Strong QCD from Hadron Structure Experiments
Nov. 6 - 9, 2019
Jefferson Lab
Newport News, VA USA

Electromagnetic and transition form factors of the Baryon Decuplet

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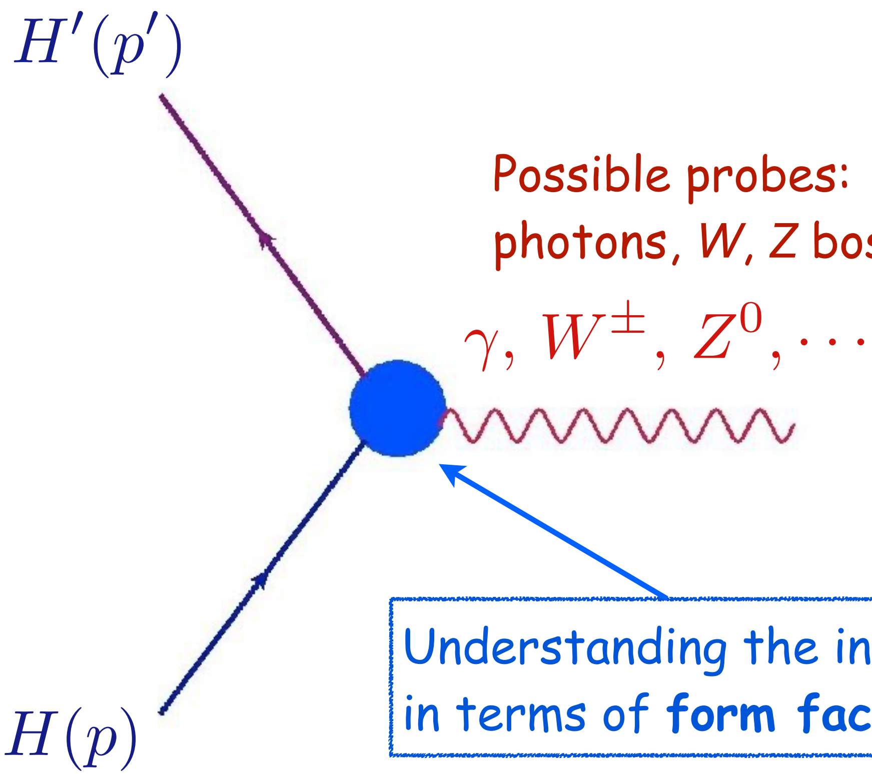
Modern Understanding of Hadron structures

Traditional way of a hadron structure

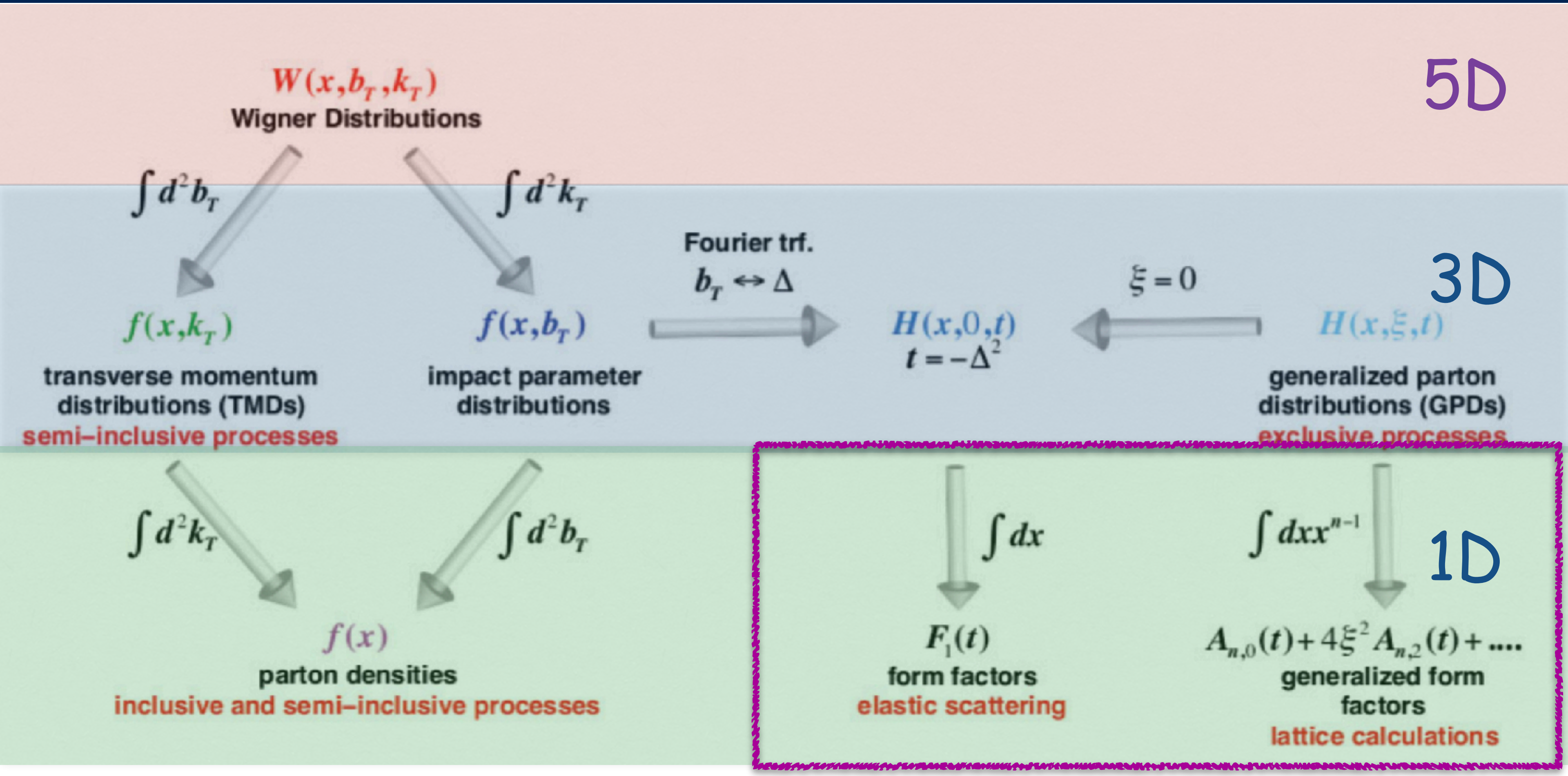
Traditional way of studying structures of hadrons



Possible probes:
photons, *W*, *Z* bosons, mesons, nucleons……

Understanding the internal structures of hadrons *H* in terms of **form factors**

Modern understanding of a baryon structure



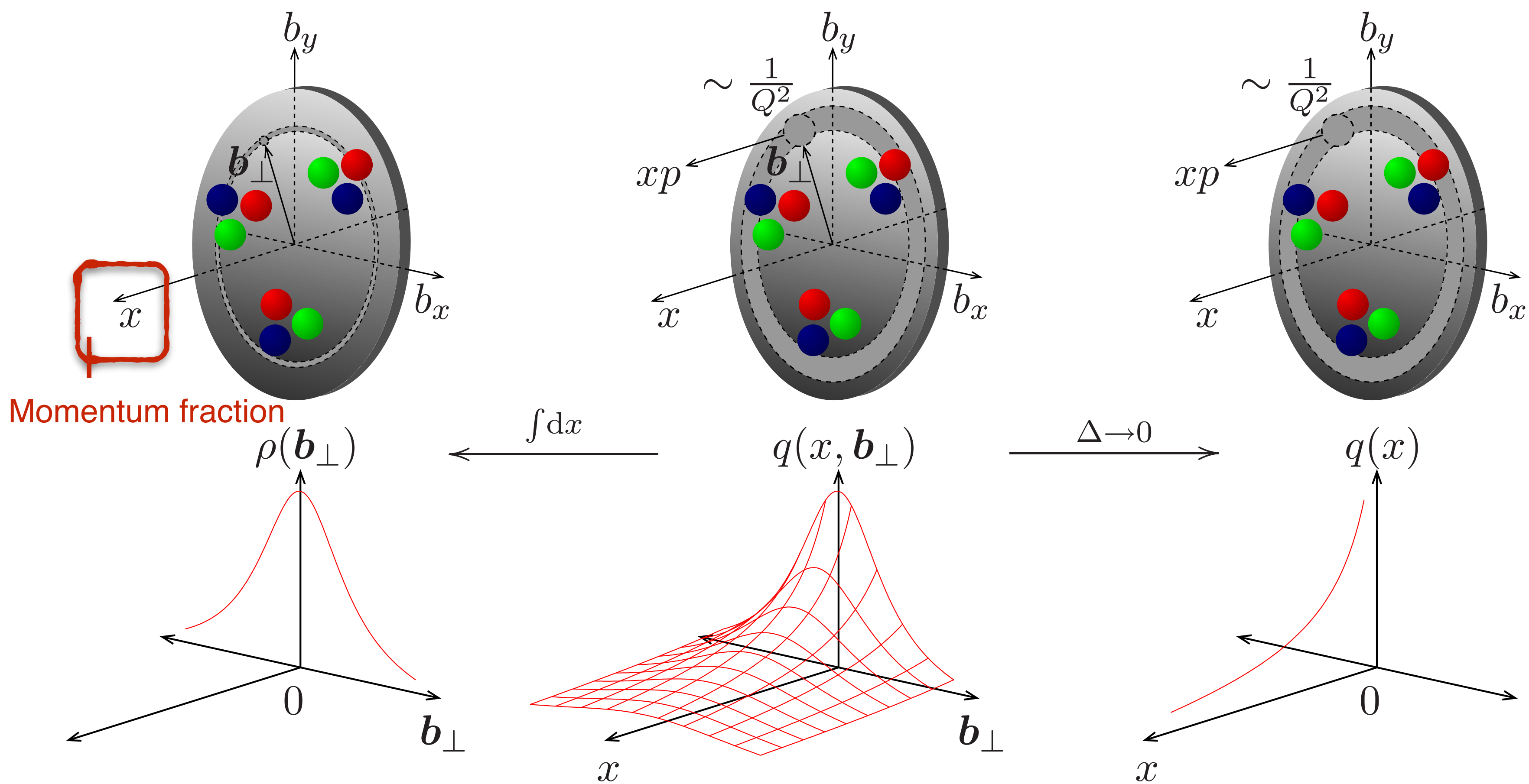
Today's topic to discuss

State of the art of the nucleon tomography

Figure taken from Eur. Phys. J. A (2016) 52: 268

Modern understanding of a baryon structure

3D Nucleon Tomography



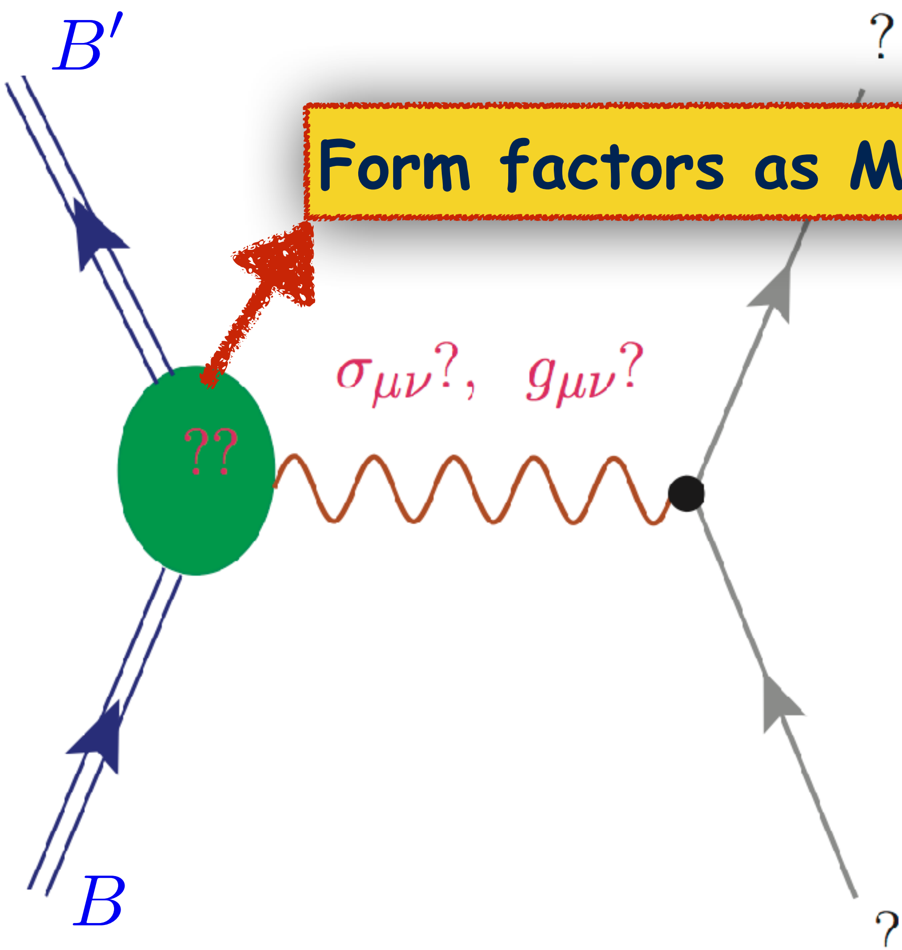
Transverse densities of Form factors

GPDs Nucleon Tomography

Structure functions Parton distributions

Modern understanding of a baryon structure

Probes are unknown for **Tensor form factors**
and the **Energy-Momentum Tensor form factors!**

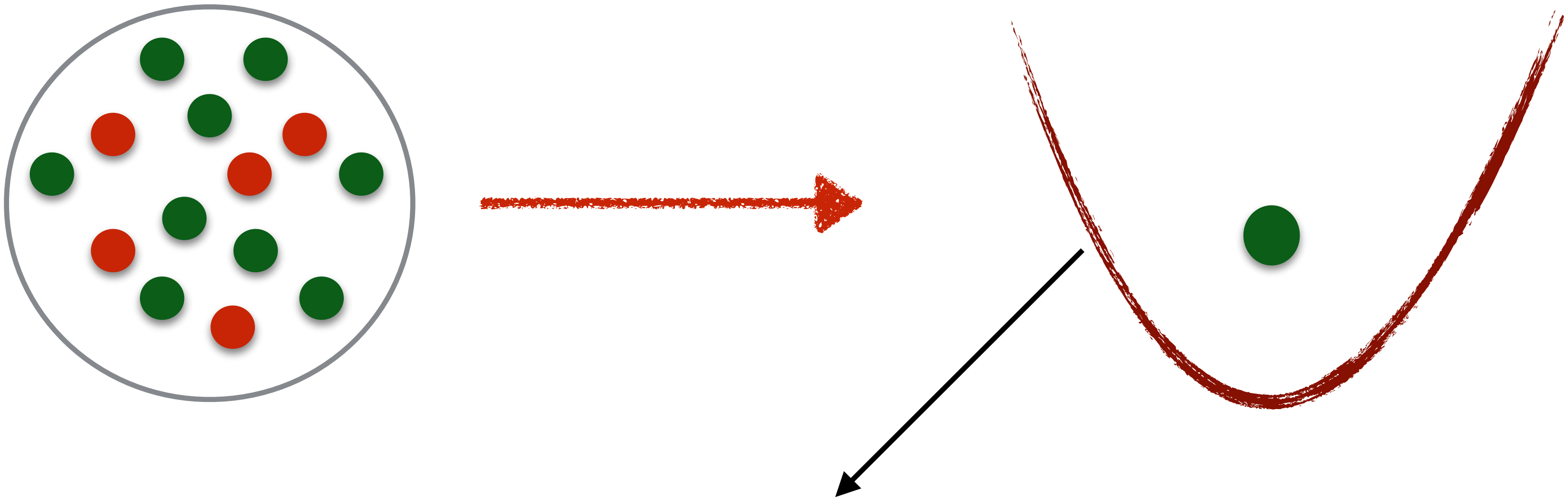


Form factors as Mellin moments of the GPDs

Baryon as Nc quarks bound by the pion mean fields

Mean-Field Approximation

Simple picture of a mean-field approximation



Mean-field potential that is produced by all other particles.

- Nuclear shell models
- Ginzburg-Landau theory for superconductivity
- Quark potential models for baryons

Mean-Field Approximation

More theoretically defined mean fields

Given action, $S[\phi]$

$$\left.\frac{\delta S}{\delta \phi}\right|_{\phi=\phi_0} = 0$$: Solution of this saddle-point equation ϕ_0

Key point: Ignore the quantum fluctuation.

→ How to understand the structure of Baryons, based on this pion mean field approach.

Baryon in pion mean fields

* A baryon can be viewed as a state of Nc quarks bound by mesonic **mean fields** (E. Witten, NPB, 1979 & 1983).

→ Its mass is proportional to *Nc*, while its width is of order *O(1)*.
Mesons are weakly interacting (Quantum fluctuations are suppressed by 1/Nc: *O(1/Nc)*.

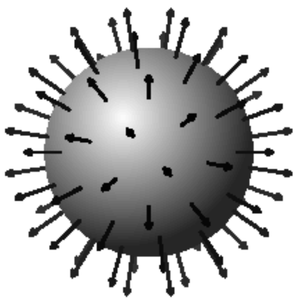
Meson mean-field approach (Chiral Quark-Soliton Model)

* Baryons as a state of Nc quarks bound by mesonic mean fields.

$$S_{\text{eff}} = -N_c \mathrm{Tr}\ln\left(i\partial\!\!\!/ + iMU^{\gamma_5} + i\hat{m}\right)$$

* Key point: Hedgehog Ansatz

$$\pi^a(\boldsymbol{r}) = \begin{cases} n^a F(r),\ n^a = x^a/r, & a = 1,\ 2,\ 3 \\ 0, & a = 4,\ 5,\ 6,\ 7,\ 8. \end{cases}$$



hedgehog

→ It breaks spontaneously $\mathrm{SU}(3)_{\text{flavor}} \otimes \mathrm{O}(3)_{\text{space}} \to \mathrm{SU}(2)_{\text{isospin+space}}$

Baryon in pion mean fields

✱ Merits of the Chiral Quark-Soliton Model

- It is directly related to nonperturbative QCD via the Instanton vacuum.

