

Advances in Coupled Channel Approaches for N^* Parameter Extraction

Michael Döring, C. Granados, H. Haberzettl, Deborah Rönchen, R. Workman;
D. Glazier, J. Haidenbauer, D. Ireland, Maxim Mai, U.-G. Meißner

Strong QCD from Hadron Structure Experiments

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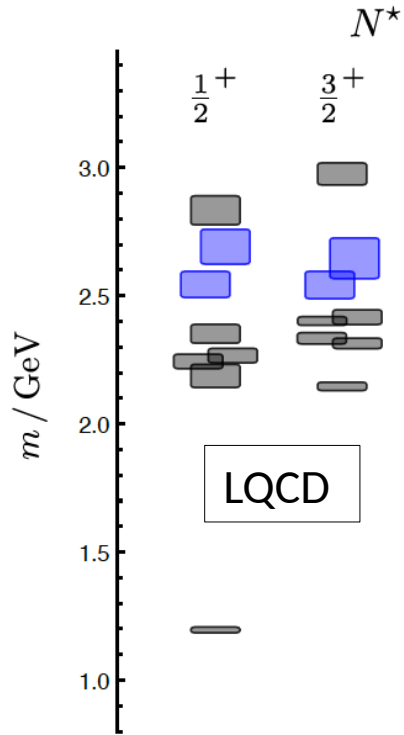


HPC support by JSC grant *jikp07*

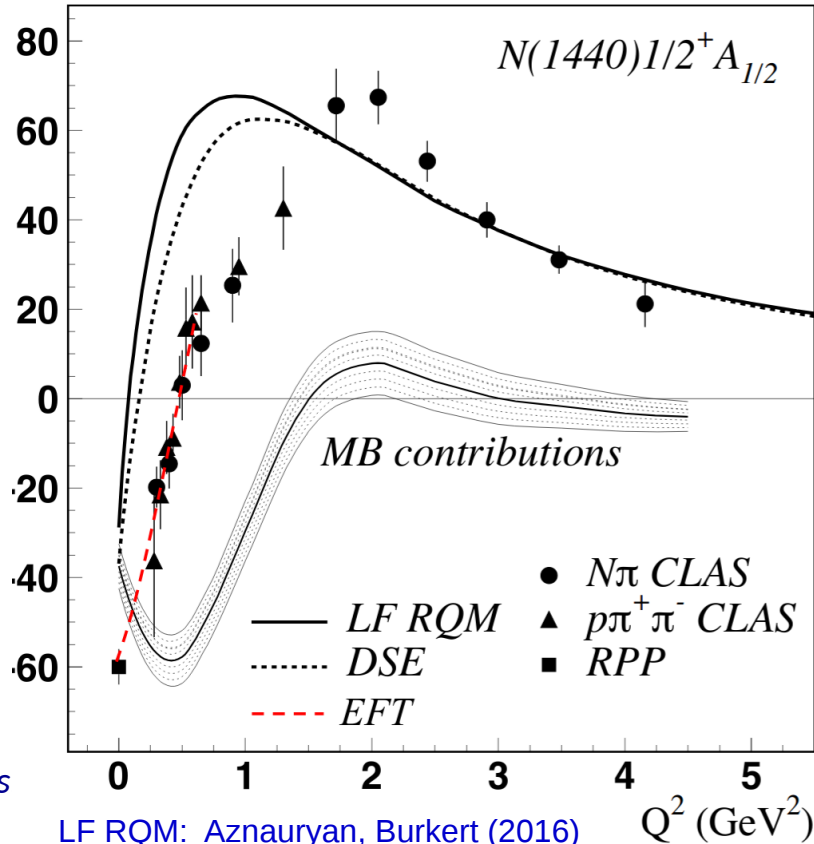
[several slides by
D. Rönchen and M. Mai]

Degrees of freedom: Quarks or hadrons?

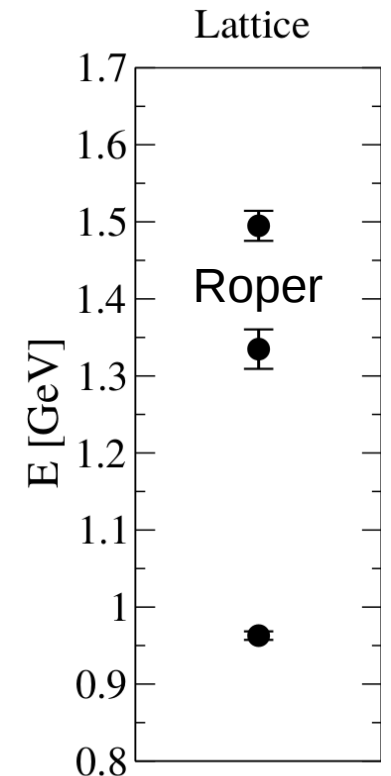
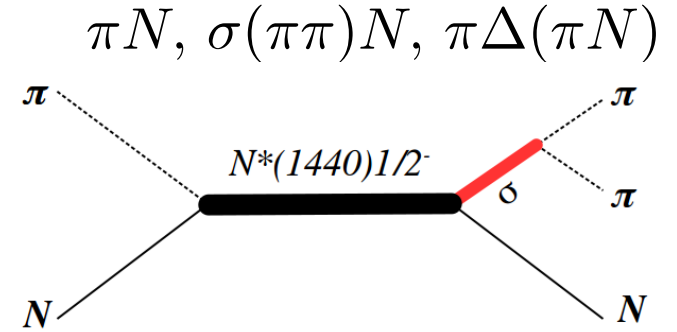
Hybrid Baryons



J.J. Dudek and R.G. Edwards
PRD85 (2012)



LF RQM: Aznauryan, Burkert (2016)
DSE: Segovia, Roberts, PRC (2016)
EFT: Bauer, Scherer, Tiator, PRC (2014)



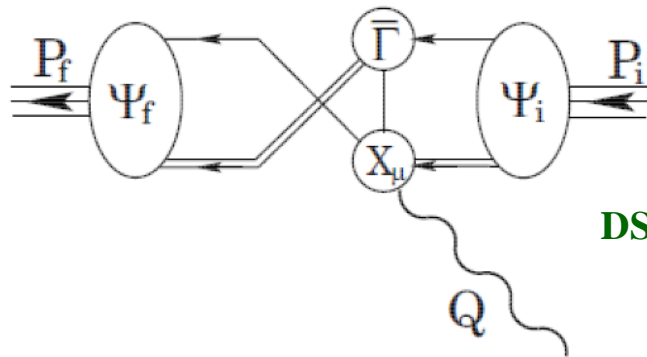
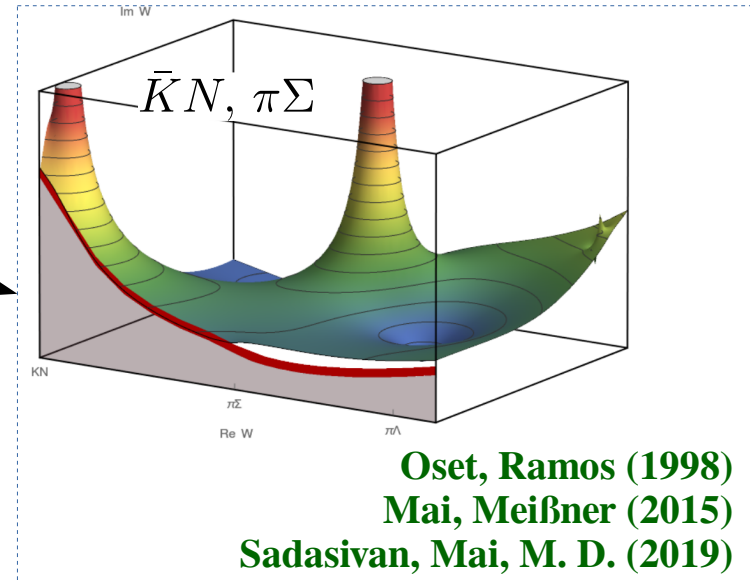
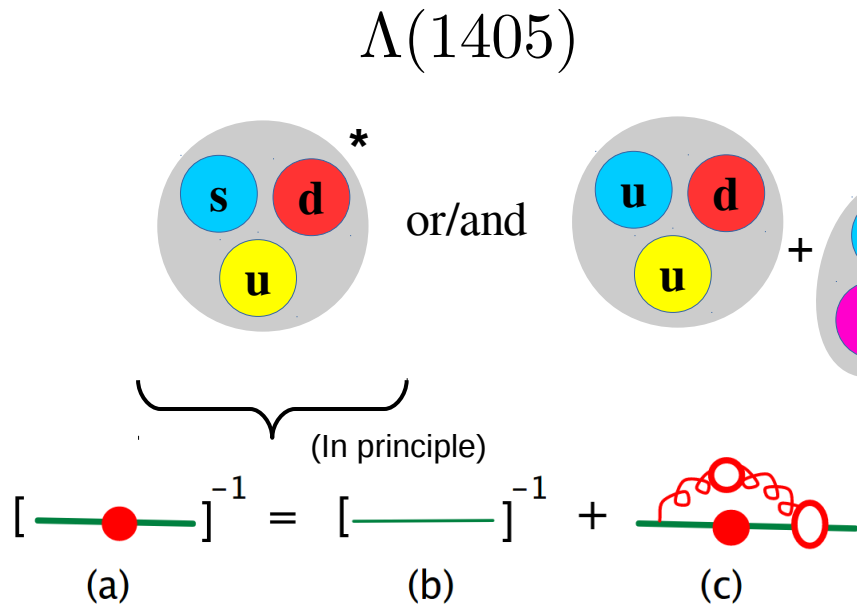
Data: [Lang et al., Phys.Rev. D95 (2017), 014510]

Hybrid states: same J^P values as q^3 baryons.
Identification? Measure Q^2 dependence of
electro-couplings (**CLAS 12**)

[parts of slide courtesy of V. Burkert]

- **QCD** at low energies
- Non-perturbative dynamics
 - Q1:** how many are there?
 - Q2:** what are they?

- *mass generation & confinement*
- rich spectrum of excited states
(missing resonance problem)
(2-quark/3-quark, hadron molecules, exotics,...)

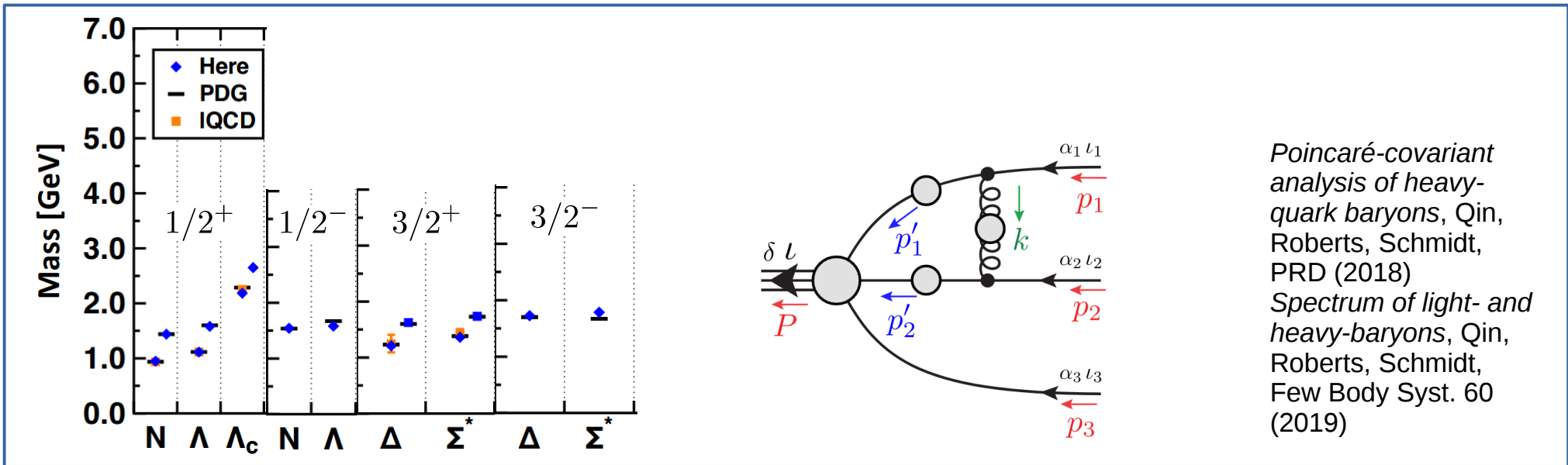
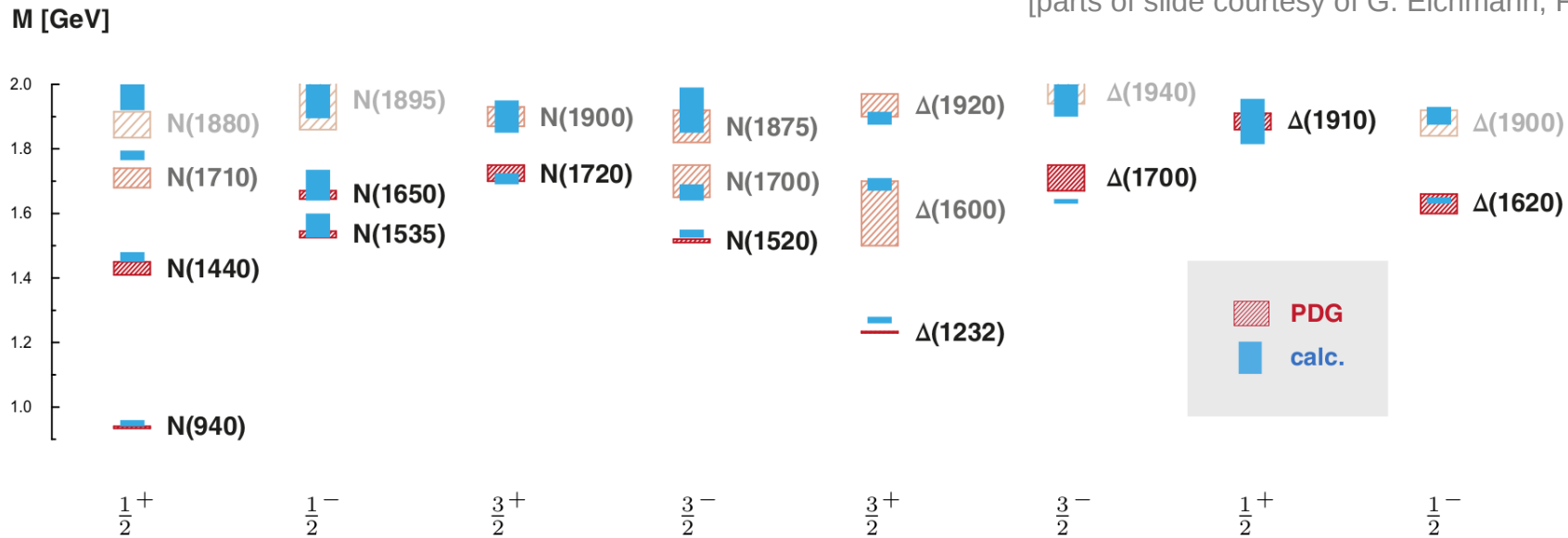


DSE (Wilson, Cloet, Chang, Roberts)

New results in dynamical quark picture

Quark-diquark with reduced pseudoscalar + vector diquarks: [GE, Fischer, Sanchis-Alepuz, PRD 94 \(2016\)](#)

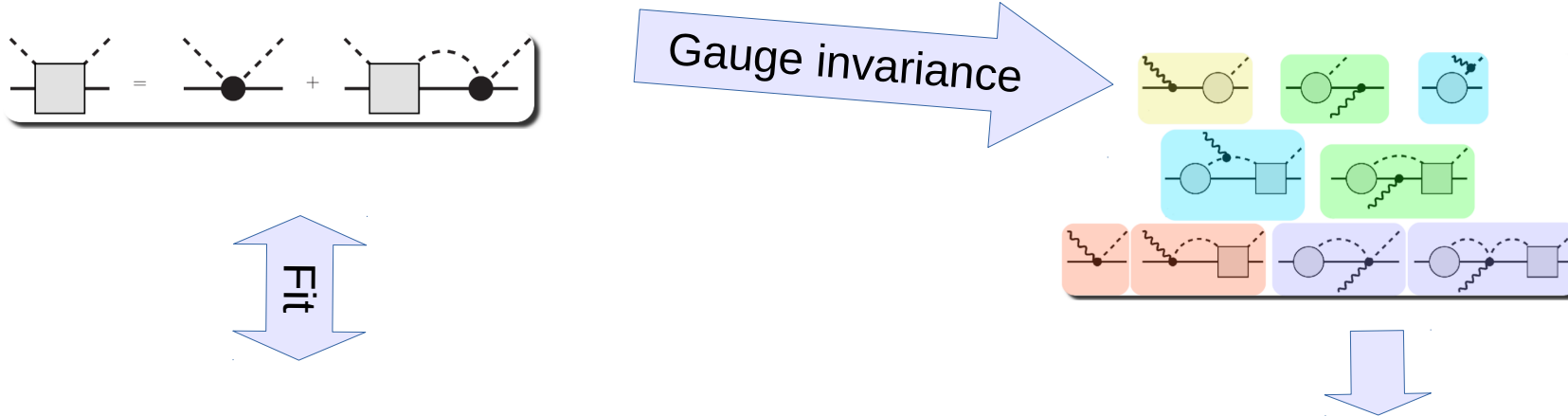
[parts of slide courtesy of G. Eichmann, Few Body 2018]



