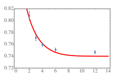

Novel Concepts for Evaluation of the Parton Distributions from QCD

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Workshop “Strong QCD from Hadron Structure Experiments”
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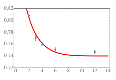
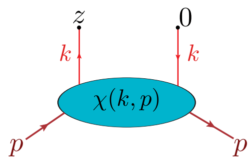


Parton Distributions

- Experimentally, we work with hadrons
- Theoretically, we works with quarks

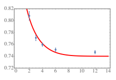
Hadron to Parton Transition



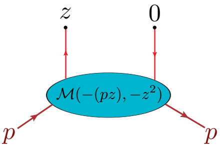
- Can be described in coordinate or momentum space

$$\langle p|\phi(0)\phi(z)|p\rangle = \frac{1}{\pi^2}\int d^4k\, e^{-ikz}\,\chi(k,p)$$

- Concept of PDFs does not rely on spin complications



Ioffe-time distributions



- Basic matrix element (ignoring spin)

$$\langle p|\phi(0)\phi(z)|p\rangle = \mathcal{M}(-(pz),-z^2)$$

- Lorentz invariance: $\mathcal{M}$ depends on z through $(pz) \equiv -\nu$ and z^2

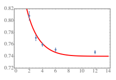
- Ioffe time ν: $\mathcal{M}(\nu,-z^2)$ = Ioffe-time pseudo-distribution (pseudo-ITD)
- Pseudo $\equiv$ off the light cone (LC), $z^2 \neq 0$
- On the light cone $z_+ = 0$: LC ITD $\mathcal{I}(\nu)$ (with $\nu = p_+ z_-$) and LC PDF $f(x)$

$$\mathcal{I}(\nu) = \int_{-1}^{1} dx\, e^{ix\nu}\, f(x) \quad , \quad f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\nu\, e^{-ix\nu}\, \mathcal{I}(\nu)$$

- Observation: ν-dependence governs x-dependence
- Using Schwinger's α-representation, it is possible to show that, for any contributing Feynman diagram, for arbitrary z^2 and arbitrary p^2

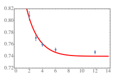
$$\mathcal{M}(\nu,-z^2) = \int_{-1}^{1} dx\, e^{ix\nu}\, \mathcal{P}(x,-z^2)$$

- Limits $-1 \leq x \leq 1$, negative x correspond to anti-particles
- Pseudo-PDF $\mathcal{P}(x,-z^2)$: Fourier transform of pseudo-ITD with respect to ν for fixed z^2



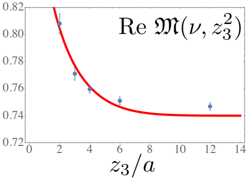
Pseudo-PDF strategy

- On the lattice: cannot take z on the light cone
  Need to take it off the light cone!
- Key observation: It does not matter if ν was obtained as $-(p_+ z_-)$ or as $p_3 z_3$: the function $\mathcal{M}(\nu, -z^2)$ is the same!
- Strategy: take $z = \{0, 0, 0, z_3\}$ (early attempts: Detmold & Lin (2006), Braun & Müller (2008); crucial attempt: X. Ji (2013))
- For $z = z_3$, we have $\nu = p_3 z_3$ and $-z^2 = z_3^2$
- Basic idea of the pseudo-PDF approach: map lattice data on $M(z_3, p)$ in terms of ν and z_3^2 and extrapolate $\mathcal{M}(\nu, z_3^2)$ to $z_3^2 = 0$



z_3^2-dependence

- Example of z_3-dependence for fixed ν

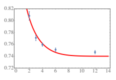


- Reduced pseudo-ITD
  $\mathfrak{M}(\nu, z_3^2) = \mathcal{M}(\nu, z_3^2)/\mathcal{M}(0, z_3^2)$
  for $u - d$ density
- Factorization:
  $\mathcal{M}(\nu, z_3^2) = Z_{\rm UV}(z_3^2)\mathcal{M}_{\rm ev}(\nu, z_3^2)$
  $\mathcal{M}(0, z_3^2) = Z_{\rm UV}(z_3^2)\mathcal{M}_{\rm ev}(0, z_3^2)$

- For vector current

$$\mathcal{M}_{\rm ev}(0, z_3^2) = \int_{-1}^{1} dx\, \mathcal{P}(x, -z^2) = 1 + \mathcal{O}(z_3^2)$$