  → Natural scale of the model given by the instanton size:

  $$\rho \approx (600\,\mathrm{MeV})^{-1}$$

- Fully relativistic quantum-field theoretic model (we have a “QCD” vacuum):

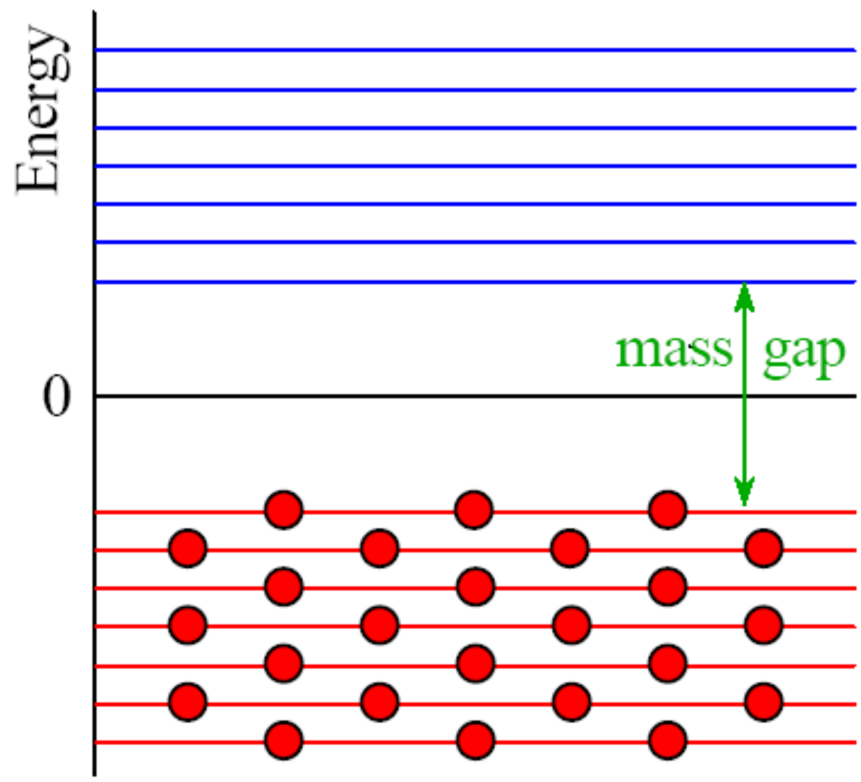
  It explains almost all properties of the lowest-lying baryons.

- It describes the light & heavy baryons on an equal footing

  (Advantage of the mean-field approach) .

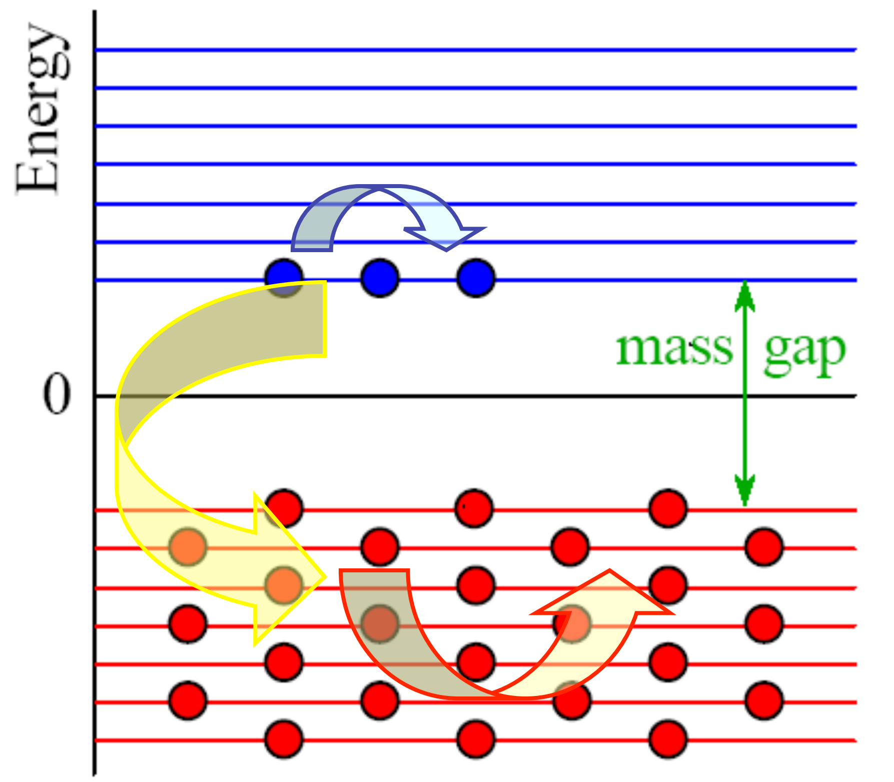
- Basically, no free parameter to fit the experimental data.
  Cutoff parameter is fixed by the pion decay constant, and
  Dynamical quark mass (M=420 MeV) is fixed by the proton
  radius.

Baryon in pion mean fields



HChK et al. Prog. Part. Nucl. Phys. Vol.95, (1995)

Baryon in pion mean fields



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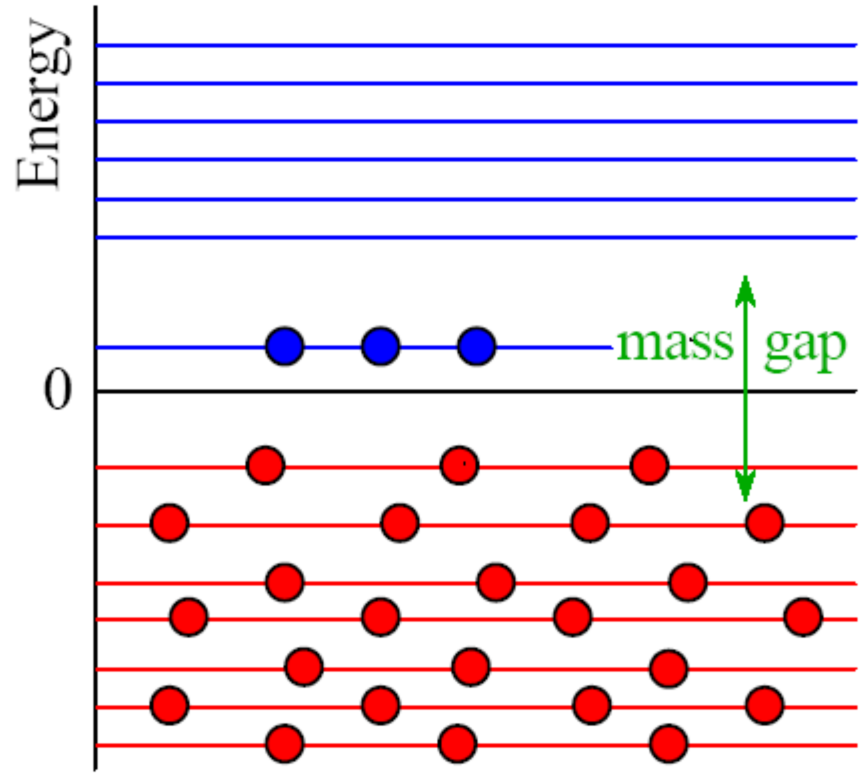
Baryon in pion mean fields

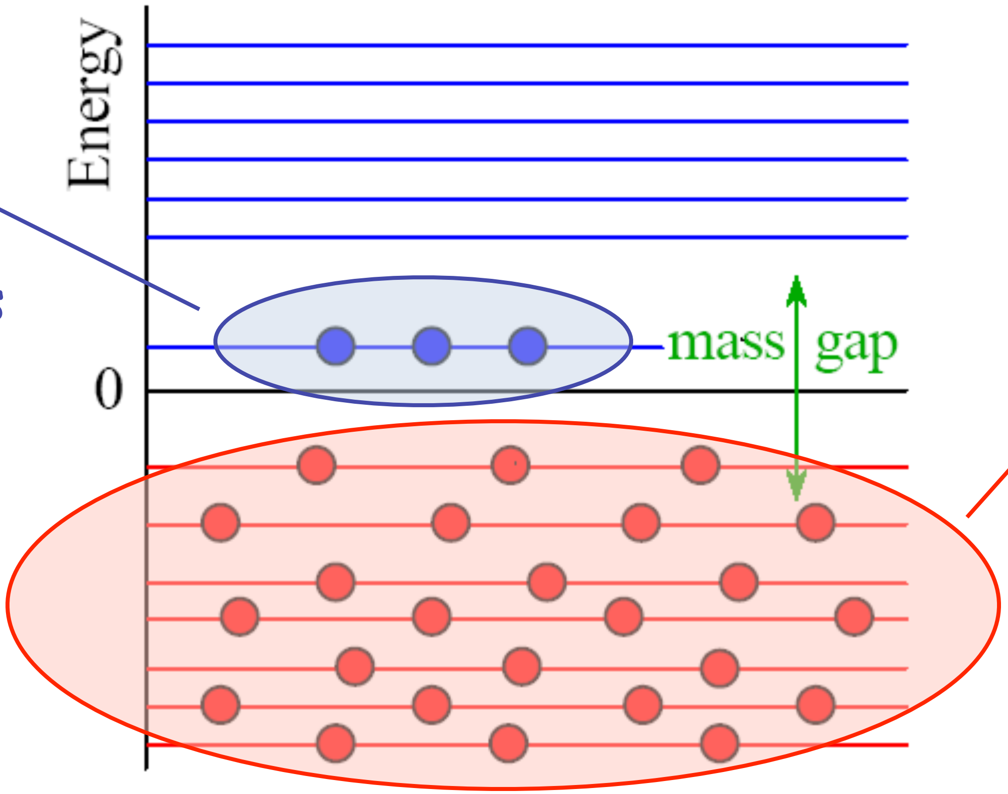


HChK et al. Prog. Part. Nucl. Phys. Vol.95, (1995)

Baryon in pion mean fields

valence level:

energy decreases



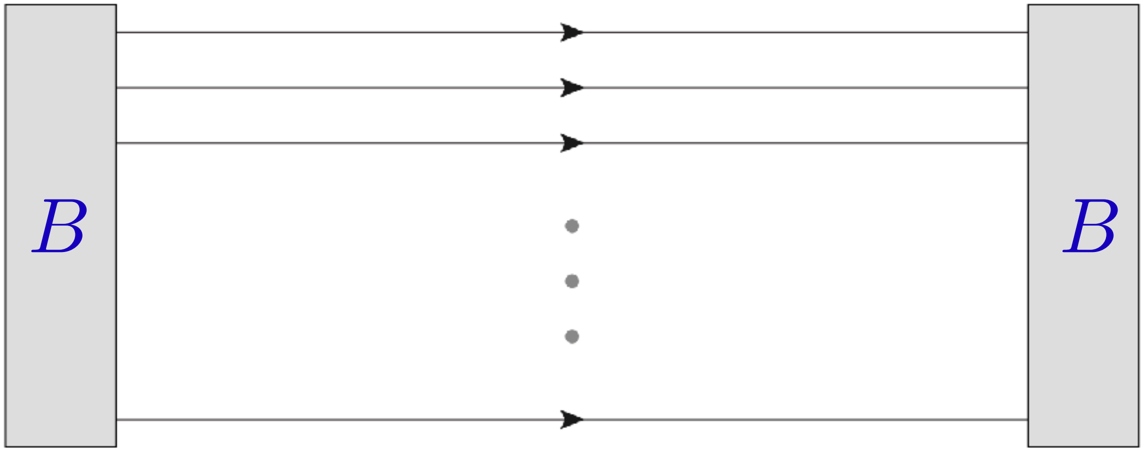
sea levels:

energy increases

system is stabilized

HChK et al. Prog. Part. Nucl. Phys. Vol.95, (1995)

A light baryon in pion mean fields

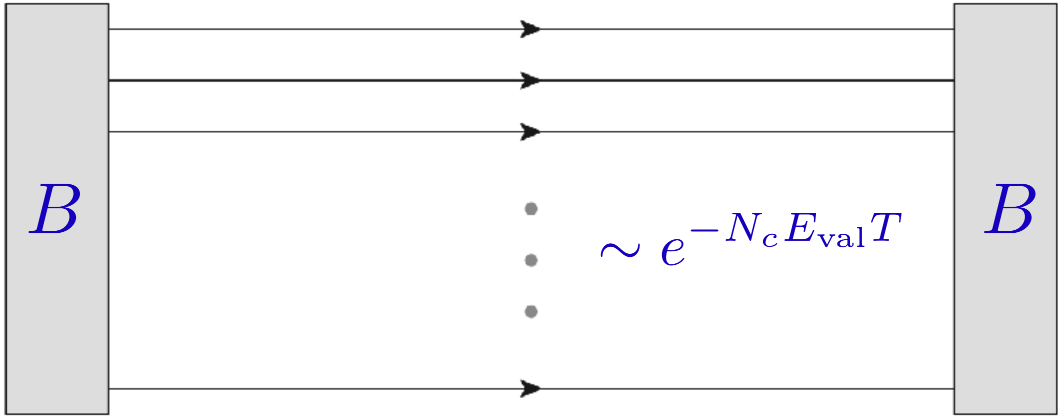


$$\langle J_B J_B^\dagger \rangle_0 \sim e^{-N_c E_{\rm val} T}$$

Presence of Nc quarks will polarize the vacuum or create mean fields.

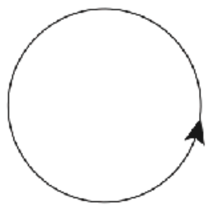
Nc valence quarks → Vacuum polarization or meson mean fields

A light baryon in pion mean fields



$$\sim e^{-N_c E_{\rm val} T}$$

$$E_{\rm cl} = N_c E_{\rm val} + E_{\rm sea}$$



$$\sim e^{-E_{\rm sea} T}$$

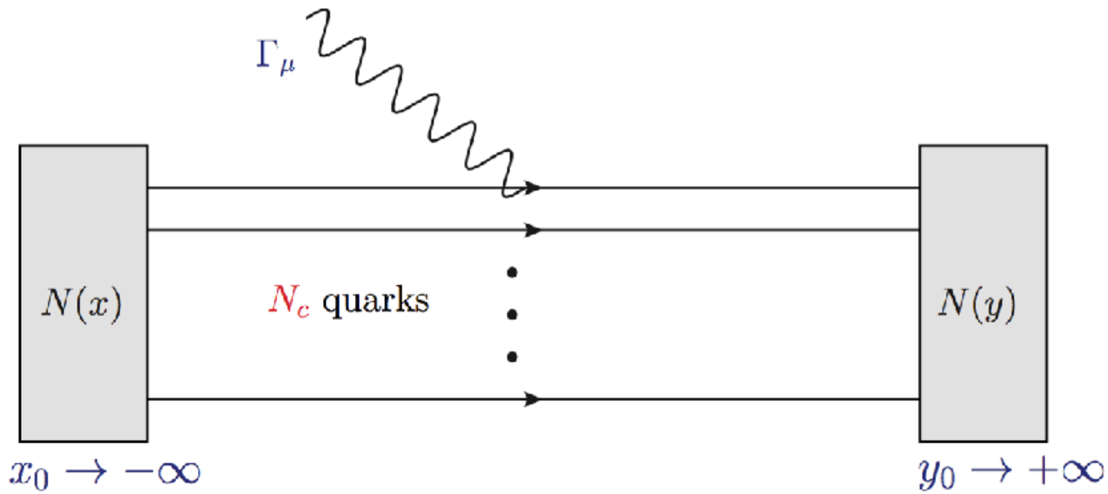
Classical Nucleon mass is described by the Nc valence quark energy and sea-quark energy.

$$\frac{\delta E_{cl}}{\delta U} = 0 \longrightarrow M_{\rm cl} \longrightarrow P(r)$$

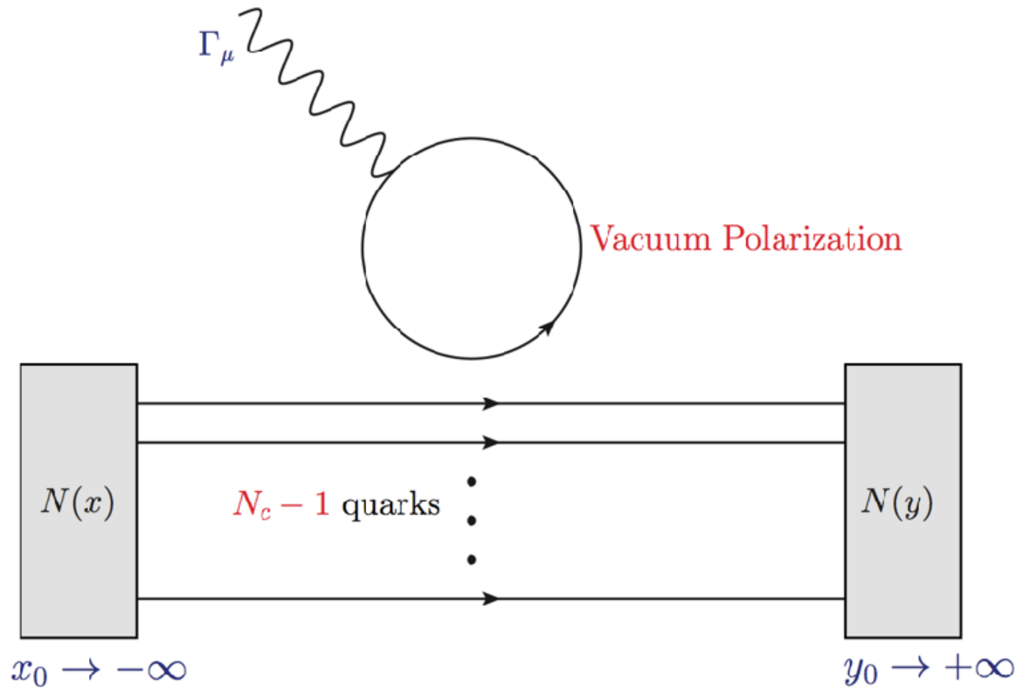
P(r): Soliton profile function or Soliton field

An observable for the light baryon

Valence part

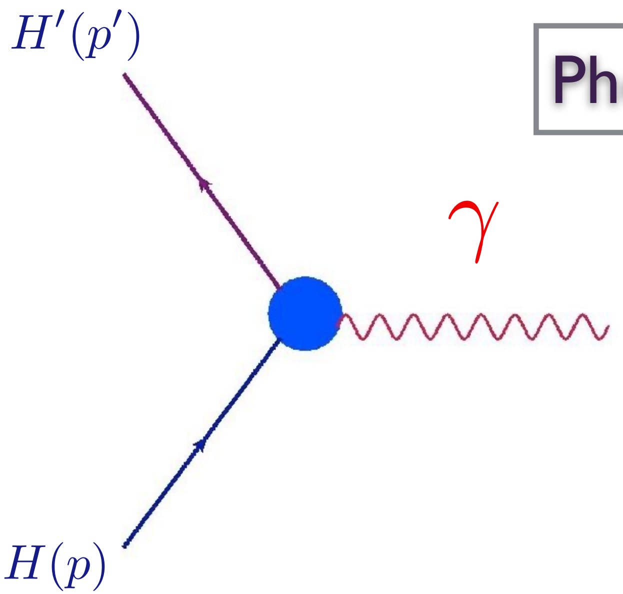


Sea part



EM Form factors
of
the Baryon decuplet

Traditional definition of form factors



Photons → Charge & magnetic densities

ν, e_{PV}

Neutrino or PV electron scattering → Axial-Vector properties

No probe for the tensor & EMT (grav.) form factors!

