The Modeling of Baryon and Meson DAs, and their Relevance for DVMP

Cédric Mezrag

CEA-Saclay, Irfu/DPhN

November 8^{th}, 2019

In collaboration with:
J. Segovia, L. Chang, M. Ding and C.D. Roberts
Phys.Lett. B783 (2018) 263-267

Hadrons seen as Fock States

- Lightfront quantization allows to expand hadrons on a Fock basis:

$$|P,\pi\rangle \propto \sum_\beta \Psi_\beta^{q\bar{q}}|q\bar{q}\rangle + \sum_\beta \Psi_\beta^{q\bar{q},q\bar{q}}|q\bar{q},q\bar{q}\rangle + \dots$$

$$|P,N\rangle \propto \sum_\beta \Psi_\beta^{qqq}|qqq\rangle + \sum_\beta \Psi_\beta^{qqq,q\bar{q}}|qqq,q\bar{q}\rangle + \dots$$

Hadrons seen as Fock States

- Lightfront quantization allows to expand hadrons on a Fock basis:

$$|P,\pi\rangle \propto \sum_\beta \Psi_\beta^{q\bar{q}}|q\bar{q}\rangle + \sum_\beta \Psi_\beta^{q\bar{q},q\bar{q}}|q\bar{q},q\bar{q}\rangle + \dots$$

$$|P,N\rangle \propto \sum_\beta \Psi_\beta^{qqq}|qqq\rangle + \sum_\beta \Psi_\beta^{qqq,q\bar{q}}|qqq,q\bar{q}\rangle + \dots$$

- Non-perturbative physics is contained in the N-particles Lightfront-Wave Functions (LFWF) Ψ^N

Hadrons seen as Fock States

- Lightfront quantization allows to expand hadrons on a Fock basis:

$$|P,\pi\rangle \propto \sum_\beta \Psi_\beta^{q\bar{q}}|q\bar{q}\rangle + \sum_\beta \Psi_\beta^{q\bar{q},q\bar{q}}|q\bar{q},q\bar{q}\rangle + \dots$$

$$|P,N\rangle \propto \sum_\beta \Psi_\beta^{qqq}|qqq\rangle + \sum_\beta \Psi_\beta^{qqq,q\bar{q}}|qqq,q\bar{q}\rangle + \dots$$

- Non-perturbative physics is contained in the N-particles Lightfront-Wave Functions (LFWF) Ψ^N
- Schematically a distribution amplitude φ is related to the LFWF through:

$$\varphi(x) \propto \int \frac{\mathrm{d}^2 k_\perp}{(2\pi)^2} \Psi(x,k_\perp)$$

S. Brodsky and G. Lepage, PRD 22, (1980)

Distribution amplitudes: definitions

$$\langle 0|O^{\alpha,\cdots}(z_1^-,\ldots,z_n^-)|P,\lambda\rangle$$

- Lightcone operator O of given number of quark and gluon fields

$$\langle 0|O^{\alpha,\cdots}(z_1^-,\ldots,z_n^-)|P,\lambda\rangle = \sum_j^N \tau_j^{\alpha,\cdots} F_j(z_i)$$

- Lightcone operator O of given number of quark and gluon fields
- Expansion in terms of scalar non-pertubative functions $F(z_i)$

$$\langle 0|O^{\alpha,\cdots}(z_1^-,\dots,z_n^-)|P,\lambda\rangle = \sum_j^N \tau_j^{\alpha,\cdots} F_j(z_i)$$

- Lightcone operator O of given number of quark and gluon fields
- Expansion in terms of scalar non-pertubative functions $F(z_i)$
- The τ_j can be chosen to have a definite twist

Distribution amplitudes: definitions

$$\langle 0|O^{\alpha,\cdots}(z_1^-,\ldots,z_n^-)|P,\lambda\rangle = \sum_j^N \tau_j^{\alpha,\cdots} F_j(z_i)$$

- Lightcone operator O of given number of quark and gluon fields
- Expansion in terms of scalar non-pertubative functions $F(z_i)$
- The τ_j can be chosen to have a definite twist
- Leading and higher twist contributions can be selected by adequate projections of O

Distribution amplitudes: definitions

$$\langle 0|O^{\alpha,\cdots}(z_1^-,\ldots,z_n^-)|P,\lambda\rangle = \sum_j^N \tau_j^{\alpha,\cdots} F_j(z_i)$$

- Lightcone operator O of given number of quark and gluon fields
- Expansion in terms of scalar non-pertubative functions $F(z_i)$
- The τ_j can be chosen to have a definite twist
- Leading and higher twist contributions can be selected by adequate projections of O

Both mesons and baryons can (in principle) have multiple independent leading twist DA, and higher-twist DA.

- pion case → a single leading twist DA:

$$\langle 0|\bar{\psi}(0)\gamma\cdot n\gamma_5\psi(z^-)|\pi;p\rangle = f_\pi \int \mathrm{d}x e^{-ixp\cdot z}\varphi_\pi(x)$$

A.V. Efremov and A.V. Radyushkin, Phys. Lett. B94 (1980) 245
G.P. Lepage and S.J. Brodsky, Phys. Rev. 022 (1980) 2157

- rho case → two leading twist DAs:

$$\langle 0|\bar{\psi}(0)\gamma\cdot n\psi(z^-)|\rho;p,\lambda\rangle = e^{(\lambda)}\cdot n\, f_\rho m_\rho \int_0^1 \mathrm{d}x e^{-ixp\cdot z}\varphi_{||}(x)$$

$$\langle 0|\bar{\psi}(0)\sigma_{\mu\nu}\psi(z^-)|\rho;p,\lambda\rangle = i(e_\mu^{(\lambda)}p_\nu - e_\nu^{(\lambda)}p_\mu)f_\rho^\perp \int_0^1 \mathrm{d}x e^{-ixp\cdot z}\varphi_\perp(x)$$

A. Ali *et al.*, Z.Phys. C63 (1994) 437-454

Nucleon Distribution Amplitudes

- 3 bodies matrix element:

$$\langle 0|\epsilon^{ijk} u_\alpha^i(z_1) u_\beta^j(z_2) d_\gamma^k(z_3)|P\rangle$$

- 3 bodies matrix element expanded at leading twist:

$$\langle 0|\epsilon^{ijk} u^i_\alpha(z_1) u^j_\beta(z_2) d^k_\gamma(z_3)|P\rangle = \frac{1}{4}\Big[\left(\not{p}C\right)_{\alpha\beta}\left(\gamma_5 N^+\right)_\gamma V(z_i^-)$$
$$+\left(\not{p}\gamma_5 C\right)_{\alpha\beta}\left(N^+\right)_\gamma A(z_i^-) - \left(ip^\mu\sigma_{\mu\nu}C\right)_{\alpha\beta}\left(\gamma^\nu\gamma_5 N^+\right)_\gamma T(z_i^-)\Big]$$

