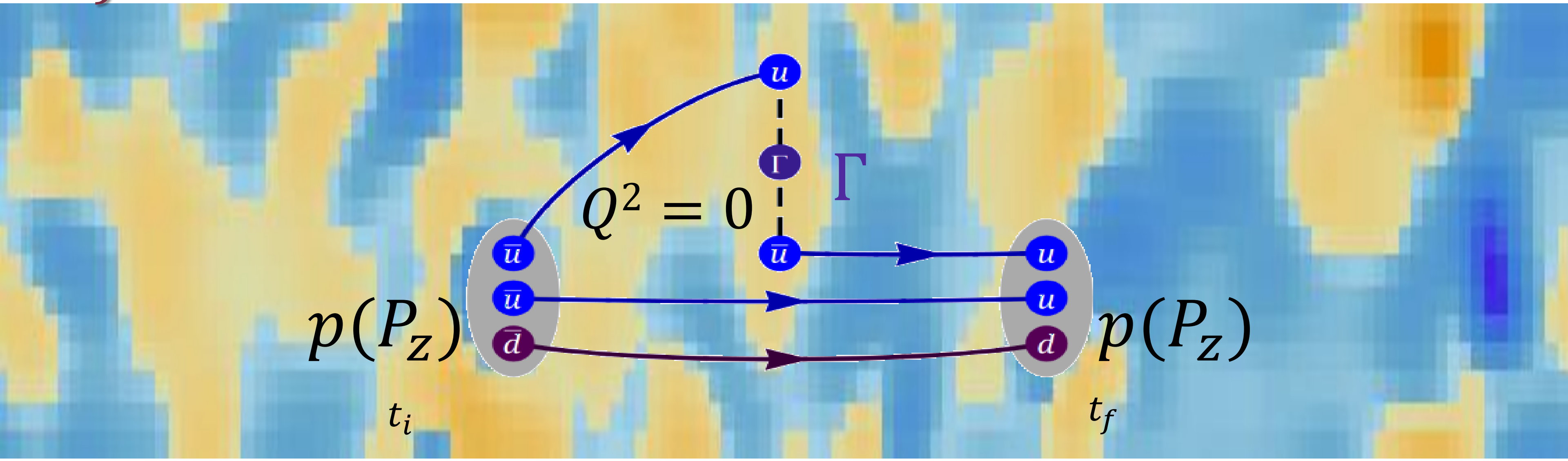

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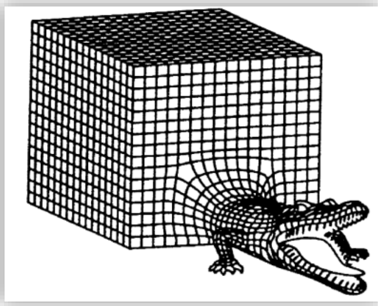

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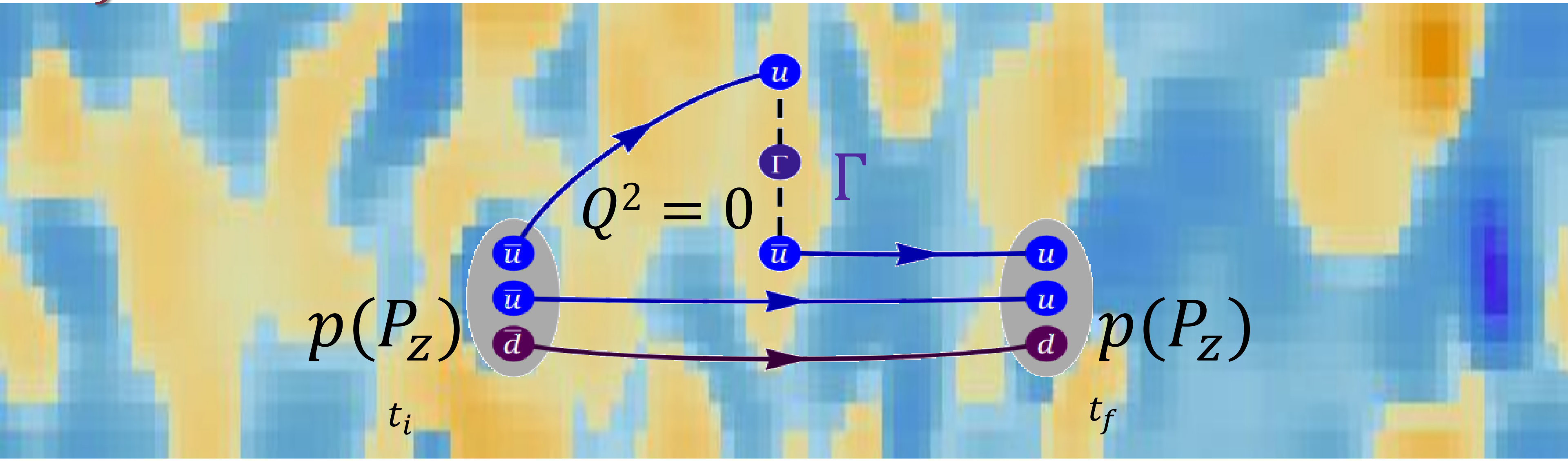
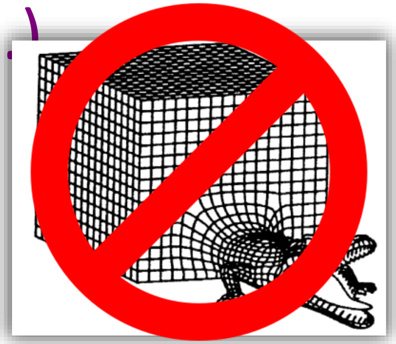


Thanks to MILC collaboration for sharing their 2+1+1 HISQ lattices

§ Systematic uncertainty (nonzero *a*, finite *L*, etc.)

- Excited-state removal; nonperturbative renorm.
- Extrapolation to the continuum limit $(m_\pi \to m_\pi^{\text{phys}}, L \to \infty, a \to 0)$



LaMET Recipe

Large-Momentum Effective Theory (LaMET)

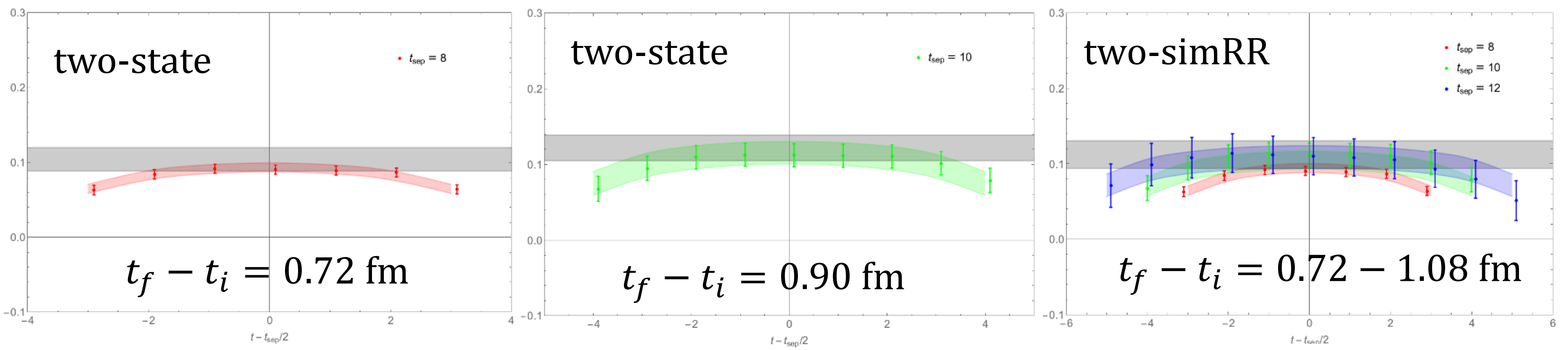
X. Ji, PRL. 111, 262002 (2013)

1) Calculate nucleon matrix elements on the lattice (*z* dependence)

Systematics: excited-state contamination

$M_\pi \approx$ **135 MeV** $a \approx 0.09$ fm

$P_z = 2.6$ GeV, $z = 4$, real matrix elements (plot by Zhouyou Fan)



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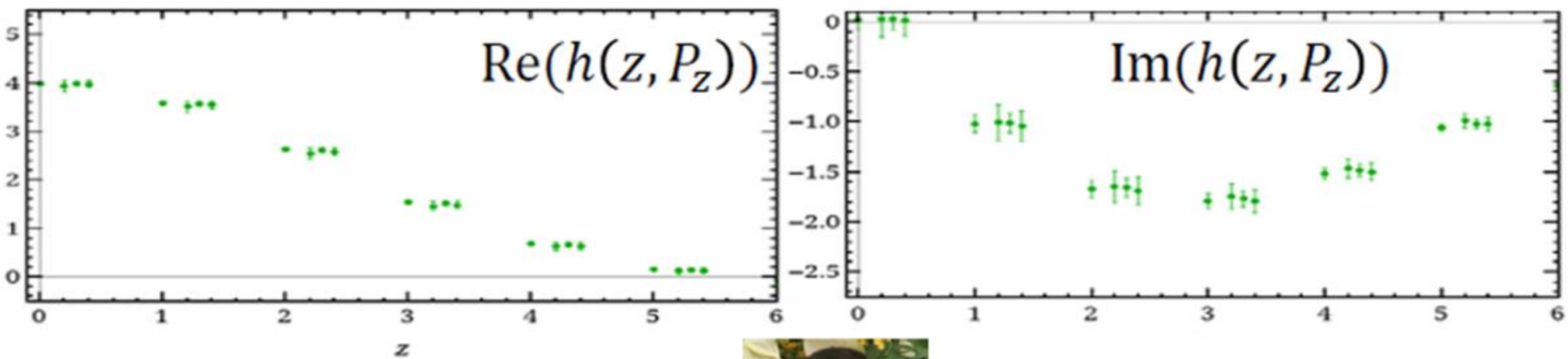


Ruizi Li

Blinded 3-state fits produced consistent results

LaMET Recipe

Large-Momentum Effective Theory (LaMET)

X. Ji, PRL. 111, 262002 (2013)

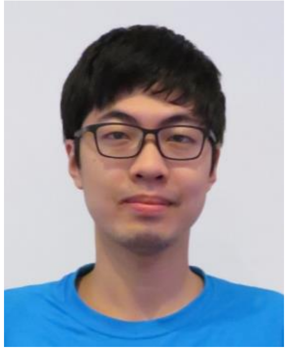
1) Calculate nucleon matrix elements on the lattice

(z dependence)

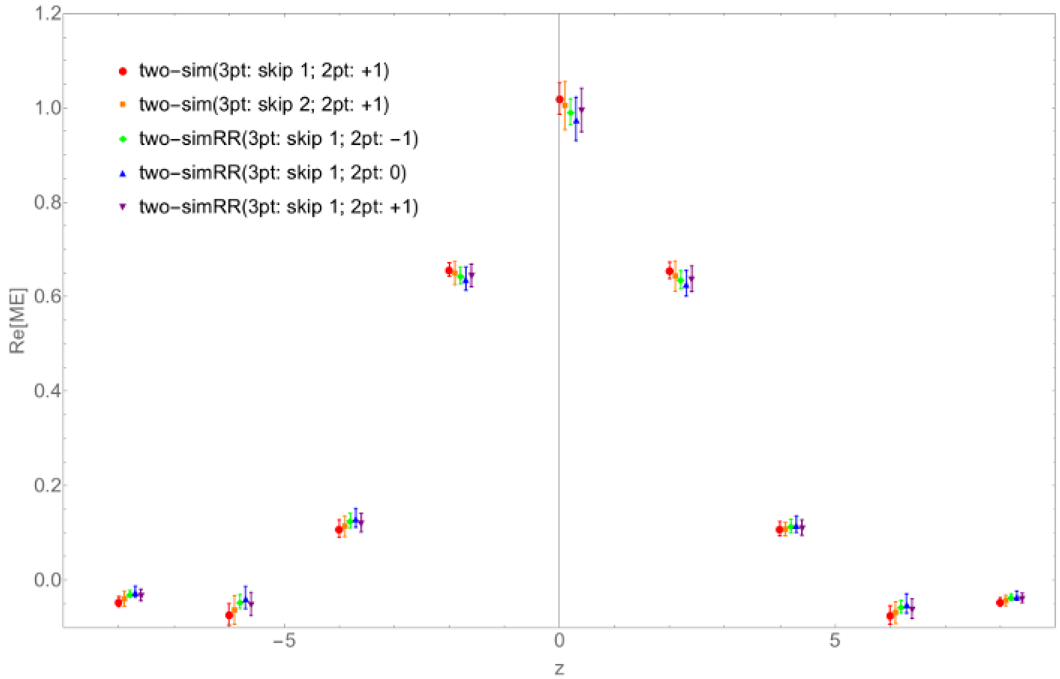
Systematics: stability in extracting matrix elements

$M_\pi \approx$ **135 MeV** $a \approx 0.09$ fm

$P_z = 2.6$ GeV,



Zhouyou Fan



LaMET Recipe

Large-Momentum Effective Theory (LaMET)

X. Ji, PRL. 111, 262002 (2013)

1) Calculate nucleon matrix elements on the lattice

(z dependence)

2) Compute quasi-distribution via

$$\tilde{q}(x,\mu,P_z) = \int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \left| \overline{\psi}(z) \, \Gamma \exp\left(-ig\int_0^z dz' \, A_z(z')\right) \psi(0) \right| P \right\rangle$$

$x = k_z/P_z$

lattice z coordinate

hadron momentum $P_\mu = \{P_t, 0, 0, P_z\}$

product of lattice gauge links

LaMET Recipe

Large-Momentum Effective Theory (LaMET)

X. Ji, PRL. 111, 262002 (2013)

1) Calculate nucleon matrix elements on the lattice (*z* dependence)

2) Compute quasi-distribution via

$$\tilde{q}(x,\mu,P_z) = \int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \middle| \overline{\psi}(z)\, \Gamma \exp\left(-ig\int_0^z dz'\, A_z(z')\right) \psi(0) \middle| P \right\rangle$$

3) Recover true distribution (take $P_z \to \infty$ limit)

$$\tilde{q}(x,\mu,P_z) = \int_{-\infty}^{\infty} \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\mu}{P_z}\right) \boldsymbol{q(y,\mu)} + \mathcal{O}\left(M_N^2/P_z^2\right) + \left(\Lambda_{\rm QCD}^2/P_z^2\right)$$

X. Xiong et al., 1310.7471; J.-W. Chen et al, 1603.06664

LaMET Recipe

Large-Momentum Effective Theory (LaMET)

X. Ji, PRL. 111, 262002 (2013)

1) Calculate nucleon matrix elements on the lattice

(z dependence)

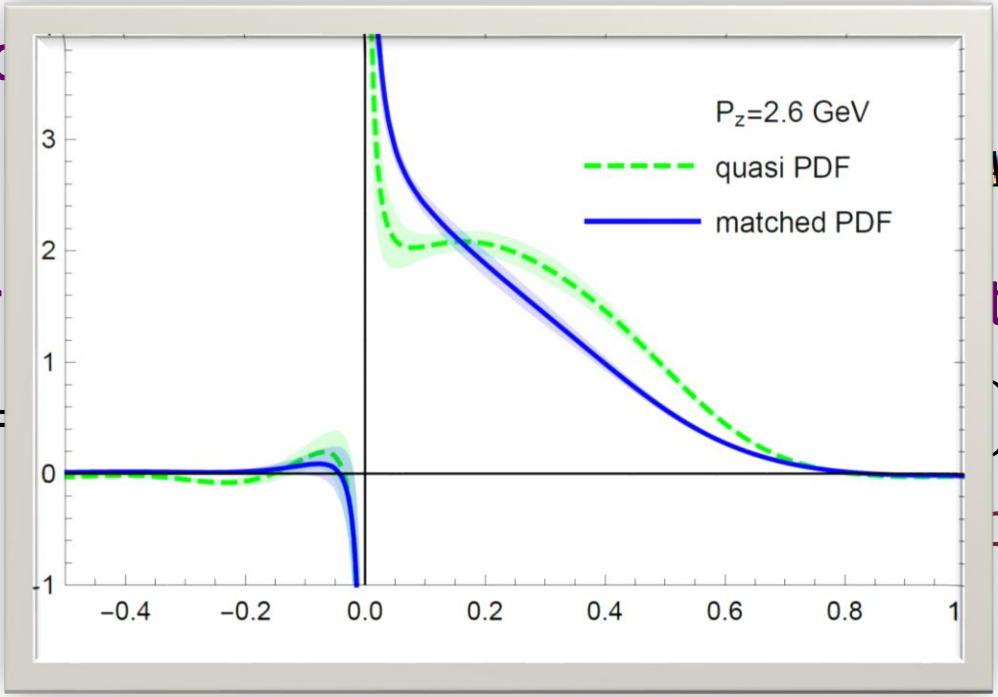
2) Compute

$\tilde{q}(x,\mu,P_z)$ ... $\Lambda_z(z')\big)\,\psi(0)\,\big|\,P\big\rangle$

3) Recover tr ... t)

$\tilde{q}(x,\mu,P_z\,) =$ ... $\big) + \big(\Lambda^2_{\mathrm{QCD}}/P_z^2\big)$

Chen et al, 1603.06664



§ Matching is a crucial step in recovering the true lightcone distribution

Progress in the theoretical development of LaMET

- **Renormalization:**

Ji and Zhang, 2015; Ishikawa et al., 2016, 2017; Chen, Ji and Zhang, 2016;

Xiong, Luu and Meiβner, 2017; Constantinou and Panagopoulos, 2017; Ji, Zhang, and Y.Z., 2017; J. Green et al., 2017; Ishikawa et al. (LP3), 2017; Wang, Zhao and Zhu, 2017; Spanoudes and Panagopoulos, 2018.

