



JOHANNES GUTENBERG  
UNIVERSITÄT MAINZ



# Strong QCD from Hadron Structure Experiments

Nov. 6 - 9, 2019  
Jefferson Lab  
Newport News, VA USA

**Topics:**

- 1-D and 3-D structure of ground/excited hadrons and atomic nuclei;
- Mass, momentum, and pressure distributions in hadrons;
- Hadron spectroscopy and new hadron states;
- QCD-based frameworks for the description of hadron spectroscopy and structure;
- Science opportunities at an Electron-Ion Collider

This workshop will focus on the properties of hadrons and nuclei, and their emergence from Strong QCD. The goal is to explore new horizons in the structure of ground and excited hadrons, 3-D femto-imaging, and spectroscopy.

**Local Organizing Committee:**  
 V.I. Mokeev (Chair), Jefferson Lab  
 D.S. Carman, Jefferson Lab  
 J.R. Chen, Jefferson Lab  
 L. Elouadihi, Jefferson Lab  
 K. Joo, University of Connecticut  
 D.O. Richards, Jefferson Lab  
 C.D. Roberts, Argonne National Lab

<https://www.jlab.org/conference/QCD2019>

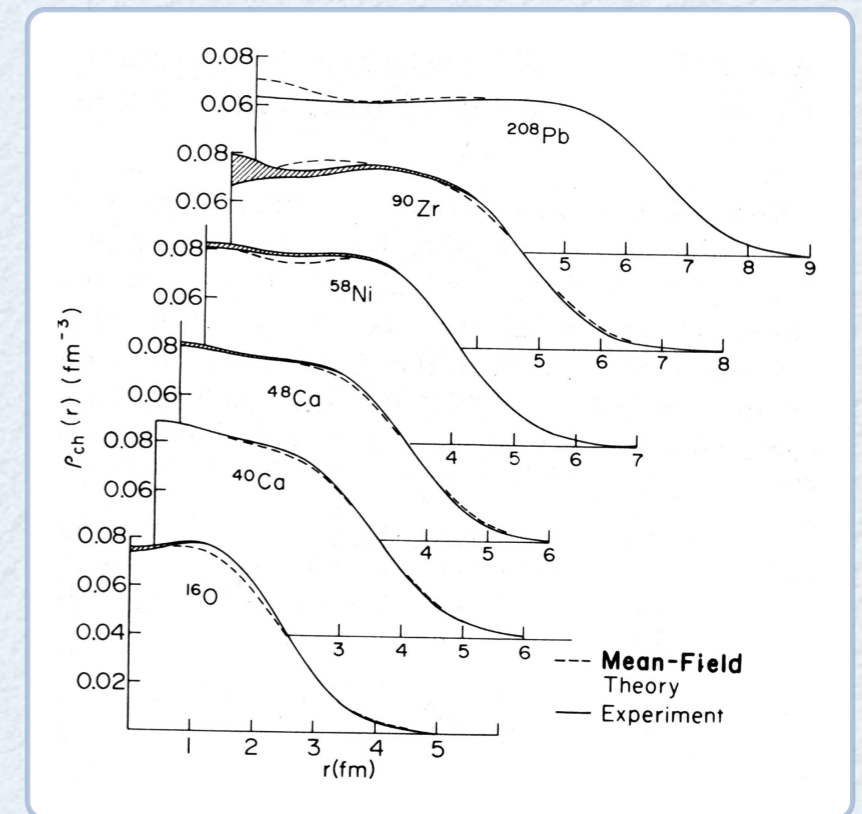
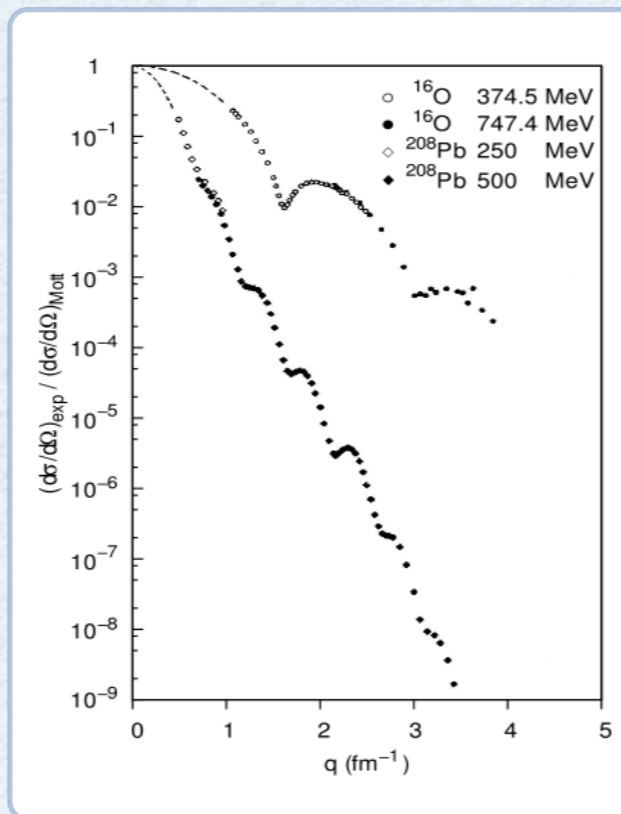
# Ground and Excited Nucleon Structure in 3D

Marc Vanderhaeghen

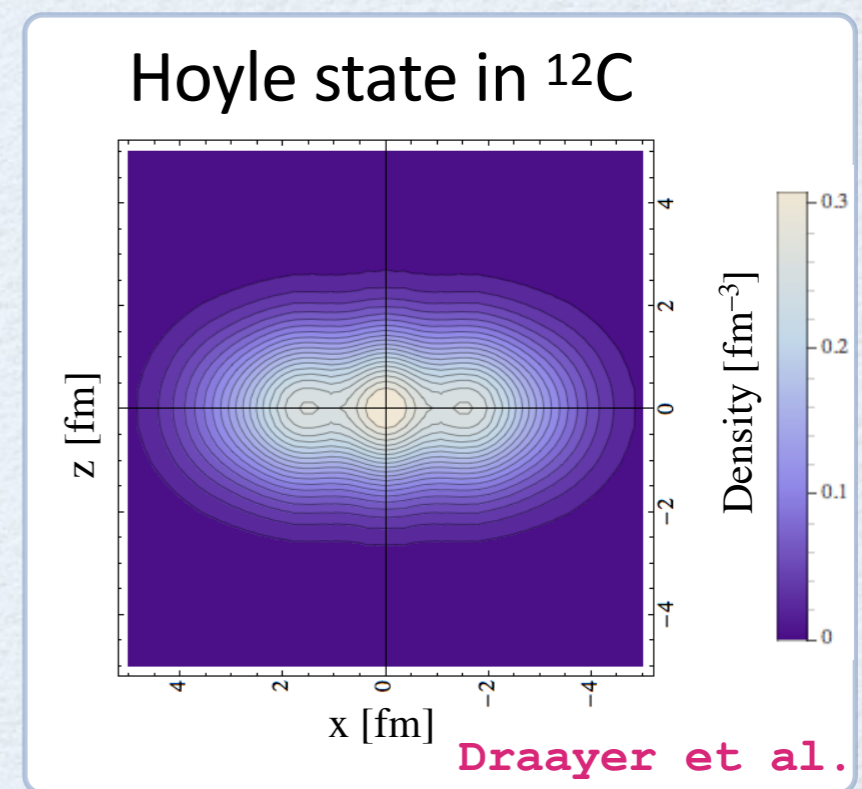
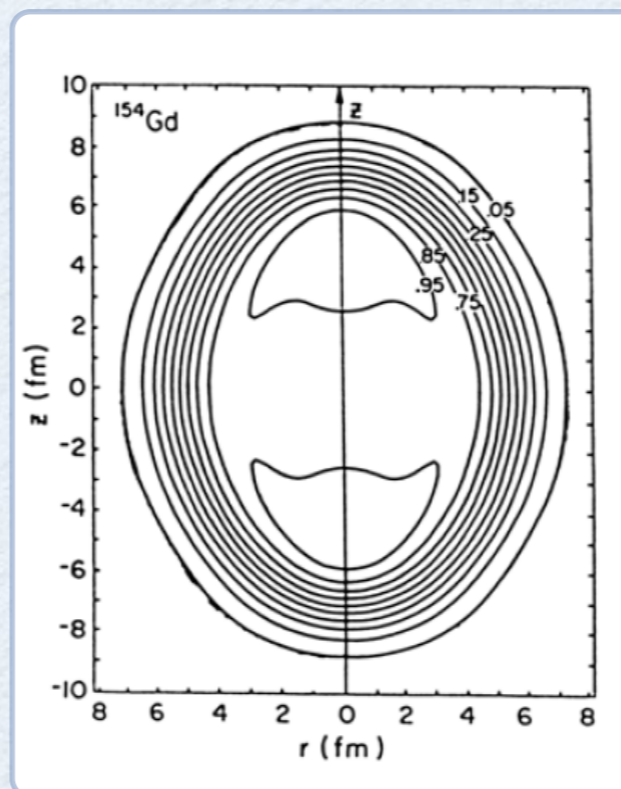
Johannes Gutenberg University Mainz

# Imaging of atomic nuclei

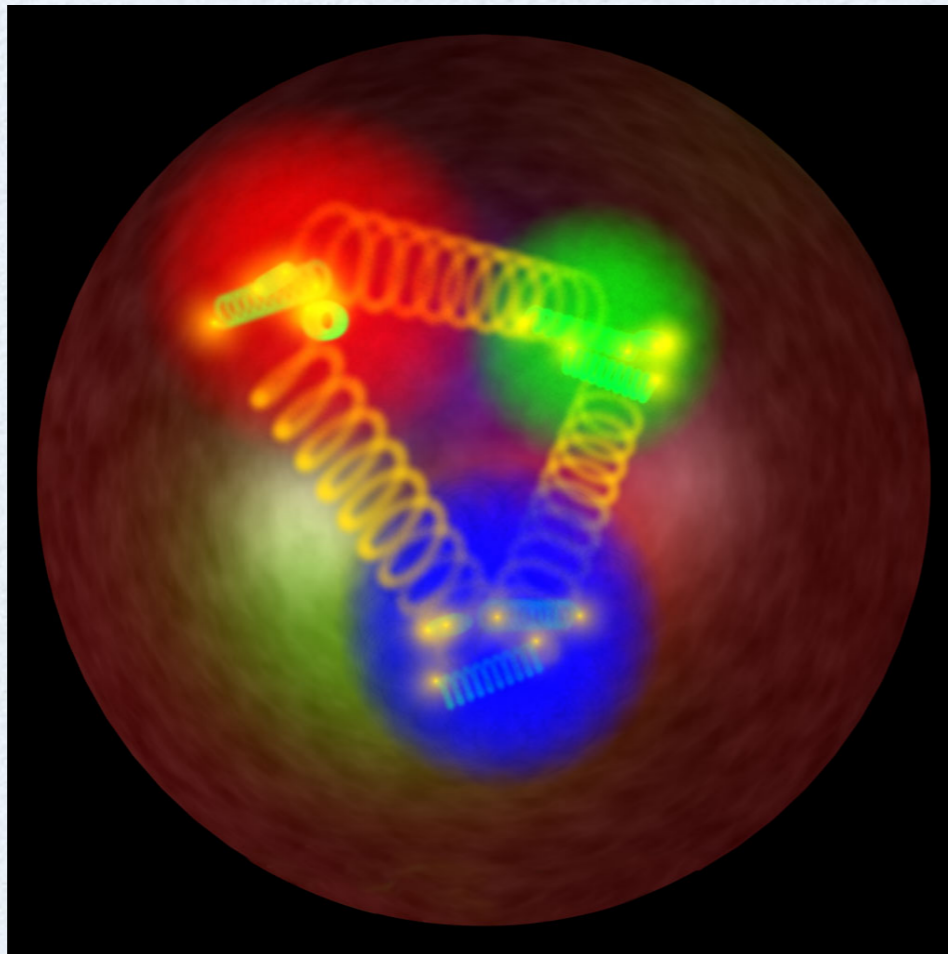
**sizes** of nuclei:  
as revealed through  
**elastic** electron scattering



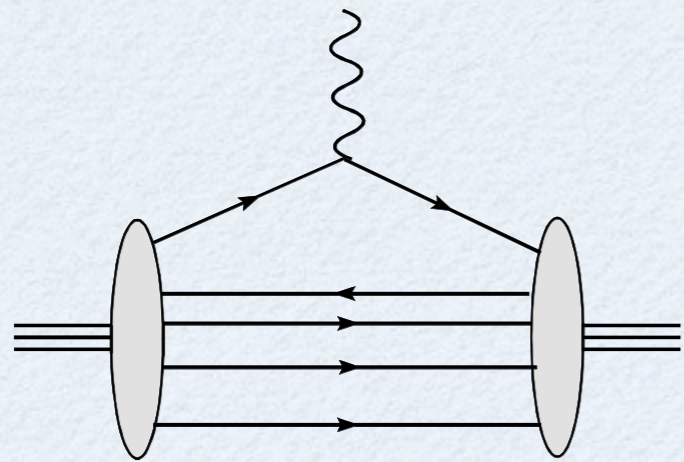
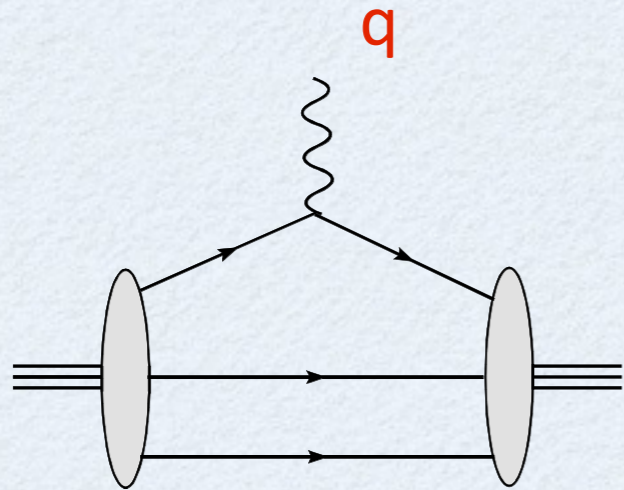
**shapes** of nuclei:  
as revealed through  
**inelastic** electron scattering  
deformations, coherent states



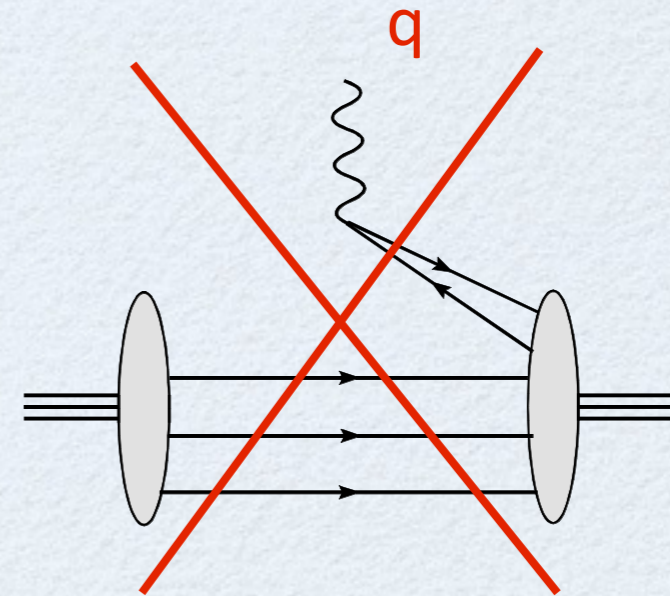
# Imaging of protons, neutrons, and nucleon resonances



# Interpretation of form factor as quark density



overlap of wave function  
Fock components  
with **same** number of quarks



overlap of wave function  
Fock components  
with **different** number of quarks  
**NO probability / charge density**  
interpretation

absent in a light-front frame!

$$q^+ = q^0 + q^3 = 0$$

# quark transverse charge densities in nucleon

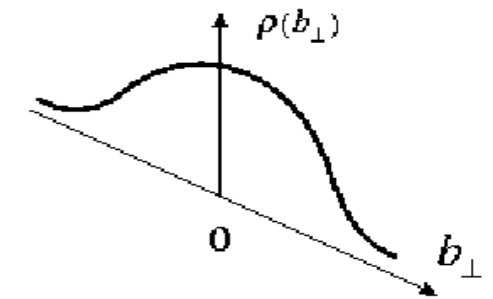
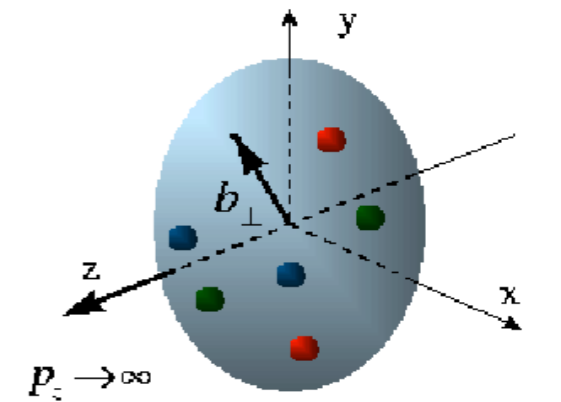
→ longitudinally polarized nucleon

$$\begin{aligned} \rho_0^N(\vec{b}) &\equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp \cdot \vec{b}} \frac{1}{2P^+} \langle P^+, \frac{\vec{q}_\perp}{2}, \lambda | J^+(0) | P^+, -\frac{\vec{q}_\perp}{2}, \lambda \rangle \\ &= \int_0^\infty \frac{dQ}{2\pi} Q J_0(bQ) F_1(Q^2) \end{aligned}$$

Soper (1997)

Burkardt (2000)

Miller (2007)

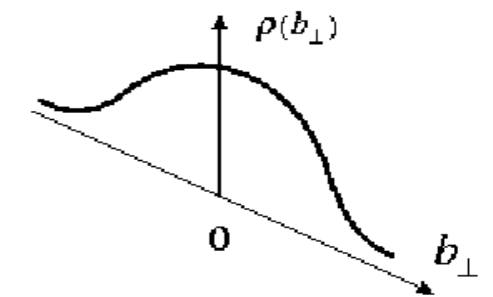
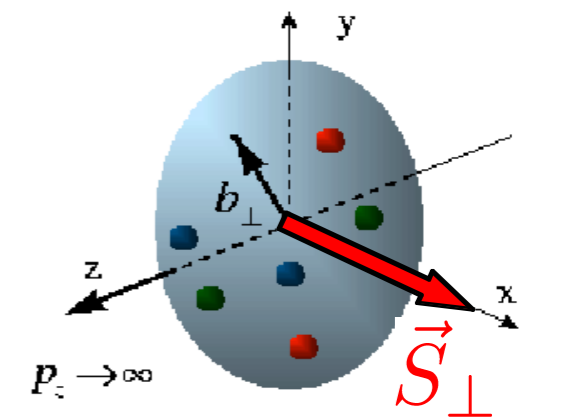


→ transversely polarized nucleon

$$\begin{aligned} \rho_T^N(\vec{b}) &\equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp \cdot \vec{b}} \frac{1}{2P^+} \langle P^+, \frac{\vec{q}_\perp}{2}, s_\perp = +\frac{1}{2} | J^+(0) | P^+, -\frac{\vec{q}_\perp}{2}, s_\perp = +\frac{1}{2} \rangle \\ &= \rho_0^N(b) + \sin(\phi_b - \phi_S) \int_0^\infty \frac{dQ}{2\pi} \frac{Q^2}{2M} J_1(bQ) F_2(Q^2) \end{aligned}$$

dipole field pattern

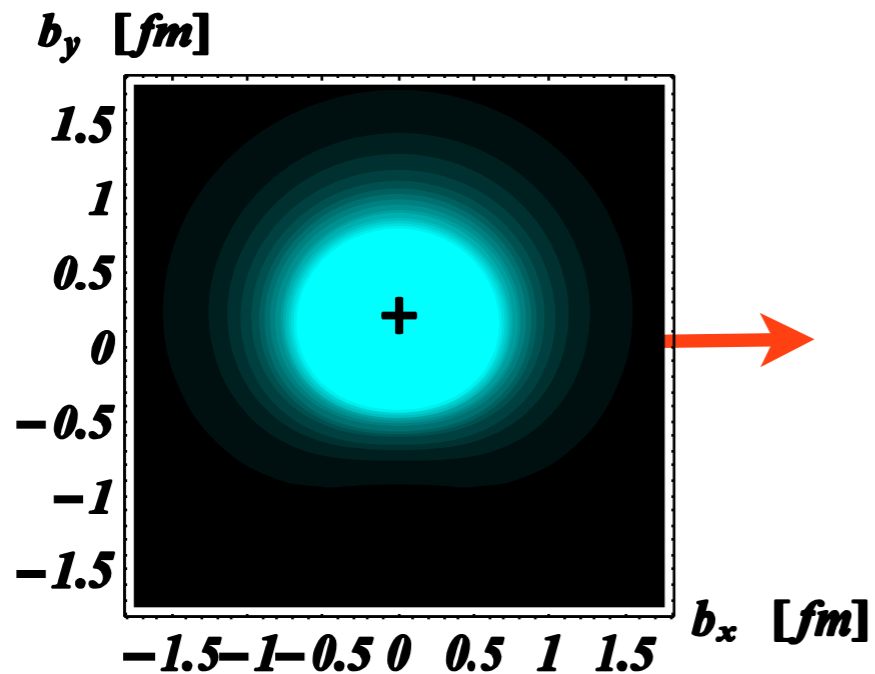
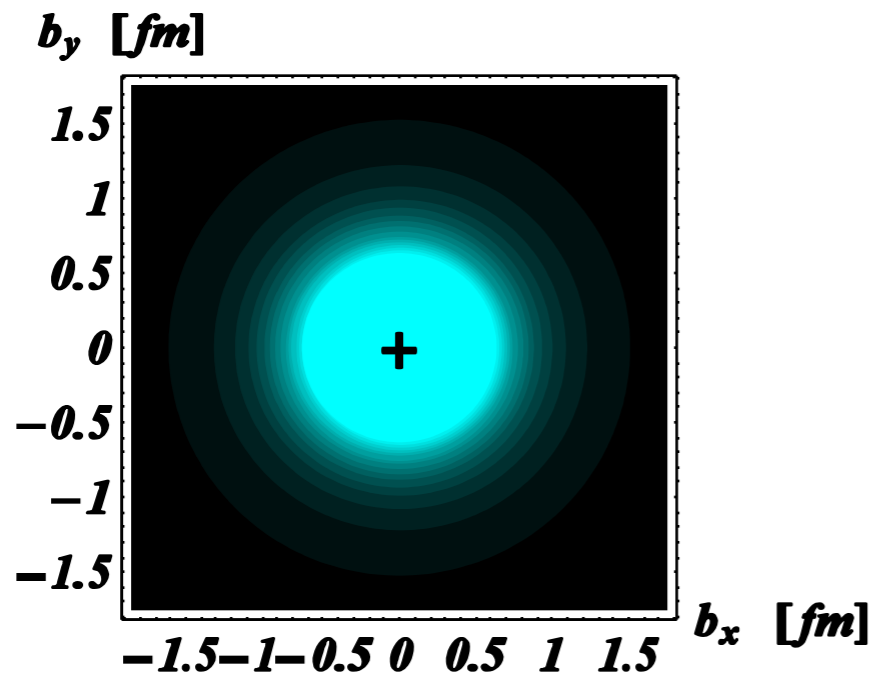
Carlson, Vdh (2007)