V. Chernyak and I. Zhitnitsky, Nucl. Phys. B 246, (1984)

- 3 bodies matrix element expanded at leading twist:

$$\langle 0|\epsilon^{ijk} u^i_\alpha(z_1) u^j_\beta(z_2) d^k_\gamma(z_3)|P\rangle = \frac{1}{4}\Big[\left(\not{p}C\right)_{\alpha\beta}\left(\gamma_5 N^+\right)_\gamma V(z_i^-)$$
$$+\left(\not{p}\gamma_5 C\right)_{\alpha\beta}\left(N^+\right)_\gamma A(z_i^-) - \left(ip^\mu\sigma_{\mu\nu}C\right)_{\alpha\beta}\left(\gamma^\nu\gamma_5 N^+\right)_\gamma T(z_i^-)\Big]$$

V. Chernyak and I. Zhitnitsky, Nucl. Phys. B 246, (1984)

- Usually, one defines $\varphi = V - A$

- 3 bodies matrix element expanded at leading twist:

$$\langle 0|\epsilon^{ijk} u^i_\alpha(z_1) u^j_\beta(z_2) d^k_\gamma(z_3)|P\rangle = \frac{1}{4}\left[\left(\not{p}C\right)_{\alpha\beta}\left(\gamma_5 N^+\right)_\gamma V(z_i^-)\right.$$
$$\left.+\left(\not{p}\gamma_5 C\right)_{\alpha\beta}\left(N^+\right)_\gamma A(z_i^-) - \left(ip^\mu \sigma_{\mu\nu} C\right)_{\alpha\beta}\left(\gamma^\nu \gamma_5 N^+\right)_\gamma T(z_i^-)\right]$$

V. Chernyak and I. Zhitnitsky, Nucl. Phys. B 246, (1984)

- Usually, one defines $\varphi = V - A$
- 3 bodies Fock space interpretation (leading twist):

$$|P,\uparrow\rangle = \int \frac{[\mathrm{d}x]}{8\sqrt{6x_1x_2x_3}}|uud\rangle \otimes [\varphi(x_1,x_2,x_3)|\uparrow\downarrow\uparrow\rangle$$
$$+\varphi(x_2,x_1,x_3)|\downarrow\uparrow\uparrow\rangle - 2T(x_1,x_2,x_3)|\uparrow\uparrow\downarrow\rangle]$$

- 3 bodies matrix element expanded at leading twist:

$$\langle 0|\epsilon^{ijk} u^i_\alpha(z_1) u^j_\beta(z_2) d^k_\gamma(z_3)|P\rangle = \frac{1}{4}\left[\left(\not{p}C\right)_{\alpha\beta}\left(\gamma_5 N^+\right)_\gamma V(z_i^-)\right.$$
$$\left. + \left(\not{p}\gamma_5 C\right)_{\alpha\beta}\left(N^+\right)_\gamma A(z_i^-) - \left(ip^\mu \sigma_{\mu\nu} C\right)_{\alpha\beta}\left(\gamma^\nu\gamma_5 N^+\right)_\gamma T(z_i^-)\right]$$

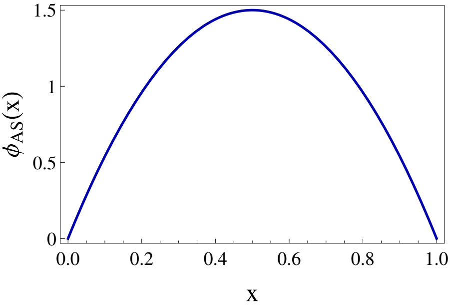
V. Chernyak and I. Zhitnitsky, Nucl. Phys. B 246, (1984)

- Usually, one defines $\varphi = V - A$
- 3 bodies Fock space interpretation (leading twist):

$$|P,\uparrow\rangle = \int \frac{[\mathrm{d}x]}{8\sqrt{6x_1x_2x_3}}|uud\rangle \otimes [\varphi(x_1,x_2,x_3)|\uparrow\downarrow\uparrow\rangle$$
$$+\varphi(x_2,x_1,x_3)|\downarrow\uparrow\uparrow\rangle - 2T(x_1,x_2,x_3)|\uparrow\uparrow\downarrow\rangle]$$

- Isospin symmetry:

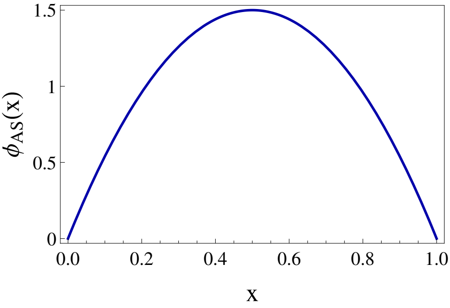
$$2T(x_1,x_2,x_3) = \varphi(x_1,x_3,x_2) + \varphi(x_2,x_3,x_1)$$



- DA are scale dependent objects, they obey Efremov-Radyushkin-Brodsky-Lepage (ERBL) evolution equations $\varphi(x) \rightarrow \varphi(x, \zeta)$
- Evolution equations known at least at NLO, and diagonalized at LO.
- the asymptotic limit is known $\varphi_{AS}(x) = 6x(1-x)$

Evolution and Asymptotic results

The meson case



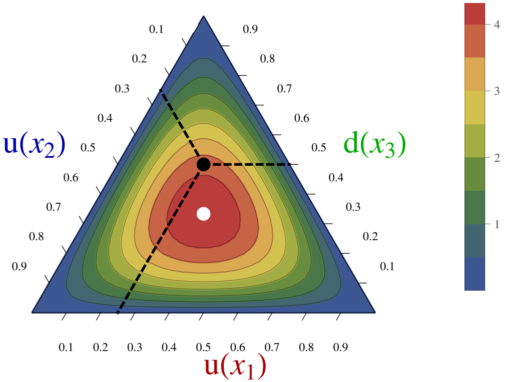
- DA are scale dependent objects, they obey Efremov-Radyushkin-Brodsky-Lepage (ERBL) evolution equations $\varphi(x) \to \varphi(x, \zeta)$
- Evolution equations known at least at NLO, and diagonalized at LO.
- the asymptotic limit is known $\varphi_{AS}(x) = 6x(1-x)$

There is no reason to believe that the asymptotic DA is a good approximation of the DA at a typical scale of $\zeta = 2GeV$.