$$\langle B(p',s)|e_B J^\mu(0)|B(p,s)\rangle = -e_B \overline{u}^\alpha(p',s)\left[\gamma^\mu \left\{F_1^B(q^2)\eta_{\alpha\beta} + F_3^B(q^2)\frac{q_\alpha q_\beta}{4M_B^2}\right\}\right.$$
$$\left. + i\frac{\sigma^{\mu\nu}q_\nu}{2M_B}\left\{F_2^B(q^2)\eta_{\alpha\beta} + F_4^B(q^2)\frac{q_\alpha q_\beta}{4M_B^2}\right\}\right] u^\beta(p,s),$$

New Definition

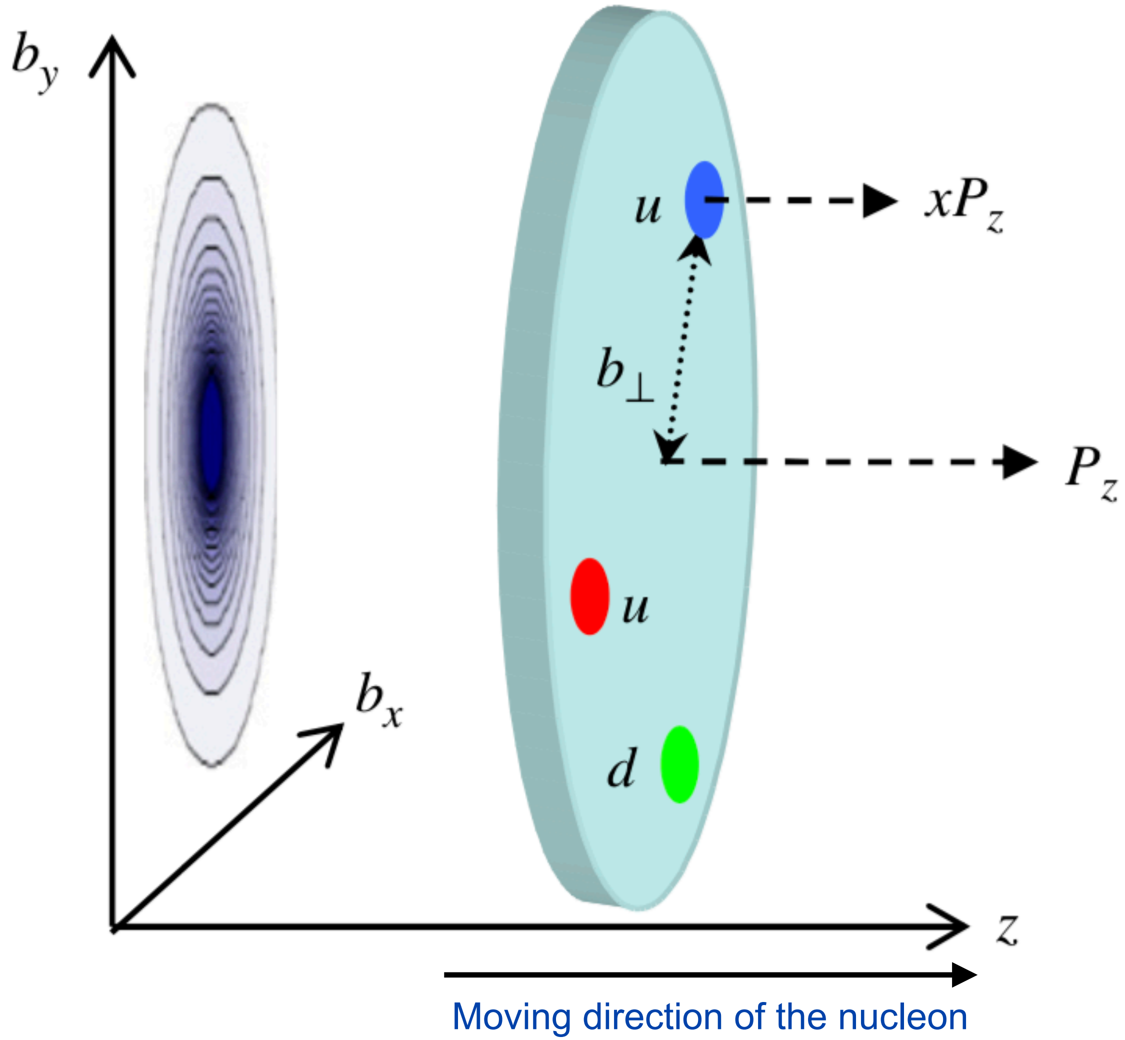
Generalized Parton Distributions

Melin transform

Generalized Form factors

2D Fourier transform

Transverse charge densities



Quark probabilities inside a nucleon

Transverse charge density

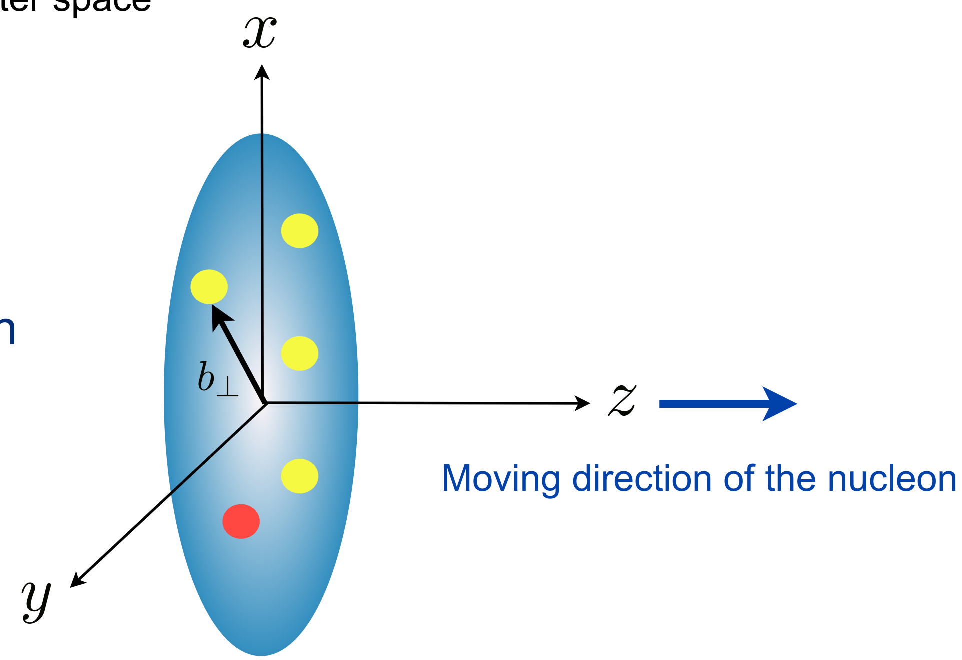
Why transverse charge densities?

2-D Fourier transform of the GPDs in impact-parameter space

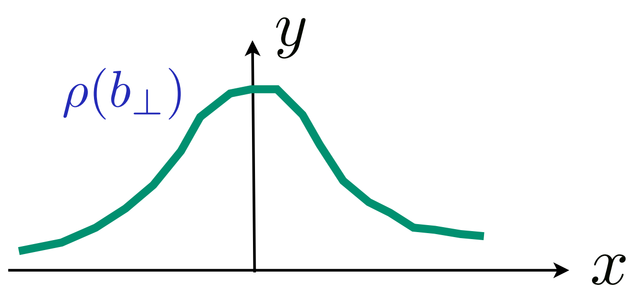
$$q(x,\, \mathbf{b}) = \int \frac{d^2q}{(2\pi)^2} e^{i\mathbf{q}\cdot\mathbf{b}} H_q(x, -\mathbf{q}^2)$$

⇨ It can be interpreted as the probability distribution of a quark in the transverse plane.

M. Burkardt, PRD **62**, 071503 (2000); Int. J. Mod. Phys. A **18**, 173 (2003).



$$\rho(\mathbf{b}) := \sum_q e_q \int dx q(x,\, \mathbf{b})$$
$$= \int \frac{d^2q}{(2\pi)^2} F_1(Q^2) e^{i\mathbf{q}\cdot\mathbf{b}}$$



EM Form factors of the baryon decuplet

- Matrix Elements of the EM current in terms of four independent form factors

$$\langle B(p',s)|J^\mu(0)|B(p,s)\rangle = -\overline{u}^\alpha(p',s)\left[\gamma^\mu\left\{F_1^B(q^2)\eta_{\alpha\beta} + F_3^B(q^2)\frac{q_\alpha q_\beta}{4M_B^2}\right\}\right.$$
$$\left. + i\frac{\sigma^{\mu\nu}q_\nu}{2M_B}\left\{F_2^B(q^2)\eta_{\alpha\beta} + F_4^B(q^2)\frac{q_\alpha q_\beta}{4M_B^2}\right\}\right]u^\beta(p,s),$$

- Sachs-type form factors: Multipole form factors

$$G_{E0}^B(Q^2) = \left(1+\frac{2}{3}\tau\right)[F_1^B - \tau F_2^B] - \frac{1}{3}\tau(1+\tau)[F_3^B - \tau F_4^B],$$
$$G_{E2}^B(Q^2) = [F_1^B - \tau F_2^B] - \frac{1}{2}(1+\tau)[F_3^B - \tau F_4^B],$$
$$G_{M1}^B(Q^2) = \left(1+\frac{4}{5}\tau\right)[F_1^B + F_2^B] - \frac{2}{5}\tau(1+\tau)[F_3^B + F_4^B],$$
$$G_{M3}^B(Q^2) = [F_1^B + F_2^B] - \frac{1}{2}(1+\tau)[F_3^B + F_4^B]$$

J.-Y. Kim & HChK, EPJC, 79:570 (2019)

EM Form factors of the baryon decuplet

- Physical meanings of the multipole form factors

$$
\begin{aligned}
e_B &= eG^B_{E0}(0) = eF^B_1(0), \\
\mu_B &= \frac{e}{2M_B}G^B_{M1} = \frac{e}{2M_B}\left[e_B + F^B_2(0)\right], \\
Q_B &= \frac{e}{M_B^2}G^B_{E2}(0) = \frac{e}{M_B^2}\left[e_B - \frac{1}{2}F^B_3(0)\right], \\
O_B &= \frac{e}{M_B^3}G^B_{M3}(0) = \frac{e}{M_B^3}\left[e_B + F^B_2(0) - \frac{1}{2}(F^B_3(0) + F^B_4(0))\right]
\end{aligned}
$$

J.-Y. Kim & HChK, EPJC, 79:570 (2019)

EM Form factors of the baryon decuplet

- Expressions for the multipole form factors

$$G^B_{E0}(Q^2) = \int \frac{d\Omega_q}{4\pi} \langle B(p',3/2)|J^0(0)|B(p,3/2)\rangle,$$

$$G^B_{E2}(Q^2) = -\int d\Omega_q \sqrt{\frac{5}{4\pi}}\frac{3}{2}\frac{1}{\tau}\langle B(p',3/2)|Y^*_{20}(\Omega_q)J^0(0)|B(p,3/2)\rangle,$$

$$G^B_{M1}(Q^2) = \frac{3M_B}{4\pi}\int \frac{d\Omega_q}{i|\boldsymbol{q}|^2} q^i\epsilon^{ik3}\langle B(p',3/2)|J^k(0)|B(p,3/2)\rangle,$$

$$G^B_{M3}(Q^2) = -\frac{35M_B}{8}\sqrt{\frac{5}{\pi}}\int \frac{d\Omega_q}{i|\boldsymbol{q}|^2\tau} q^i\epsilon^{ik3}\langle B(p',3/2)|\left(Y^*_{20}(\Omega_q)+\sqrt{\frac{1}{5}}Y^*_{00}(\Omega_q)\right)J^k(0)|B(p,3/2)\rangle$$

- Note that in any chiral solitonic model M3 form factors turn out to vanish. It implies that M3 form factors must be tiny.

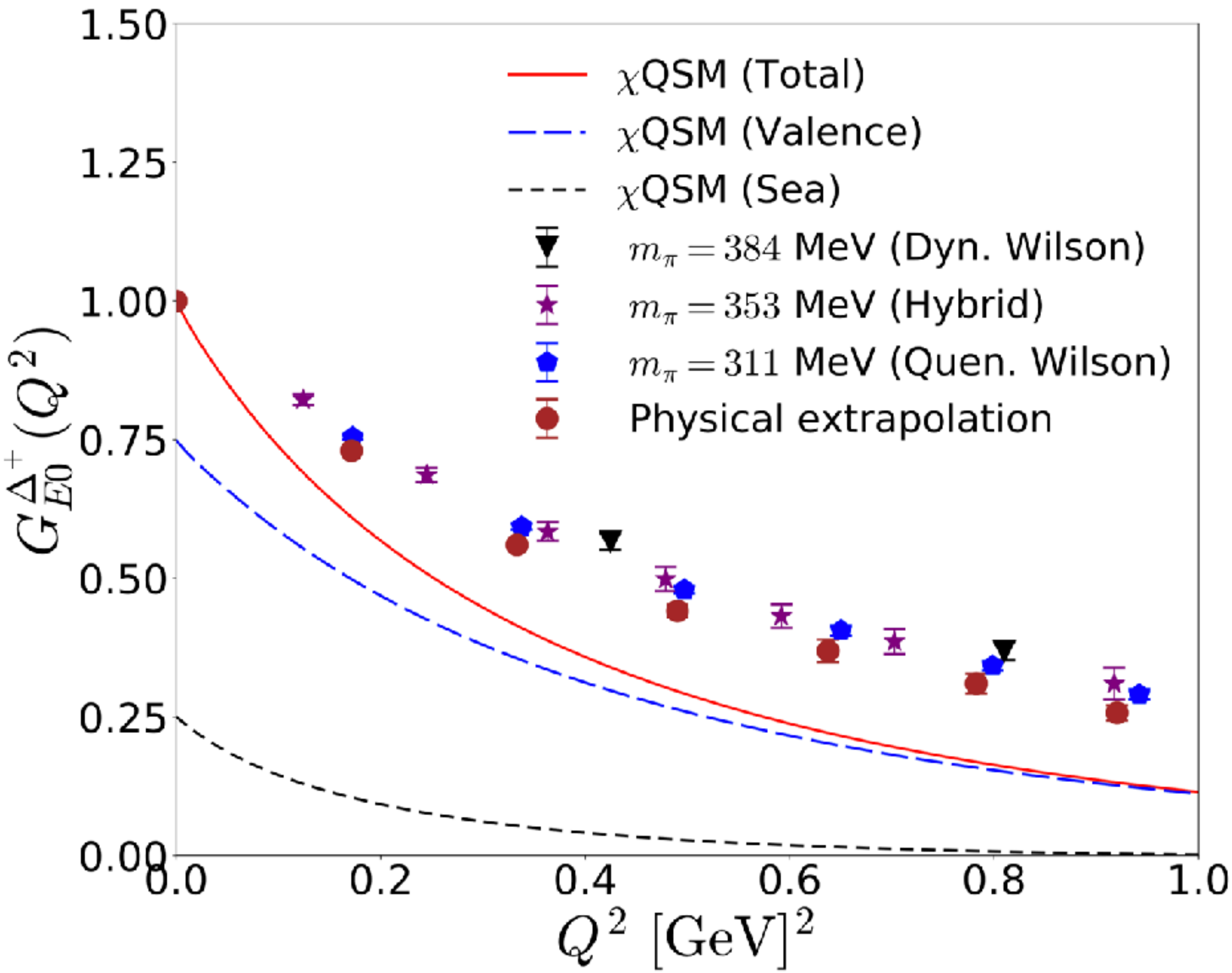
» T. Ledwig & M. Vanderhaeghen, Phys.Rev. D79 (2009) 094025
in an SU(3) symmetric case within the same framework.

J.-Y. Kim & HChK, EPJC, 79:570 (2019)

Valence & Sea contributions

E0 form factor of the Delta+

Lattice data: Alessandro et al.



