Towards a combined analysis of inclusive/exclusive electroproduction

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in collaboration with
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JPAC and Hall B

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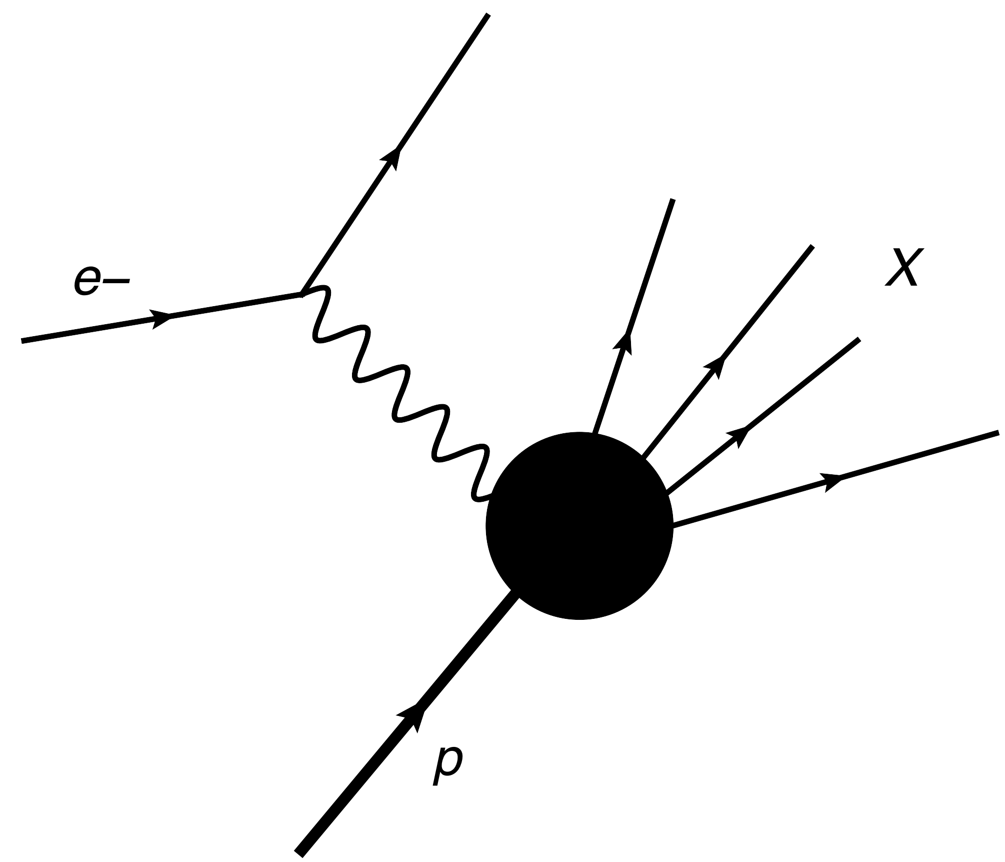
arXiv:1904.08016 [hep-ph]

STRONG QCD FROM HADRON STRUCTURE EXPERIMENTS 2019

2019-11-08

Nucleon excitation structure



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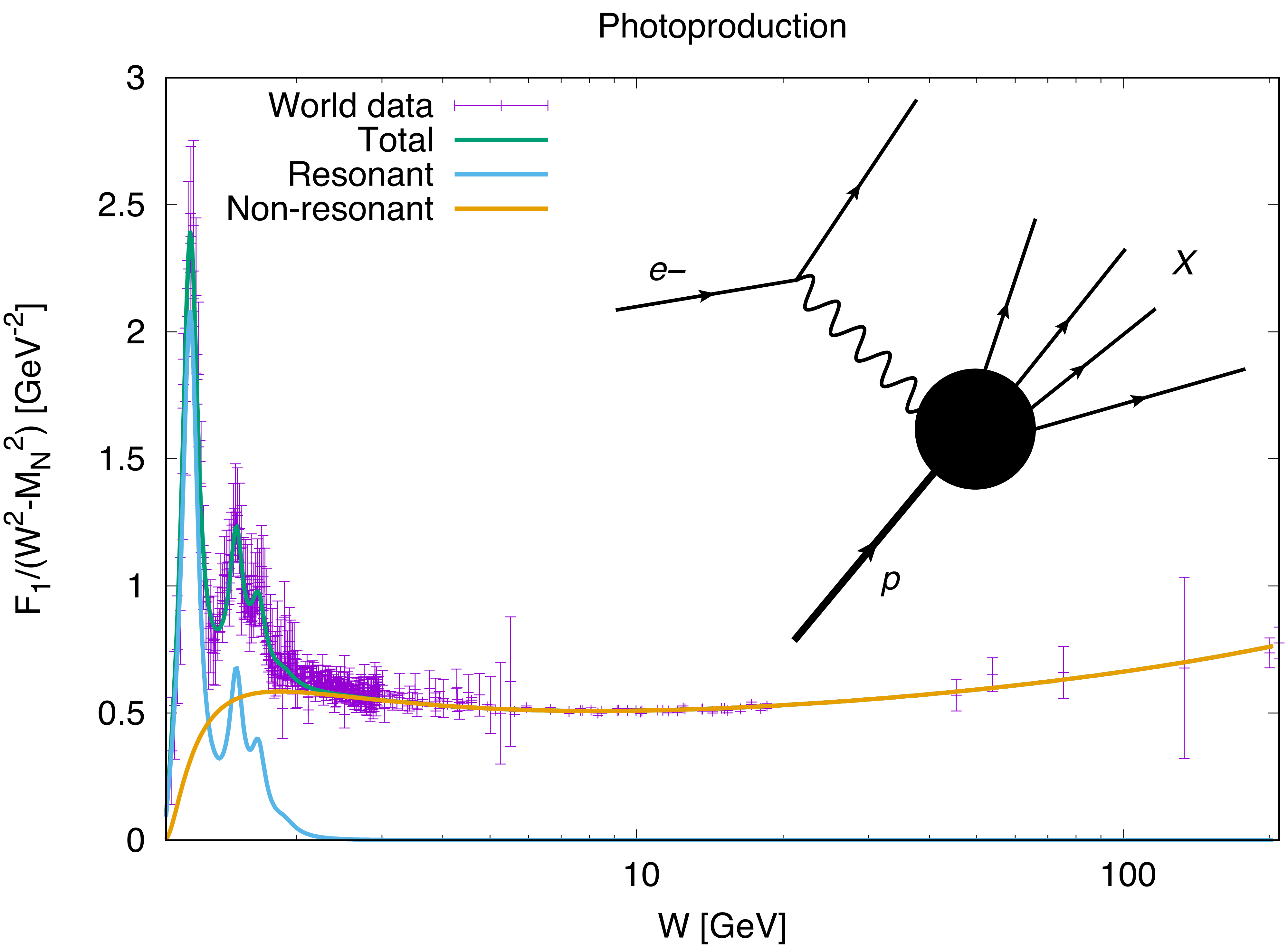

Inclusive and exclusive reactions: can we use one for the other?

Results of resonant contributions to inclusive observables

Non-resonant piece: low and high energies communicate

Results of combined description (preliminary)

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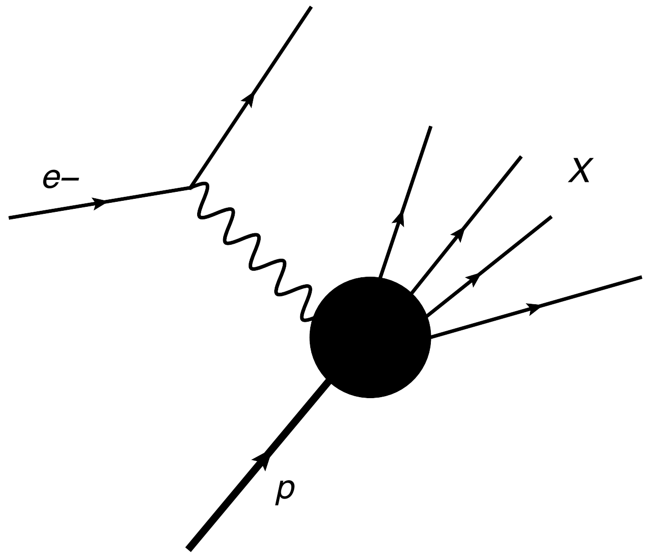
Inclusive electron scattering



$$F_1 \propto \sigma_T(W, Q^2)$$

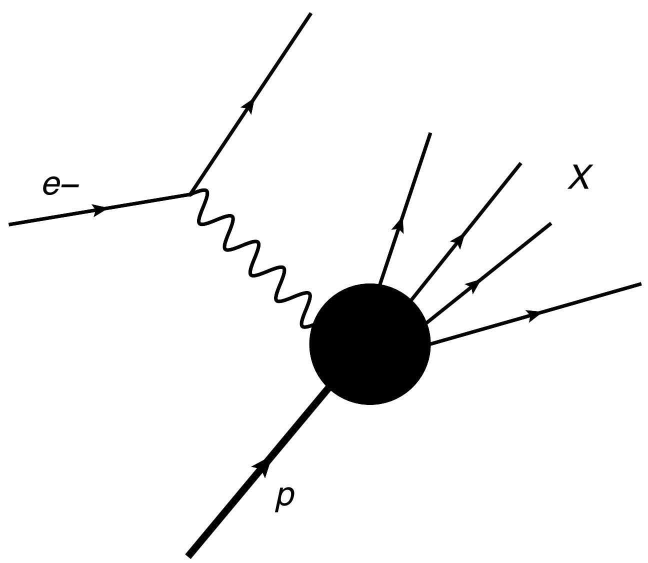
$$F_2 \propto \sigma_T(W, Q^2) + \sigma_L(W, Q^2)$$

$$\sigma_U(W, Q^2) = \sigma_T(W, Q^2) + \epsilon_T \sigma_L(W, Q^2)$$

- Gives access to structure functions

- Here: unpolarized structure functions

Inclusive electron scattering



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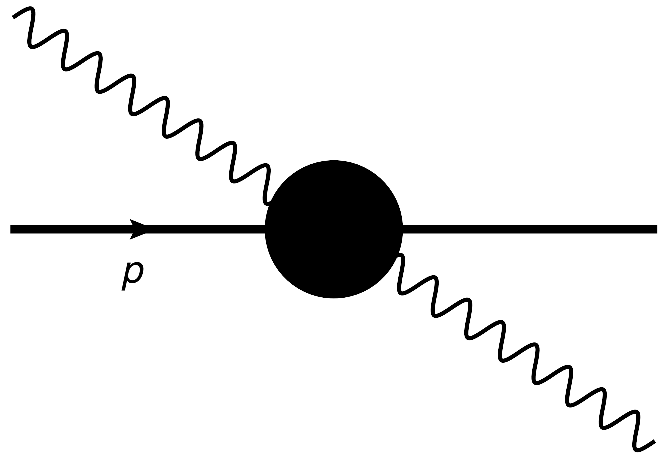


- Gives access to structure functions

- Here: unpolarized structure functions

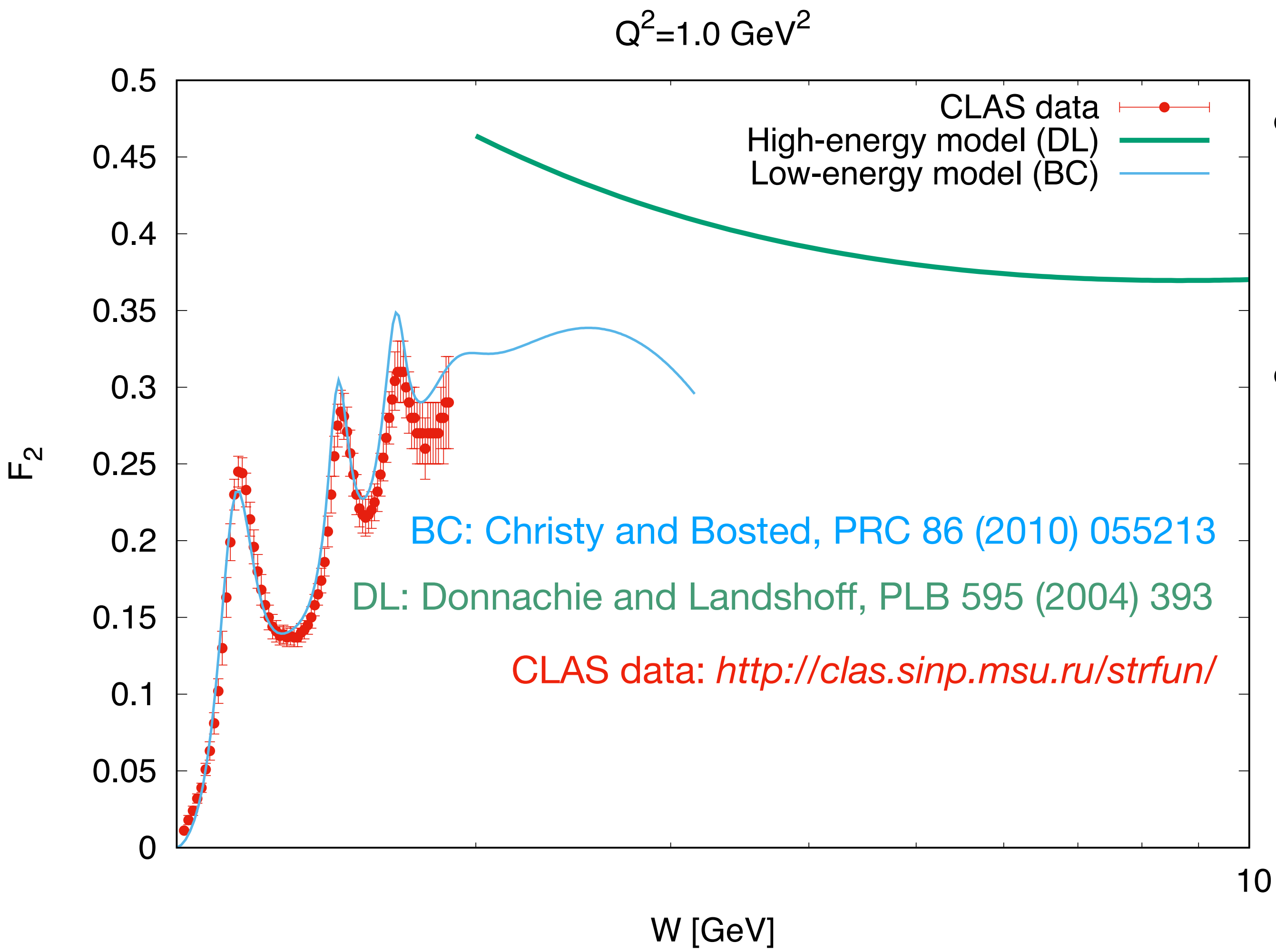
$$\Im T_1(\nu, Q^2) = \frac{\pi\alpha}{M} F_1(\nu, Q^2)$$

$$\Im T_2(\nu, Q^2) = \frac{\pi\alpha}{\nu} F_2(\nu, Q^2)$$

  - Related to forward virtual Compton scattering (VVCS) amplitudes T_1 and T_2

  - Information about hadronic contribution to **muonic hydrogen Lamb shift!**

Inclusive data and models on the market



- **At low energies: precise CLAS data**
  Well covered up to 3rd resonance regime: towards parton distributions at large x
- CLAS12 is to reach
  0.05 GeV^2 < Q^2 < 12 GeV^2, W up to 4 GeV

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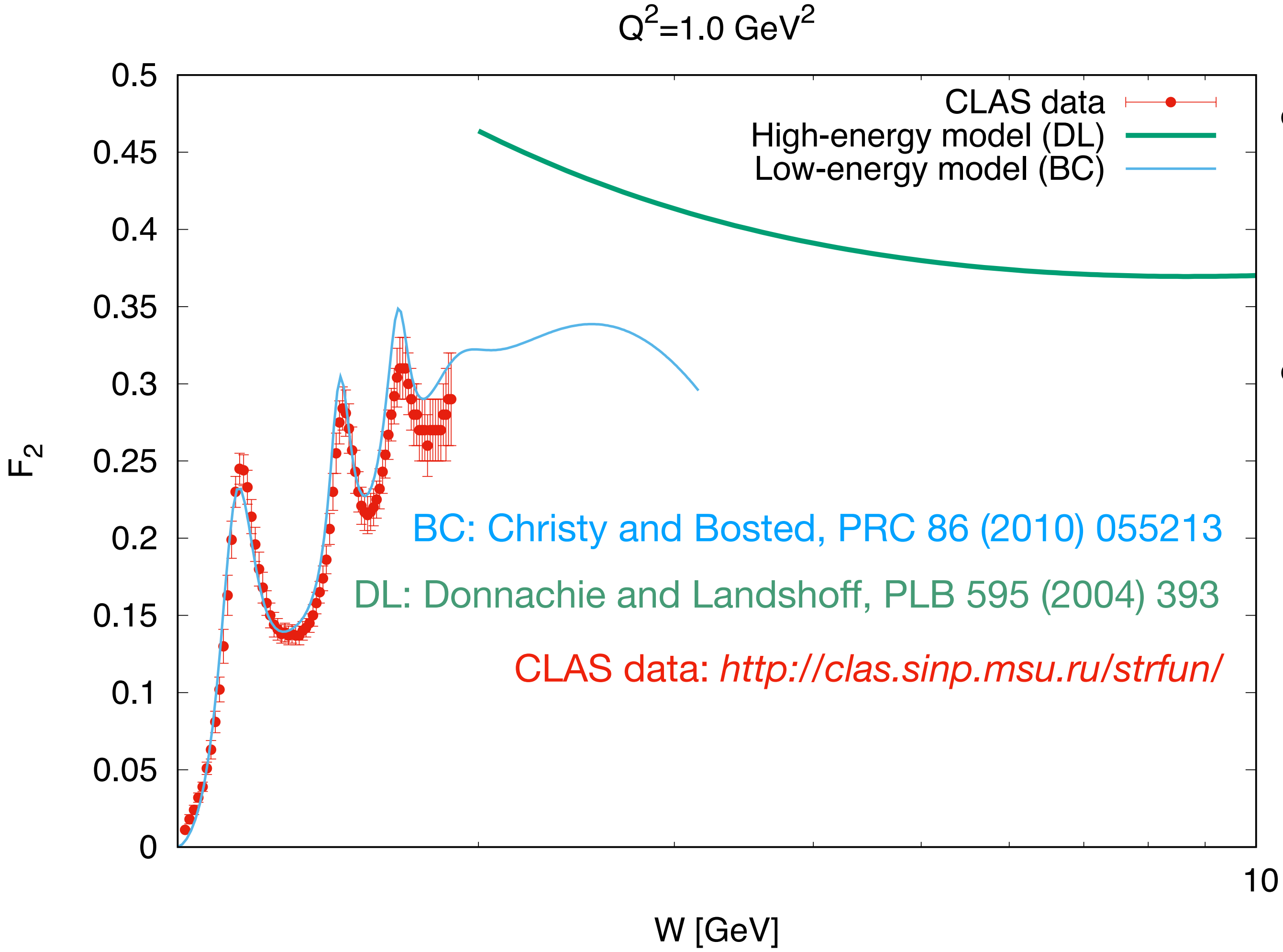

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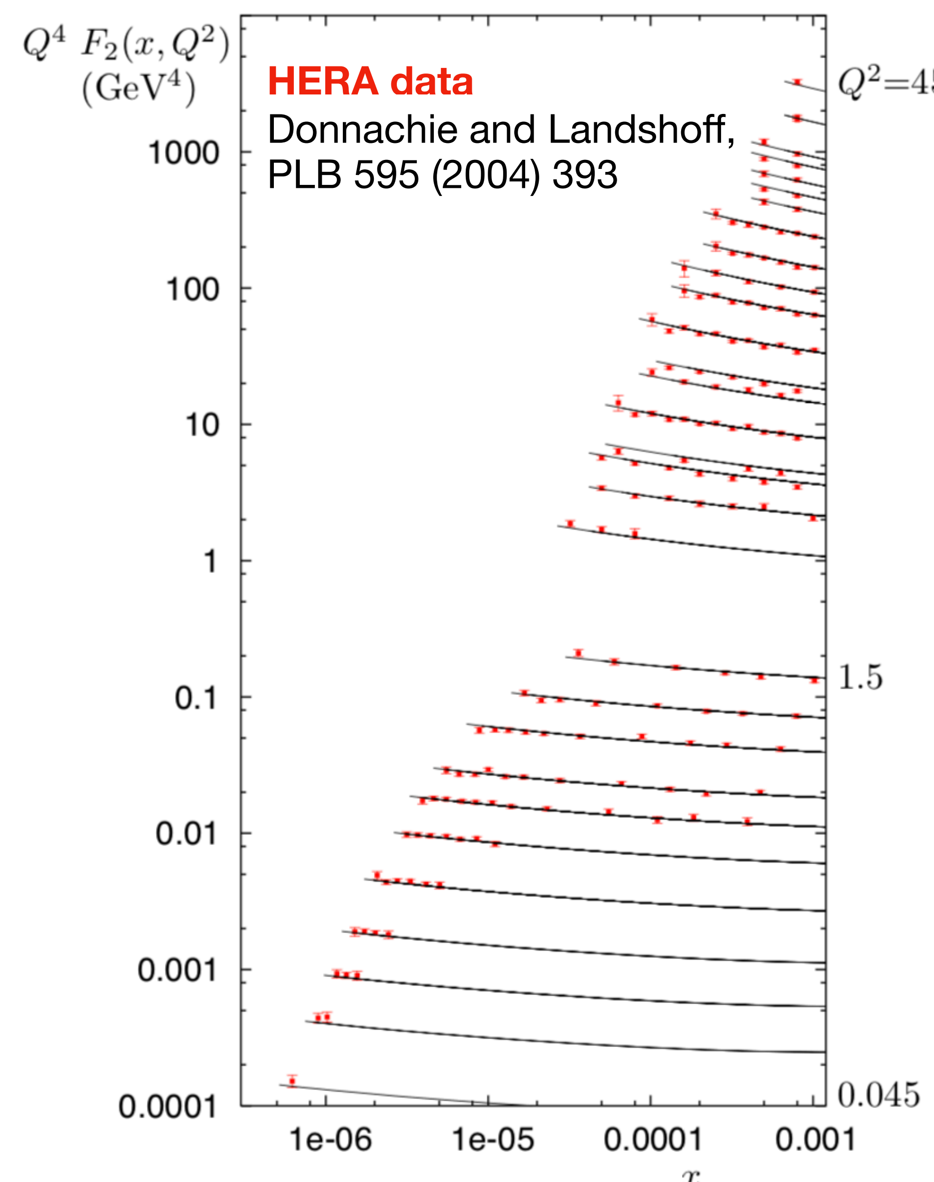
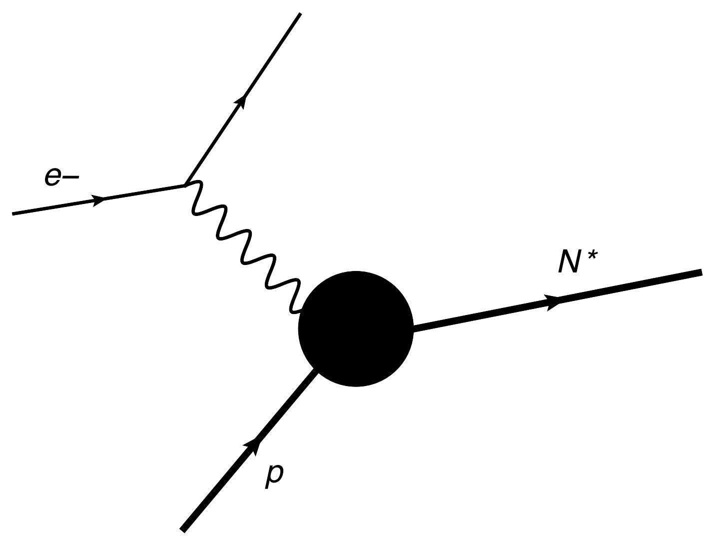