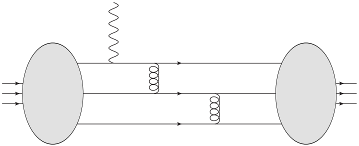
- Baryon DAs obey evolution equations (only LO case is known and is not diagonalised)

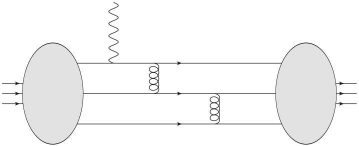
- Baryon DAs obey evolution equations (only LO case is known and is not diagonalised)
- At large scale, they both yield the so-called asymptotic DA $\varphi_{\mathrm{as}} = 120x_1x_2x_3$:

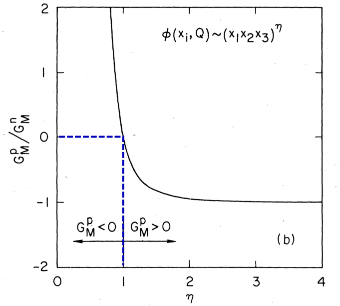
- Baryon DAs obey evolution equations (only LO case is known and is not diagonalised)
- At large scale, they both yield the so-called asymptotic DA $\varphi_{\rm as} = 120 x_1 x_2 x_3$:



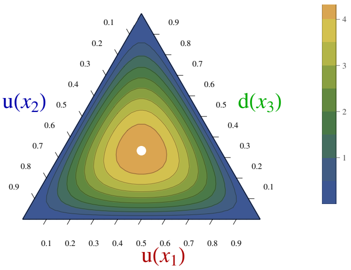
Form Factors: Nucleon case





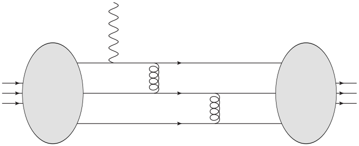


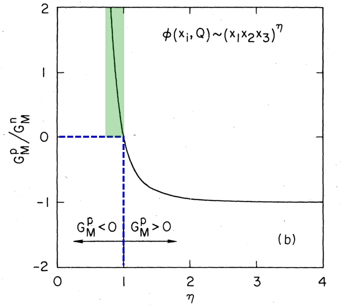
S. Brodsky and G. Lepage, PRD 22, (1980)

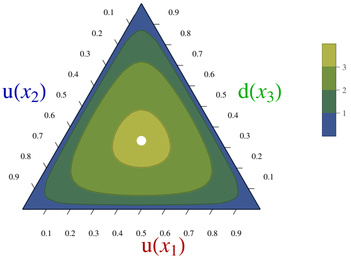


$\eta = 1$

Form Factors: Nucleon case

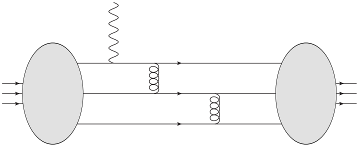


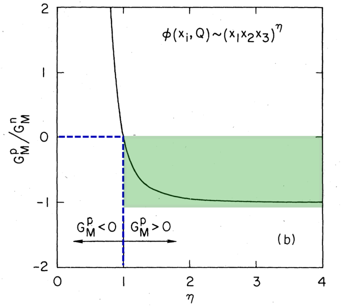


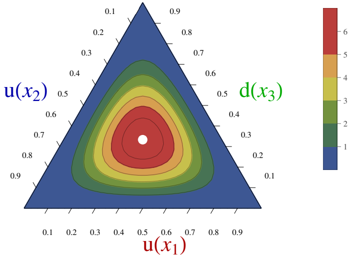


$\eta = 0.5$

S. Brodsky and G. Lepage, PRD 22, (1980)







$\eta = 2$

S. Brodsky and G. Lepage, PRD 22, (1980)

Asymptotic DA and vanishing FF

When $Q^2 \to \infty$, $\varphi \to \varphi_{\mathrm{as}}$ and become fully symmetric under permutations. One obtains:

$$F_p^1 \propto \int \frac{[\mathrm{d}x_i][\mathrm{d}y_i]}{Q^4} \varphi_{\mathrm{as}}(x_i)\varphi_{\mathrm{as}}(y_i) \left[(5e_u + e_d)H_1(x_i, y_i) + (e_u + 2e_d)H_2(x_i, y_i)\right]$$

Asymptotic DA and vanishing FF

When $Q^2 \to \infty$, $\varphi \to \varphi_{\rm as}$ and become fully symmetric under permutations. One obtains:

$$F_p^1 \propto \int \frac{[\mathrm{d}x_i][\mathrm{d}y_i]}{Q^4} \varphi_{\rm as}(x_i)\varphi_{\rm as}(y_i) \left[(5e_u + e_d)H_1(x_i, y_i) + (e_u + 2e_d)H_2(x_i, y_i)\right]$$

- H_1 contributions vanishes due to the integration procedure when $\varphi = \varphi_{\rm as}$. This is also the case for the neutron.

Asymptotic DA and vanishing FF

When $Q^2 \to \infty$, $\varphi \to \varphi_{\rm as}$ and become fully symmetric under permutations. One obtains:

$$F_p^1 \propto \int \frac{[\mathrm{d}x_i][\mathrm{d}y_i]}{Q^4} \varphi_{\rm as}(x_i)\varphi_{\rm as}(y_i) \left[(5e_u + e_d)H_1(x_i, y_i) + (e_u + 2e_d)H_2(x_i, y_i)\right]$$

- H_1 contributions vanishes due to the integration procedure when $\varphi = \varphi_{\rm as}$. This is also the case for the neutron.
- H_2 contribution vanishes in the proton case due to the specfic charge combination.

Asymptotic DA and vanishing FF

When $Q^2 \to \infty$, $\varphi \to \varphi_{\rm as}$ and become fully symmetric under permutations. One obtains:

$$F_p^1 \propto \int \frac{[\mathrm{d}x_i][\mathrm{d}y_i]}{Q^4} \varphi_{\rm as}(x_i)\varphi_{\rm as}(y_i)\left[(5e_u + e_d)H_1(x_i,y_i) + (e_u + 2e_d)H_2(x_i,y_i)\right]$$

- H_1 contributions vanishes due to the integration procedure when $\varphi = \varphi_{\rm as}$. This is also the case for the neutron.
- H_2 contribution vanishes in the proton case due to the specfic charge combination.

Data available at large Q^2 today are incompatible with φ_{as}

Asymptotic DA and vanishing FF

When $Q^2 \to \infty$, $\varphi \to \varphi_{\rm as}$ and become fully symmetric under permutations. One obtains:

$$F_p^1 \propto \int \frac{[\mathrm{d}x_i][\mathrm{d}y_i]}{Q^4} \varphi_{\rm as}(x_i)\varphi_{\rm as}(y_i) \left[(5e_u + e_d)H_1(x_i, y_i) + (e_u + 2e_d)H_2(x_i, y_i)\right]$$

- H_1 contributions vanishes due to the integration procedure when $\varphi = \varphi_{\rm as}$. This is also the case for the neutron.
- H_2 contribution vanishes in the proton case due to the specfic charge combination.