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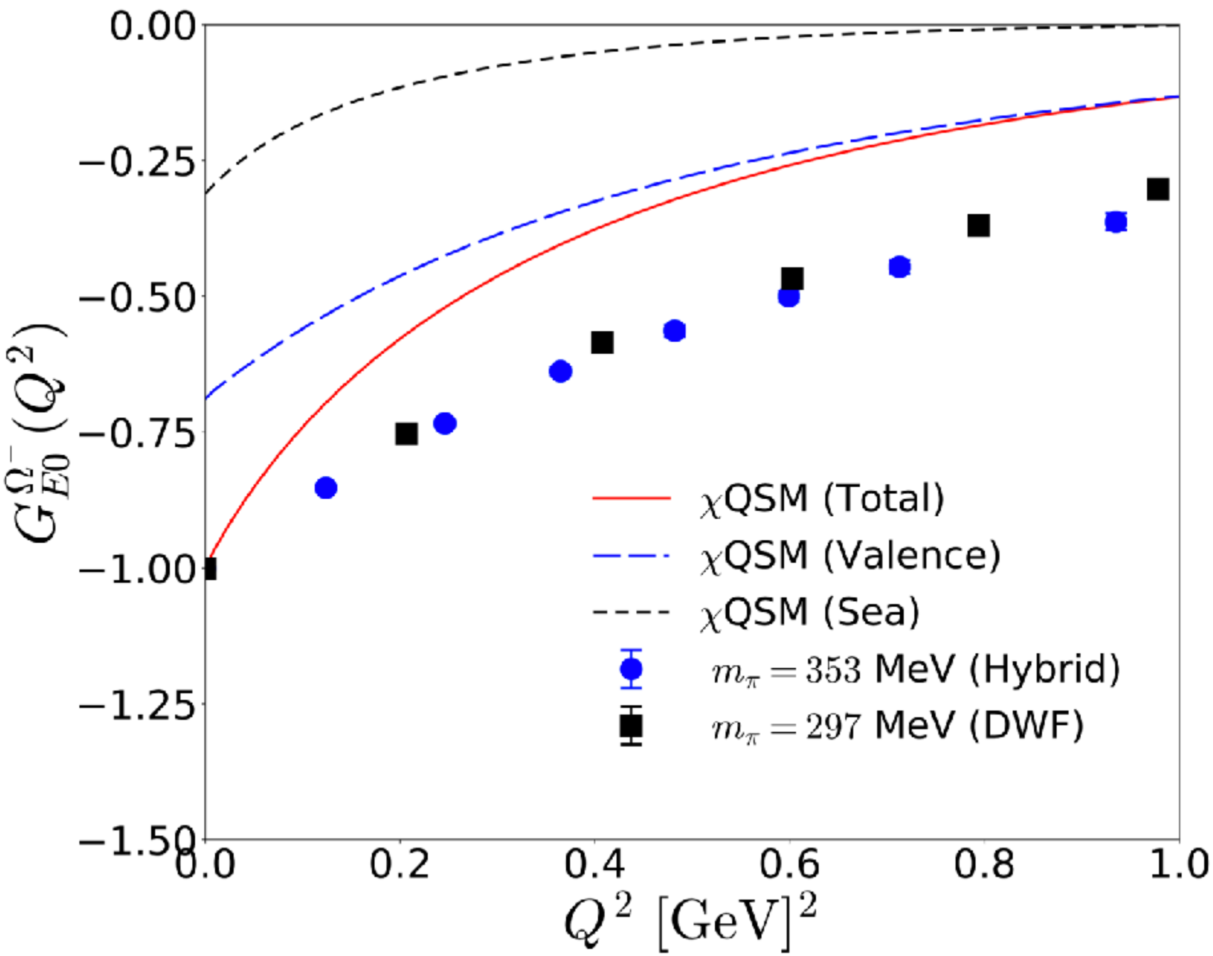

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Valence & Sea contributions

M1 form factor of the Delta+

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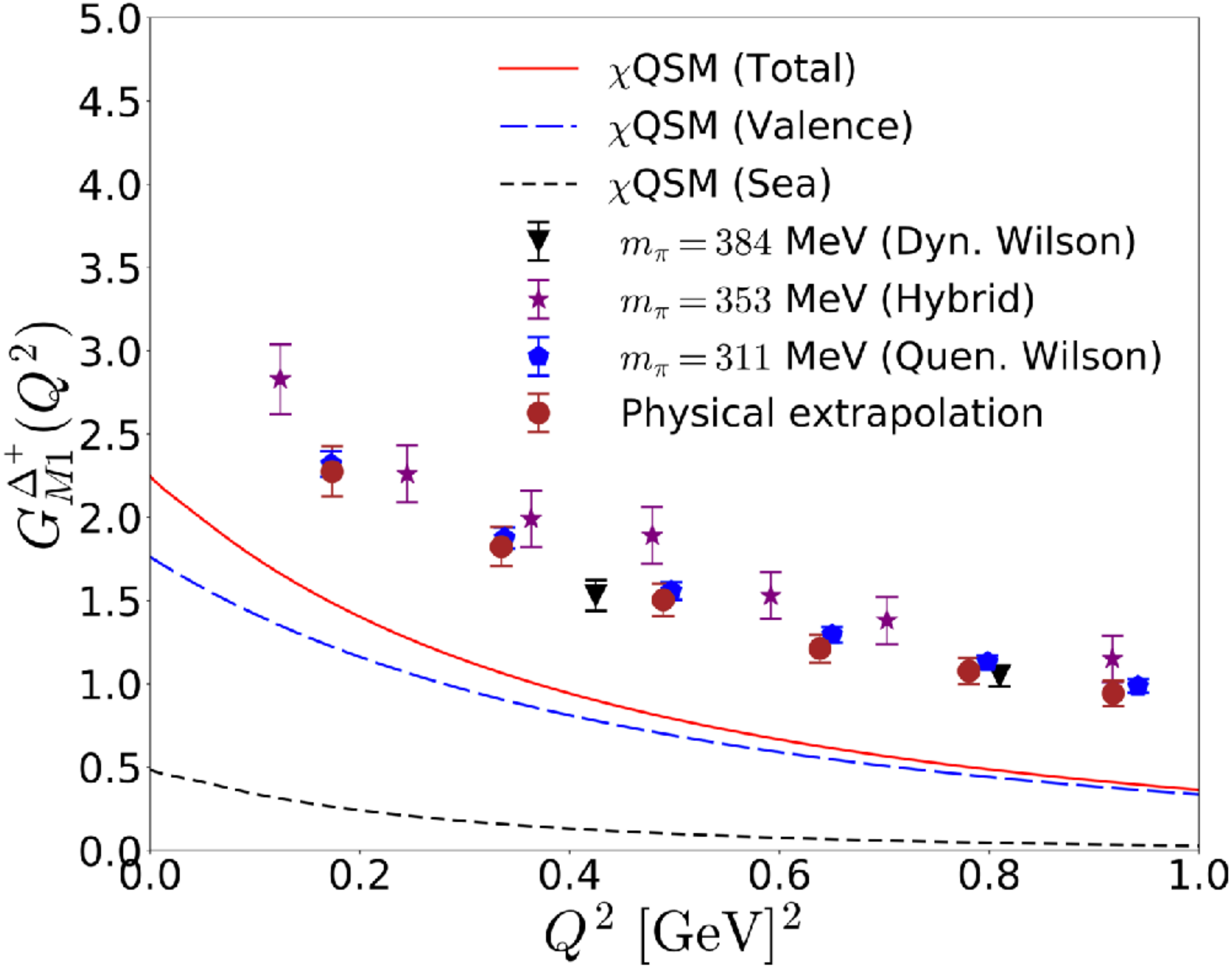

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M1 form factor of the Omega-

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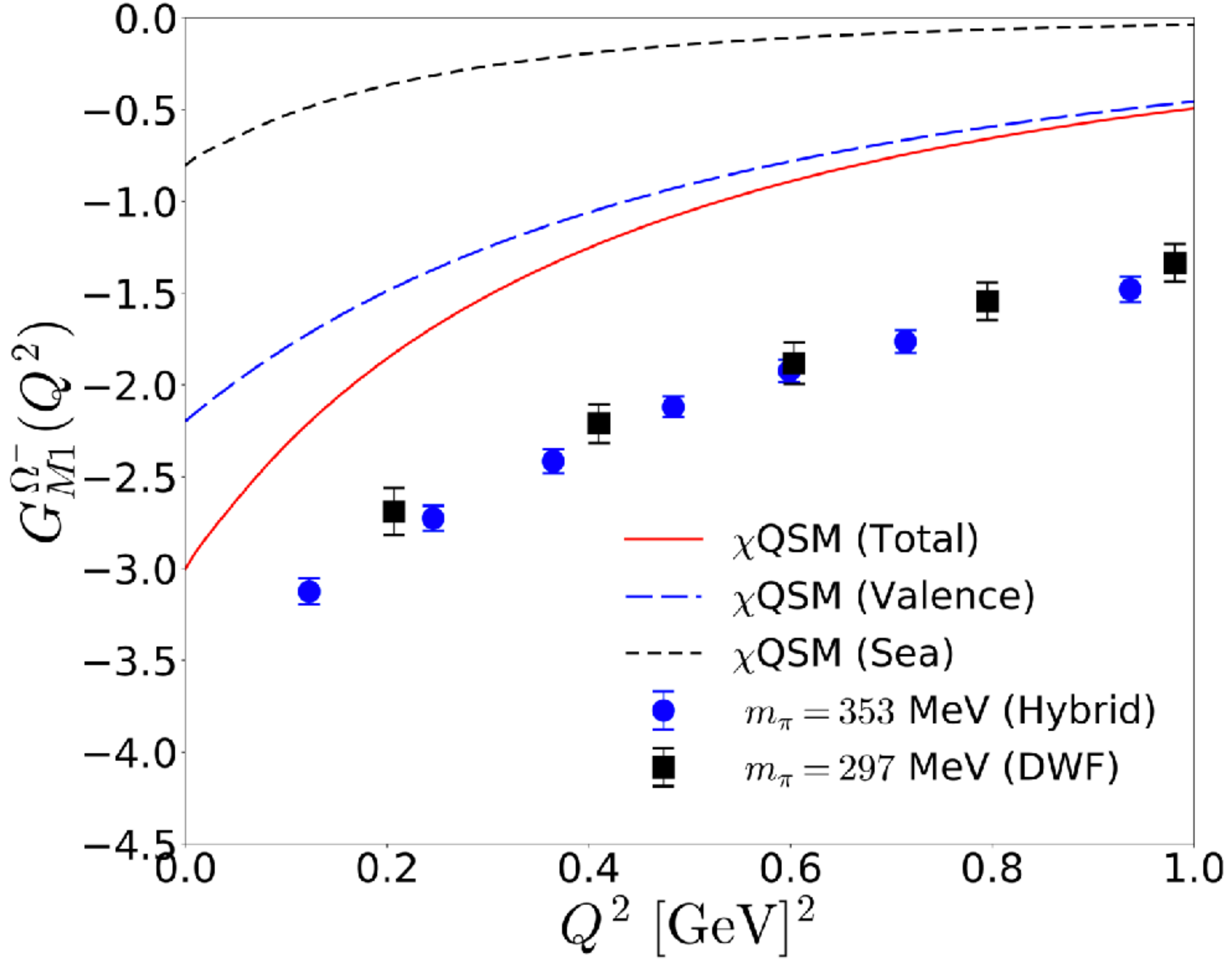
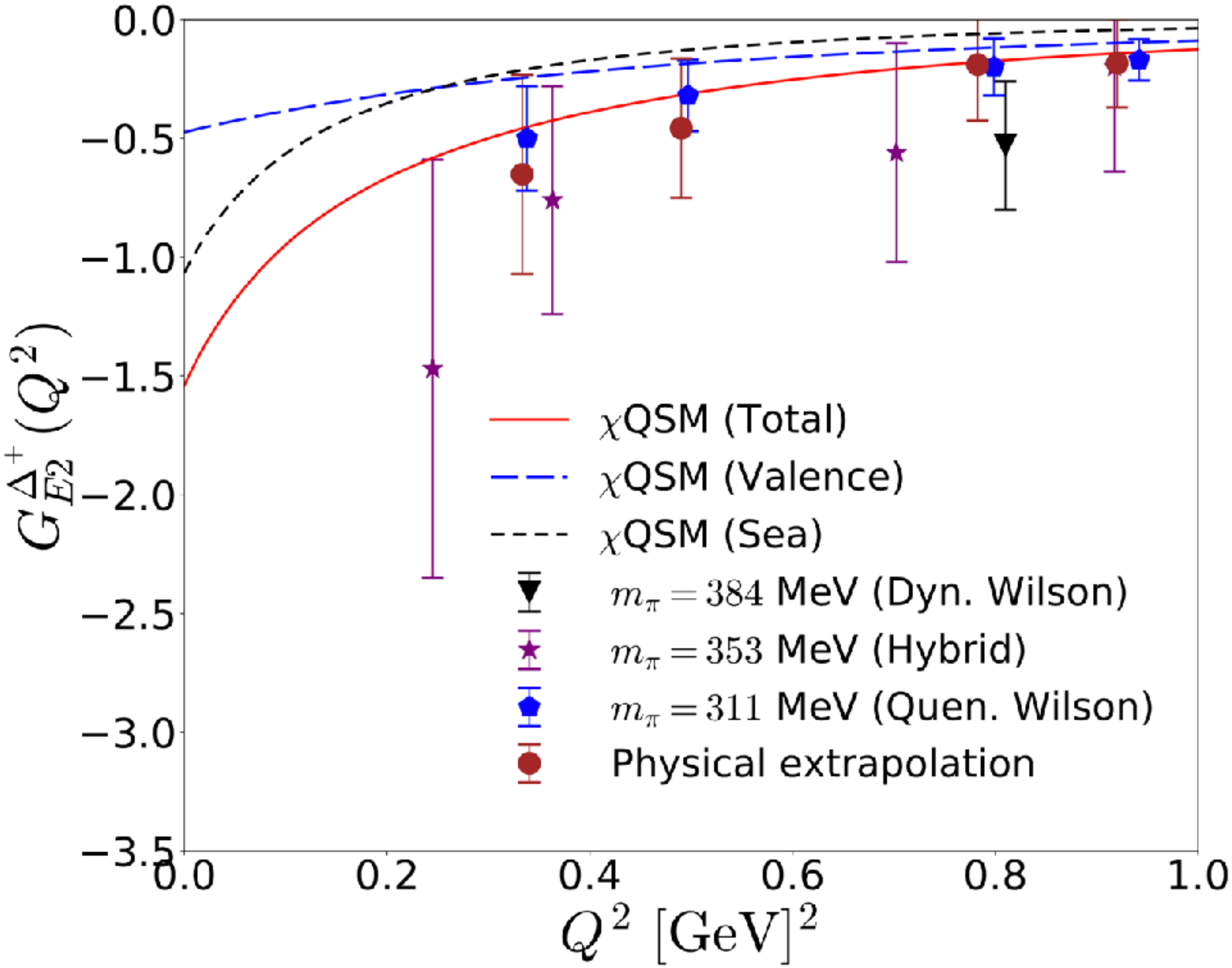


J.-Y. Kim & HChK, EPJC, 79:570 (2019)

Valence & Sea contributions

E2 form factor of the Delta+

Lattice data: Alessandro et al.



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Valence & Sea contributions

E2 form factor of the Omega-

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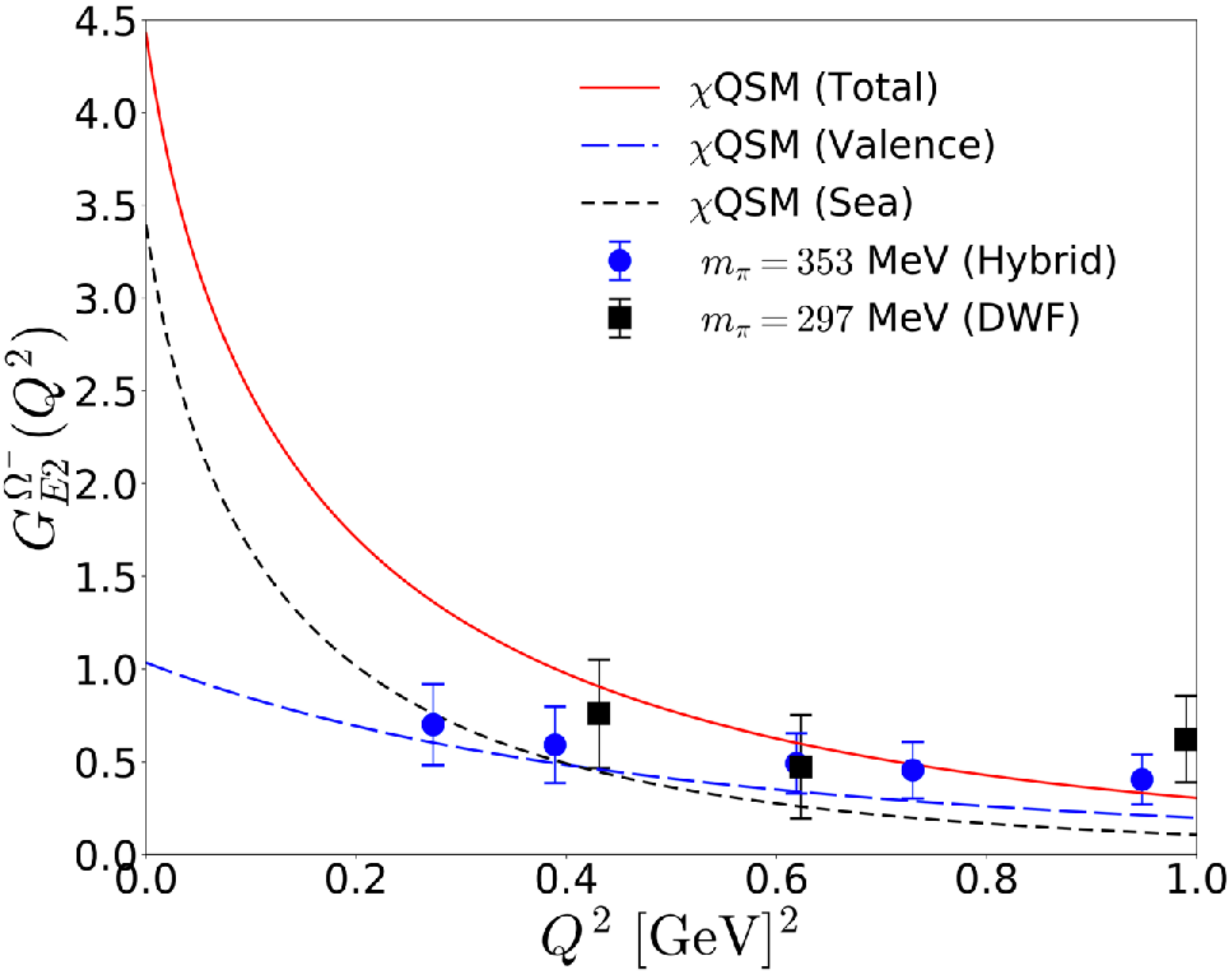
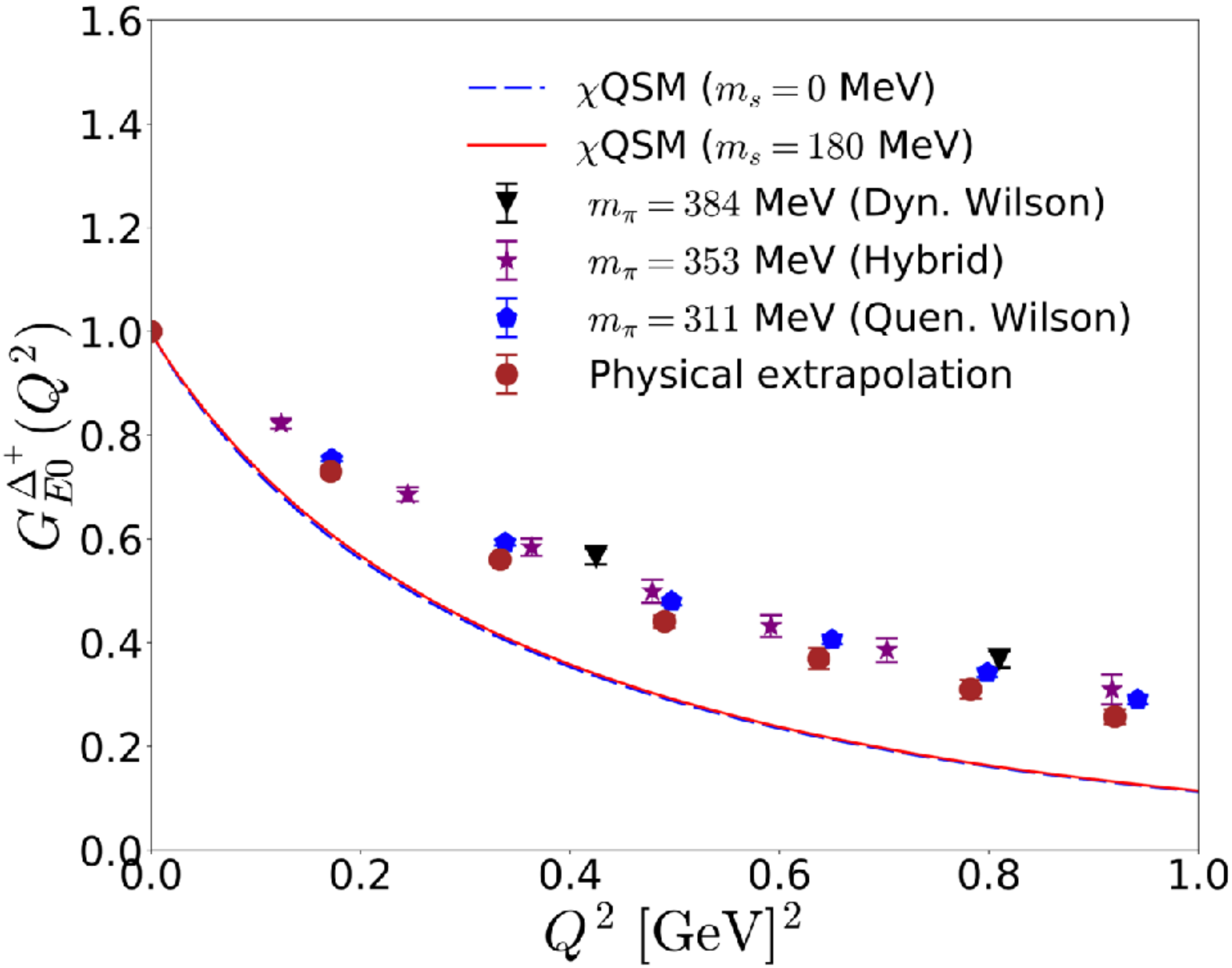


J.-Y. Kim & HChK, EPJC, 79:570 (2019)

Effects of SU(3) symmetry breaking

E0 form factor of the Delta+

Lattice data: Alessandro et al.