- **At high energies: HERA and ZEUS data**
  Extremely good Q^2 coverage at low x
- **Goal:** combining high and low-energy models
- Description of observables integrated over x: Cottingham formula, subtraction function in VVCS, **Lamb shift**, …
- Tests on quark-hadron duality

Exclusive electron scattering



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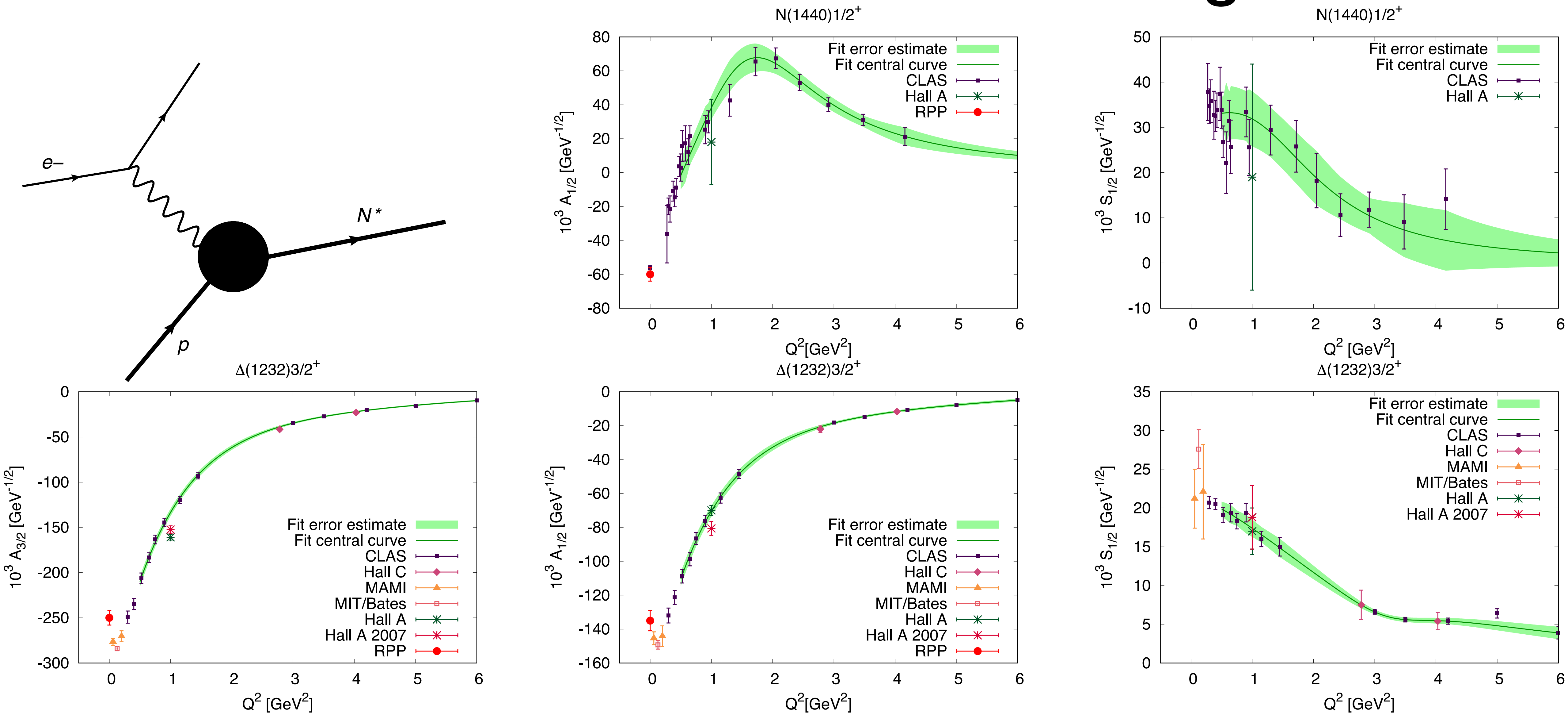
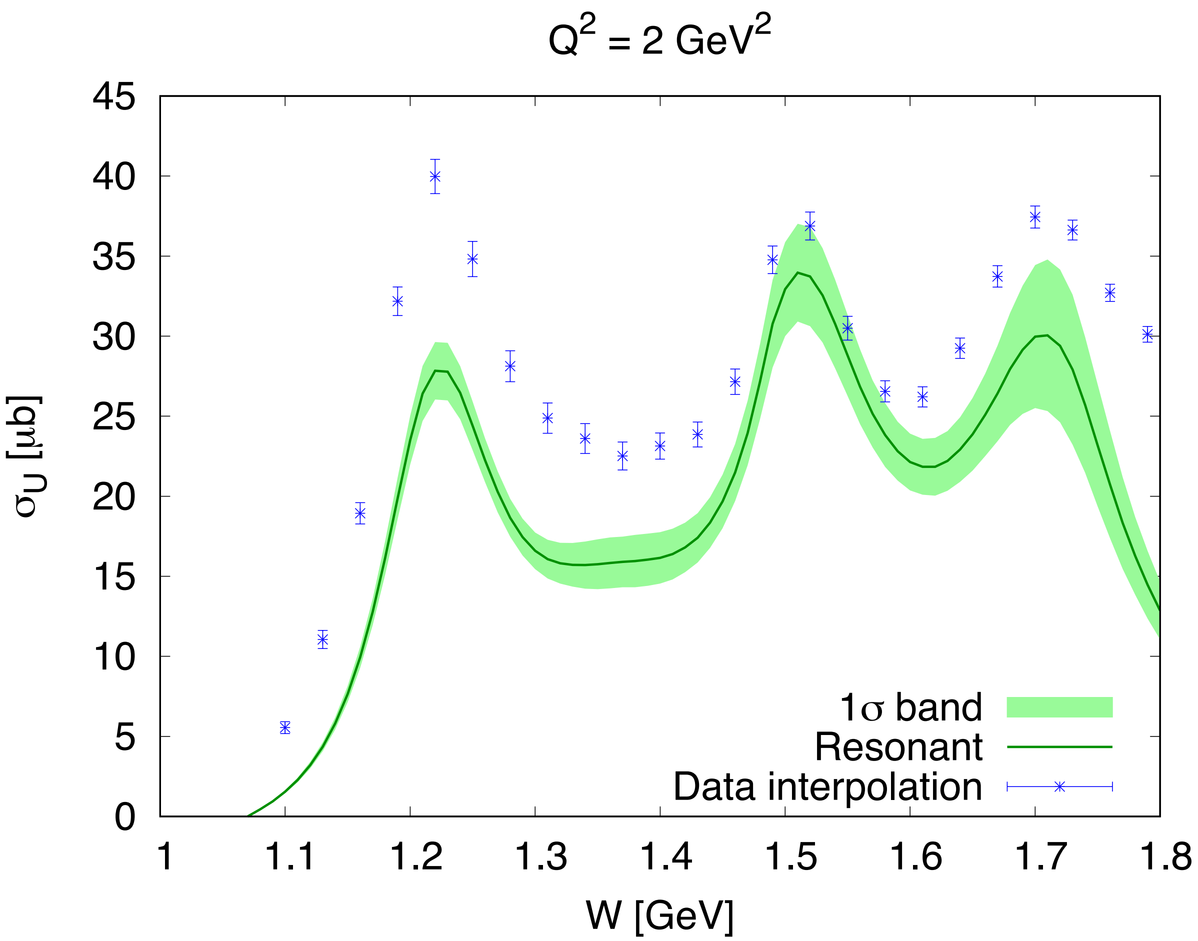


- Longitudinal and transverse electrocouplings
- Allows us to determine each of the resonant contributions separately
- Good world data on some of the main resonances;
  CLAS data up to 3rd resonance region
  (and soon full resonance region coverage with CLAS12)

From exclusive to inclusive electron scattering

$$\sigma_{T,L}(W,Q^2) = \sigma^R_{T,L}(W,Q^2) + \sigma^{NR}_{T,L}(W,Q^2)$$



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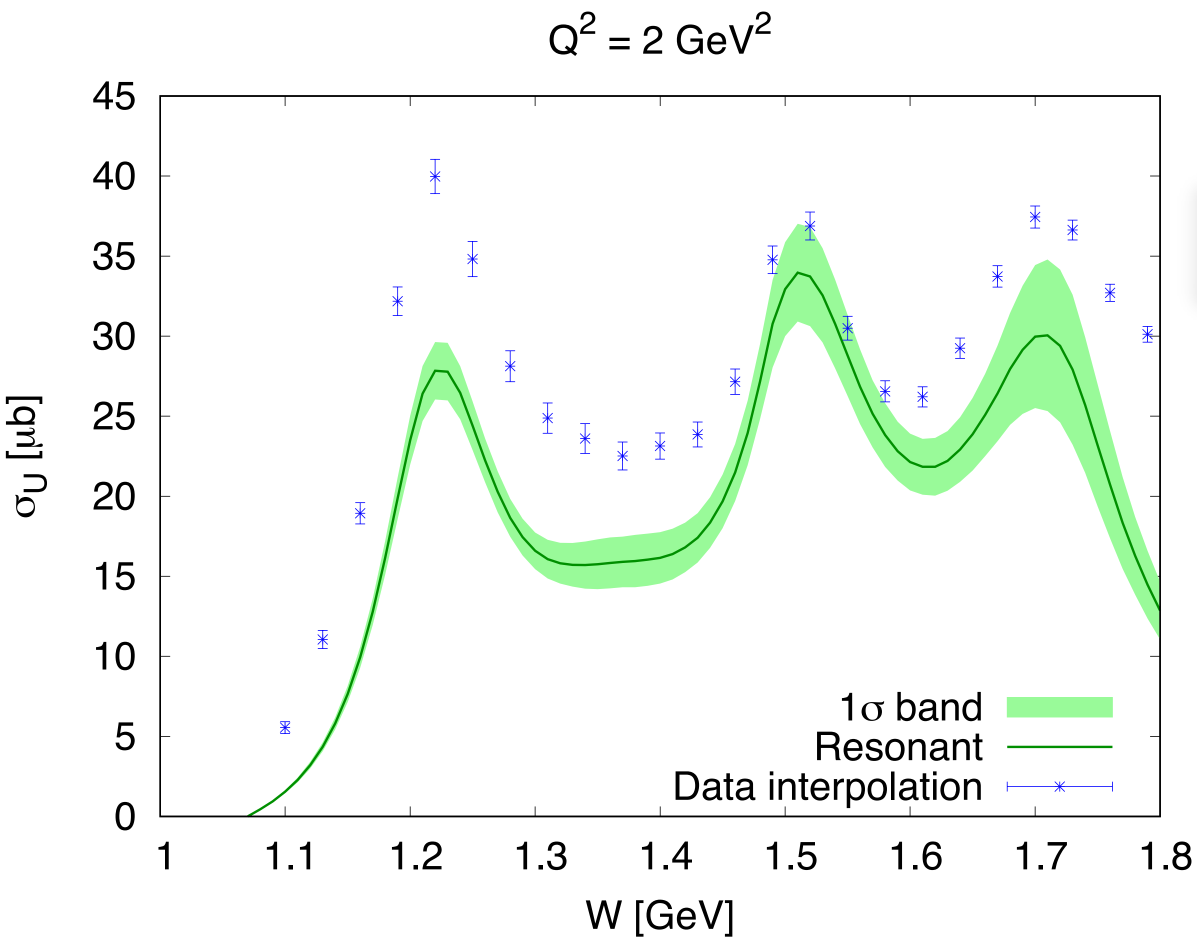

Breit-Wigner resonance model

Mokeev et al., PRC 86 (2012) 035203

$$\sigma^{R}_{T,L}(W,Q^2) = \frac{\pi}{q_\gamma^2}\sum_{N^*,\Delta^*}(2J_r+1)\frac{M_r^2\Gamma_{\rm tot}(W)\Gamma_\gamma^{T,L}(M_r)}{(M_r^2-W^2)^2+M_r^2\Gamma^2_{\rm tot}(W)}$$

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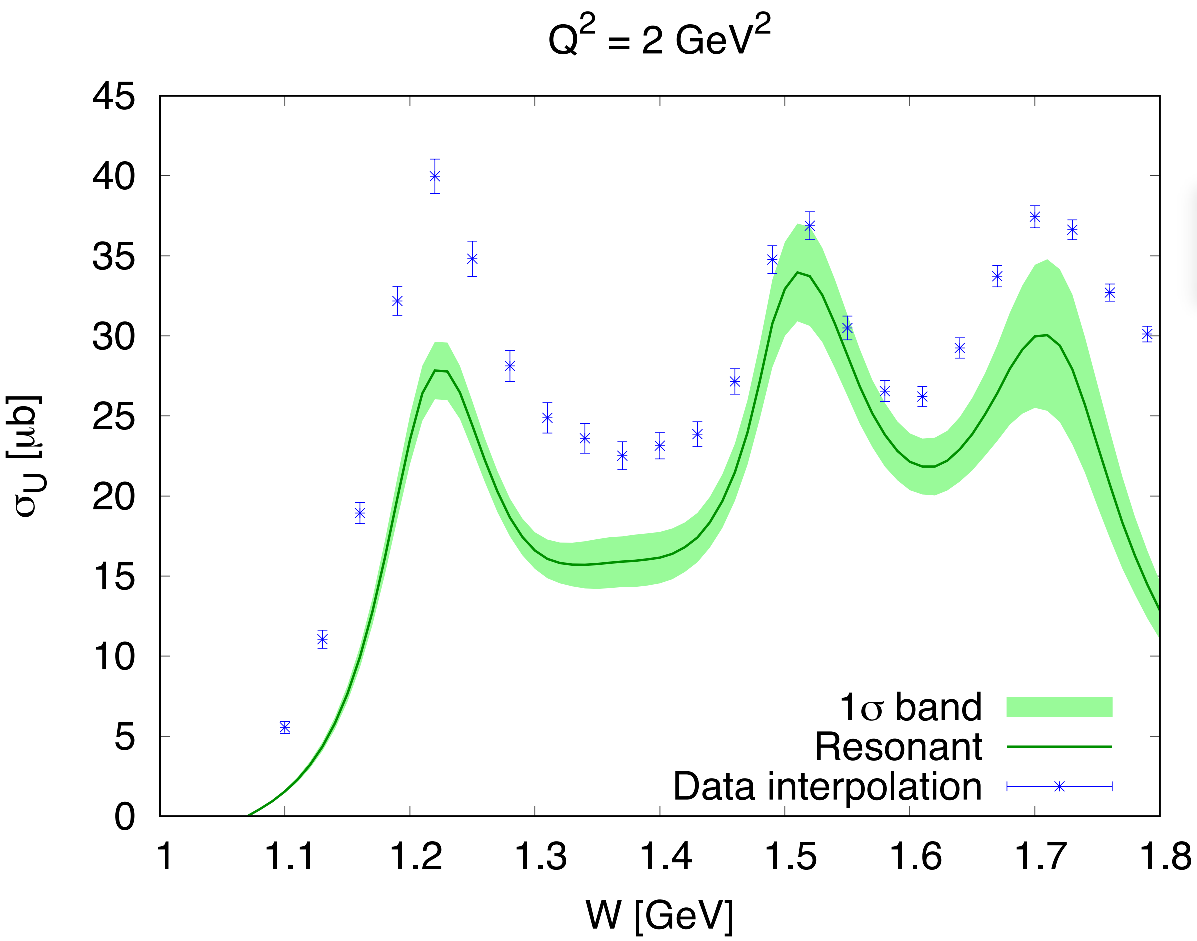

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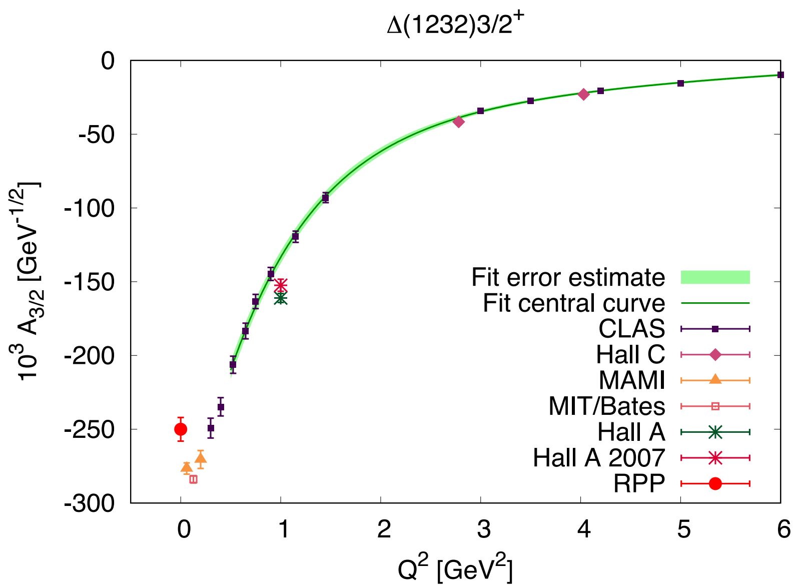
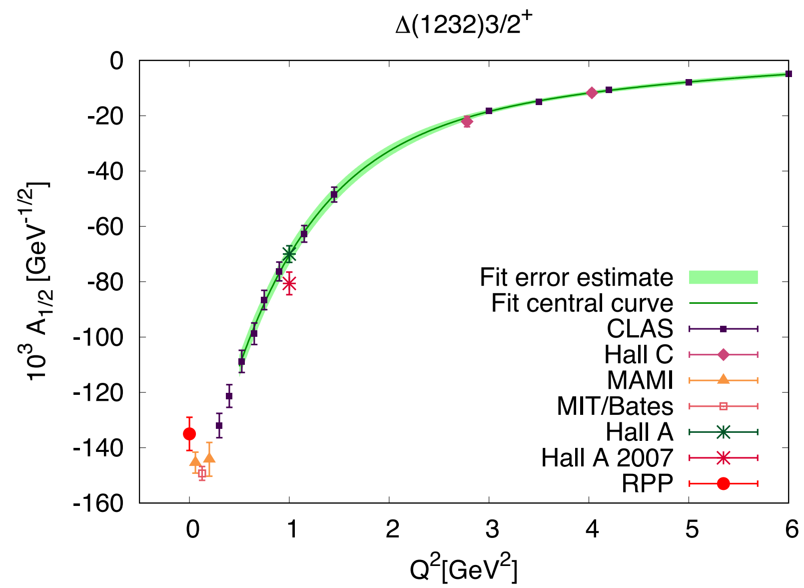
$$\Gamma_\gamma^T(M_r, Q^2) \sim \left| A_{1/2}(Q^2) \right|^2 + \left| A_{3/2}(Q^2) \right|^2$$

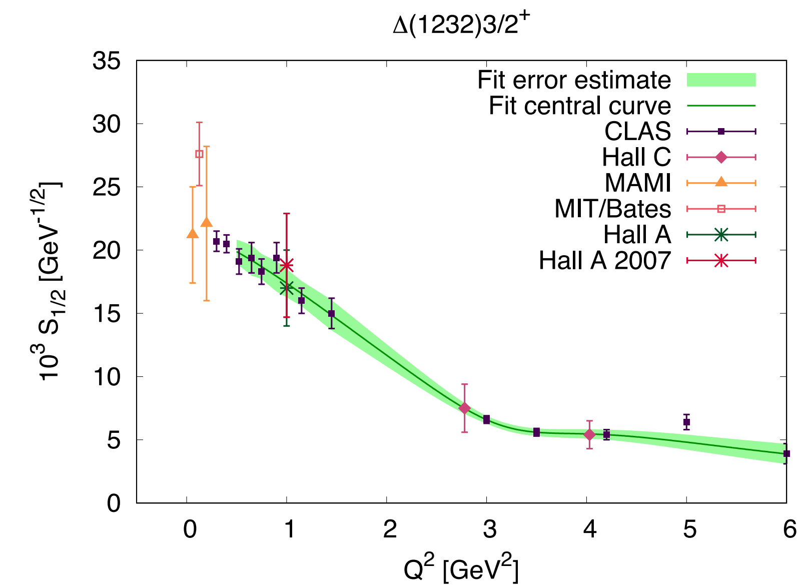
$$\Gamma_\gamma^L(M_r, Q^2) \sim \left| S_{1/2}(Q^2) \right|^2$$