Data available at large Q^2 today are incompatible with φ_{as}

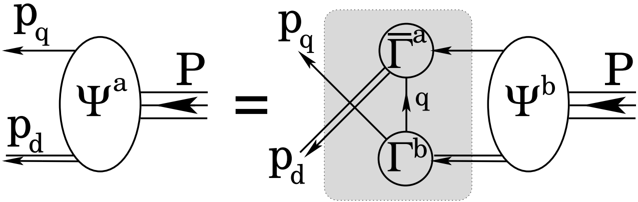
Caveat: Leading Order analysis only

- QCD Sum Rules
  - V. Chernyak and I. Zhitnitsky, Nucl. Phys. B 246 (1984)
- Relativistic quark model
  - Z. Dziembowski, PRD 37 (1988)
- Scalar diquark clustering
  - Z. Dziembowski and J. Franklin, PRD 42 (1990)
- Phenomenological fit
  - J. Bolz and P. Kroll, Z. Phys. A 356 (1996)
- Lightcone quark model
  - B. Pasquini *et al.*, PRD 80 (2009)
- Lightcone sum rules
  - I. Anikin *et al.*, PRD 88 (2013)
- Lattice Mellin moment computation
  - G. Bali *et al.*, EPJ. A55 (2019)

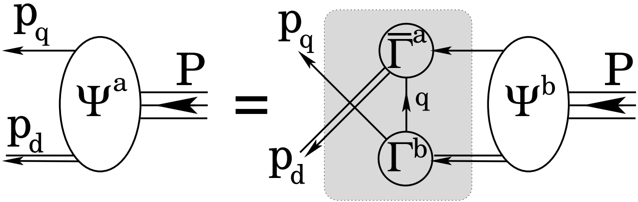
Baryon and Diquarks

- The Faddeev equation provides a covariant framework to describe the nucleon as a bound state of three dressed quarks.

- The Faddeev equation provides a covariant framework to describe the nucleon as a bound state of three dressed quarks.
- It predicts the existence of strong diquarks correlations inside the nucleon.

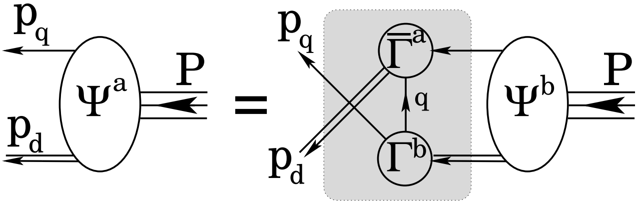


- The Faddeev equation provides a covariant framework to describe the nucleon as a bound state of three dressed quarks.
- It predicts the existence of strong diquarks correlations inside the nucleon.



- Mostly two types of diquark are dynamically generated by the Faddeev equation:
  - Scalar diquarks,
  - Axial-Vector (AV) diquarks.

- The Faddeev equation provides a covariant framework to describe the nucleon as a bound state of three dressed quarks.
- It predicts the existence of strong diquarks correlations inside the nucleon.



- Mostly two types of diquark are dynamically generated by the Faddeev equation:
  - Scalar diquarks,
  - Axial-Vector (AV) diquarks.
- Can we understand the nucleon structure in terms of quark-diquarks correlations?

Faddeev WF Model

- Algebraic parametrisation inspired by the results obtained from DSEs and Faddeev equations.
- It is based on Nakanishi representation, which is proved to be a good parametrisation of Green functions at all order of perturbation theory.
- We also assume the dynamical diquark correlations, both scalar and AV, and compare in the end with Lattice QCD results.
- This is an exploratory work.

- Operator point of view for every DA (and at every twist):

$$\langle 0|\epsilon^{ijk}\left(u_{\uparrow}^{i}(z_1)C\not{n}u_{\downarrow}^{j}(z_2)\right)\not{n}d_{\uparrow}^{k}(z_3)|P,\lambda\rangle \to \varphi(x_1,x_2,x_3),$$

Braun *et al.*, Nucl.Phys. B589 (2000)

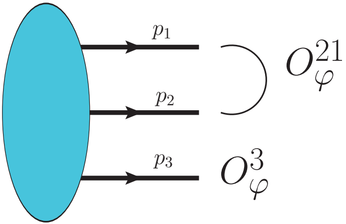
Nucleon Distribution Amplitude

- Operator point of view for every DA (and at every twist):

$$\langle 0|\epsilon^{ijk}\left(u_\uparrow^i(z_1)C\not{n}u_\downarrow^j(z_2)\right)\not{n}d_\uparrow^k(z_3)|P,\lambda\rangle \to \varphi(x_1,x_2,x_3),$$

Braun *et al.*, Nucl.Phys. B589 (2000)

- We can apply it on the wave function:

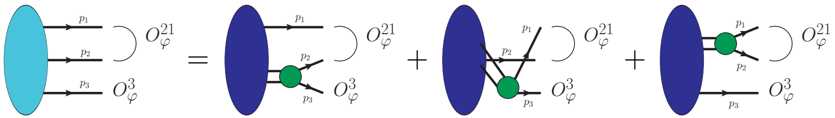


- Operator point of view for every DA (and at every twist):

$$\langle 0|\epsilon^{ijk}\left(u^i_\uparrow(z_1)C\not{n}u^j_\downarrow(z_2)\right)\not{n}d^k_\uparrow(z_3)|P,\lambda\rangle \to \varphi(x_1,x_2,x_3),$$

Braun *et al.*, Nucl.Phys. B589 (2000)

- We can apply it on the wave function:



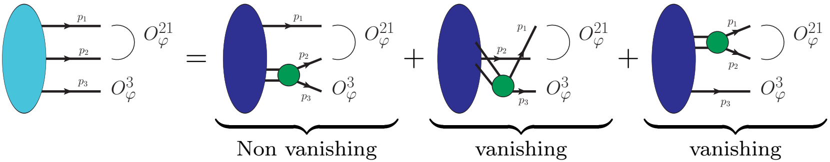
Nucleon Distribution Amplitude

- Operator point of view for every DA (and at every twist):

$$\langle 0|\epsilon^{ijk}\left(u_\uparrow^i(z_1)C\not{n}u_\downarrow^j(z_2)\right)\not{n}d_\uparrow^k(z_3)|P,\lambda\rangle \to \varphi(x_1,x_2,x_3),$$

Braun *et al.*, Nucl.Phys. B589 (2000)

- We can apply it on the wave function:



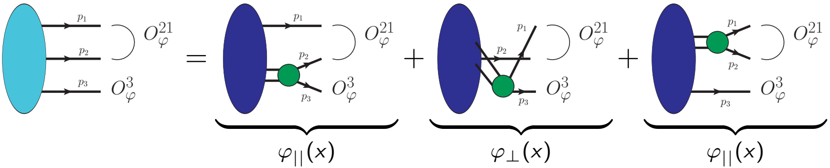
- The operator then selects the relevant component of the wave function.

- Operator point of view for every DA (and at every twist):

$$\langle 0|\epsilon^{ijk}\left(u^i_\uparrow(z_1)C\not{n}u^j_\downarrow(z_2)\right)\not{n}d^k_\uparrow(z_3)|P,\lambda\rangle \rightarrow \varphi(x_1,x_2,x_3),$$

Braun *et al.*, Nucl.Phys. B589 (2000)

- We can apply it on the wave function:



- The operator then selects the relevant component of the wave function.