Using ONLY meson-baryon degrees of freedom (no explicit quark dynamics):

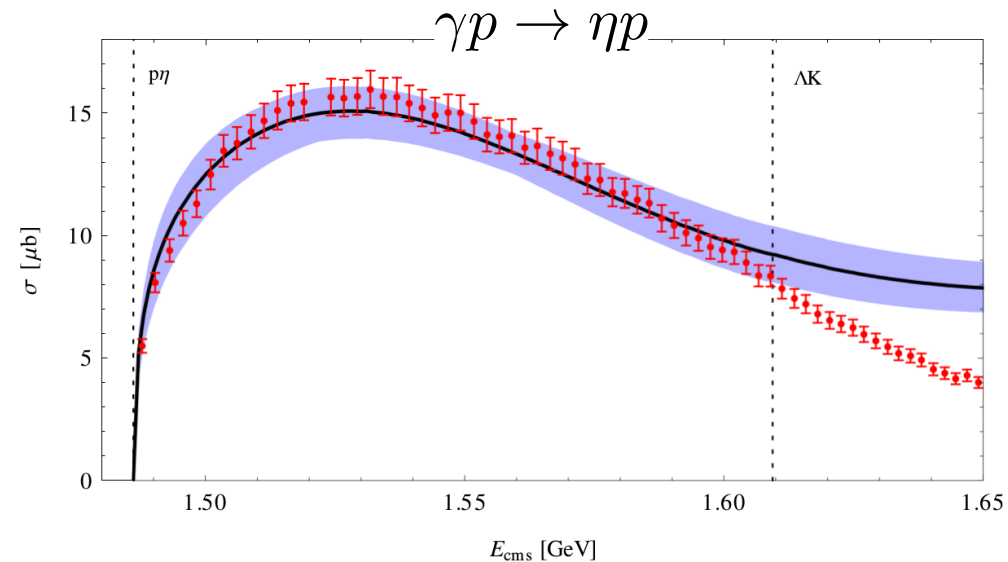
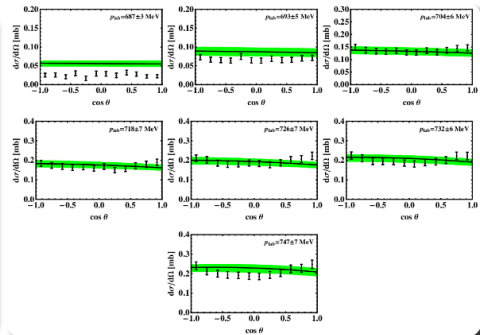
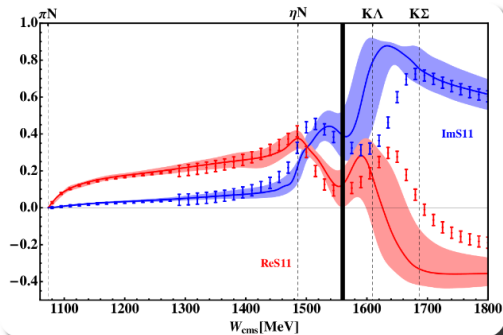
Manifestly gauge invariant approach based on full BSE solution

[Ruic, M. Mai, U.-G. Meissner PLB 704 (2011)]



► Exact unitary meson-baryon scattering amplitude T with parameters, fixed to reproduce:

- πN -partial wave S_{11} and S_{31} for $\sqrt{s} < 1560$ MeV Arndt et al. (2012)
- $\pi^- p \rightarrow \eta n$ differential cross sections Prakhov et al. (2005)



→ Making the “Missing resonance problem” worse ?!

Phenomenology

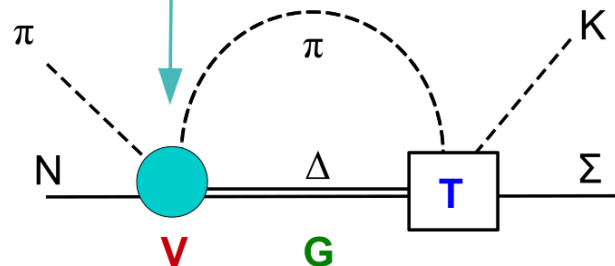
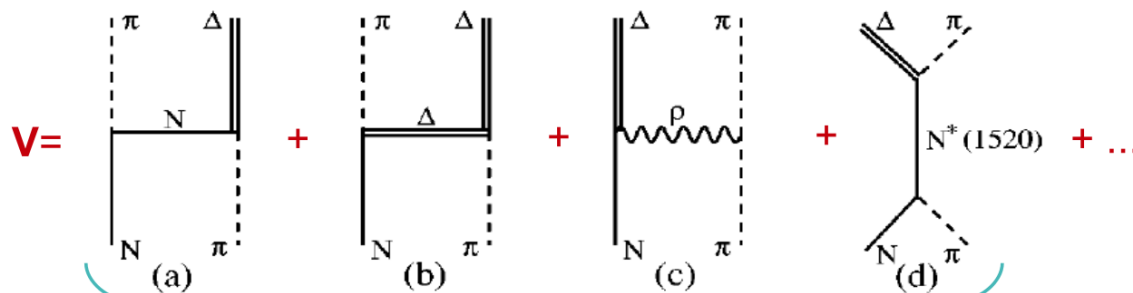
The Julich-Bonn Dynamical Coupled-Channel Approach

e.g. EPJ A 49, 44 (2013)

Dynamical coupled-channels (DCC): simultaneous analysis of different reactions

The scattering equation in partial-wave basis

$$\langle L' S' p' | T_{\mu\nu}^{JJ} | L S p \rangle = \langle L' S' p' | V_{\mu\nu}^{JJ} | L S p \rangle + \sum_{\gamma, L'' S''} \int_0^{\infty} dq \, q^2 \langle L' S' p' | V_{\mu\gamma}^{JJ} | L'' S'' q \rangle \frac{1}{E - E_{\gamma}(q) + i\epsilon} \langle L'' S'' q | T_{\gamma\nu}^{JJ} | L S p \rangle$$



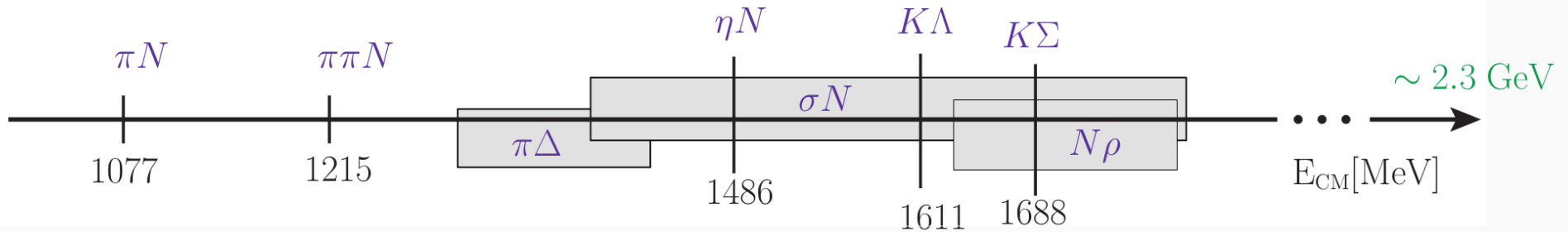
- potentials V constructed from effective \mathcal{L}

- s -channel diagrams: T^P
genuine resonance states

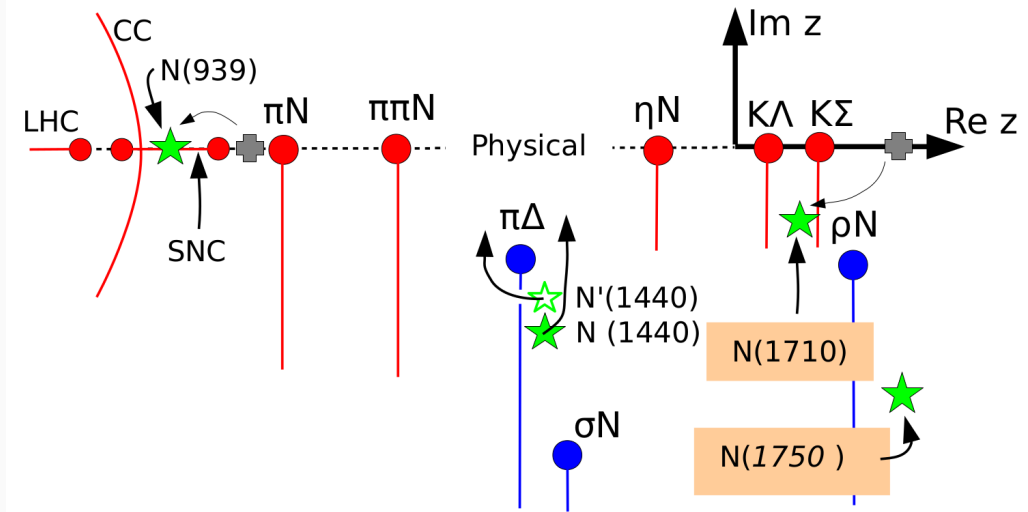
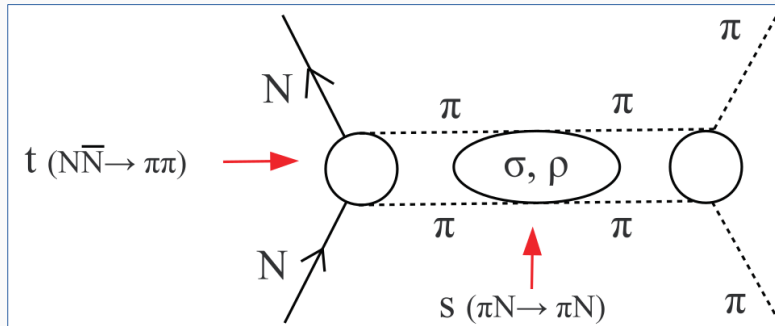
- t - and u -channel: T^{NP}
dynamical generation of poles
partial waves strongly correlated

JuBo: Channels and Analytic Structure

Channels included:



- (2-body) unitarity and analyticity respected
 - 3-body $\pi\pi N$ channel:
 - parameterized effectively as $\pi\Delta$, σN , ρN
 - $\pi N/\pi\pi$ subsystems fit the respective phase shifts
- ↳ branch points move into complex plane



JuBo: Data base

- $\pi N \rightarrow X$: > **7,000** data points ($\pi N \rightarrow \pi N$: GW-SAID WI08 (ED solution))
- $\gamma N \rightarrow X$:

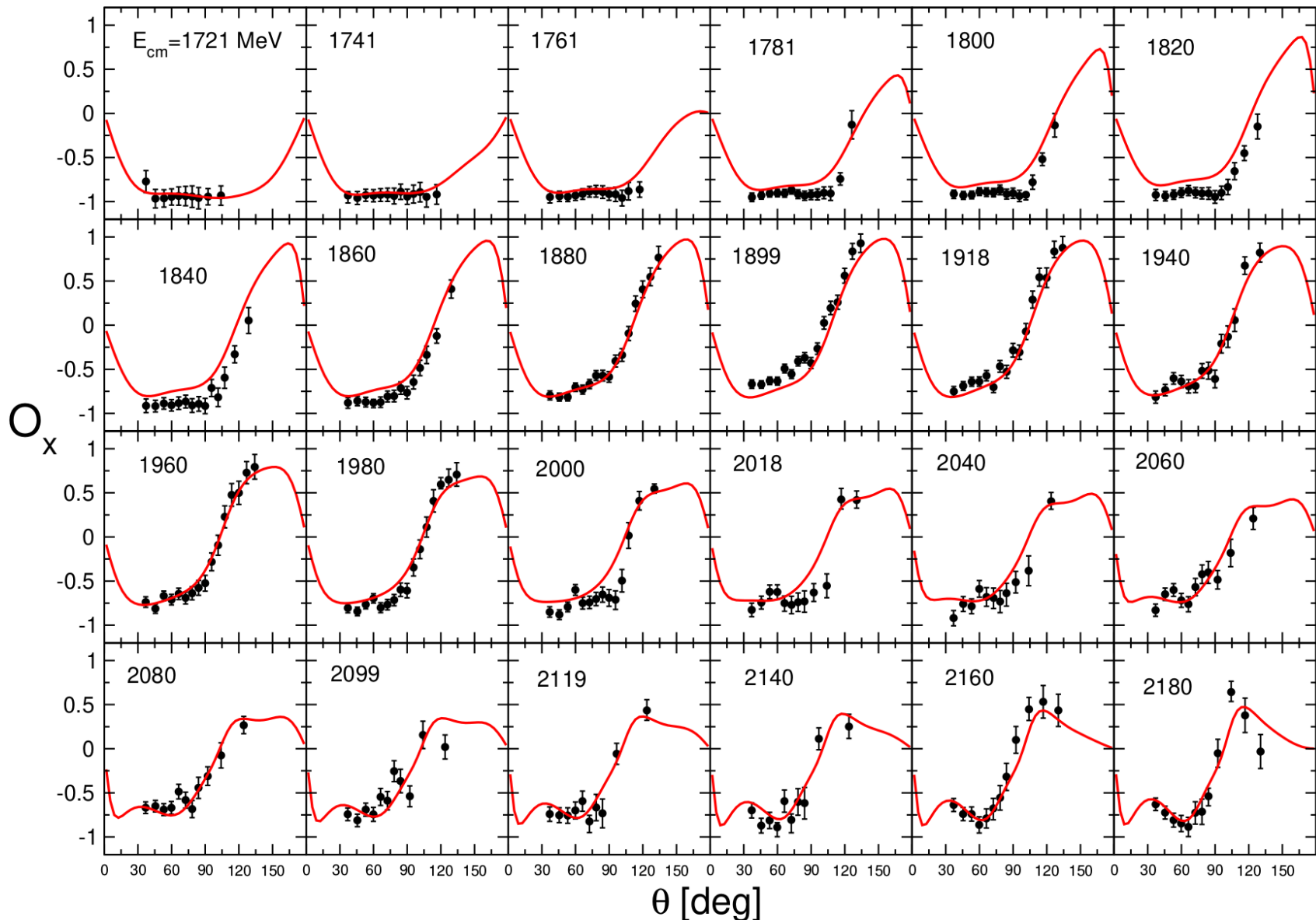
Reaction	Observables (# data points)	p./channel
$\gamma p \rightarrow \pi^0 p$	$d\sigma/d\Omega$ (18721), Σ (2927), P (768), T (1404), $\Delta\sigma_{31}$ (140), G (393), H (225), E (467), F (397), $C_{x'}$ (74), $C_{z'}$ (26)	25,542
$\gamma p \rightarrow \pi^+ n$	$d\sigma/d\Omega$ (5961), Σ (1456), P (265), T (718), $\Delta\sigma_{31}$ (231), G (86), H (128), E (903)	9,748
$\gamma p \rightarrow \eta p$	$d\sigma/d\Omega$ (9112), Σ (403), P (7), T (144), F (144), E (129)	9,939
$\gamma p \rightarrow K^+ \Lambda$	$d\sigma/d\Omega$ (2478), P (1612), Σ (459), T (383), $C_{x'}$ (121), $C_{z'}$ (123), $O_{x'}$ (66), $O_{z'}$ (66), O_x (314), O_z (314),	5,936
$\gamma p \rightarrow K^+ \Sigma^0$	$d\sigma/d\Omega$ (4271), P (422), Σ (280), T (127), $C_{x',z'}$ (188), $O_{x,z}$ (254)	5,542
$\gamma p \rightarrow K^0 \Sigma^+$	$d\sigma/d\Omega$ (242), P (78)	320
	in total	57,027

A new **SAID** interface [Video, R. Workman]

Selected Fit Results (I)

● $\gamma p \rightarrow K^+ \Lambda$:

<http://collaborations.fz-juelich.de/ikp/meson-baryon/main>



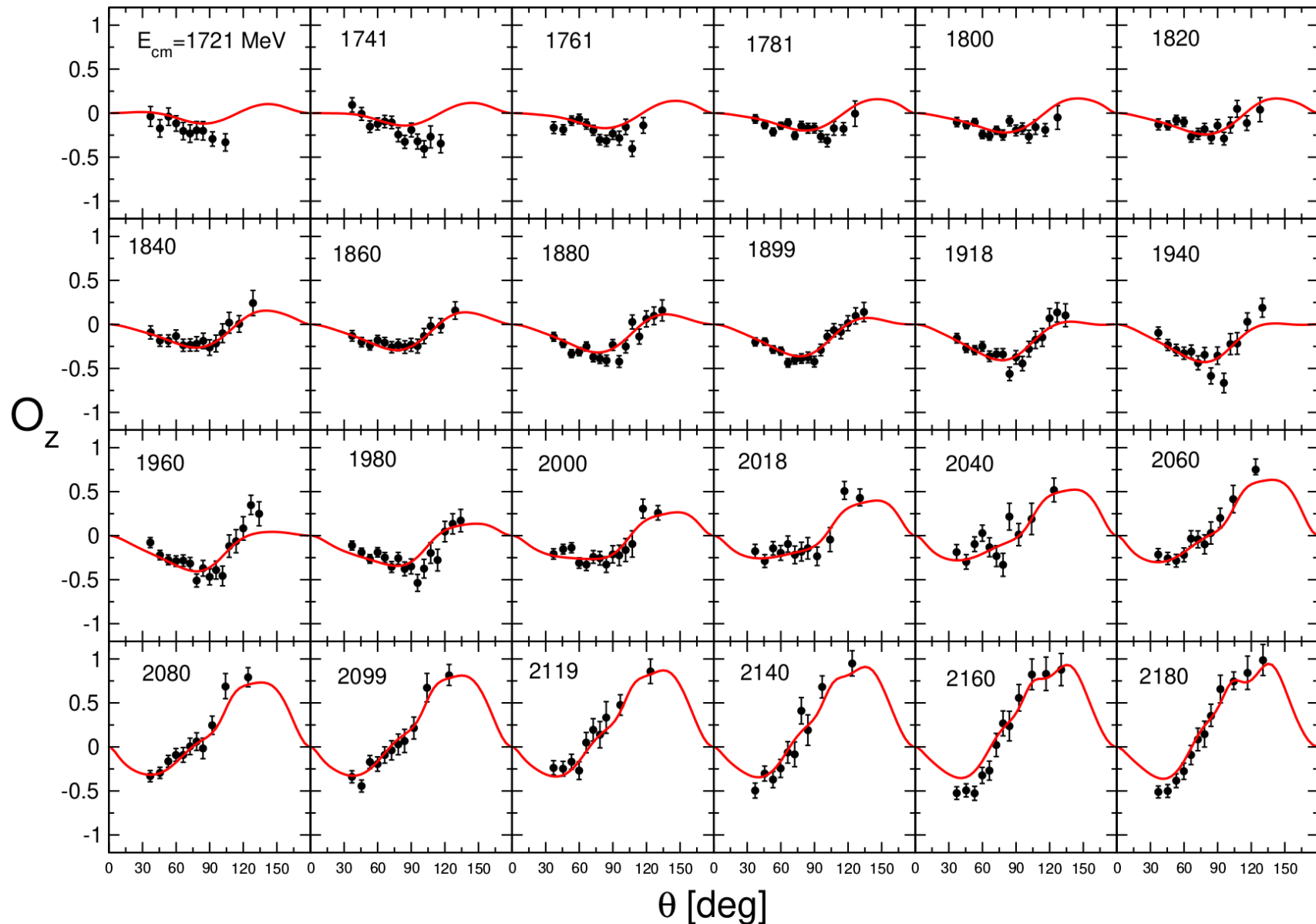
[D. Roenchen, M. D., U.-G. Meißner, EPJ A 54, 110 (2018)]

data: Paterson (CLAS) PRC 93, 065201 (2016), red line: fit JüBo2019

Selected Fit Results (II)

● $\gamma p \rightarrow K^+ \Lambda$:

<http://collaborations.fz-juelich.de/ikp/meson-baryon/main>



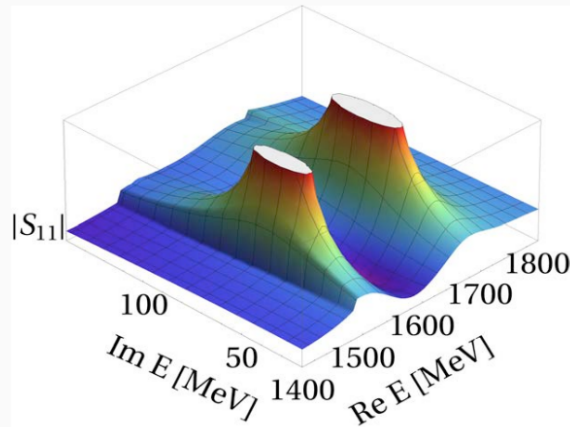
[D. Roenchen, M. D., U.-G. Meißner, EPJ A 54, 110 (2018)]

data: Paterson (CLAS) PRC 93, 065201 (2016), red line: fit JüBo2019

Resonance Couplings

Resonance states: Poles in the T -matrix on the 2^{nd} Riemann sheet

[D. Roenchen, M. D., U.-G. Meißner, EPJ A 54, 110 (2018)]



- $\text{Re}(E_0)$ = “mass”, $-2\text{Im}(E_0)$ = “width”
- elastic πN residue ($|r_{\pi N}|, \theta_{\pi N \rightarrow \pi N}$), normalized residues for inelastic channels ($\sqrt{\Gamma_{\pi N} \Gamma_{\mu}} / \Gamma_{\text{tot}}, \theta_{\pi N \rightarrow \mu}$)
- photocouplings at the pole: $\tilde{A}_{\text{pole}}^h = A_{\text{pole}}^h e^{i\vartheta^h}$, $h = 1/2, 3/2$

Inclusion of $\gamma p \rightarrow K^+ \Lambda$ in JüBo (“JuBo2017-1”): 3 additional states

	z_0 [MeV]	$\frac{\Gamma_{\pi N}}{\Gamma_{\text{tot}}}$	$\frac{\Gamma_{\eta N}}{\Gamma_{\text{tot}}}$	$\frac{\Gamma_{K\Lambda}}{\Gamma_{\text{tot}}}$	$\frac{\Gamma_{K\Sigma}}{\Gamma_{\text{tot}}}$
N(1900) $3/2^+$	1923 - i 108.4	1.5 %	0.78 %	2.99 %	69.5 %
N(2060) $5/2^-$	1924 - i 100.4	0.35 %	0.15 %	13.47 %	27.02 %
$\Delta(2190)$ $1/2^+$	2191 - i 103.0	33.12 %			3.78 %

- N(1900) $3/2^+$: s-channel resonances, seen in many other analyses of kaon photoproduction (BnGa), 3 stars in PDG
- N(2060) $5/2^-$: dynamically generated, 2 stars in PDG, seen e.g. by BnGa
- $\Delta(2190)$ $1/2^+$: dyn. gen., no equivalent PDG state

Electroproduction (preliminary)

C. Granados, M.D., H. Haberzettl, D. Rönchen, R. Workman et al.

Data & Analyses

Take advantage of multi-channel approach
 → analyze simultaneously final states πN , ηN , $K\Lambda$

~ 10^6 pion electroproduction data

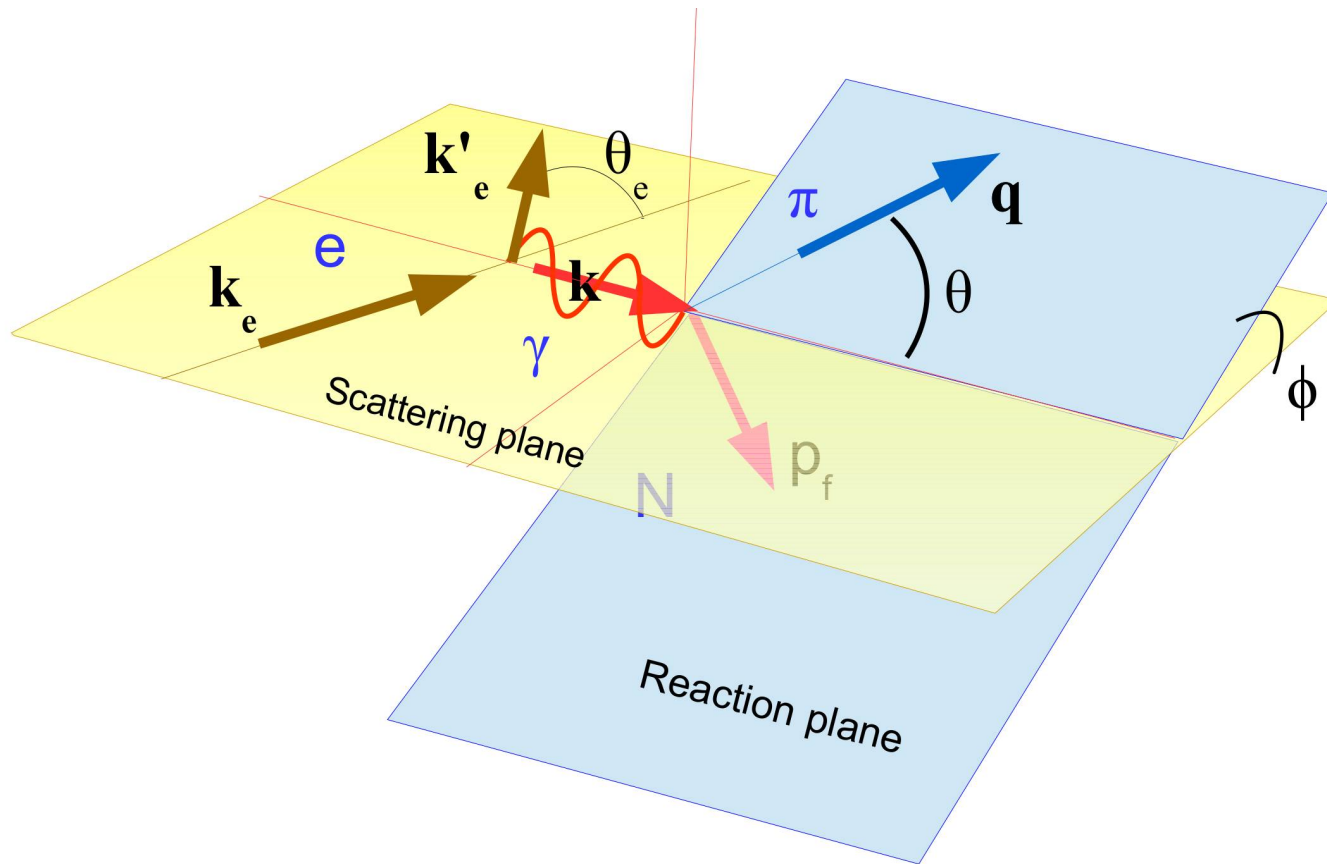
ηN , $K\Lambda$:

Reaction	Observable	Q^2 [GeV]	W [GeV]	Ref.
$ep \rightarrow e'p'\eta$	$\sigma_U, \sigma_{LT}, \sigma_{TT}$	1.6 – 4.6	2.0 – 3.0	[132]
	$\sigma_U, \sigma_{LT}, \sigma_{TT}$	0.13 – 3.3	1.5 – 2.3	[137]
	$d\sigma/d\Omega$	0.25 – 1.5	1.5 – 1.86	[138]
$ep \rightarrow e'K^+\Lambda$	P_N^0	0.8 – 3.2	1.6 – 2.7	[139]
	$\sigma_U, \sigma_{LT}, \sigma_{TT}, \sigma_{LT'}$	1.4 – 3.9	1.6 – 2.6	[140]
	P'_x, P'_z	0.7 – 5.4	1.6 – 2.6	[141]
	$\sigma_T, \sigma_L, \sigma_{LT}, \sigma_{TT}$	0.5 – 2.8	1.6 – 2.4	[142]
	P'_x, P'_z	0.3 – 1.5	1.6 – 2.15	[143]

ηp and $K^+\Lambda$ electroproduction data measured at CLAS for different photon virtualities Q^2 and total energy W . Based on material provided by courtesy of D. Carman (JLab) and I. Strakovsky (GW).

- **ANL-Osaka** PRC **80**, 025207 (2009), Few-Body Syst. 59, 24 (2018),...
- **Aznauryan, Burkert et al.**, PRC **80**, 055203 (2009), Int.J.Mod.Phys. E22, 1330015 (2013),...
- **EtaMAID2018**, EPJA 54 (2018), 210
- **MAID2007**, EPJA 34 (2007) 69
- **SAID**, PiN Newsletter 16, 150 (2002)
- **Gent group** Phys. Rev. C 89, 065202 (2014),...

Kinematics



Unpolarized differential cross section:

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma_T}{d\Omega} + \epsilon \frac{d\sigma_L}{d\Omega} + \sqrt{2\epsilon(1+\epsilon)} \frac{d\sigma_{LT}}{d\Omega} \cos \phi + \epsilon \frac{d\sigma_{TT}}{d\Omega} \cos 2\phi + h \sqrt{2\epsilon(1+\epsilon)} \frac{d\sigma_{LT'}}{d\Omega} \sin \phi$$

$$\epsilon = \frac{1}{1 + 2 \frac{k_L^2}{Q^2} \tan \theta_e}$$

Decomposed in in electric, magnetic, and longitudinal multipoles

Formalism

- Photoproduction solution as constraint
- Constraints from (Pseudo)-threshold:

$$\begin{aligned}
 (E_{l+}^I, L_{l+}^I) &\rightarrow k^l q^l \quad (l \geq 0) \\
 (M_{l+}^I, M_{l-}^I) &\rightarrow k^l q^l \quad (l \geq 1) \\
 (L_l^I) &\rightarrow kq \quad (l = 1) \\
 (E_{l-}^I, L_{l-}^I) &\rightarrow k^{l-2} q^l \quad (l \geq 2)
 \end{aligned}
 \quad
 \begin{aligned}
 k = |\mathbf{k}| &= \frac{\sqrt{\left((W - M_N)^2 + Q^2\right) \left((W + M_N)^2 + Q^2\right)}}{2W} \\
 q = |\mathbf{q}| &= \frac{\sqrt{\left((W - M_N)^2 - M_m^2\right) \left((W + M_N)^2 - M_m^2\right)}}{2W}
 \end{aligned}$$

- Siegert's theorem

$$\frac{E_{l+}}{L_{l+}} \rightarrow 1, \quad \frac{E_{l-}}{L_{l-}} \rightarrow \frac{-l}{l-1}$$

Amaldi, Fubini, Furlan,
 Springer Tracts Mod. Phys. 83, 1 (1979)
 Tiator, Few-body Systems 57, 1087 (2016)

at pseudo-threshold

- Watson's theorem, multi-channel unitarity

$$M_{\mu\gamma^*}(q, W, Q^2) = V_{\mu\gamma^*}(q, W, Q^2) + \sum_{\kappa} \int dp p^2 T_{\mu\kappa}(q, p, W) G_{\kappa}(p, W) V_{\nu\gamma^*}(p, W, Q^2)$$

$$V_{\mu\gamma^*}(p, W, Q^2) = \alpha_{\mu\gamma^*}^{NP}(p, W, Q^2) + \sum_i \frac{\gamma_{\mu;i}^a(p) \gamma_{\gamma^*;i}^c(W, Q^2)}{W - m_i^b}$$

Very preliminary results (before fit)