- $\mathcal{M}(0, z_3^2)$ cancels (renormalizes) UV-divergent factor $Z_{\rm UV}(z_3^2)$ generated by gauge link
- Still $\mathfrak{M}(\nu, z_3^2)$ has perturbative evolution $\ln z_3^2$-dependence for small z_3^2
- Data from quenched calculation of $u_v - d_v$ by Orginos et al. (2017)

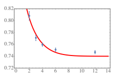


Renormalization

- Link-related UV divergences have the same structure as in HQET
- They are multiplicatively renormalizable
  (Qiu et al. , Ji et al. , Green et al. 2017)
- UV regulator a appears only in the combination z_3/a
- UV-sensitive terms form a factor $Z(z_3^2/a^2)$
- This factor is an artifact of having a non-lightlike z
- It has nothing to do with the lightcone PDFs
- We should build modified function $Z^{-1}(z_3^2/a^2)\mathcal{M}(\nu, z_3^2; a)$
- To do this, one should know the $Z(z_3^2/a^2)$ factor
- Easier way out: consider reduced pseudo-ITD

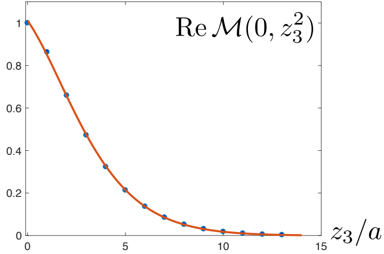
$$\mathfrak{M}(\nu, z_3^2) \equiv \frac{\mathcal{M}(\nu, z_3^2)}{\mathcal{M}(0, z_3^2)} = \lim_{a \to 0} \frac{\mathcal{M}(\nu, z_3^2; a)}{\mathcal{M}(0, z_3^2; a)}$$

- $Z(z_3^2/a^2)$ factors cancel, and $\mathfrak{M}(\nu, z_3^2)$ has finite $a \to 0$ limit

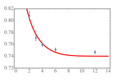


Rest-frame density and Z factor

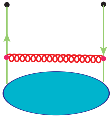
- Rest-frame density $\mathcal{M}(0, z_3^2)$ is produced by data at $P = 0$
- Advantage: Z-factor $\mathcal{M}(0, z_3^2)$ is calculated together with $\mathcal{M}(\nu, z_3^2)$, this reduces errors
- Results for imaginary part are compatible with zero, as required



- Visible linear component in $|z_3|$ for small and middle values of $|z_3|$
- Reflects linear exponential factor $Z(z_3^2) \sim e^{-c|z_3|/a}$



Logarithmic singularities and evolution



- Still, $\mathfrak{M}(\nu, z_3^2)$ has logarithmic singularity $\ln\left(z_3^2\right)$
- At one loop,

$$\mathfrak{M}^{\mathrm{hard}}(\nu, z_3^2) = -\frac{\alpha_s}{2\pi} C_F \ln\left(z_3^2\right) \int_0^1 du\, B(u)\, \mathfrak{M}^{\mathrm{soft}}(u\nu)$$

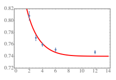
- Generates perturbative evolution with Altarelli-Parisi (AP) evolution kernel

$$B(u) = \left[\frac{1+u^2}{1-u}\right]_+$$

- Since $z^2 \to 0$ limit is singular, regularization (like $\overline{\mathrm{MS}}$) is needed for LC PDFs
- Thus, $\mathfrak{M}(\nu, z_3^2) \to \mathcal{I}(\nu, \mu^2) \equiv$ ITD for $\overline{\mathrm{MS}}$ PDF $f(x, \mu^2)$

$$\mathfrak{M}(\nu, 0)|_{\mu^2} \equiv \mathcal{I}(\nu, \mu^2) = \int_{-1}^1 dx\, e^{ix\nu}\, f(x, \mu^2)$$

- Implemented by “matching” between “z_3^2” and $\overline{\mathrm{MS}}$ schemes



Matching relations

- Basically, matching converts $\mathfrak{M}(\nu, z_3^2)$ into $\mathcal{I}(\nu, \mu^2)$, i.e. z_3^2-dependence of $\mathfrak{M}(\nu, z_3^2)$ into μ^2-dependence of $\mathcal{I}(\nu, \mu^2)$
- Matching condition between reduced pseudo-ITD and $\overline{\text{MS}}$ ITD (Y. Zhao 2017, A.R. 2017)