- **Factorization:**

Ma and Qiu, 2014, 2015, 2017; Izubuchi, Ji, Jin, Stewart and Y.Z., 2018.

- **One-loop matching:**

Xiong, Ji, Zhang and Y.Z., 2014; Ji, Schaefer, Xiong and Zhang, 2015; Xiong and Zhang, 2015; Constantinou and Panagopoulos, 2017; I. Stewart and Y. Z., 2017; Wang, Zhao and Zhu, 2017; Izubuchi, Ji, Jin, Stewart and Y.Z., 2018.

- **Power corrections:**

J.-W. Chen et al., 2016; A. Radyushkin, 2017.

- **Transvers momentum dependent parton distribution function:**

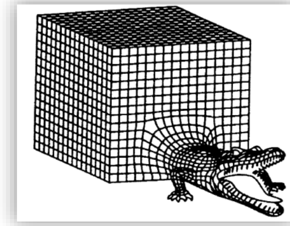
Ji, Xiong, Sun, Yuan, 2015; Ji, Jin, Yuan, Zhang and Y.Z., 2018; Ebert, Stewart and Y.Z., in progress.

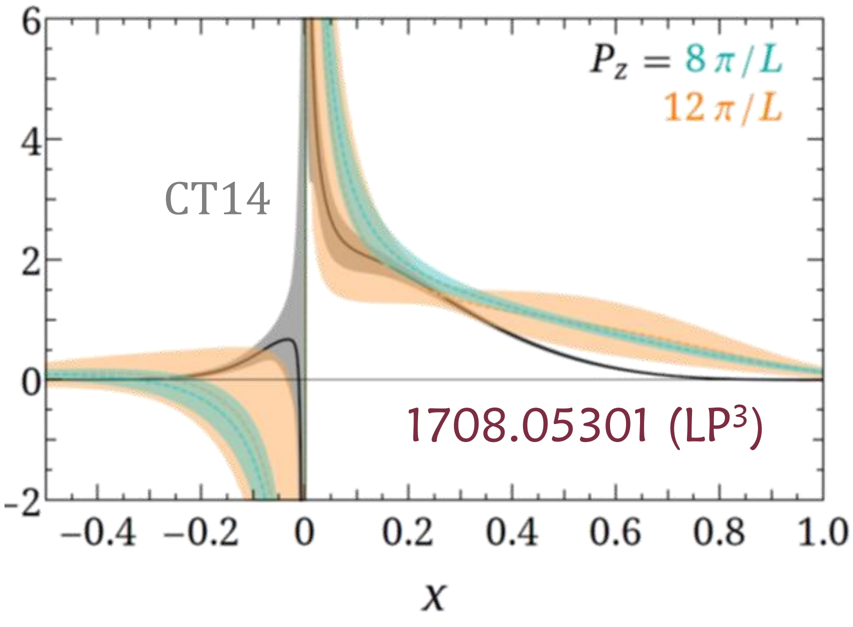
Slide credit: Yong Zhao, CIPANP 2018 Plenary talk; also see Y. Zhao's Lattice 2019 talk

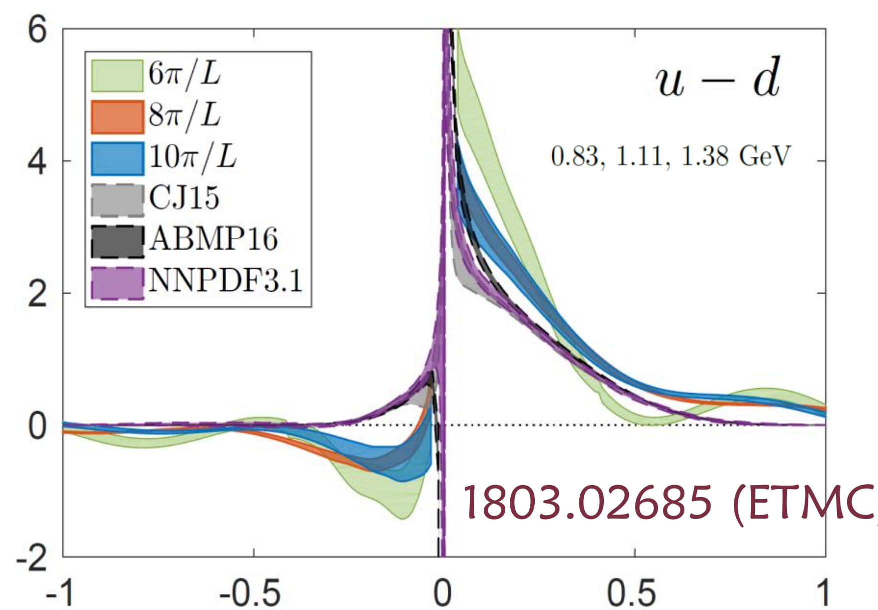
Physical Pion Mass Results

§ Exciting! Two collaborations' results at physical pion mass

- Boost momenta $P_z \leq 1.4$ GeV
- Study of systematics still needed





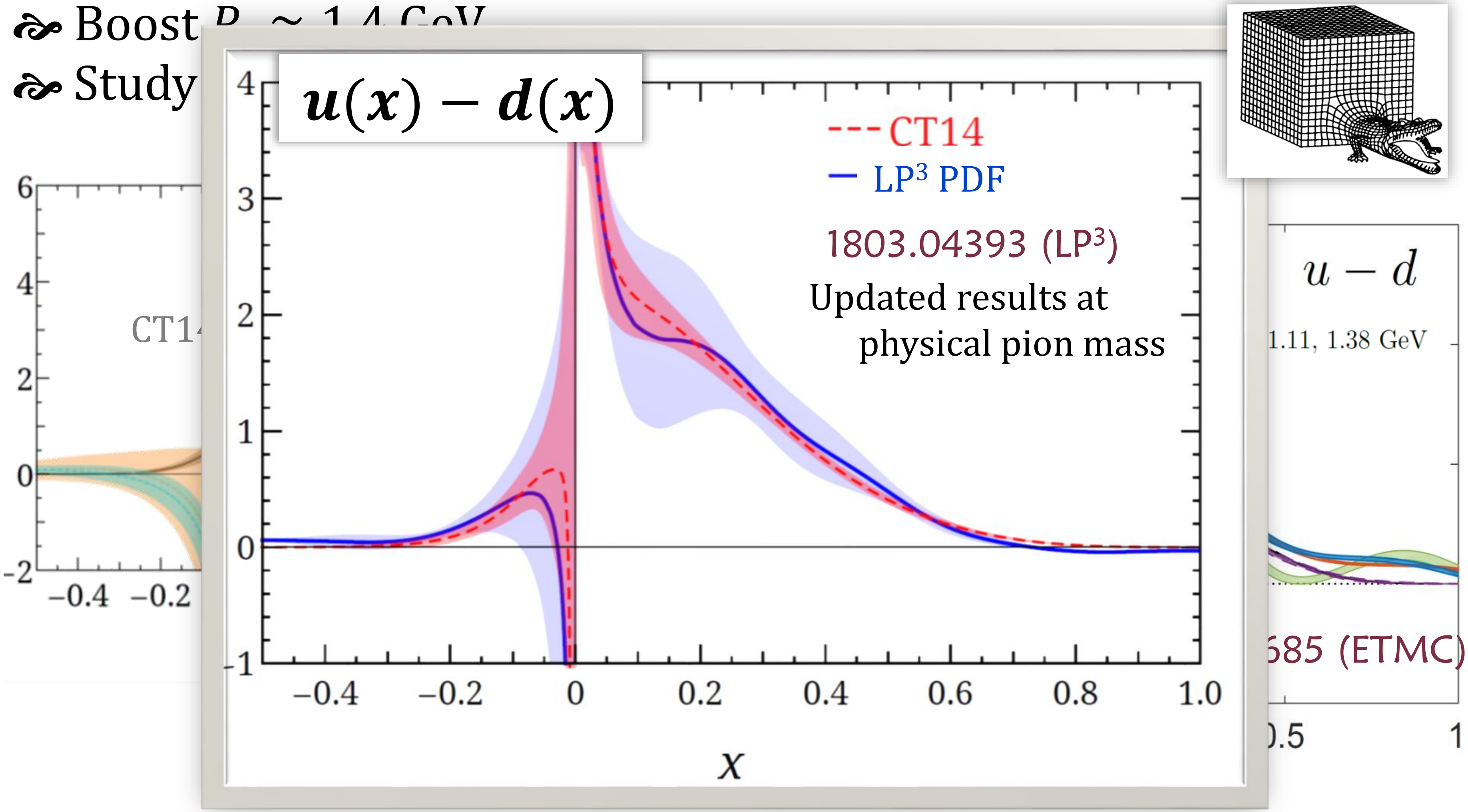


No parametrization! (e.g. $xf(x,\mu_0) = a_0 x^{a_1}(1-x)^{a_2}P(x)$)
Less pretty results;
less likely to exactly coincide with global fits.

Physical Pion Mass Results

§ Exciting! Two collaborations' results at physical pion mass

- Boost P ~ 1.4 GeV
- Study

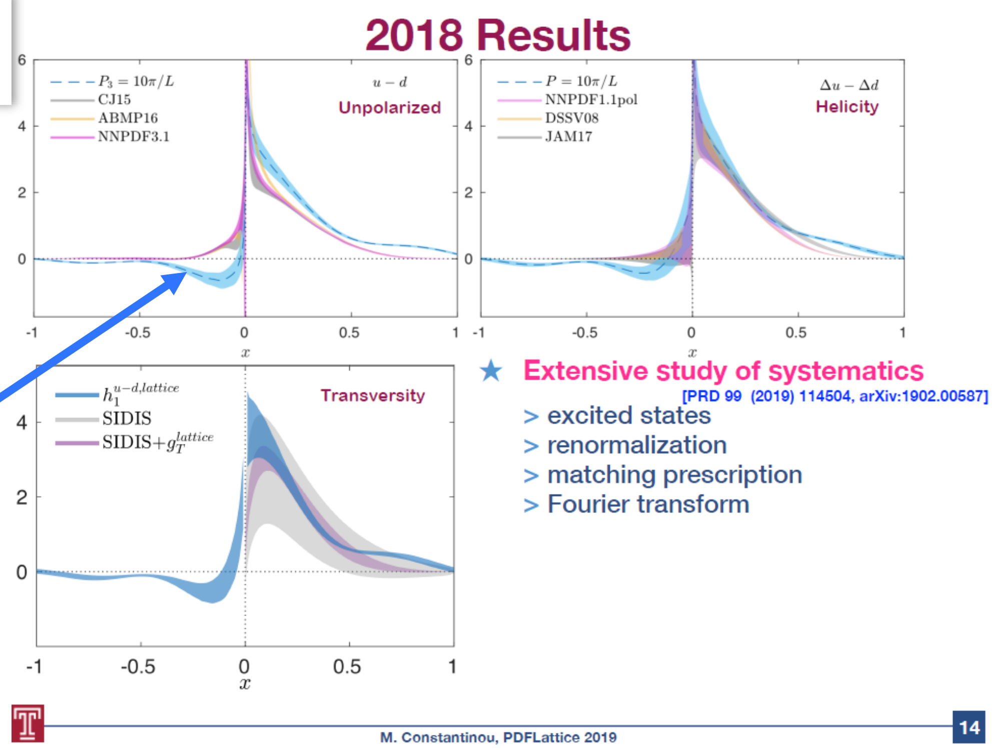


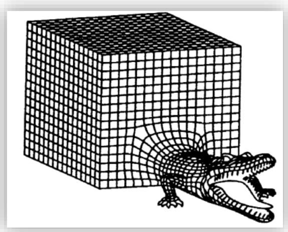
Different Errors

$u(x) - d(x)$

- Systematics *studied* but **NOT** included

Huge underestimate of the errors!