$W = 1232 \text{ MeV}$

$\Delta W = 2 \text{ MeV}$

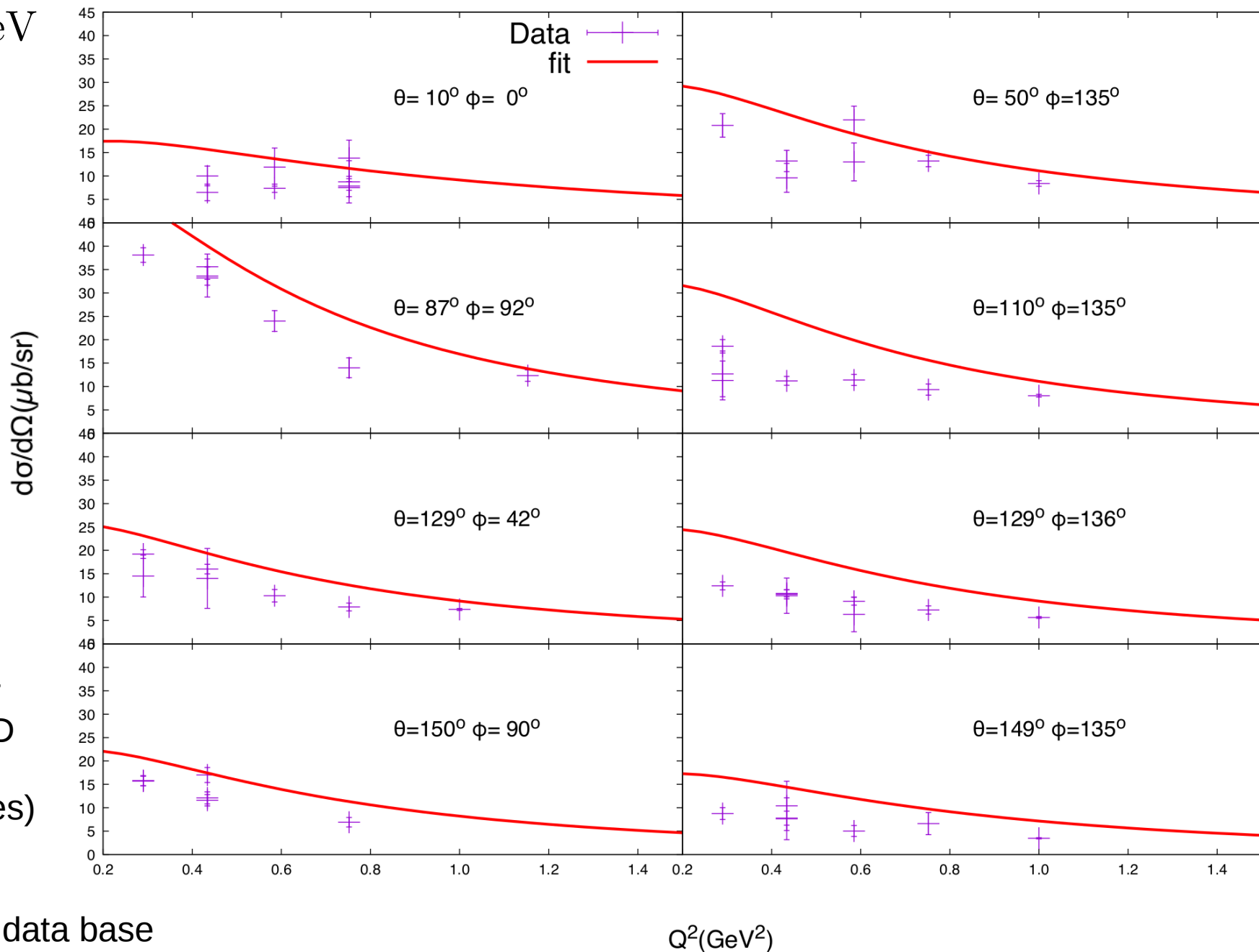
$\Delta\theta \approx 1^\circ$

$\Delta\phi \approx 1^\circ$

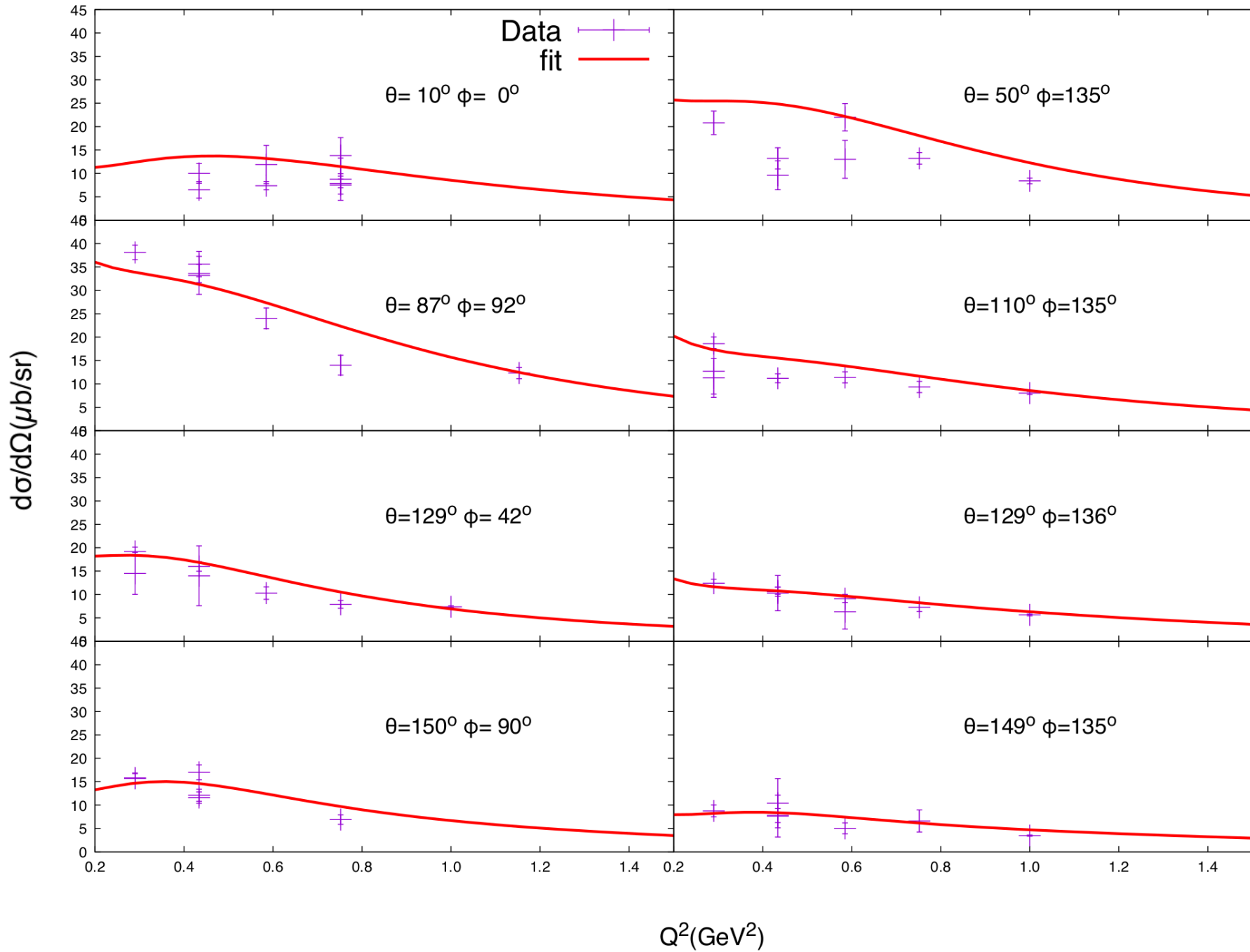
$\epsilon = 0.95$

About
1,500 data

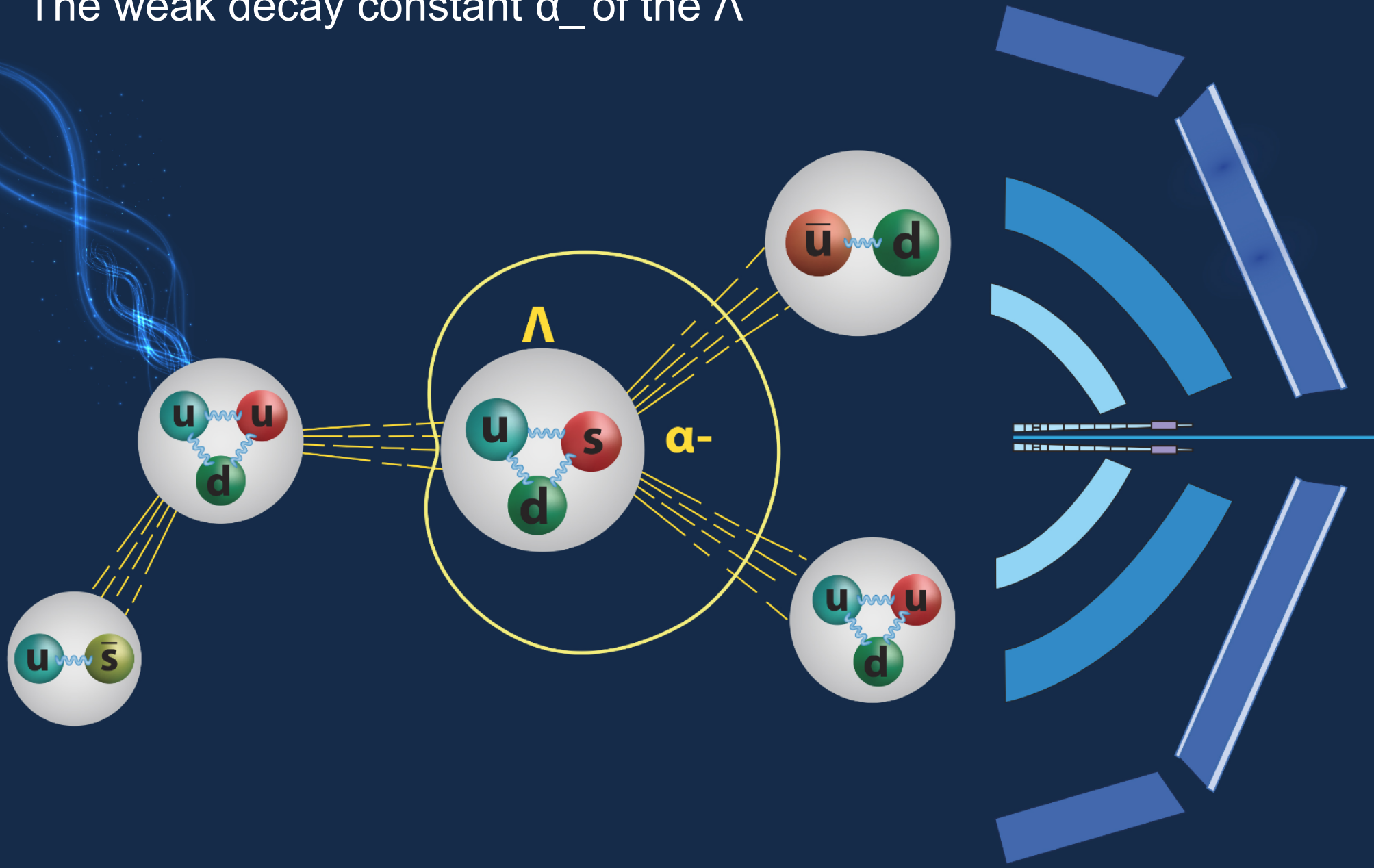
Solution after
fitting to MAID
multipoles
(up to p-waves)



Selected Fit Results (after fit)



The weak decay constant α_- of the Λ



Affects

- Baryon spectroscopy
- Polarization experiments at STAR, ATLAS,...
- Decay properties of other hyperons (chain decays)

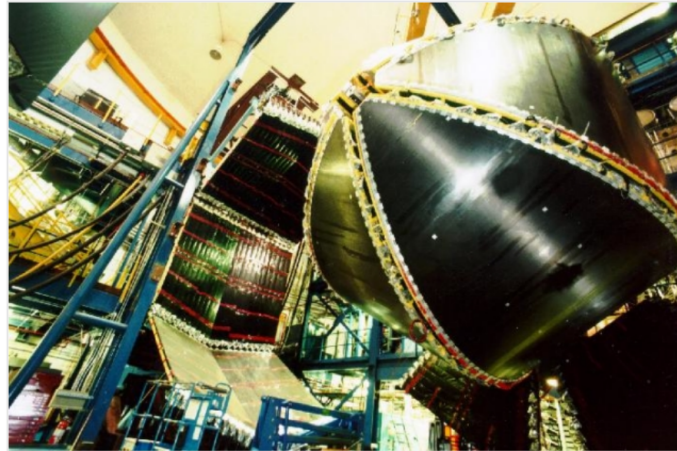


CERN COURIER

HIGGS AND ELECTROWEAK | NEWS

Λ -hyperon anomaly confirmed

1 November 2019



The CLAS detector at Jefferson Laboratory in the US. Credit: Jefferson Laboratory

BESIII (2018) & this work

nature

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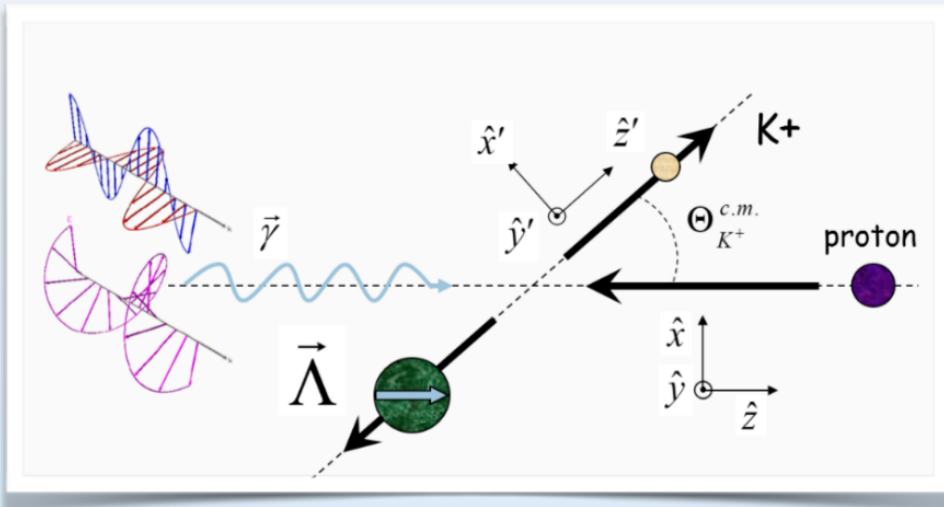
Table 1 | Summary of the results

Parameters	This work (BES III)	Previous results (PDG)
α_ψ	$0.461 \pm 0.006 \pm 0.007$	0.469 ± 0.027 (ref. ¹⁴)
$\Delta\Phi$	$42.4 \pm 0.6 \pm 0.5^\circ$	-
α_-	$0.750 \pm 0.009 \pm 0.004$	0.642 ± 0.013 (ref. ⁶)
α_+	$-0.758 \pm 0.010 \pm 0.007$	-0.71 ± 0.08 (ref. ⁶)
$\bar{\alpha}_0$	$-0.692 \pm 0.016 \pm 0.006$	-
A_{CP}	$-0.006 \pm 0.012 \pm 0.007$	0.006 ± 0.021 (ref. ⁶)
$\bar{\alpha}_0/\alpha_+$	$0.913 \pm 0.028 \pm 0.012$	-

> 5σ difference between new result and PDG¹².

Kaon photoproduction (this work)

Experimental setup



Intensity

$$(LP) : 1 + \alpha_- \cos \theta_y \mathbf{P}$$

$$- p_L^\gamma \cos 2\phi \mathbf{\Sigma}$$

$$- \alpha_- p_L^\gamma \cos 2\phi \cos \theta_y \mathbf{T}$$

$$- \alpha_- p_L^\gamma \sin 2\phi \cos \theta_x \mathbf{O}_x$$

$$- \alpha_- p_L^\gamma \sin 2\phi \cos \theta_z \mathbf{O}_z$$

$$(CP) : 1 + \alpha_- \cos \theta_y \mathbf{P}$$

$$+ p_C^\gamma \alpha_- \cos \theta_x \mathbf{C}_x$$

$$+ p_C^\gamma \alpha_- \cos \theta_z \mathbf{C}_z$$

● 7 polarization observables: $\mathbf{P}, \mathbf{\Sigma}, \mathbf{T}, \mathbf{O}_x, \mathbf{O}_z, \mathbf{C}_x, \mathbf{C}_z$

● Kinematic variables: θ_i, W_i

● 1 fundamental: α_- , and 2 calibration parameters: p_L^γ, p_C^γ

[CLAS] McCracken et al.(2010)

[CLAS] Bradford et al.(2007)

[CLAS] Paterson et al. (2016)

BUT: observables are not independent \longrightarrow **FIERZ IDENTITIES**

Chiang, Tabakin (1997)

Sandorfi et al. (2011)

Kaon photoproduction and Fierz identities

Helicity space maps on Clifford algebra \blacktriangleright Fierz identities:

Chiang, Tabakin (1997)

$$\Sigma\mathbf{P} - \mathbf{C}_x\mathbf{O}_z + \mathbf{C}_z\mathbf{O}_x - \mathbf{T} = 0 \quad \& \quad \mathbf{O}_x^2 + \mathbf{O}_z^2 + \mathbf{C}_x^2 + \mathbf{C}_z^2 + \Sigma^2 - \mathbf{T}^2 + \mathbf{P}^2 = 1$$

A-priori:

\Rightarrow Observables are not independent

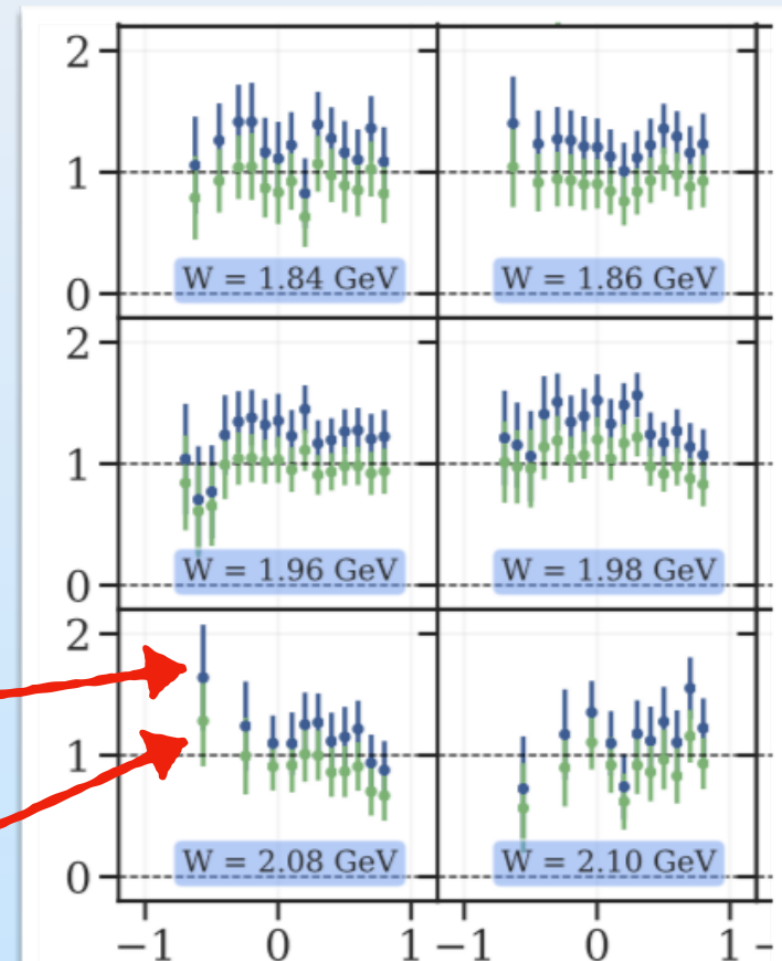
\Rightarrow determine α_- such that FI are fulfilled