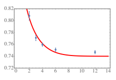
$$\begin{aligned}\mathfrak{M}(\nu, z_3^2) =& \mathcal{I}(\nu, \mu^2) - \frac{\alpha_s}{2\pi} C_F \int_0^1 dw\, \mathcal{I}(w\nu, \mu^2) \\ & \times \left\{ B(w) \left[\ln\left(z_3^2 \mu^2 \frac{e^{2\gamma_E}}{4} \right) + 1 \right] + \left[4 \frac{\ln(1-w)}{1-w} - 2(1-w) \right]_+ \right\}\end{aligned}$$

$$\mathfrak{M}(\nu, z_3^2) = \int_{-1}^{1} dx \left[e^{ix\nu} - \frac{\alpha_s}{2\pi} C_F R(x\nu, z_3^2\mu^2) \right] f(x, \mu^2)$$

- Direct connection between lattice $\mathfrak{M}(\nu, z_3^2)$ and LC PDF $f(x, \mu^2)$
- Real part of $R(\nu x, z_3^2 \mu^2)$ connects $\text{Re}\,\mathfrak{M}(\nu, z_3^2)$ with valence PDFs. Explicit form:

$$\begin{aligned}\text{Re}\, R(\nu x, z_3^2 \mu^2) =& \left\{ \frac{1-\cos(\nu x)}{\nu^2 x^2} - \frac{2\sin(\nu x)}{\nu x} + 2\, \sin(\nu x)\, \text{Si}(\nu x) \right. \\ & \left. + 2\, \cos(\nu x) \left(\text{Ci}(\nu x) - \log(\nu x) - \gamma_E + \frac{3}{4} \right) \right\} \ln\left(z_3^2 \mu^2 \frac{e^{2\gamma_E + 1}}{4} \right) \\ & + 2\text{Re} \left[i\nu x e^{i\nu x}\, {}_3F_3(1,1,1;2,2,2;-i\nu x) \right] \\ & + \cos(\nu x) - 2 \frac{1-\cos(\nu x)}{\nu^2 x^2}\end{aligned}$$

- No intermediaries, like pseudo-PDFs or quasi-PDFs, are needed
- Work in progress on getting kernels for RI/MOM renormalization used in quasi-PDF applications



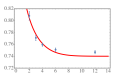
Pseudo-PDF strategy in action

- Exploratory lattice study of reduced pseudo-ITD $\mathfrak{M}(\nu, z_3^2)$ for the valence $u_v - d_v$ parton distribution in the nucleon [Orginos et al. 2017]
- Lattice QCD calculations in quenched approximation
- $32^3 \times 64$ lattices, lattice spacing $a = 0.093$ fm
- Pion mass $601(1)$ MeV and nucleon mass $1411(4)$MeV
- Six lattice momenta $p_i\ (2\pi/L)$, with 2.5 GeV maximal momentum
- Real part of lightcone ITD $\mathcal{I}(\nu)$ corresponds to cosine Fourier transform of $q_v(x) = u_v(x) - d_v(x)$

$$\mathcal{R}(\nu) \equiv \mathrm{Re}\,\mathcal{I}(\nu) = \int_0^1 dx\ \cos(\nu x)\, q_v(x)$$

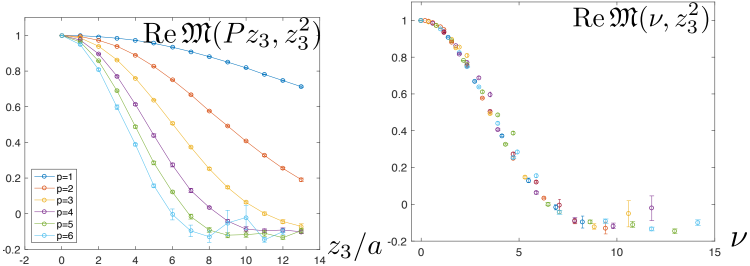
- On the lattice, we extract the reduced pseudo-ITD

$$\mathfrak{M}(\nu, z_3^2) \equiv \frac{\mathcal{M}(\nu, z_3^2)}{\mathcal{M}(0, z_3^2)}$$

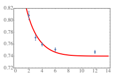


Reduced Ioffe-time distributions

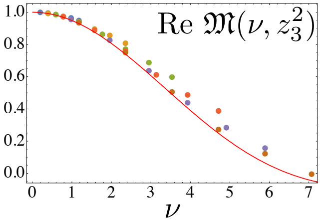
- Left: Real part of the ratio $\mathcal{M}(Pz_3, z_3^2)/\mathcal{M}(0, z_3^2)$ as a function of z_3
- Taken at six values of P $\Rightarrow$ curves have Gaussian-like shape
- $\Rightarrow$ $Z(z_3^2)$ link factor cancels in the ratio