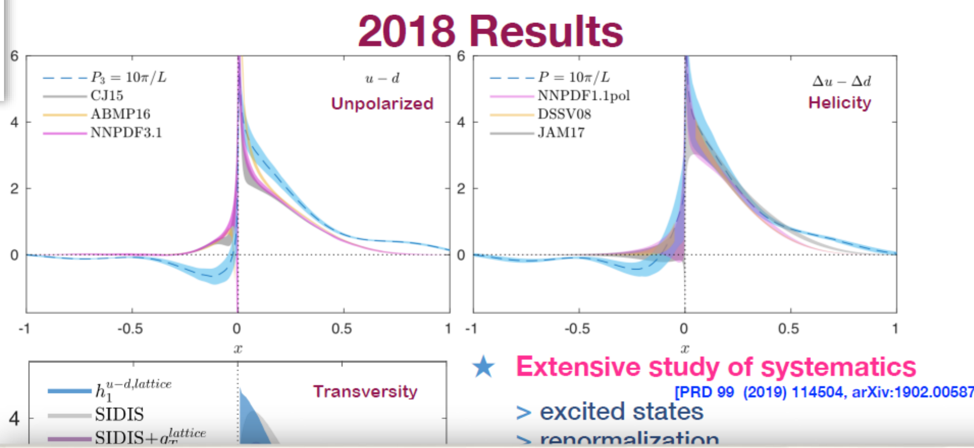




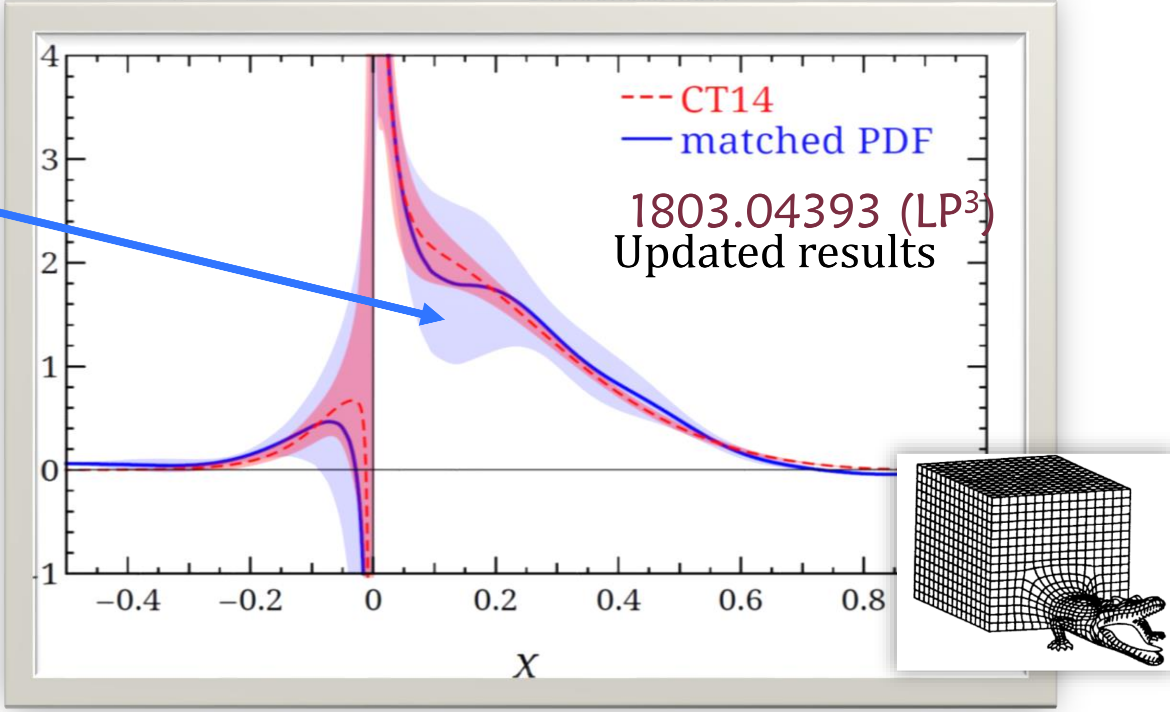
Different Errors

$$u(x) - d(x)$$

- Systematics *studied* but **NOT** included



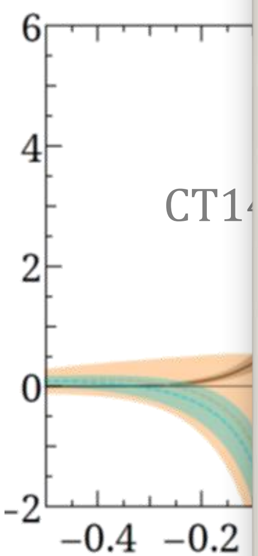
Includes listed errors

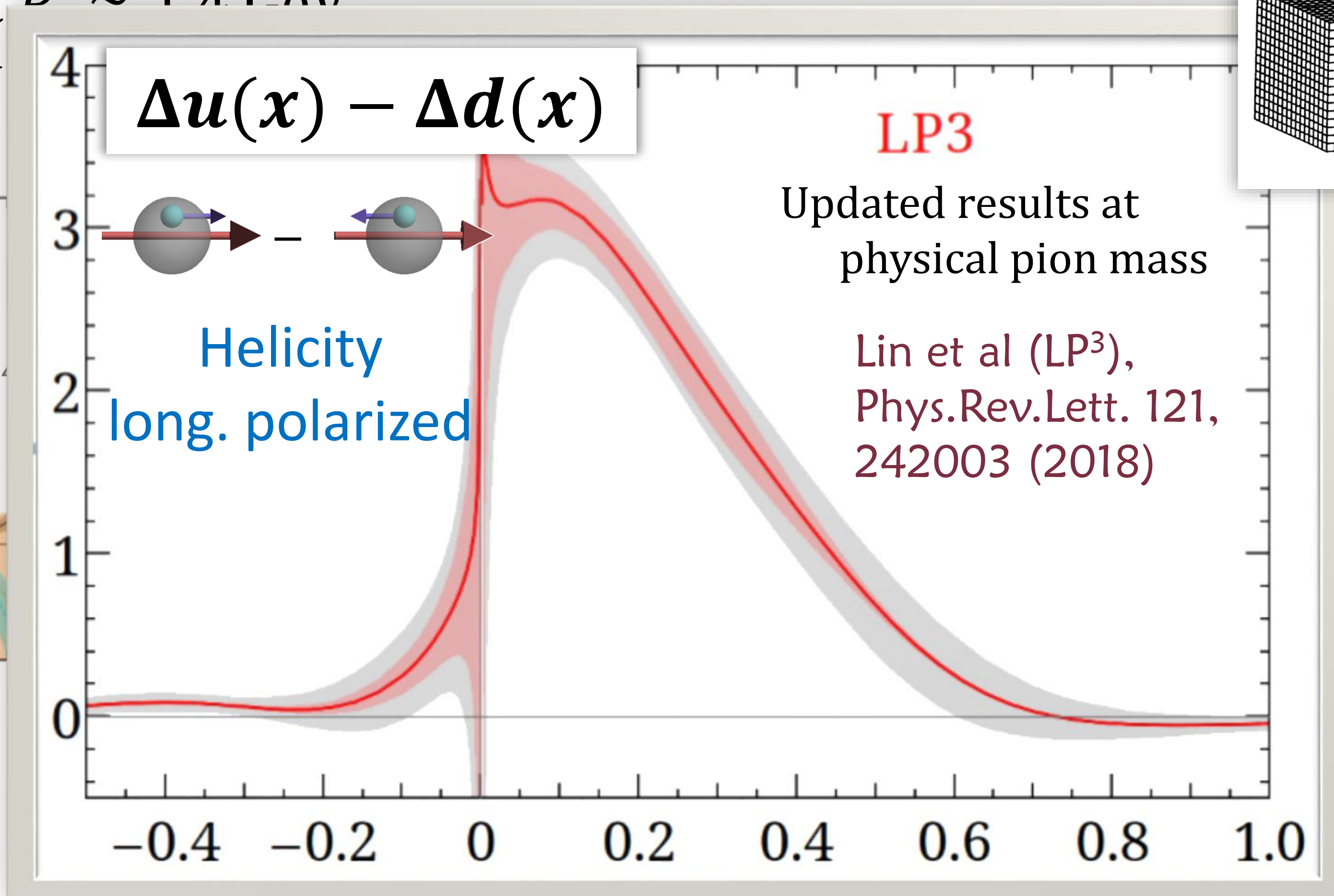


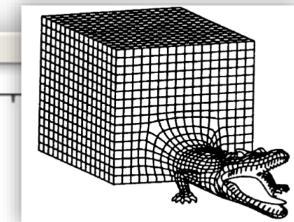
Physical Pion Mass Results

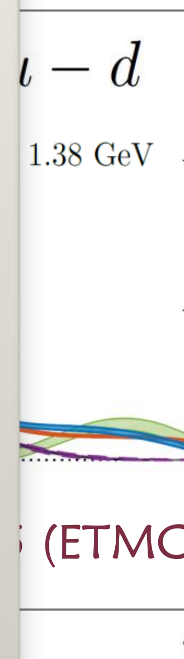
§ Exciting! Two collaborations' results at physical pion mass