Almost no breaking effects

J.-Y. Kim & HChK, EPJC, 79:570 (2019)

Effects of SU(3) symmetry breaking

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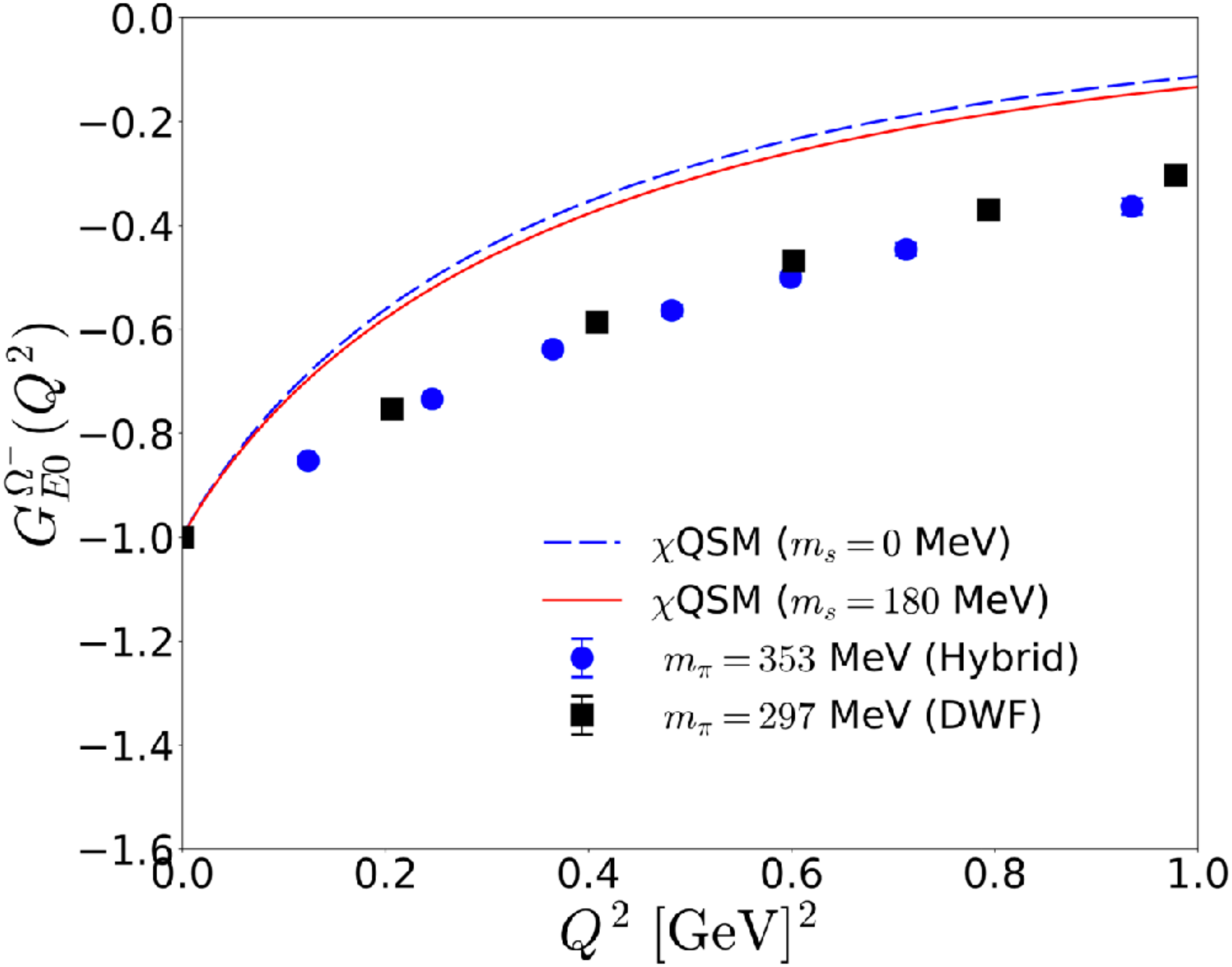

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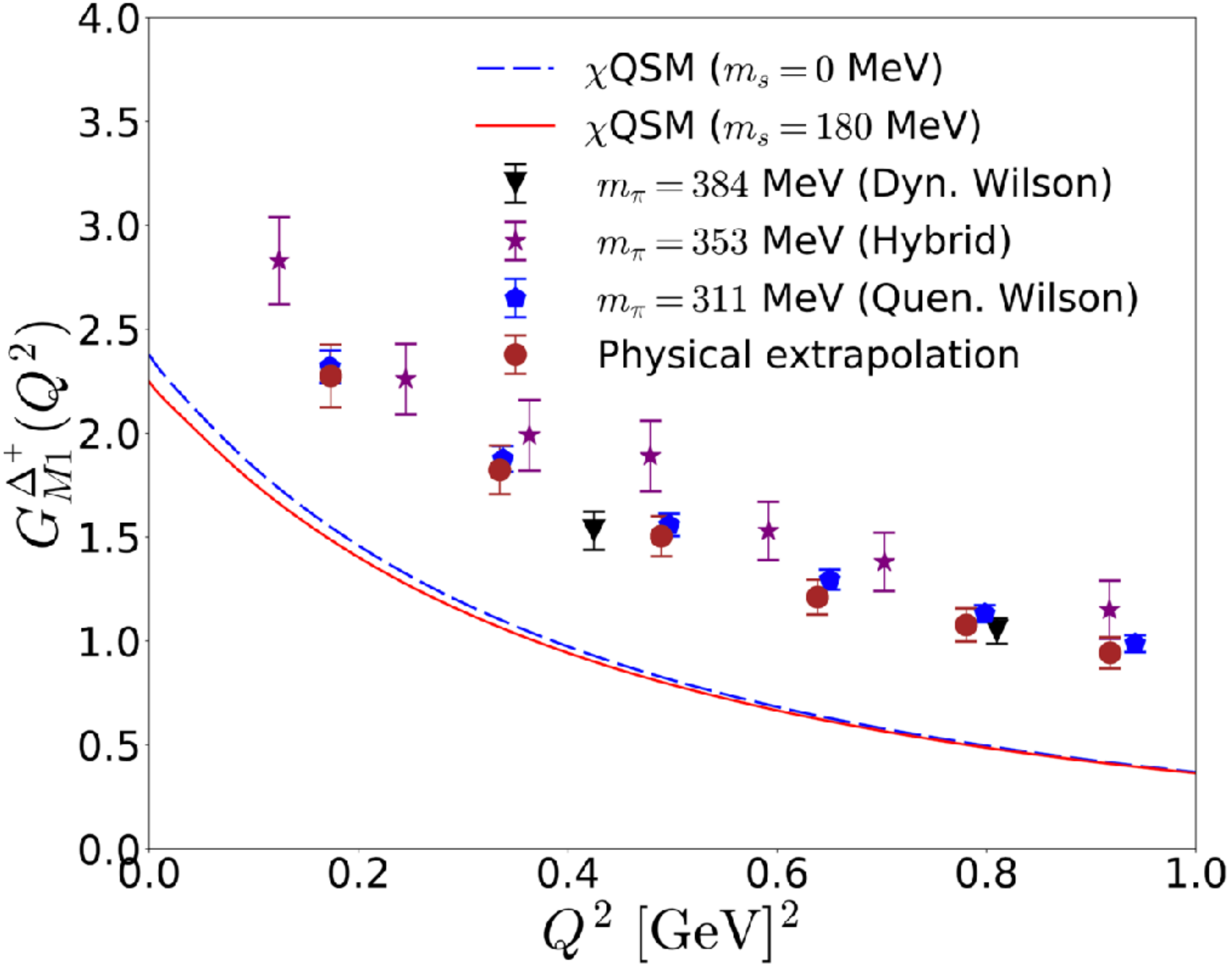

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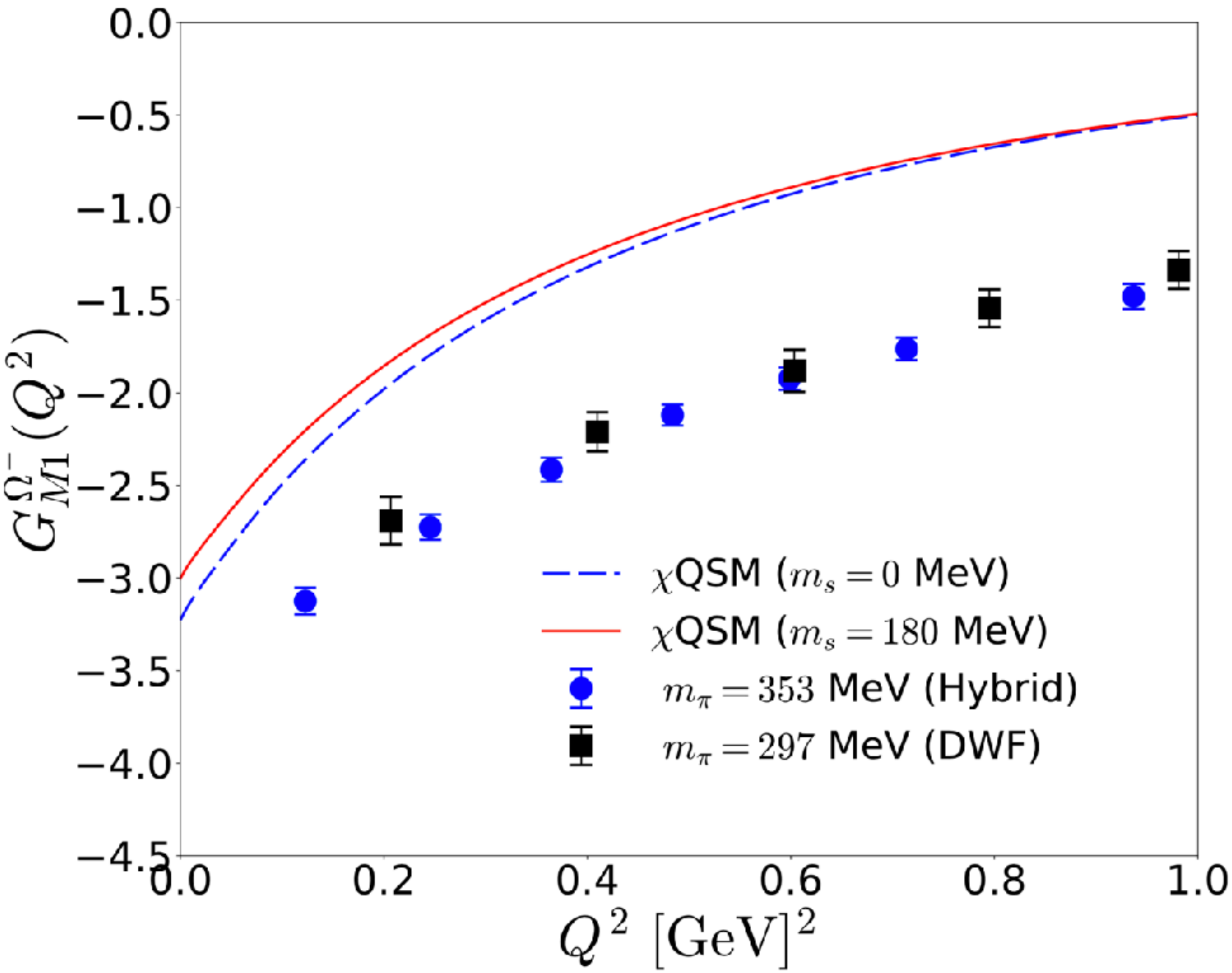
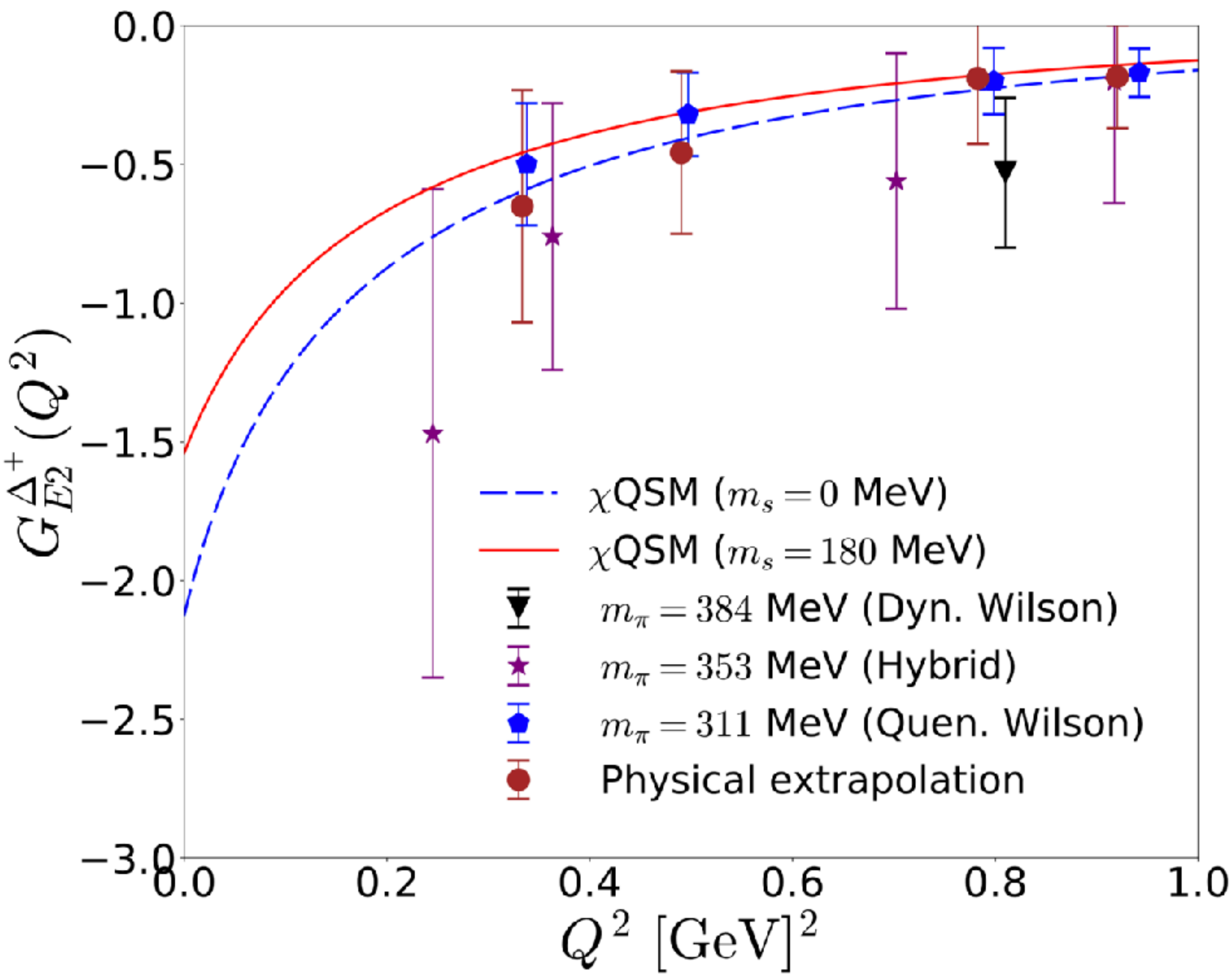


J.-Y. Kim & HChK, EPJC, 79:570 (2019)

Effects of SU(3) symmetry breaking

E2 form factor of the Delta+

Lattice data: Alessandro et al.