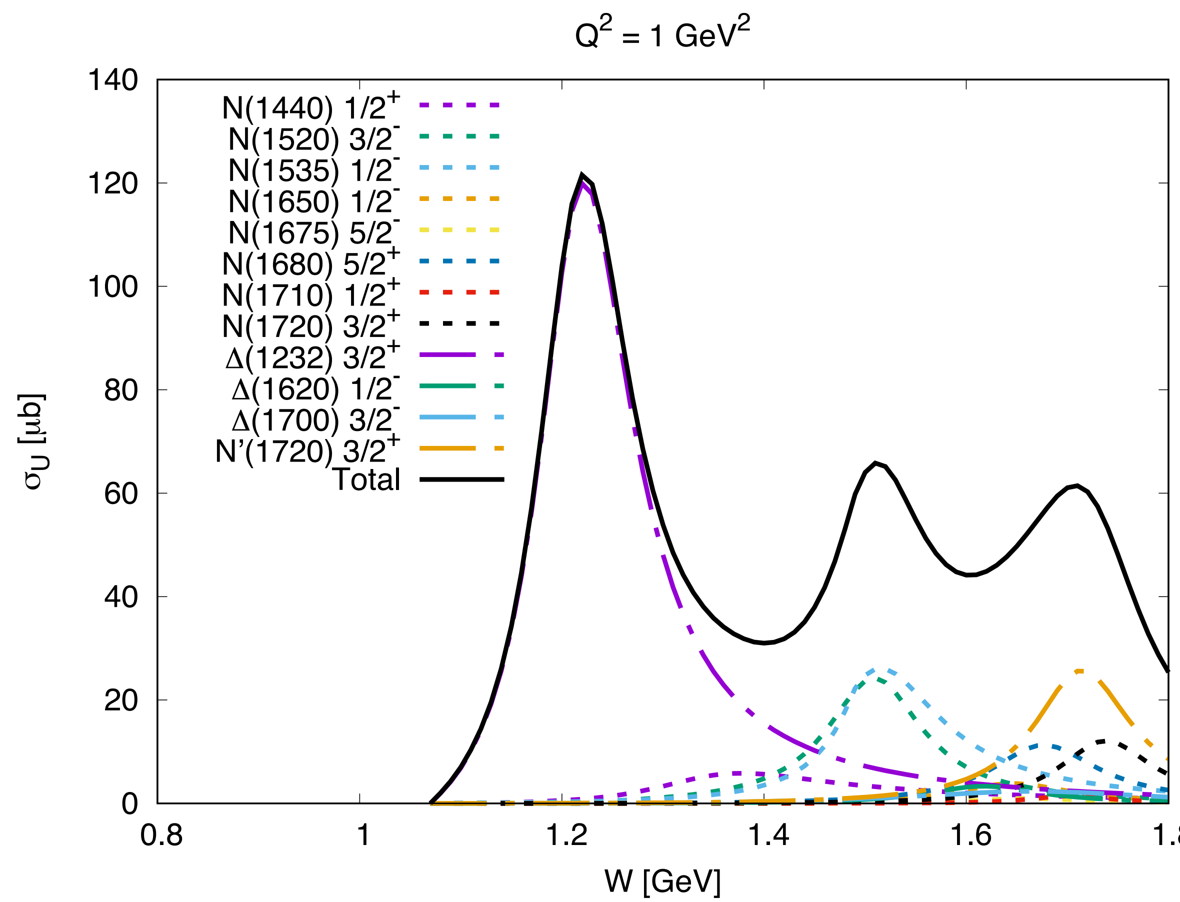
Electrocouplings from data

How electrocouplings enter inclusive data









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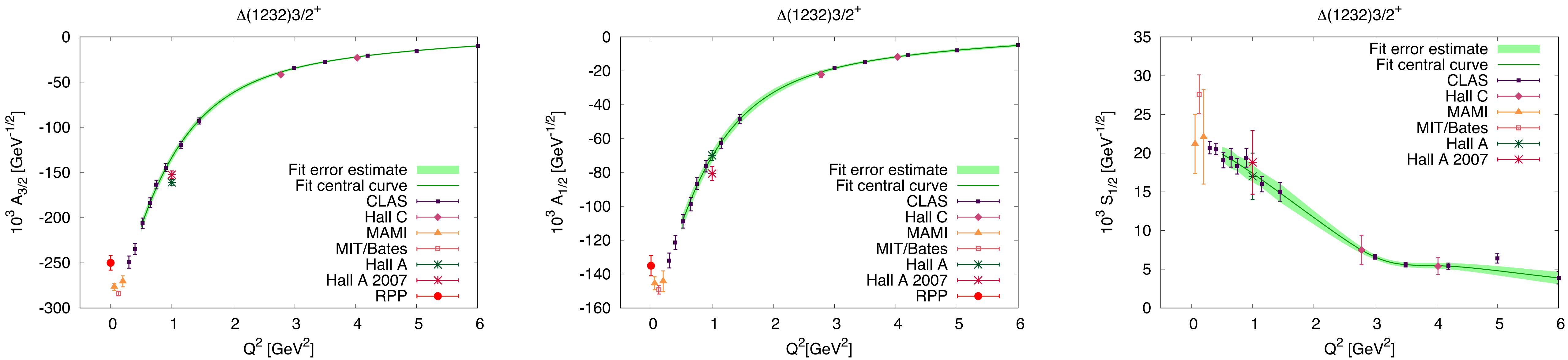
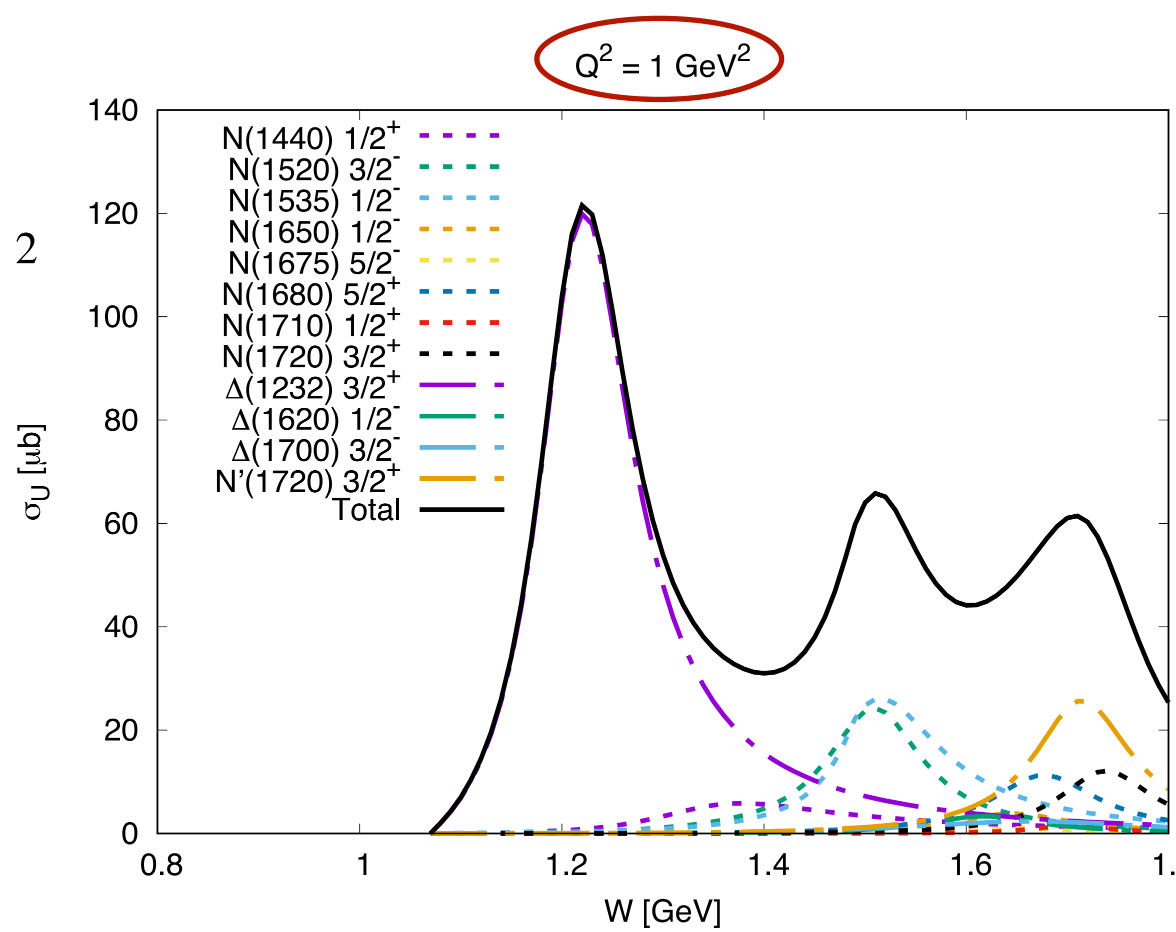


$$\sigma^R_{T,L}(W,Q^2) \sim \left|A_{1/2}(Q^2)\right|^2,\ \left|A_{3/2}(Q^2)\right|^2,\ \left|S_{1/2}(Q^2)\right|^2$$



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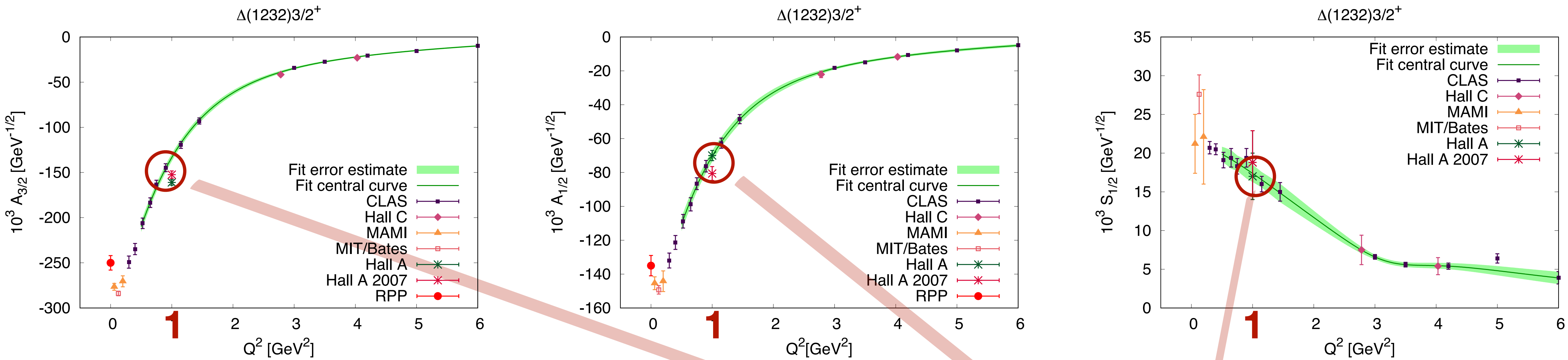

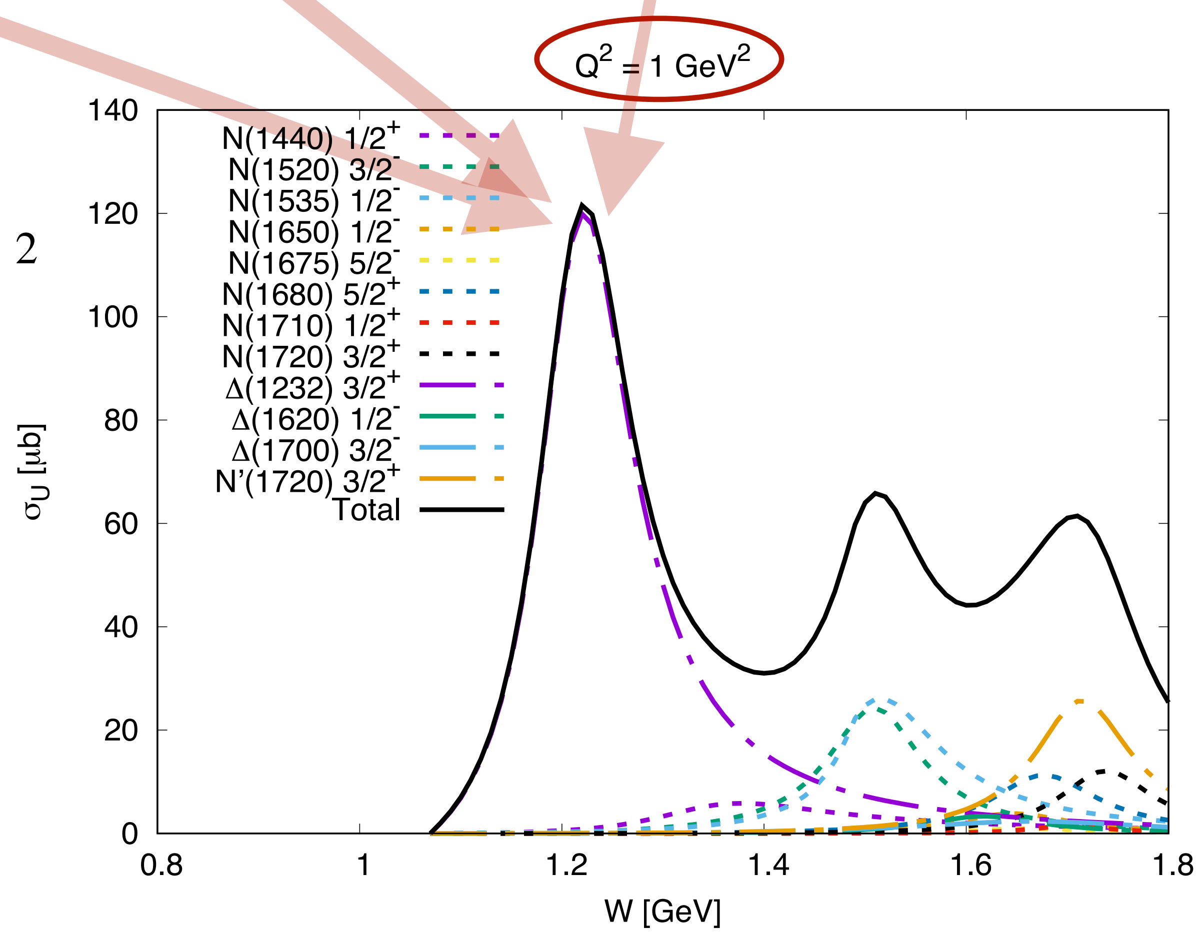

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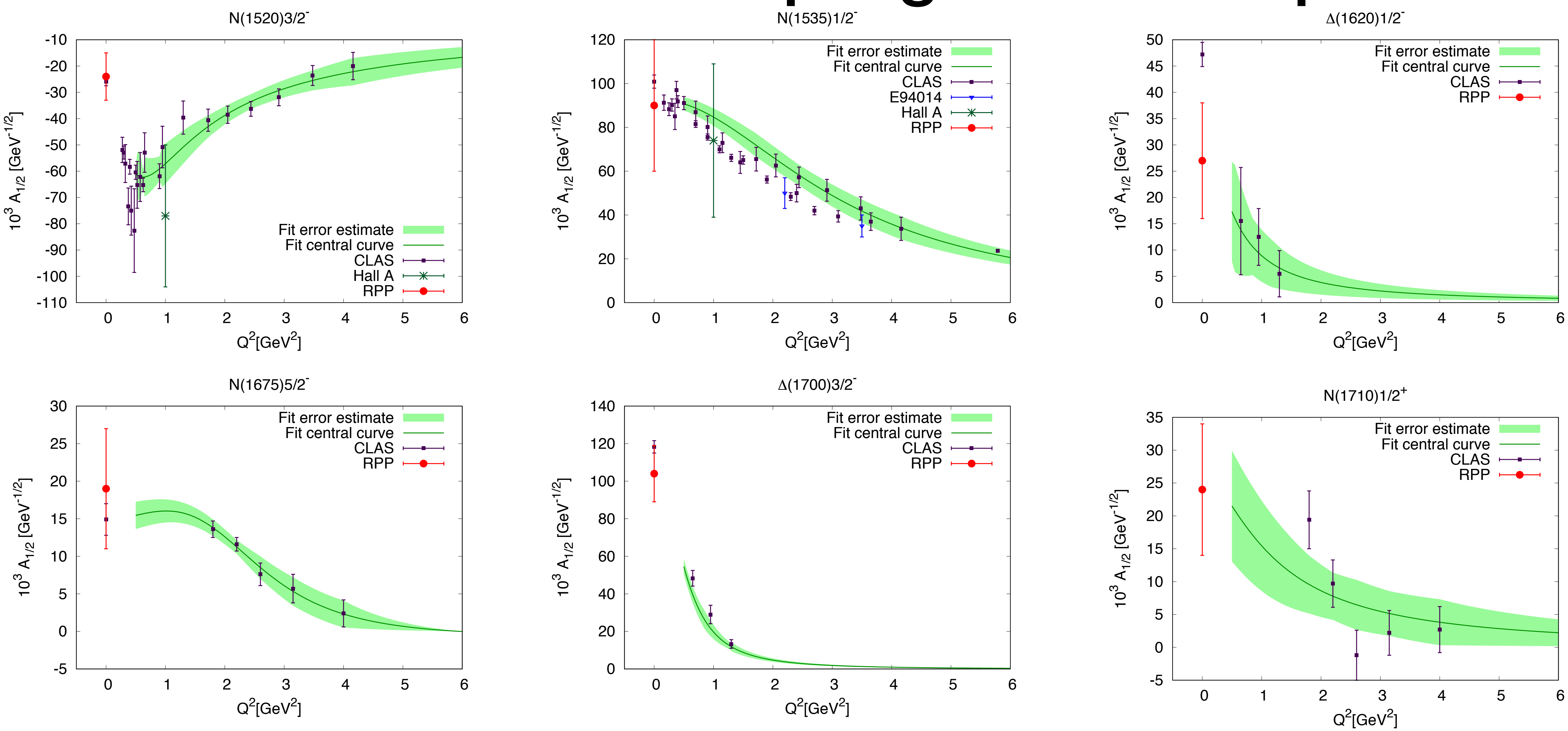
Further electrocoupling data examples



https://userweb.jlab.org/~mokeev/resonance_electrocouplings/

- Interpolation/extrapolation functions: *https://userweb.jlab.org/~isupov/couplings/*
  in good agreement with world data and preliminary CLAS12 results at higher Q^2
- Error bands estimated from data uncertainties
  and scaled with coupling size in extrapolation region

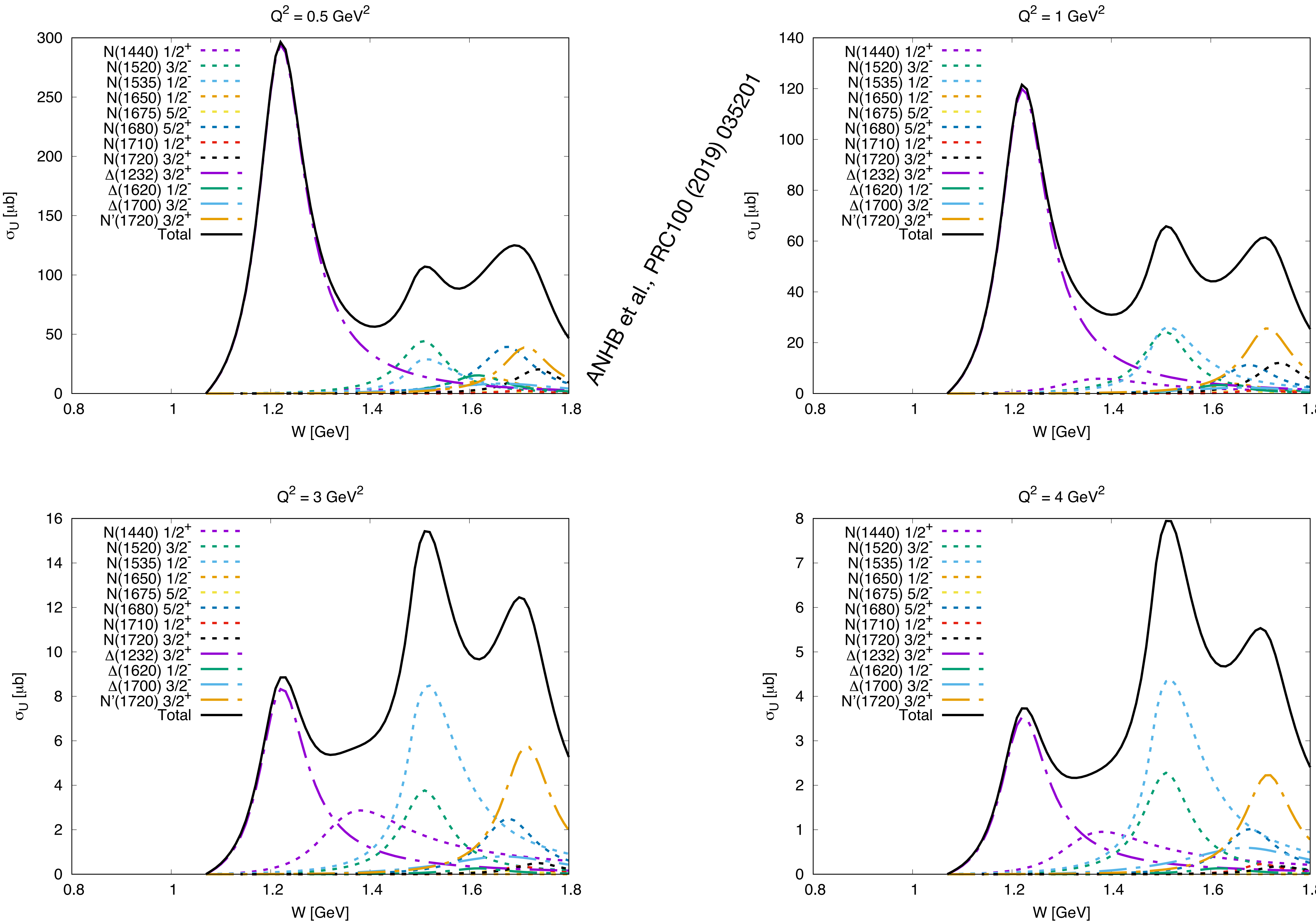
Inclusive and exclusive reactions: can we use one for the other?