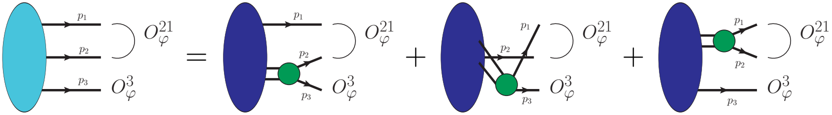
Nucleon Distribution Amplitude

- Operator point of view for every DA (and at every twist):

$$\langle 0|\epsilon^{ijk}\left(u^i_\uparrow(z_1)C\not{n}u^j_\downarrow(z_2)\right)\not{n}d^k_\uparrow(z_3)|P,\lambda\rangle \rightarrow \varphi(x_1,x_2,x_3),$$

Braun *et al.*, Nucl.Phys. B589 (2000)

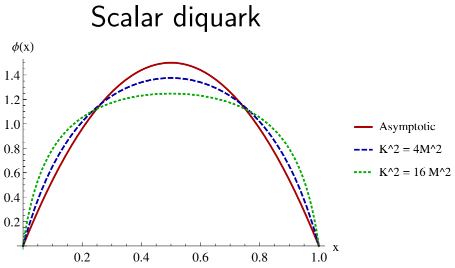
- We can apply it on the wave function:



- The operator then selects the relevant component of the wave function.
- Our ingredients are:
  - Perturbative-like quark and diquark propagator
  - Nakanishi based diquark Bethe-Salpeter-like amplitude (green disks)
  - Nakanishi based quark-diquark amplitude (dark blue ellipses)

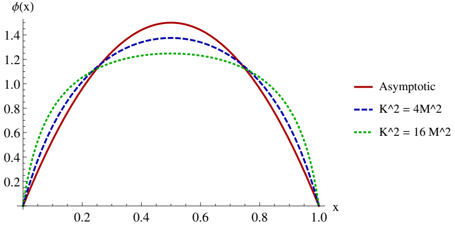
Scalar Diquark DA

$$\phi(x) \propto 1 - \frac{M^2}{K^2}\frac{\ln\left[1+\frac{K^2}{M^2}x(1-x)\right]}{x(1-x)}$$

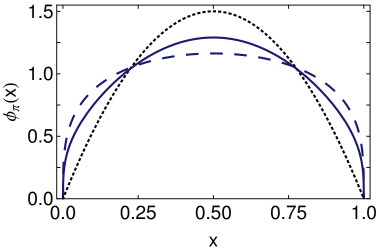


$$\phi(x) \propto 1 - \frac{M^2}{K^2} \frac{\ln\left[1 + \frac{K^2}{M^2} x(1-x)\right]}{x(1-x)}$$

Scalar diquark



Pion

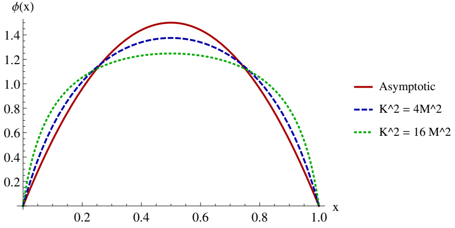


Pion figure from L. Chang *et al.*, PRL 110 (2013)

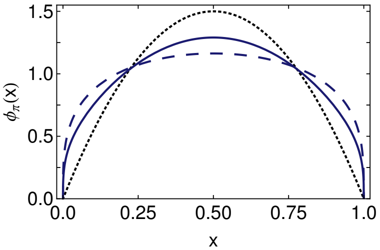
Scalar Diquark DA

$$\phi(x) \propto 1 - \frac{M^2}{K^2}\frac{\ln\left[1+\frac{K^2}{M^2}x(1-x)\right]}{x(1-x)}$$

Scalar diquark



Pion

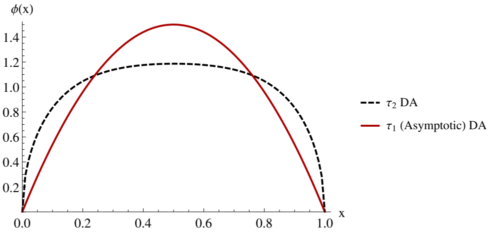


Pion figure from L. Chang *et al.*, PRL 110 (2013)

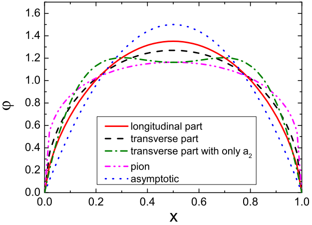
- This results provide a broad and concave meson DA parametrisation
- The endpoint behaviour remains linear

Comparison with the ρ meson

AV diquark

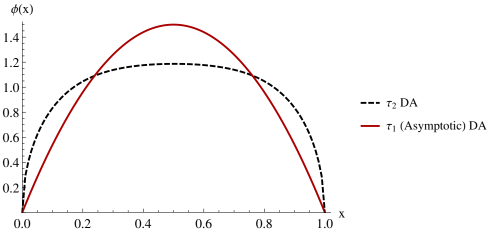


ρ meson

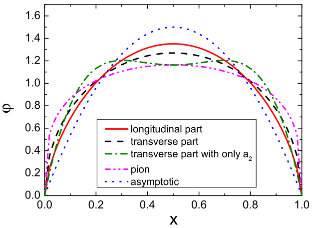


ρ figure from F. Gao *et al.*, PRD 90 (2014)

AV diquark



ρ meson



ρ figure from F. Gao *et al.*, PRD 90 (2014)

- Same "shape ordering" $\rightarrow \phi_\perp$ is flatter in both cases.
- Farther apart compared to the ρ meson case.

Mellin Moments

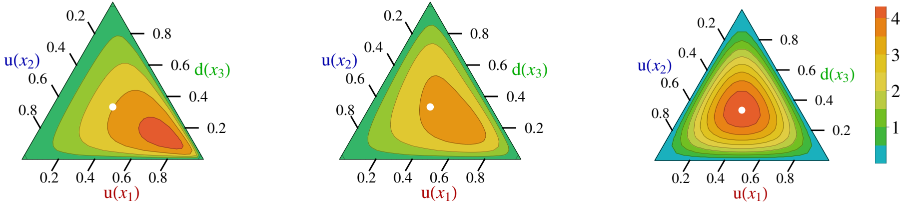
- We do not compute the PDA directly but Mellin moments of it:

$$\langle x_1^m x_2^n \rangle = \int_0^1 \mathrm{d}x_1 \int_0^{1-x_1} \mathrm{d}x_2 \; x_1^m x_2^n \varphi(x_1, x_2, 1 - x_1 - x_2)$$

- For a general moment $\langle x_1^m x_2^n \rangle$, we change the variable in such a way to write down our moments as:

$$\langle x_1^m x_2^n \rangle = \int_0^1 \mathrm{d}\alpha \int_0^{1-\alpha} \mathrm{d}\beta \; \alpha^m \beta^n f(\alpha, \beta)$$

- f is a complicated function involving the integration on 6 parameters
- Uniqueness of the Mellin moments of continuous functions allows us to identify f and φ