\Rightarrow statistically non-trivial question

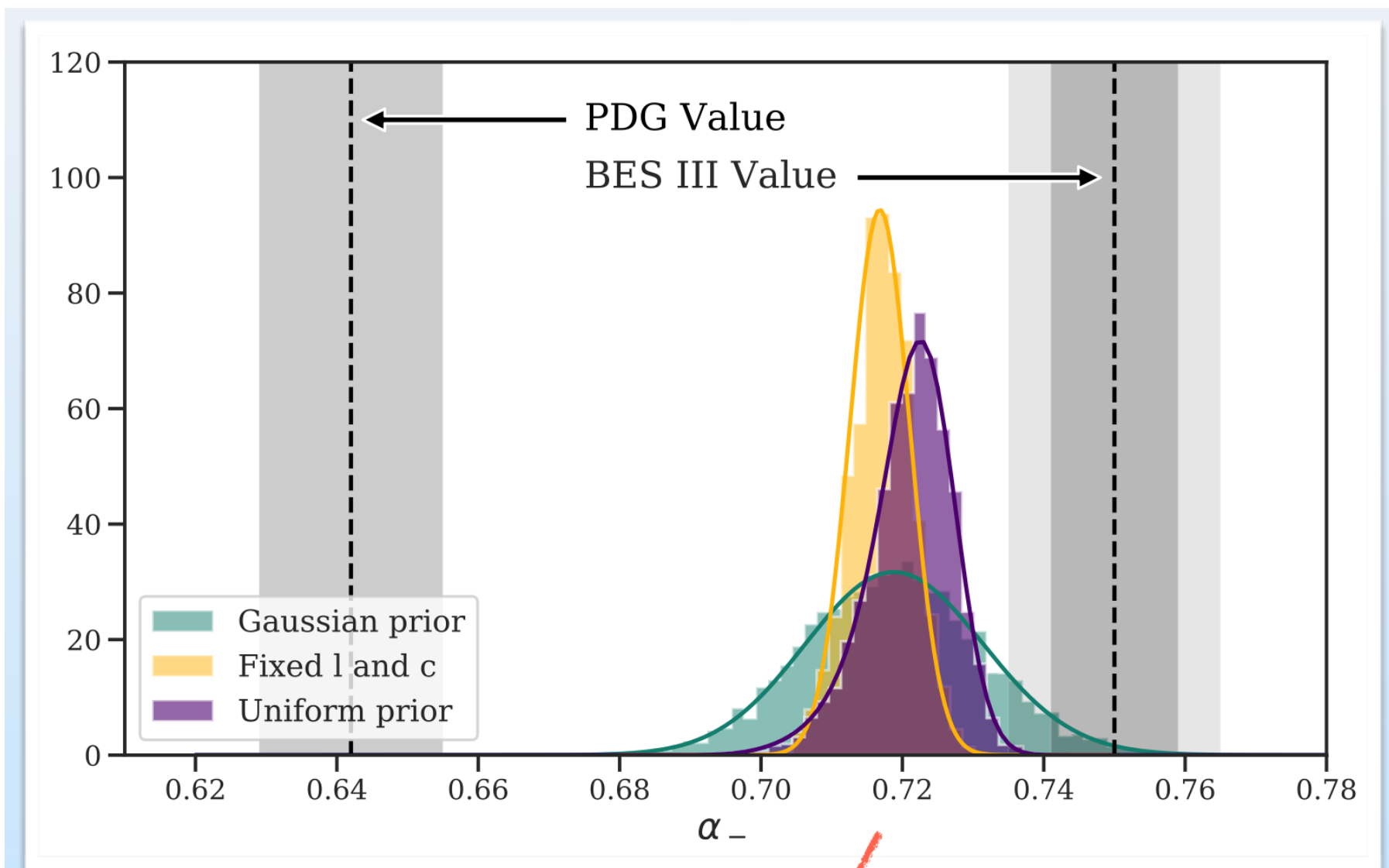
- Non-linear random variables
- Fit three data normalization constants (polarizations and α_-)
 \rightarrow Avoid biases!
- Observable sampling to solve full problem \rightarrow Partial linearization possible

$\alpha_-[\text{PDG}]$

$\alpha_-[\text{PDG}] / a$



Overall result



$\alpha_- = 0.721 \pm 0.006 \pm 0.005$

More observables for a preciser α_-

- Sandorfi, Hoblit, Kamano, J.Phys. G38 (2011) 053001:

$$G^2 + H^2 + E^2 + F^2 + \Sigma^2 + T^2 - P^2 = 1.$$

$$O_{x'}^2 + O_{z'}^2 + C_{x'}^2 + C_{z'}^2 + \Sigma^2 - T^2 + P^2 = 1.$$

$$T_{x'}^2 + T_{z'}^2 + L_{x'}^2 + L_{z'}^2 - \Sigma^2 + T^2 + P^2 = 1.$$

$$G^2 + H^2 - E^2 - F^2 - O_{x'}^2 - O_{z'}^2 + C_{x'}^2 + C_{z'}^2 = 0.$$

$$G^2 - H^2 + E^2 - F^2 + T_{x'}^2 + T_{z'}^2 - L_{x'}^2 - L_{z'}^2 = 0.$$

$$O_{x'}^2 - O_{z'}^2 + C_{x'}^2 - C_{z'}^2 - T_{x'}^2 + T_{z'}^2 - L_{x'}^2 + L_{z'}^2 = 0.$$

+ many more

$$\Sigma = +TP + T_{x'}L_{z'} - T_{z'}L_{x'}.$$

$$T = +\Sigma P - C_{x'}O_{z'} + C_{z'}O_{x'}.$$

$$P = +\Sigma T + GF + EH.$$

$$G = +PF + O_{x'}L_{x'} + O_{z'}L_{z'}.$$

$$H = +PE + O_{x'}T_{x'} + O_{z'}T_{z'}.$$

$$E = +PH - C_{x'}L_{x'} - C_{z'}L_{z'}.$$

$$F = +PG + C_{x'}T_{x'} + C_{z'}T_{z'}.$$

$$O_{x'} = +TC_{z'} + GL_{x'} + HT_{x'}.$$

$$O_{z'} = -TC_{x'} + GL_{z'} + HT_{z'}.$$

$$C_{x'} = -TO_{z'} - EL_{x'} + FT_{x'}.$$

$$C_{z'} = +TO_{x'} - EL_{z'} + FT_{z'}.$$

$$T_{x'} = +\Sigma L_{z'} + HO_{x'} + FC_{x'}.$$

$$T_{z'} = -\Sigma L_{x'} + HO_{z'} + FC_{z'}.$$

$$L_{x'} = -\Sigma T_{z'} + GO_{x'} - EC_{x'}.$$

$$L_{z'} = +\Sigma T_{x'} + GO_{z'} - EC_{z'}.$$

Summary

- Phenomenology of excited baryons through coupled-channels, two- and three-body effects
- Global analyses of pion and photon-induced reactions
 - Jülich-Bonn analysis finds/confirms new states in analysis of photoproduction data
 - Extension to analysis of electroproduction data in progress
 - Statistical aspects: How to find a minimal resonance spectrum
- Data-driven new value for α_1 determined. Changes polarization measurements at CLAS (baryon spectroscopy) but has impact in wide areas of hadron physics

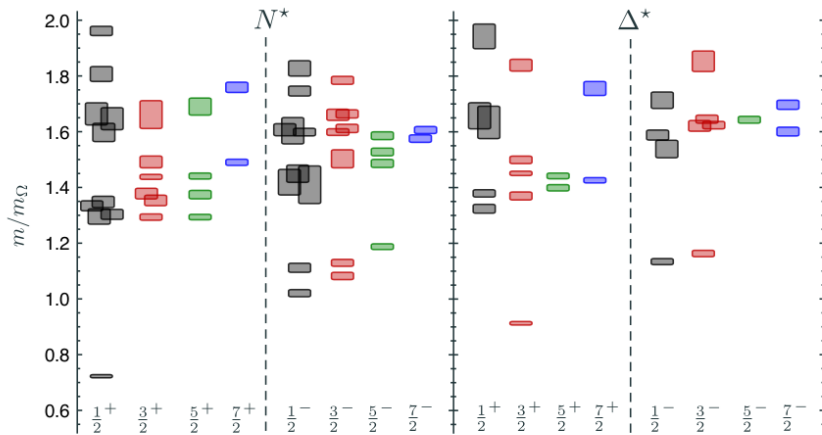
Spare slides

The Missing Resonance Problem

- above 1.8 GeV much more states are predicted than observed,

“Missing resonance problem”

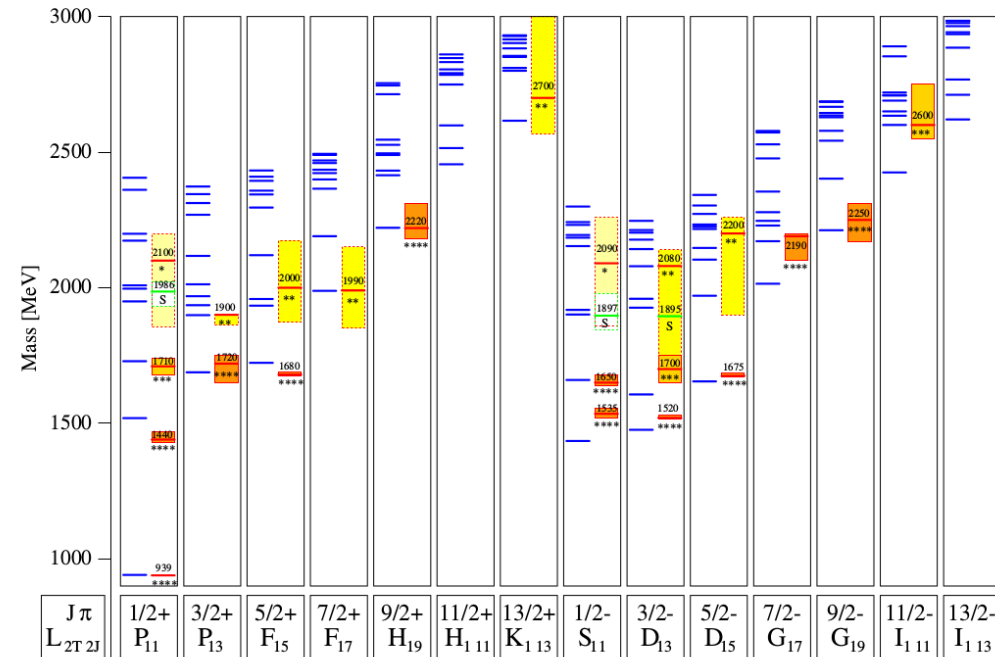
Lattice calculation (single hadron approximation):



[Edwards *et al.*, Phys.Rev. D84 (2011)]

- only 15 established N^* states (PDG 2015)
- $\sim 48\%$ of the states have **** or *** status (PDG 1982: 58% with **** or ***)

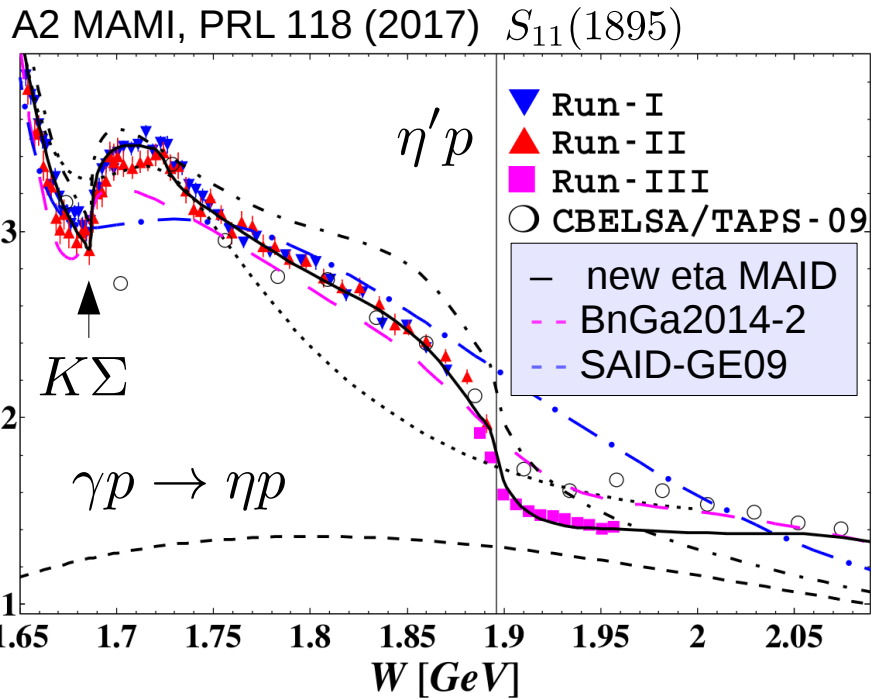
N^* spectrum in a relativistic quark model:



Löring *et al.* EPJ A 10, 395 (2001), experimental spectrum: PDG 2000

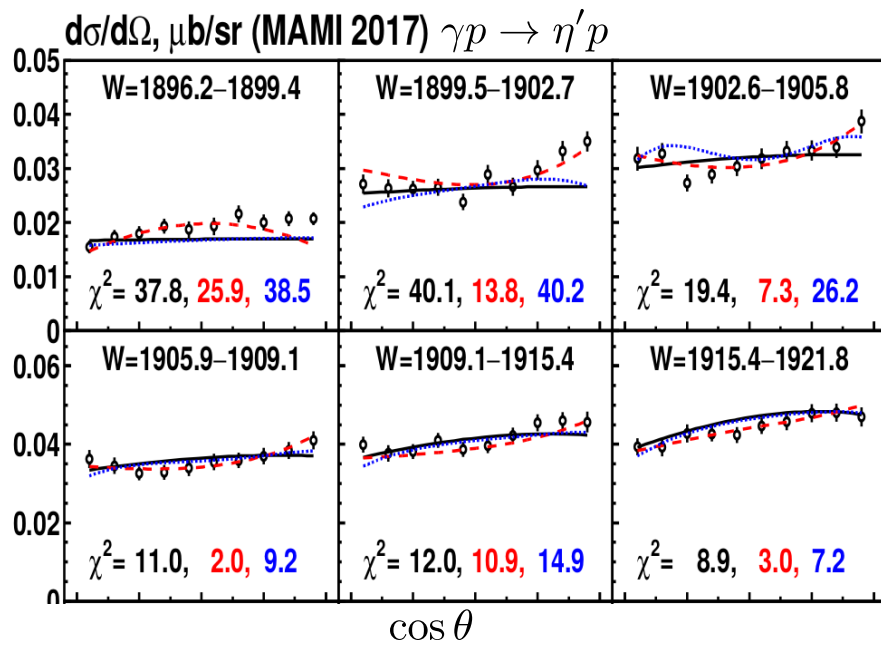
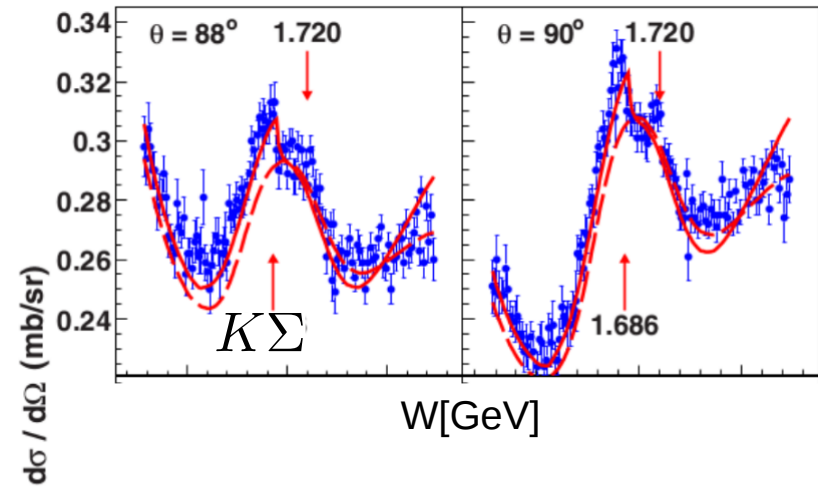
Overviews: Crede, Roberts, Rep. Prog. Phys. 76 (2013)
Aznauryan *et al.*, Int. J. Mod. Phys. E 22 (2013)

Resonances or not?



$$\pi N \rightarrow \pi N$$

EPECUR/SAID PRC 93 (2016)



BnGa
PLB785 (2018):