# spatial imaging of nucleons

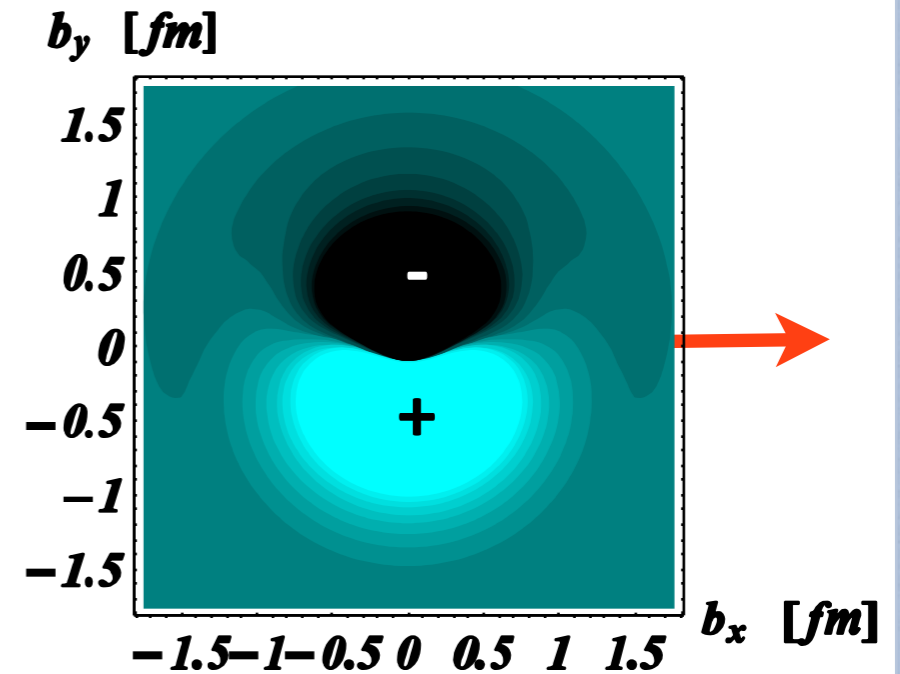
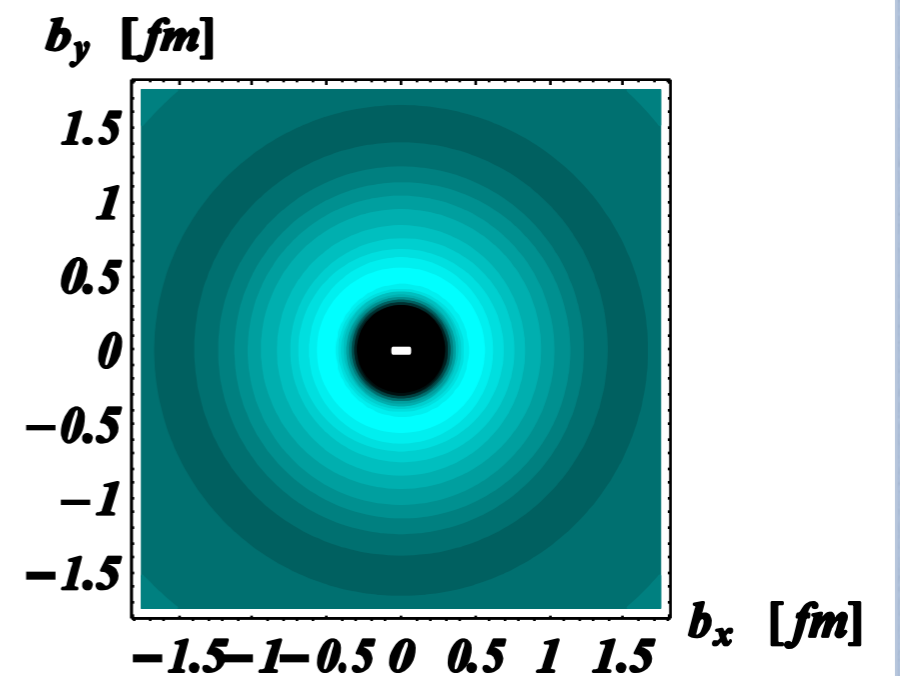
proton

neutron



induced  
electric dipole  
moment:

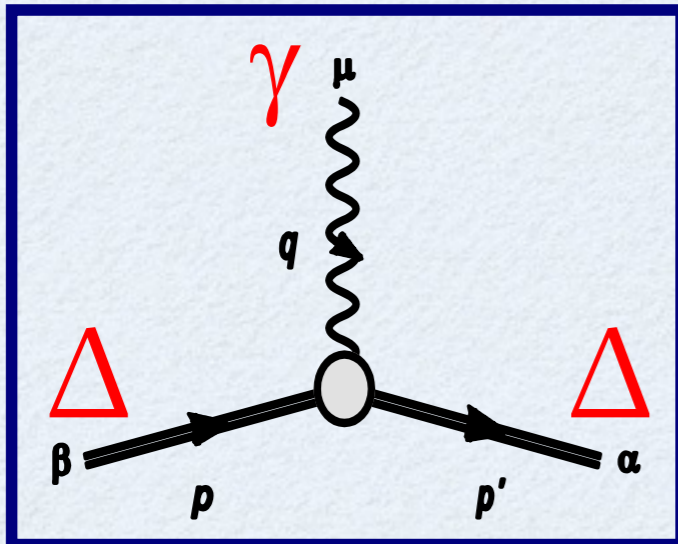
$$d_y = \kappa \frac{e}{2M}$$



Miller (2007)

Carlson, Vdh (2007)

# $\Delta(1232)$ electromagnetic transitions



$$\begin{aligned} & \langle \Delta(p', \lambda') | J^\mu(0) | \Delta(p, \lambda) \rangle \\ &= -\bar{u}_\alpha(p', \lambda') \left\{ \left[ F_1^*(Q^2) g^{\alpha\beta} + F_3^*(Q^2) \frac{q^\alpha q^\beta}{(2M)^2} \right] \gamma^\mu \right. \\ & \quad \left. + \left[ F_2^*(Q^2) g^{\alpha\beta} + F_4^*(Q^2) \frac{q^\alpha q^\beta}{(2M)^2} \right] \frac{i\sigma^{\mu\nu} q_\nu}{2M} \right\} u_\beta(p, \lambda) \end{aligned}$$

4 multipole form factors

Electric charge FF:

$$G_{E0}(Q^2)$$

Magnetic dipole FF:

$$G_{M1}(Q^2)$$

Electric quadrupole FF:

$$G_{E2}(Q^2)$$

Magnetic octupole FF:

$$G_{M3}(Q^2)$$

multipole moments

$$e_\Delta = G_{E0}(0)$$

$$\mu_\Delta = \frac{e_\Delta}{2M} G_{M1}(0)$$

$$Q_\Delta = \frac{e_\Delta}{M^2} G_{E2}(0)$$

$$O_\Delta = \frac{e_\Delta}{2M^3} G_{M3}(0)$$

# Quark charge densities in $\Delta(1232)$

$$\begin{aligned}
 \rho_{T s_{\perp} = \frac{3}{2}}^{\Delta}(\vec{b}) &\equiv \int \frac{d^2 \vec{q}_{\perp}}{(2\pi)^2} e^{-i \vec{q}_{\perp} \cdot \vec{b}} \frac{1}{2P^+} \langle P^+, \frac{\vec{q}_{\perp}}{2}, s_{\perp}^{\Delta} = \frac{3}{2} | J^+(0) | P^+, -\frac{\vec{q}_{\perp}}{2}, s_{\perp}^{\Delta} = \frac{3}{2} \rangle \\
 &= \int_0^{\infty} \frac{dQ}{2\pi} Q \left\{ J_0(bQ) \frac{1}{4} \left( A_{\frac{3}{2} \frac{3}{2}} + 3A_{\frac{1}{2} \frac{1}{2}} \right) \longrightarrow G_{E0}(0) + \mathcal{O}(Q^2) \right. \\
 &\quad - \sin(\phi_b - \phi_S) J_1(bQ) \frac{1}{4} \left( 2\sqrt{3}A_{\frac{3}{2} \frac{1}{2}} + 3A_{\frac{1}{2} -\frac{1}{2}} \right) \longrightarrow \frac{Q}{2M} \{ 3G_{E0}(0) - G_{M1}(0) + \mathcal{O}(Q^2) \} \\
 &\quad - \cos 2(\phi_b - \phi_S) J_2(bQ) \frac{\sqrt{3}}{2} A_{\frac{3}{2} -\frac{1}{2}} \longrightarrow \frac{Q^2}{8M^2} \{ 3G_{E0}(0) - 2G_{M1}(0) - G_{E2}(0) + \mathcal{O}(Q^2) \} \\
 &\quad \left. + \sin 3(\phi_b - \phi_S) J_3(bQ) \frac{1}{4} A_{\frac{3}{2} -\frac{3}{2}} \right\} \longrightarrow \frac{Q^3}{32M^3} \{ G_{E0}(0) - G_{M1}(0) - G_{E2}(0) + G_{M3}(0) + \mathcal{O}(Q^2) \}
 \end{aligned}$$

Quadrupole moment:

$$Q_{s_{\perp}}^{\Delta} \equiv e \int d^2 \vec{b} (b_x^2 - b_y^2) \rho_{T s_{\perp}}^{\Delta}(\vec{b})$$

$$Q_{\frac{3}{2}}^{\Delta} = -Q_{\frac{1}{2}}^{\Delta} = \frac{1}{2} \{ 2 [G_{M1}(0) - 3e_{\Delta}] + [G_{E2}(0) + 3e_{\Delta}] \} \left( \frac{e}{M^2} \right)$$

for spin 3/2 point particle: transverse density =  $\delta$ -function

leads to “natural values” of multipole moments

$$G_{E0}(0) = e_{\Delta} \quad G_{M1}(0) = 3e_{\Delta}, \quad G_{E2}(0) = -3e_{\Delta}, \quad G_{M3}(0) = -e_{\Delta}$$



# Natural values of hadron e.m. moments

Transverse charge densities depend only on anomalous values of e.m. moments  $\Rightarrow$  determine hadron internal structure

Spin  $j$  :  $2j+1$  multipoles

$j$	$G_{E0}(0)$ ( $e$ )	$G_{M1}(0)$ ( $e/2M$ )	$G_{E2}(0)$ ( $e/M^2$ )	$G_{M3}(0)$ ( $e/2M^3$ )	$G_{E4}(0)$ ( $e/M^4$ )	$G_{M5}(0)$ ( $e/2M^5$ )
0	1	1				
1/2	1	1				
1	1	2	-1			
3/2	1	3	-3	-1		
2	1	4	-6	-4	1	
$\vdots$						
$j$	$C_{2j}^0$	$C_{2j}^1$	$-C_{2j}^2$	$-C_{2j}^3$	$C_{2j}^4$	$C_{2j}^5$

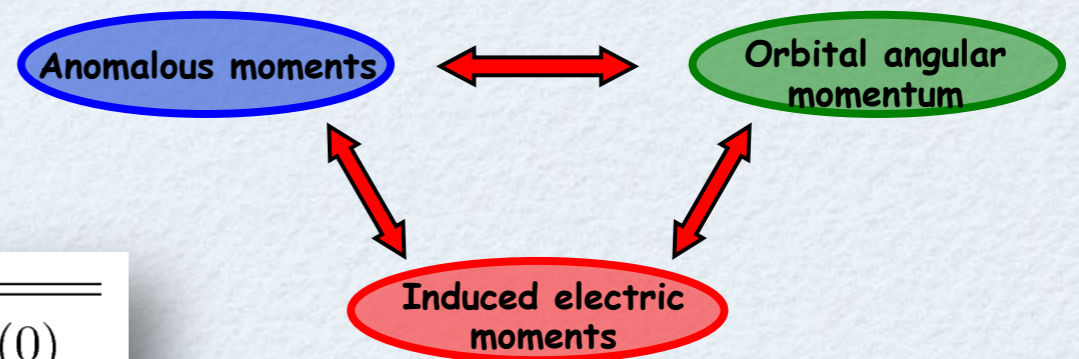
Standard Model

Supergravity

Charge normalization

Universal  $g=2$  factor

$$G_{M1}(0) = 2j$$



Structureless particle

No OAM

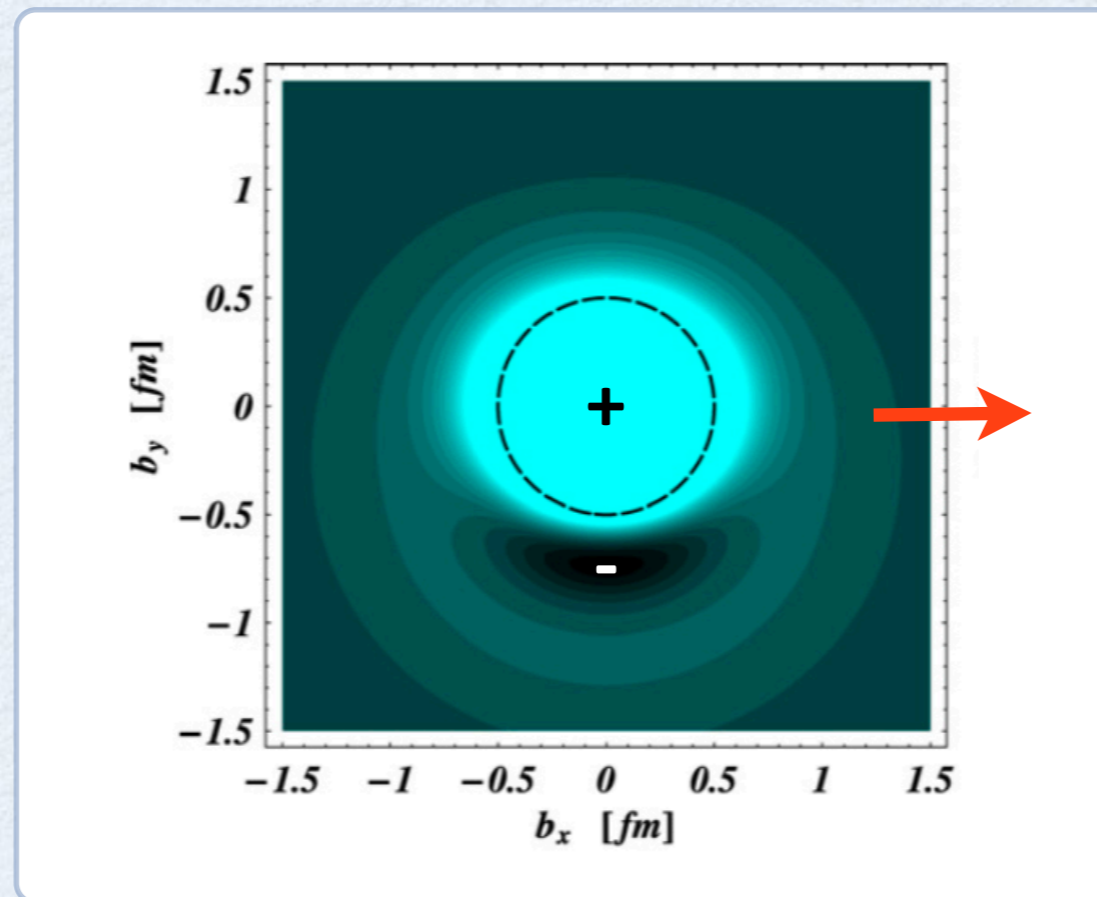
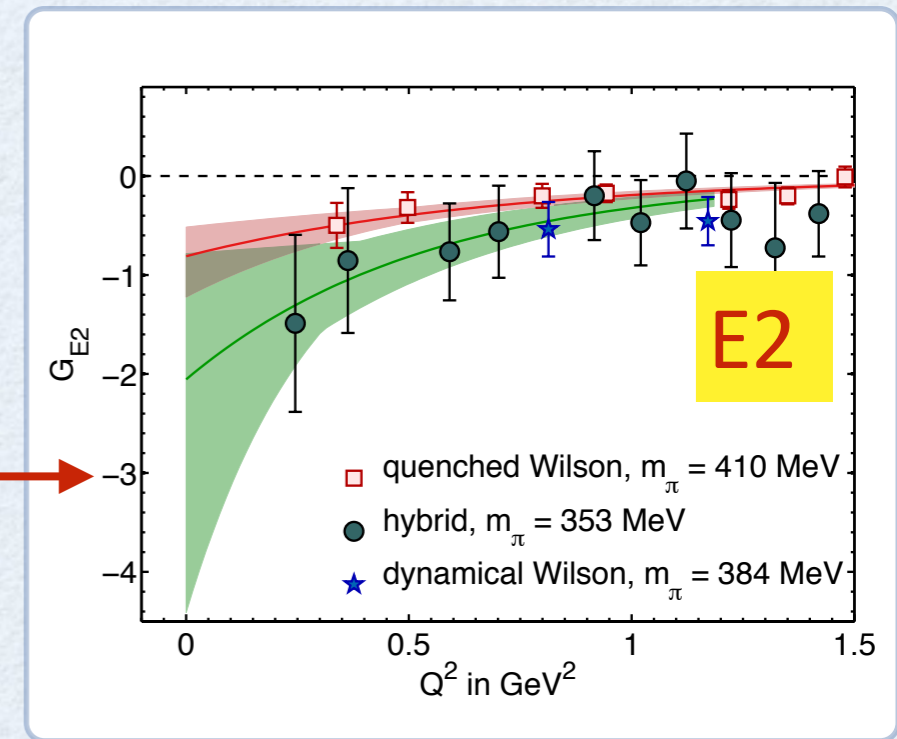
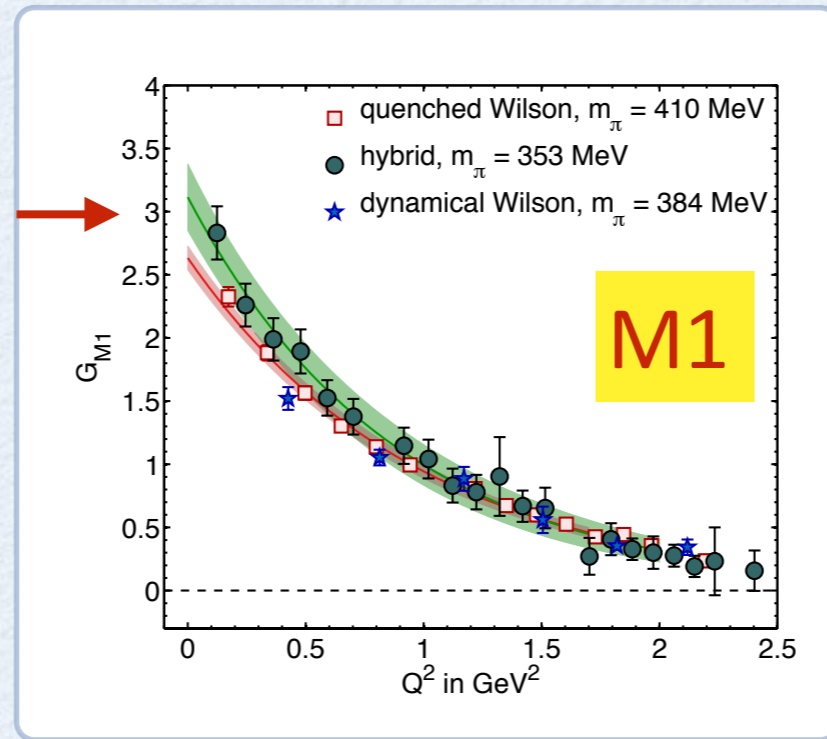
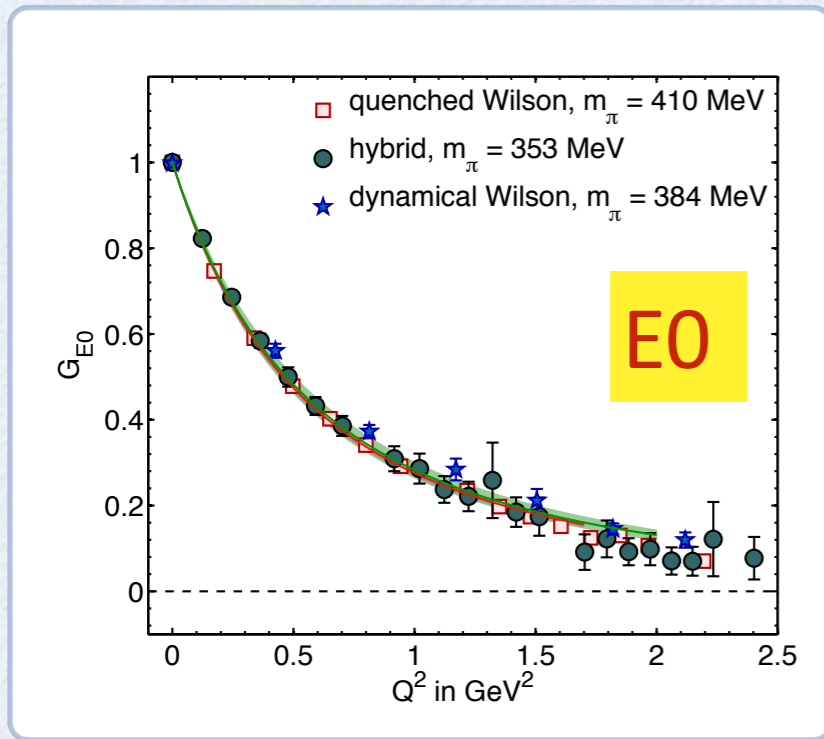
No distortions

No anomalous moments

Natural EM moments

Lorcé (2008)

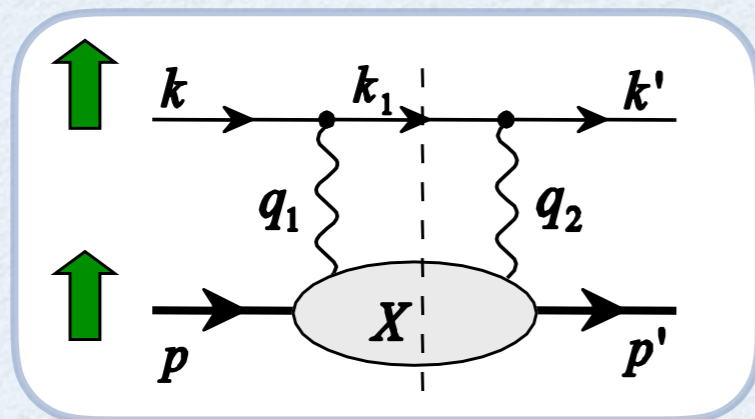
# Quark charge densities in $\Delta^+(1232)$ : lattice QCD



Alexandrou et al. (2008)

# Access $\Delta$ e.m. form factors in experiment: normal spin asymmetries

Beam or target normal spin asymmetries:



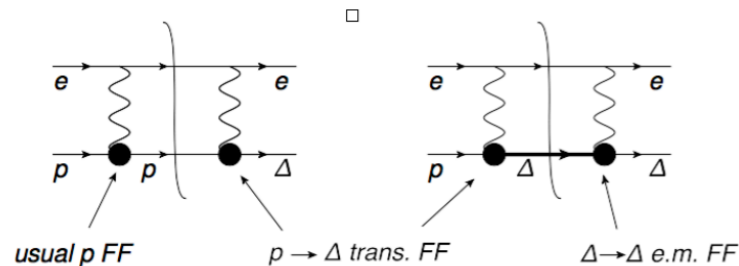
directly proportional to **Im part** of TPE

target:  $A_n \sim \alpha_{em} \sim 10^{-2}$

beam:  $B_n \sim \alpha_{em} \frac{m_e}{E_e} \sim 10^{-6} - 10^{-5}$

Beam for  $ep \rightarrow e\Delta$  accesses  $\Delta$  e.m. FFs

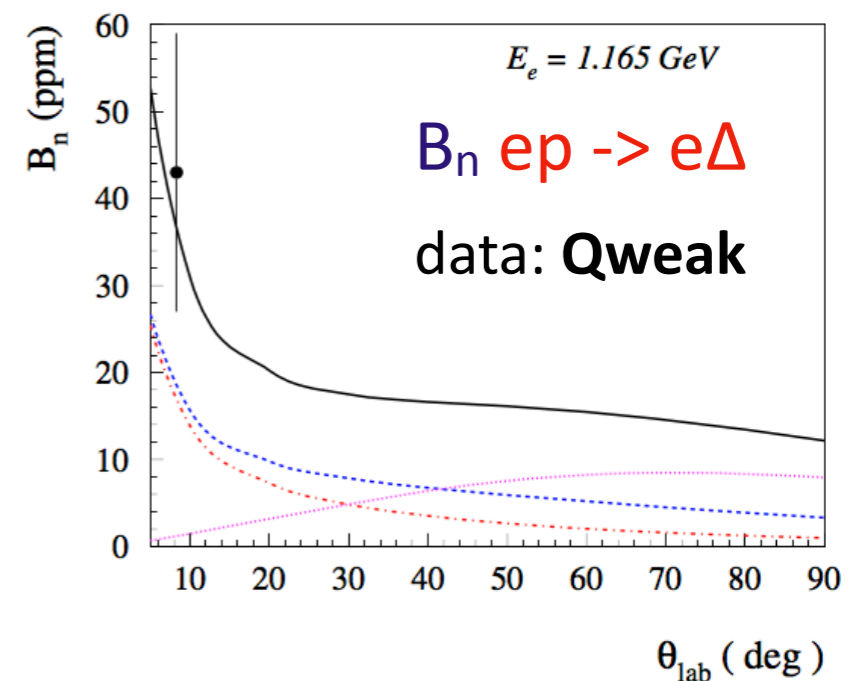
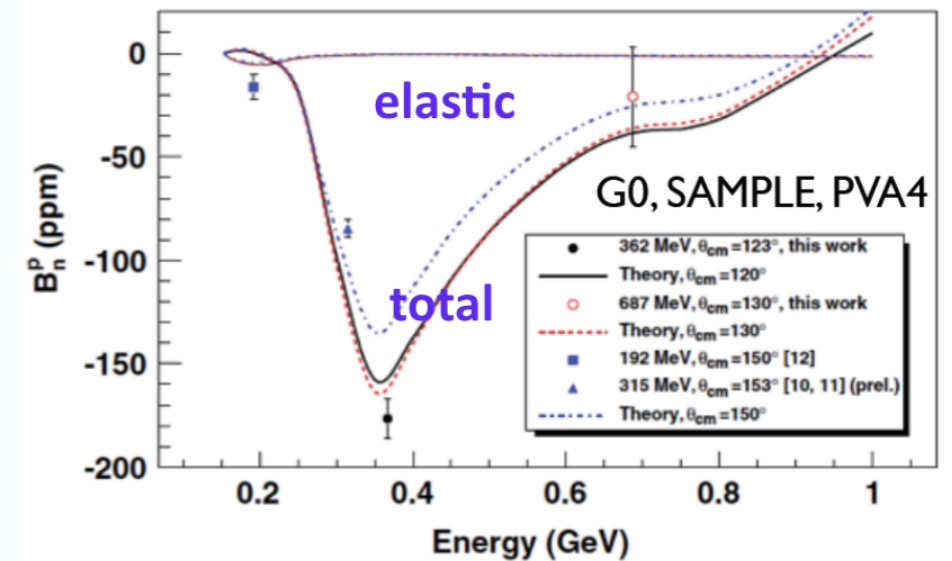
Carlson, Pasquini, Pauk, Vdh (2017)