<u>Results of resonant contributions to inclusive observables</u>

Non-resonant piece: low and high energies communicate

Results of combined description (preliminary)

Resonant contributions at different Q^2



ANHB et al., PRC100 (2019) 035201

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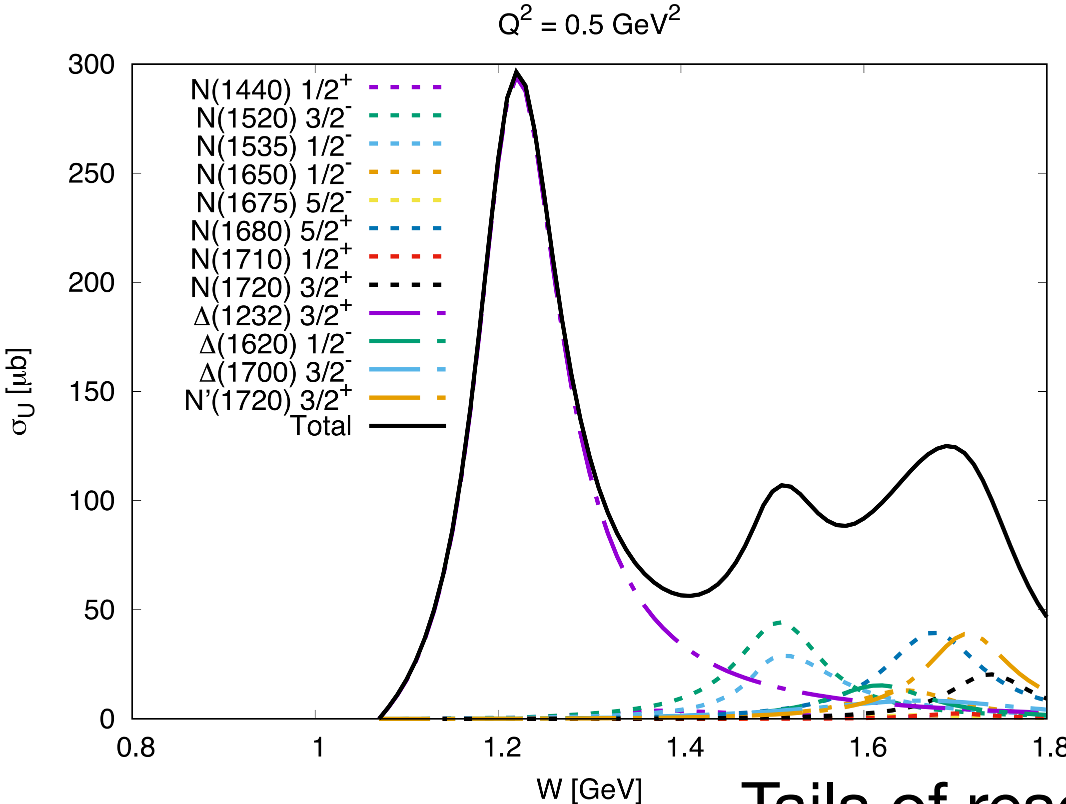

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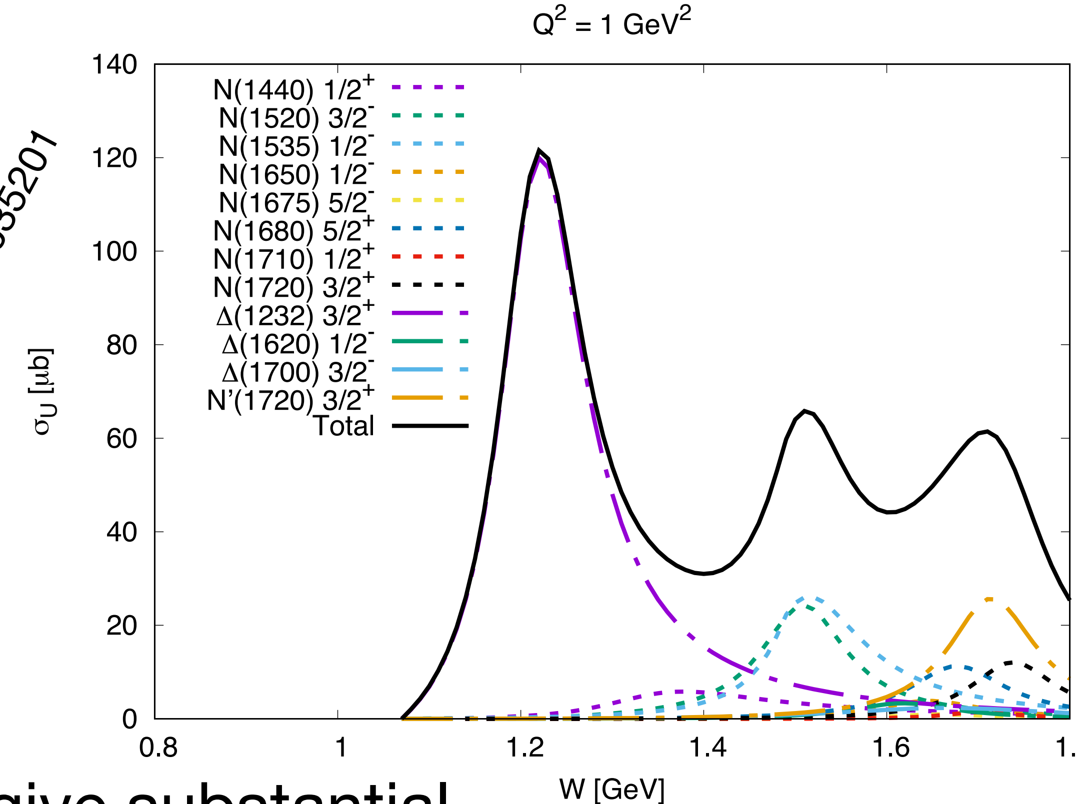
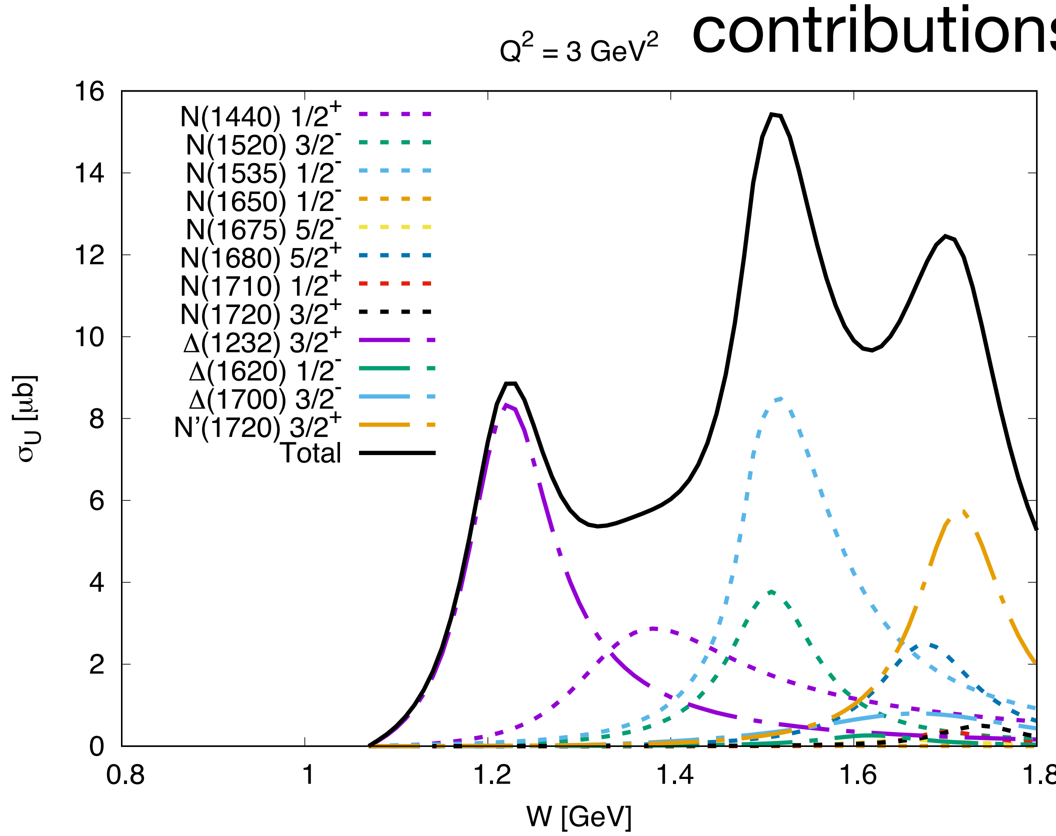
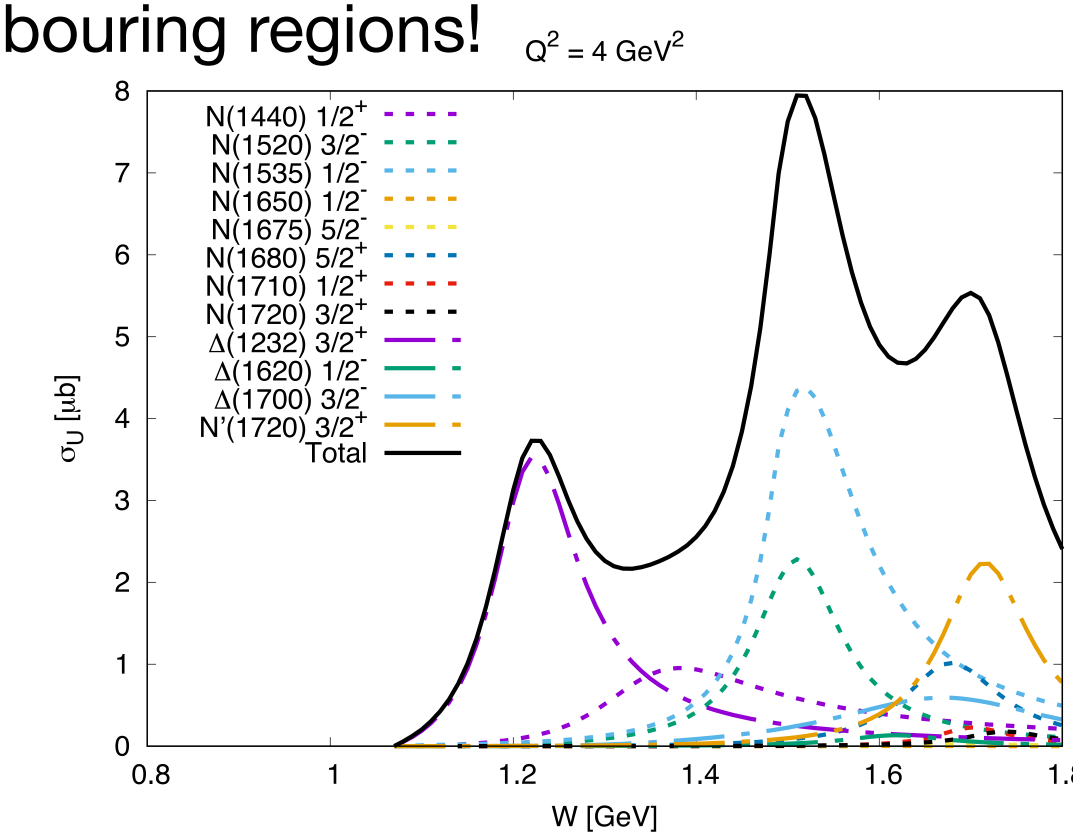
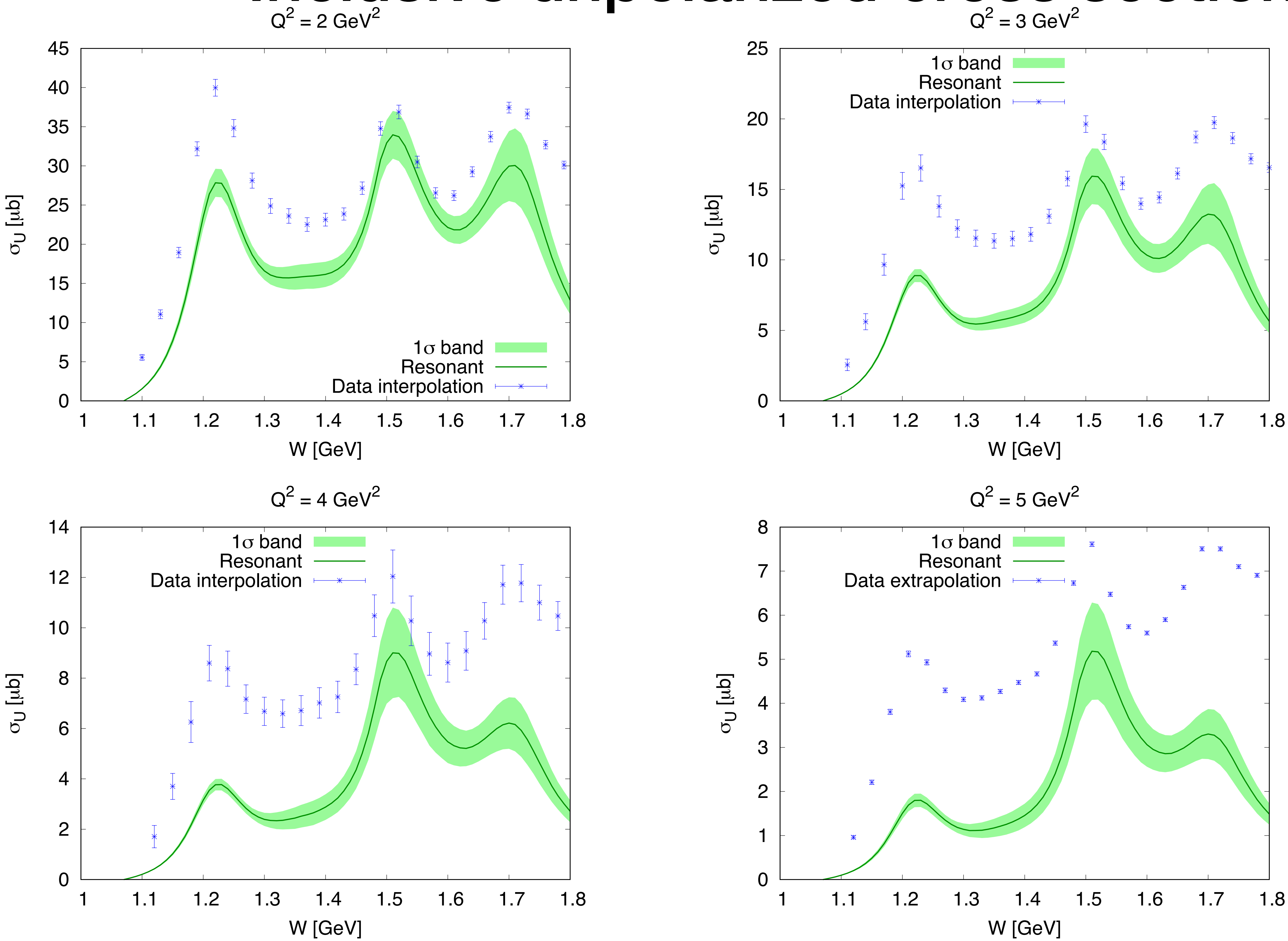


Tails of resonances give substantial contributions to neighbouring regions!





Inclusive unpolarized cross sections



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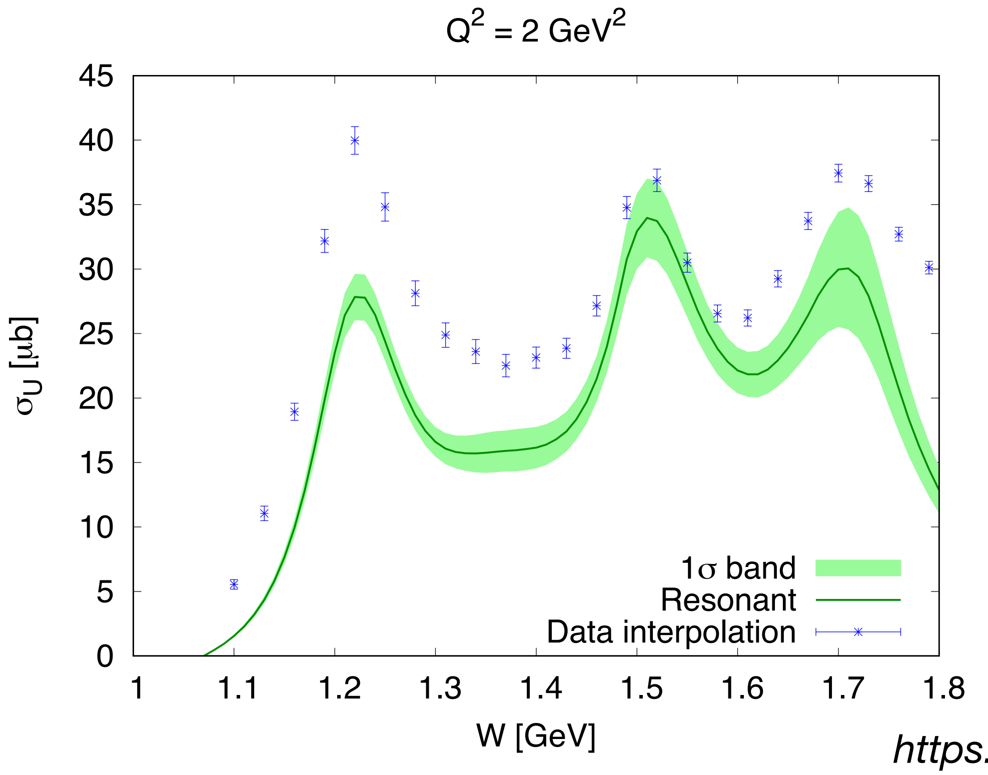
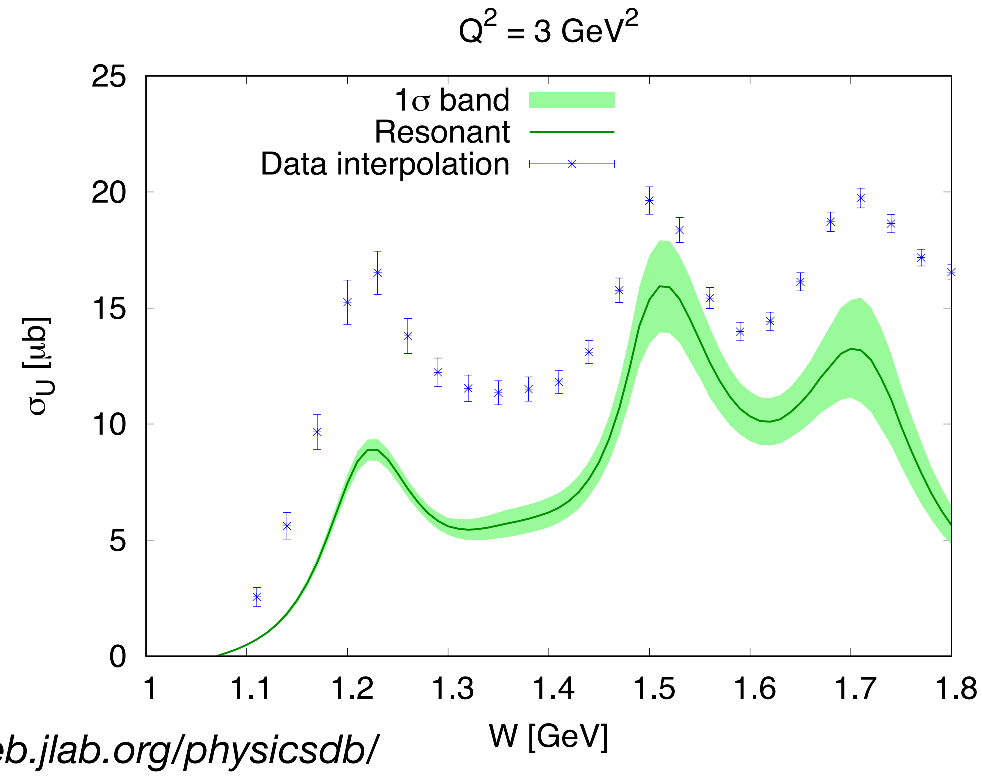
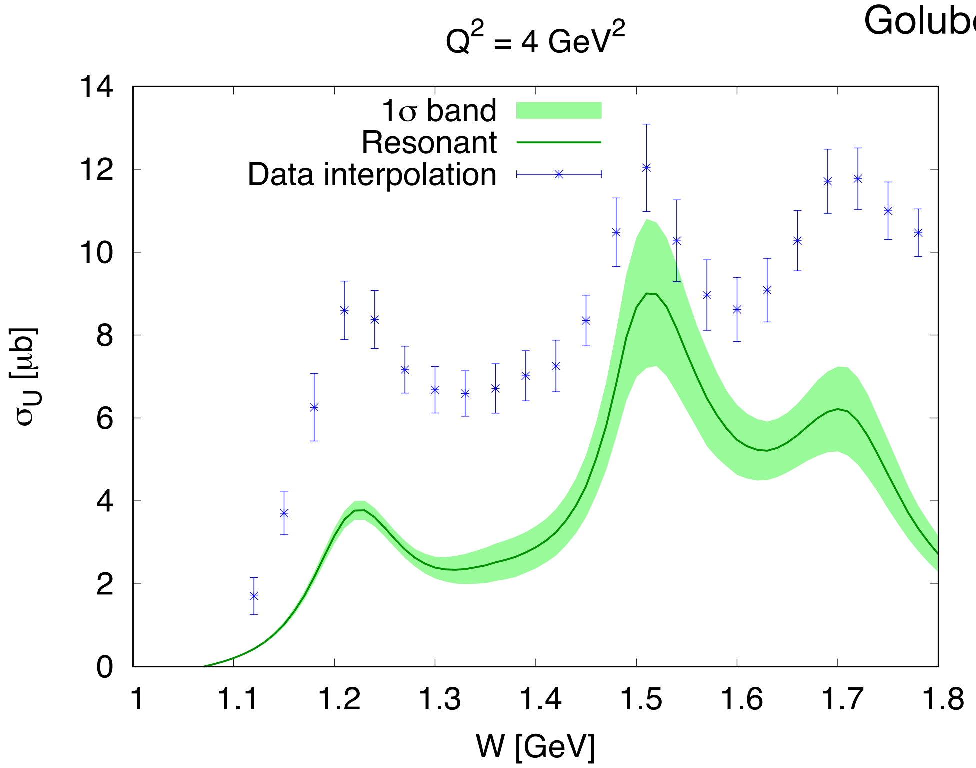
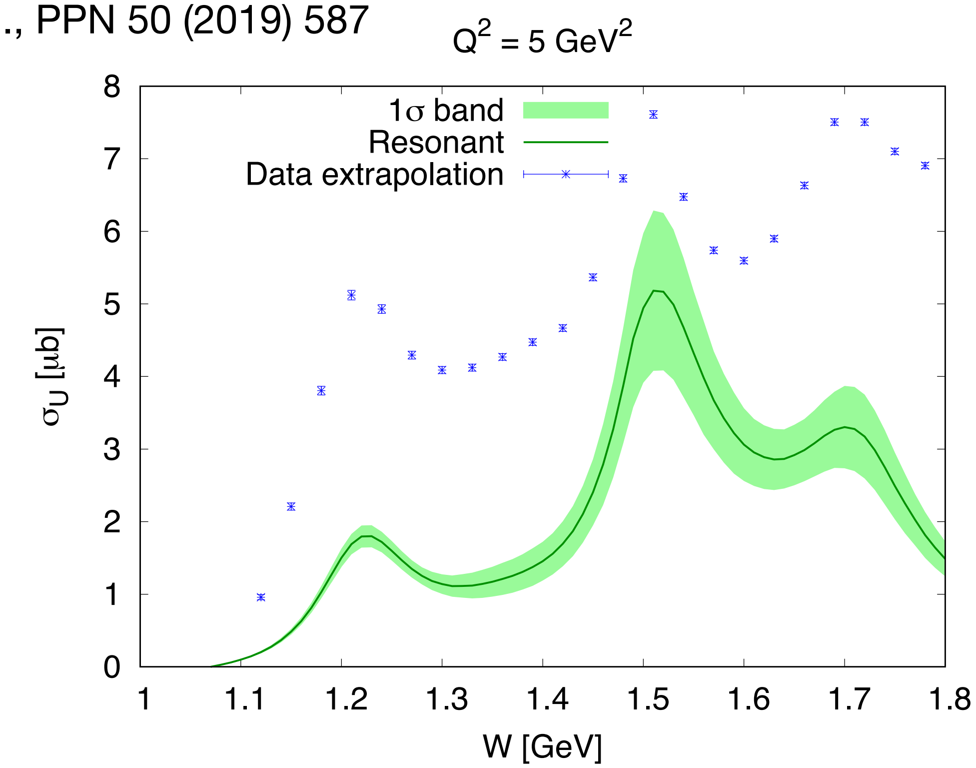




https://clasweb.jlab.org/physicsdb/

Golubenko et al., PPN 50 (2019) 587





ANHB et al., PRC100 (2019) 035201

Second resonance region decreases less with Q^2: intricate differences in Q^2 evolution of electrouplings