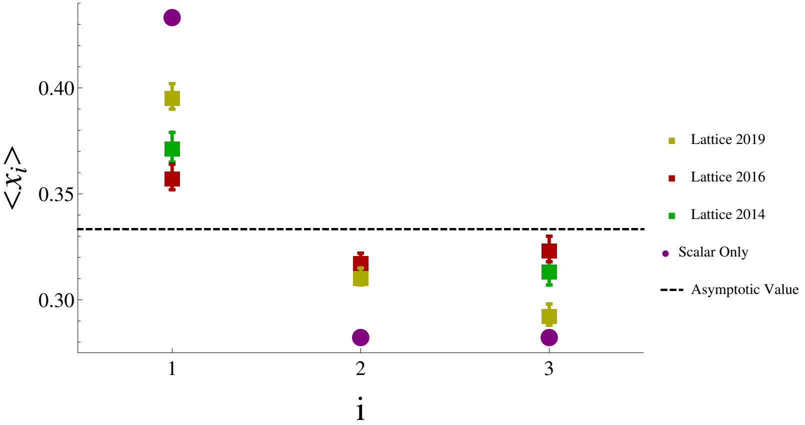


Scalar diquark Only | Nucleon DA | Asymptotic DA

- Typical symmetry in the pure scalar case
- Results evolved from 0.51 to 2 GeV with both scalar and AV diquark
- Nucleon DA is skewed compared to the asymptotic one
- It is also broader than the asymptotic results
- These properties are consequences of our quark-diquark picture

C.Mezrag *et al.*, Phys.Lett. B783 (2018) 263-267

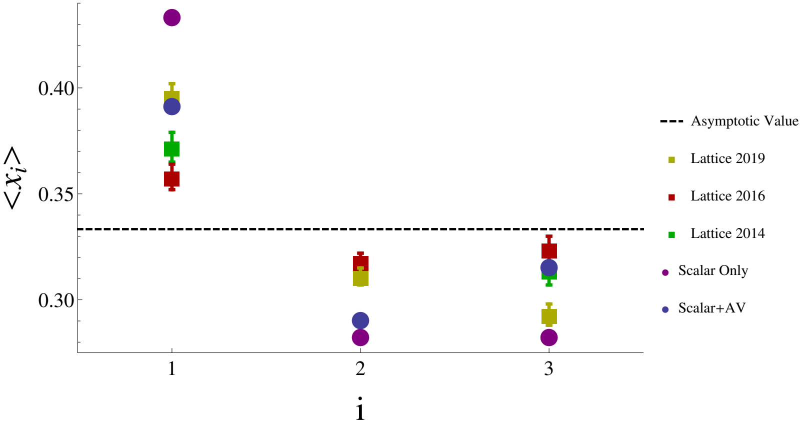
$$< x_i >_\varphi = \int \mathcal{D}x \; x_i \varphi(x_1, x_2, x_3)$$



Lattice data from V.Braun *et al*, PRD 89 (2014)
G. Bali *et al.*, JHEP 2016 02
G. Bali *et al.*, EPJ. A55 (2019)

Comparison with lattice

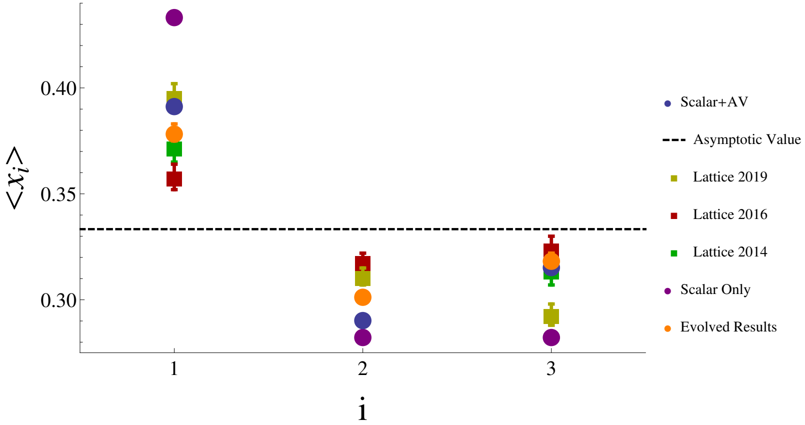
$$< x_i >_\varphi = \int \mathcal{D}x \; x_i \varphi(x_1, x_2, x_3)$$



Lattice data from V.Braun *et al*, PRD 89 (2014)
G. Bali *et al.*, JHEP 2016 02
G. Bali *et al.*, EPJ. A55 (2019)

Comparison with lattice

$$< x_i >_\varphi = \int \mathcal{D}x \; x_i \varphi(x_1, x_2, x_3)$$



Lattice data from V.Braun *et al*, PRD 89 (2014)
G. Bali *et al.*, JHEP 2016 02
G. Bali *et al.*, EPJ. A55 (2019)

Summary so far

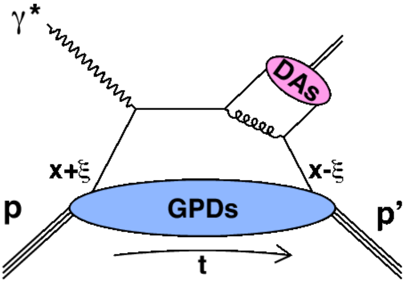
Achievements

- **DSE compatible** framework for Baryon PDAs.
- Based on the Nakanishi representation.
- First results from exploratory work (2017).

Work in progress/future work

- Improvement of the Nakanishi Ansätze.
- Calculation of the Dirac form factor
- Higher-twist PDA (completely unknown)

DVMP and Distribution Amplitudes



DVMP factorisation at LO

$$\mathcal{F}^q(\xi, t, Q^2) \propto \frac{\alpha_s(\mu_R)}{Q} \int_{-1}^{1} \mathrm{d}x \frac{F^q(x, \xi, t, \mu_F^2)}{\xi - x - i\epsilon} \int_0^1 \mathrm{d}z \frac{\varphi(z, \mu_\varphi)}{(1 - z)}$$

see e.g. D. Mueller *et al.* Nucl.Phys. B884 (2014) 438-546

- DVMP amplitude depends on the meson DA

DVMP factorisation at LO

$$\mathcal{F}^q(\xi, t, Q^2) \propto \frac{\alpha_s(\mu_R)}{Q} \int_{-1}^{1} \mathrm{d}x \frac{F^q(x,\xi,t,\mu_F^2)}{\xi - x - i\epsilon} \int_0^1 \mathrm{d}z \frac{\varphi(z,\mu_\varphi)}{(1-z)}$$

see e.g. D. Mueller *et al.* Nucl.Phys. B884 (2014) 438-546

- DVMP amplitude depends on the meson DA
- At LO, the x and z convolutions are fully factorised
- The DA contributes to the absolute normalisation

DVMP factorisation at LO

$$\mathcal{F}^q(\xi, t, Q^2) \propto \frac{\alpha_s(\mu_R)}{Q} \int_{-1}^{1} \mathrm{d}x \frac{F^q(x, \xi, t, \mu_F^2)}{\xi - x - i\epsilon} \int_0^1 \mathrm{d}z \frac{\varphi(z, \mu_\varphi)}{(1-z)}$$

see e.g. D. Mueller *et al.* Nucl.Phys. B884 (2014) 438-546

- DVMP amplitude depends on the meson DA
- At LO, the x and z convolutions are fully factorised
- The DA contributes to the absolute normalisation
- At NLO the situation is more complex, contributions from DA and GPDs are not fully separated anymore.