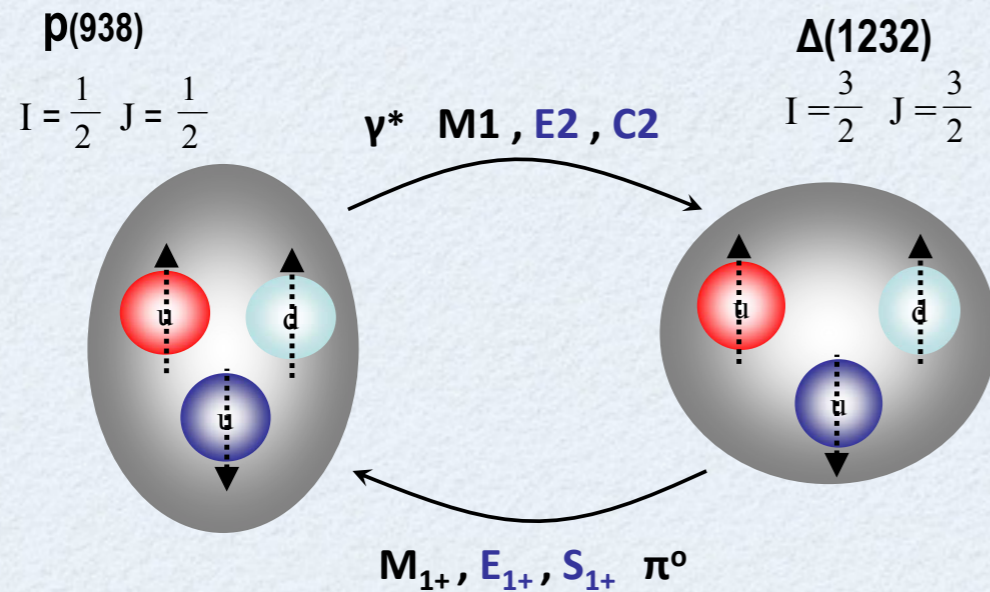
Results for QWeak kinematics

- Nucleon = dash-dot red line
- $\Delta$  = dashed blue line
- $S_{11} + D_{13}$  = dotted purple line
- Total = solid black line

$B_n ep \rightarrow ep$  Phys.Rev.Lett. 107 (2011) 022501

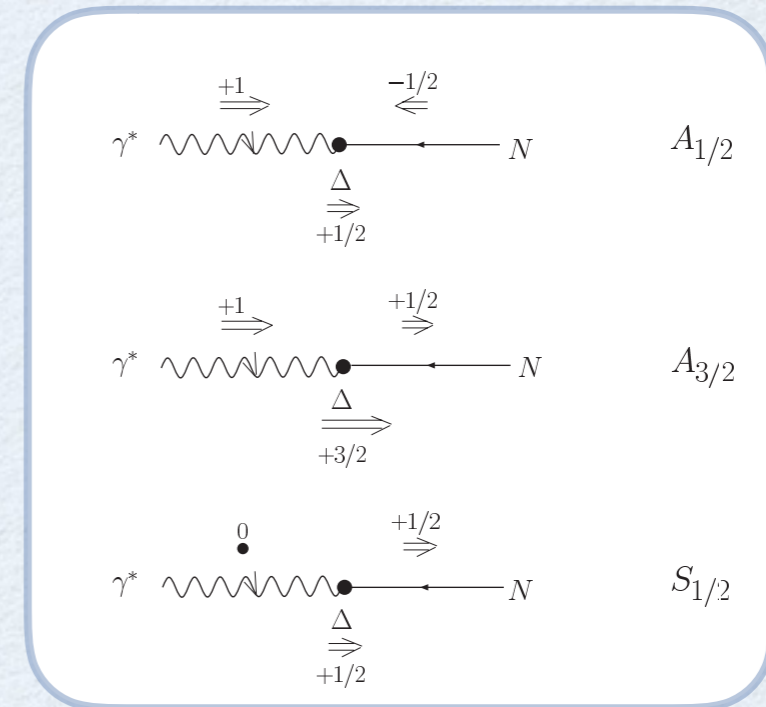


# N $\rightarrow$ $\Delta(1232)$ e.m. transition densities



Spherical  $\Rightarrow$  M1

Deformed  $\Rightarrow$  M1, E2, C2



experiment measures **multipoles**

$$\bar{M}_{1+}^{(3/2)}(Q^2) \equiv \sqrt{\frac{2}{3}} a_{\Delta} \text{Im} M_{1+}^{(3/2)}(Q^2, W = M_{\Delta})$$

theory calculates **helicity amplitudes**

$$A_{3/2} \equiv -\frac{e}{\sqrt{2q_{\Delta}}} \frac{1}{(4M_N M_{\Delta})^{1/2}} \langle \Delta(\vec{0}, +3/2) | \mathbf{J} \cdot \epsilon_{\lambda=+1} | N(-\vec{q}, +1/2) \rangle$$

$$A_{1/2} \equiv -\frac{e}{\sqrt{2q_{\Delta}}} \frac{1}{(4M_N M_{\Delta})^{1/2}} \langle \Delta(\vec{0}, +1/2) | \mathbf{J} \cdot \epsilon_{\lambda=+1} | N(-\vec{q}, -1/2) \rangle$$

$$S_{1/2} \equiv \frac{e}{\sqrt{2q_{\Delta}}} \frac{1}{(4M_N M_{\Delta})^{1/2}} \langle \Delta(\vec{0}, +1/2) | J^0 | N(-\vec{q}, +1/2) \rangle.$$

$$A_{3/2} = -\frac{\sqrt{3}}{2} \left\{ \bar{M}_{1+}^{(3/2)} - \bar{E}_{1+}^{(3/2)} \right\}$$

$$A_{1/2} = -\frac{1}{2} \left\{ \bar{M}_{1+}^{(3/2)} + 3 \bar{E}_{1+}^{(3/2)} \right\}$$

$$S_{1/2} = -\sqrt{2} \bar{S}_{1+}^{(3/2)}$$

# N → Δ(1232) e.m. multipoles

large  $N_c$  limit of QCD: N and Δ(1232) degenerate  
 Quadrupole moment related to neutron rms radius

$$Q_{p \rightarrow \Delta^+} = \frac{1}{\sqrt{2}} r_n^2 \frac{N_c}{N_c + 3} \sqrt{\frac{N_c + 5}{N_c - 1}}$$

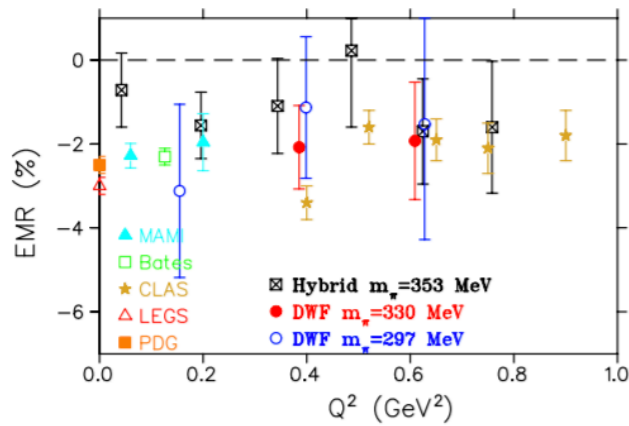
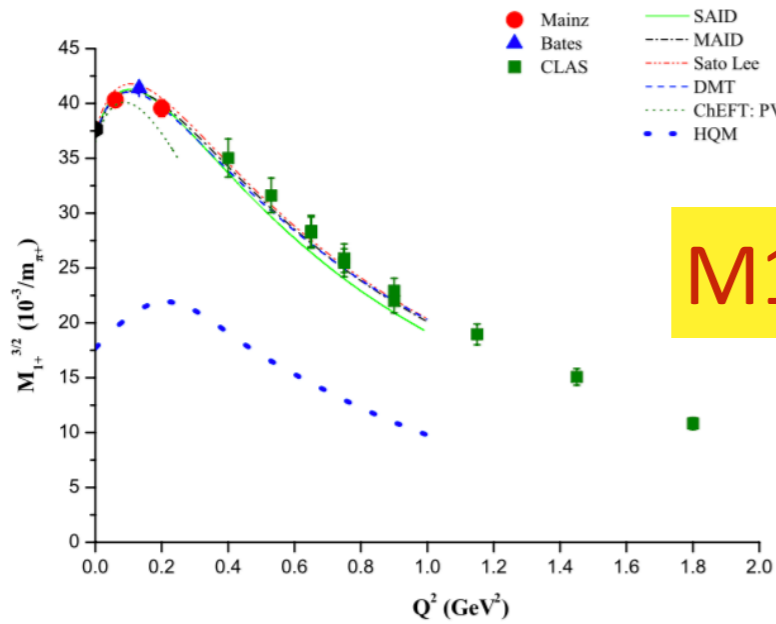
Buchmann, Hester,  
 Lebed (2002)

Exp.:  $r_n^2 = -0.113(3) \text{ fm}^2$

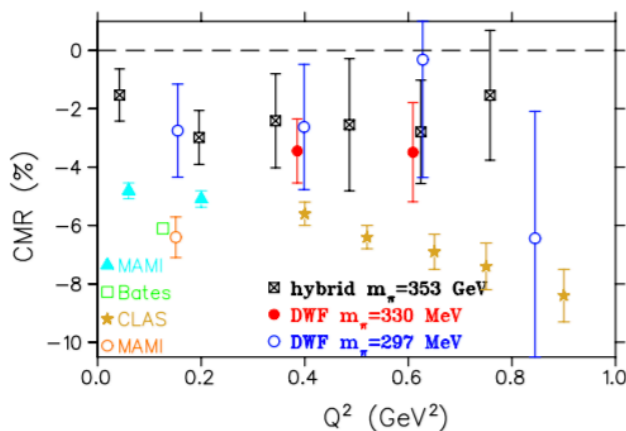
large  $N_c$ :  $Q_{p \rightarrow \Delta^+} = -0.080 \text{ fm}^2$

Exp.:  $Q_{p \rightarrow \Delta^+} = -0.085(3) \text{ fm}^2$

very good  
 agreement !



E2/M1



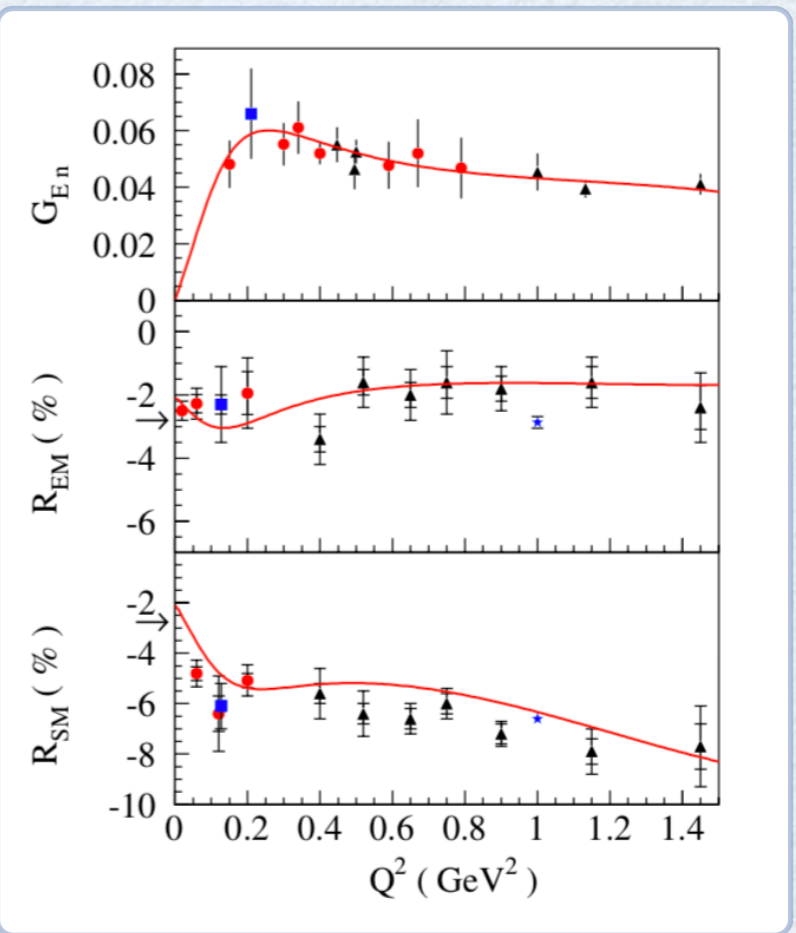
C2/M1

low  $Q^2$  relations

$$G_E^*(Q^2) = \left(\frac{M_N}{M_\Delta}\right)^{3/2} \frac{M_\Delta^2 - M_N^2}{2\sqrt{2}Q^2} G_{En}(Q^2)$$

$$G_C^*(Q^2) = \frac{4M_\Delta^2}{M_\Delta^2 - M_N^2} G_E^*(Q^2)$$

Pascalutsa, Vdh (2007)



# N $\rightarrow$ $\Delta(1232)$ transition densities

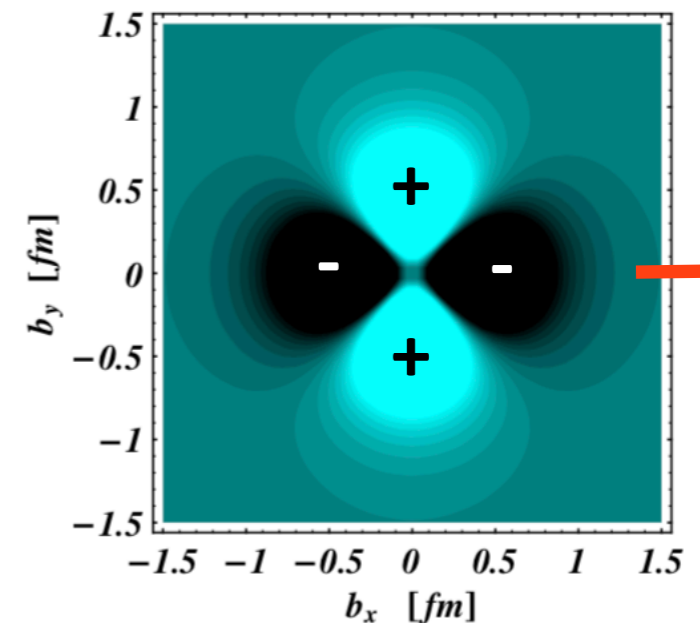
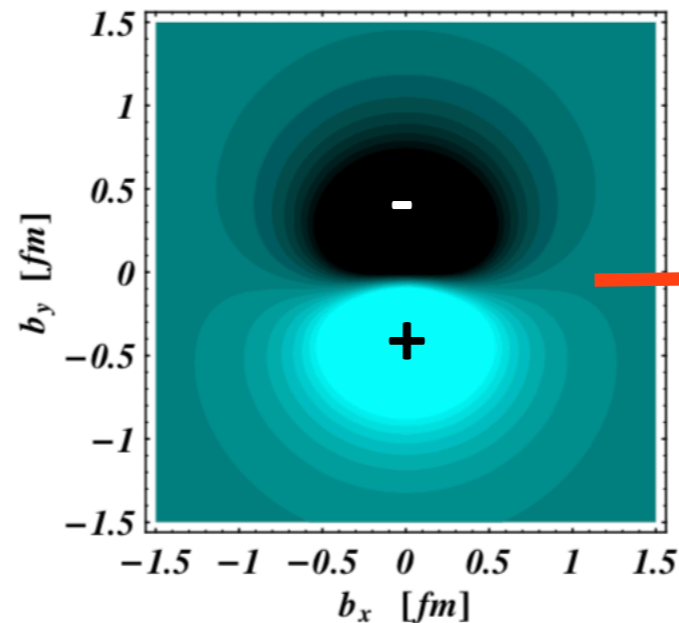
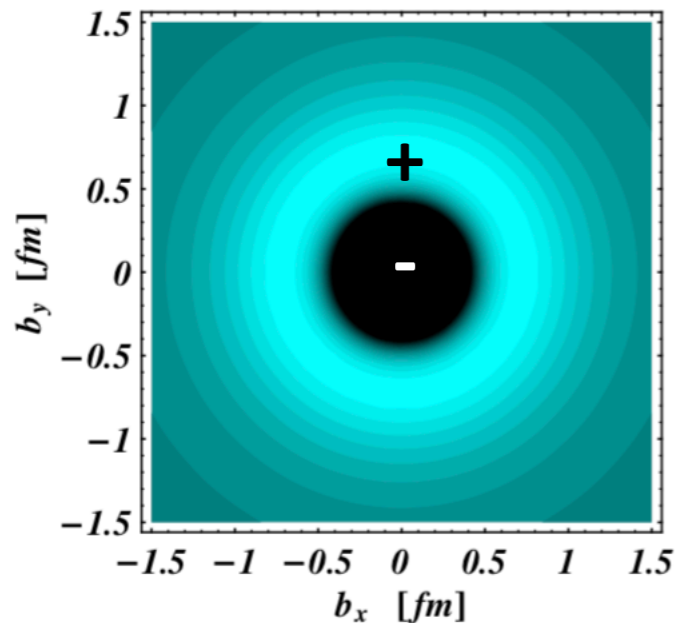
$$\begin{aligned} \rho_T^{N\Delta}(\vec{b}) &\equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp \cdot \vec{b}} \frac{1}{2P^+} \langle P^+, \frac{\vec{q}_\perp}{2}, s_\perp^\Delta = +\frac{1}{2} | J^+(0) | P^+, -\frac{\vec{q}_\perp}{2}, s_\perp^N = +\frac{1}{2} \rangle \\ &= \int_0^\infty \frac{dQ}{2\pi} \frac{Q}{2} \left\{ J_0(bQ) G_{+\frac{1}{2}+\frac{1}{2}}^+ \longrightarrow \text{monopole} \right. \\ &\quad \left. - \sin(\phi_b - \phi_S) J_1(bQ) \left[ \sqrt{3} G_{+\frac{3}{2}+\frac{1}{2}}^+ + G_{+\frac{1}{2}-\frac{1}{2}}^+ \right] \longrightarrow \text{dipole} \right. \\ &\quad \left. - \cos 2(\phi_b - \phi_S) J_2(bQ) \sqrt{3} G_{+\frac{3}{2}-\frac{1}{2}}^+ \right\} \longrightarrow \text{quadrupole} \end{aligned}$$

$\rho_0$

P  $\rightarrow$   $\Delta^+$

$\rho_T$

quadrupole term



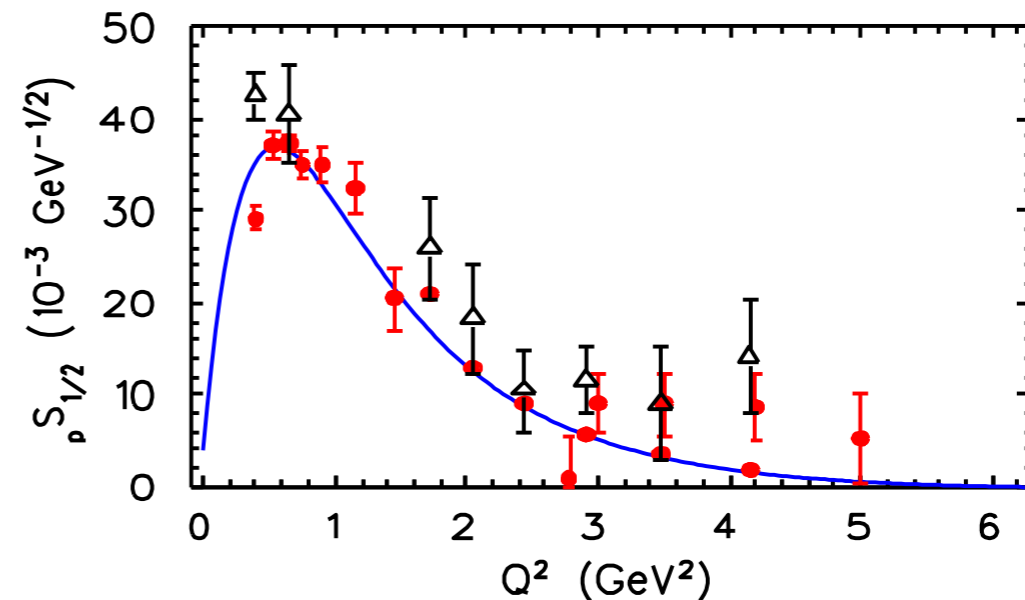
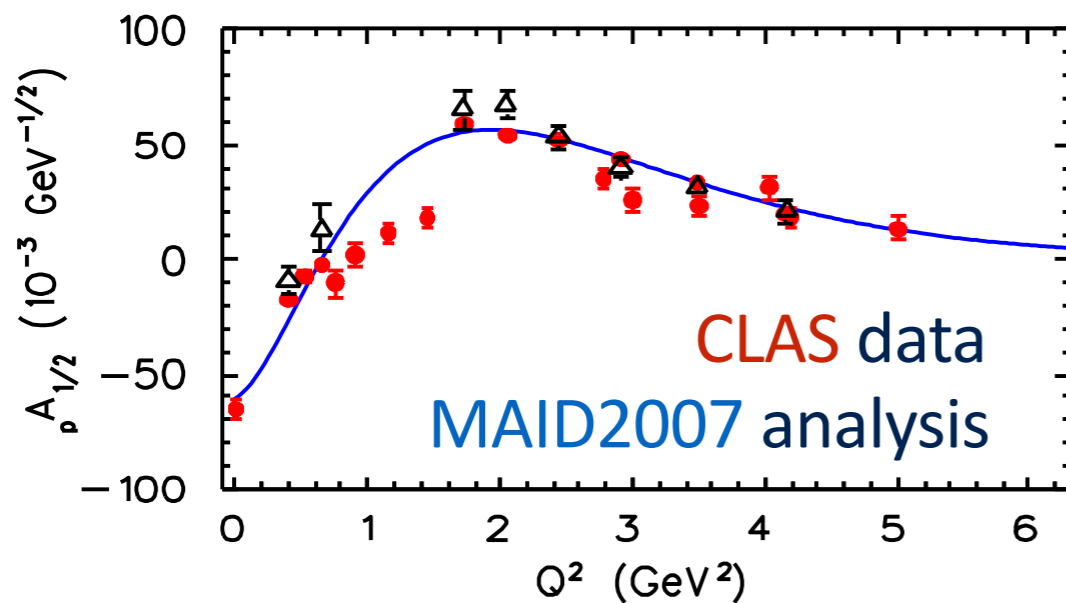
# $N \rightarrow P_{11}(1440)$ transition densities

$$\langle N^*(p', \lambda') | J^\mu(0) | N(p, \lambda) \rangle = \bar{u}(p', \lambda') \left\{ F_1^{NN^*}(Q^2) \left( \gamma^\mu - \gamma \cdot q \frac{q^\mu}{q^2} \right) + F_2^{NN^*}(Q^2) \frac{i\sigma^{\mu\nu} q_\nu}{(M^* + M_N)} \right\} u(p, \lambda)$$

helicity  
amplitudes

$$A_{1/2} = e \frac{Q_-}{\sqrt{K} (4M_N M^*)^{1/2}} \left\{ F_1^{NN^*} + F_2^{NN^*} \right\}$$

$$S_{1/2} = e \frac{Q_-}{\sqrt{2K} (4M_N M^*)^{1/2}} \left( \frac{Q_+ + Q_-}{2M^*} \right) \frac{(M^* + M_N)}{Q^2} \left\{ F_1^{NN^*} - \frac{Q^2}{(M^* + M_N)^2} F_2^{NN^*} \right\}$$