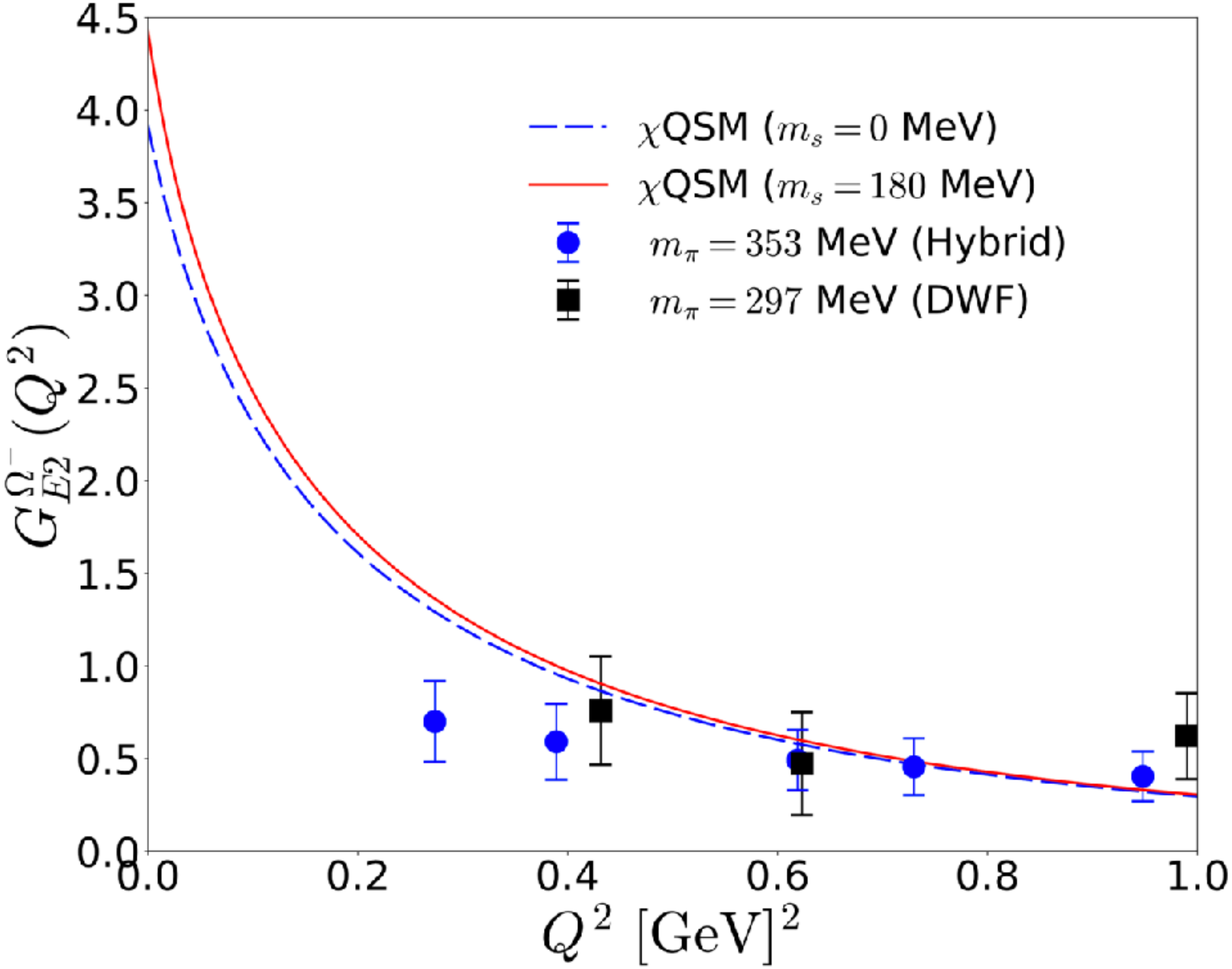
Sizable effects from SU(3) symmetry breaking

J.-Y. Kim & HChK, EPJC, 79:570 (2019)

Effects of SU(3) symmetry breaking

E2 form factor of the Omega-

Lattice data: Alessandro et al.

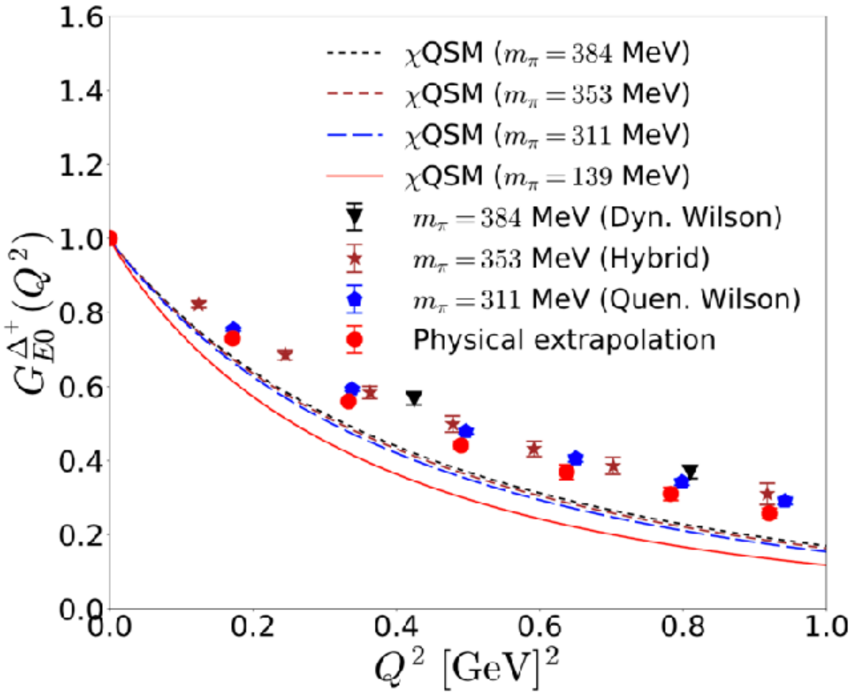


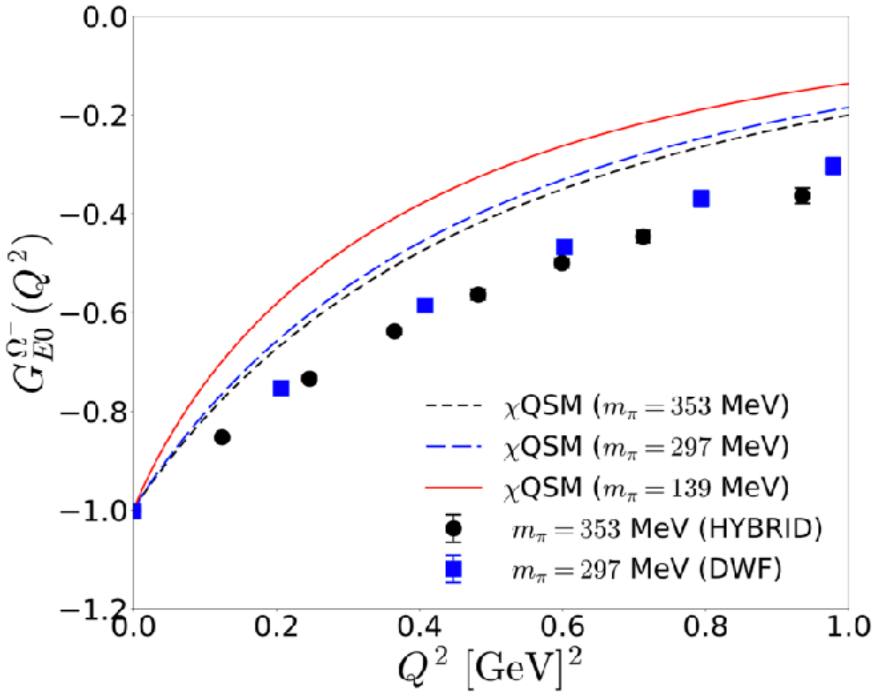
J.-Y. Kim & HChK, EPJC, 79:570 (2019)

Comparison with the lattice data

E0 form factors

Lattice data: Alessandro et al.



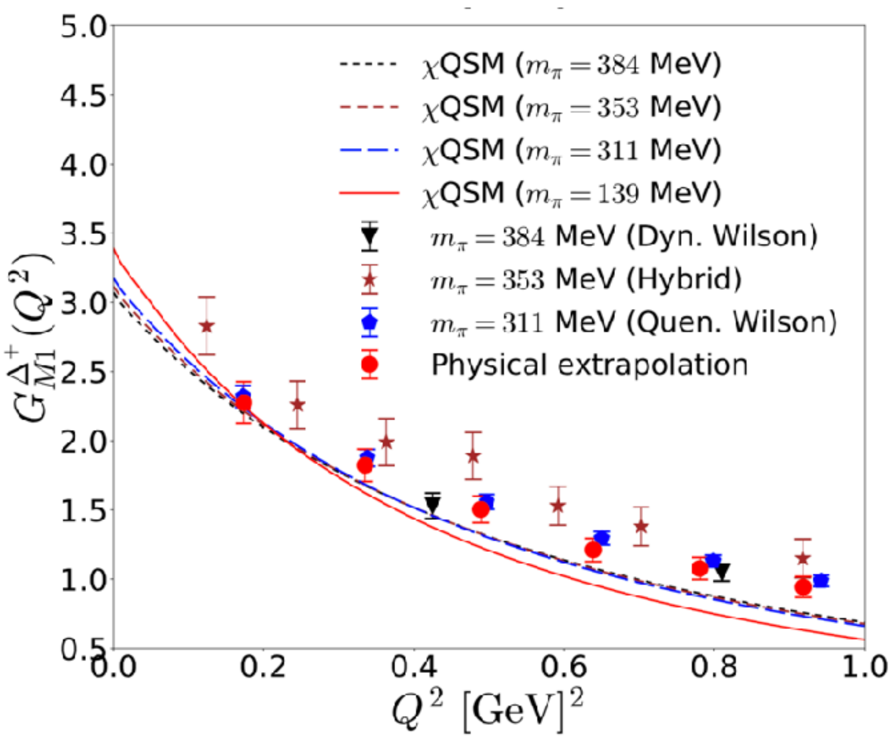


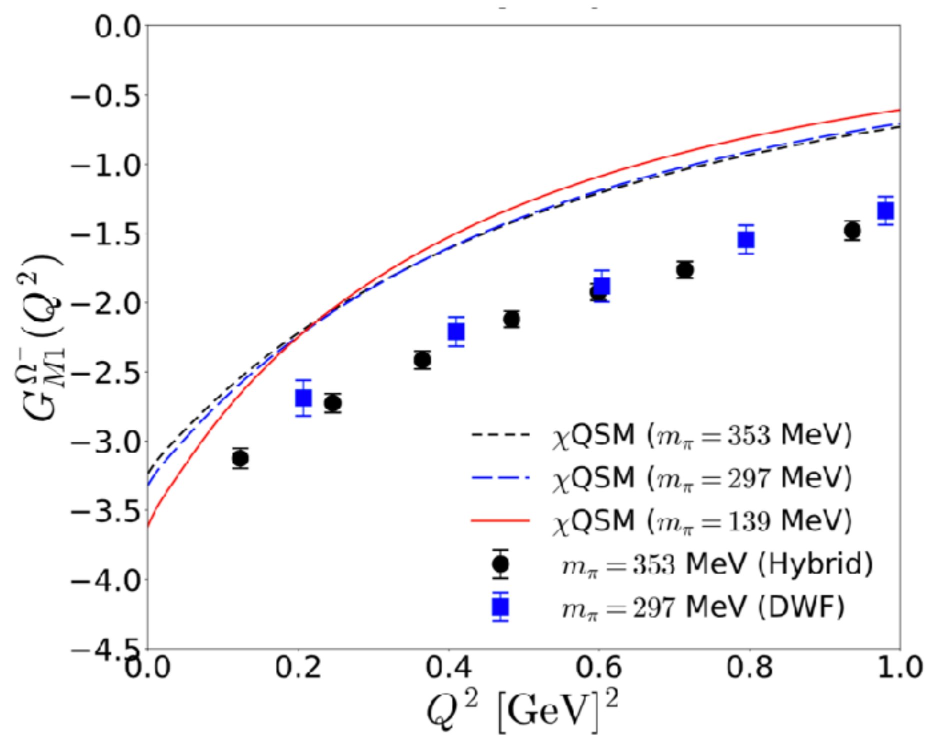
J.-Y. Kim & HChK, EPJC, 79:570 (2019)

Comparison with the lattice data

M1 form factors

Lattice data: Alessandro et al.



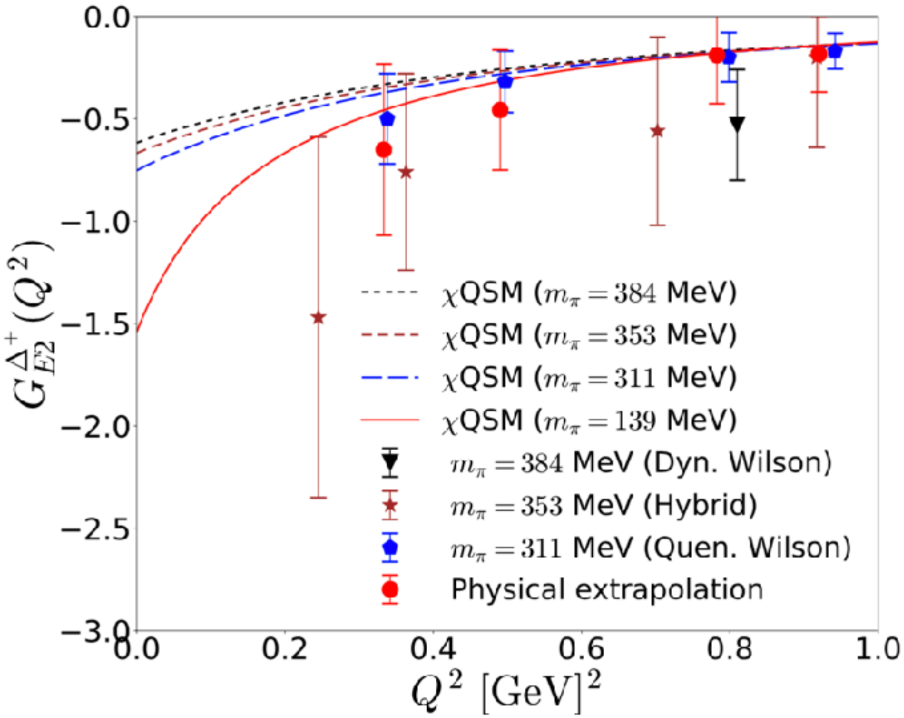


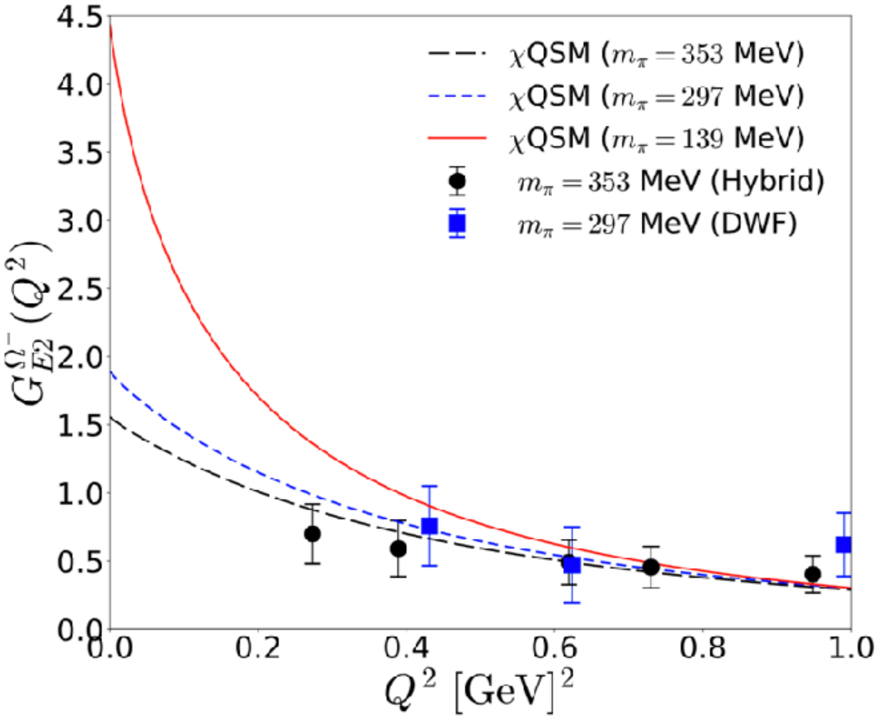
J.-Y. Kim & HChK, EPJC, 79:570 (2019)

Comparison with the lattice data

E2 form factors

Lattice data: Alessandro et al.





J.-Y. Kim & HChK, EPJC, 79:570 (2019)

Multipole pattern in the transverse plane

Δ^+

Carlson & Vanderhaeghen, PRD 100 (2008) 032004

Transverse charge density

$$\rho^{\Delta}_{T\frac{3}{2}}(\vec{b}) = \int_0^\infty \frac{dQ}{2\pi} Q[J_0(Qb)\frac{1}{4}(A_{\frac{3}{2}\frac{3}{2}} + 3A_{\frac{1}{2}\frac{1}{2}}) - \sin(\phi_b - \phi_S)J_1(Qb)\frac{1}{4}(2\sqrt{3}A_{\frac{3}{2}\frac{1}{2}} + 3A_{\frac{1}{2}-\frac{1}{2}})$$
$$- \cos(2(\phi_b - \phi_S))J_2(Qb)\frac{\sqrt{3}}{2}A_{\frac{3}{2}-\frac{1}{2}} + \sin(3(\phi_b - \phi_S))J_3(Qb)\frac{1}{4}A_{\frac{3}{2}-\frac{3}{2}}]$$

Transverse spin of the Delta

$$\boldsymbol{S}_\perp = \cos\phi_S \hat{e}_x + \sin\phi_S \hat{e}_y$$

Radial vector in the transverse plane

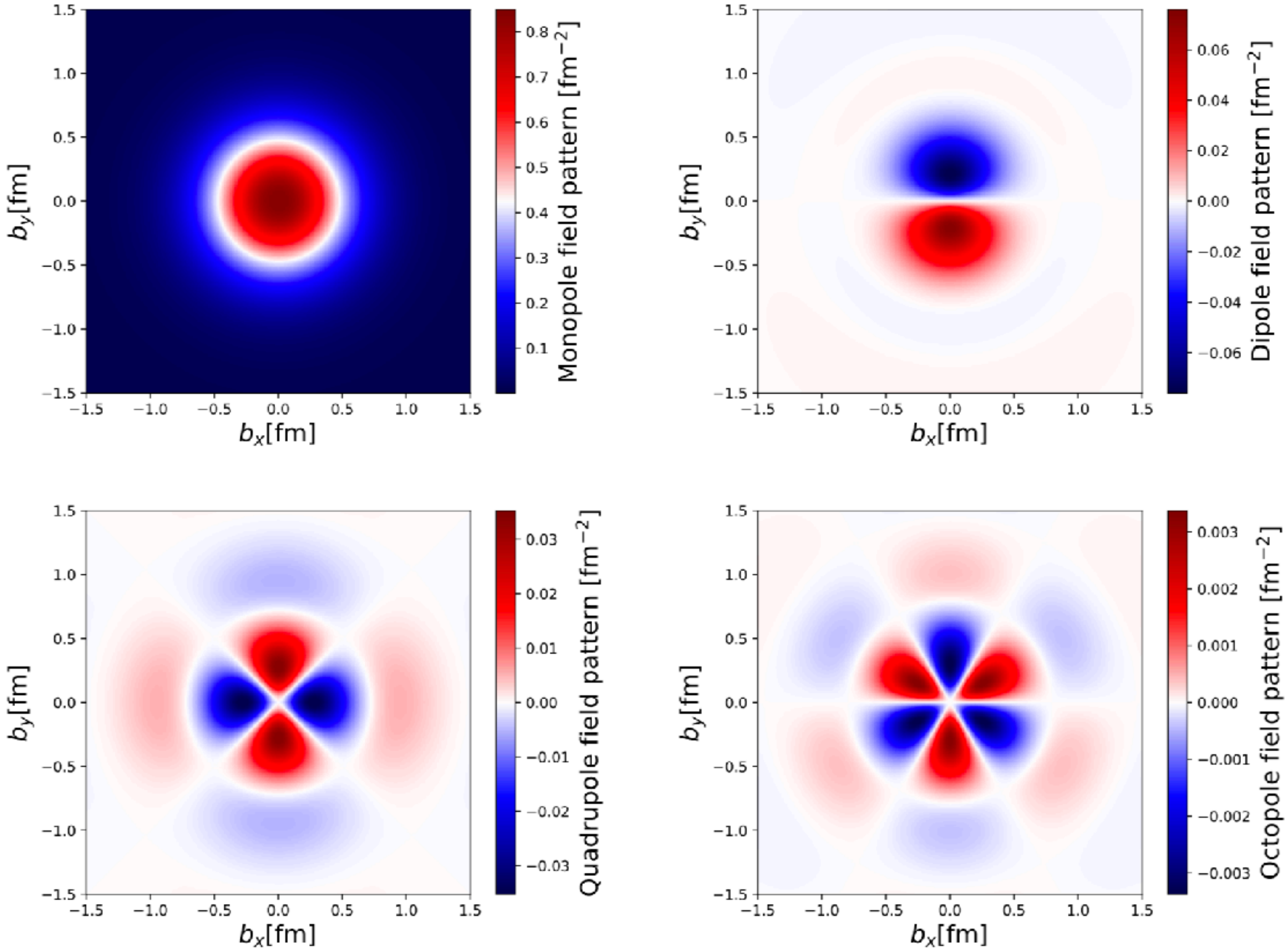
$$\boldsymbol{b} = b(\cos\phi_b \hat{e}_x + \sin\phi_b \hat{e}_y)$$