- Boost $P_z \sim 1.4$ GeV
- Study





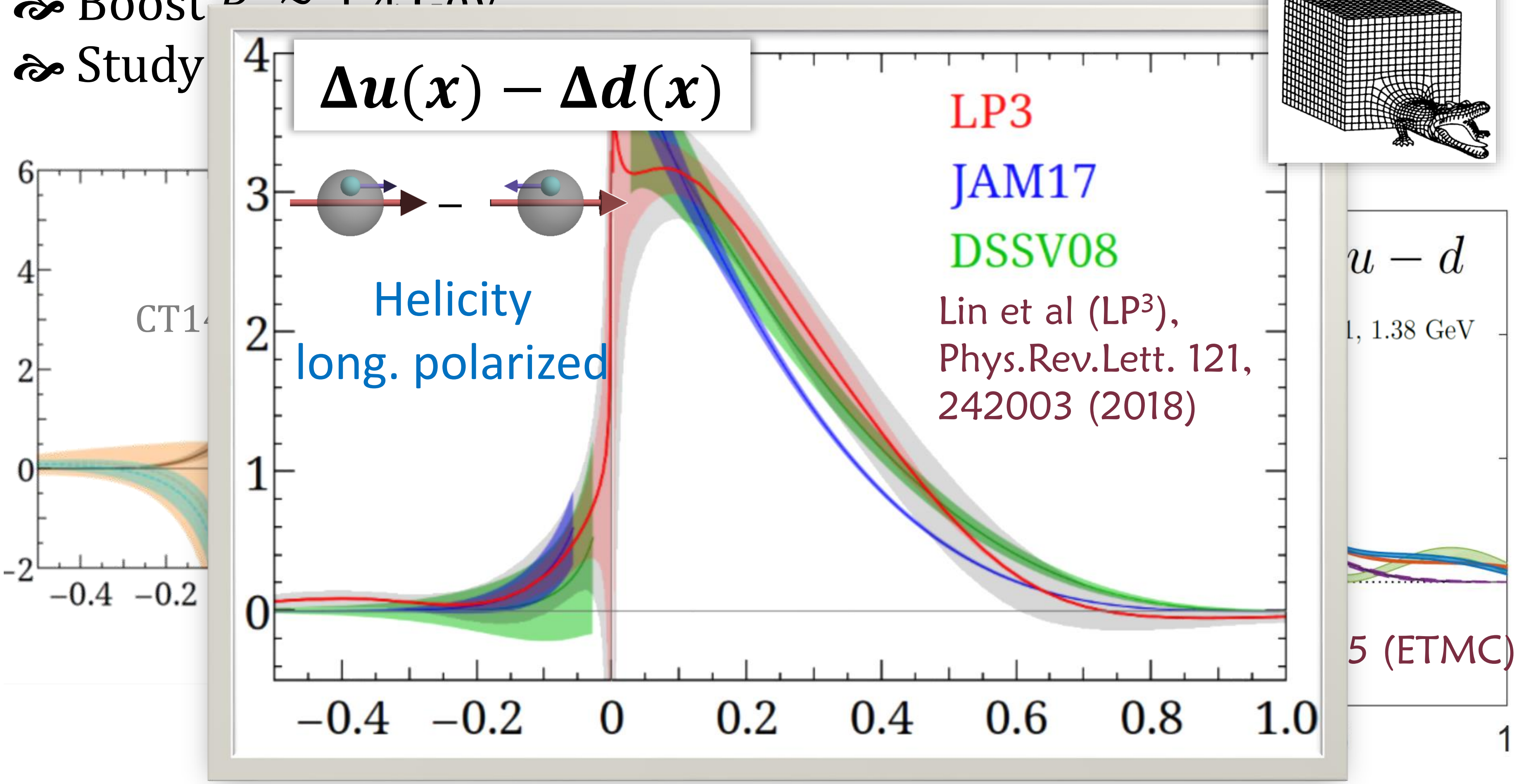




Physical Pion Mass Results

§ Exciting! Two collaborations' results at physical pion mass

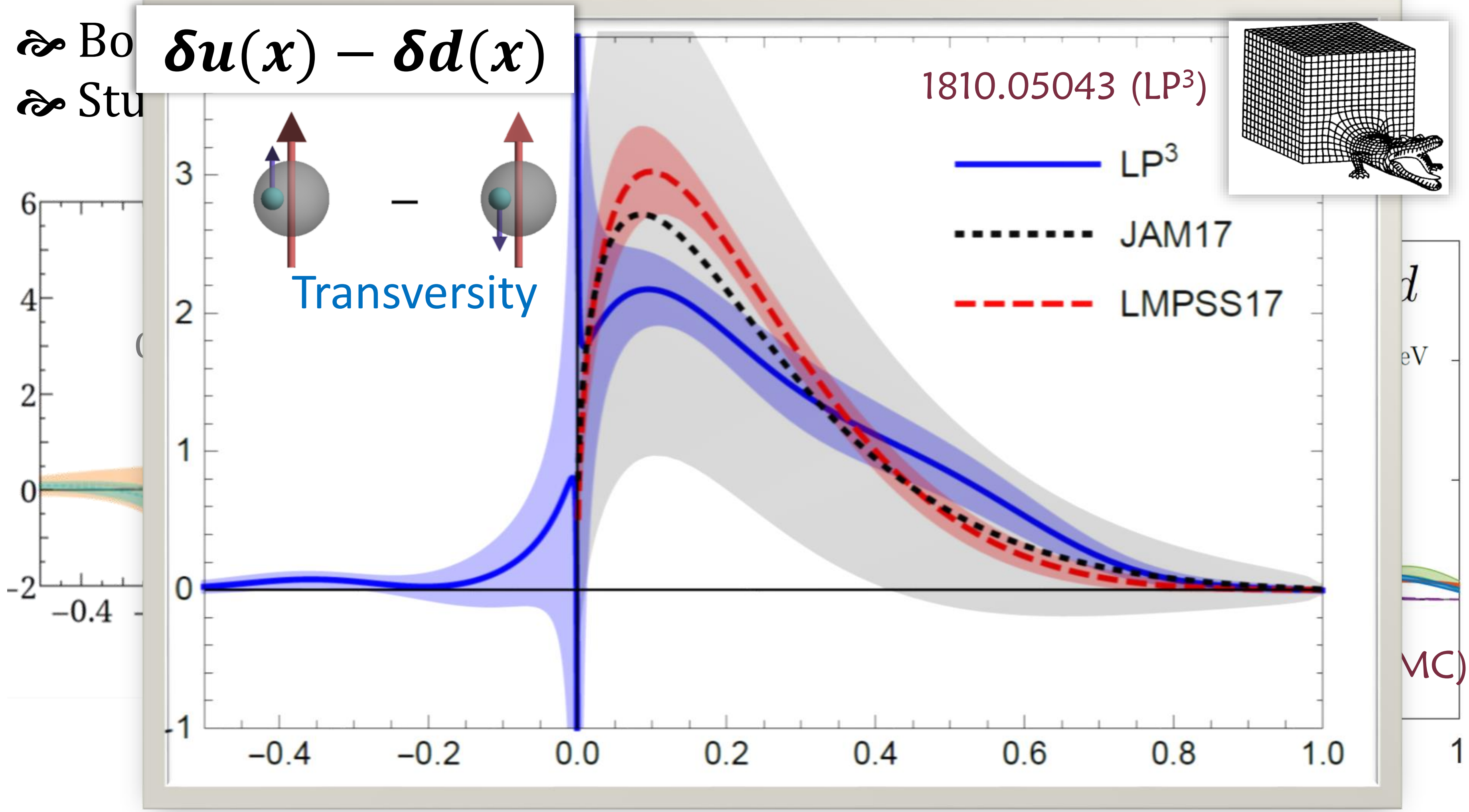
- Boost $P_z \sim 1.4$ GeV
- Study



Physical Pion Mass Results

§ Exciting! Two collaborations' results at physical pion mass

- Bo
- Stu



Physical Pion Mass Results

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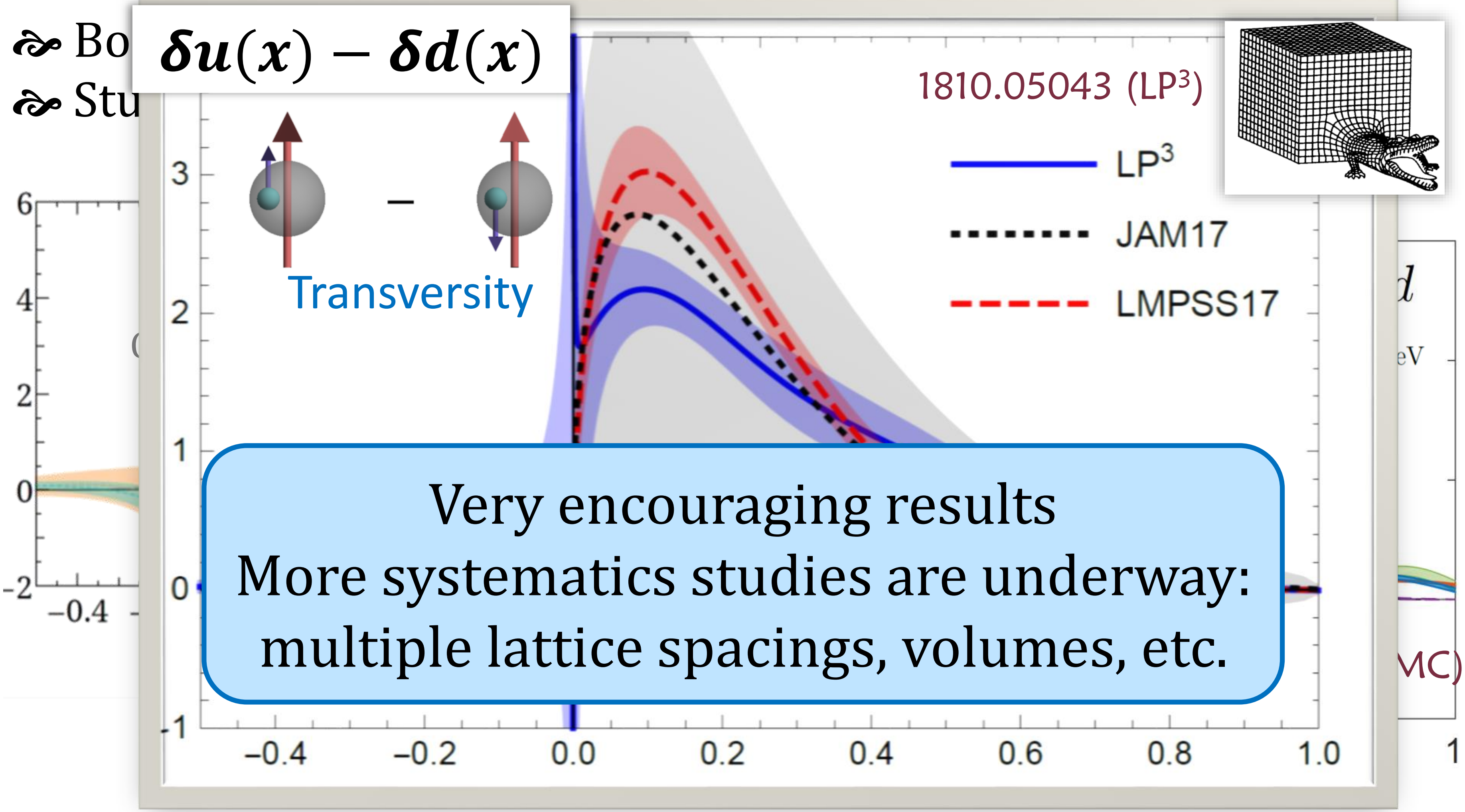
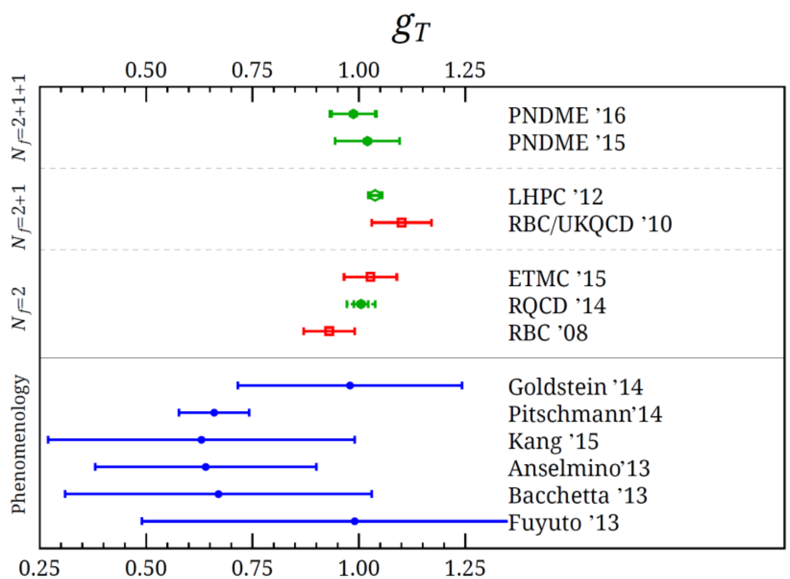


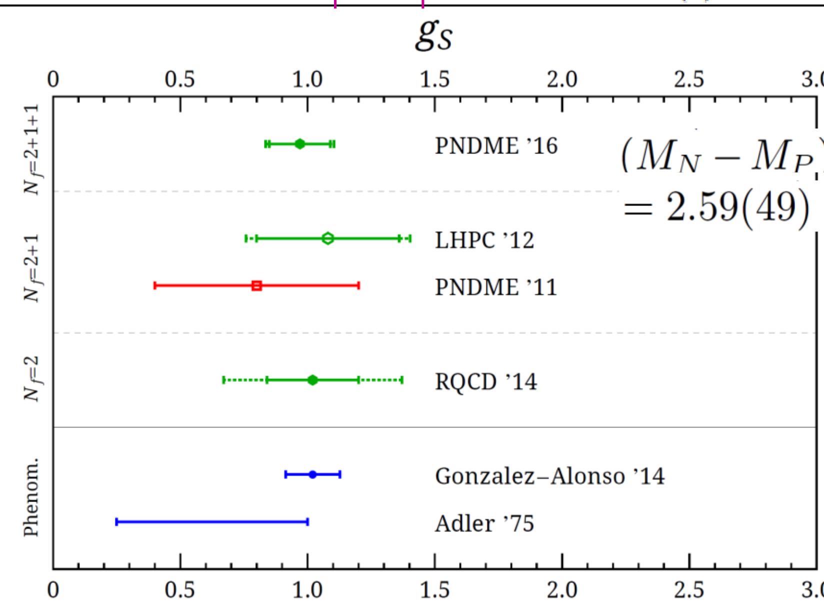
State-of-the-Art Moments

- § FLAG rating system
- § New: excited-state rating

PNDME, 1506.06411; 1606.07049

Collaboration	Ref.	publication status	N_f	chiral extrapolation	continuum extrapolation	finite volume	excited state	renormalization	g_T
PNDME'15	This work	P	2+1+1	★	★	★	★	★	1.020(76)[a]
ETMC'13	[30]	C	2+1+1	■	○	○	■	★	1.11(3)[b]
LHPC'12	[28]	A	2+1	★	○	★	○	★	1.037(20)[c]
RBC/UKQCD'10	[29]	A	2+1	○	■	★	★	★	1.10(7)[d]
RQCD'14	[31]	P	2	★	★	★	○	★	1.005(17)(29))[e]
ETMC'13	[30]	C	2	★	■	○	■	○	1.114(46)[f]
RBC'08	[32]	P	2	■	■	★	■	★	0.93(6)[g]



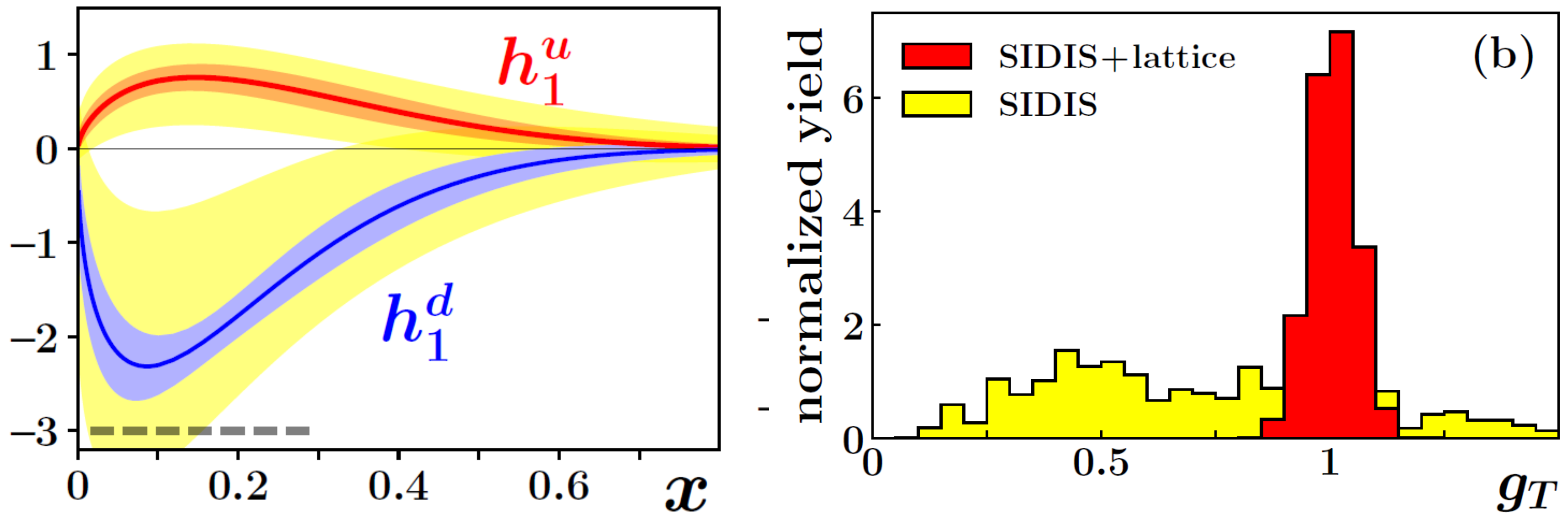


$$(M_N - M_P)^{\mathrm{QCD}} = 2.59(49)\ \mathrm{MeV}$$

Lattice Constraints to PDFs

§ Improved **transversity** distribution with LQCD g_T

- Global analysis with 12 extrapolation forms: $g_T = 1.006(58)$
  - Including only multiple lattice spacing, volumes works
- Use to constrain the global analysis fits to SIDIS $\pi^{\pm}$ production data from proton and deuteron targets



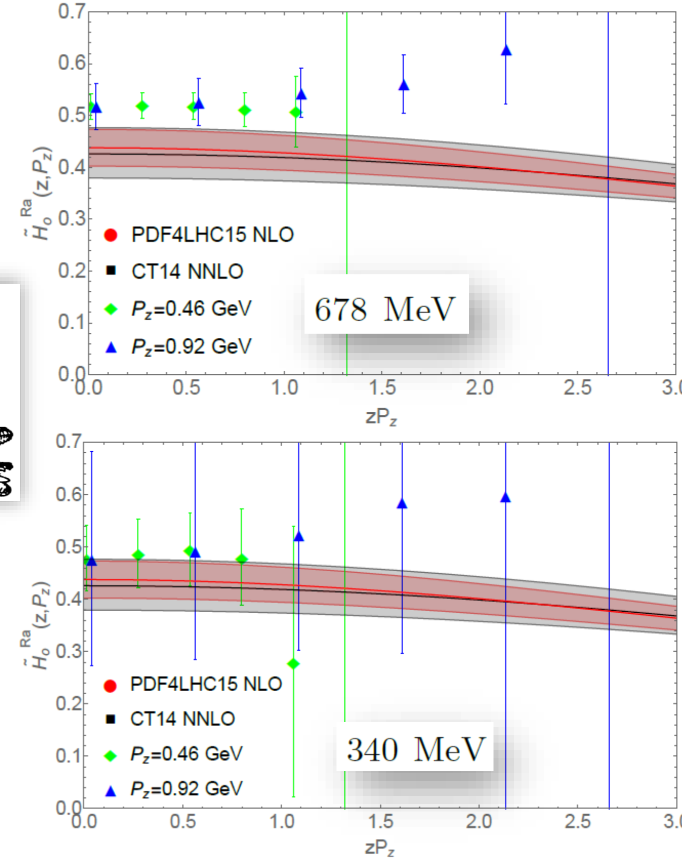
Lin, Melnitchouk, Prokudin, Sato, 1710.09858, Phys. Rev. Lett. 120, 152502 (2018)

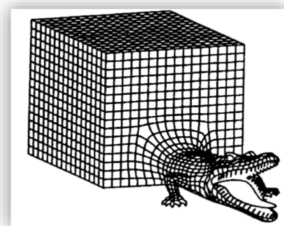
Gluon PDF

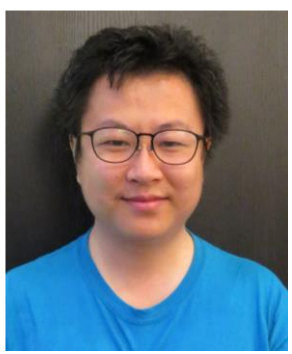
§ Pioneering first glimpse into gluon PDF using LaMET

- Lattice details: overlap/2+1DWF, 0.16fm, 340-MeV sea pion mass
- Study strange/light-quark
- Promising results using coordinate-space comparison,
  but signal does not go far in z
- Hard numerical problem to be solved

Fan. et al, Phys.Rev.Lett. 121, 242001 (2018)







Zhouyou Fan   Yi-Bo Yang

First Lattice GPD

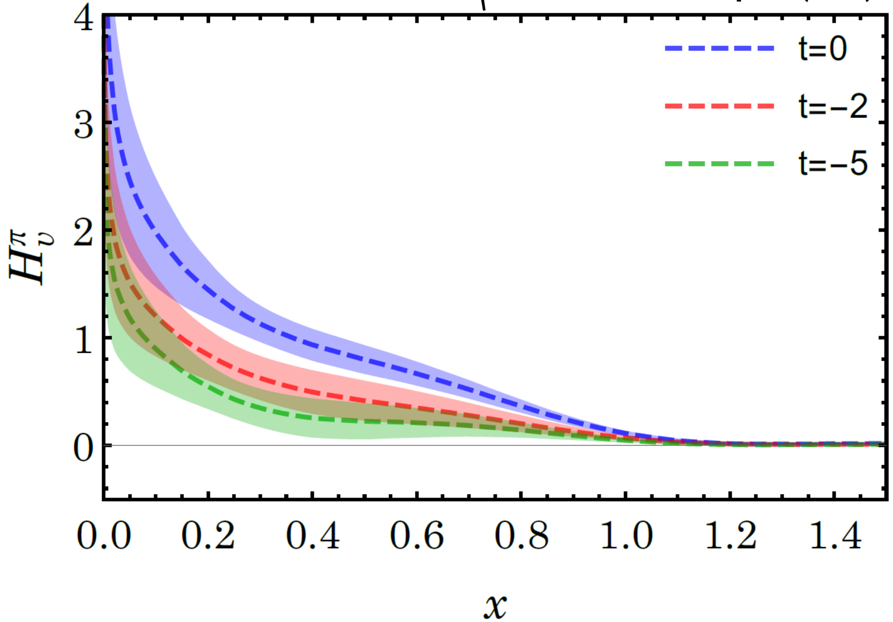
§ Pioneering first glimpse into pion GPD using LaMET