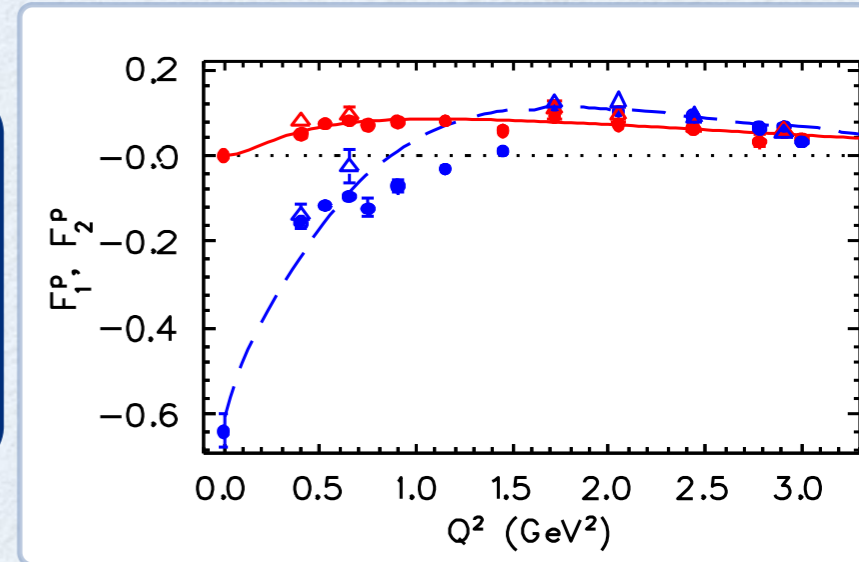


Tiator, Vdh (2008)

# $N \rightarrow P_{11}(1440)$ transition densities

$$\rho_0^{NN^*}(\vec{b}) = \int_0^\infty \frac{dQ}{2\pi} Q J_0(bQ) F_1^{NN^*}(Q^2)$$

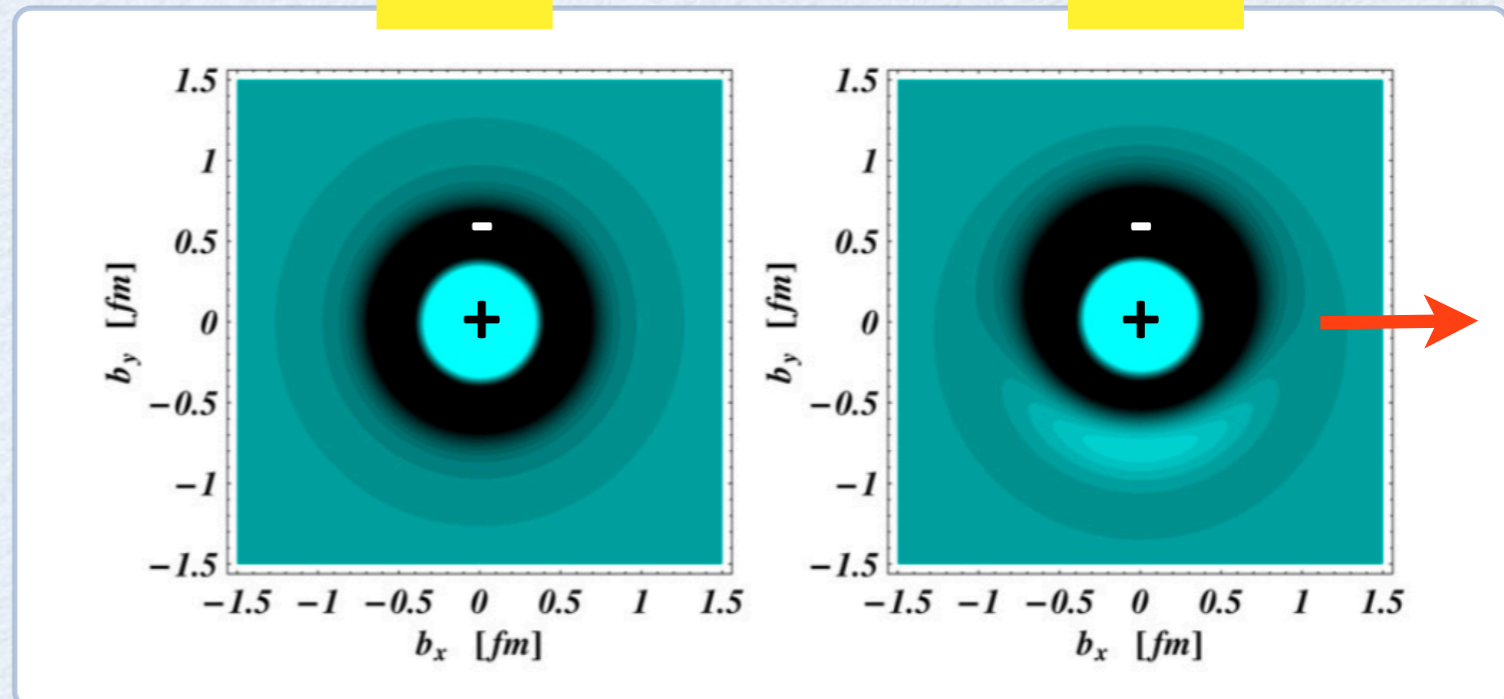
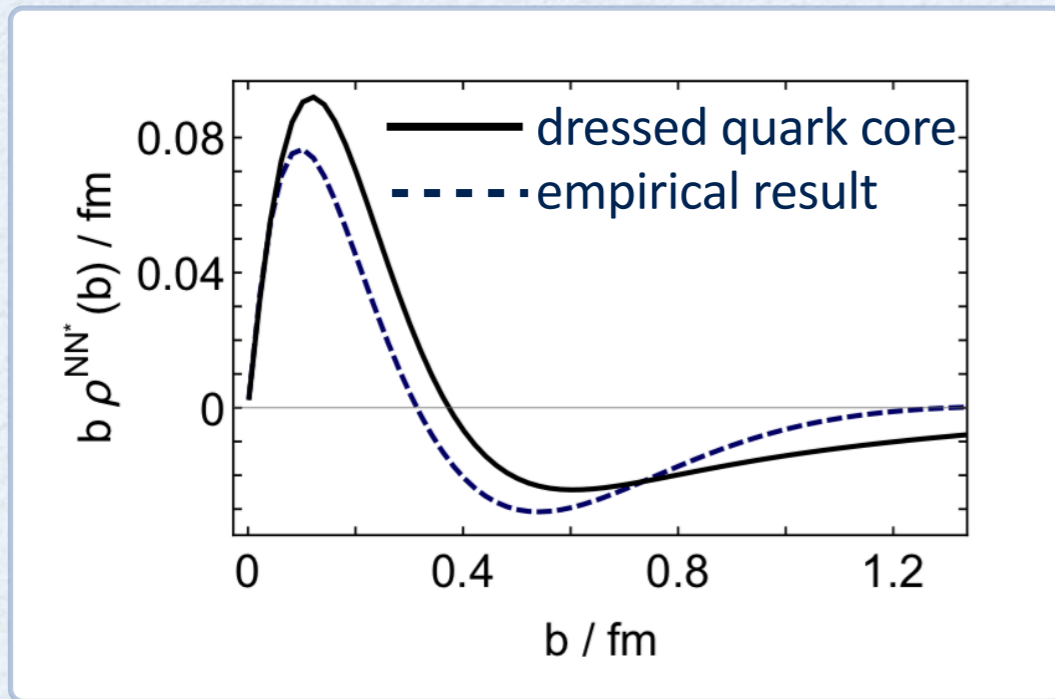
$$\rho_T^{NN^*}(\vec{b}) = \rho_0^{NN^*}(b) + \sin(\phi_b - \phi_S) \int_0^\infty \frac{dQ}{2\pi} \frac{Q^2}{(M^* + M_N)} J_1(bQ) F_2^{NN^*}(Q^2)$$



DSE: dressed quark core calculation

$\rho_0$

$\rho_T$



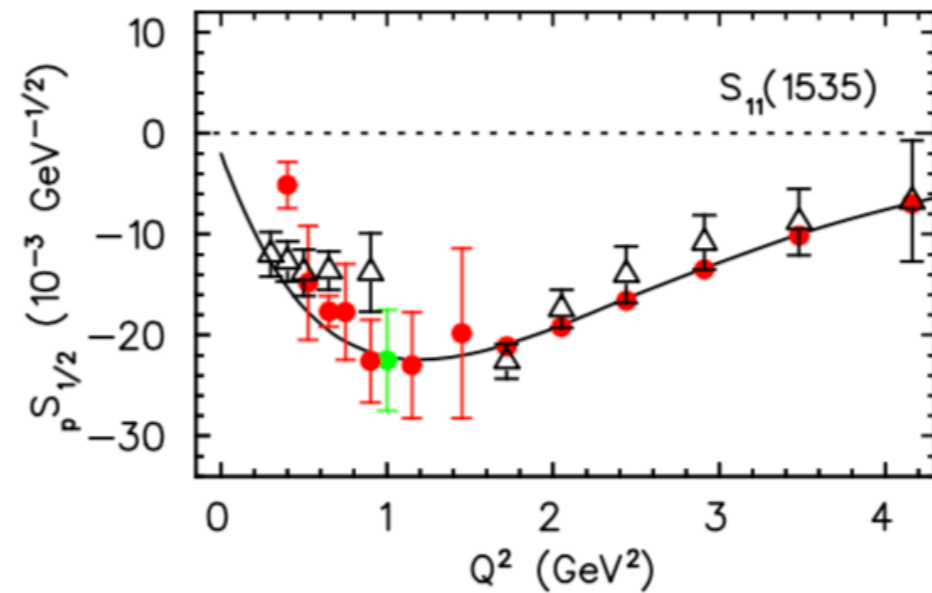
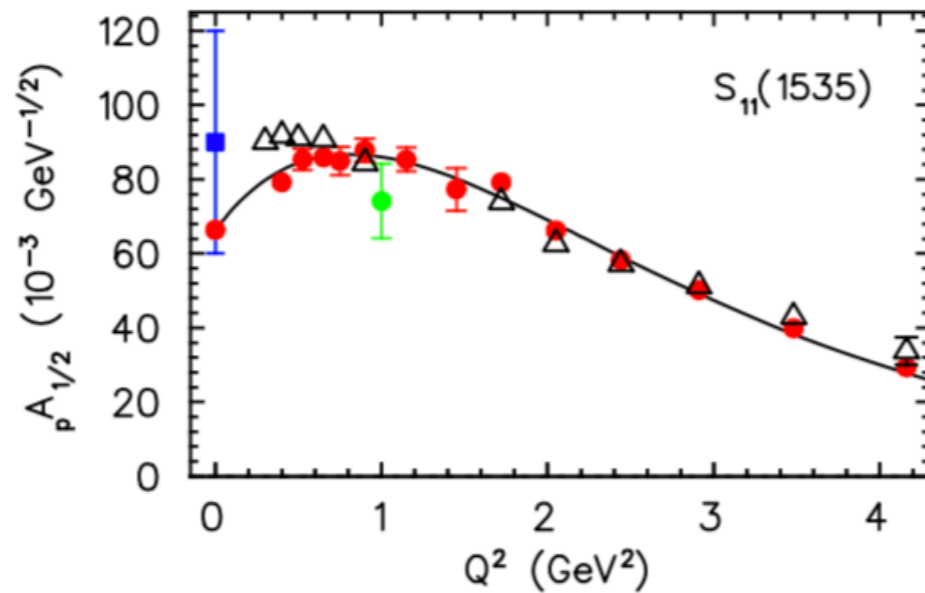
Roberts, Segovia et al. (2016,2018)

Tiator, Vdh (2008)

At large distances: u-quark core screened by mesonic tail (MB FSI)



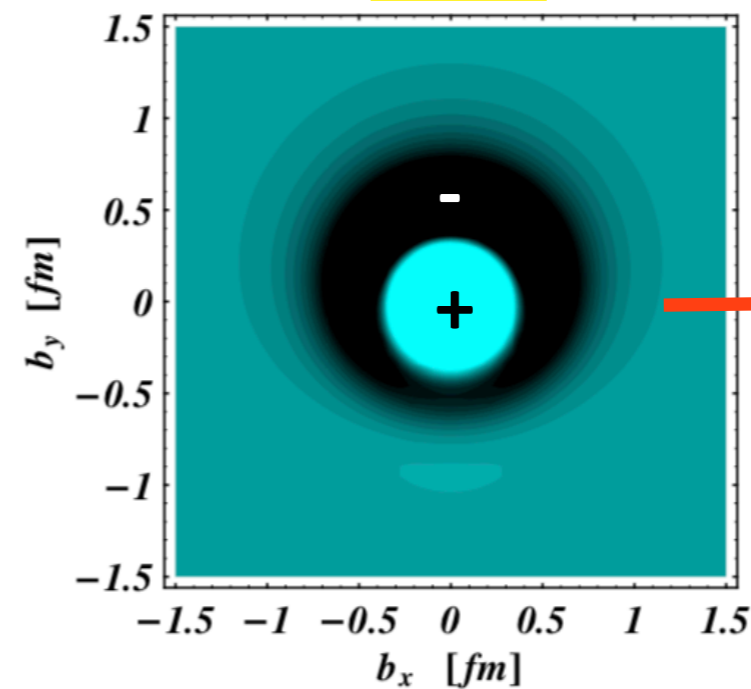
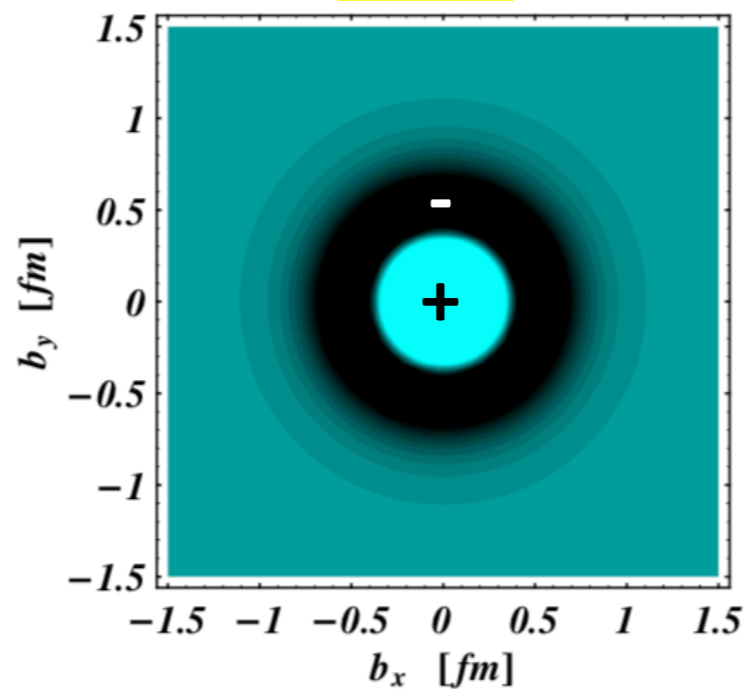
# $N \rightarrow S_{11}(1535)$ transition densities



CLAS data  
MAID2007  
analysis

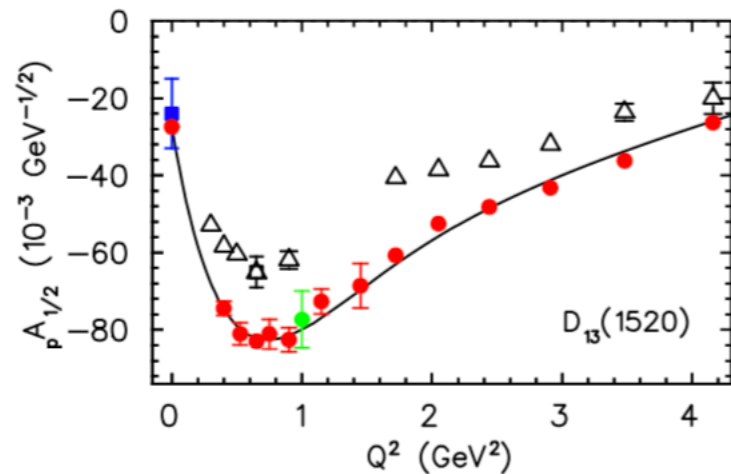
$\rho_0$

$\rho_T$



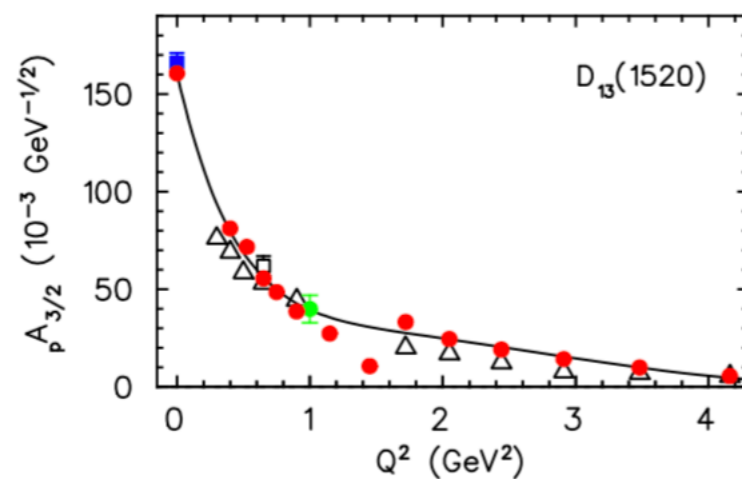
$$s_{\perp} = -s'_{\perp} = +1/2$$

# $N \rightarrow D_{13}(1520)$ transition densities



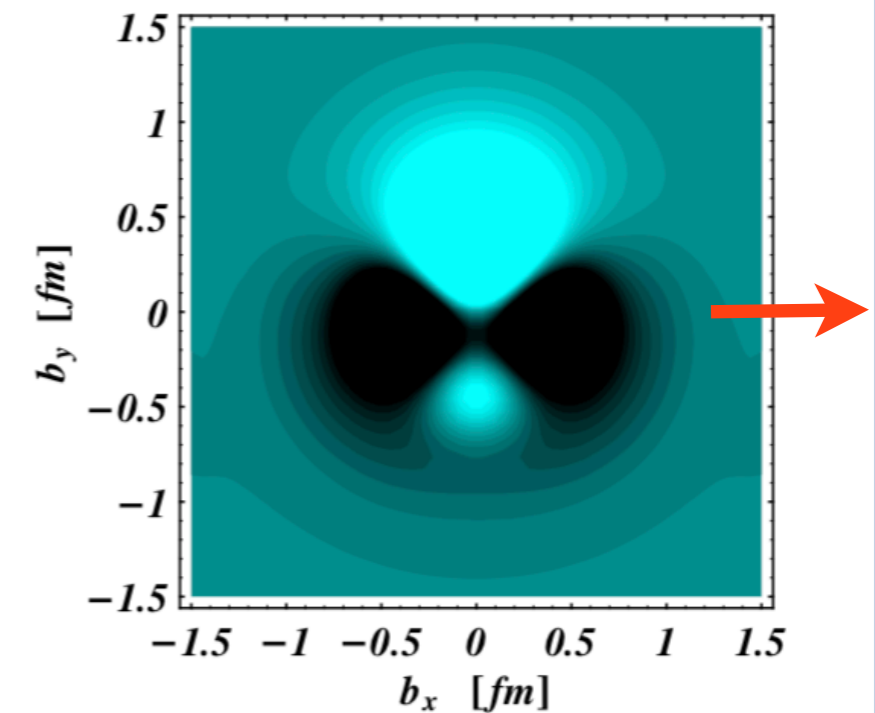
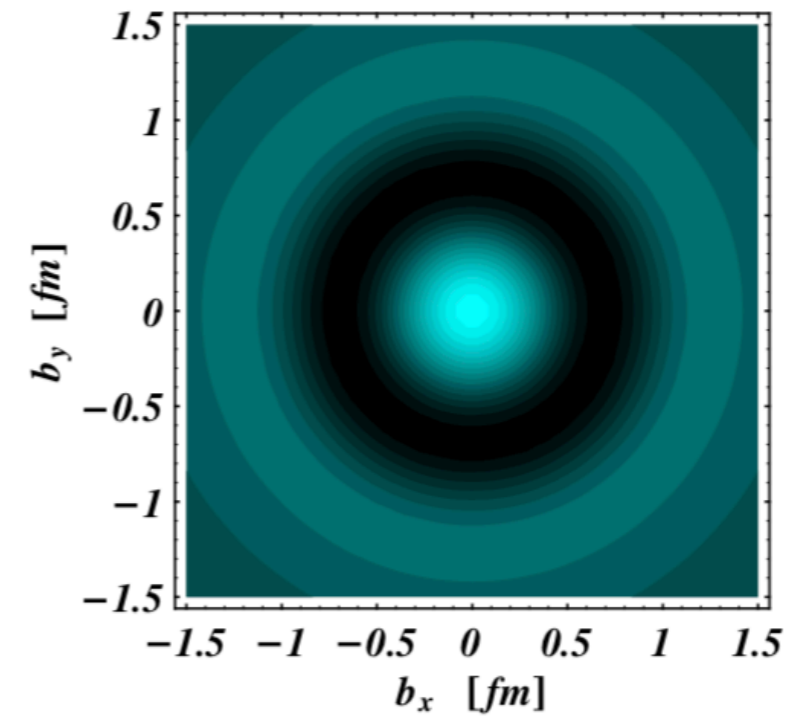
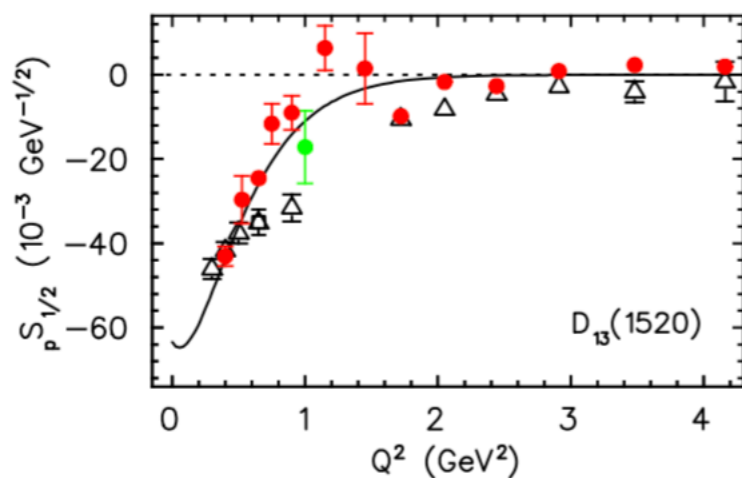
$\rho_0$

$$\lambda = \lambda' = +1/2$$



$\rho_T$

$$s_{\perp} = -s'_{\perp} = +1/2$$



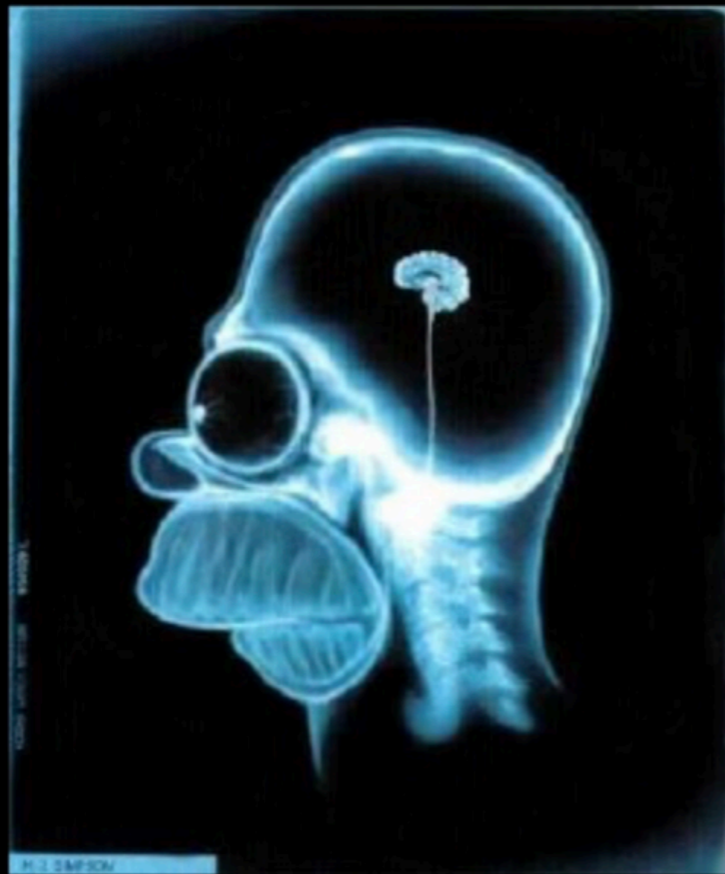
Tiator, Vdh (2011)

large quadrupole

# Structure vs dynamics:

## Quark spatial vs momentum distributions

MRI studies brain anatomy.