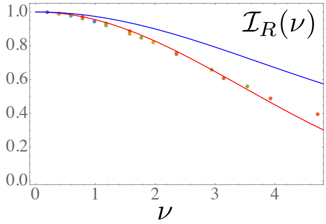


- Right: Same data, as functions of $\nu = Pz_3$ (z_3^2 varies from point to poiint)
- Data practically fall on the same universal curve

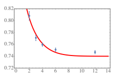


Building $\overline{\mathrm{MS}}$ ITD





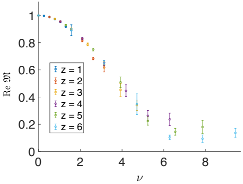
- Points in $a \leq z_3 \leq 6a$ region
- All points lie higher than curve based on fit to all $a \leq z_3 \leq 14a$ data
- Higher values of $\mathfrak{R}$ for smaller-z_3 points are a consequence of evolution
- $\mu = 1/a$ at lattice spacing of 0.093 fm is $\approx$ 2.15 GeV
- Using $\alpha_s/\pi = 0.1$ and $z_3 \leq 4a$ data, generate the points for $\mathcal{I}_R(\nu, (1/a)^2)$
- Evolved points have a rather small scatter
- The curve corresponds to the cosine transform of a normalized $\sim x^a(1-x)^b$ distribution with $a = 0.35$ and $b = 3$
- Upper curve: ITD of the CJ15 global fit PDF for $\mu =$2.15 GeV



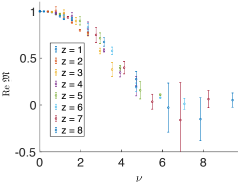
Dynamic fermions (Joo et al., arXiv:1908.09771)

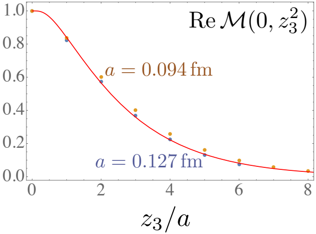
Reduced ITD for two lattice spacings

- $a = 0.127$ fm

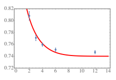


- $a = 0.094$ fm



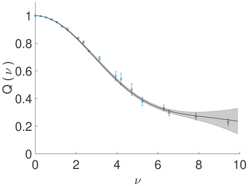


- Z-factor Re $\mathcal{M}(0, z_3^2)$ for two lattice spacings
- Essentially universal function of z/a
- Curve is given by perturbative formula for the link $Z(z/a)$ factor with $\alpha_s = 0.26$

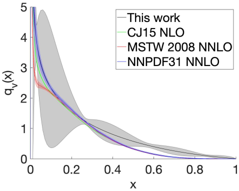


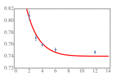
PDF from dynamic fermions

- Light-cone ITD for $\mu = 2$ GeV extracted from $a = 0.127$ fm data

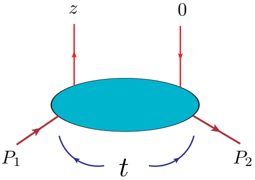


- PDF compared to global fits





GPDs in pseudo-PDF approach



- GPD for pion (e.g.) (X. Ji, 1996)

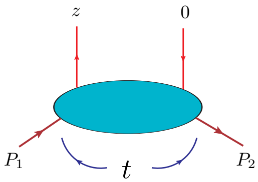
$$\langle p_2|\bar{\psi}(0)\ldots\psi(z)|p_1\rangle = 2\mathcal{P}^{\alpha} e^{-i(p_1 z)/2+i(p_2 z)/2} \int_{-1}^{1} dx\, e^{-ix(\mathcal{P}z)}\, H(x,\xi,t;\mu^2)$$

- Average momentum $\mathcal{P} = (p_1 + p_2)/2$, momentum transfer $t = (p_1 - p_2)^2$
- Two Ioffe times $\nu_1 = -(p_1 z)$ and $\nu_2 = -(p_2 z)$
- On the lattice, $z = z_3$ and $p_1 = \{E_1, \Delta_{1,\perp}, P_1\}$, $p_2 = \{E_2, \Delta_{2,\perp}, P_2\}$
- Skewness

$$\xi = \frac{(p_1 z_3) - (p_2 z_3)}{(p_1 z_3) + (p_2 z_3)} = \frac{P_1 - P_2}{P_1 + P_2} = \frac{\nu_1 - \nu_2}{\nu_1 + \nu_2}$$