- Lattice details: clover/HISQ, 0.12fm, 310-MeV pion mass

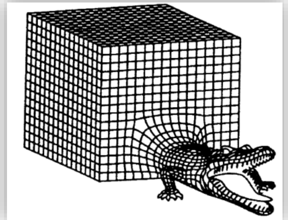
  $P_z \approx 1.3, 1.6$ GeV

  J. Chen, HL, J. Zhang, 1904.12376

$$H_q^\pi(x,\xi,t,\mu) = \int \frac{d\eta^-}{4\pi} e^{-ix\eta^- P^+} \left\langle \pi(P+\Delta/2) \left| \bar{q}\left(\frac{\eta^-}{2}\right) \gamma^+ \Gamma\left(\frac{\eta^-}{2}, -\frac{\eta^-}{2}\right) q\left(-\frac{\eta^-}{2}\right) \right| \pi(P-\Delta/2) \right\rangle$$



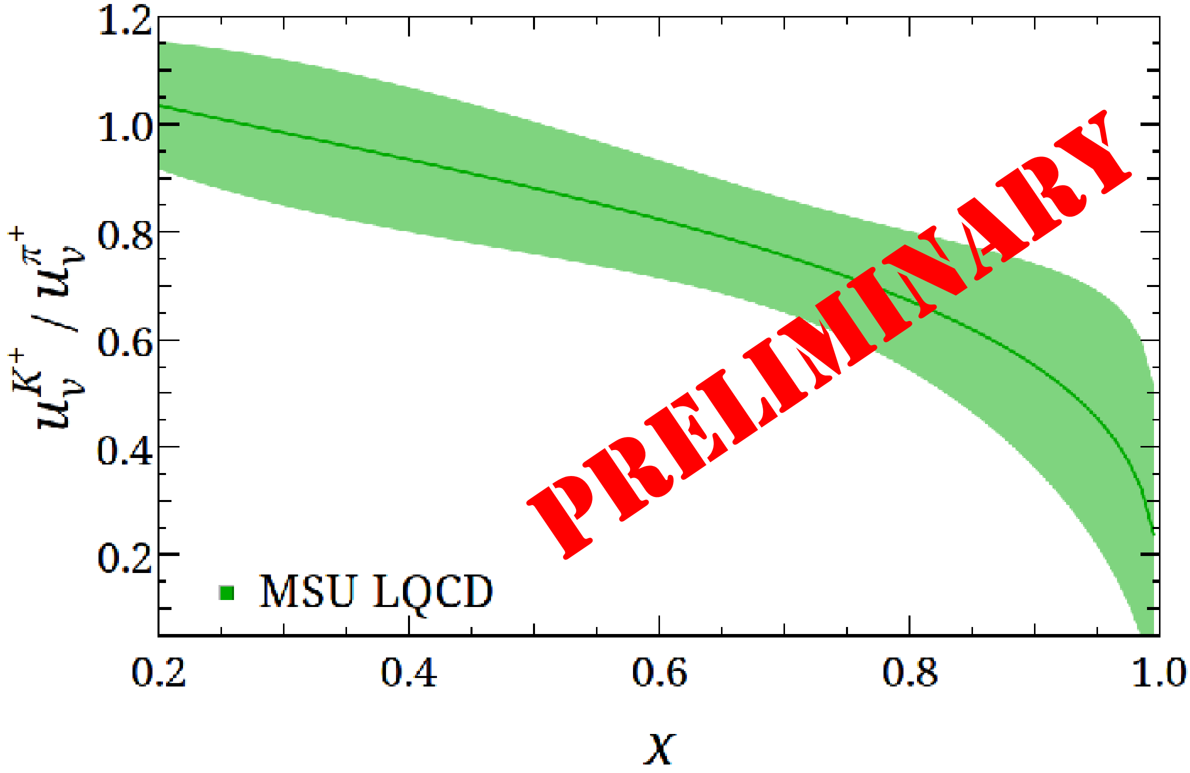
$\xi = 0$

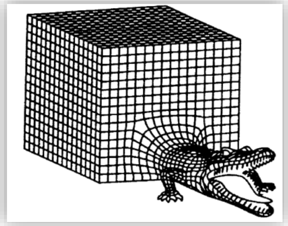


First Kaon PDF

§ We also investigate the kaon PDF with LaMET

- $a = 0.12$ fm, $m_\pi = 220$ MeV, $40^3 \times 64$ MILC HISQ lattice
- $t_{\text{sep}} = 0.6$– 1.08 fm





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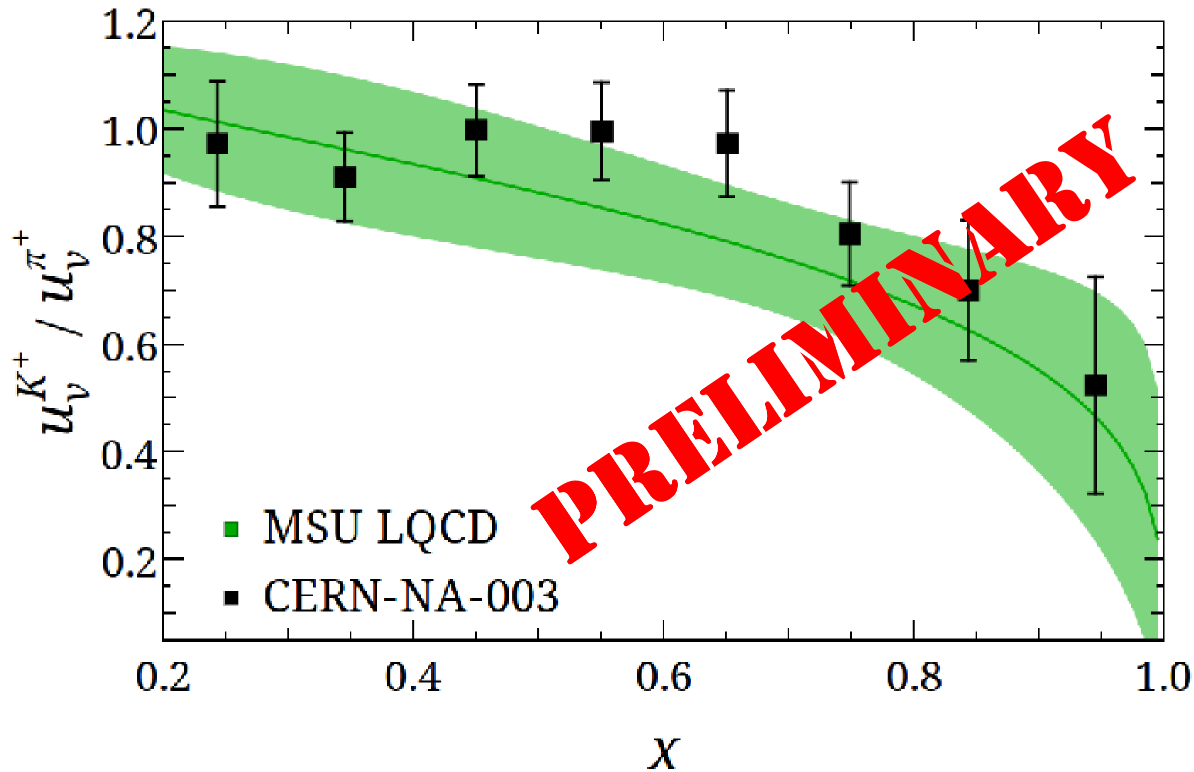
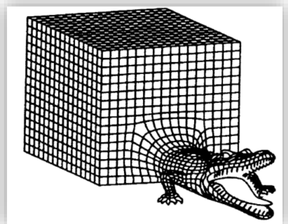
- $a = 0.12$ fm, $m_\pi = 220$ MeV, $40^3 \times 64$ MILC HISQ lattice
- $t_{\text{sep}} = 0.6$– 1.08 fm

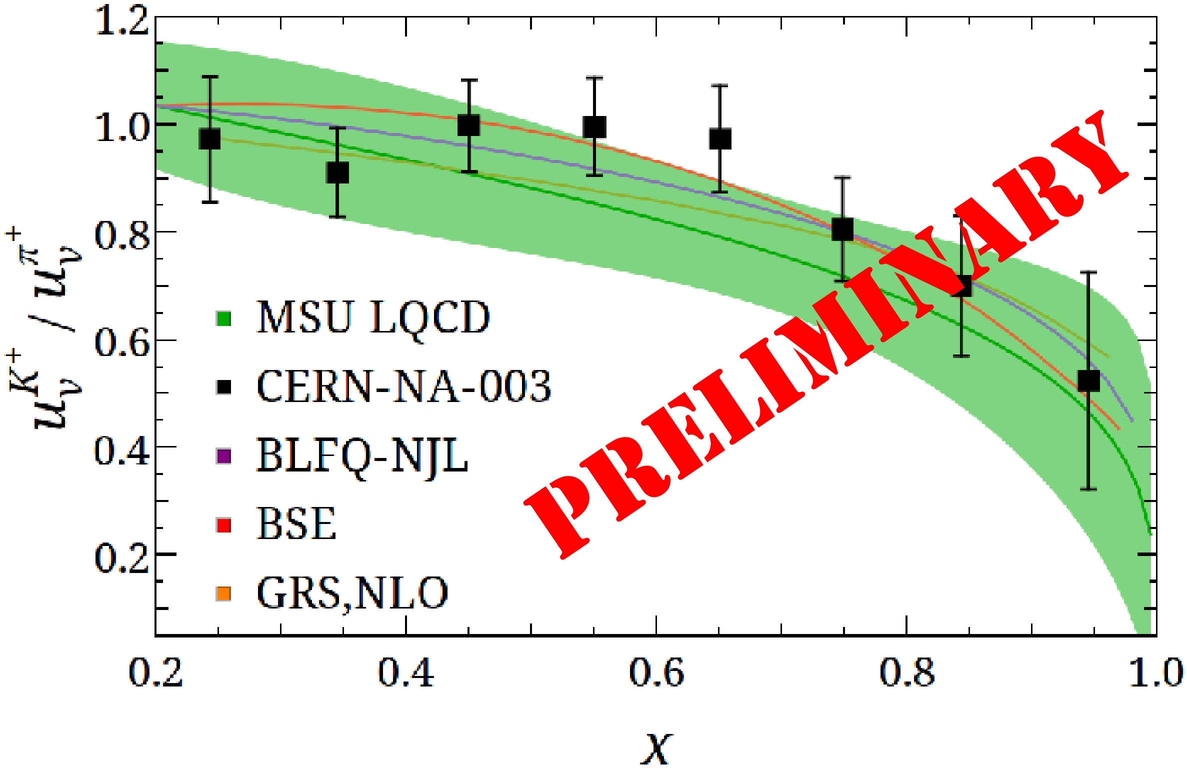


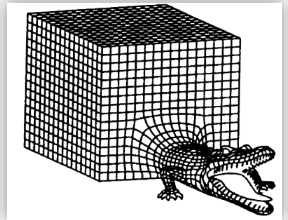


First Kaon PDF

§ We also investigate the kaon PDF with LaMET

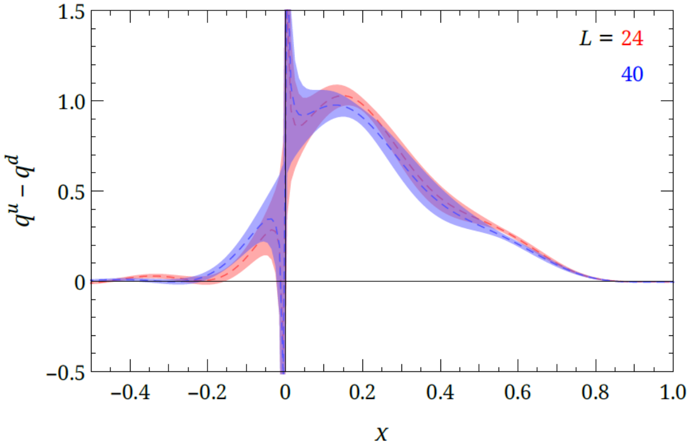
- $a = 0.12$ fm, $m_\pi = 220$ MeV, $40^3 \times 64$ MILC HISQ lattice
- $t_{\text{sep}} = 0.6$– 1.08 fm

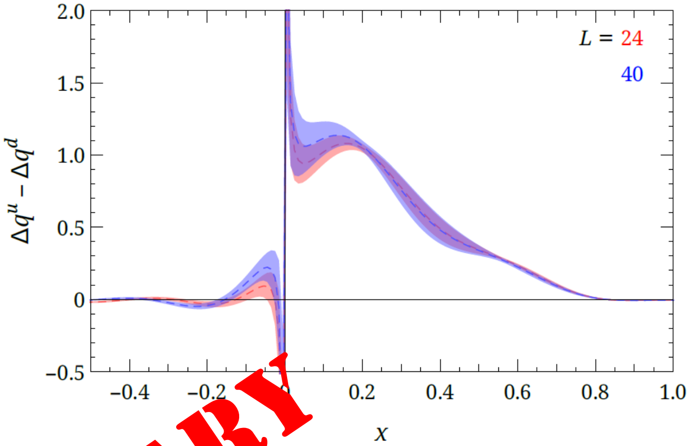


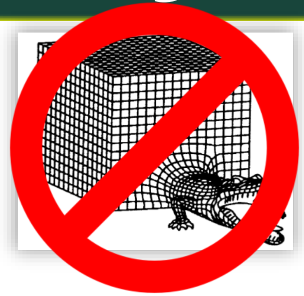


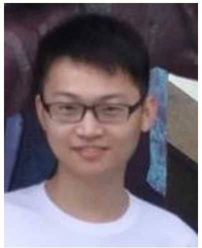
Systematics Study Underway

§ Finite-volume study





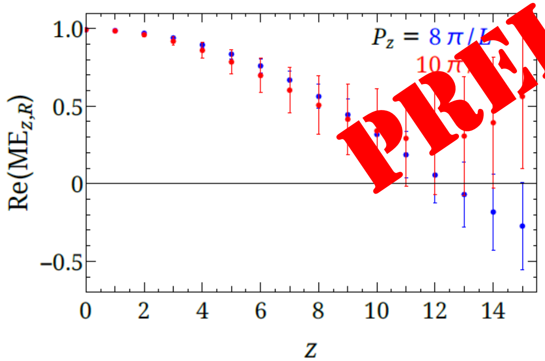


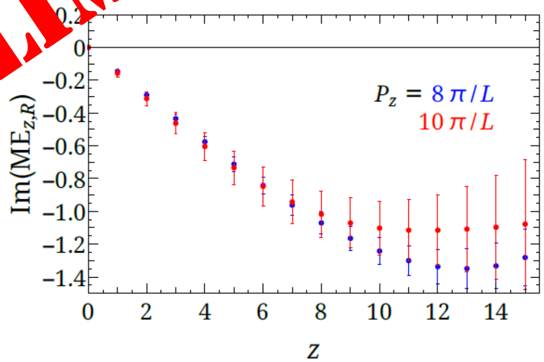


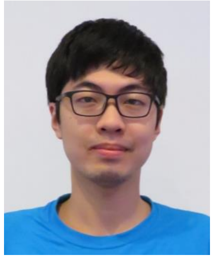
R. Zhang

§ Looking into fine lattice spacing a = 0.042 fm

PRELIMINARY







Zhouyou Fan

+BNL group

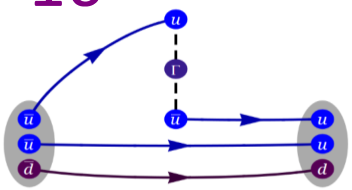
How about the small-x region for the upcoming EIC era?