Functional MRI (fMRI) studies brain function.



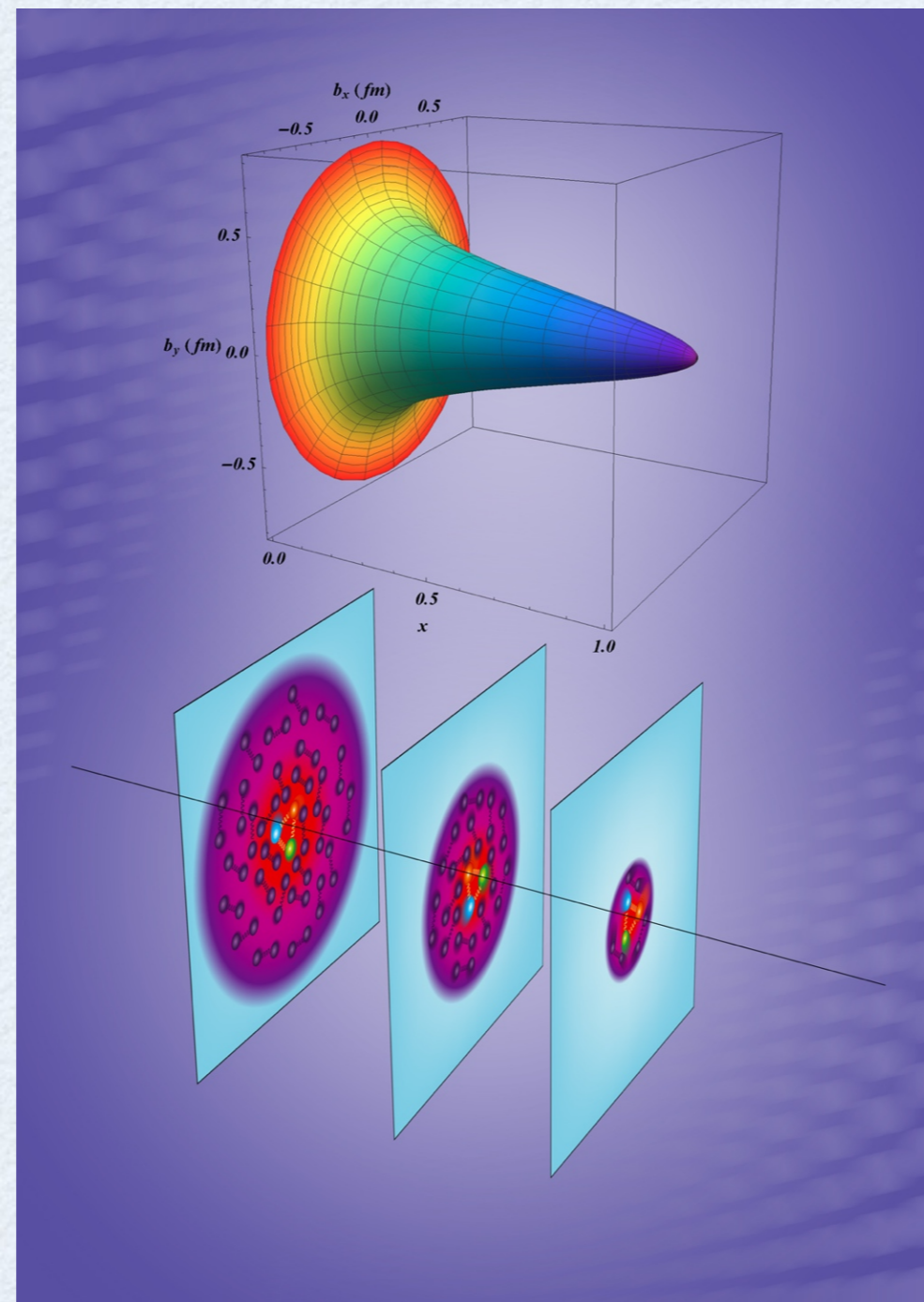
# Correlations in transverse position/longitudinal momentum

**elastic  
scattering**

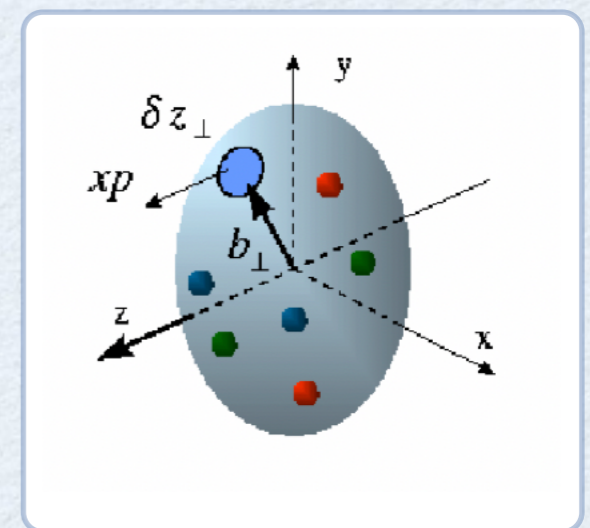


**DIS**

quark  
distributions in  
**transverse  
position space**



quark  
distributions in  
**longitudinal  
momentum**

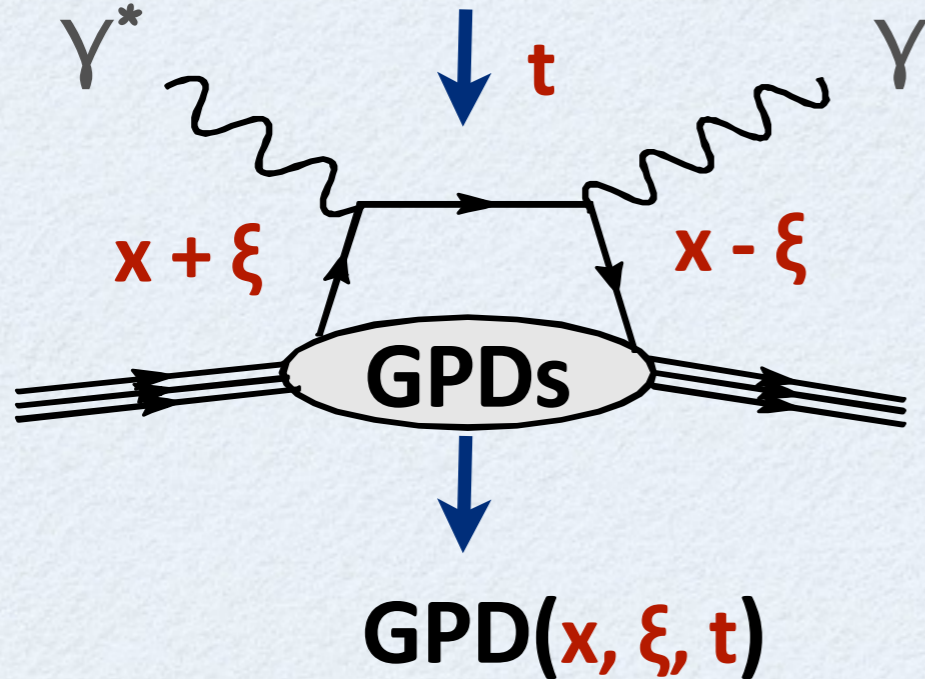


Burkardt (2000, 2003)

Belitsky, Ji, Yuan  
(2004)

# DVCS: tool to access GPDs

$Q^2 \gg 1 \text{ GeV}^2$



➔ at large  $Q^2$ : **QCD factorization theorem**

Müller et al (1994)

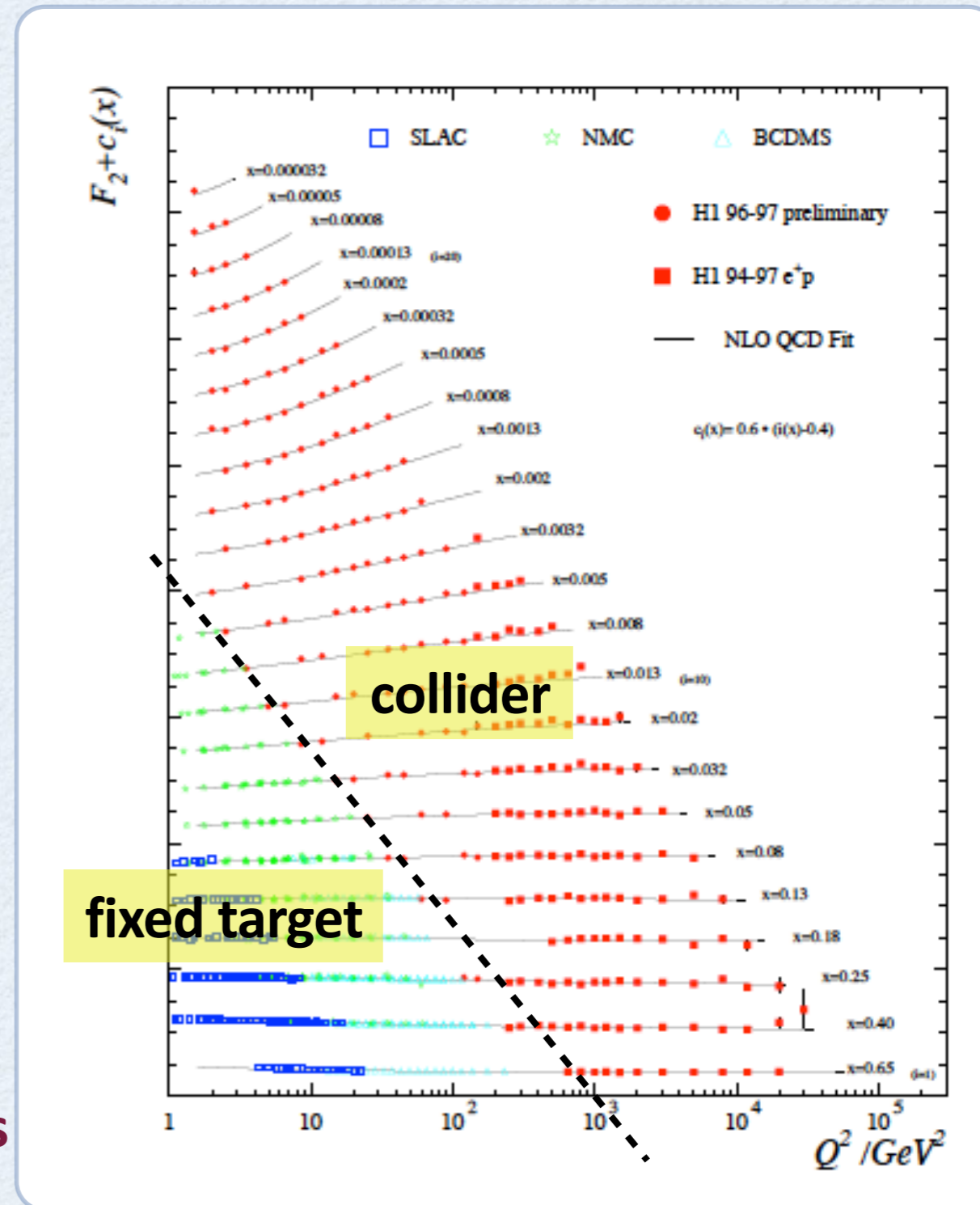
Ji (1995)      Radyushkin (1996)

Collins, Frankfurt, Strikman (1996)

at twist-2: **4 quark helicity conserving GPDs**

➔ key:  $Q^2$  leverage needed to test **QCD scaling**

world data on proton  $F_2$



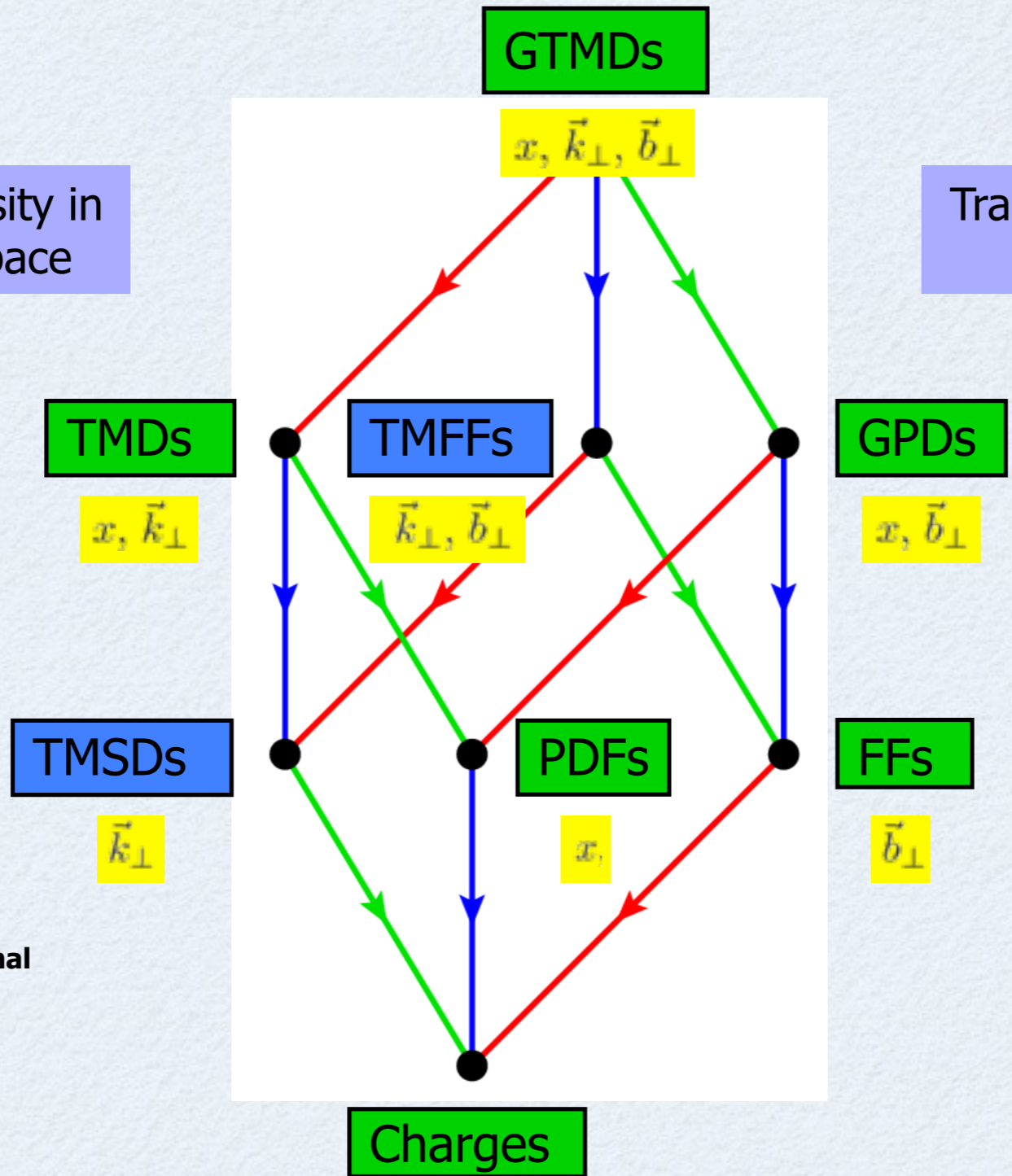
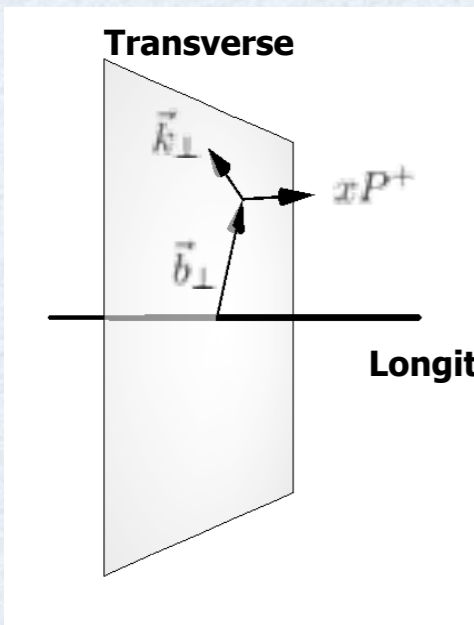
# GTMDs

<b>Momentum space</b>	$\vec{k}_\perp \leftrightarrow \vec{z}_\perp$	<b>Position space</b>
	$\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$	

Transverse density in momentum space

Transverse density in position space

Lorcé (2011)



	$\int d^2b_\perp$
	$\int dx$
	$\int d^2k_\perp$

# GPDs: known limits

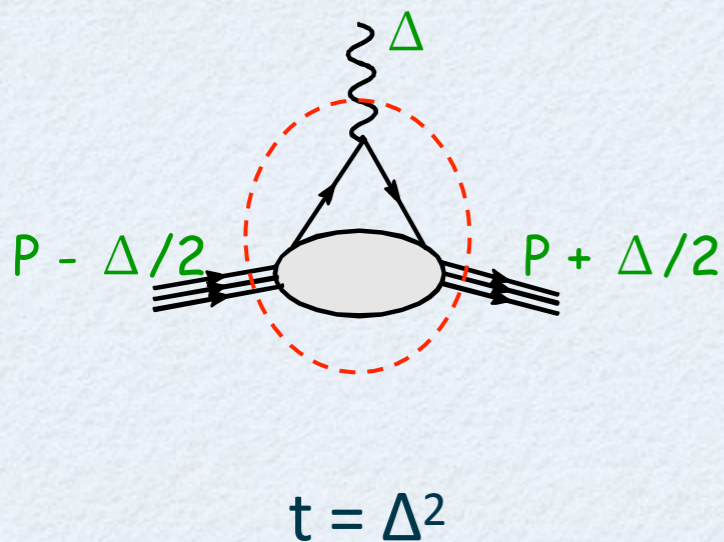
➔ in forward kinematics ( $\xi=0, t=0$ ) : **PDF limit**

$$H^q(x, \xi = 0, t = 0) = q(x)$$

$$\tilde{H}^q(x, \xi = 0, t = 0) = \Delta q(x)$$

$E, \tilde{E}^q$  do not appear in forward kinematics (DIS) ➔ **new information**

➔ first moments of GPDs : **elastic form factor limit**



$$\int_{-1}^{+1} dx H^q(x, \xi, t) = F_1^q(t)$$

➔ Dirac FF

$$\int_{-1}^{+1} dx E^q(x, \xi, t) = F_2^q(t)$$

➔ Pauli FF

$$\int_{-1}^{+1} dx \tilde{H}^q(x, \xi, t) = G_A^q(t)$$

➔ axial FF

$$\int_{-1}^{+1} dx \tilde{E}^q(x, \xi, t) = G_P^q(t)$$

➔ pseudoscalar FF

# GPDs: moments, total angular momentum

$$\int_{-1}^{+1} dx x H^q(x, \xi, t) = A(t) + \xi^2 C(t)$$

$$\int_{-1}^{+1} dx x E^q(x, \xi, t) = B(t) - \xi^2 C(t)$$

form factors of energy-momentum tensor

Polyakov, Weiss (1999)

Polyakov (2003)

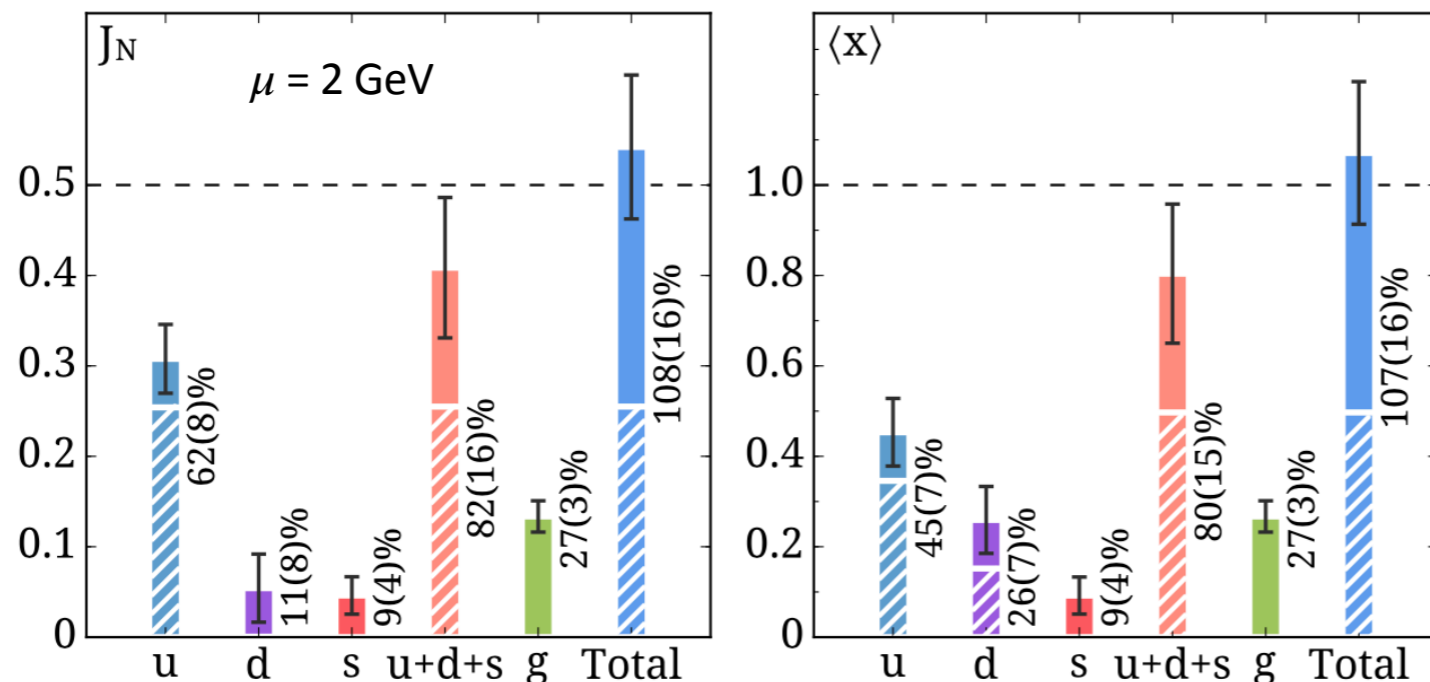
Goeke, Schweitzer et al. (2007)

Ji's angular momentum sum rule

$$\int_{-1}^{+1} dx x \{ H^q(x, \xi, 0) + E^q(x, \xi, 0) \} = A(0) + B(0) = 2J^q$$

lattice QCD calculations at the physical point

Alexandrou et al. (2017)



d, s-quarks carry very small total J in proton, u-quark carries around 60%, gluons around 30%

Sharing of momentum and total angular momentum between quarks and gluons identical in proton !