- $P_1 = (1+\xi)P$ and $P_2 = (1-\xi)P$

Lattice implementation for pseudo-GPDs

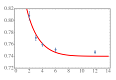


- Discrete set of coordinates $z_3 = n_z a$ and longitudinal momenta $P_1 = 2\pi N_1/L$, $P_2 = 2\pi N_2/L$
- Possible values for skewness are given by a set of rational numbers

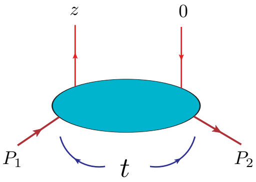
$$\xi = \frac{P_1 - P_2}{P_1 + P_2} = \frac{N_1 - N_2}{N_1 + N_2}$$

- Changing N_1 and N_2 from 0 to 6, gives 13 possible values for ξ ranging from 0 to 1 and rather well representing the whole $0 \leq \xi \leq 1$ segment
- Varying ξ also changes t. For purely longitudinal momenta

$$t_0 = -\frac{8\xi^2 M^2}{1 - \xi^2 + \frac{M^2}{P^2} + \sqrt{(1 - \xi^2 + \frac{M^2}{P^2})^2 + 4\xi^2 \frac{M^2}{P^2}}}$$

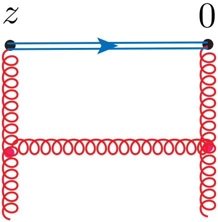


Mapping pseudo-GPDs



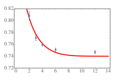
- Choose particular values of P_1 and P_2 $\Longrightarrow$ fixes value of ξ and, for chosen z_3, also $\nu = (\nu_1 + \nu_2)/2$
- Add transverse component $\Delta_\perp$: $p_1 = \{E_1, 0_\perp, P_1\}$ and $p_2 = \{E_2, \Delta_\perp, P_2\}$
- Take several values of $\Delta_\perp$ to change t $\Longrightarrow$ gives t-dependence for fixed ξ, ν
- Changing z_3, we will change ν leaving ξ and t unchanged
- Using matching conditions, convert ν-dependence into x-dependence
- End up with $H(x, \xi, t; \mu^2)$ for a fixed ξ as a function of x and t
- Huge number of points: $\sim 6 \times 6 \times 6$ for $P_1 \times P_2 \times \Delta_\perp$ and $\geq \times 6$ for z_3
- But, if successful, gives much more detailed information about GPDs than experiments
- Keeping ν_1 and ν_2 vs. ν and ξ $\Longrightarrow$ double distributions

Gluons in Ioffe-time approach



- Gluons present challenges:
  1) to get lattice signal
  2) to get matching relations
  3) to extract PDFs, etc.
- Strict requirement on gauge invariance

- Use Balitsky-Braun method to calculate in coordinate representation in operator form (Balitsky, Morris, A.R., arXiv:1910:13963)
- Start with operator $\mathcal{O}_{\mu\alpha;\lambda\beta}(z) = G_{\mu\alpha}(z)\,\widetilde{W}(z,0;A)G_{\lambda\beta}(0)$
- Get operator $\delta\mathcal{O}_{\mu\alpha;\lambda\beta}(z)$ resulting from modification by gluon exchanges



Structure of gluon matrix elements

- 6 invariant amplitudes in forward case

$$
\begin{aligned}
M_{\mu\alpha;\lambda\beta}(z,p) &= \langle p|\mathcal{O}_{\mu\alpha;\lambda\beta}(z)|p\rangle \\
&= \left(g_{\mu\lambda}p_\alpha p_\beta - \ldots\right)\mathcal{M}_{pp} \\
&+ \left(g_{\mu\lambda}z_\alpha z_\beta - \ldots\right)\mathcal{M}_{zz} \\
&+ \left(g_{\mu\lambda}z_\alpha p_\beta - \ldots\right)\mathcal{M}_{zp} \\
&+ \left(g_{\mu\lambda}p_\alpha z_\beta - \ldots\right)\mathcal{M}_{pz} \\
&+ \left(p_\mu z_\alpha - \ldots\right)\mathcal{M}_{ppzz} \\
&+ \left(g_{\mu\lambda}g_{\alpha\beta} - g_{\mu\beta}g_{\alpha\lambda}\right)\mathcal{M}_{gg}
\end{aligned}
$$