Difficulties at Small-x

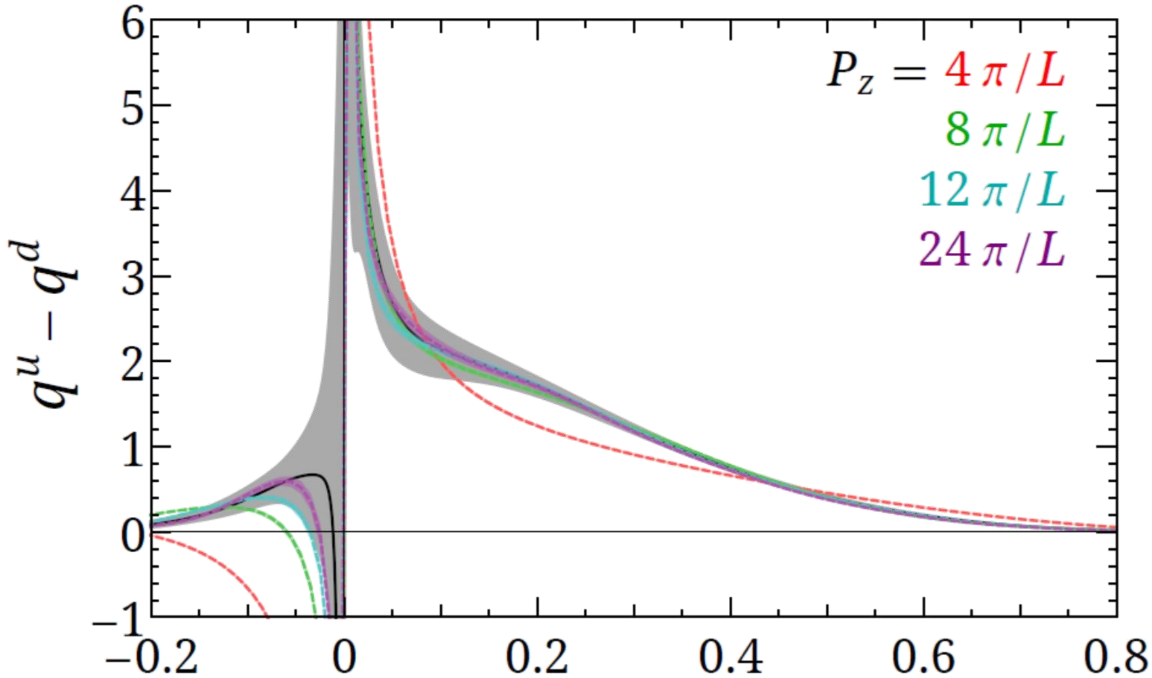
§ Keeping moderate length of displacement $z_{max}=16$

Derivative approach

$$q(x) = \int_{-z_{\max}}^{+z_{\max}} dz \frac{-P_z}{2\pi} \frac{e^{ixP_z z}}{iP_z x} h'(z)$$



1708.05301 (HL et al.)

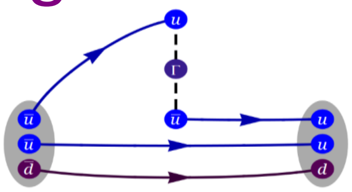


§ Reaching $x<0.1$ for antiquark remains challenging

- Without replying on assumed parametrization

Difficulties at Small-x

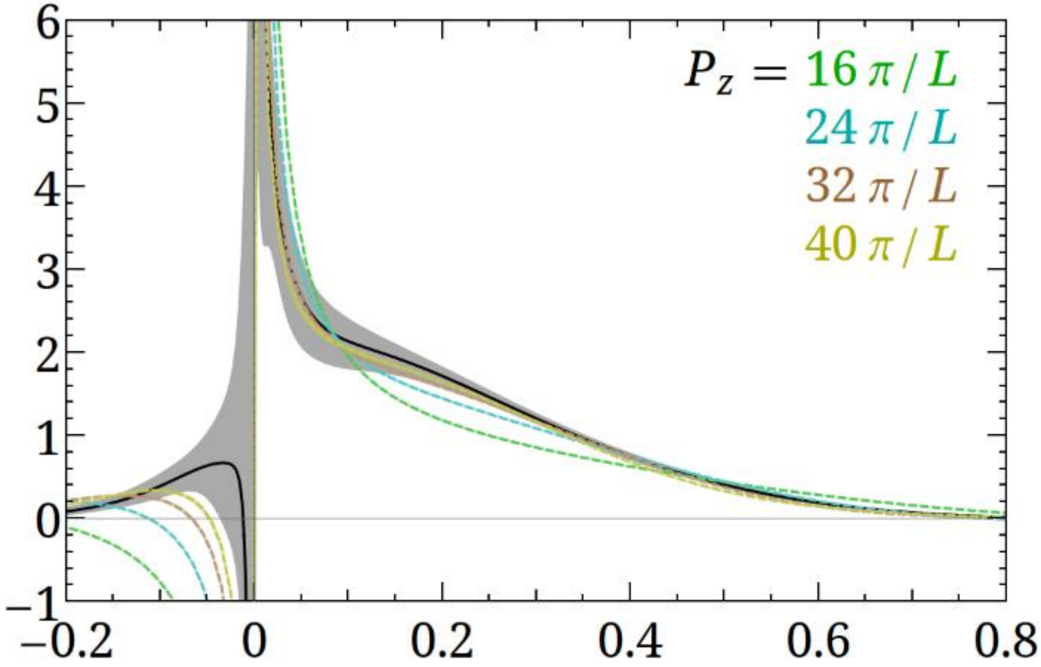
§ Keeping moderate length of displacement z_{max}=8



Derivative approach

$$q(x) = \int_{-z_{\max}}^{+z_{\max}} dz \frac{-P_z}{2\pi} \frac{e^{ixP_z z}}{iP_z x} h'(z)$$

1708.05301 (LP[3])



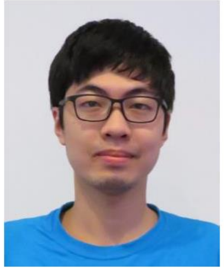
§ Reaching *x*<0.1 for antiquark remains challenging

- Without replying on assumed parametrization

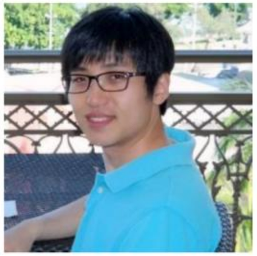
What can AI do for us?

Machine-Learning Prediction for Quasi-PDF Matrix Elements


Rui Zhang


Zhouyou Fan


Ruizi Li


Boram Yoon

+ HWL

arXiv:1909.10990 [hep-lat]

Slides modified from Rui Zhang's
CTEQ FALL 2019 Meeting presentation

Machine Learning Prediction

Input
$X_i = (O_i^1, O_i^2, \ldots)$

ML Model →

Output
$\hat{O}_i$

All Data
- Labeled (lb) Data
  - Training (tr) Data
  - Bias-Correction (BC) Data
- Unlabeled (ul) Data

Prediction with bias correction Yoon et al., PRD 2018:

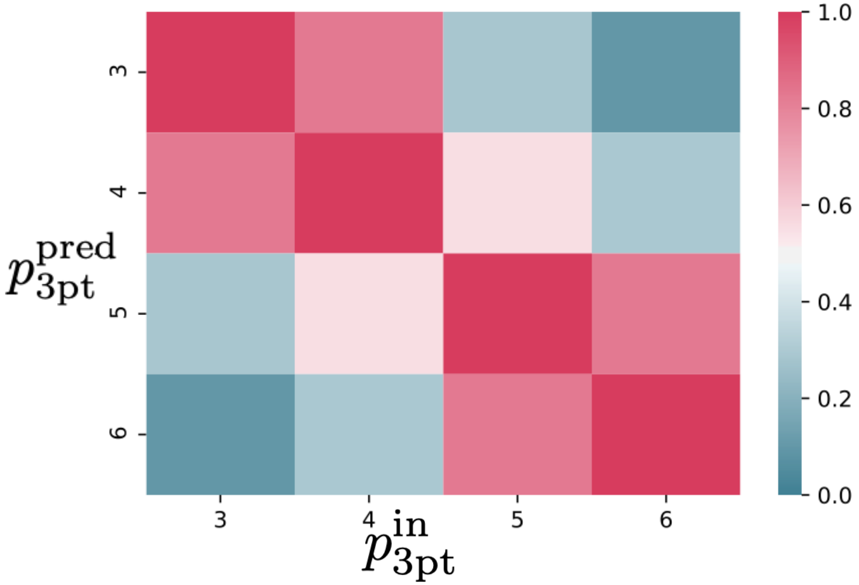
$$\langle C_{\text{pred,BC}} \rangle = \langle C_{\text{pred}} \rangle_{\text{ul}} + \langle C_{\text{BC}} - C_{\text{pred}} \rangle_{\text{BC}}$$

Data and Its Correlations

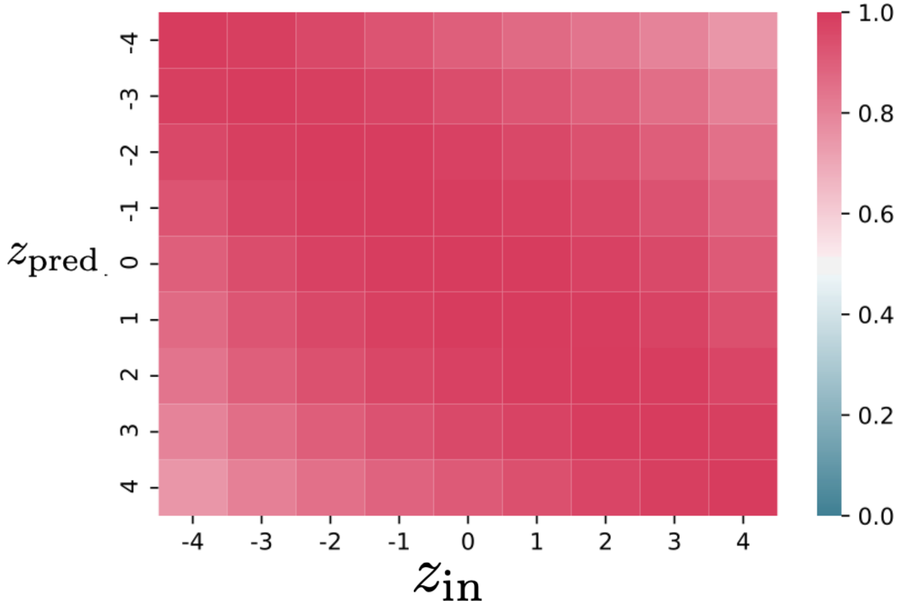
§ Kaon quasi-PDF correlators, with insertion on strange

- a=0.12fm, $m_\pi = 220\text{MeV}$ $40^3 \times 64$ MILC HISQ lattice
- $t_{\text{sep}} = 0.6$ fm

p-correlation



z-correlation



More *z*-inputs can be used for the prediction.

Supervised Regression Models

- Gradient boosting tree (GBT)
  Natekin and Knoll, Front. Neurorobot. 2013

$$f = f_{N_{\text{est}}} = \sum_{i}^{N_{\text{est}}} r_i h_i(x)$$

$$h_i(x) = \underset{h}{\operatorname{argmin}} \sum_j L\left(y_j, f_{i-1}(x_j) + h(x_j)\right)$$

$$f_0 = r \cdot h_0 \qquad \text{a decision tree (DT) toward } y$$

$$f_1 = f_0 + r \cdot h_1 \qquad \text{a DT correcting } f_0 \text{ toward } y$$

$$f_2 = f_1 + r \cdot h_2 \qquad \text{a DT correcting } f_1 \text{ toward } y$$

$$\vdots \qquad\qquad \vdots$$

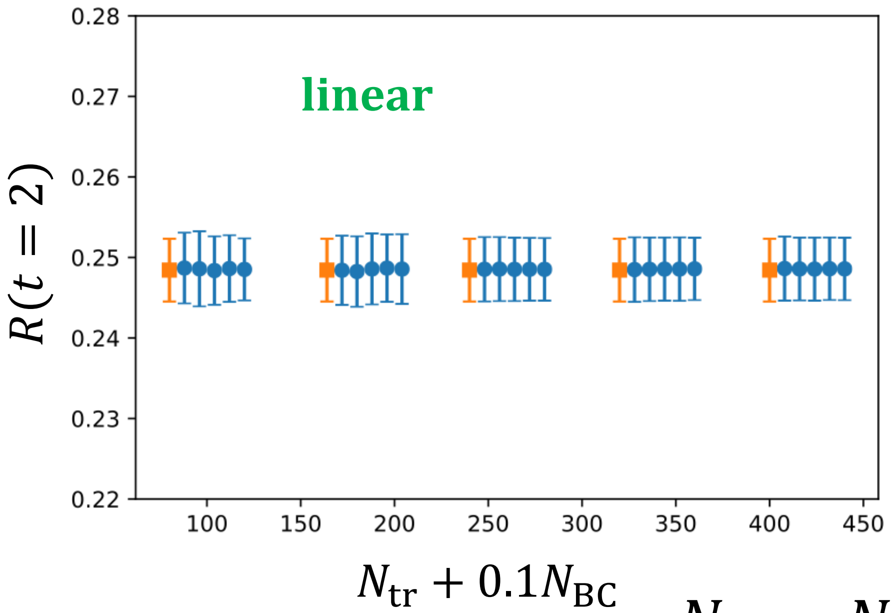
- Linear regressor

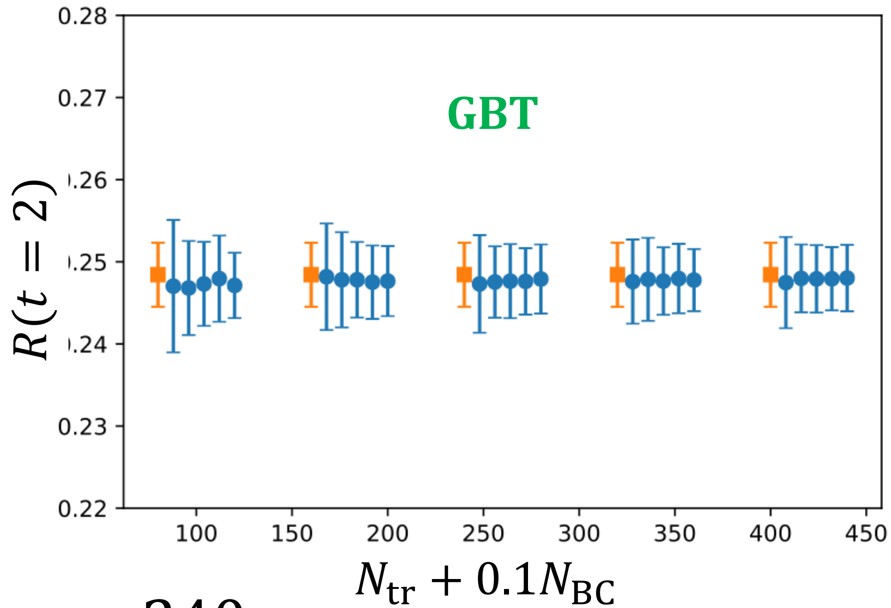
$$f^{\text{lin}}(\vec{x}) = \theta_0 + \vec{\theta} \cdot \vec{x}$$