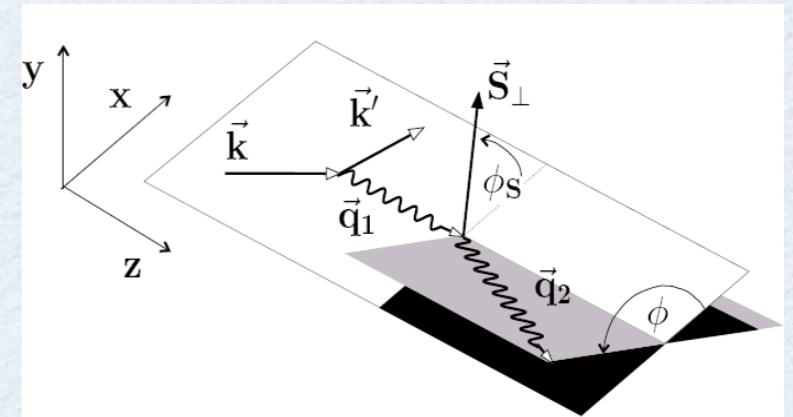


# DVCS observables: path towards accessing GPDs

$$A = \frac{\sigma^+ - \sigma^-}{\sigma^+ + \sigma^-} = \frac{\Delta\sigma}{2\sigma}$$

$$\xi \sim x_B / (2 - x_B)$$

$$k = t / 4M^2$$



Polarized beam, unpolarized target:

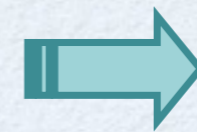
$$\Delta\sigma_{LU} \sim \sin\phi \{F_1 \mathbf{H} + \xi(F_1 + F_2) \tilde{\mathbf{H}} + kF_2 \mathbf{E}\} d\phi$$



$$\mathbf{H}(\xi, t)$$

Unpolarized beam, longitudinal target:

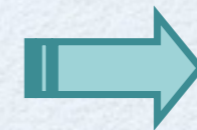
$$\Delta\sigma_{UL} \sim \sin\phi \{F_1 \tilde{\mathbf{H}} + \xi(F_1 + F_2) (\mathbf{H} + \xi / (1 + \xi) \mathbf{E})\} d\phi$$



$$\tilde{\mathbf{H}}(\xi, t)$$

Unpolarized beam, transverse target:

$$\Delta\sigma_{UT} \sim \cos\phi \sin(\phi_s - \phi) \{k(F_2 \mathbf{H} - F_1 \mathbf{E})\} d\phi$$



$$\mathbf{E}(\xi, t)$$

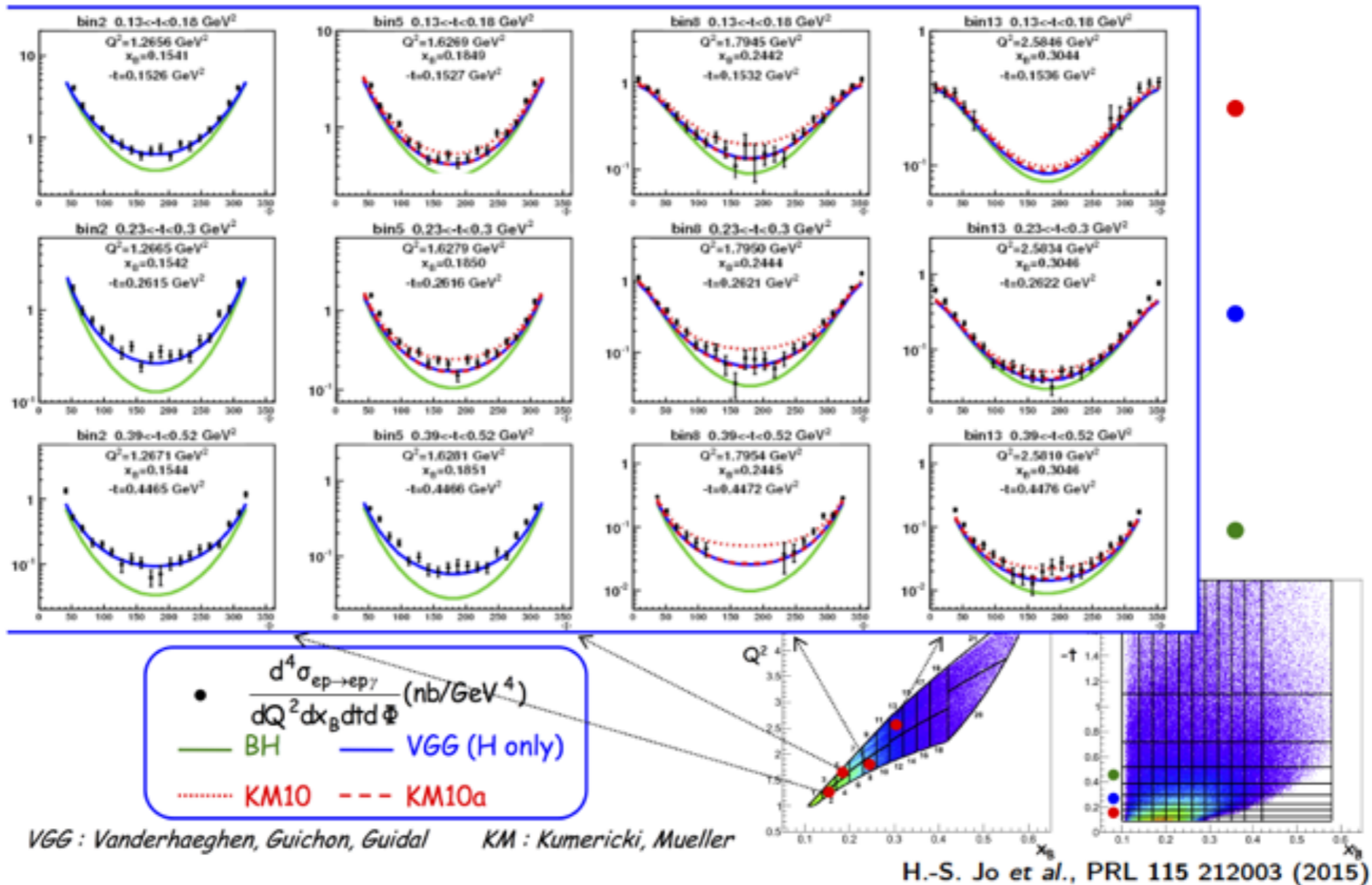
Unpolarized total cross section:

Separates h.t. contributions to DVCS



$$\text{Re}(T^{\text{DVCS}})$$

# DVCS unpolarized cross sections

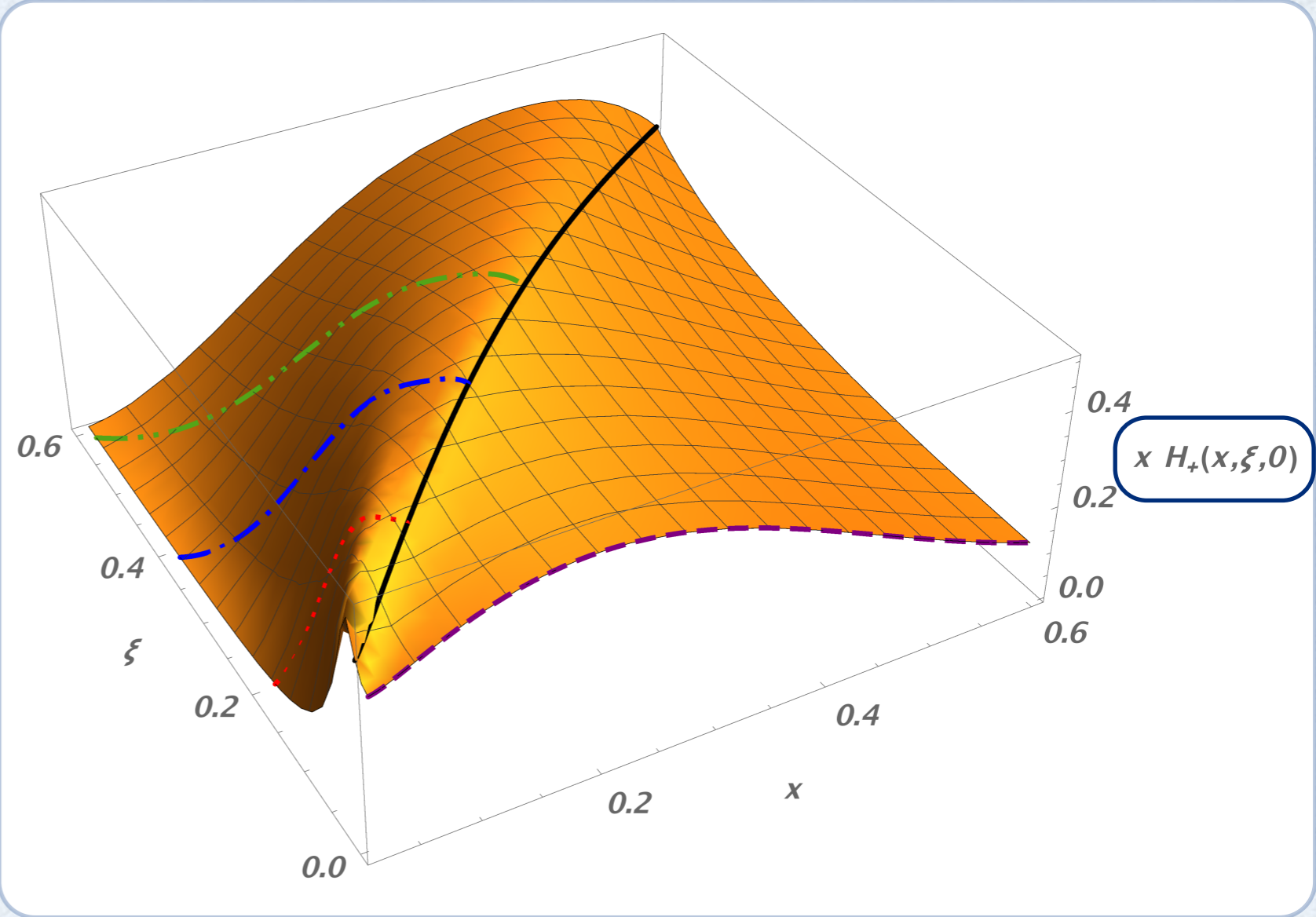


- ➔ CFF fit extractions from data:
- Guidal (2008, ...)
  - Guidal, Moutarde (2009, ...)
  - Kumericki, Mueller, Passek-Kumericki (2008, ...)
  - Goldstein, Hernandez, Liuti (2011, ...)
  - e.g. review: Kumericki, Liuti, Moutarde: EPJA52 (2016), no.6, 157

# GPD $H_+$

$$H_+(x, \xi, t) \equiv H(x, \xi, t) - H(-x, \xi, t)$$

- **DIS:**  $\xi=0$  limit  
momentum  
distribution  $x q_+(x)$
- **DVCS:** CFF  
 $\mathcal{H}_{Im}(x, 0) = H_+(x, x, 0)$   
accesses GPD for  $x = \xi$   
DD model with  $b_v = 1, b_v = 5$
- - - **DDVCS:**  $e p \rightarrow e p l^+ l^-$   
 $BSA \sim H_+(x, \xi, 0)$   
accesses GPD for  $x < \xi$

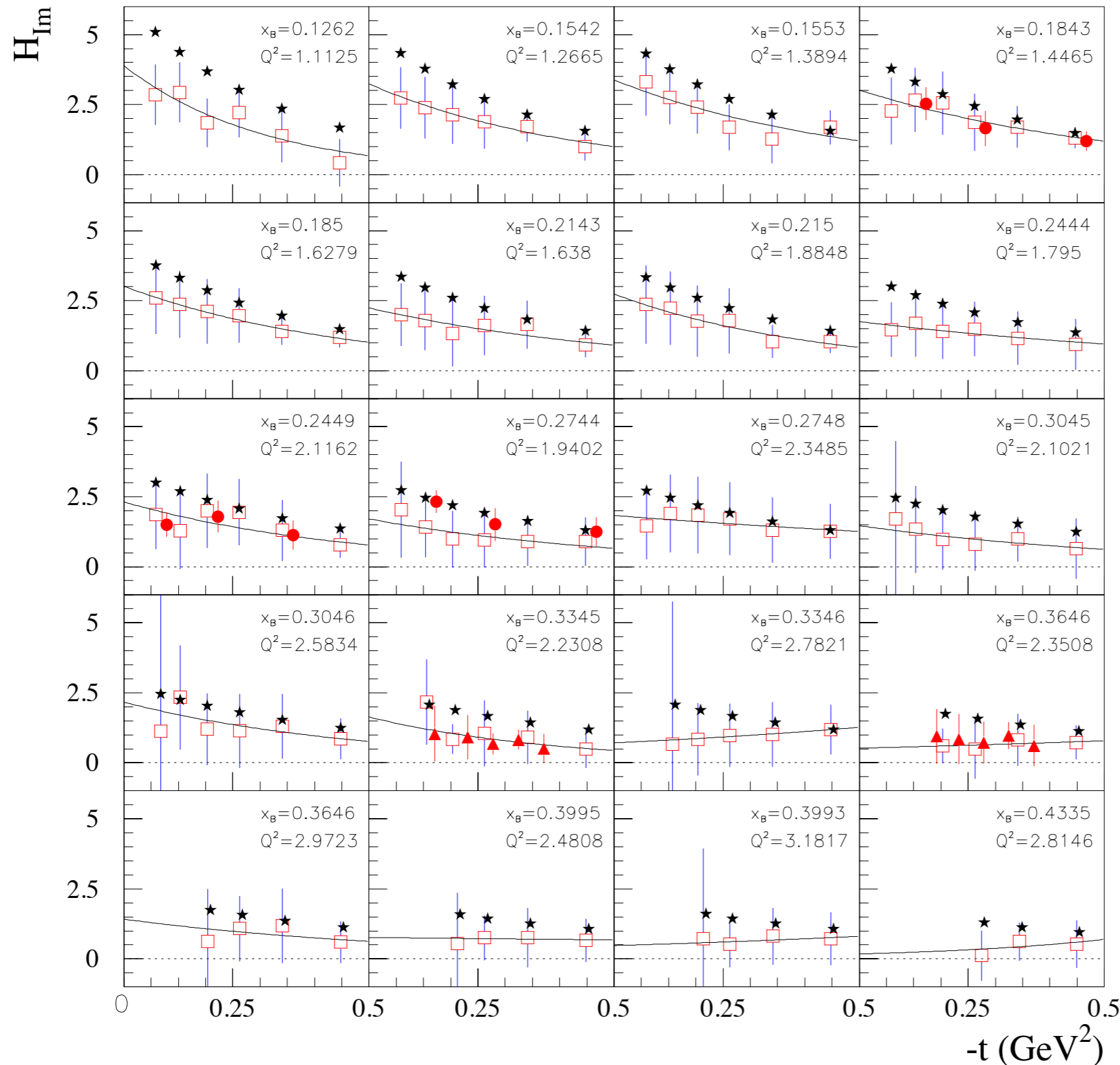


DVCS process accesses  
Compton Form Factors

$$\mathcal{H}_{Re}(\xi, t) \equiv \mathcal{P} \int_0^1 dx \left\{ \frac{1}{x - \xi} + \frac{1}{x + \xi} \right\} H_+(x, \xi, t)$$

$$\mathcal{H}_{Im}(\xi, t) \equiv H_+(\xi, \xi, t)$$

# global analysis of JLab 6 GeV data



$$\mathcal{H}_{Im}(\xi, t)$$

red solid circles:  
CLAS:  $\sigma, A_{LU}, A_{UL}, A_{LL}$

red open squares:  
CLAS:  $\sigma, A_{LU}$

red triangles:  
Hall A:  $\sigma, A_{LU}$

black stars  
VGG model values

Dupré, Guidal,  
Vdh (2017)

CFF  $\mathcal{H}_{Im}$ :

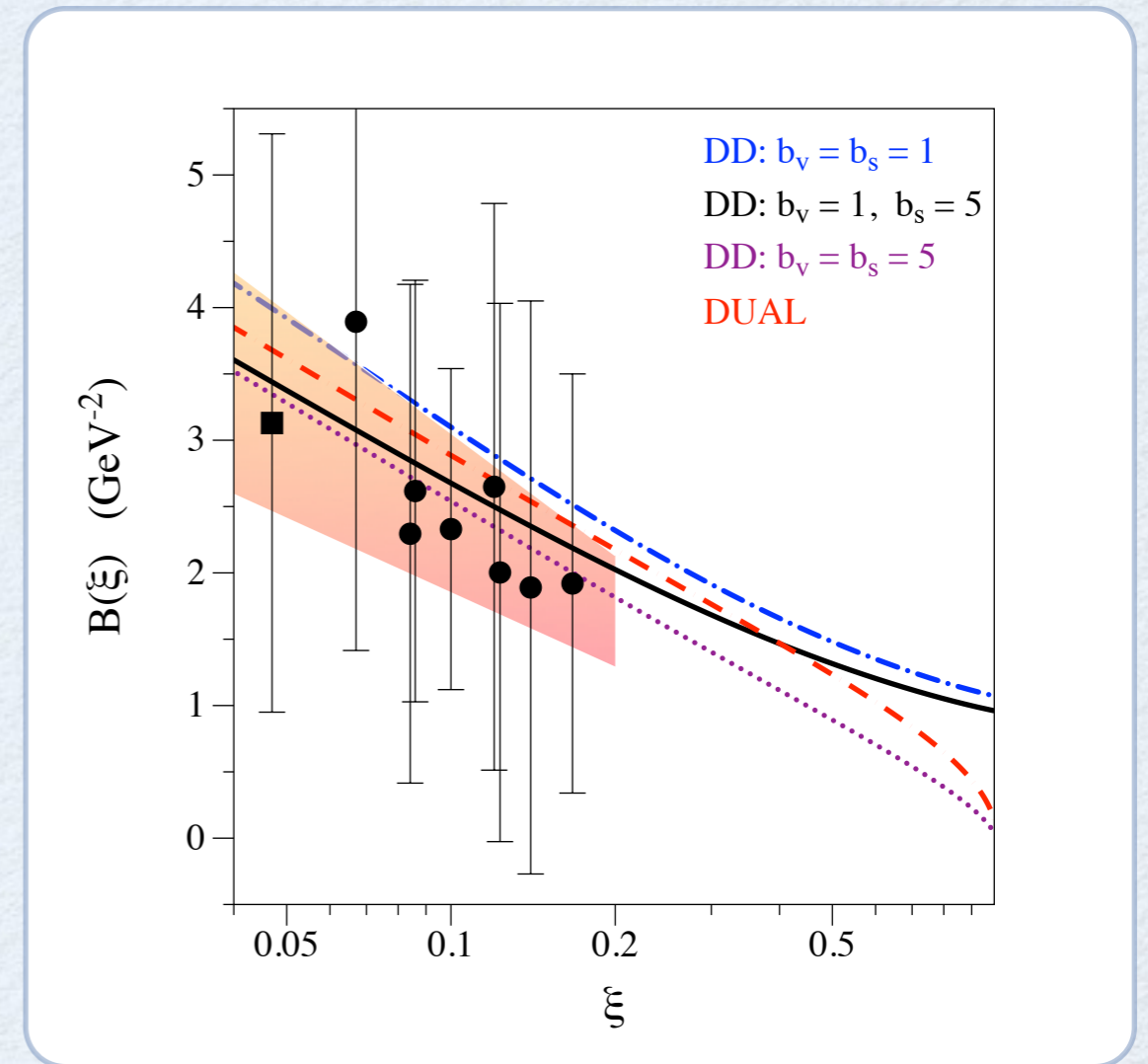
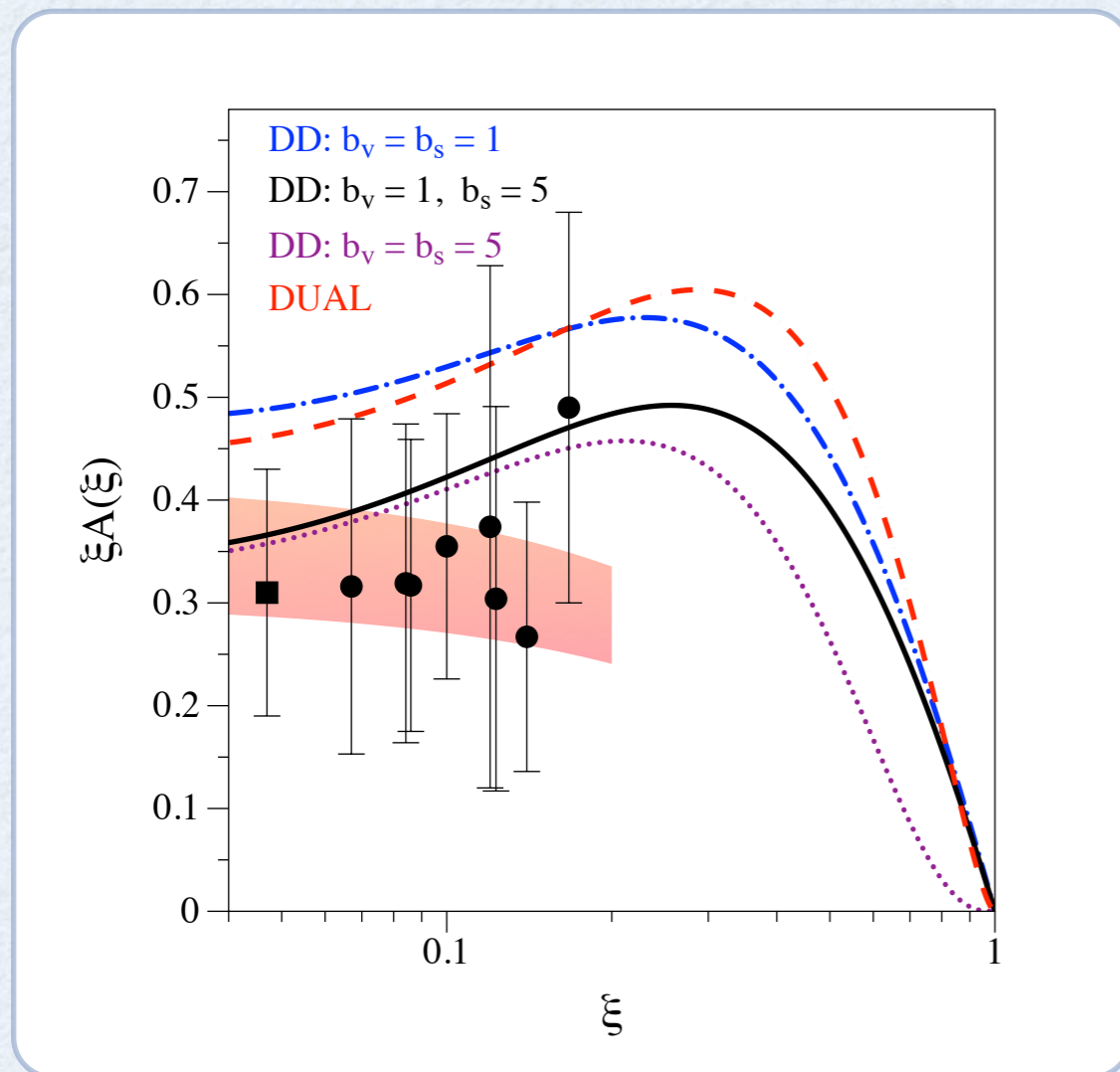
$$\mathcal{H}_{Im}(\xi, t) = A(\xi)e^{B(\xi)t}$$

black circles: CFF fit of JLab data

Dupré, Guidal, Vdh (2017)

black squares: CFF fit of HERMES data

Guidal, Moutarde (2009)



$$A(\xi) = a_A(1 - \xi)/\xi$$

red bands:  
1- parameter  
fits of data

$$B(\xi) = a_B \ln(1/\xi)$$

# 3D imaging



$$\rho^q(x, \mathbf{b}_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\mathbf{b}_\perp \cdot \Delta_\perp} H_-^q(x, \xi = 0, -\Delta_\perp^2)$$

Burkardt (2000)

number density of quarks (q) with longitudinal momentum x at a transverse distance  $\mathbf{b}_\perp$  in proton



non-singlet (valence quark) GPDs:  $H_-^q(x, 0, t) \equiv H^q(x, 0, t) + H^q(-x, 0, t)$



x-dependent radius

$$\langle b_\perp^2 \rangle^q(x) \equiv \frac{\int d^2 \mathbf{b}_\perp \mathbf{b}_\perp^2 \rho^q(x, \mathbf{b}_\perp)}{\int d^2 \mathbf{b}_\perp \rho^q(x, \mathbf{b}_\perp)} = -4 \frac{\partial}{\partial \Delta_\perp^2} \ln H_-^q(x, 0, -\Delta_\perp^2) \Big|_{\Delta_\perp=0}$$

$$H_-^q(x, 0, t) = q_v(x) e^{B_0(x)t} \longrightarrow \langle b_\perp^2 \rangle^q(x) = 4B_0(x)$$



x-independent radius

$$\langle b_\perp^2 \rangle^q = \frac{1}{N_q} \int_0^1 dx q_v(x) \langle b_\perp^2 \rangle^q(x)$$