Preliminary results (J.-Y. Kim & HChK)

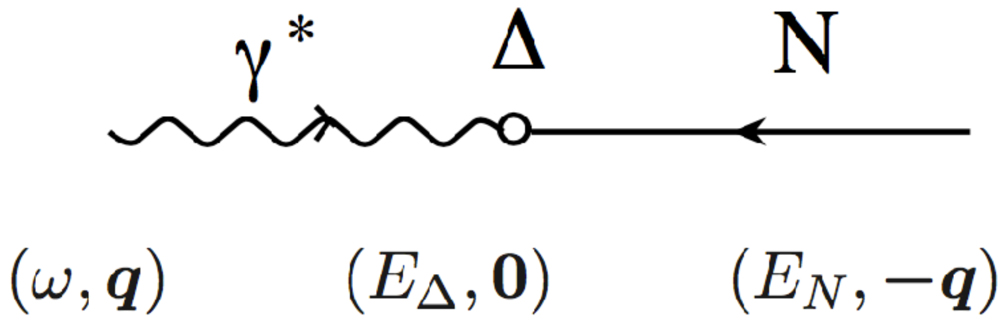
Multipole pattern in the transverse plane

Δ^+

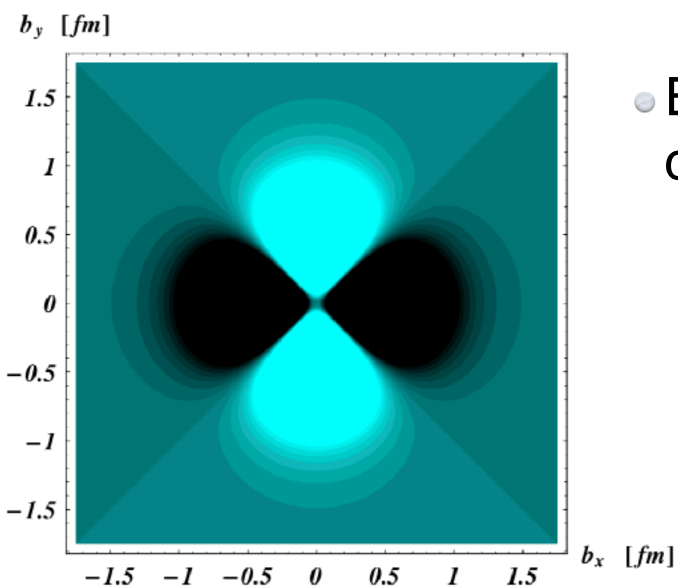
Preliminary results (J.-Y. Kim & HChK)



EM transition form factors of the decuplet



- EM transition FFs provide information on how the Delta looks like.



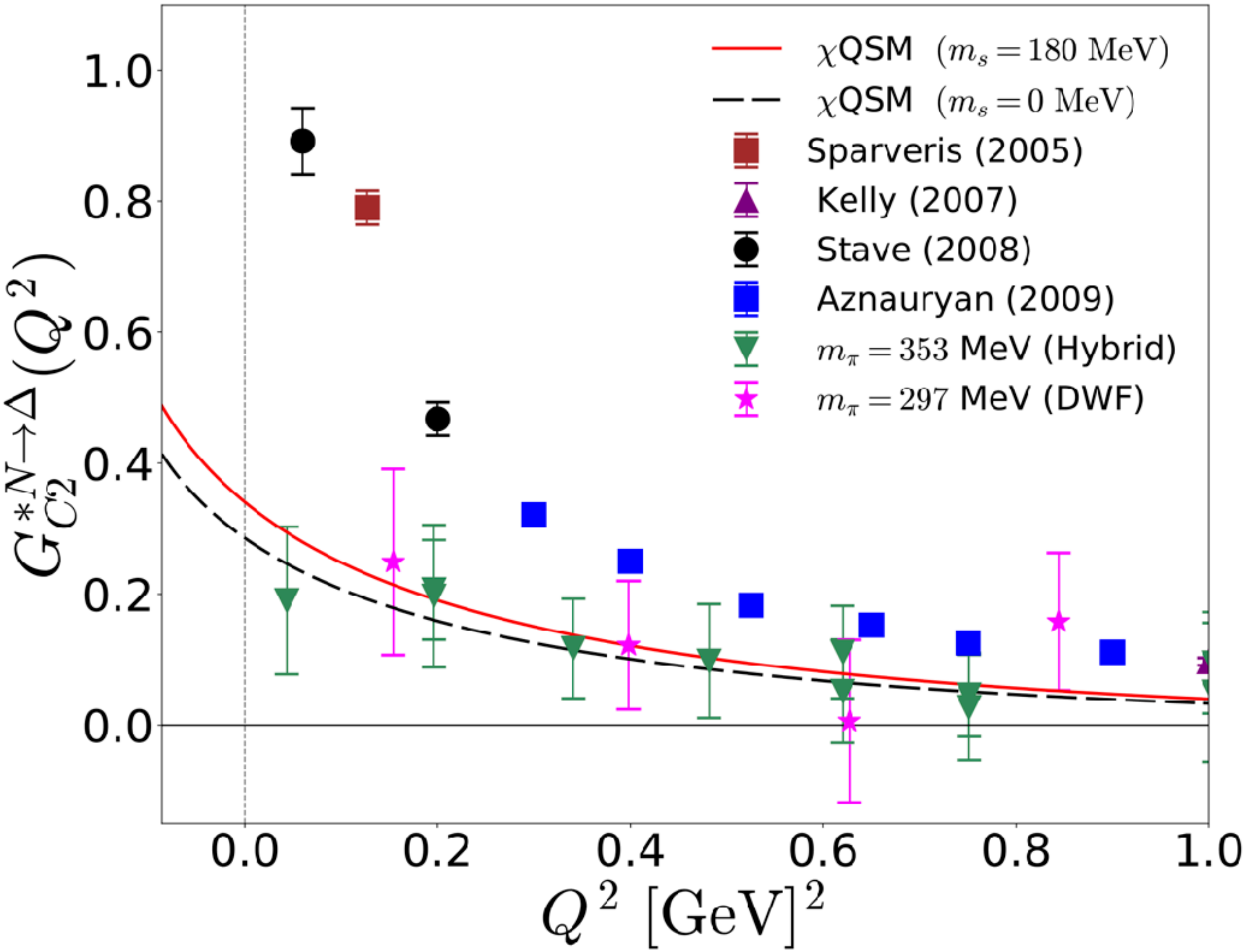
- EM transition FFs are related to the VBB coupling constants through VDM & CFI.

→ Essential to understand a production mechanism of hadrons.