Training-BC Splitting

§ We vary the number of training/BC samples to find the optimal combination

- *P=4* z-prediction: $z_{\text{in}} \in [0,3], z_{\text{pred}} = 4$



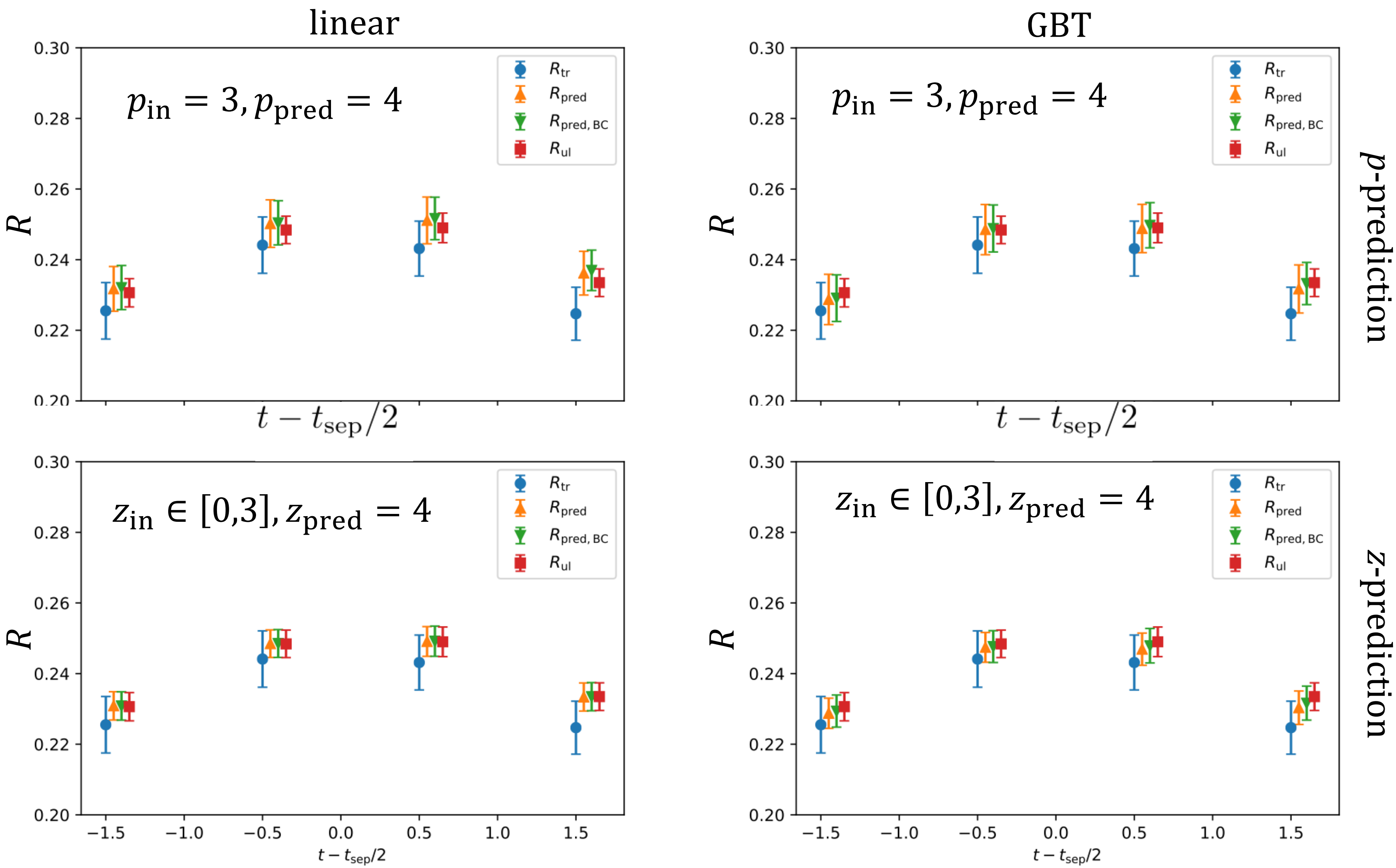


$N_{\text{ul}} = 1180$

$N_{\text{tr}} = N_{\text{BC}} = 240$

- Linear model has a better performance for consistency of direct measurement and errors

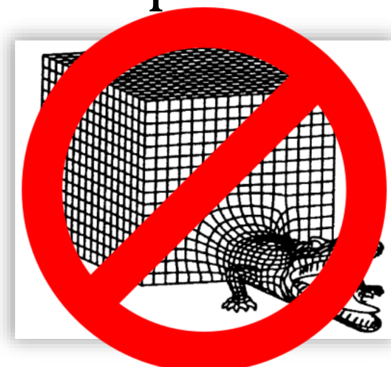
Kaon PDF Results



Summary & Outlook

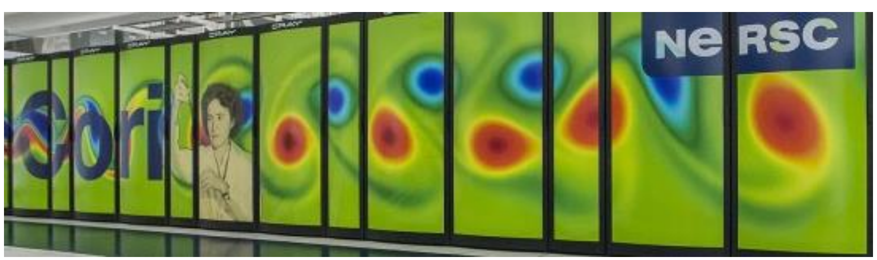
Exciting time for studying structure on the lattice

- § Overcoming longstanding obstacle to full x-distribution
  - Most importantly, this can be done with today's computer
  - Nucleon PDFs at physical pion mass
  - Extended to GPD, and first look of kaon PDF



- § Systematic uncertainty study underway
  - Finite-volume, lattice spacing, etc.
- § Small-*x* structure functions? Larger momentum boost?
  - Machine learning prediction and other ideas?

Challenge = Opportunity



Backup Slides

Global Analysis

§ Experiments cover diverse kinematics of parton variables

- Global analysis takes advantage of all data sets

Theory Input → **Global Analysis of PDFs** ← **Exp't Input**

§ Some choices made for the analysis

- Choice of data sets and kinematic cuts
- Strong coupling constant $\alpha_s(M_Z)$
- How to parametrize the distribution

$$xf(x,\mu_0) = a_0 x^{a_1}(1-x)^{a_2} P(x)$$

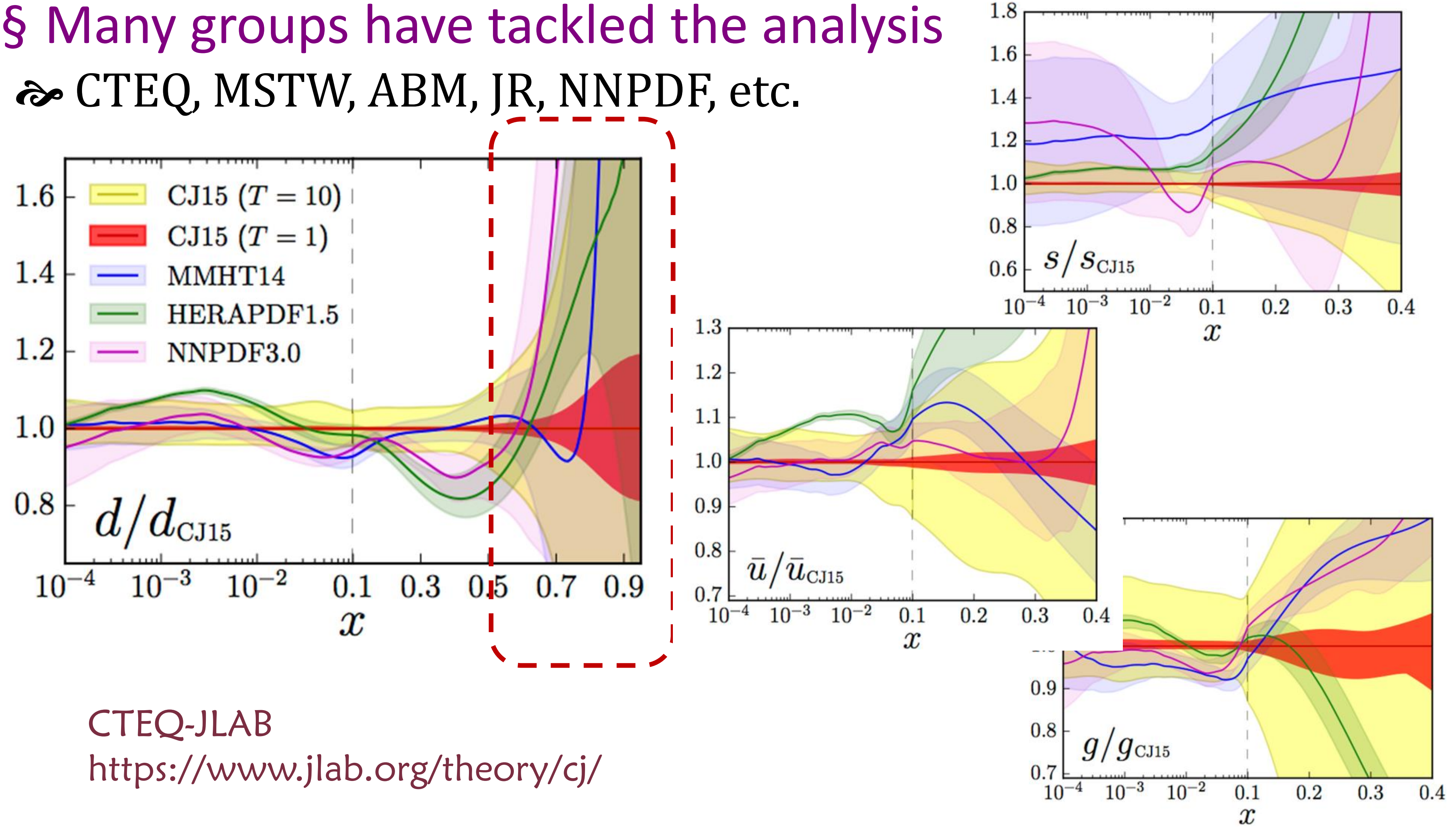
- Assumptions imposed

  SU(3) flavor symmetry, charge symmetry, strange and sea distributions

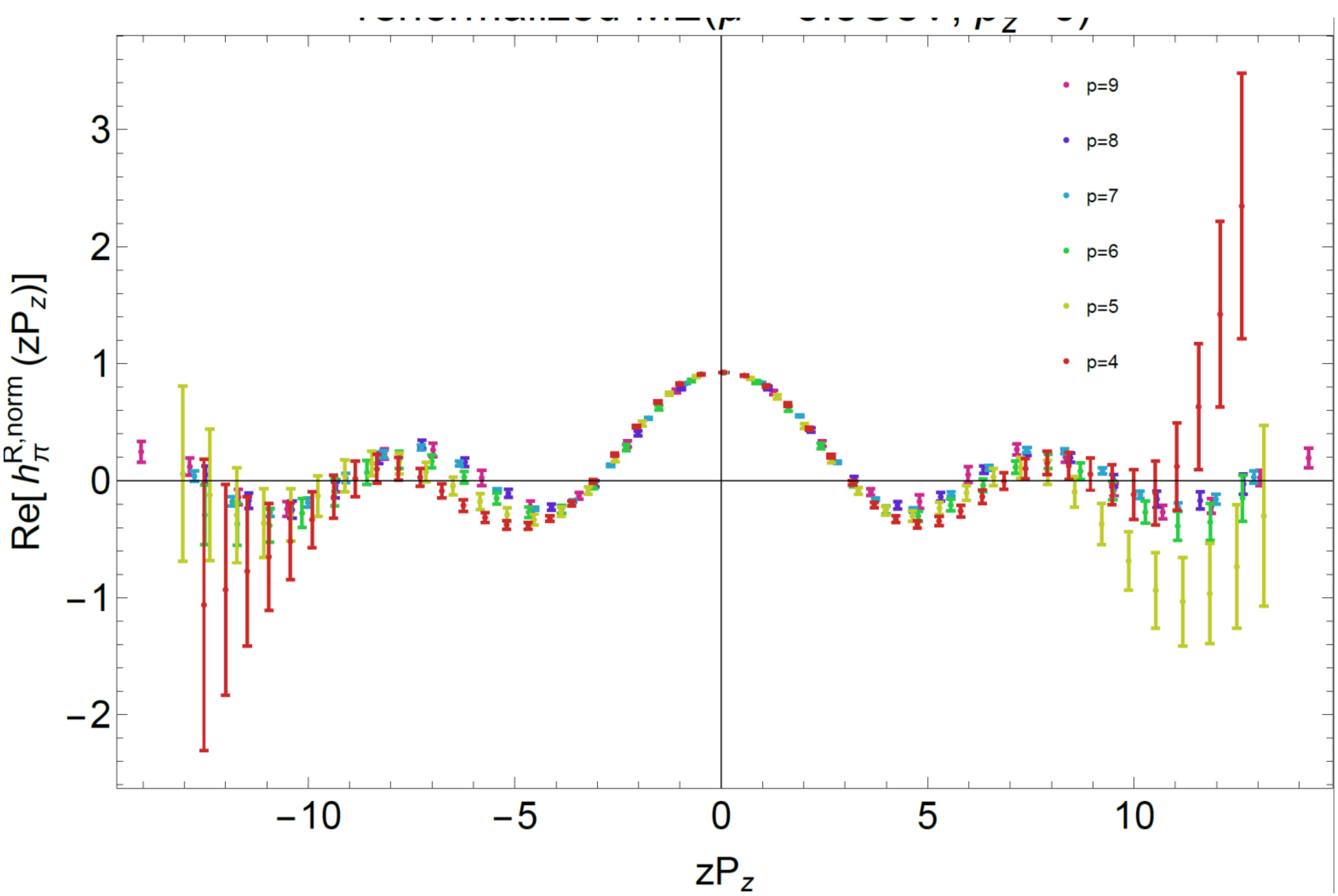
  $$s = \bar{s} = \kappa\left(\bar{u} + \bar{d}\right)$$

Global Analysis

- Discrepancies appear when data is scarce
- Many groups have tackled the analysis
  - CTEQ, MSTW, ABM, JR, NNPDF, etc.



CTEQ-JLAB
https://www.jlab.org/theory/cj/



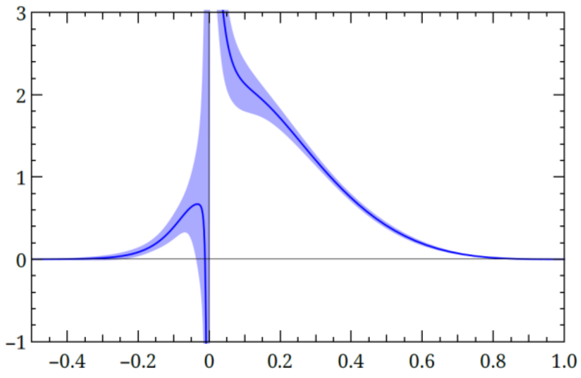
Re[$h_\pi^{R,norm}$ (zP$_z$)]
3
2
1
0
−1
−2
−10
−5
0
5
10
zP$_z$
p=9
p=8
p=7
p=6
p=5
p=4

Truncated Fourier Transform

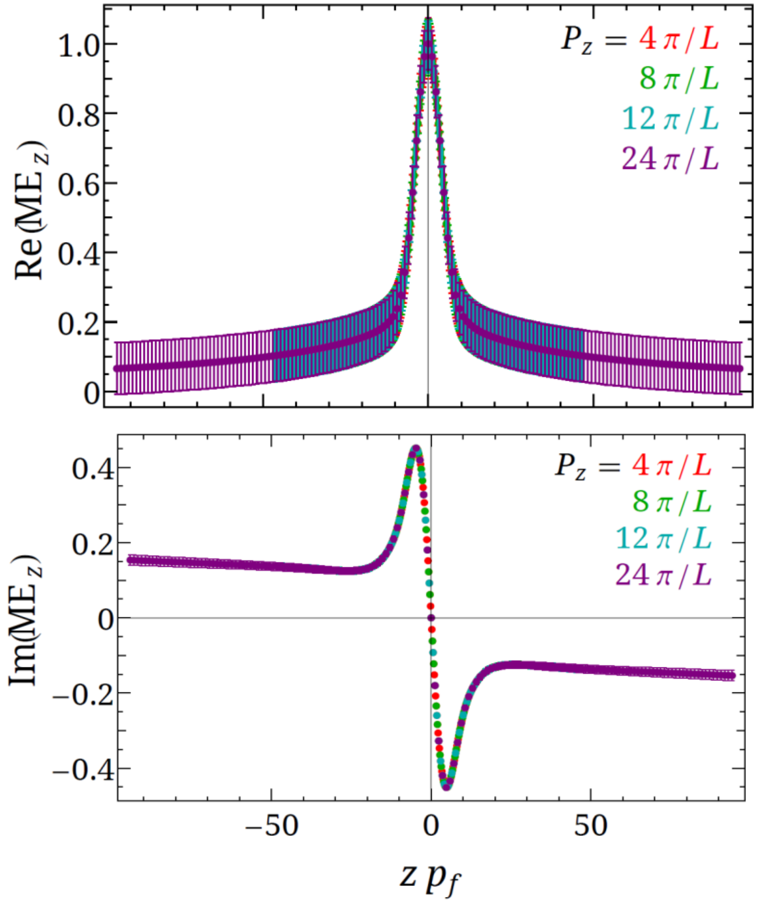
§ Simple exercise with CT14 PDF

1708.05301 (LP3)

Pick your favorite PDF
(CT14 here) 1506.07443



Fourier transform to
coordinate space
at lattice momentum



?