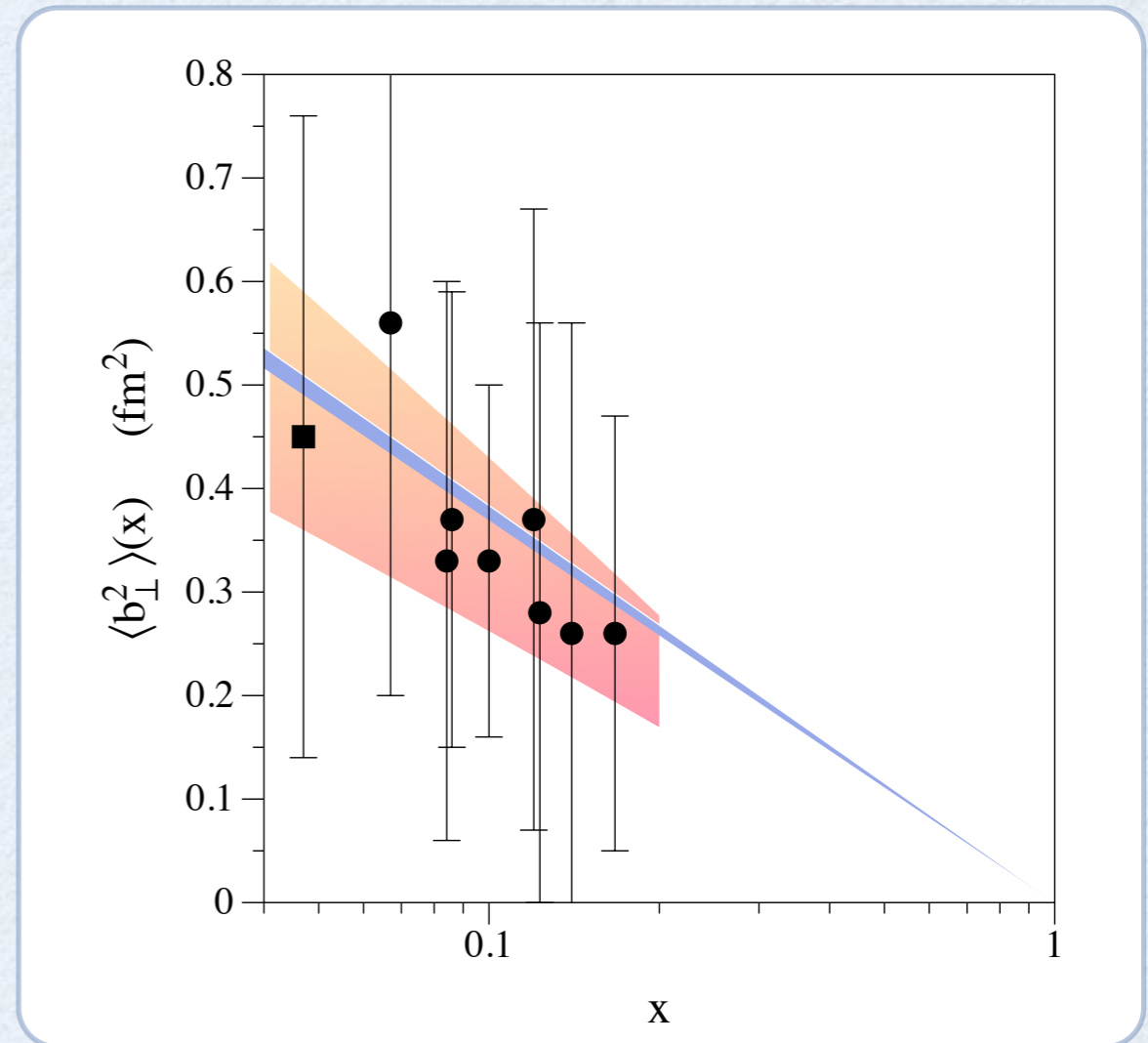
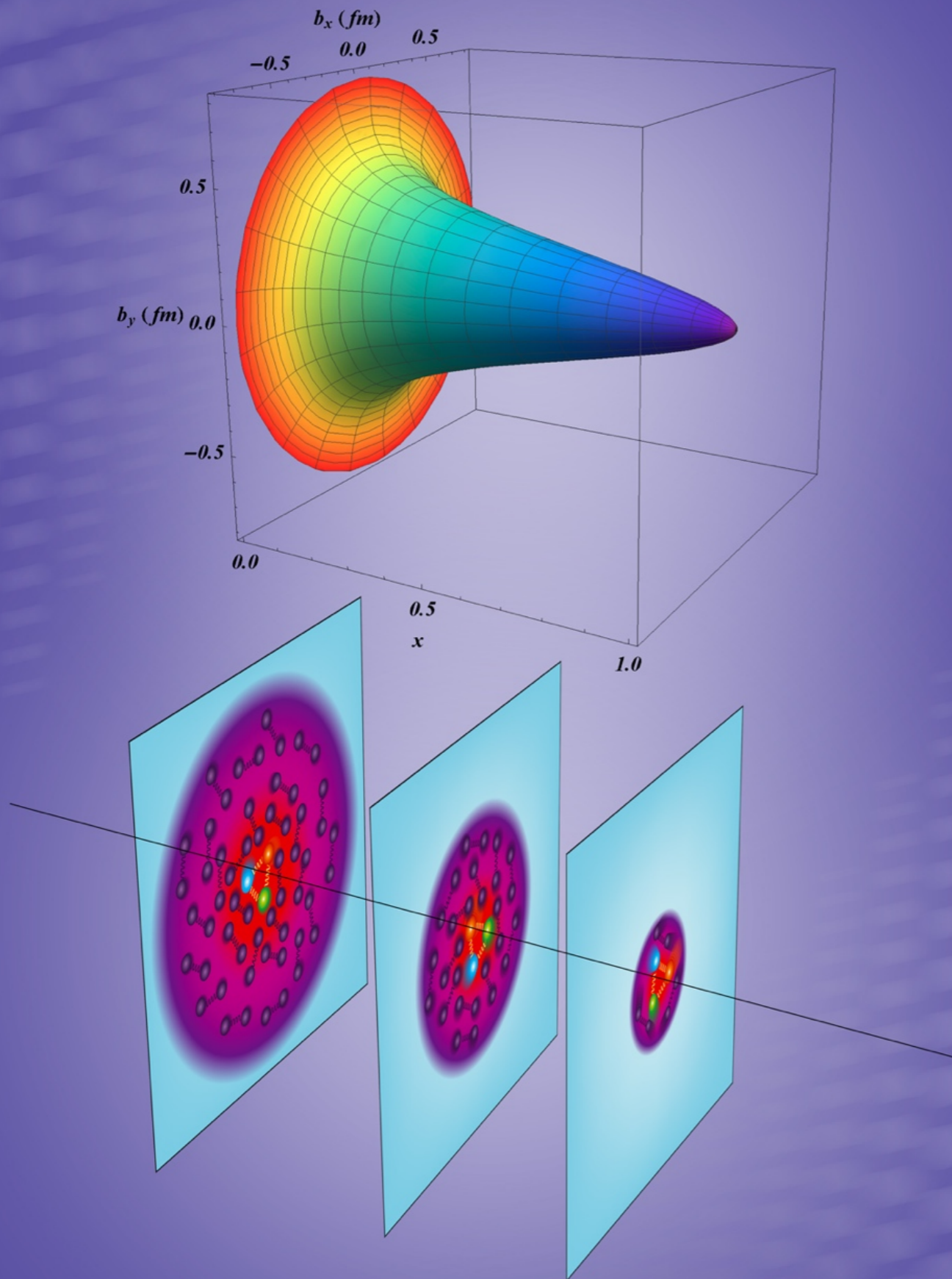
$N_u=2, N_d=1$

$$\langle b_\perp^2 \rangle = 2e_u \langle b_\perp^2 \rangle^u + e_d \langle b_\perp^2 \rangle^d = 2/3 \langle r_1^2 \rangle = 0.43 \pm 0.01 \text{ fm}^2$$

Bernauer (2014)

# 3D imaging of proton

black circles: CFF fit of JLab data



narrow band:  $B_0(x) = a_{B_0} \ln(1/x)$

$a_{B_0}$  fixed from elastic scattering

# CFF $\mathcal{H}_{Re}$ : dispersion relation formalism

Anikin, Teryaev (2007)

Diehl, Ivanov (2007)

Polyakov, Vdh (2008)

Kumericki-Passek, Mueller, Passek (2008)

Goldstein, Liuti (2009)

Guidal, Moutarde, Vdh (2013)

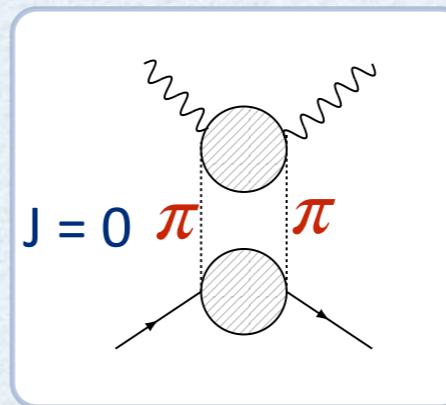
➔ once-subtracted fixed-t dispersion relation

$$\mathcal{H}_{Re}(\xi, t) = -\Delta(t) + \mathcal{P} \int_0^1 dx H_+(x, x, t) \left[ \frac{1}{x - \xi} + \frac{1}{x + \xi} \right]$$

$\xi$ -independent  
subtraction function

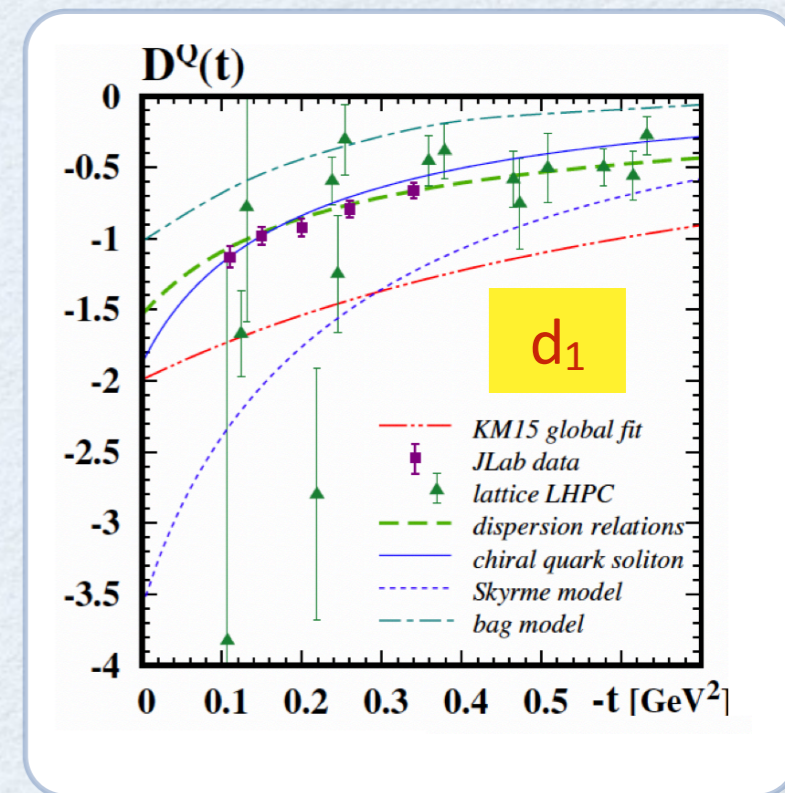
$$\Delta(t) \equiv \frac{2}{N_f} \int_{-1}^1 dz \frac{D(z, t)}{1 - z}$$

known from CFF  $\mathcal{H}_{Im}(x, t)$



dispersive  
estimate

Pasquini,  
Polyakov,  
Vdh (2014)



Burkert, Elouadrhiri,  
Girod (2018)

D-term  
Polyakov,  
Weiss  
(1999)

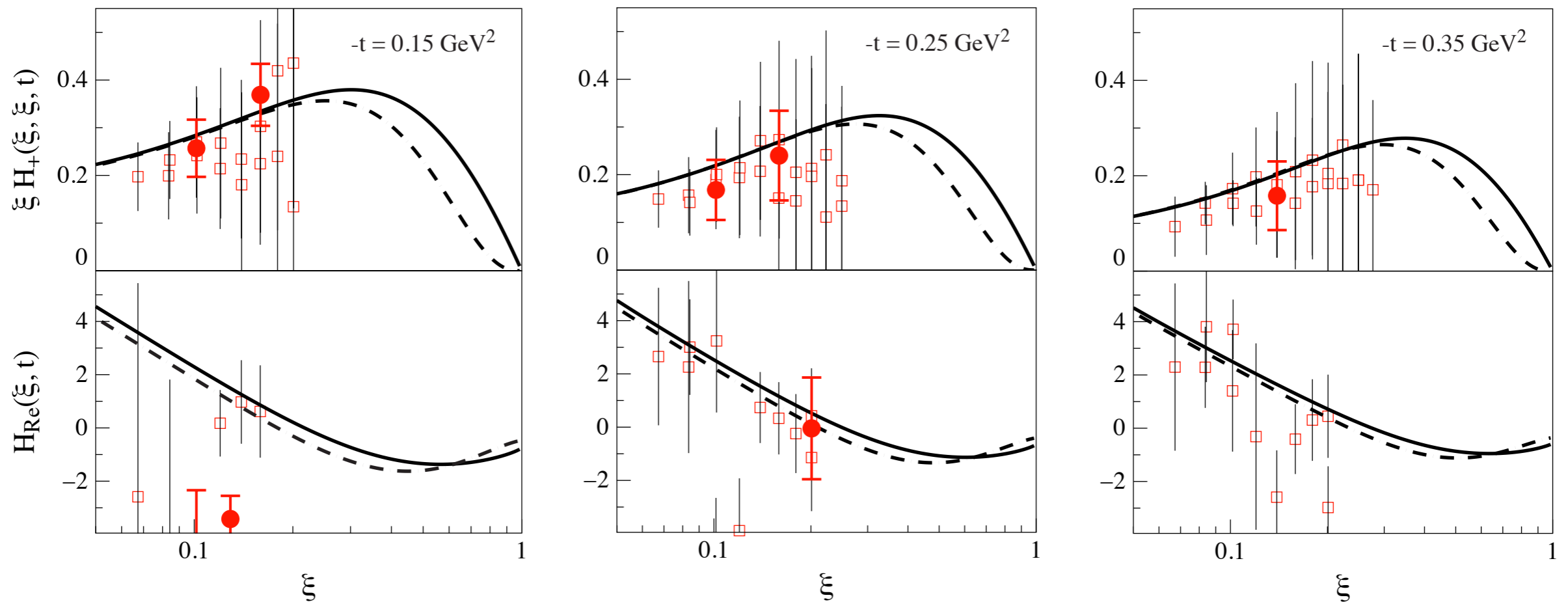
$$D(z, t) = (1 - z^2) \sum_{\substack{n=1 \\ n \text{ odd}}}^{\infty} d_n(t) C_n^{3/2}(z)$$



# experimental strategy for CFF $\mathcal{H}_{\text{Re}}$ : direct extraction vs dispersion formalism

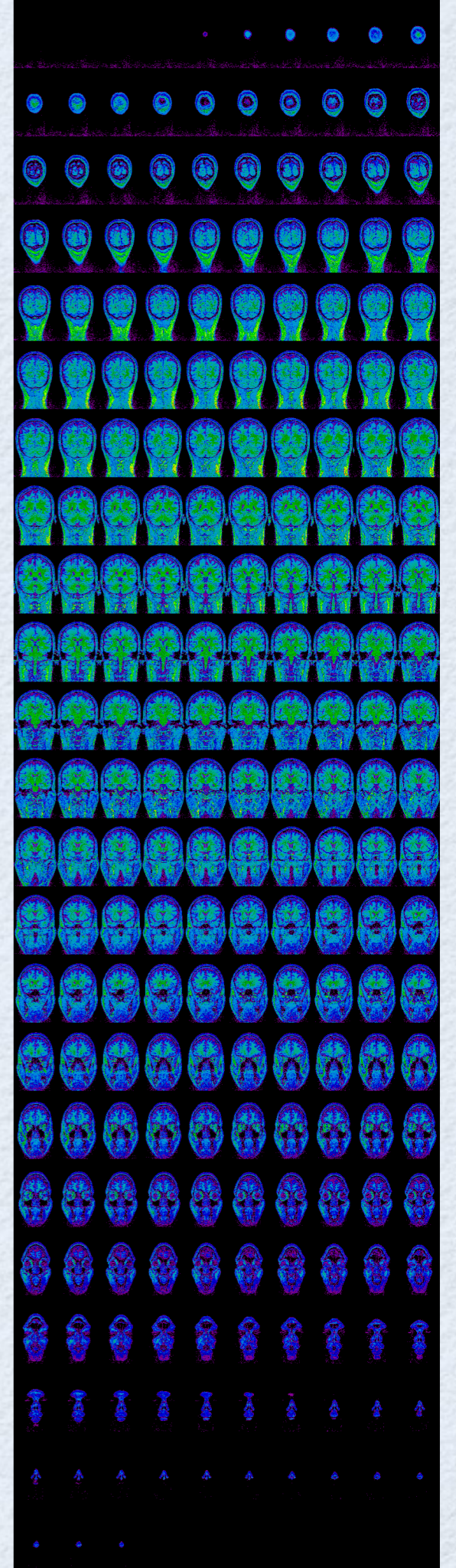
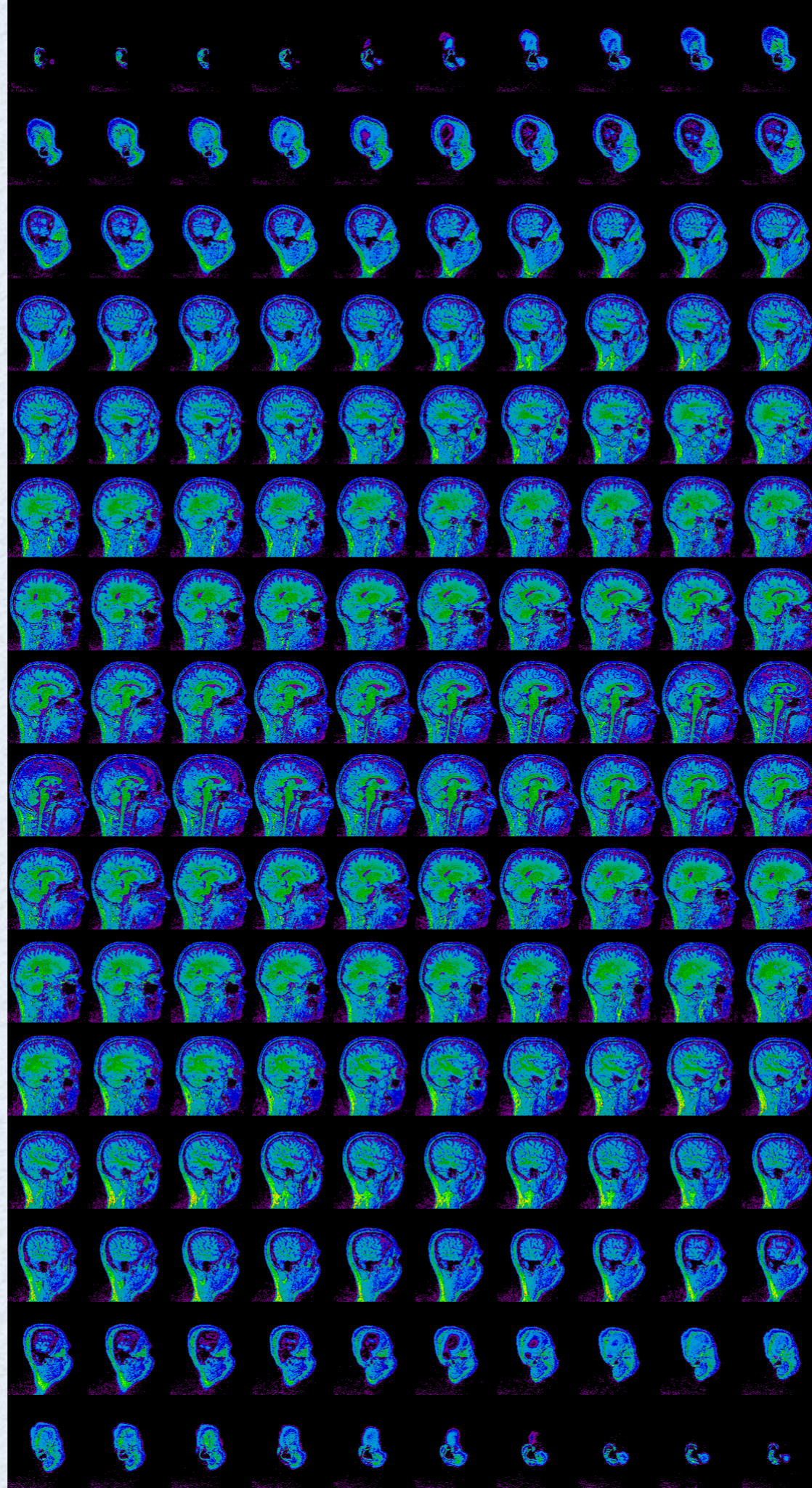
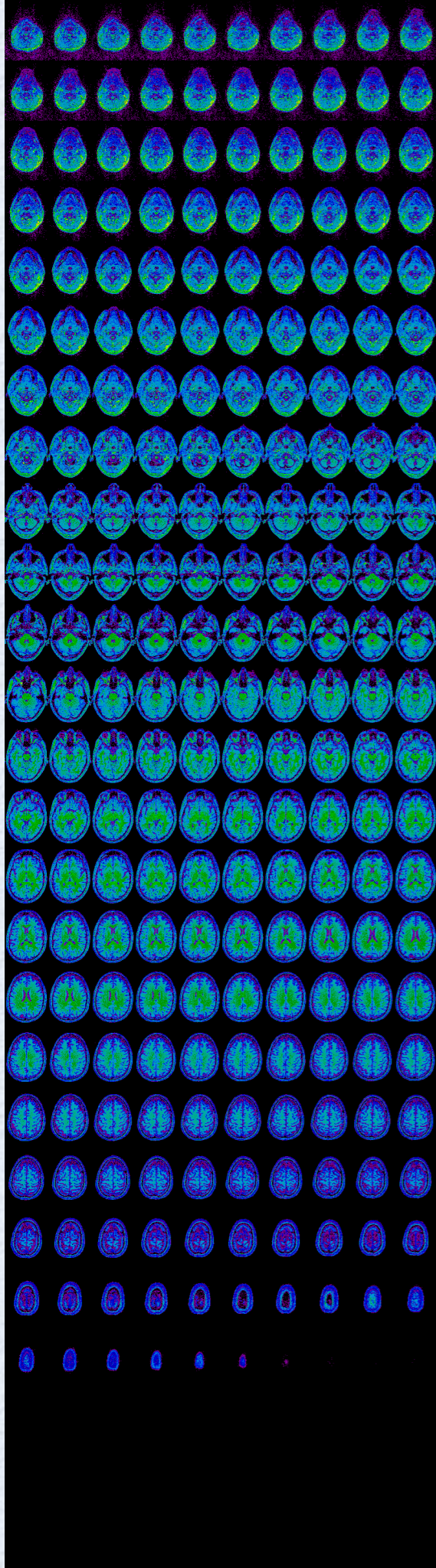
red solid circles: CLAS:  $\sigma$ ,  $A_{\text{LU}}$ ,  $A_{\text{UL}}$ ,  $A_{\text{LL}}$

red open squares: CLAS:  $\sigma$ ,  $A_{\text{LU}}$



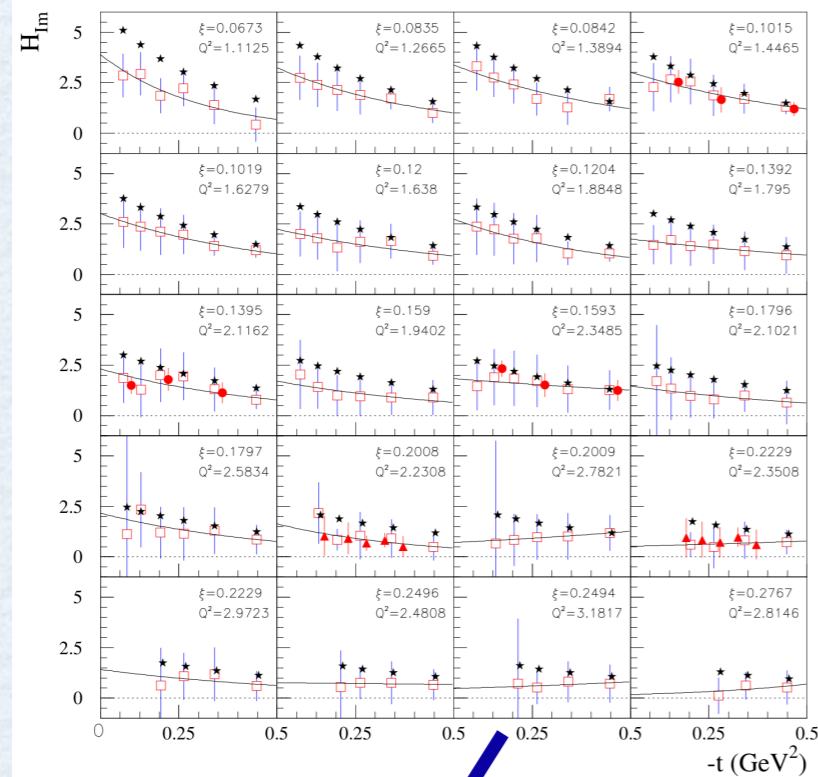
Curves for  $\Delta(t) = 0$ ;  $\Delta(t) < 0$  would shift  $H_{\text{Re}}$  curves up !