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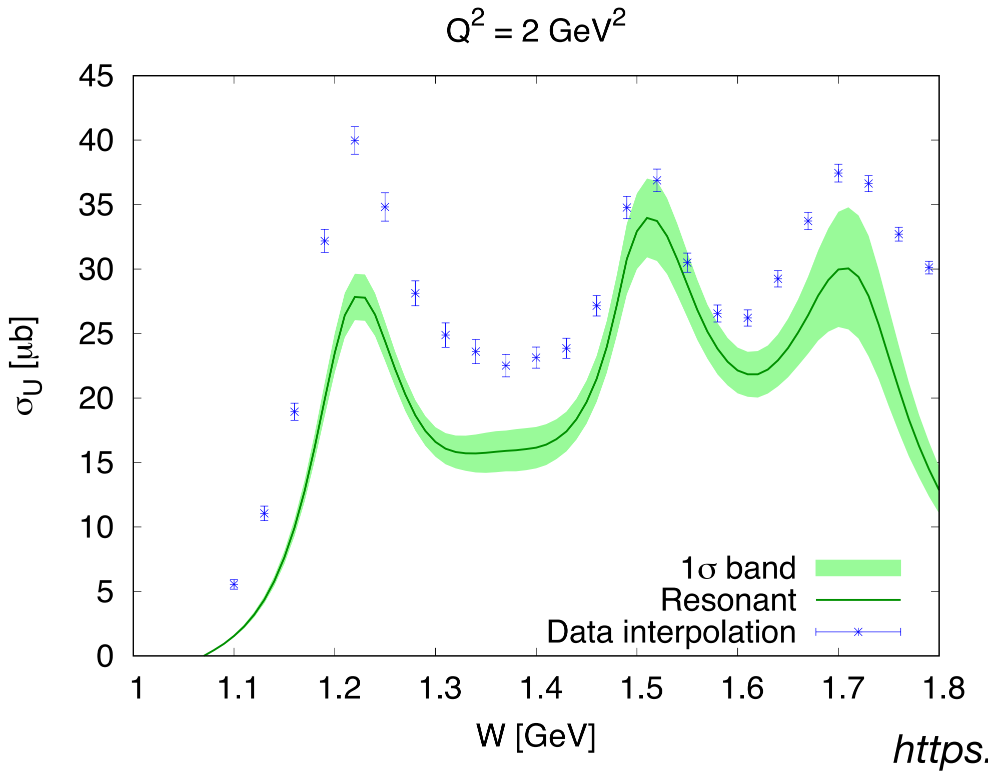

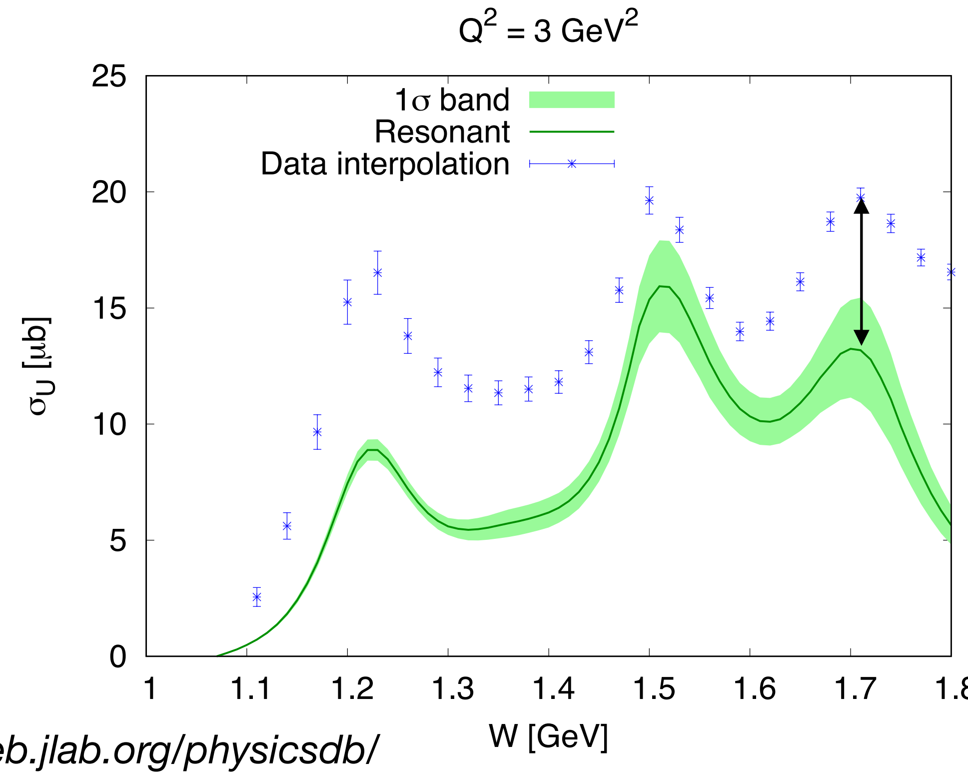

Time to figure out the non-resonant contributions!

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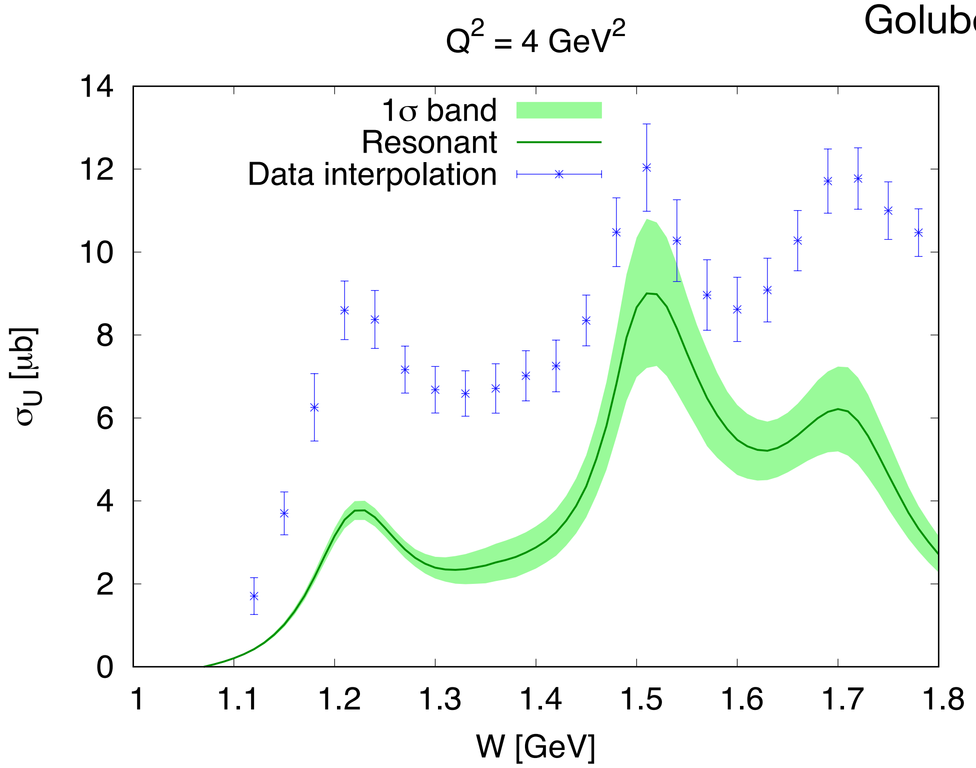
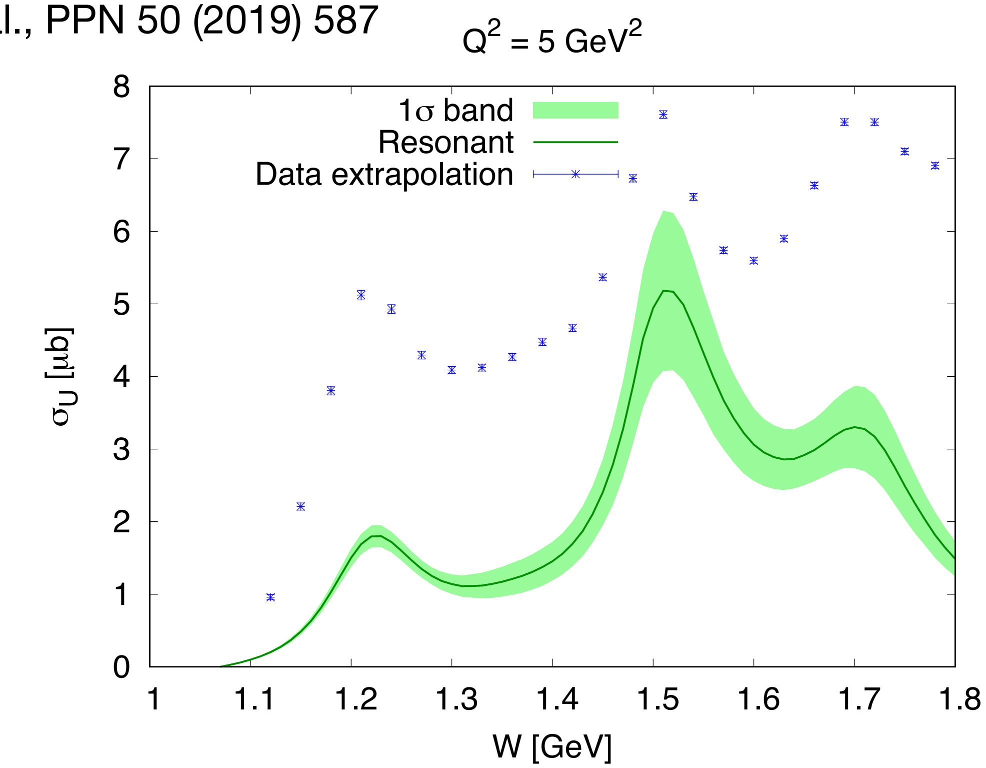
Golubenko et al., PPN 50 (2019) 587





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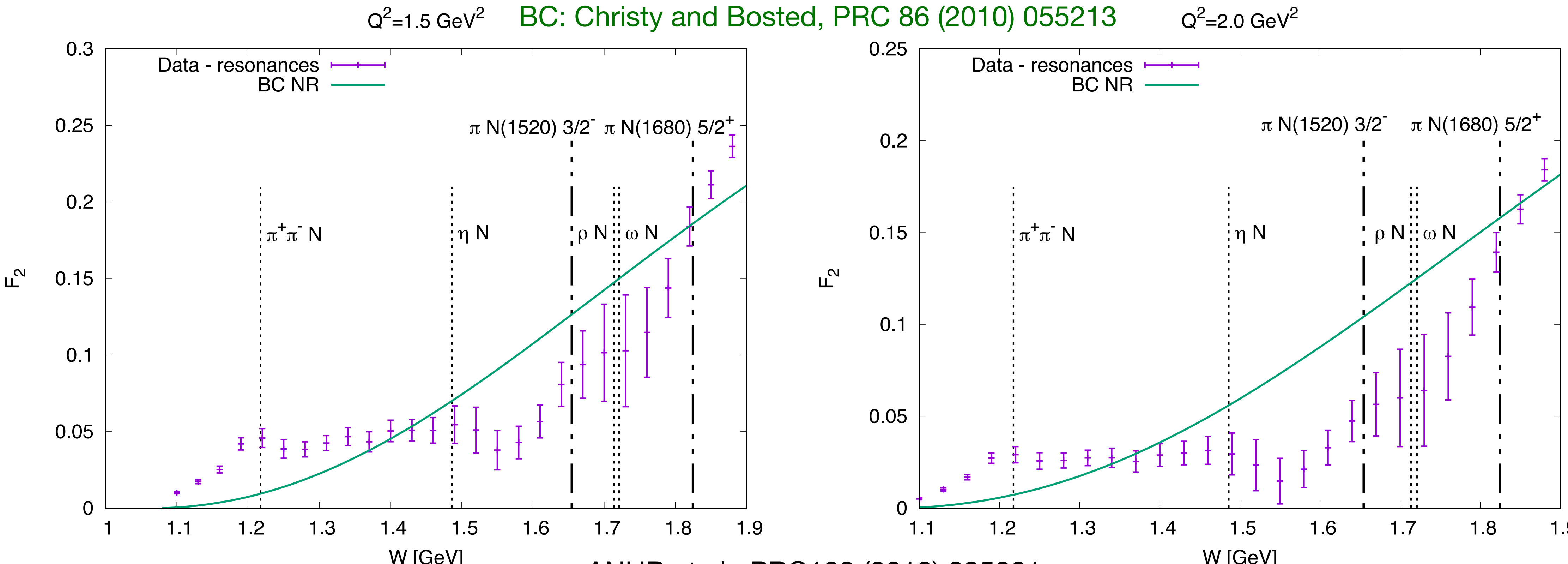
Results of resonant contributions to inclusive observables

<u>Non-resonant piece: low and high energies communicate</u>

Results of combined description (preliminary)

First estimates of non-resonant contribution

BC: Christy and Bosted, PRC 86 (2010) 055213



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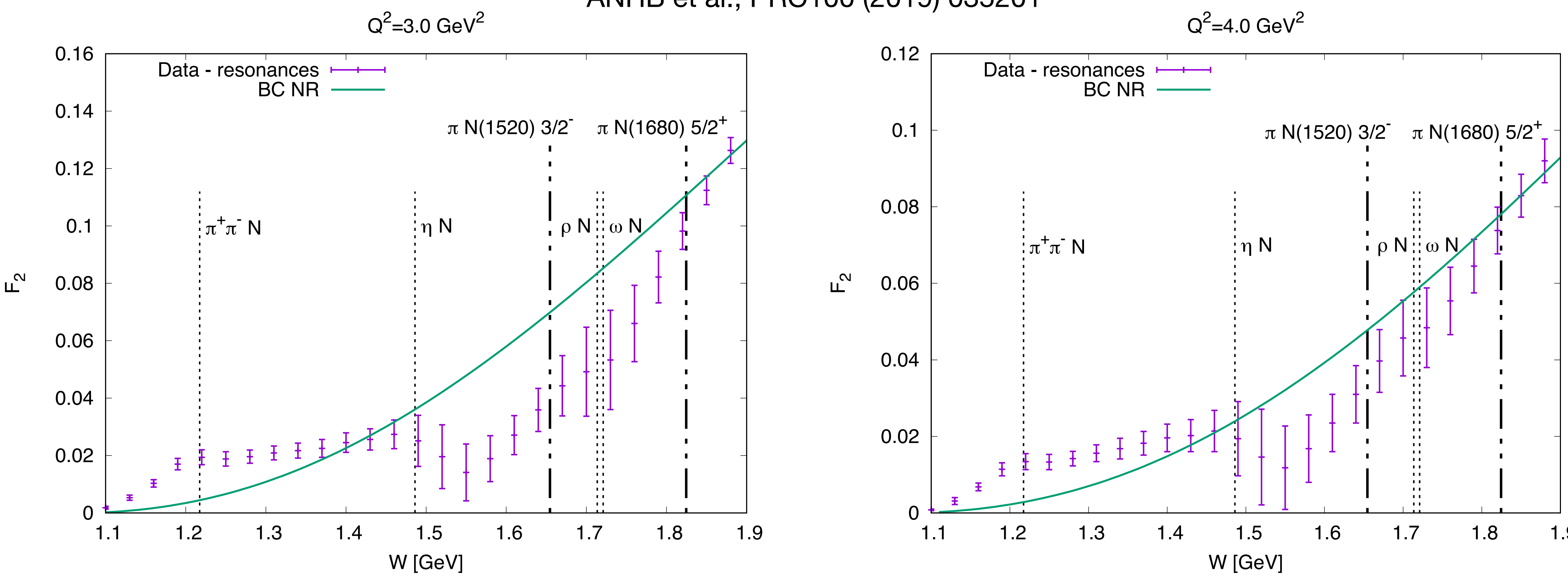

Continuation to high energies

$$\sigma_{T,L}(W,Q^2) = \sigma^R_{T,L}(W,Q^2) + \sigma^{NR}_{T,L}(W,Q^2)$$

Non-resonant
Regge piece

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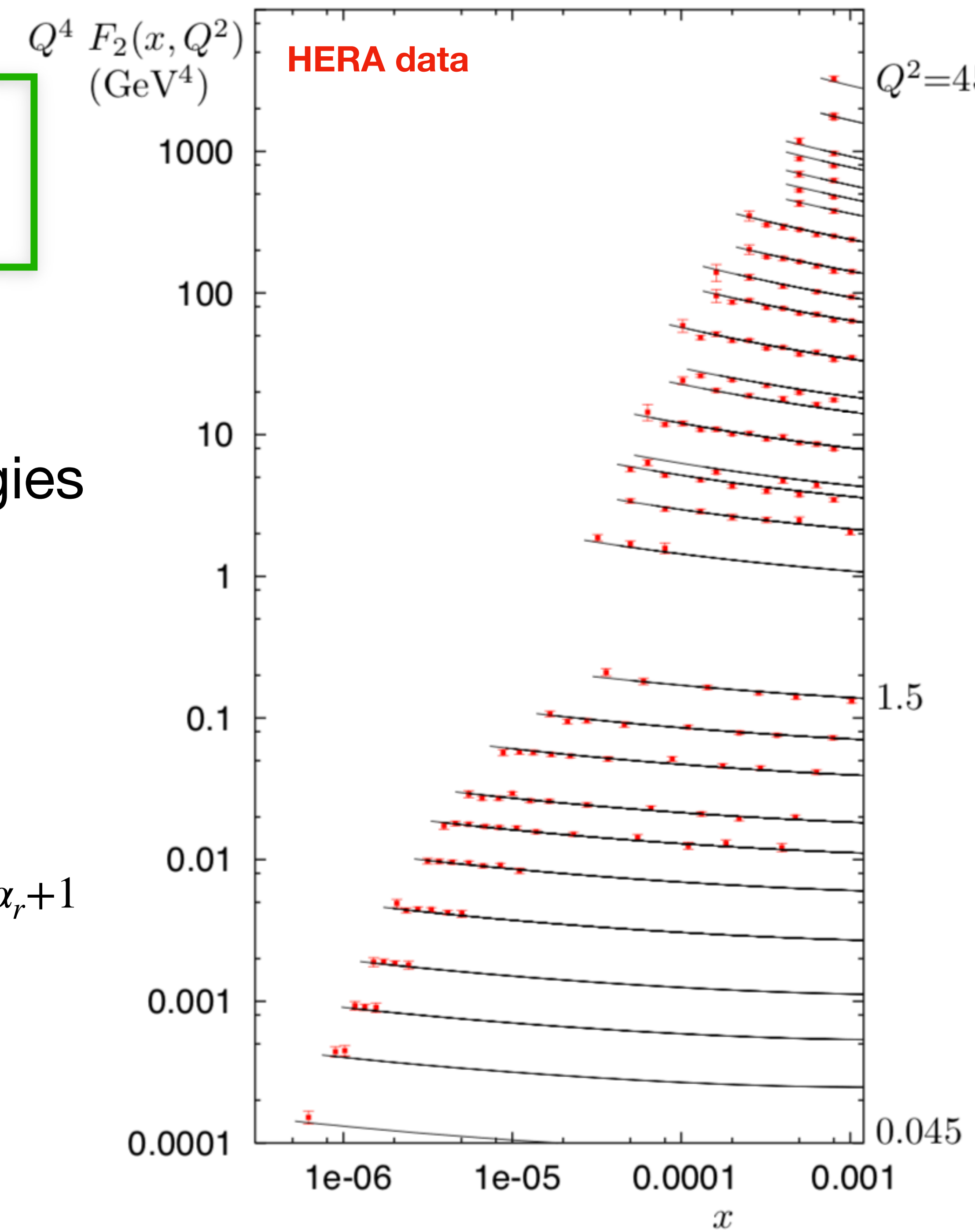