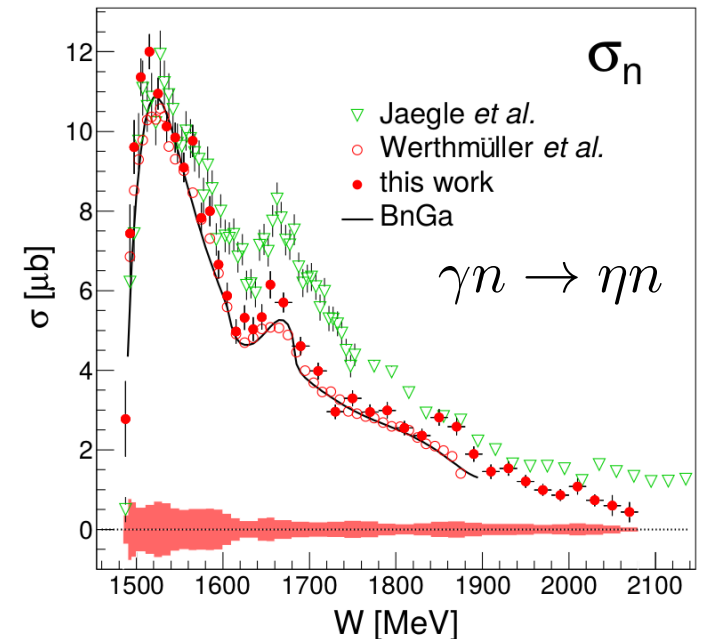
No narrow
resonance

$3/2^-$ narrow
Resonance

$5/2^-$ narrow
Resonance

Data: A2.Mami
PRL 118 (2017)

[CBELSA/TAPS EPJA 53 (2017)]



Current state in η photoproduction: Multipoles from different groups

From: **EtaMAID2018**
[Tiator et al., EPJA54 (2018)]
Analyzes:

$$\gamma p \rightarrow \eta p$$

$$\gamma p \rightarrow \eta' p$$

$$\gamma n \rightarrow \eta n$$

$$\gamma n \rightarrow \eta' n$$

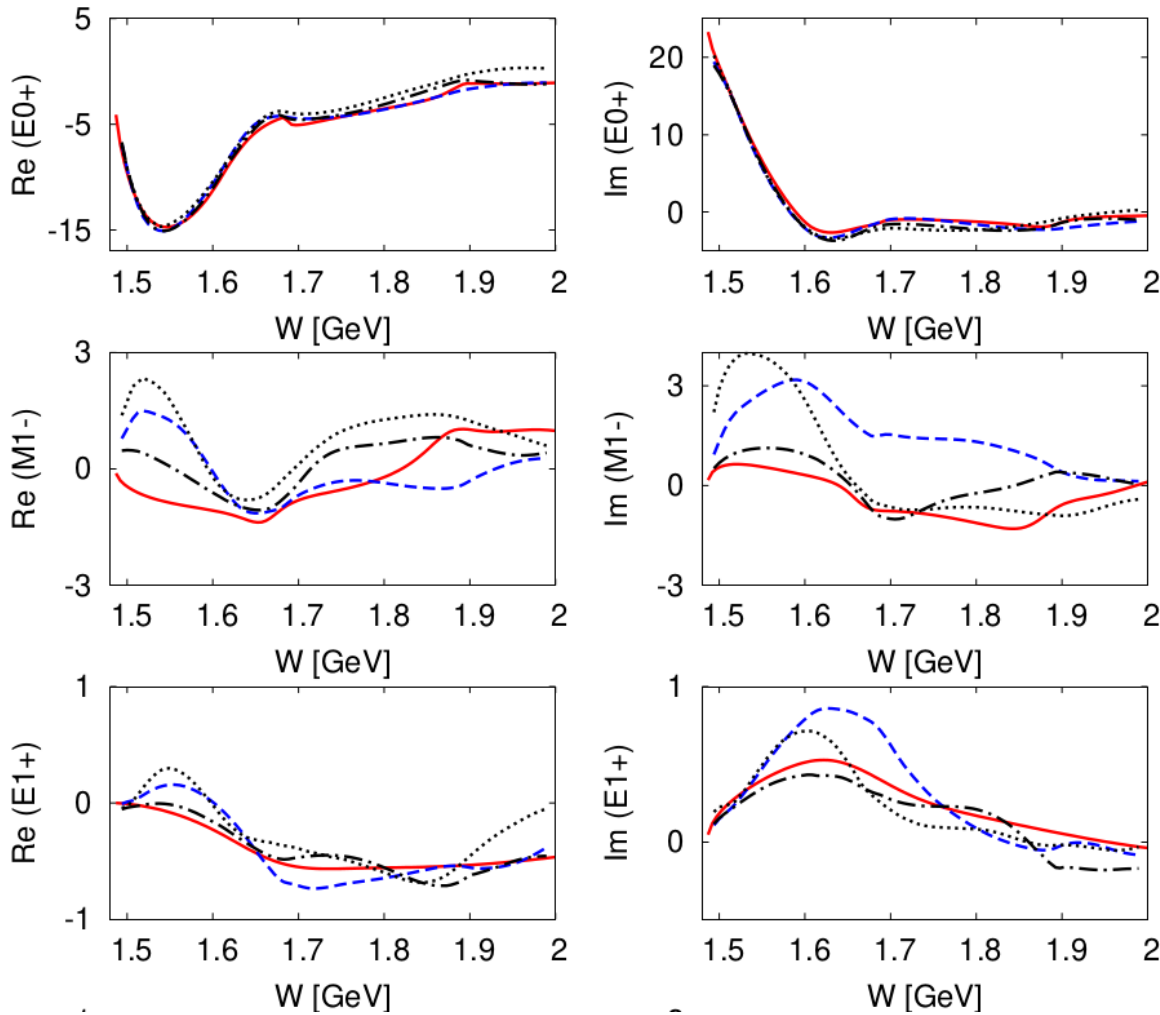
EtaMAID2018

BnGa [PLB 772 (2017)]

JuBo (dotted) [EPJA 54 (2018)]

KSU [1804.06031]

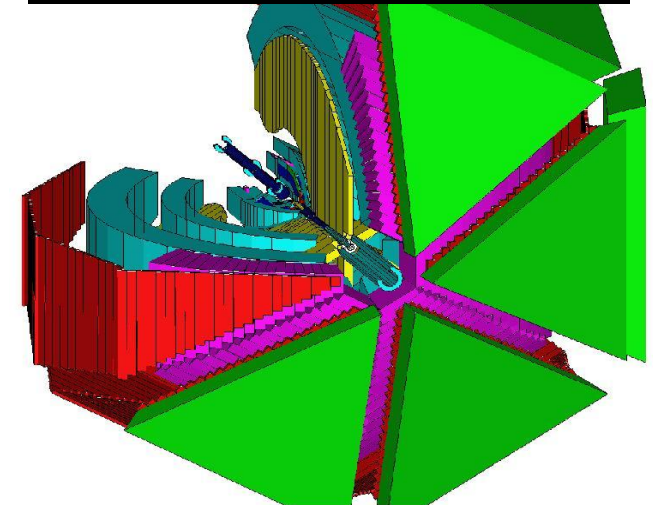
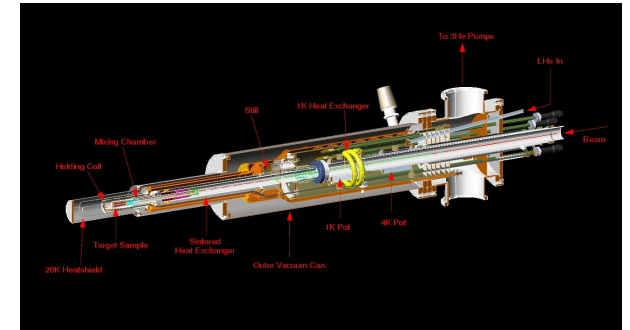
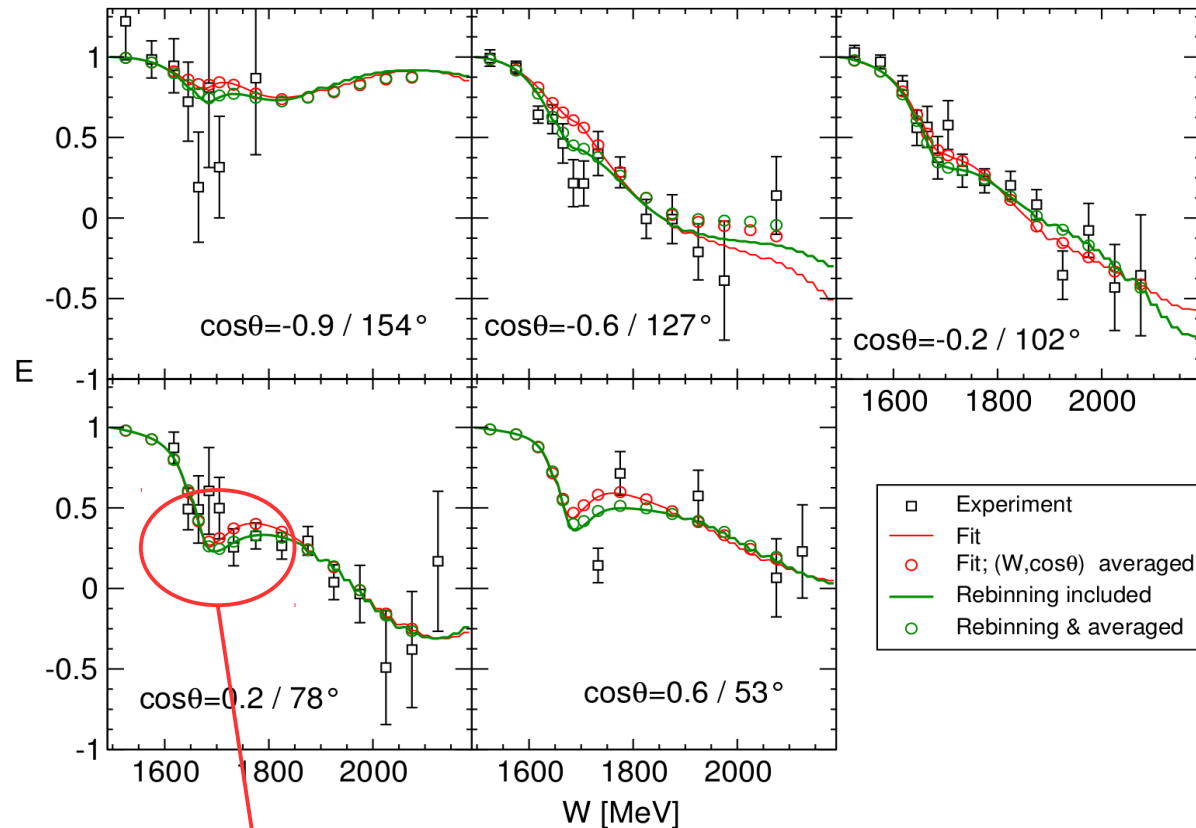
Review: Krusche, Wilkins,
[Prog.Part.Nucl.Phys. 80 (2014)]



Resonances and other structures

CLAS/JuBo (M. D., D. Rönchen), Phys.Lett. B755 (2016)

- First-ever measurement of observable E in η photo-production, enabled through the CLAS FROST target



Is this a new narrow baryonic resonance?

→ Conventional explanation in terms of interference effects.
 Systematic elimination of resonance through model selection
 (LASSO and other methods, Phys.Rev. D99 (2019) 016001)

Observable	σ	Σ	T	P	E	F	G	H	T_x	T_z	L_x	L_z	O_x	O_z	C_x	C_z
$\rho\pi^0$	✓	✓	✓		✓	✓	✓	✓								
$\eta\pi^+$	✓	✓	✓		✓	✓	✓	✓								
$\rho\eta$	✓	✓	✓		✓	✓	✓	✓								
$\rho\eta'$	✓	✓	✓		✓	✓	✓	✓								
$K^+\Lambda$	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
$K^+\Sigma^0$	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
$\rho\omega/\phi$	✓	✓	✓		✓	✓	✓	✓	✓ SDME							
$K^{*+}\Lambda$	✓			✓					SDME							
$K^{0*}\Sigma^+$	✓	✓								✓	✓	SDME				
$\rho\pi^-$	✓	✓			✓	✓	✓									
$\rho\rho^-$	✓	✓			✓	✓	✓									
$K^-\Sigma^+$	✓	✓			✓	✓	✓									
$K^0\Lambda$	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓
$K^0\Sigma^0$	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓
$K^{0*}\Sigma^0$	✓	✓									✓	✓				

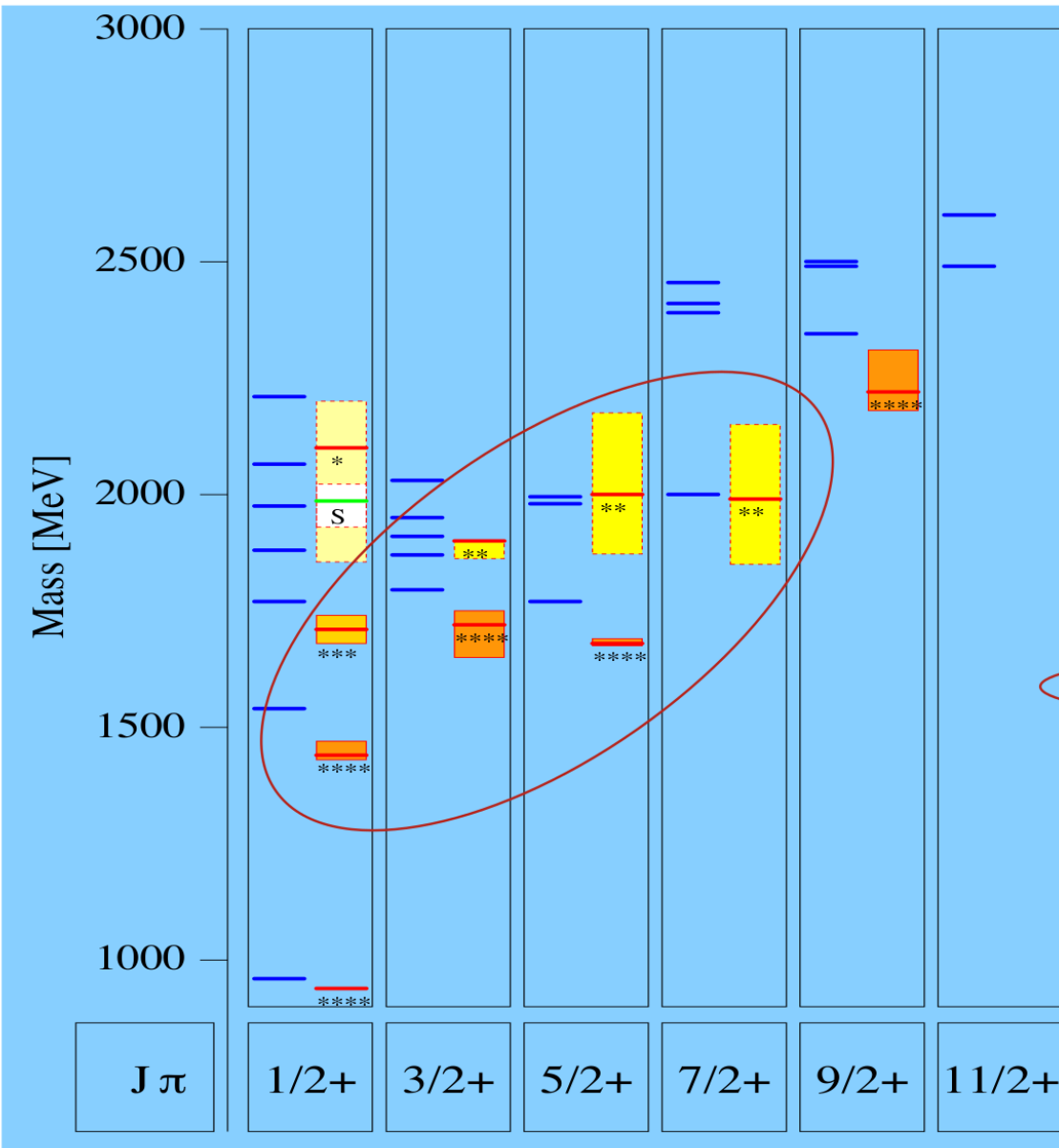
Phys.Lett. B771 (2017)
Phys.Lett. B755 (2016)



$\gamma p \rightarrow X$

$\gamma n \rightarrow X$

Spectrum of N* resonances



N^*	$J^P (L_{2l,2J})$	2010	2014
$N(1440)$	$1/2^+ (P_{11})$	* * *	* * *
$N(1520)$	$3/2^- (D_{13})$	* * *	* * *
$N(1535)$	$1/2^- (S_{11})$	* * *	* * *
$N(1650)$	$1/2^- (S_{11})$	* * *	* * *
$N(1675)$	$5/2^- (D_{15})$	* * *	* * *
$N(1680)$	$5/2^+ (F_{15})$	* * *	* * *
$N(1685)$			*
$N(1700)$	$3/2^- (D_{13})$	* * *	* * *
$N(1710)$	$1/2^+ (P_{11})$	* * *	* * *
$N(1720)$	$3/2^+ (P_{13})$	* * *	* * *
$N(1860)$	$5/2^+$		**
$N(1875)$	$3/2^-$		* * *
$N(1880)$	$1/2^+$		**
$N(1895)$	$1/2^-$		**
$N(1900)$	$3/2^+ (P_{13})$	**	* * *
$N(1990)$	$7/2^+ (F_{17})$	**	**
$N(2000)$	$5/2^+ (F_{15})$	**	**
$N(2080)$	D_{13}	**	
$N(2090)$	S_{11}	*	
$N(2040)$	$3/2^+$		*
$N(2060)$	$5/2^-$		**
$N(2100)$	$1/2^+ (P_{11})$	*	*
$N(2120)$	$3/2^-$		**
$N(2190)$	$7/2^- (G_{17})$	* * *	* * *
$N(2200)$	D_{15}	**	