Carlson & Vanderhaeghen, PRD 100 (2008) 032004

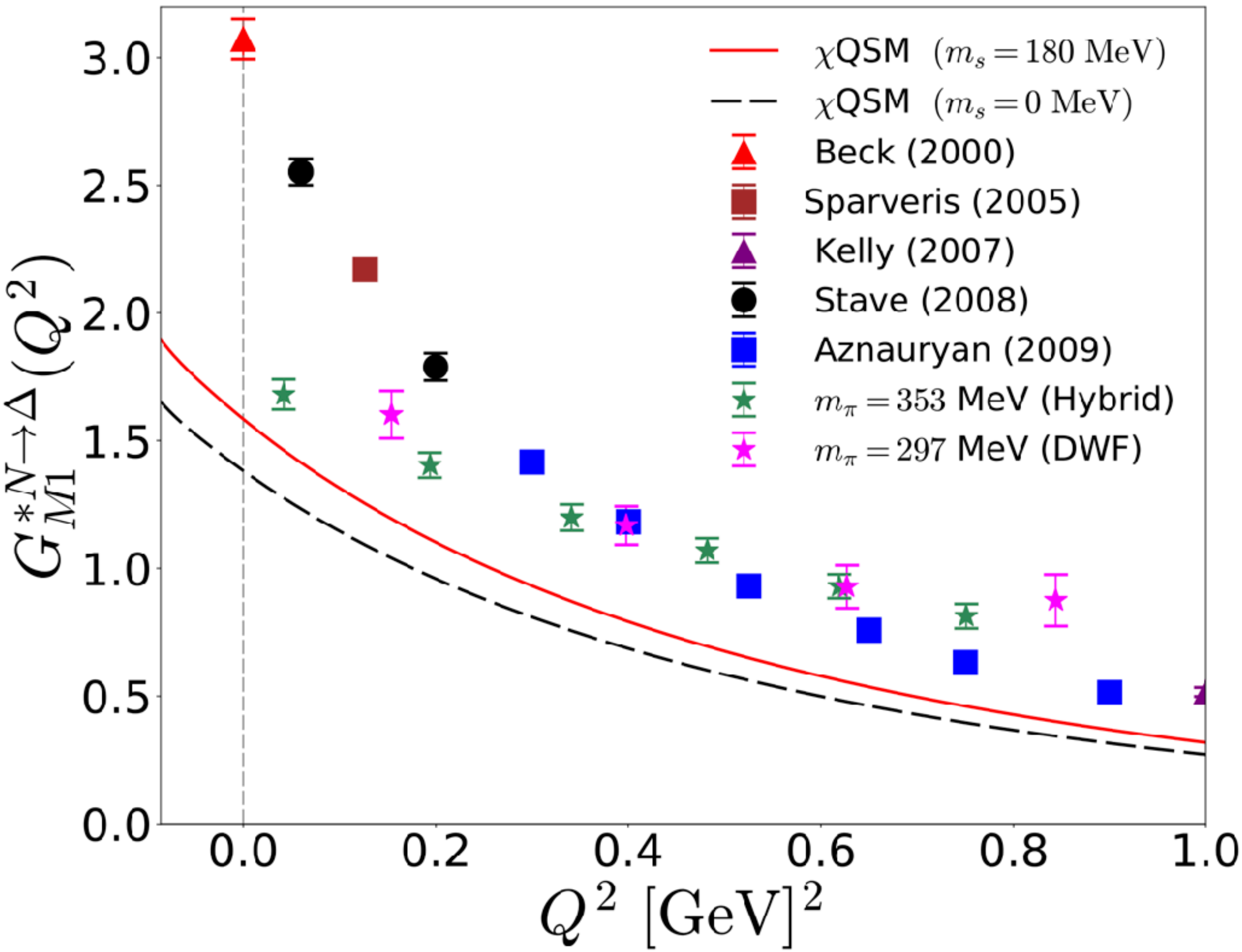
Delta-N transitions

Coulomb form factors



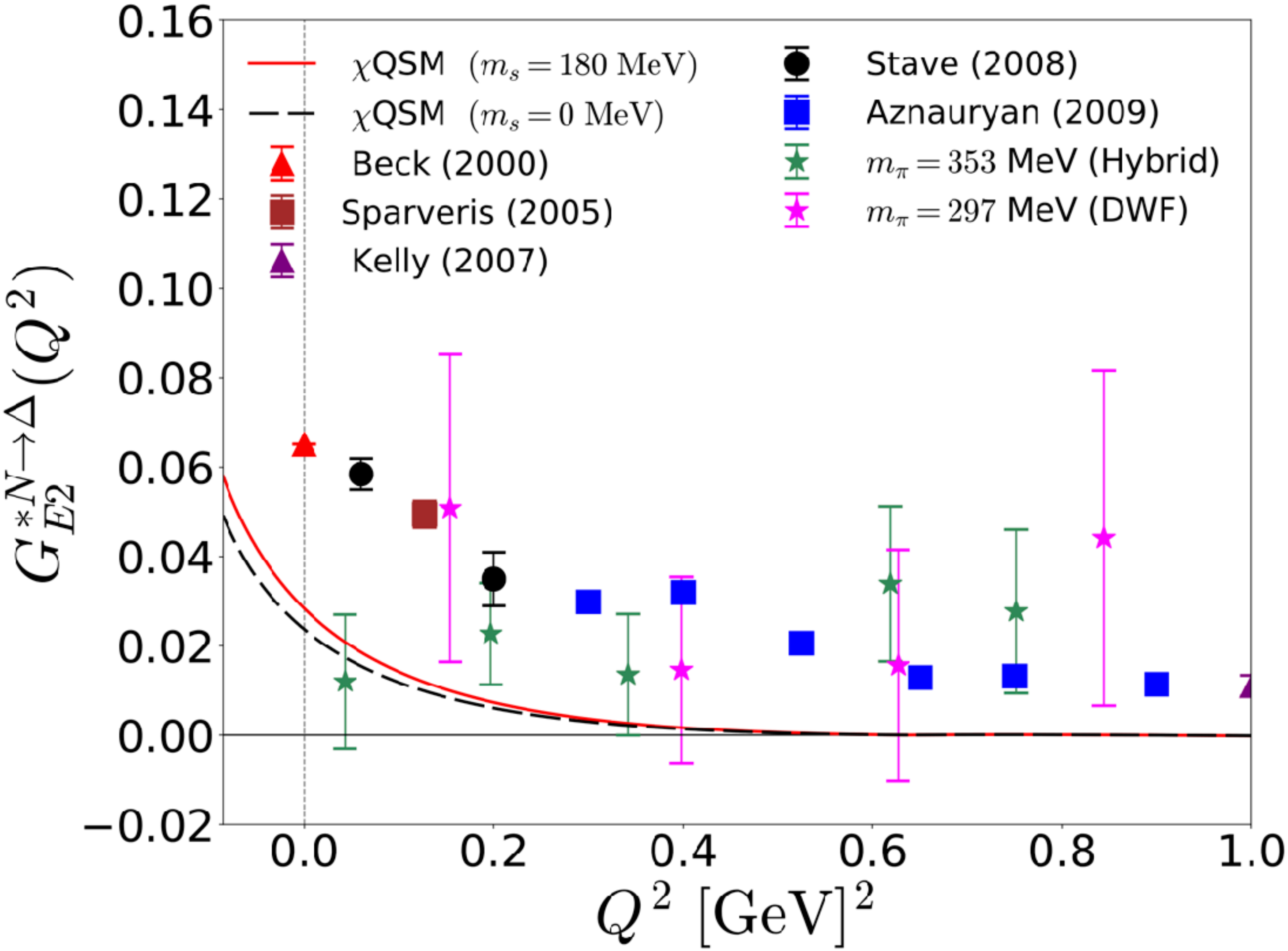
Delta-N transitions

M1 form factors



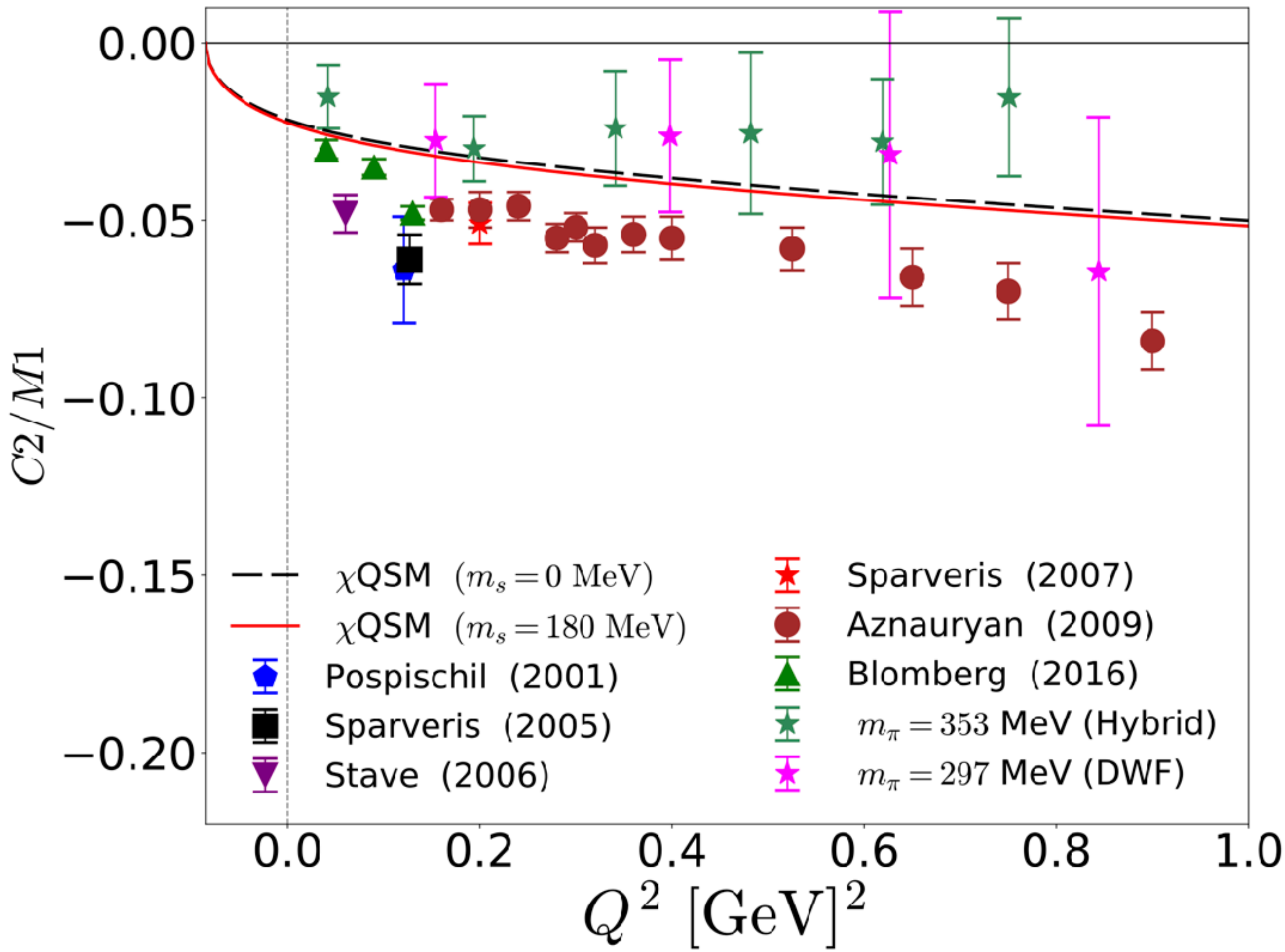
Delta-N transitions

E2 form factors



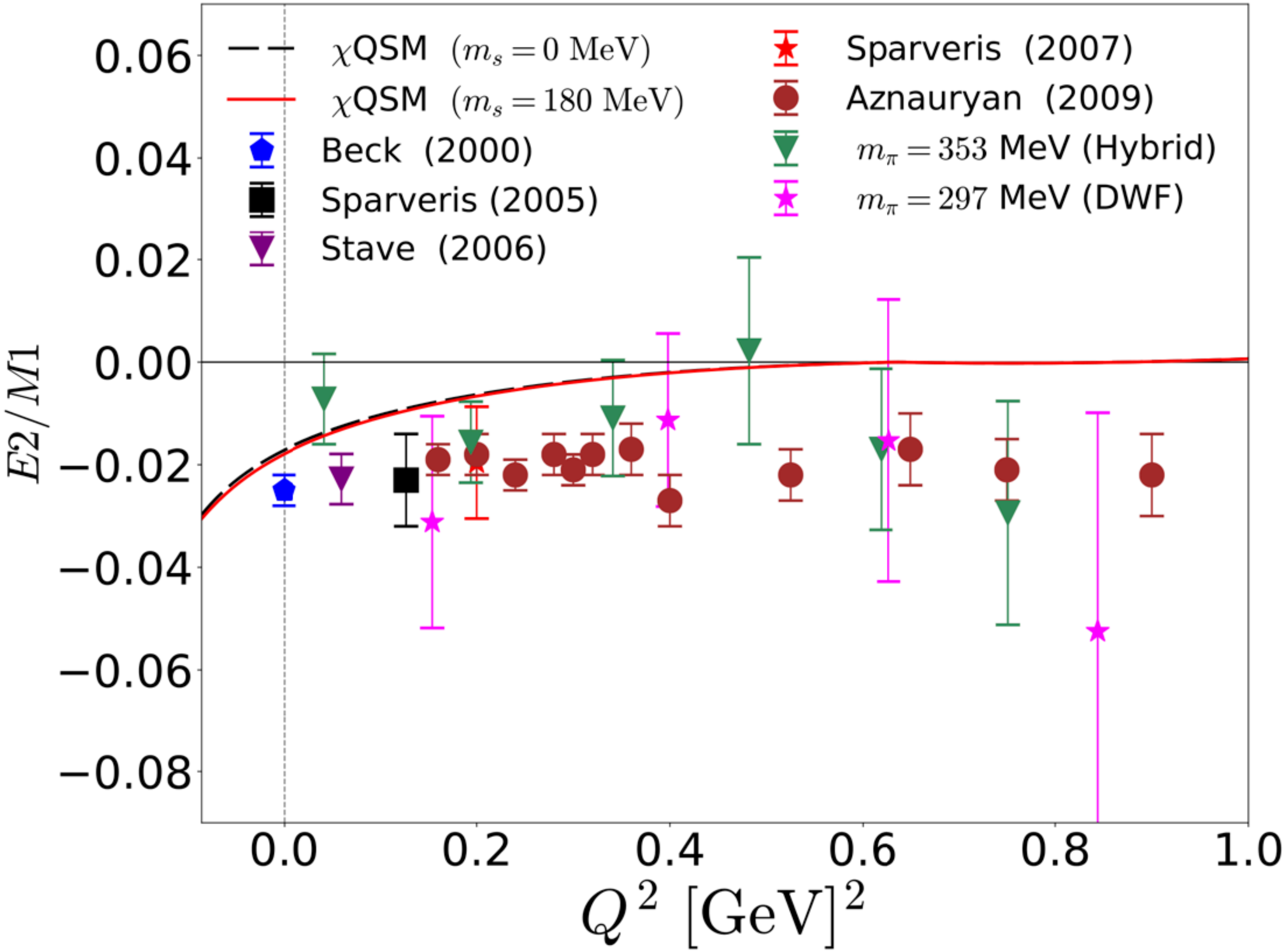
Delta-N transitions

C2/M1



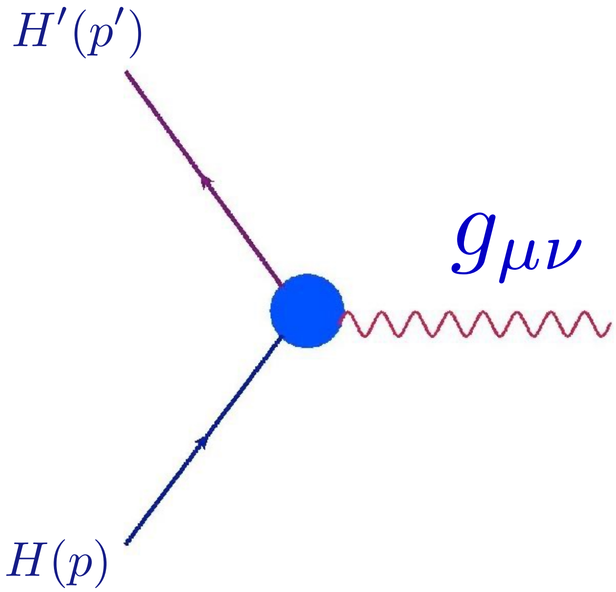
Delta-N transitions

E2/M1



Gravitational Form factors
of
the pion & Nucleon

Gravitational form factors



Graviton: To weak to probe the EMT structure of a hadron

Given an action,

$$\frac{\delta S}{\delta g_{\mu\nu}} = 0 \quad \text{or}$$

$\delta S = 0$ under Poincaré transform

$$\langle \pi^a(p')|T_{\mu\nu}(0)|\pi^b(p)\rangle = \frac{\delta^{ab}}{2}[(tg_{\mu\nu} - q_\mu q_\nu)\Theta_1(t) + 2P_\mu P_\nu \Theta_2(t)]$$

Gravitational form factors

$$2\delta^{ab} H_\pi^{I=0}(x,\xi,t) = \frac{1}{2}\int \frac{d\lambda}{2\pi} e^{ix\lambda(P\cdot n)} \langle \pi^a(p')|\bar\psi(-\lambda n/2)\not{n}[-\lambda n/2, \lambda n/2]\psi(\lambda n/2)|\pi^b(p)\rangle$$

Gravitational or EMT form factors
as the second Melin moments of the EM GPD

$$\int dx x H_\pi^{I=0}(x,\xi,t) = A_{2,0}(t) + 4\xi^2 A_{2,2}(t) \qquad \Theta_1 = -4A_{2,2}^{I=0} \qquad \Theta_2 = A_{2,0}^{I=0}$$

$$\langle \pi^a(p')|T_{\mu\nu}(0)|\pi^b(p)\rangle = \frac{\delta^{ab}}{2}[(tg_{\mu\nu} - q_\mu q_\nu)\Theta_1(t) + 2P_\mu P_\nu \Theta_2(t)]$$

T^{00} : Mass form factor

T^{i0} : Angular momentum

T^{ij} : Shear force and Pressure → Stability of a particle: von Laue condition

Mechanics of a particle

M.V. Polyakov & P. Schweitzer, Int.J.Mod.Phys. A33 (2018) 1830025.

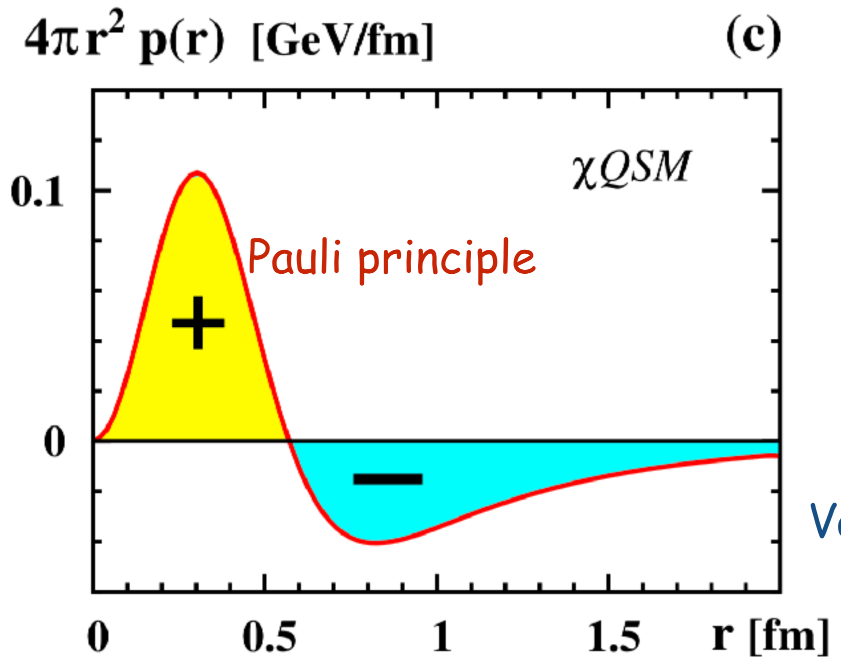
Stability

- Pion: The stability is guaranteed by the chiral symmetry and its spontaneous breakdown

H.D. Son & HChK, PRD 90 (2014) 111901

$$\mathcal{P} = \frac{3M}{f_\pi^2 \bar{M}} (m\langle\bar{\psi}\psi\rangle + m_\pi^2 f_\pi^2) = 0$$

- Nucleon: The stability is guaranteed by the balance between the core valence quarks and the sea quarks (XQSM).

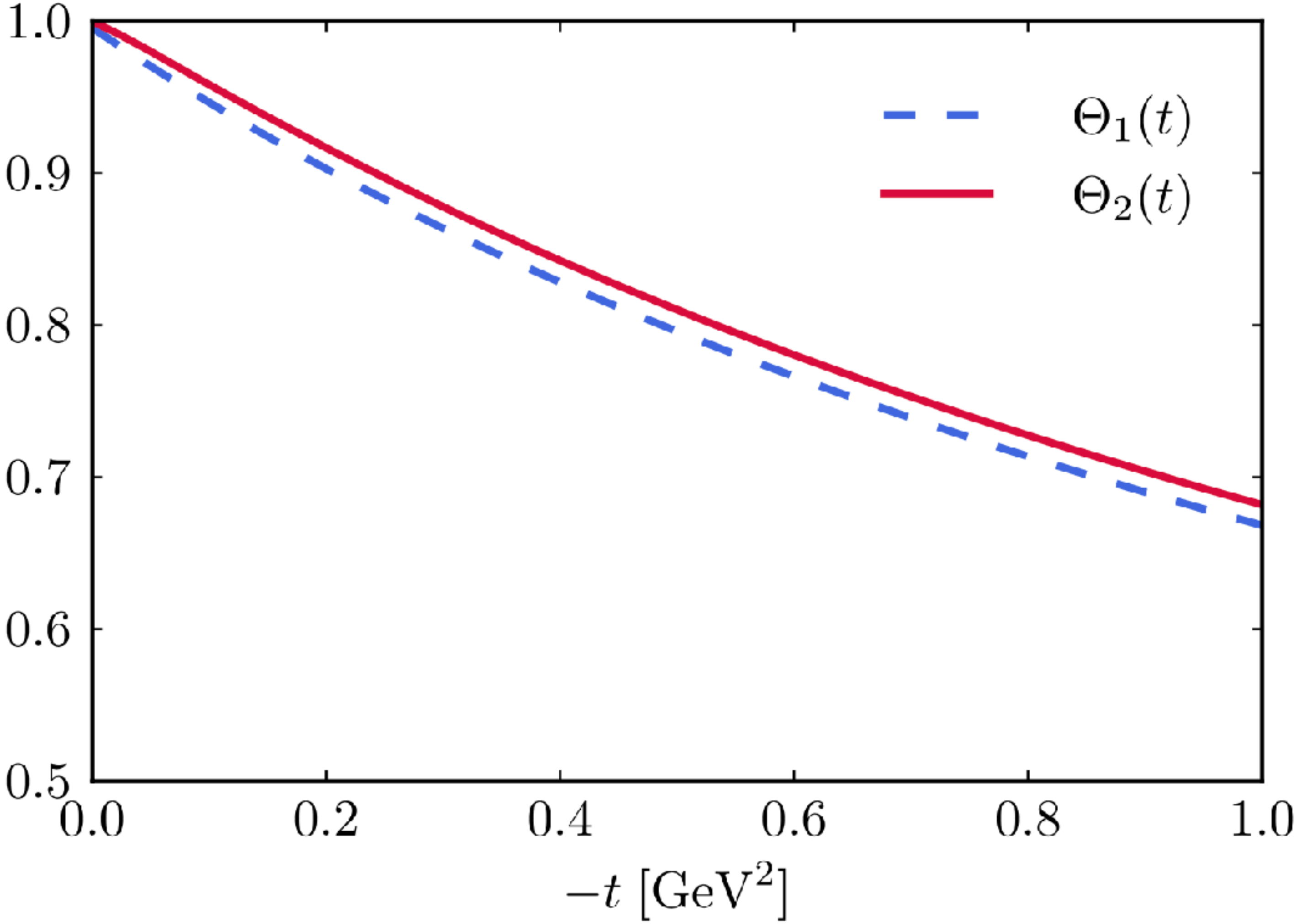


K. Goeke et al., PRD75 (2007) 094021

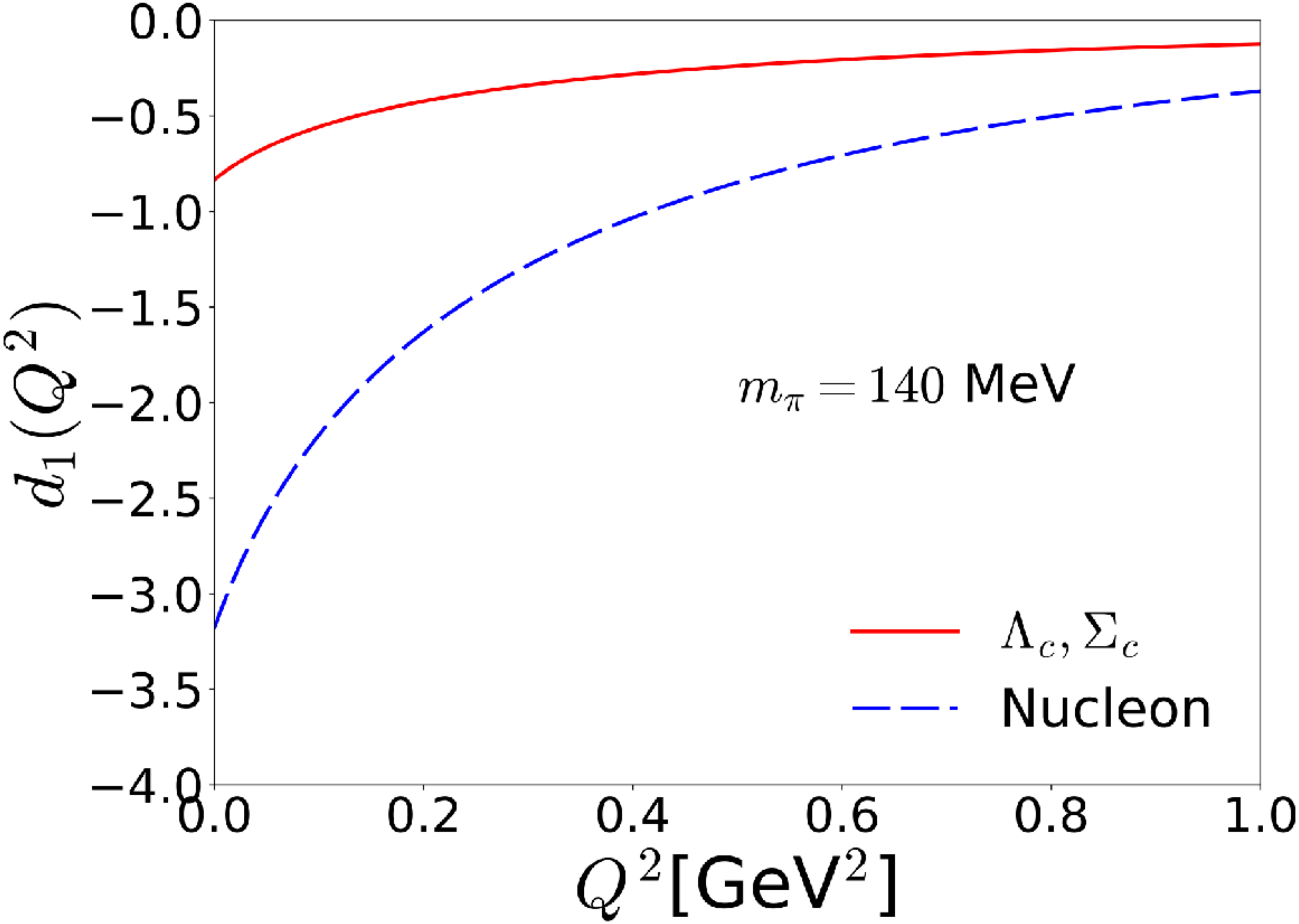
Vacuum polarization (pion clouds)

EMT form factors of the pion

With effects of SU(3) symmetry breaking included



d1 form factors of heavy baryons



Summary & Outlook

Summary & Outlook

- In this talk, we have presented results of series of recent works on the EM form factors of the baryon decuplet.
- We briefly have discussed the gravitational form factors of the pion, nucleon, and heavy baryons.

*** Pion mean-field approaches indeed work for the lowest-lying baryons.**

Outlook

* Theoretical Extension:

- How to go beyond the mean-field approximation:
  Meson-loop corrections (RPA-like)
- Momentum-dependent dynamical quark mass (relatively easy)
- How to introduce the quark confinement as a background field.

* Phenomenological Extension:

- Describing excited baryons with new symmetry
  (hedgehog symmetry): smaller groups than SU(6) x O(3).
- GPDs and TMDs for excited baryons?

Though this be madness, yet there is method in it.

Hamlet Act 2, Scene 2

by Shakespeare

Thank you very much for the attention!