Excellent fit to data at high energies

Donnachie and Landshoff, PLB 595 (2004) 393

$$F_2(x,Q^2) = f_h(Q^2)x^{-\alpha_h+1} + f_s(Q^2)x^{-\alpha_s+1} + f_r(Q^2)x^{-\alpha_r+1}$$

$$x = \frac{Q^2}{2M_N\nu}$$

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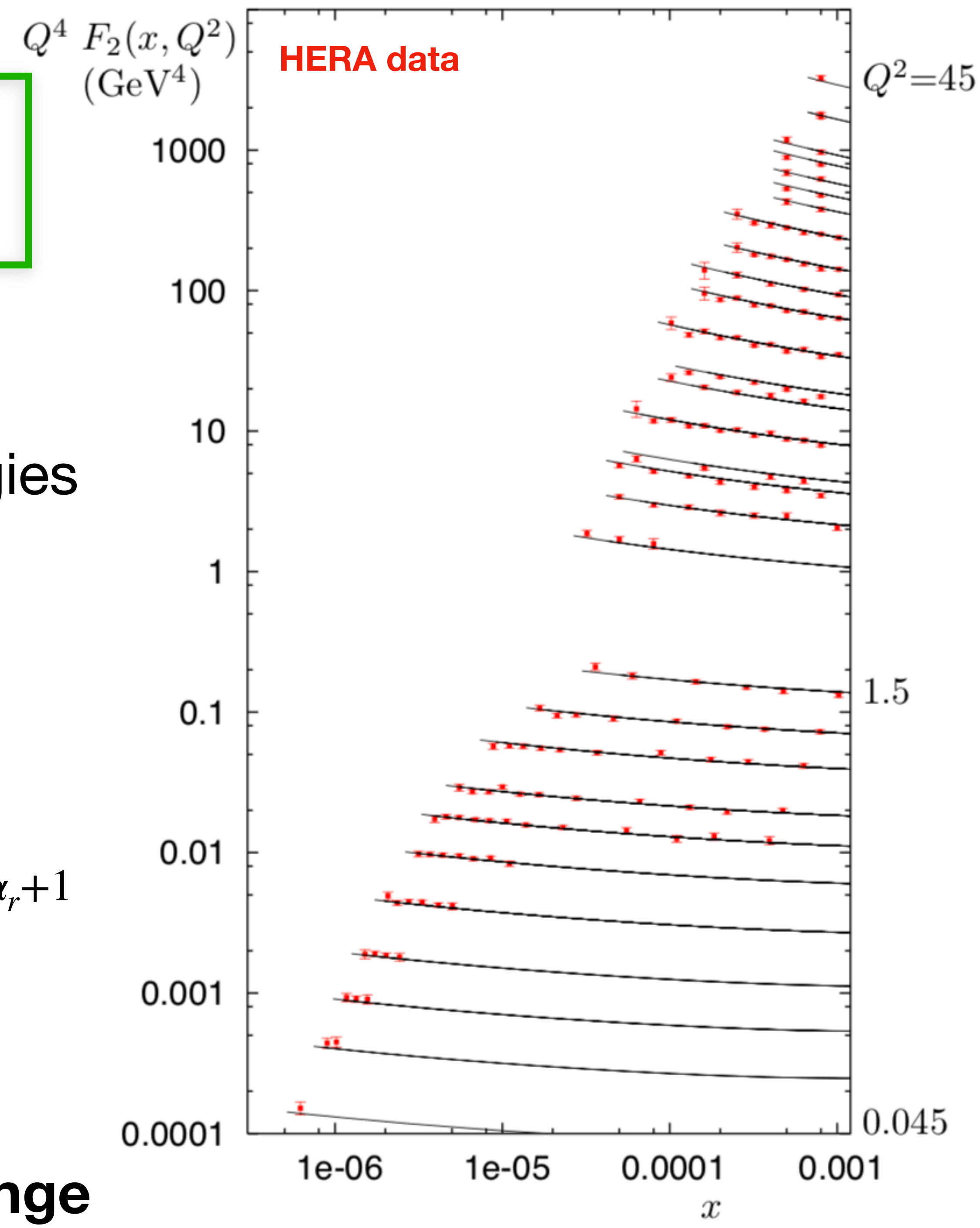

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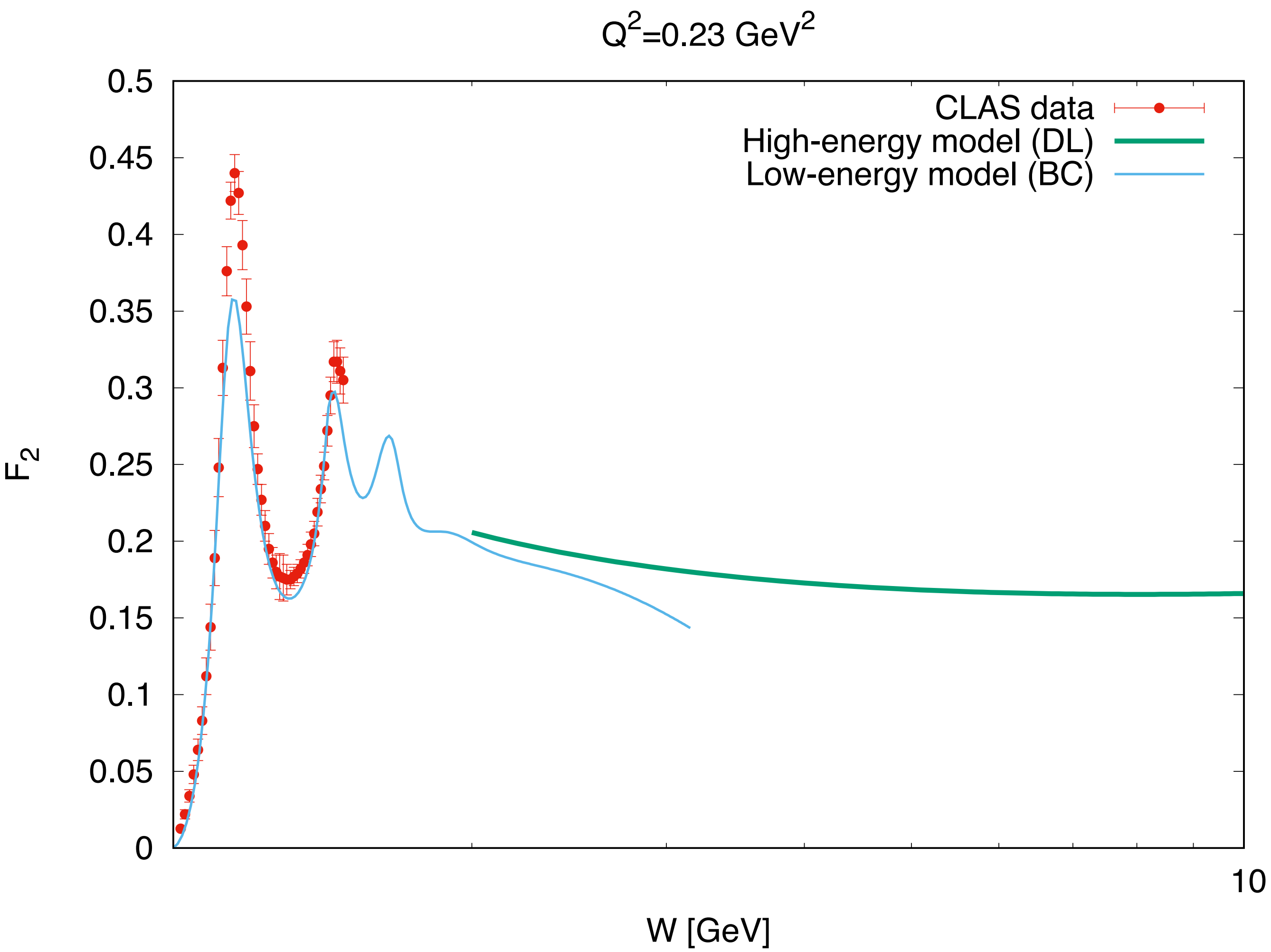
Hard pomeron
$\alpha_h = 1.452$

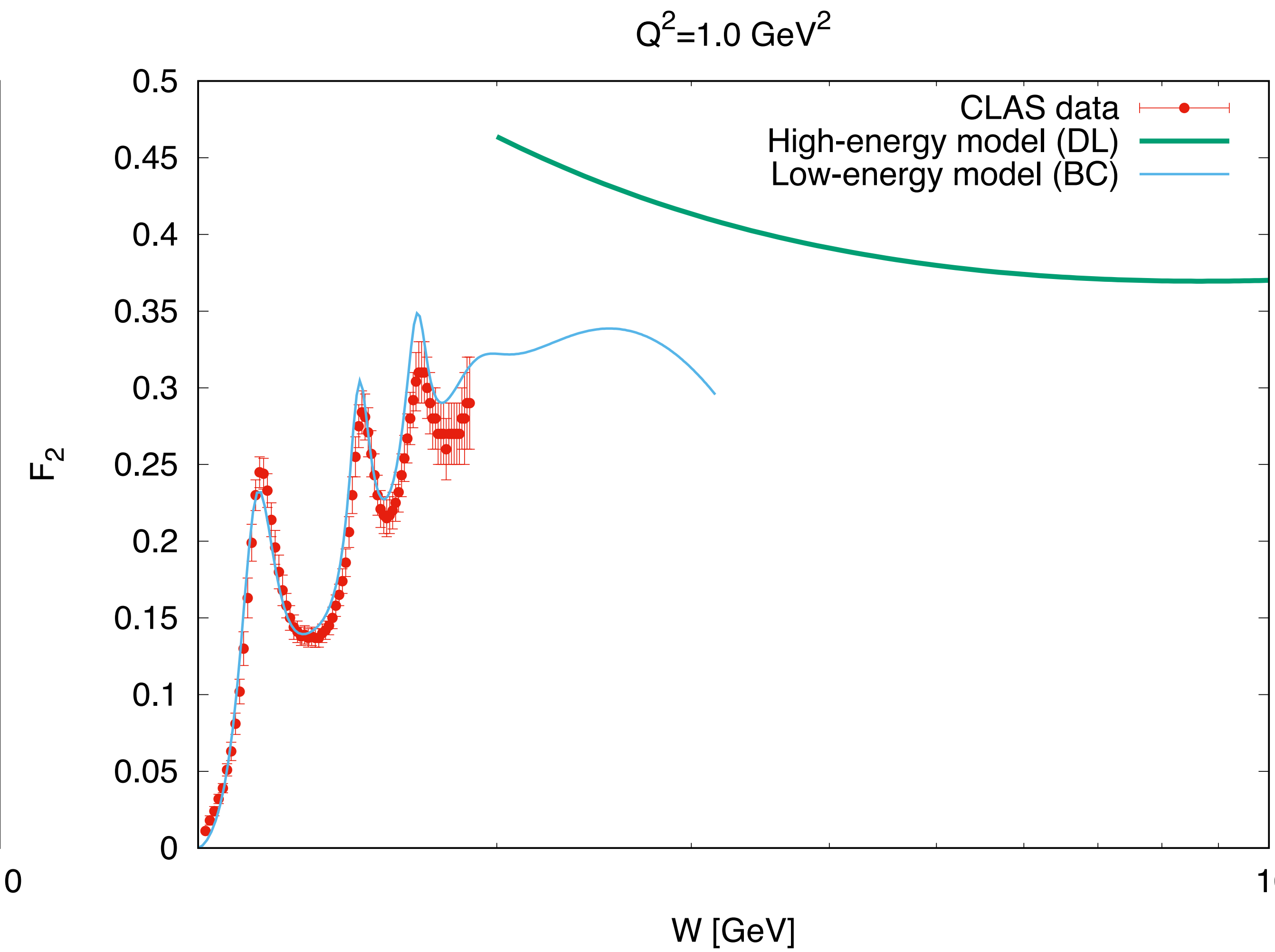
Soft pomeron
$\alpha_s = 1.0667$

Meson exchange
$\alpha_r = 0.524$

High and low energies: previous works

BC: Christy and Bosted, PRC 86 (2010) 055213 DL: Donnachie and Landshoff, PLB 595 (2004) 393 CLAS data: *http://clas.sinp.msu.ru/strfun/*





- Excellent description of data at low and high energies, separately
- At low Q^2: high- and low-energy theories compatible in overlap region
- At slightly **higher Q^2: huge gap** between high and low-energy models
- Leads to difficulties in extracting observables integrated over x (or energies)

Proposal for updated parametrization

$$F_1(\nu, Q^2) = F_1^{\text{res}}(\nu, Q^2) + \sum_{i=0}^{2} \gamma_{\alpha_i}(Q^2)(\nu - \nu_{\text{thr}})^{\alpha_i} \left(1 - \frac{\nu_{\text{thr}}}{\nu}\right)^{a_i(Q^2)} \left(1 + \frac{\nu_{\text{thr}}}{\nu}\right)^{b_i(Q^2)}$$

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- Automatically recovers Regge behaviour at high energies for $\alpha_i < 1$

$$F_1(\nu, Q^2) \longrightarrow \sum_{i=0}^{2} \gamma_{\alpha_i}(Q^2)\nu^{\alpha_i}$$

- For the Pomerons we have the requirement

$$b_i(Q^2) = a_i(Q^2) + \alpha_i, \quad 1 \leq \alpha_i < 2$$

- Implements threshold and resonant behaviour

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Last and definitely not least: obeys dispersion relations!

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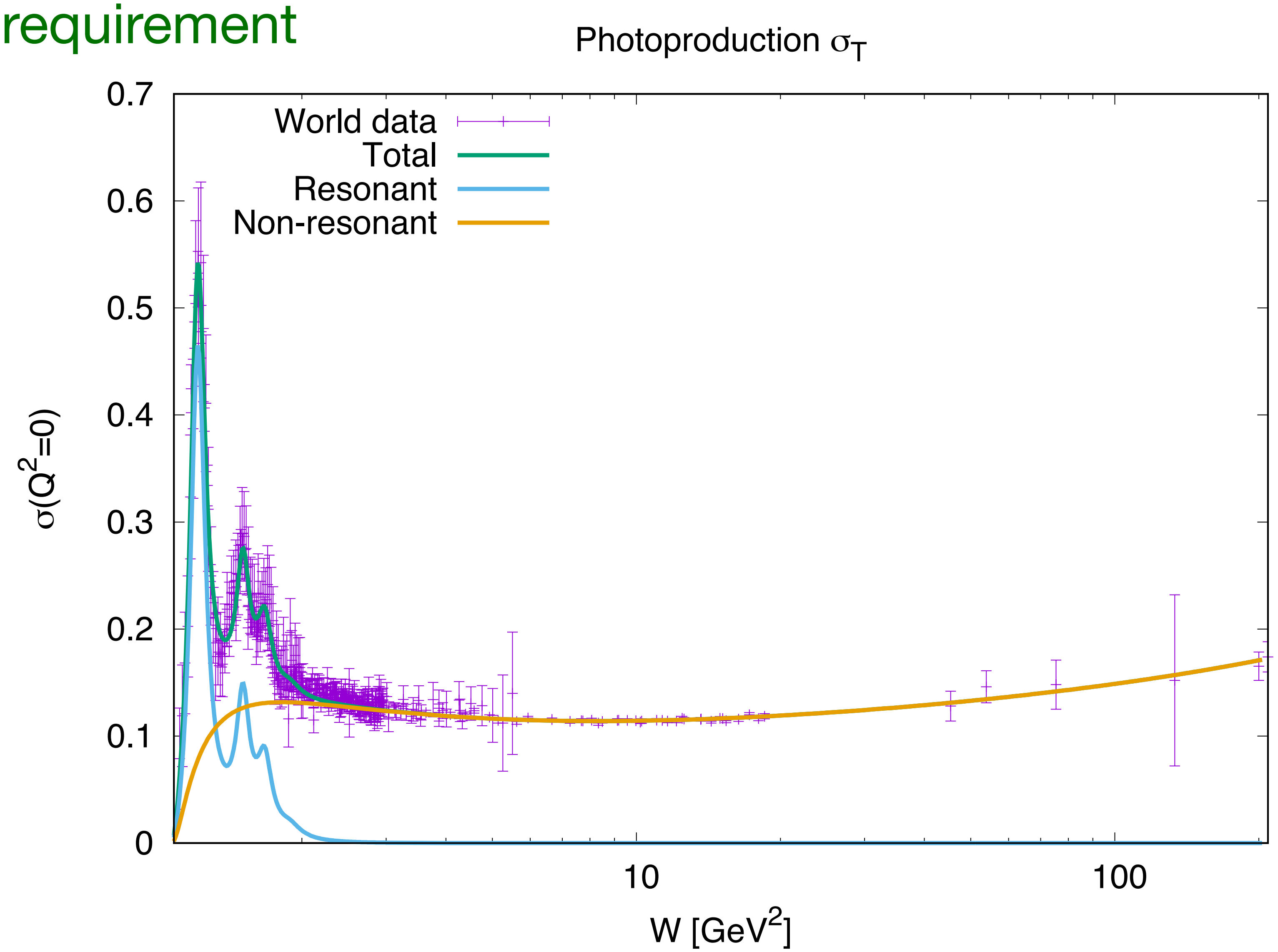

Dispersion relation and Lamb shift

VVCS amplitude T_1 obeys once-subtracted dispersion relation

$$\Re T_1(\nu, Q^2) = \qquad \frac{2\alpha\nu^2}{\pi M}\int_{\nu_0}^{\infty} \mathrm{d}\nu' \frac{F_1(\nu', Q^2)}{\nu'(\nu'^2 - \nu^2)}$$

Dispersion relation and Lamb shift

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$$\Re T_1(\nu, Q^2) = \frac{2\alpha\nu^2}{\pi M}\int_{\nu_0}^{\infty} \mathrm{d}\nu' \frac{F_1(\nu', Q^2)}{\nu'(\nu'^2 - \nu^2)}$$

$$F_1(\nu', Q^2) \rightarrow \propto \Im T_1(\nu, Q^2)$$

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Our formalism will allow the phenomenological determination of the subtraction contribution to the hydrogen Lamb shift for the first time!

$$\Delta E^{\text{subtr.}}(nS) = \frac{2e^2 m\phi_n^2}{\pi}\int_0^{\infty} \frac{\mathrm{d}Q}{Q^3} \frac{v_l + 2}{(1+v_l)^2}\bar{T}_1(0,Q^2)$$

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Source of the largest uncertainty of hadronic contribution to Lamb shift
relevant to improve muonic spectroscopy in proton charge radius extraction

Birse and Govern, EPJ A 48 (2012) 120

Alarcón et al., EPJ C 74 (2014) 2852

Tomalak and Vanderhaeghen, EPJ C 76 (2016) 125

Fit constraints

$$F_1(\nu, Q^2) = F_1^{\text{res}}(\nu, Q^2) + \sum_{i=0}^{2} \gamma_{\alpha_i}(Q^2)(\nu - \nu_{\text{thr}})^{\alpha_i} \left(1 - \frac{\nu_{\text{thr}}}{\nu}\right)^{a_i(Q^2)} \left(1 + \frac{\nu_{\text{thr}}}{\nu}\right)^{b_i(Q^2)}$$

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- F1 ~ imaginary part of T1:
  additional constraints obtained by imposing for subtraction function

$$\mathbf{\overline{T}_1(0, 0) = 0}$$

  and using the experimental value of the magnetic polarizability:

$$\beta_{M1} = \frac{\mathrm{d}}{\mathrm{d}Q^2} \overline{T}_1(0, Q^2) \bigg|_{Q^2=0}$$

- Constrains the fit parameter space even further

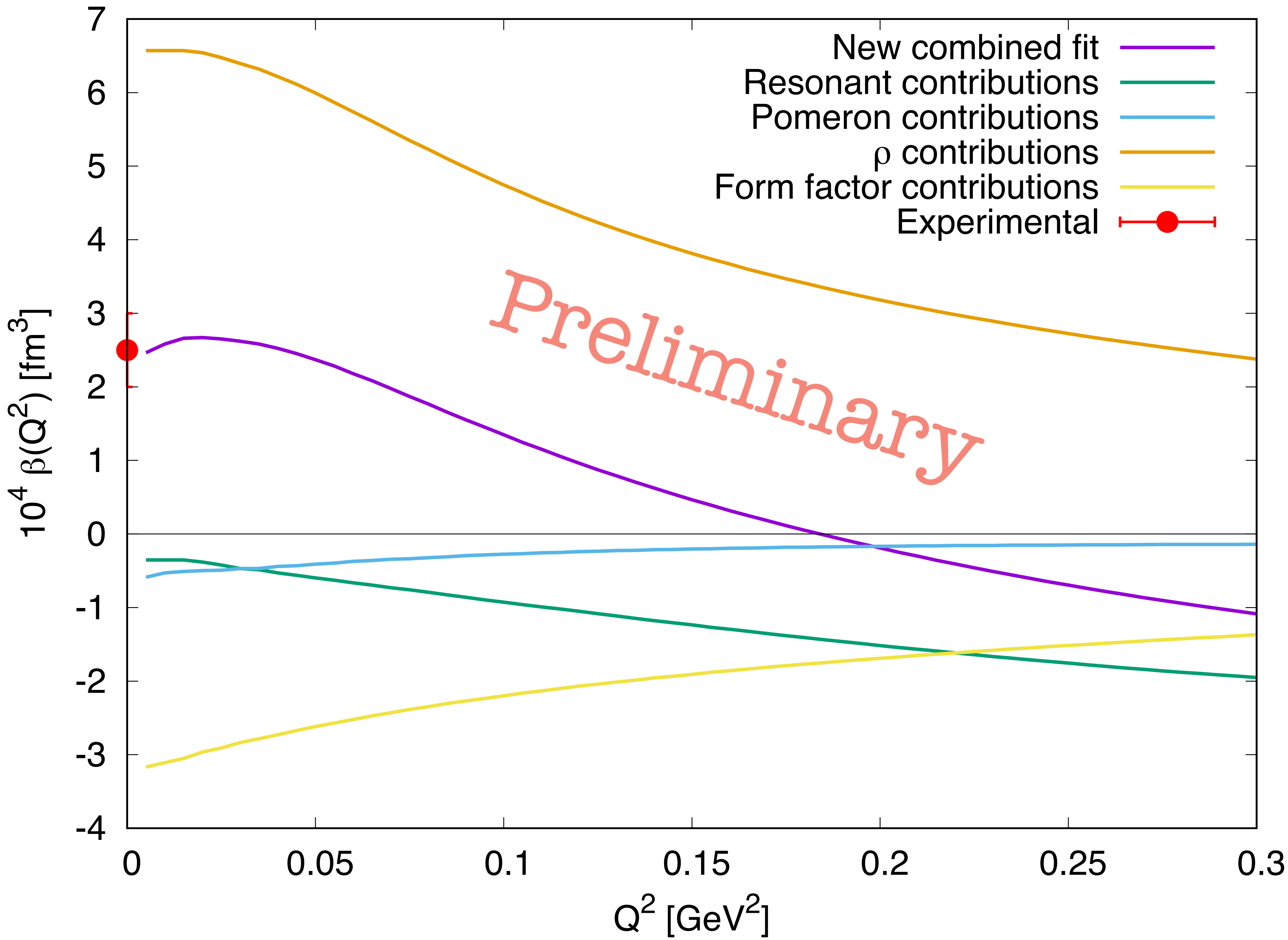
Inclusive and exclusive reactions: can we use one for the other?