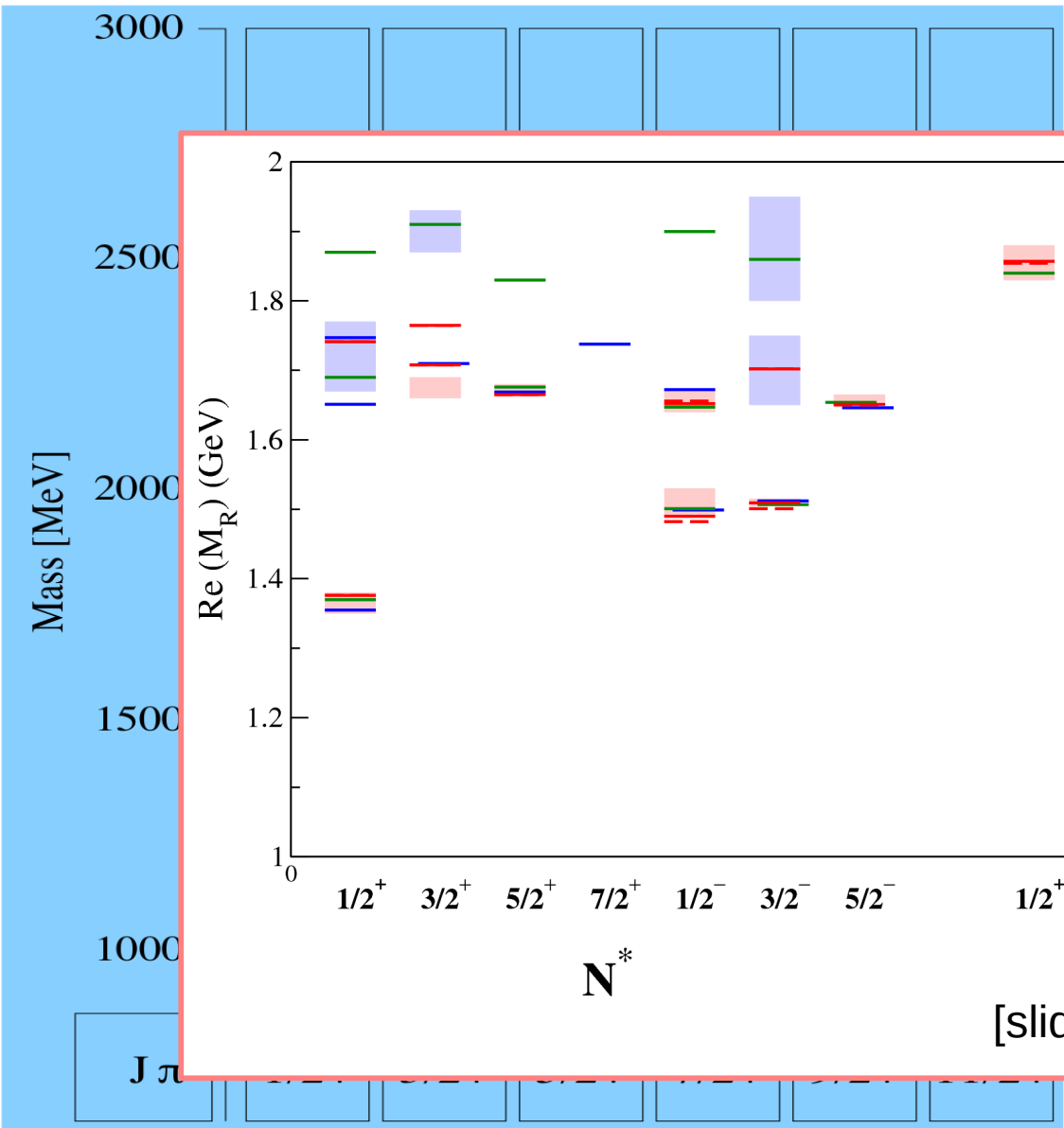
- Most new resonances by Bonn-Gatchina group;
- Many from kaon photoproduction

[Slide: V. Crede/Nstar 2017, slight modifications]

[See also: Crede, Roberts, Rep. Prog. Phys. 76 (2013)]

Spectrum of N^* resonances

N^*	$J^P (L_{2l,2J})$	2010	2014
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N^*	$J^P (L_{2l,2J})$	2010	2014
$N(2190)$	$1/2^- (G_{17})$	* * *	* * *
$N(2200)$	D_{15}	**	* * *

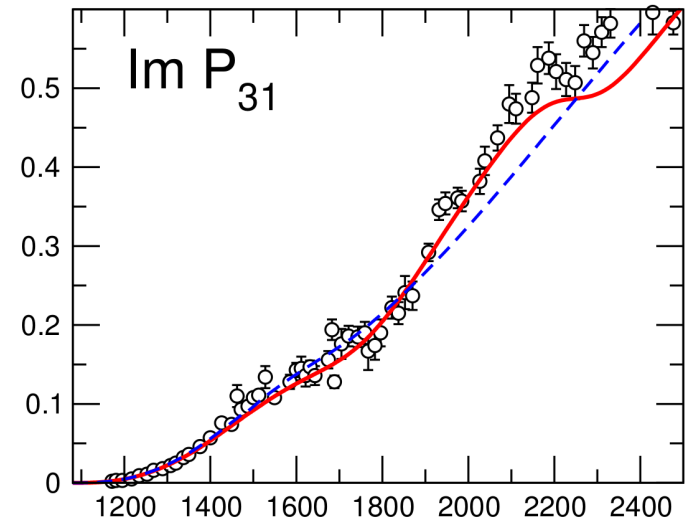
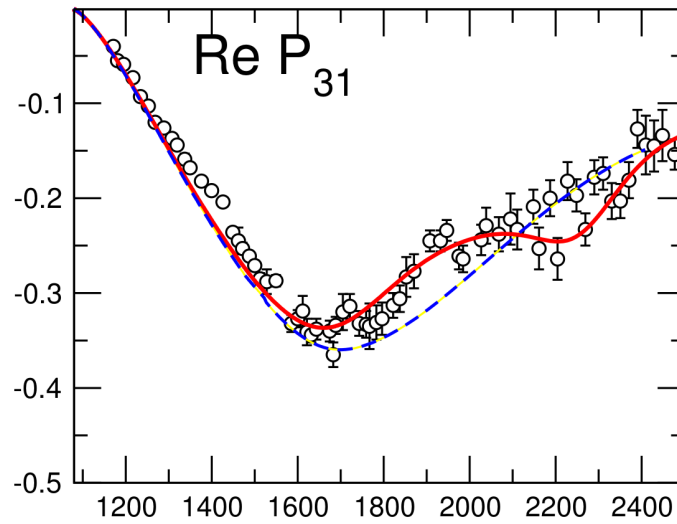
[slide: ANL/Osaka Kamano@N*2017]

- Most new resonances by Bonn-Gatchina group;
- Many from kaon photoproduction

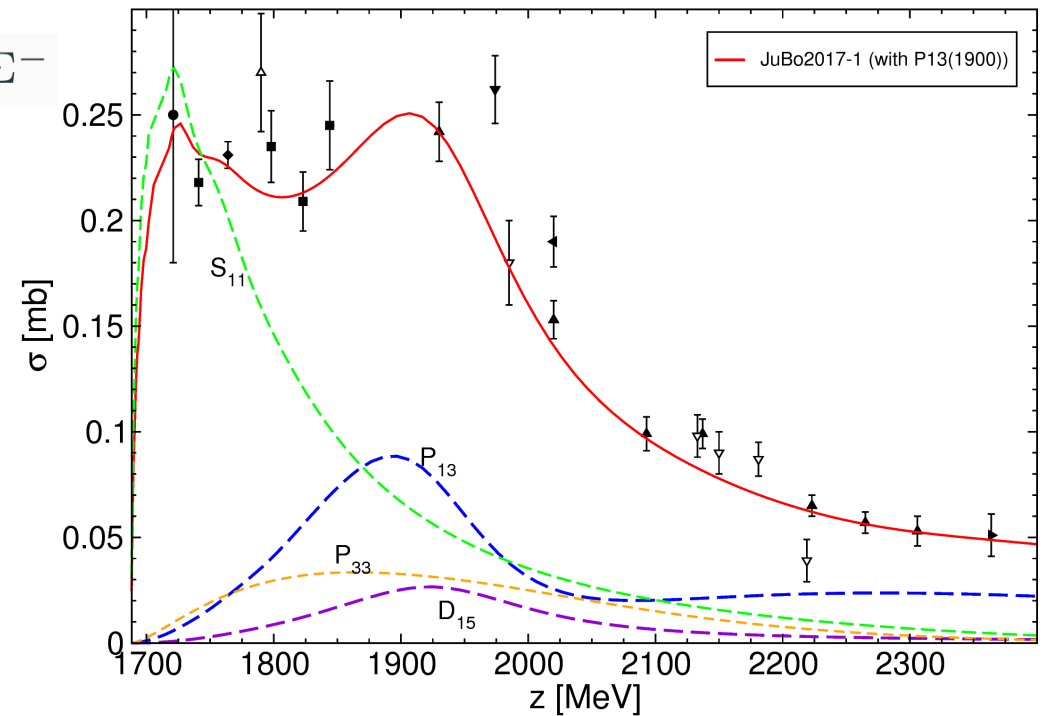
[Slide: V. Crede/Nstar 2017, slight modifications]
 [See also: Crede, Roberts, Rep. Prog. Phys. 76 (2013)]

Visible influence of new states

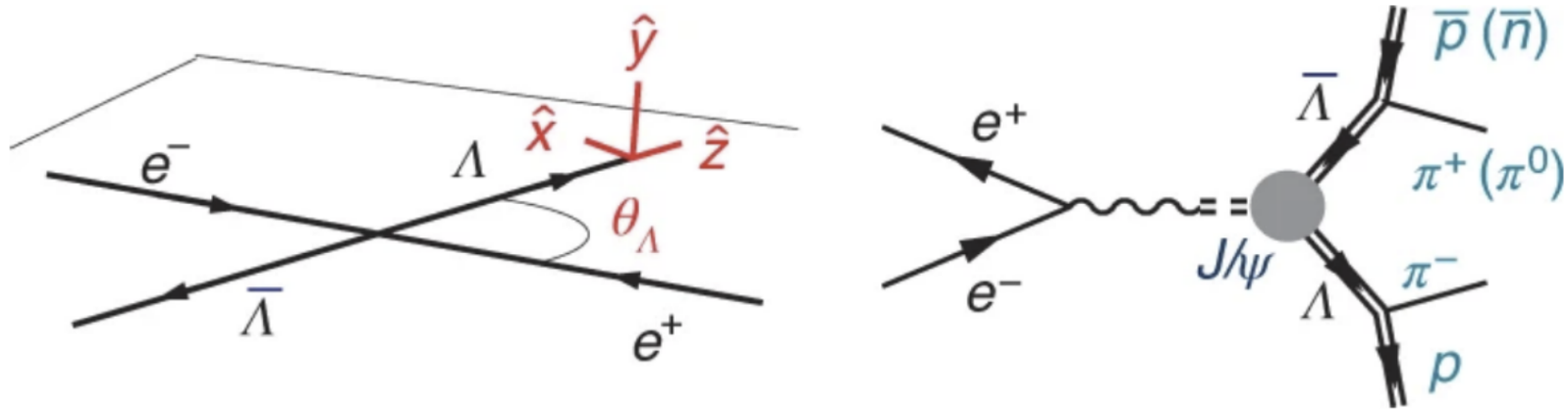
$\Delta(2190)3/2^+$ in πN PW



$N(1900)3/2^+$, $N(2060)5/2^-$ in σ_{tot} in $\pi^- p \rightarrow K^+ \Sigma^-$



BES III: Direct measurement



$\Lambda\bar{\Lambda}$ production process.

$$\begin{aligned}
 & \mathcal{W}(\xi; \alpha_\psi, \Delta\Phi, \alpha_-, \alpha_+) \\
 & = 1 + \alpha_\psi \cos^2 \theta_\Lambda + \alpha_- \alpha_+ \left[\sin^2 \theta_\Lambda (n_{1,x} n_{2,x} - \alpha_\psi n_{1,y} n_{2,y}) \right. \\
 & \quad \left. + (\cos^2 \theta_\Lambda + \alpha_\psi) n_{1,z} n_{2,z} \right] \\
 & + \alpha_- \alpha_+ \sqrt{1 - \alpha_\psi^2} \cos(\Delta\Phi) \sin \theta_\Lambda \cos \theta_\Lambda (n_{1,x} n_{2,z} + n_{1,z} n_{2,x}) \\
 & + \sqrt{1 - \alpha_\psi^2} \sin(\Delta\Phi) \sin \theta_\Lambda \cos \theta_\Lambda (\alpha_- n_{1,y} + \alpha_+ n_{2,y})
 \end{aligned}$$

$\Lambda\bar{\Lambda}$ intensity distribution

● Λ decays weakly to $p\pi^+$

● The decay parameter: α_-

- essential for many modern experiments

e.g. LEAR@CERN, STAR@BNL, ATLAS@CERN

- affects decay parameters of other hyperons

e.g. Trippe et al. (1967), Bono et al. (CLAS) (2018)

Ω^- DECAY PARAMETERS		Ξ^0 DECAY PARAMETERS	
$\alpha(\Omega^-) \alpha_-(\Lambda)$ FOR $\Omega^- \rightarrow \Lambda K^-$		$\alpha(\Xi^0) \alpha_-(\Lambda)$	
Some early results have been omitted.		This is a product of the $\Xi^0 \rightarrow \Lambda \pi^0$ and $\Lambda \rightarrow p \pi^-$ as)	
VALUE	EVTS	VALUE	EVTS
0.0115 ± 0.0015	OUR AVERAGE	-0.261 ± 0.006	OUR AVERAGE

PDG live (2019)

- impacts LO parameters of SU(3) baryon ChPT

Holstein (2000)

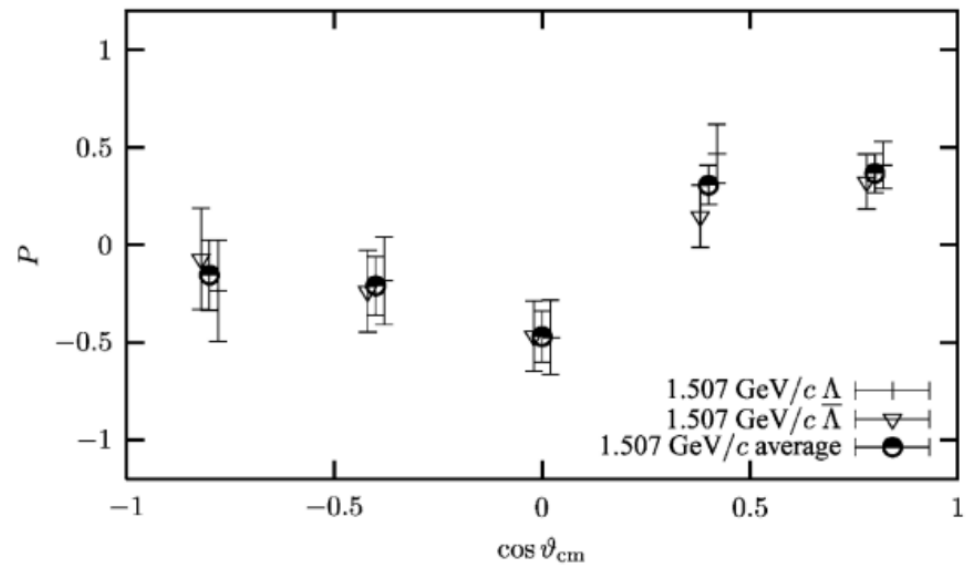
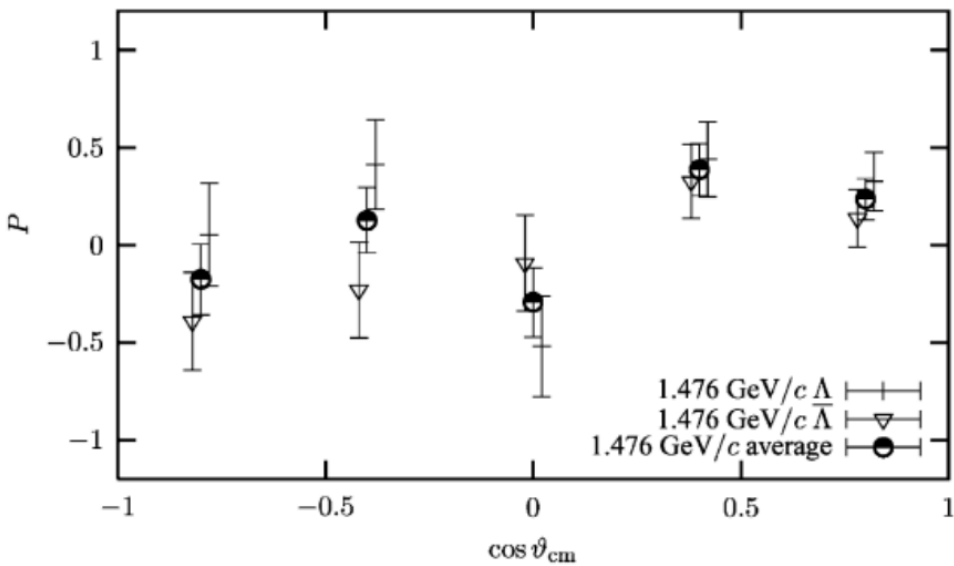
- essential for $(\gamma p \rightarrow K^+ \Lambda)$ — new measurement by (CLAS)