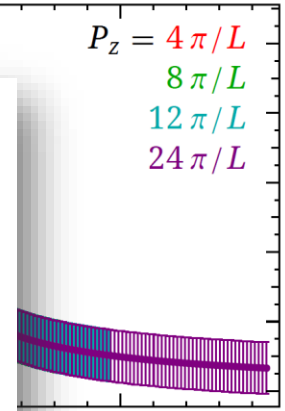
Fourier transform
back to momentum space

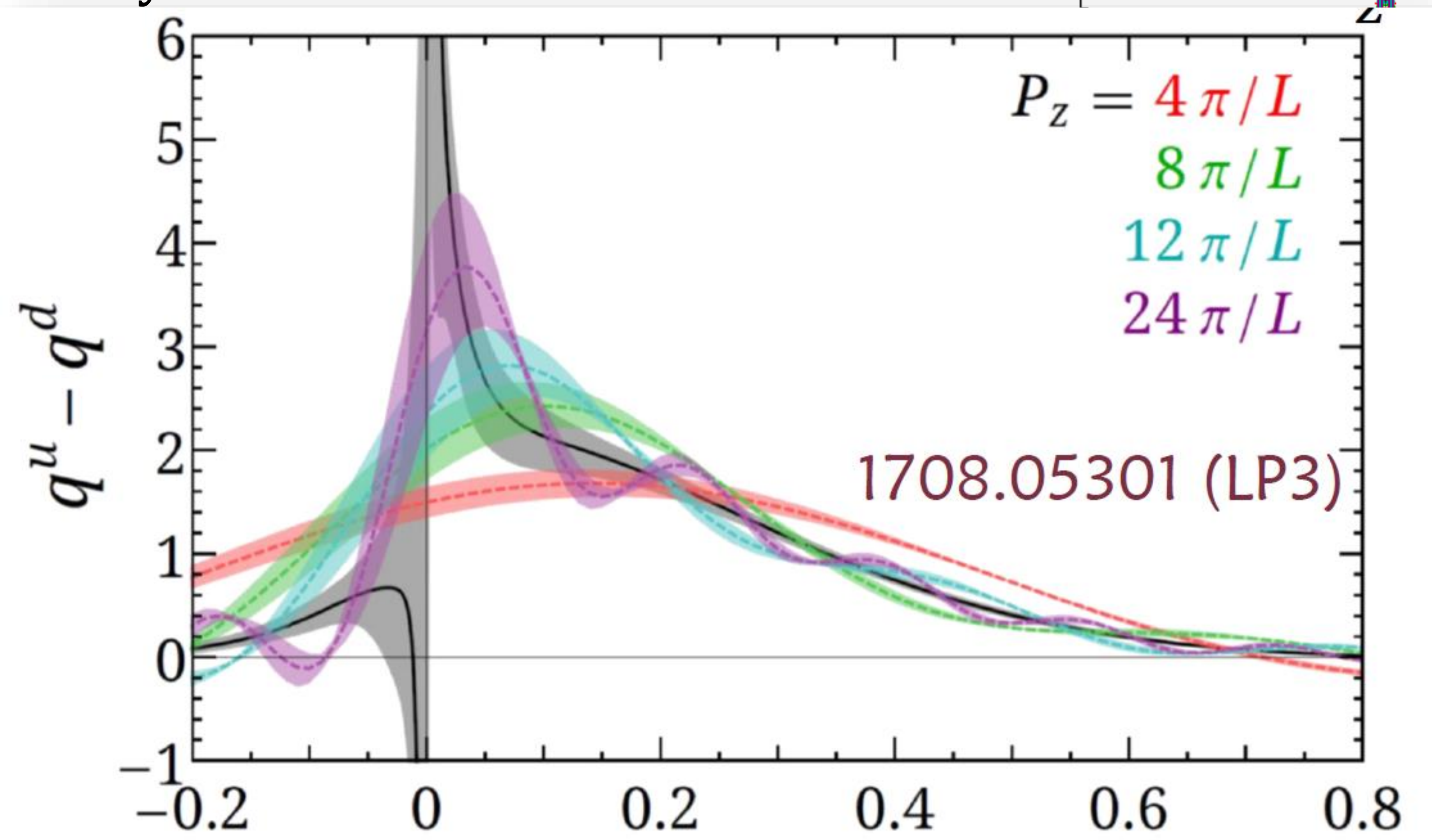
Truncated Fourier Transform

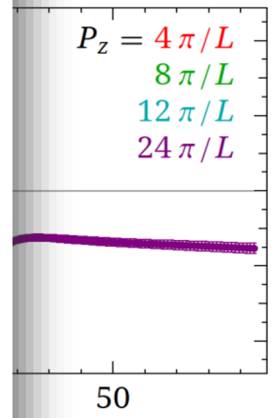
§ Simple exercise with CT14 PDF

1708.05301 (LP3)

Pick your favorite PDF







back to momentum space

Fourier Truncation Issues

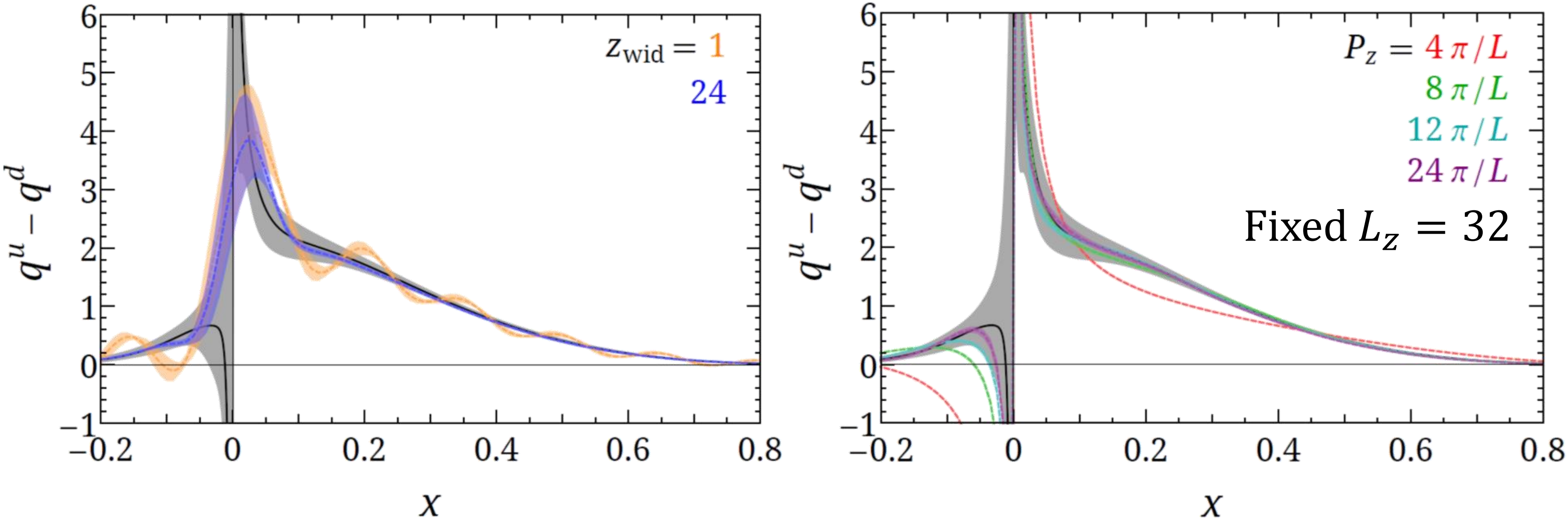
- Luckily, we know the answer
- Two possible solutions proposed (likely more) 1708.05301 (LP[3])

Filter approach

$$F(z_{\rm lim}, z_{\rm wid}) = \frac{1+\mathrm{erf}\left(\frac{z+z_{\rm lim}}{z_{\rm wid}}\right)}{2}\,\frac{1-\mathrm{erf}\left(\frac{z-z_{\rm lim}}{z_{\rm wid}}\right)}{2}$$

Derivative approach

$$q(x) = \int_{-z_{\rm max}}^{+z_{\rm max}} dz\,\frac{-P_z}{2\pi}\,\frac{e^{ixP_z z}}{iP_z x}\,h'(z)$$

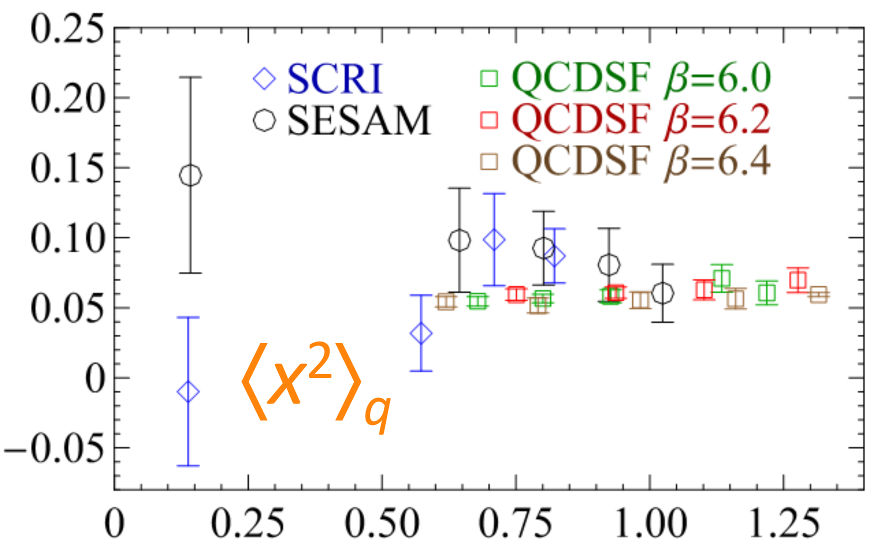


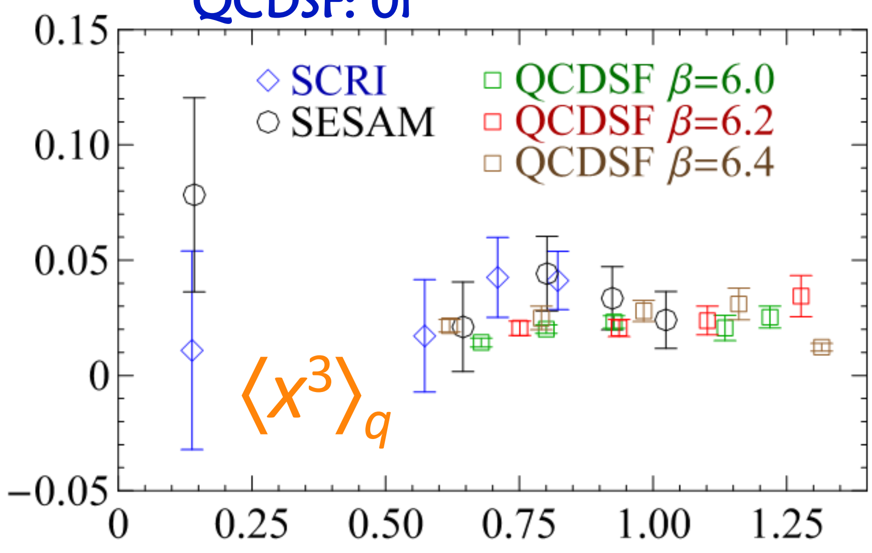
Problem with Moments

- § For higher moments, ops mix with lower-dimension ops
  - Renormalization is difficult too
- § Relative error grows in higher moments
  - Calculation would be costly and difficult

Dolgov et al. PRD66, 034506 (2002)
Göckeler et al. PRD71, 114511 (2005)

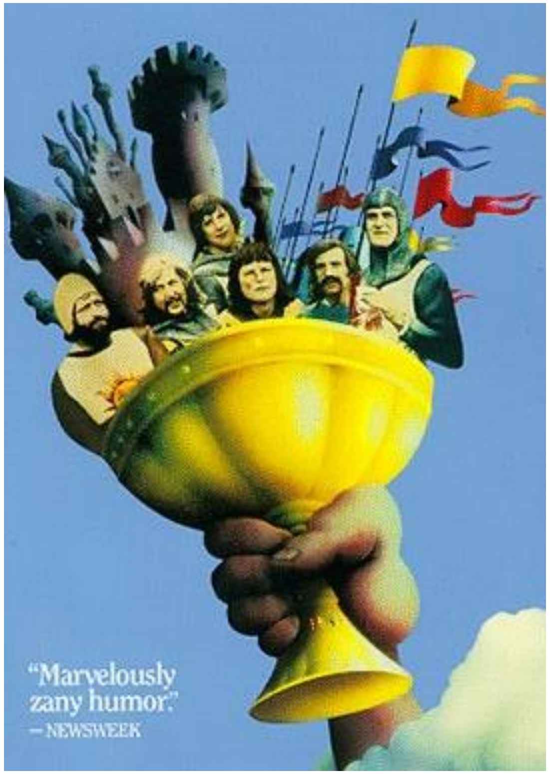
LHPC (SCRI, SESAM):
2f, Wilson and clover
QCDSF: 0f





Beyond Traditional Moments?

- Longstanding obstacle!
- Holy grail of structure calculations
- Applies to many structure quantities:
  - Generalized parton distributions (GPDs)
  - Transverse-momentum distributions (TMD)
  - Meson distribution amplitudes…
  - Wigner distribution



Beyond Traditional Moments?

- Reaching for higher moments
  - Fictitious heavy quarks (Detmold and Lin, hep-lat/0507007)
  - Smeared lat. ops (Davoudi et al. 1204.4146)
- Direct calculation of *x* dependence
  - Hadronic tensor currents (Liu et al., hep-ph/9806491, … 1603.07352)
  - Inversion method/OPE without OPE (QCDSF, hep-lat/9809171, …1703.01153)
  - Euclidean correlation functions (RQCD, 1709.04325)
  - Lattice cross-section method (Y.-Q. Ma and J. Qiu, 2014, 2017)
  - Large-momentum effective theory (LaMET) and variations
    - Original LaMET ("quasi-PDF" method) This talk
    - Pseudo-PDF method: differs in FT (A. Radyushkin, 2017)
    - Smeared quasi-PDF (C. Monahan and K. Orginos, 2017)