DVMP factorisation at LO

$$\mathcal{F}^q(\xi, t, Q^2) \propto \frac{\alpha_s(\mu_R)}{Q} \int_{-1}^{1} \mathrm{d}x \frac{F^q(x, \xi, t, \mu_F^2)}{\xi - x - i\epsilon} \int_0^1 \mathrm{d}z \frac{\varphi(z, \mu_\varphi)}{(1-z)}$$

see e.g. D. Mueller *et al.* Nucl.Phys. B884 (2014) 438-546

- DVMP amplitude depends on the meson DA
- At LO, the x and z convolutions are fully factorised
- The DA contributes to the absolute normalisation
- At NLO the situation is more complex, contributions from DA and GPDs are not fully separated anymore.

What is the impact of various models on DVMP?

- Asymptotic DA : $\varphi_{AS} = 6x(1-x)$
- Square-root DA : $\varphi_{SR} = \frac{8}{\pi}\sqrt{x(1-x)}$

A. Radyushkin, Nucl.Phys. A532 (1991) 141-154
S. Brodsky *et al.* Int.J.Mod.Phys.Conf.Ser. 39 (2015) 1560081

- Fits on Lattice second moment of DA

V. Braun *et al.* Phys.Rev. D92 (2015) no.1, 014504

  - Power model : $\varphi_p(x) \propto (x(1-x))^\nu$

J. Segovia *et al.*, Phys.Lett. B731 (2014) 13-18

  - Log model : $\varphi_{\ln}(x) \propto 1 - \frac{\ln[1+\kappa x(1-x)]}{\kappa x(1-x)}$

C. Mezrag *et al.*, Phys.Lett. B783 (2018) 263-267

Models of Pion DA

- Asymptotic DA : $\varphi_{AS} = 6x(1-x)$
- Square-root DA : $\varphi_{SR} = \frac{8}{\pi}\sqrt{x(1-x)}$

A. Radyushkin, Nucl.Phys. A532 (1991) 141-154
S. Brodsky *et al.* Int.J.Mod.Phys.Conf.Ser. 39 (2015) 1560081

- Fits on Lattice second moment of DA

V. Braun *et al.* Phys.Rev. D92 (2015) no.1, 014504

  - Power model : $\varphi_p(x) \propto (x(1-x))^\nu$

J. Segovia *et al.*, Phys.Lett. B731 (2014) 13-18

  - Log model : $\varphi_{\ln}(x) \propto 1 - \frac{\ln[1+\kappa x(1-x)]}{\kappa x(1-x)}$

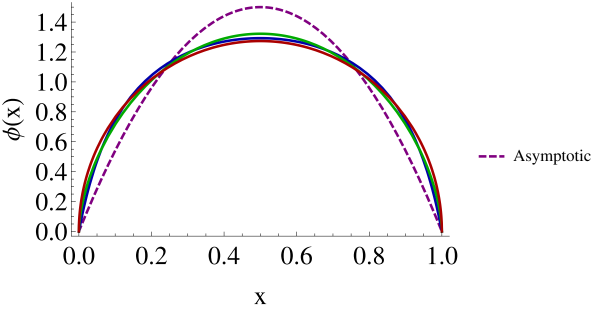
C. Mezrag *et al.*, Phys.Lett. B783 (2018) 263-267

Bottom line

- 4 different *concave* pion DA models
- 2 tuned to Lattice QCD results of the second moment

	$x(1-x)$	$\varphi_{\ln}(x)$	$(x(1-x))^{\nu}$	$\sqrt{x(1-x)}$
$\langle x^{-1}\rangle$	3	3.38	3.61	4
$\frac{\langle x^{-1}\rangle}{\langle x^{-1}\rangle_{As}}$	1	1.13	1.20	1.33

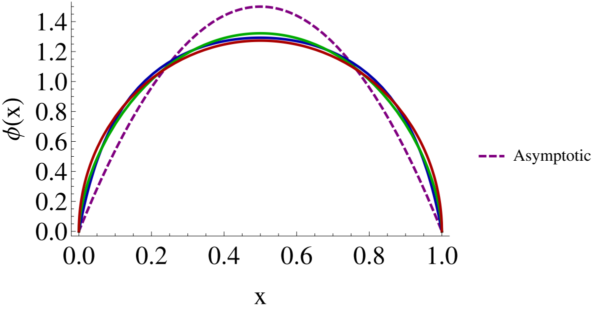
$$\langle x^{-1}\rangle = \int_0^1 \mathrm{d}x \frac{\varphi(x)}{1-x}$$



$n = -1$ Mellin Moment

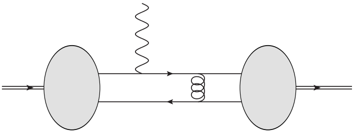
	$x(1-x)$	$\varphi_{\ln}(x)$	$(x(1-x))^\nu$	$\sqrt{x(1-x)}$
$\langle x^{-1} \rangle$	3	3.38	3.61	4
$\frac{\langle x^{-1} \rangle}{\langle x^{-1} \rangle_{As}}$	1	1.13	1.20	1.33

$$\langle x^{-1} \rangle = \int_0^1 \mathrm{d}x \frac{\varphi(x)}{1-x}$$

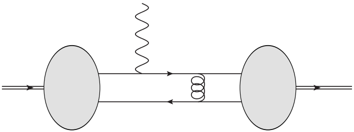


Additionnal complication : evolution and scale setting

Form Factors



$$Q^2F(Q^2) = \mathcal{N}\int[\mathrm{d}x_i][\mathrm{d}y_i]\varphi(x,\zeta_x^2)T(x,y,Q^2,\zeta_x^2,\zeta_y^2)\varphi(y,\zeta_y^2)$$



$$Q^2F(Q^2) = \mathcal{N}\int [\mathrm{d}x_i][\mathrm{d}y_i]\varphi(x,\zeta_x^2)T(x,y,Q^2,\zeta_x^2,\zeta_y^2)\varphi(y,\zeta_y^2)$$

- LO Kernel and NLO kernels are known
- $T_0 \propto \frac{\alpha_S(\mu_R^2)}{(1-x)(1-y)}$
- $T_1 \propto \frac{\alpha_s^2(\mu_R^2)}{(1-x)(1-y)}(f_{UV}(\mu_R^2) + f_{IR}(\zeta^2) + f_{finite})$