Results of resonant contributions to inclusive observables

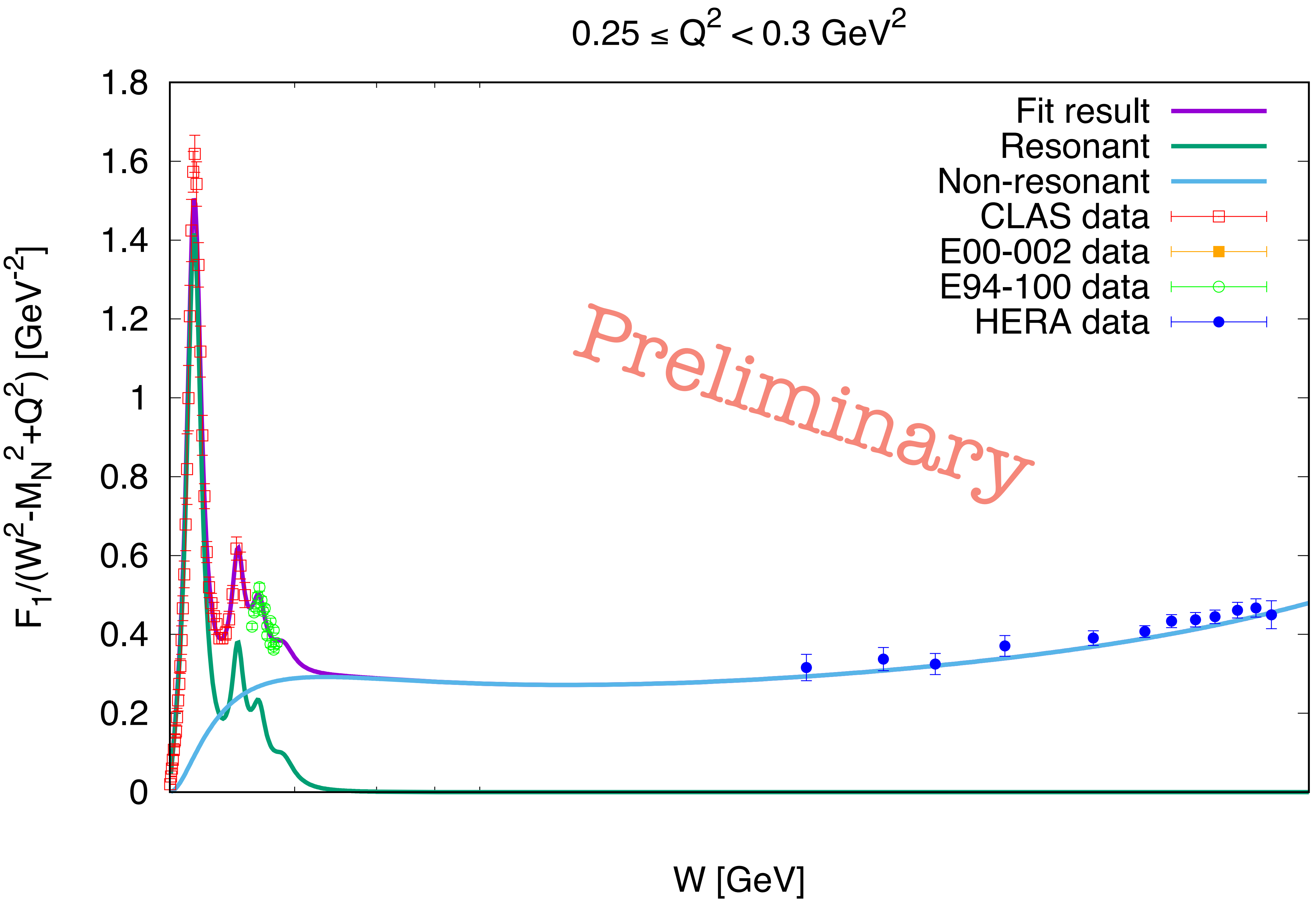
Non-resonant piece: low and high energies communicate

Results of combined description (preliminary)

Magnetic polarizability: our fit results



Structure functions at non-vanishing Q²



First combined model for low and high energies!

Summary

- New resonance model: **N* electrocouplings** from CLAS(12) allow to describe the **resonant contributions** to inclusive electron-scattering
- Intricate behaviour with W and Q^2 in the resonance regime
- First estimates for the non-resonant behaviour

Summary

- New resonance model: **N* electrocouplings** from CLAS(12) allow to describe the **resonant contributions** to inclusive electron-scattering

- Intricate behaviour with W and Q^2 in the resonance regime

- First estimates for the non-resonant behaviour

Work in progress

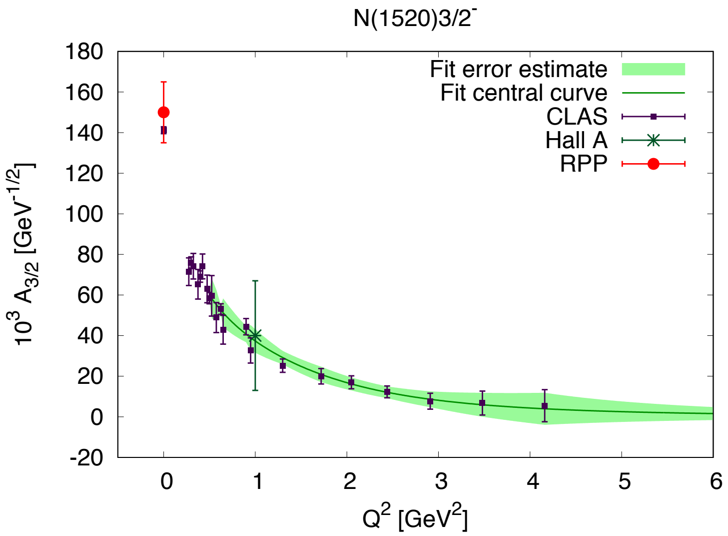
- Analytical transition between low and high *x* (or energies): important for VVCS subtraction function

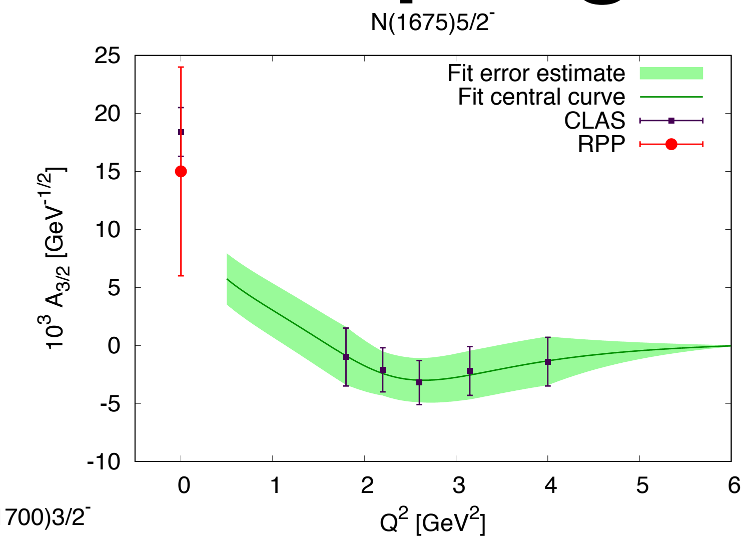
- First phenomenological determination of subtraction contribution to Lamb shift

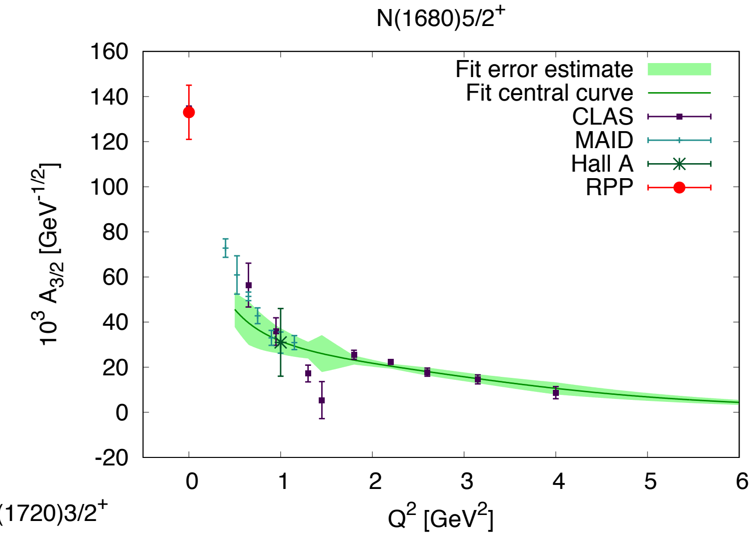
- Updates on inclusive and exclusive electron scattering CLAS12: coming soon!