Dupré, Guidal, Niccolai, Vdh (2017)

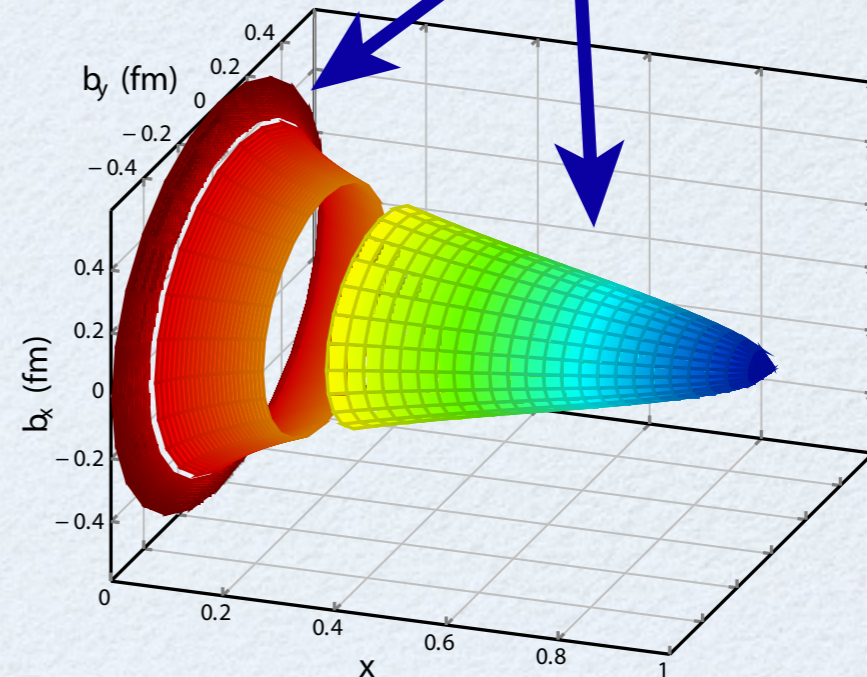
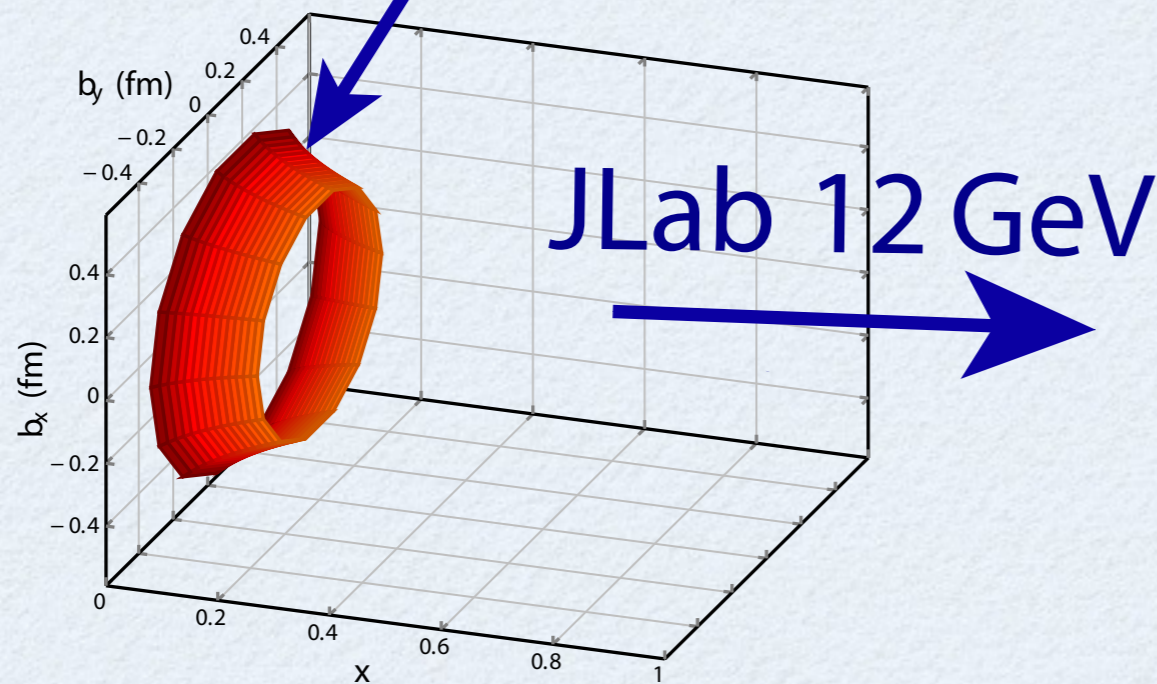
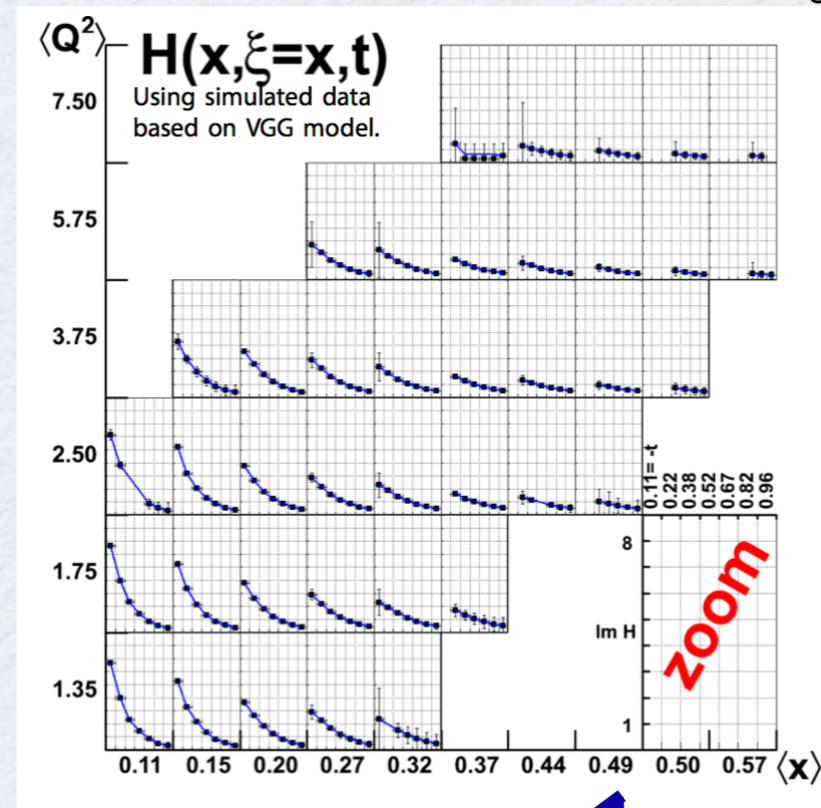


# Projections for CFFs at JLab 12 GeV

Düpré-Guidal-Vanderhaeghen-PRD **95** 011501 (R) (2017)



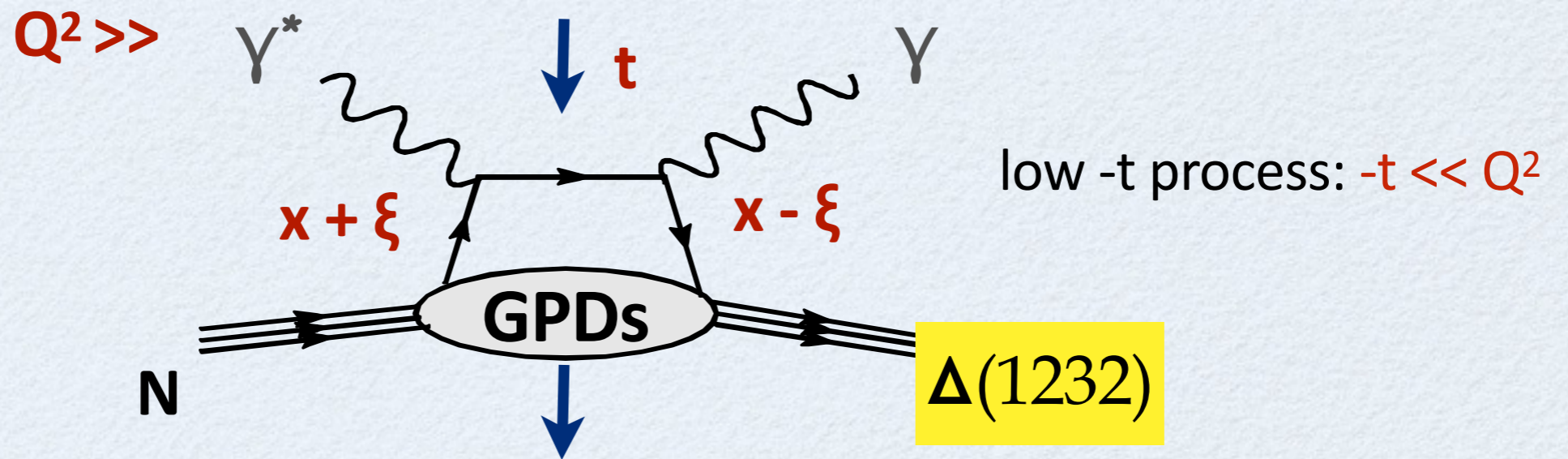
CLAS12 projections E12-06-119 with DVCS  $A_{UL}$  and  $A_{LU}$



JLab 12 GeV

courtesy of Z.E. Meziani

# $N \rightarrow \Delta(1232)$ DVCS and GPDs



8 twist-2 **GPDs**( $x, \xi, t$ ): 4 unpolarized, 4 polarized

➔ unpolarized GPDs:  $H_M, H_E, H_C, H_4$  Frankfurt, Polyakov, Strikman, Vdh (2000)

$$\int_{-1}^{+1} H_M(x, \xi, t) = 2G_M^*(t)$$

$$\int_{-1}^{+1} H_E(x, \xi, t) = 2G_E^*(t)$$

$$\int_{-1}^{+1} H_C(x, \xi, t) = 2G_C^*(t)$$

$$\int_{-1}^{+1} H_4(x, \xi, t) = 0$$

Jones-Scadron e.m. FFs for  $N \rightarrow \Delta$

Similar relations for polarized GPDs

# N $\rightarrow$ $\Delta(1232)$ magnetic dipole GPD

large  $N_c$  :

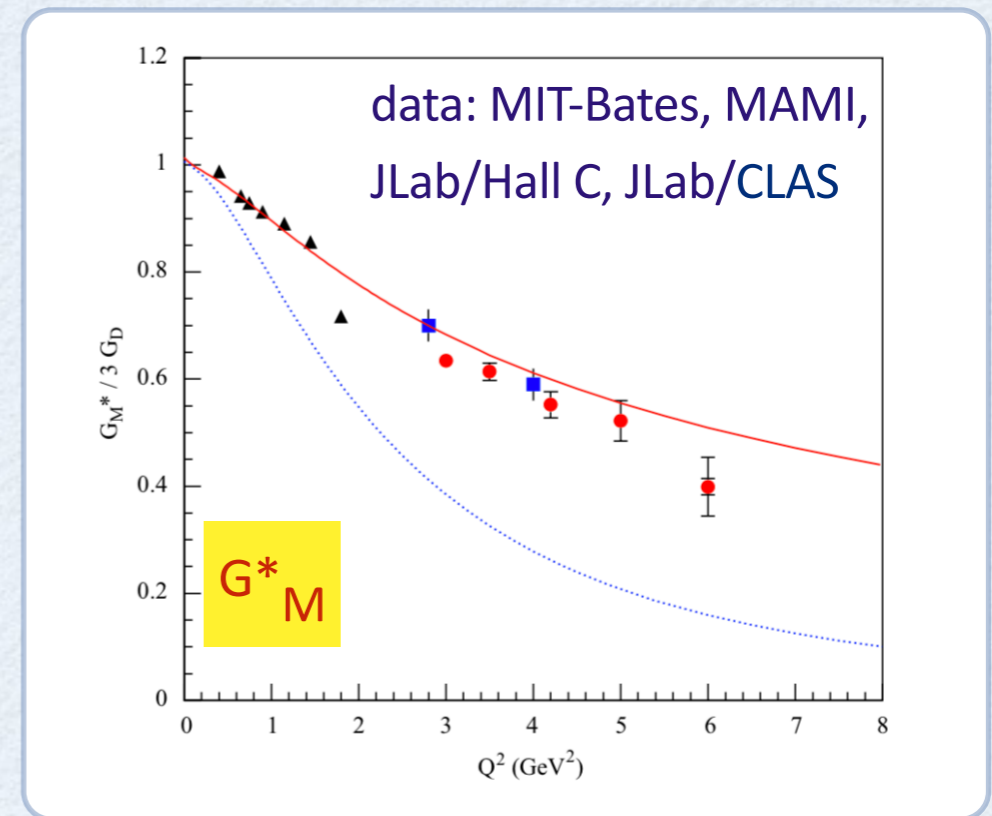
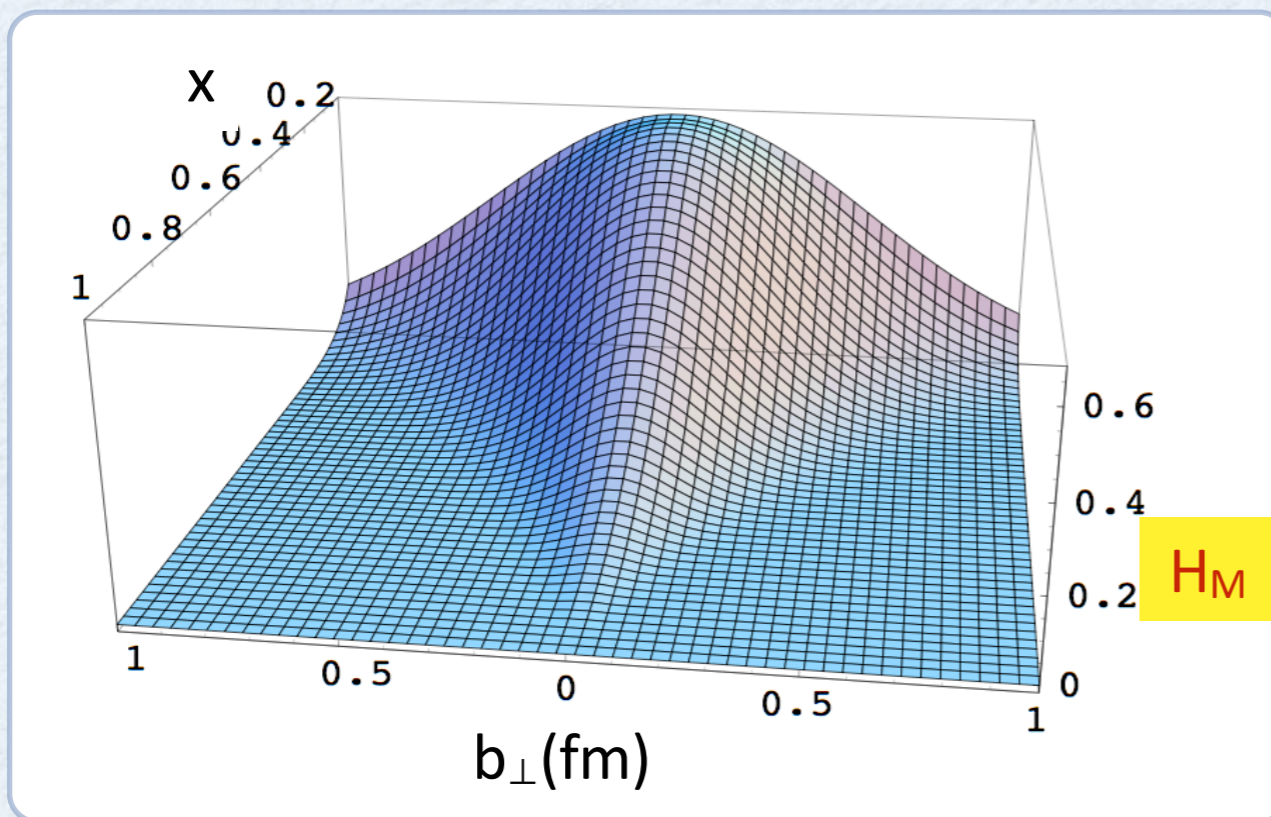
$$H_M(x, \xi, t) = 2 \frac{G_M^*(0)}{\kappa_V} \{E^u(x, \xi, t) - E^d(x, \xi, t)\}$$

Frankfurt, Polyakov,  
Strikman, Vdh (2000)

$$G_M^*(t) = \frac{G_M^*(0)}{\kappa_V} \int_{-1}^{+1} \{E^u(x, \xi, t) - E^d(x, \xi, t)\} = \frac{G_M^*(0)}{\kappa_V} \{F_2^p(t) - F_2^n(t)\}$$

large  $N_c$  :  $G_M^*(0) = \kappa_V / \sqrt{2} \simeq 2.62$   
exp :  $G_M^*(0) \simeq 3.02$

large  $N_c$  + nucleon Regge GPD model



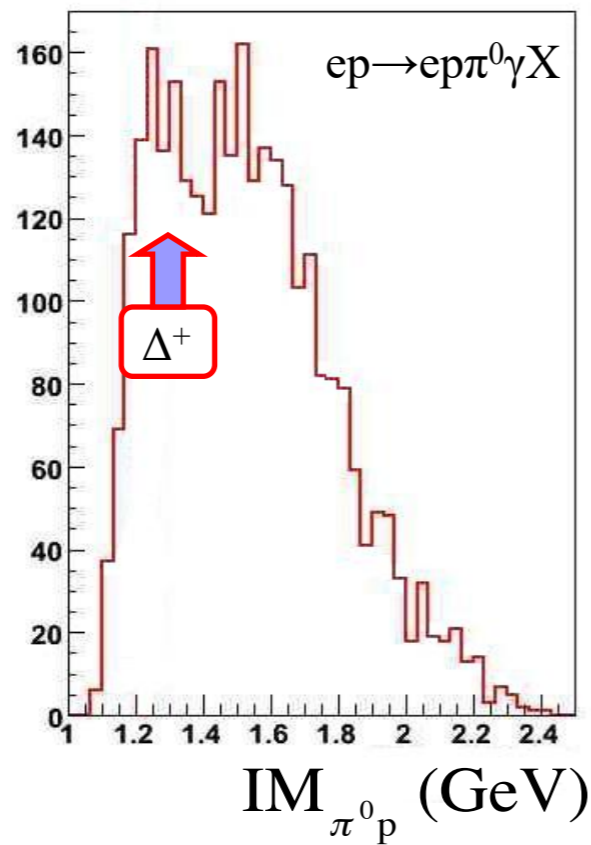
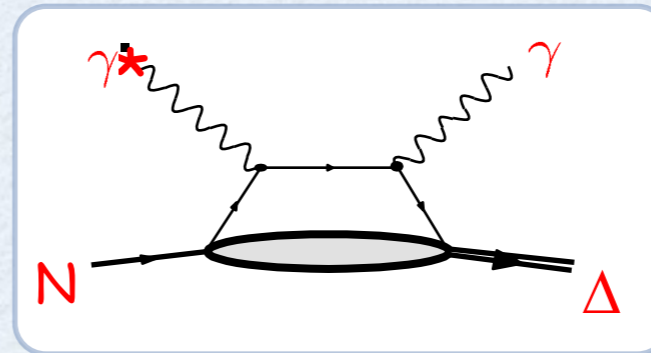
Guidal, Polyakov, Radyushkin, Vdh (2005)

# $N \rightarrow \Delta, N^*$ DVCS: experiment

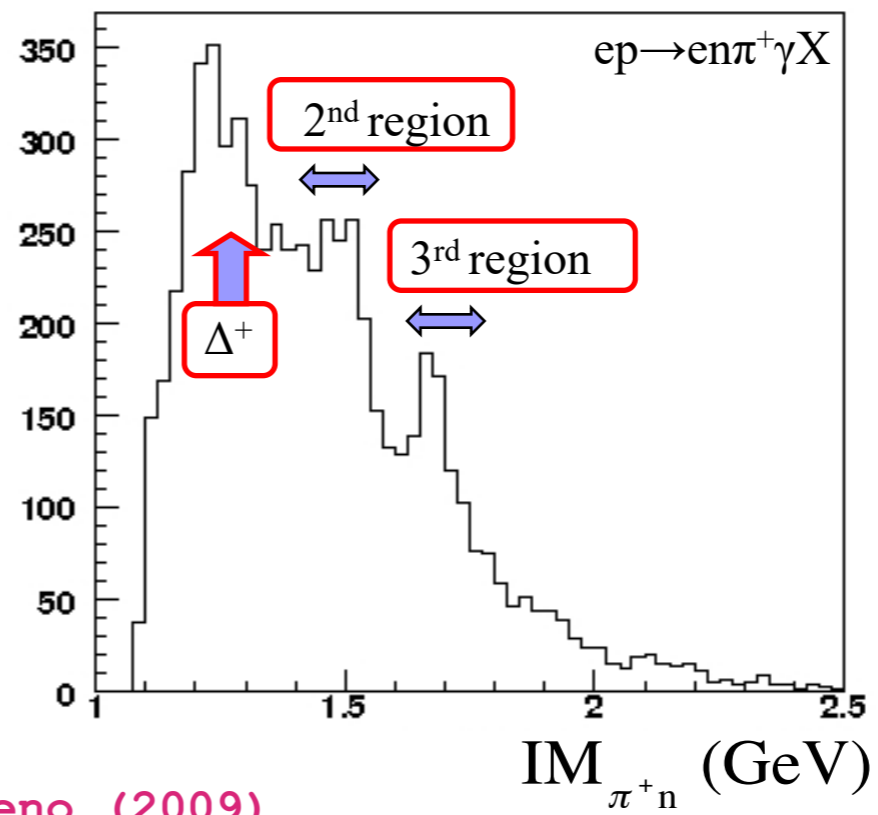
events seen in CLAS6

$W > 2 \text{ GeV}$

$Q^2 \approx 2.5 \text{ GeV}^2$



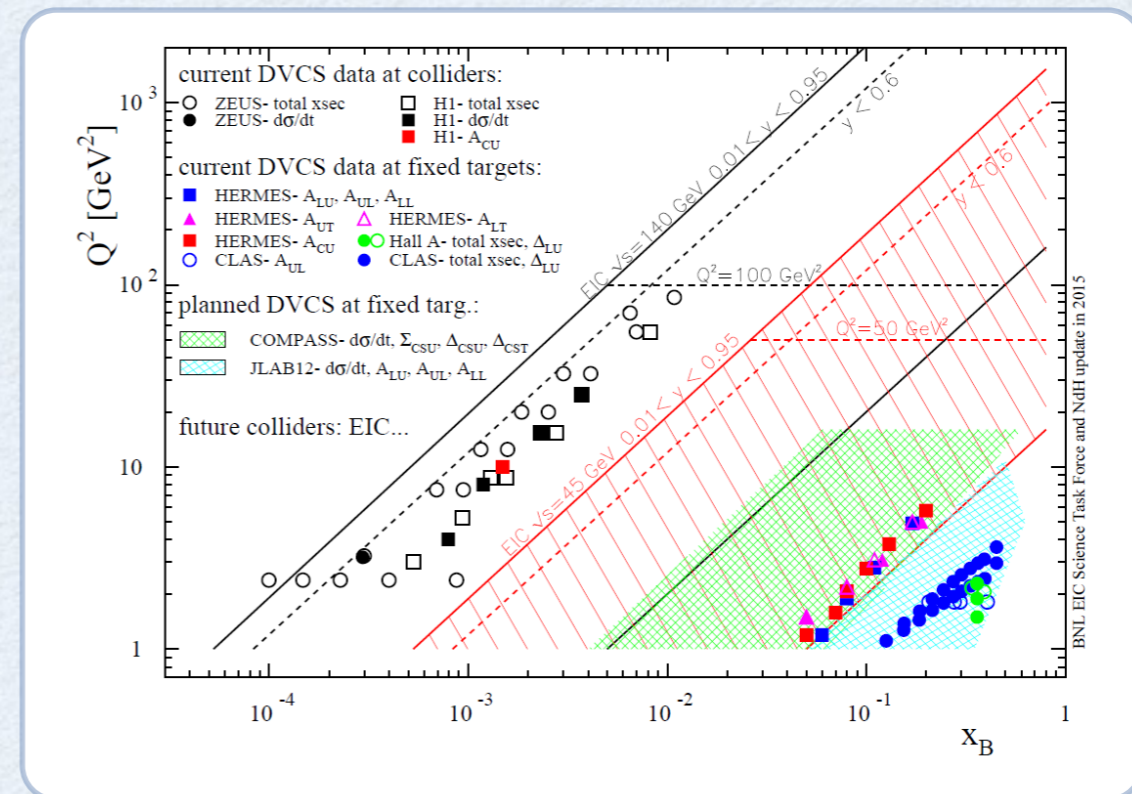
Moreno (2009)



unique opportunity for CLAS12

# Outlook

- ➔ elastic / transition FFs have allowed to get a first glimpse at the spatial distributions of quarks in nucleons
- ➔ GPDs allow for a proton imaging in longitudinal momentum and transverse position: established for nucleon, new opportunities on quark structure in nucleon resonance excitations
- ➔ global analysis of JLab 6 GeV data have shown a proof of principle of such 3D imaging (tools available: fitters, neural network, dispersive techniques)
- ➔ systematic 3D imaging is in reach: COMPASS, JLab 12 GeV,...EIC



**imaging and visualisation at the femtoscale just started**