R Field *et al.*, NPB 186 429 (1981)
F. Dittes and A. Radyushkin, YF 34 529 (1981)
B. Melic *et al.*, PRD 60 074004 (1999)

Pion FF

- The UV scale dependent term behaves like:

$$f_{UV}(\mu_R^2) \propto \beta_0 \left(5/3 - \ln((1-x)(1-y)) + \ln\left(\frac{\mu_R^2}{Q^2}\right)\right)$$

- Here I take two examples:
    - the standard choice of $\zeta_x^2 = \zeta_y^2 = \mu^2 = Q^2/4$
    - the regularised BLM-PMC scale $\zeta_x^2 = \zeta_y^2 = \mu^2 = e^{-5/3}Q^2/4$

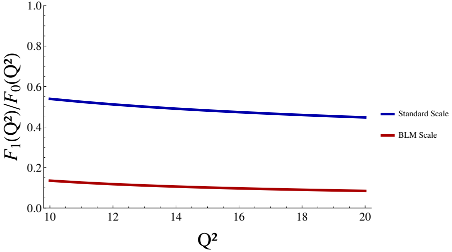
S. Brodsky *et al.*, PRD 28 228 (1983)
S. Brodsky and L. Di Giustino, PRD 86 085026 (2011)

- Take the PDA model coming from the scalar diquark:

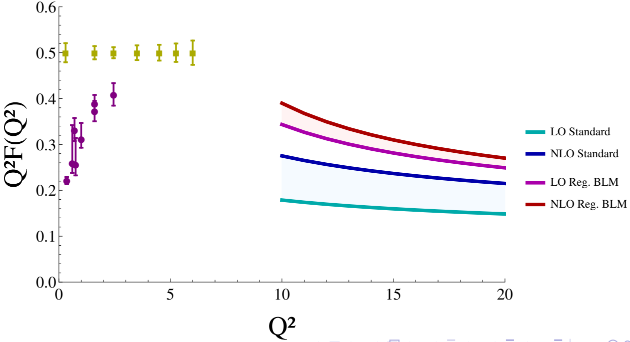
$$\varphi_{\ln}(x) \propto 1 - \frac{\ln\left[1 + \kappa x(1-x)\right]}{\kappa x(1-x)}$$

κ is fitted to the lattice Mellin Moment

Pion FF



1.0
0.8
0.6
0.4
0.2
0.0
$F_1(Q^2)/F_0(Q^2)$
10
12
14
16
18
20
Q^2
Standard Scale
BLM Scale



0.6
0.5
0.4
0.3
0.2
0.1
0.0
$Q^2F(Q^2)$
0
5
10
15
20
Q^2
LO Standard
NLO Standard
LO Reg. BLM
NLO Reg. BLM

Pion FF

- The UV scale dependent term behaves like:

$$f_{UV}(\mu_R^2) \propto \beta_0 \left(5/3 - \ln((1-x)(1-y)) + \ln\left(\frac{\mu_R^2}{Q^2}\right)\right)$$

- Here I take two examples:
  - the standard choice of $\zeta_x^2 = \zeta_y^2 = \mu^2 = Q^2/4$
  - the regularised BLM-PMC scale $\zeta_x^2 = \zeta_y^2 = \mu^2 = e^{-5/3}Q^2/4$

S. Brodsky *et al.*, PRD 28 228 (1983)
S. Brodsky and L. Di Giustino, PRD 86 085026 (2011)

- BLM scale reduces significantly the impact of the NLO corrections and increase dramatically the LO one.
- Future large Q^2 data coming from JLab 12 and the EIC might shed light on the pion DA.

DVMP and PARTONS

http://partons.cea.fr

- PARTONS $\rightarrow$ open-source software for GPDs phenomenology
- Flexible code architecture allowing GPDs studies in a broad range of assumptions.
- Discussions for the development on the DVMP branch have started (Kemal Tezgin and Pawel Sznajder).
  We would like :
  - LO and NLO perturbative kernel
  - Various models of DA
  - Evolution code for the leading twist DA

DVMP and PARTONS

http://partons.cea.fr

cea

- PARTONS → open-source software for GPDs phenomenology
- Flexible code architecture allowing GPDs studies in a broad range of assumptions.
- Discussions for the development on the DVMP branch have started (Kemal Tezgin and Pawel Sznajder).
  We would like :
  - LO and NLO perturbative kernel
  - Various models of DA
  - Evolution code for the leading twist DA

- PARTONS → first quantitative studies of the impact of the meson DA at LO and NLO on GPD extraction
- PARTONS → comparison with different non-perturbative predictions of the meson DA and the GPDs

Baryon DA and DVMP

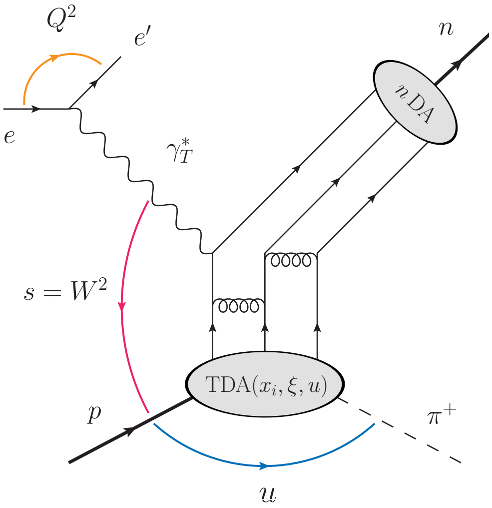


figure from K. Park *et al.*, Phys. Lett. B 780 340-345 (2018)

Summary

Summary and Conclusion

Modelling of Distribution Amplitudes

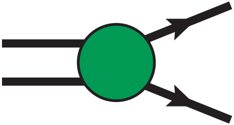
- A formalism able to handle the computation of Baryon DA
- Rely on diquark correlation with a spatial extension
- Impact of the nature and structure of the diquarks on the nucleon DA
- Good comparison with lattice-QCD results
- Improvements are in progress

DVMP and DA

- DVMP is very sensitive to the shape of DA
- Non-perturbative approaches help but still no definitive solution
- DVMP studies may need to be coupled to other processes sensitive to GPDs (DVCS) and DA (Form Factors?)
- PARTONS will be the good tool to exploit DVMP data

Thank you for your attention

Back up slides



At all order of perturbation theory, one can write (Euclidean space):

$$\Gamma(k,P) = \mathcal{N}\int_0^\infty \mathrm{d}\gamma \int_{-1}^{1} \mathrm{d}z \frac{\rho_n(\gamma,z)}{(\gamma + (k+\frac{z}{2}P)^2)^n}$$

We use a "simpler" version of the latter as follow:

$$\tilde{\Gamma}(q,P) = \mathcal{N}\int_{-1}^{1} \mathrm{d}z \frac{\rho_n(z)}{(\Lambda^2 + (q+\frac{z}{2}P)^2)^n}$$