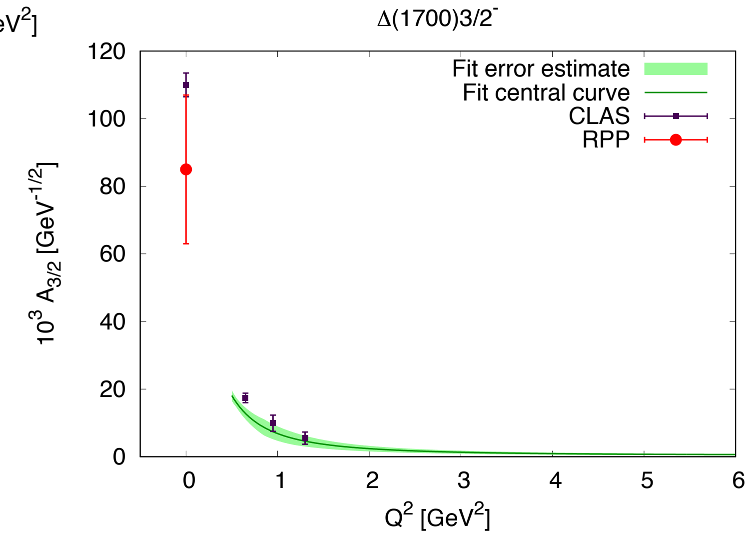
Backup

Further electrocoupling data examples

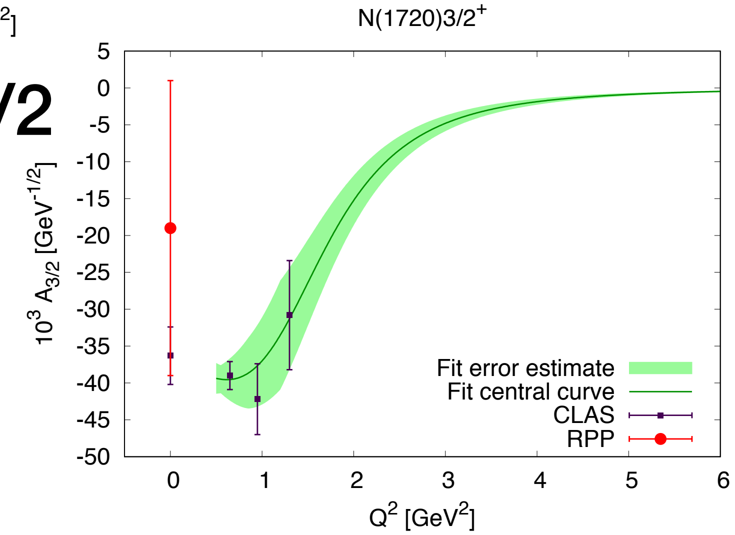




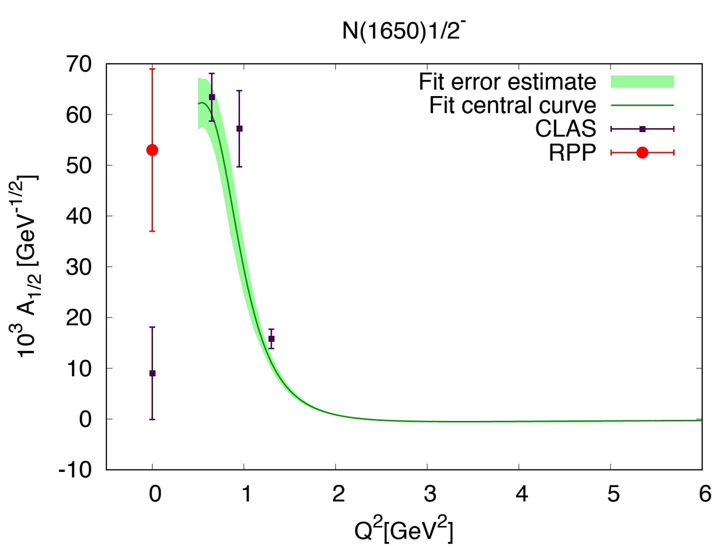


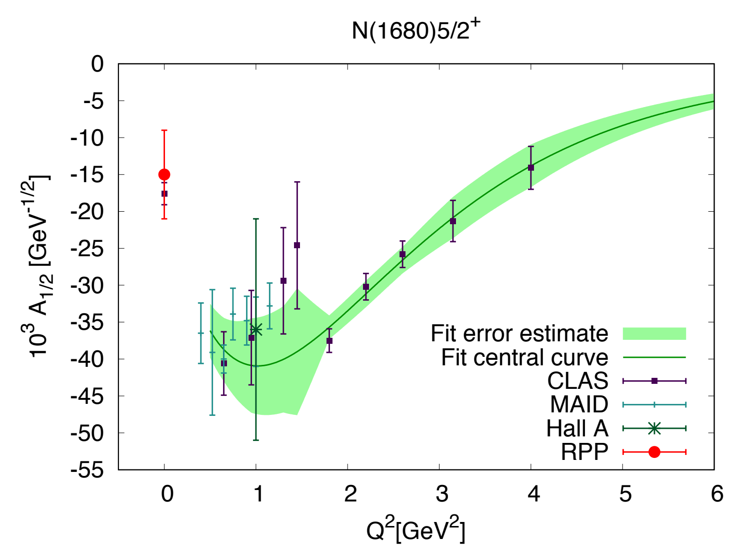


$A_{3/2}$

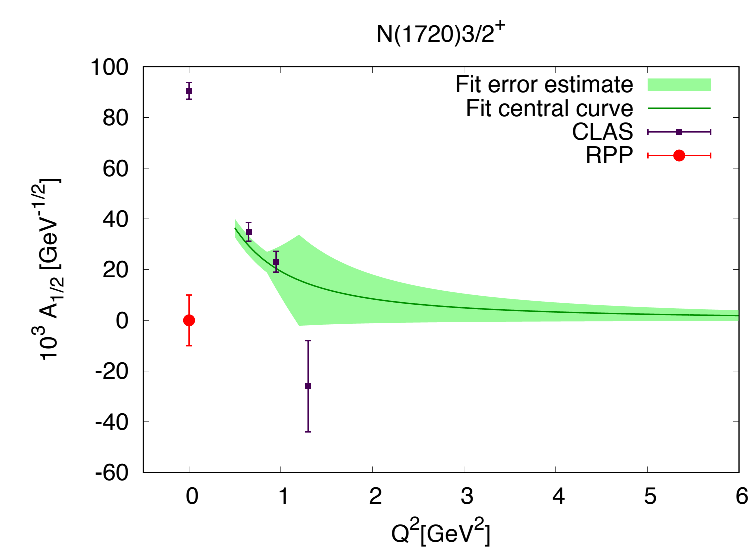


$A_{1/2}$

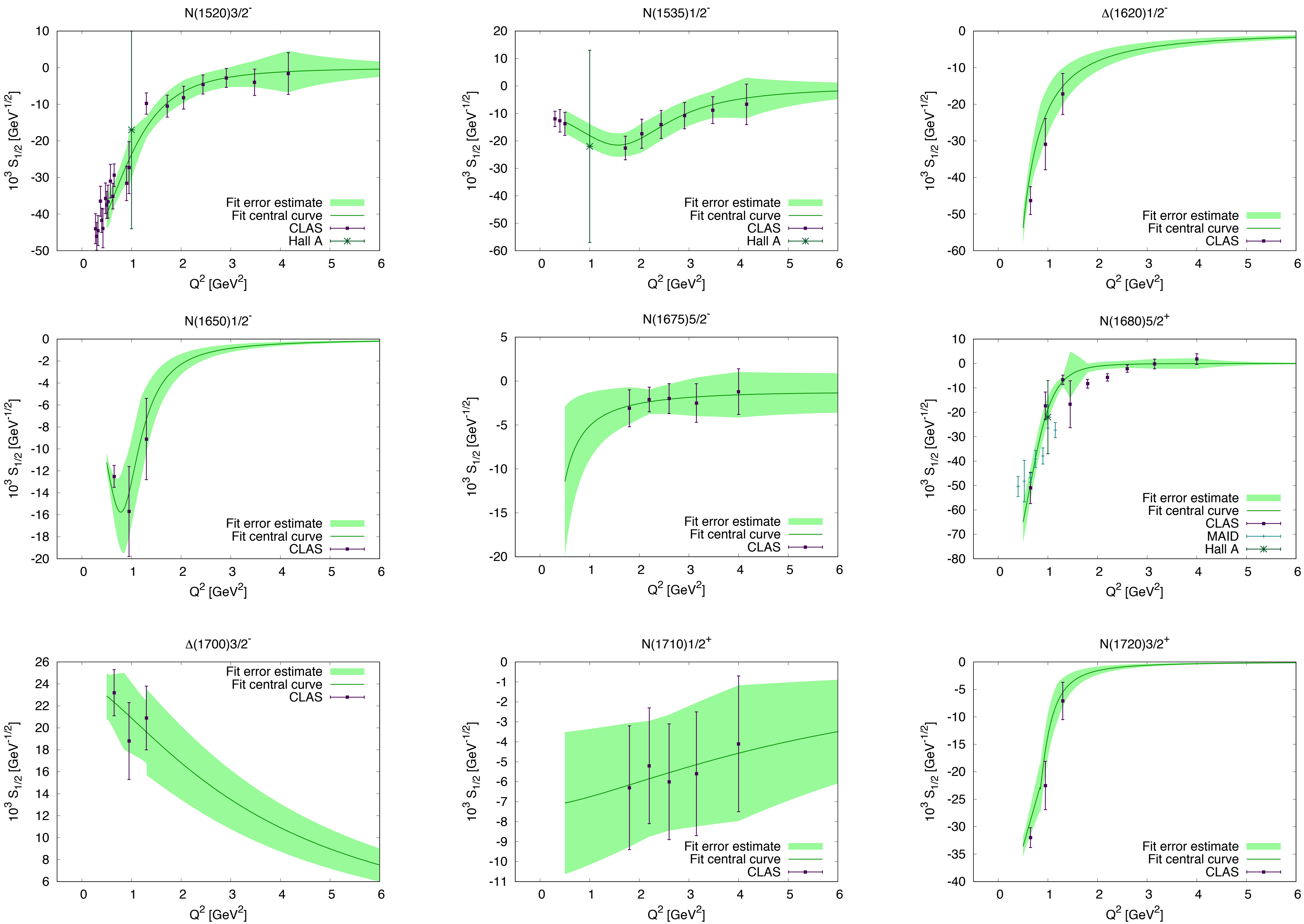




X

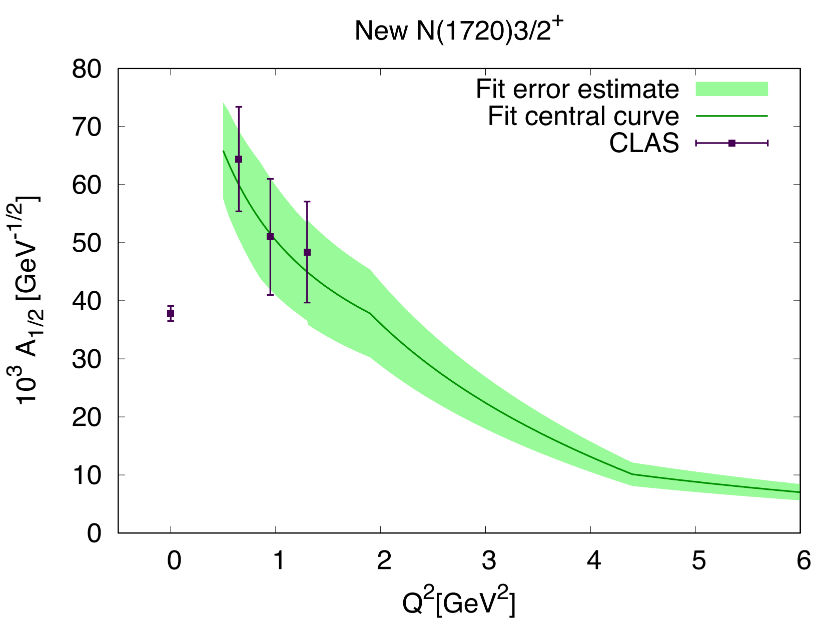


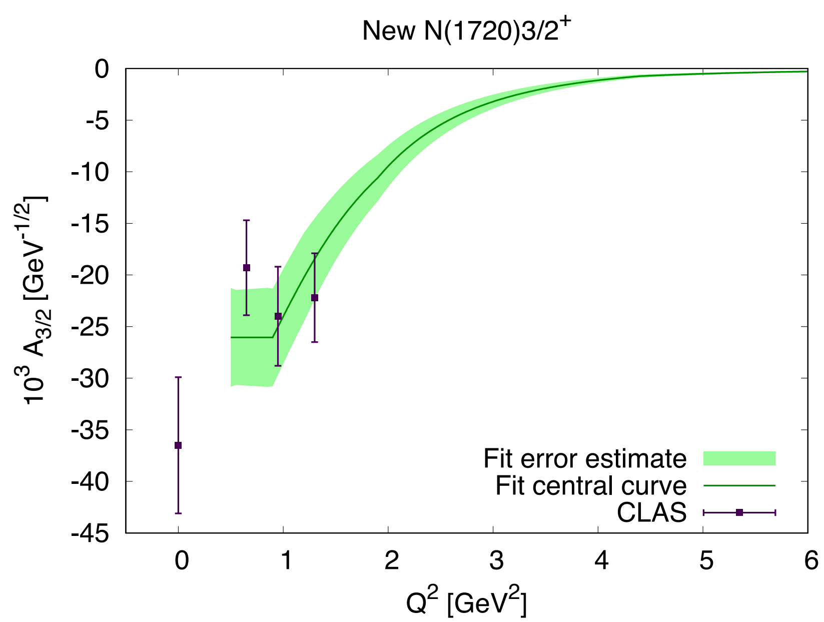
$S_{1/2}$

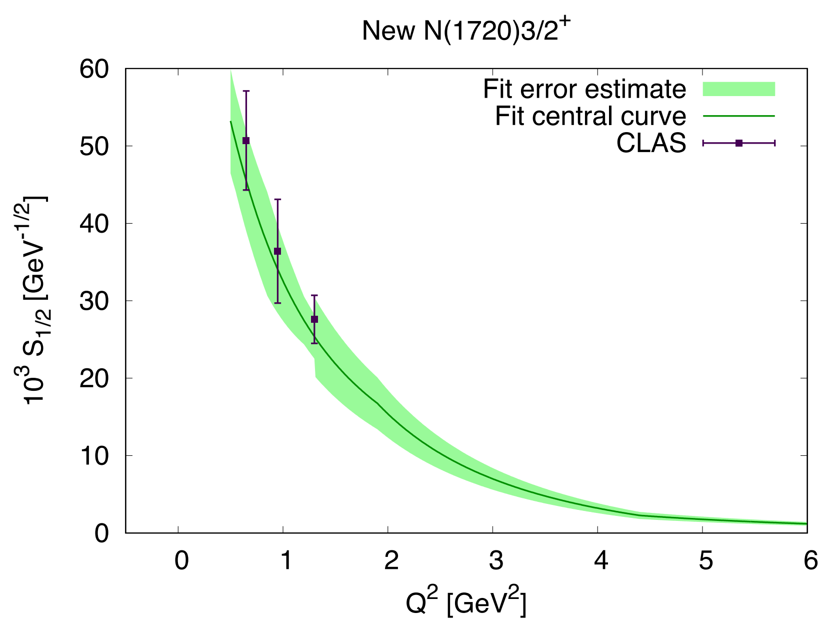


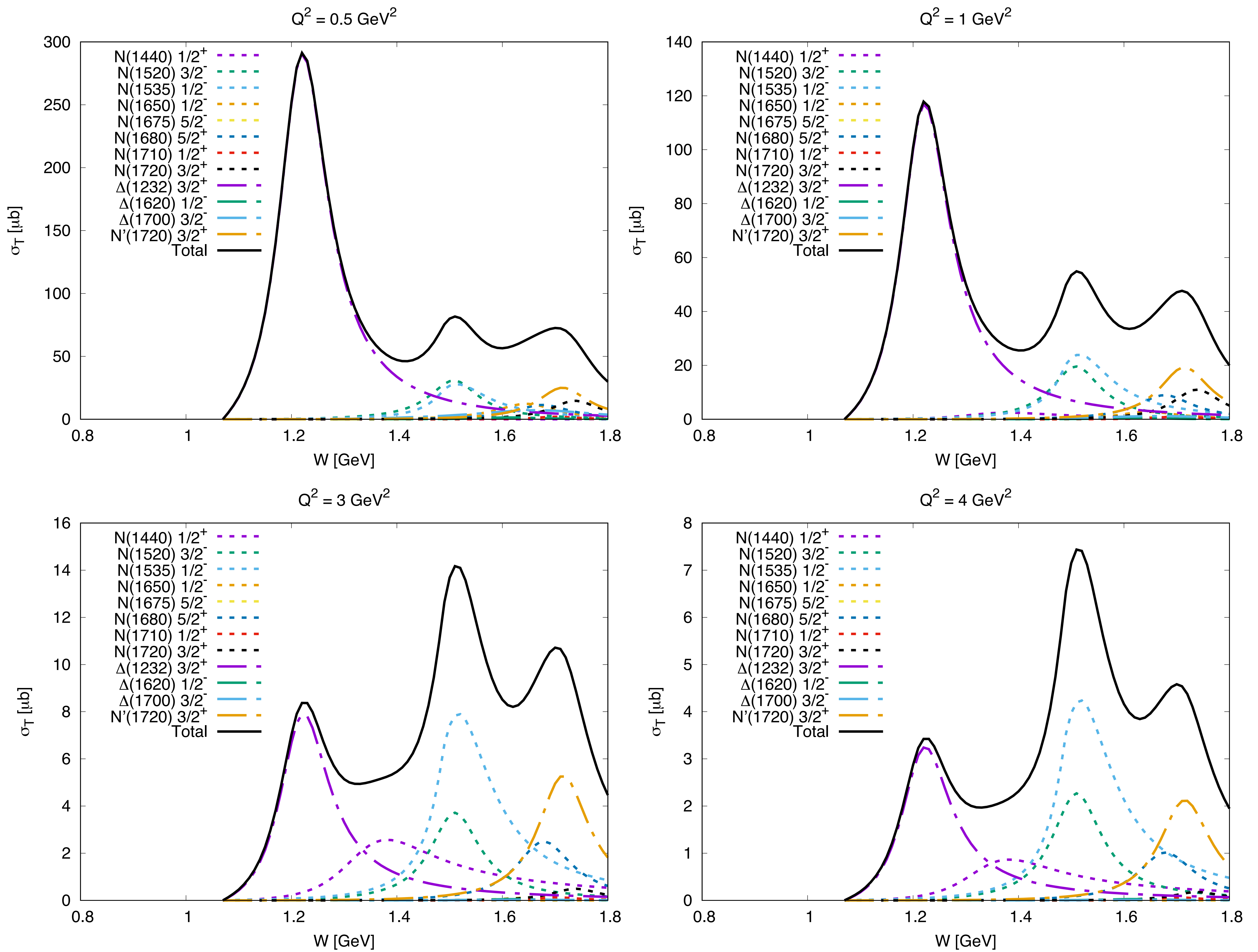
N(1520)3/2-
N(1535)1/2-
Δ(1620)1/2-
N(1650)1/2-
N(1675)5/2-
N(1680)5/2+
Δ(1700)3/2-
N(1710)1/2+
N(1720)3/2+
10^3 S_1/2 [GeV^-1/2]
Q^2 [GeV^2]
Fit error estimate
Fit central curve
CLAS
Hall A
MAID

New resonance

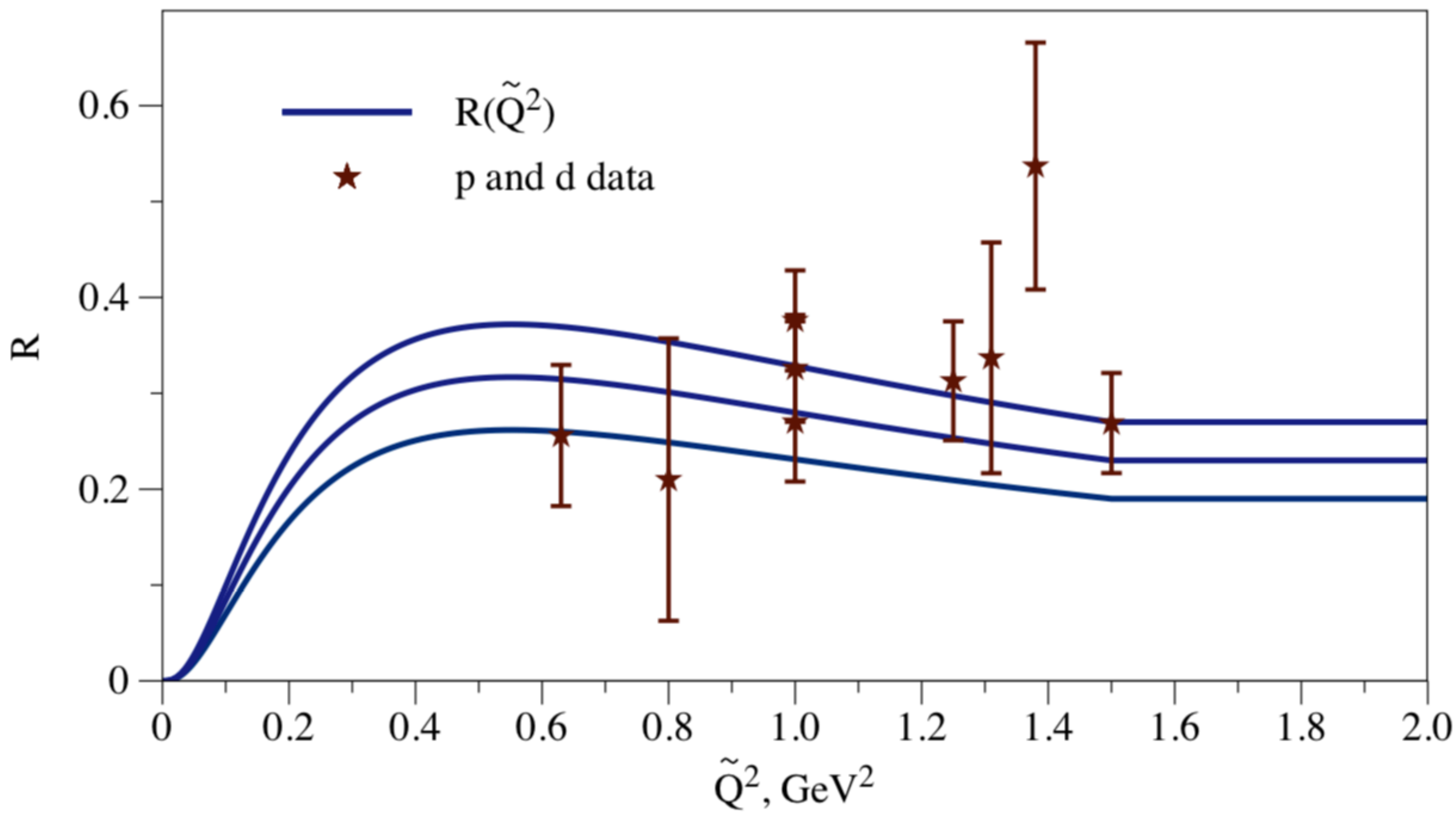






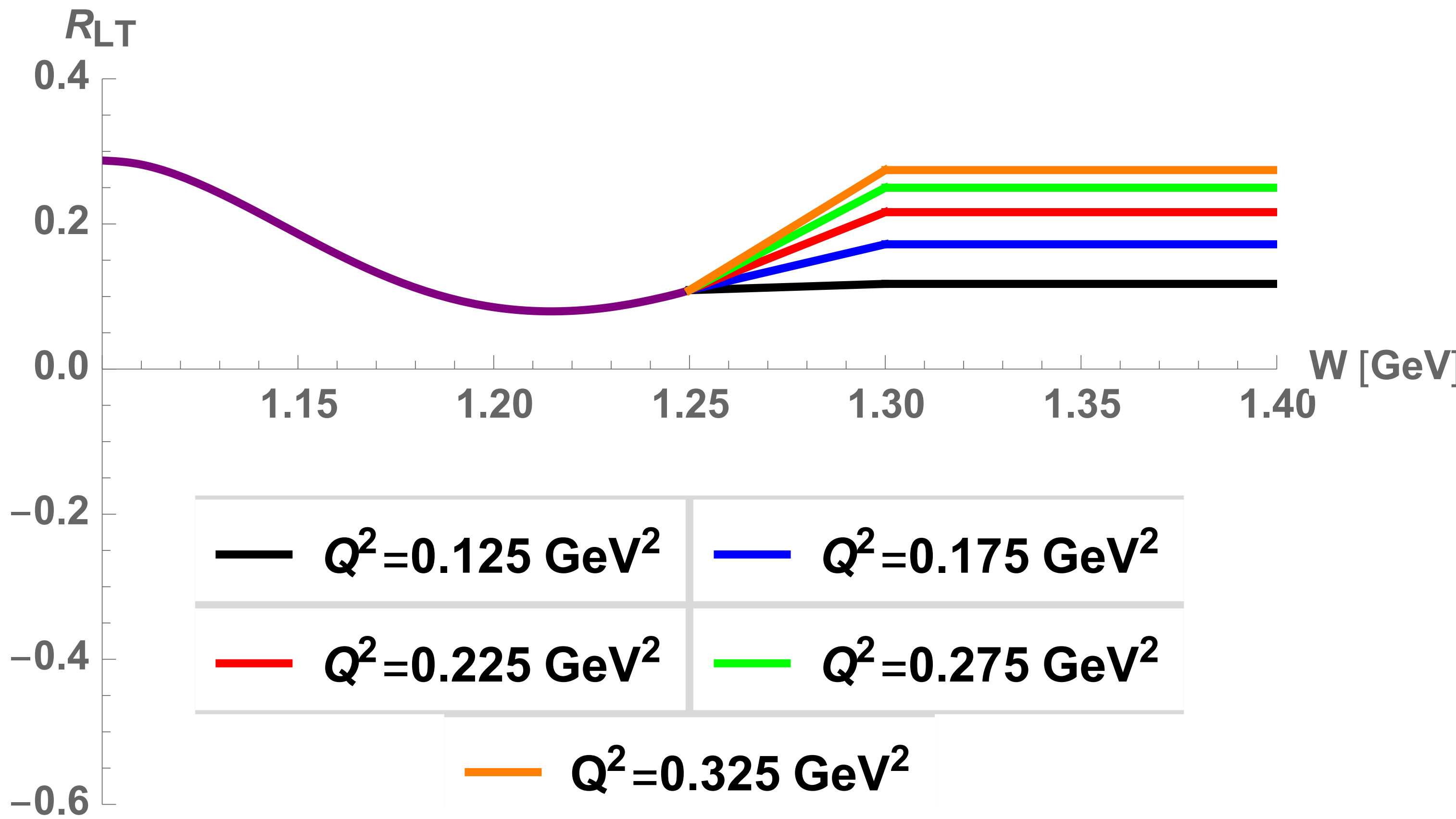


Q^2 = 0.5 GeV^2
Q^2 = 1 GeV^2
Q^2 = 3 GeV^2
Q^2 = 4 GeV^2
σ_T [μb]
W [GeV]
N(1440) 1/2$^+$
N(1520) 3/2$^-$
N(1535) 1/2$^-$
N(1650) 1/2$^-$
N(1675) 5/2$^-$
N(1680) 5/2$^+$
N(1710) 1/2$^+$
N(1720) 3/2$^+$
Δ(1232) 3/2$^+$
Δ(1620) 1/2$^-$
Δ(1700) 3/2$^-$
N'(1720) 3/2$^+$
Total

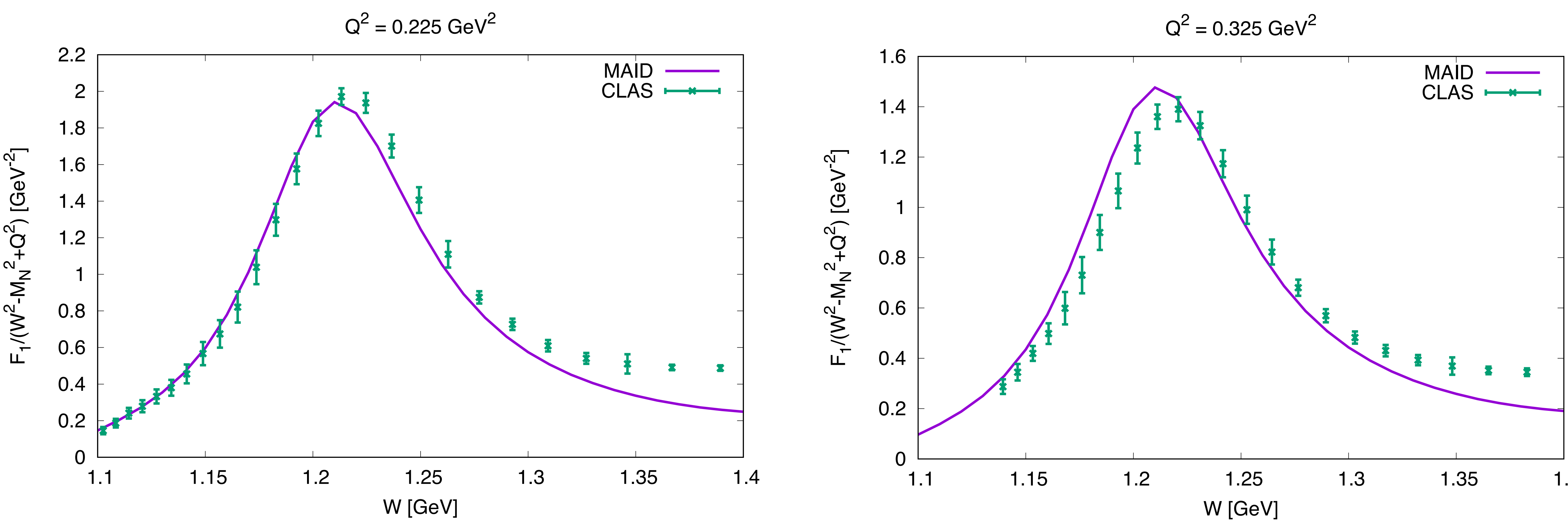


R($\tilde{Q}^2$)
p and d data
R
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
$\tilde{Q}^2$, GeV^2

X

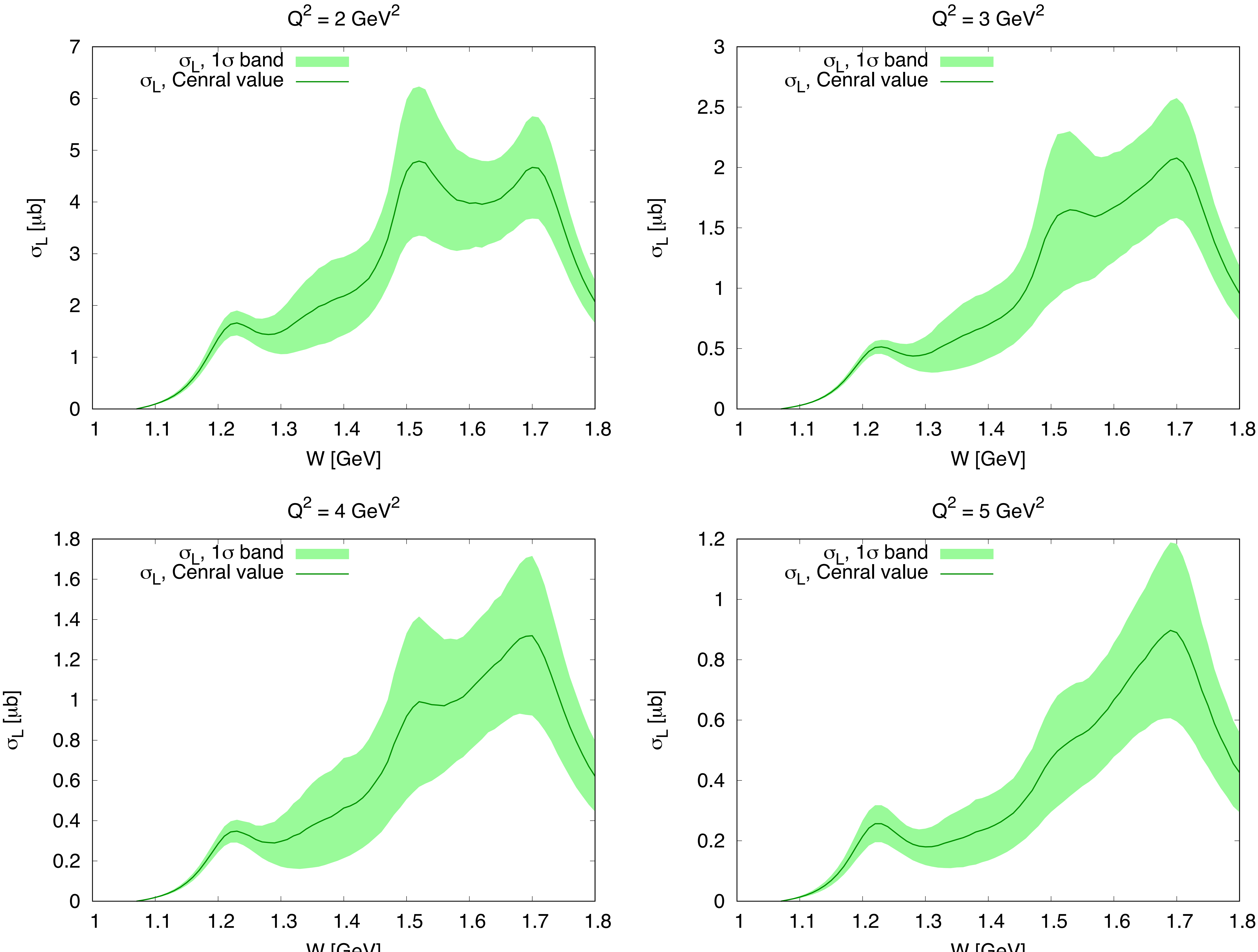


R_LT
0.4
0.2
0.0
−0.2
−0.4
−0.6
1.15
1.20
1.25
1.30
1.35
1.40
W [GeV]
Q2=0.125 GeV2
Q2=0.175 GeV2
Q2=0.225 GeV2
Q2=0.275 GeV2
Q2=0.325 GeV2



Q2 = 0.225 GeV2
MAID
CLAS
F1/(W2-MN2+Q2) [GeV-2]
W [GeV]
Q2 = 0.325 GeV2
MAID
CLAS
F1/(W2-MN2+Q2) [GeV-2]
W [GeV]

X


Q^2 = 2 GeV^2
Q^2 = 3 GeV^2
Q^2 = 4 GeV^2
Q^2 = 5 GeV^2
σ_L, 1σ band
σ_L, Cenral value
σ_L [μb]
W [GeV]

Subtraction function calculation

$$\bar{T}_1(0,Q^2) = T_1^{\mathrm{NR}}(0,Q^2) + \frac{\alpha}{M} F_D^2(Q^2) + \frac{2\alpha}{M} \int_{\nu_0}^{\infty} \mathrm{d}\nu \frac{F_1^{\mathrm{R}}(\nu, Q^2)}{\nu}$$