- To define LC PDF, take $z = z_-$

$$g^{\alpha\beta}M_{+\alpha;+\beta}(z_-,p) = 2p_+^2\mathcal{M}_{pp}(\nu,0)$$

- Only transverse indices work

$$
\begin{aligned}
M_{+i;+i} = &M_{0i;0i} + M_{3i;3i} \\
&+ \left(M_{0i;3i} + M_{3i;0i}\right)
\end{aligned}
$$

- Each term contains $\mathcal{M}_{pp}$

$$
\begin{aligned}
M_{0i;i0} &= 2p_0^2\mathcal{M}_{pp} + 2\mathcal{M}_{gg} \\
M_{3i;i3} &= 2p_3^2\mathcal{M}_{pp} + 2z_3^2\mathcal{M}_{zz} \\
&+ 2z_3p_3\left(\mathcal{M}_{zp} + \mathcal{M}_{pz}\right) - 2\mathcal{M}_{gg} \\
M_{0i;i3} &= 2p_0\left(p_3\mathcal{M}_{pp} + z_3\mathcal{M}_{pz}\right) \\
M_{3i;i0} &= 2p_0\left(p_3\mathcal{M}_{pp} + z_3\mathcal{M}_{zp}\right)
\end{aligned}
$$

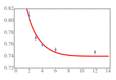
- ... with contaminations. But

$$M_{ji;ij} \equiv \langle pG_{ji}(z)G_{ij}(0)\rangle p = -2\mathcal{M}_{gg}$$

- “Clean” structure

$$M_{0i;i0} + M_{ji;ij} = 2p_0^2\mathcal{M}_{pp}$$

- Good news: combination is UV multiplicatively renormalizable



Novel Concepts to Extract PDFs

Parton Densities
Ioffe-time distributions
Pseudo-PDF strategy
Renormalization
Rest-frame density
Evolution
Matching
Lattice & pPDFs
Building $\overline{\mathrm{MS}}$ ITD

Outlook
GPDs
Gluons

Matching relation

- After lengthy calculations, we obtained matching relation

$$\mathfrak{M}(\nu, z_3^2) = \frac{\mathcal{I}(\nu,\mu^2)}{\mathcal{I}(0,\mu^2)}$$
$$- \frac{\alpha_s N_c}{\pi}\int_0^1 \mathrm{d}u \, \frac{\mathcal{I}(u\nu,\mu^2)}{\mathcal{I}(0,\mu^2)} \left\{ \left[\frac{(1-u(1-u))^2}{1-u}\right]_+ \left[\ln\left(z_3^2\mu^2 e^{2\gamma_E}/4\right)+2\right] \right.$$
$$\left. + 2\left[\frac{\log(1-u)}{1-u}\right]_+ + \frac{1}{3}\left[1-6u-u^3\right]_+ \right\}$$

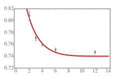
  between the "lattice function" $\mathfrak{M}(\nu, z_3^2)$ and the light-cone ITD $\mathcal{I}(\nu,\mu^2)$
- $\mathcal{I}(\nu,\mu^2)$ is connected to the gluon PDF $f_g(x,\mu^2)$ by

$$\mathcal{I}(\nu,\mu^2) = \frac{1}{2}\int_{-1}^{1} dx \; e^{ix\nu} \, x f_g(x,\mu^2) \, .$$

- $\mathcal{I}(0,\mu^2) =$ fraction of hadron momentum carried by gluons

$$\mathcal{I}(0,\mu^2) = \int_0^1 \mathrm{d}x \, x f_g(x,\mu^2) \equiv \langle x \rangle_{\mu^2}$$

- Matching relation allows to extract the shape of $f_g(x,\mu^2)$
- Its normalization, i.e., $\langle x \rangle_{\mu^2}$ should be found by an independent lattice calculation



In conclusion

- Feynman parton model uses “infinite momentum frame” (IMF) to get PDFs
- IMF idea was abandoned in later approaches based on the operator product expansion (OPE) in the coordinate space
- Quasi-PDF approach (Large momentum effective theory) revives the IMF concept
- Ioffe-time distribution approach (“pseudo-PDFs”) abandons the IMF idea and returns to OPE analysis in coordinate space
- Kernel relations directly connect lattice matrix elements and PDFs, without any need for intermediaries like quasi-PDFs and pseudo-PDFs