THIS TALK: ESTIMATE α_-

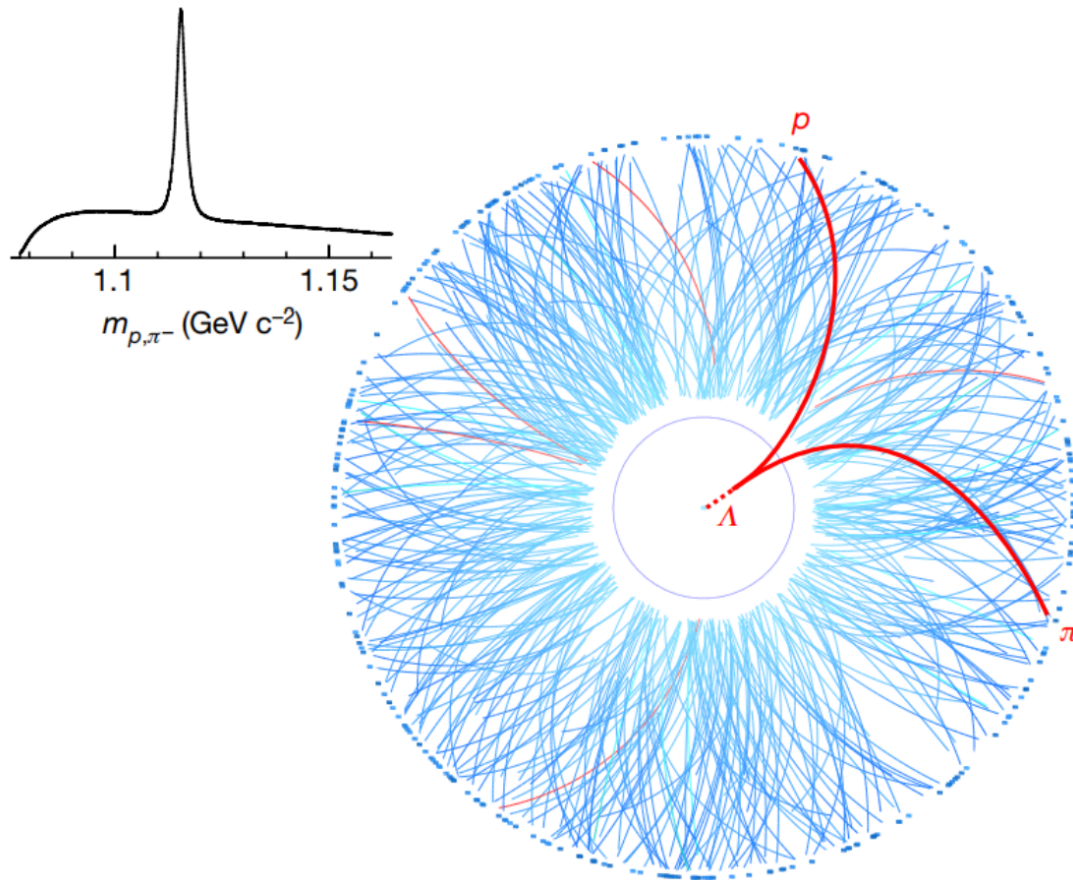
Where α matters (1):
Baryon spectroscopy

Where α matters (2):
 $\bar{p}p \rightarrow \bar{\Lambda}\Lambda$

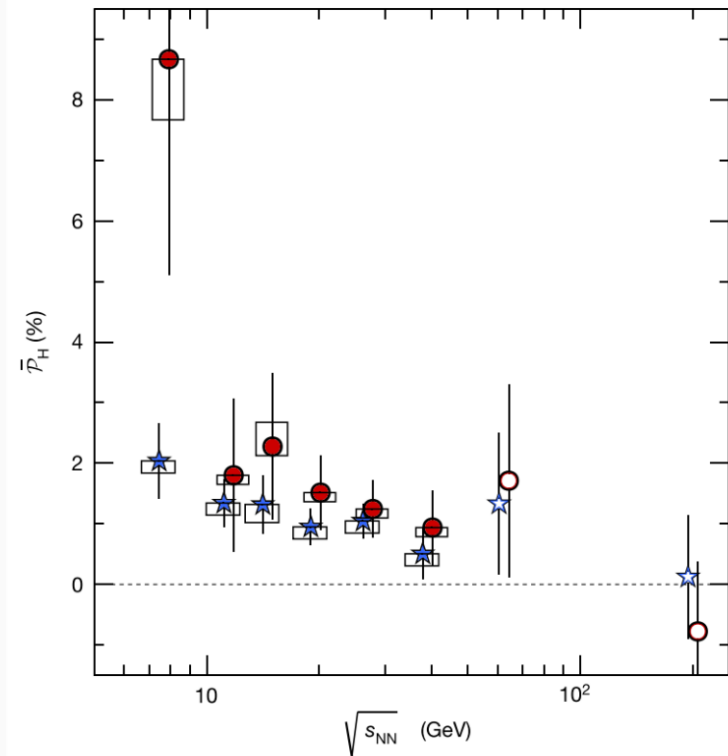


Where α matters (3): Global Λ polarization in nuclear collisions

STAR Au-Au collision



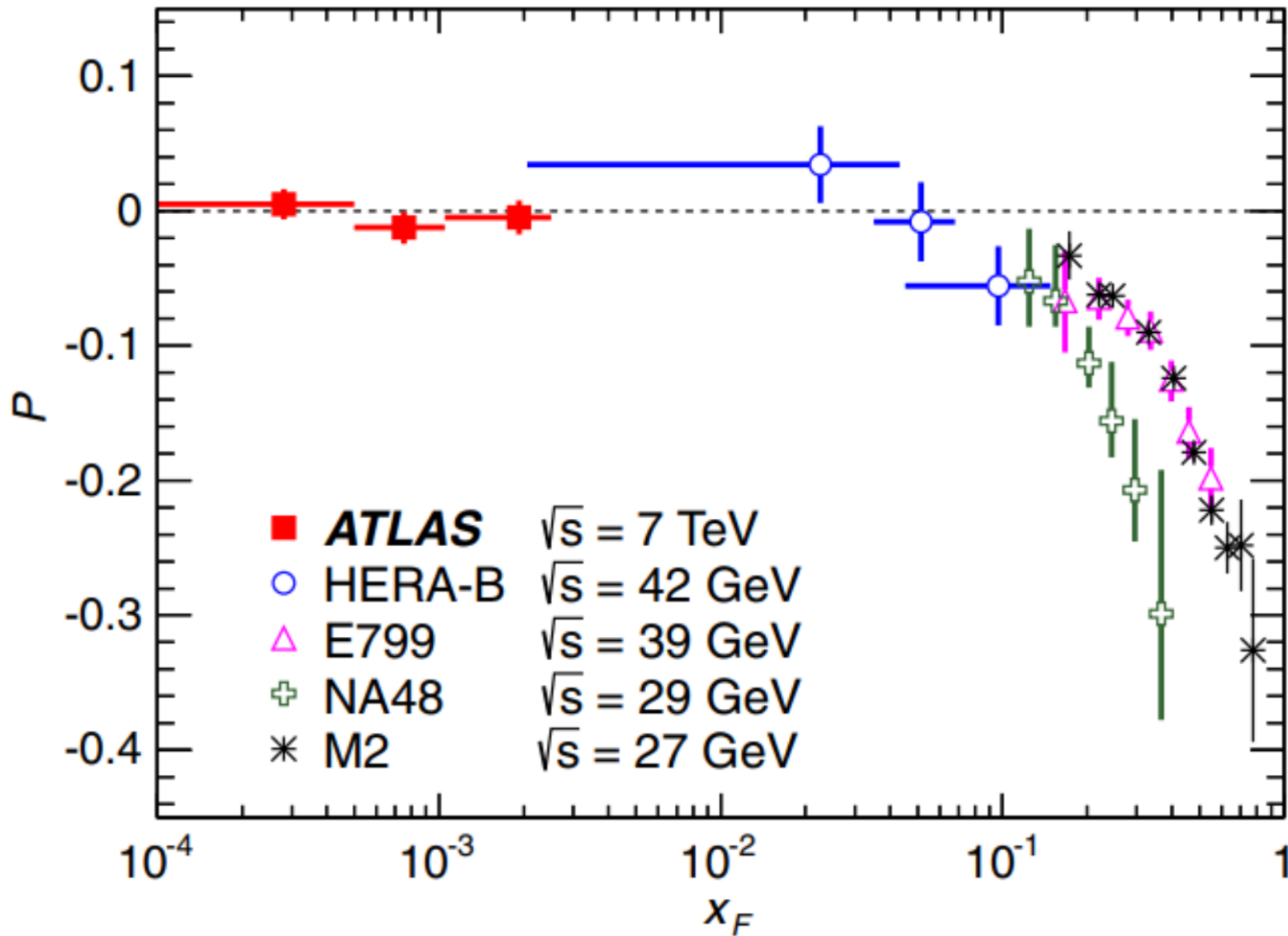
Average Λ ($\bar{\Lambda}$) polarization in collisions...



STAR coll., Nature (2017)

Where α matters (4):

Λ ($\bar{\Lambda}$) Transverse polarization with ATLAS



Definition of Fierz value and its distribution

◎ Define random variables:

$\mathcal{N}[\mu, \sigma^2]$ from CLAS measurements

$$\mathcal{F}_i^{(1)} = a^2 l^2 \left(\mathcal{O}_{x,i}^2 + \mathcal{O}_{z,i}^2 - \mathcal{T}_i^2 \right) + a^2 c^2 \left(\mathcal{C}_{x,i}^2 + \mathcal{C}_{z,i}^2 \right) + l^2 \Sigma_i^2 + a^2 \mathcal{P}_i^2$$

...similarly for second F.I.

◎ FV , a , l , c become random variables, but:

A. Scaling: $\left\{ \begin{array}{l} \text{Data and errors are scaled with } a, l, c \\ \text{Normalization of } PDF[a^2 \mathcal{P}_i^2] \end{array} \right.$

d'Agostini (1994)

B. Most “observables” and scale parameters enter quadratically

& Is there a closed form of $PDF[\mathcal{F}_i]$?

Roe (2015)

Statistical challenges (1)

A. Scaling

Imagine linear case: $\mathcal{F} := a \mathcal{O} = 1$

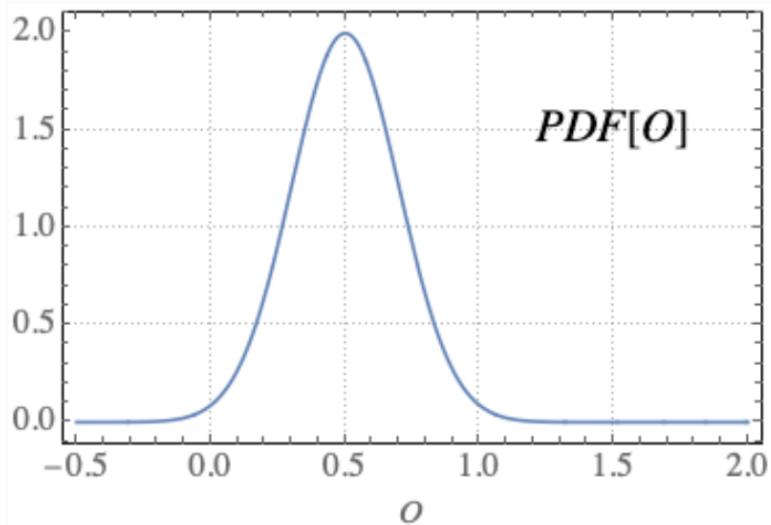
$$\mathcal{O} = \mathcal{N}[\mu, \sigma^2]$$

$$p_{\mathcal{F}}(f, a) = \int dO p(O) \delta(aO - f)$$

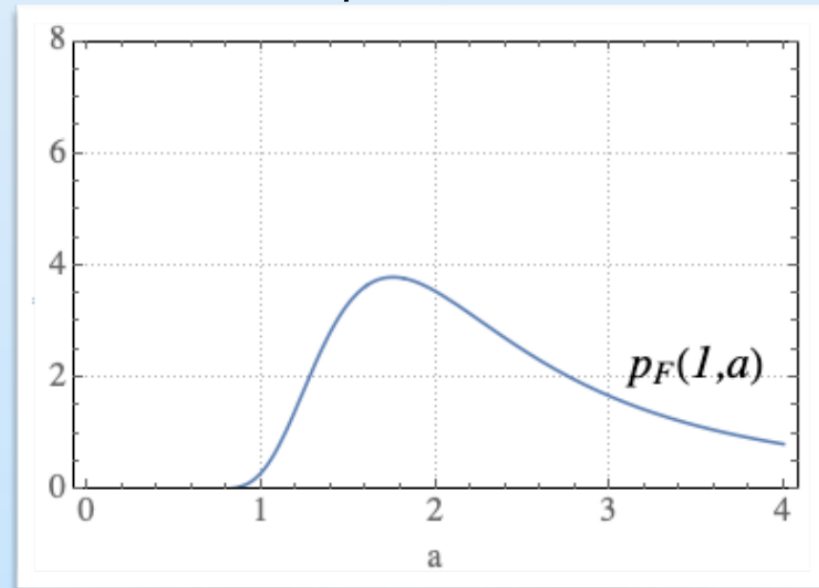
$$p_{\mathcal{F}}(1, a) = \frac{1}{a\sqrt{2\pi\mu\sigma}} e^{-\frac{(1-a\mu)^2}{2(a\sigma)^2}}$$

conditional probability

PDF of \mathcal{O} suggests $a=2$



PDF of \mathcal{F} peaks at $a < 2$



⇒ remove a -dependence from the normalization

Statistical challenges (1)

A. Scaling

Imagine linear case:

$$\mathcal{F} := a \mathcal{O} = 1$$

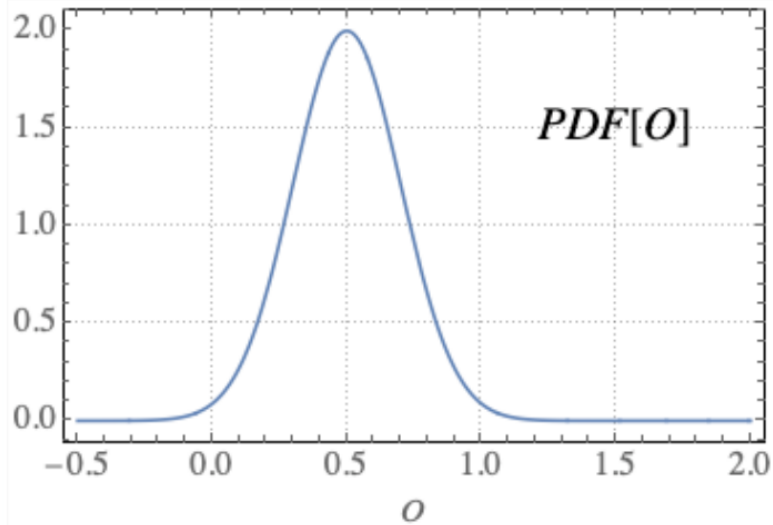
$$\mathcal{O} = \mathcal{N}[\mu, \sigma^2]$$

$$p_{\mathcal{F}}(f, a) = \int d\mathcal{O} p(\mathcal{O}) \delta(a\mathcal{O} - f)$$

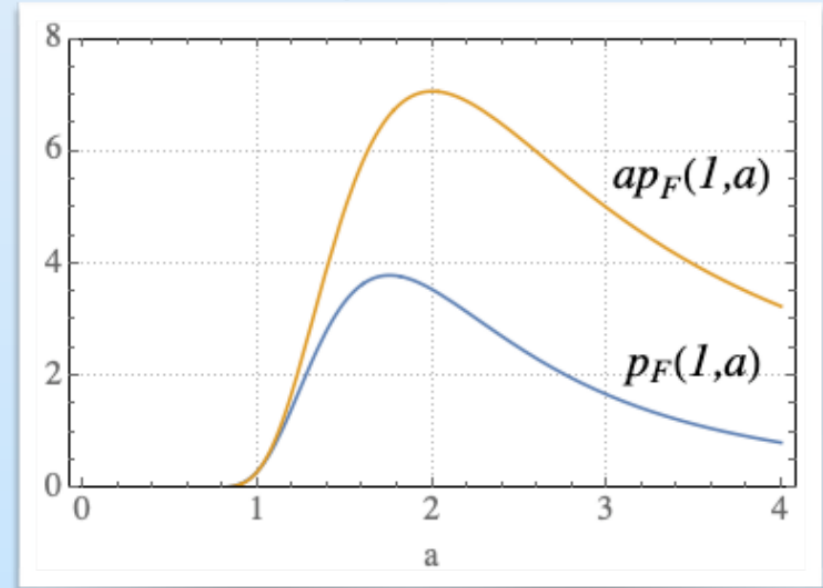
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Statistical challenges (2)

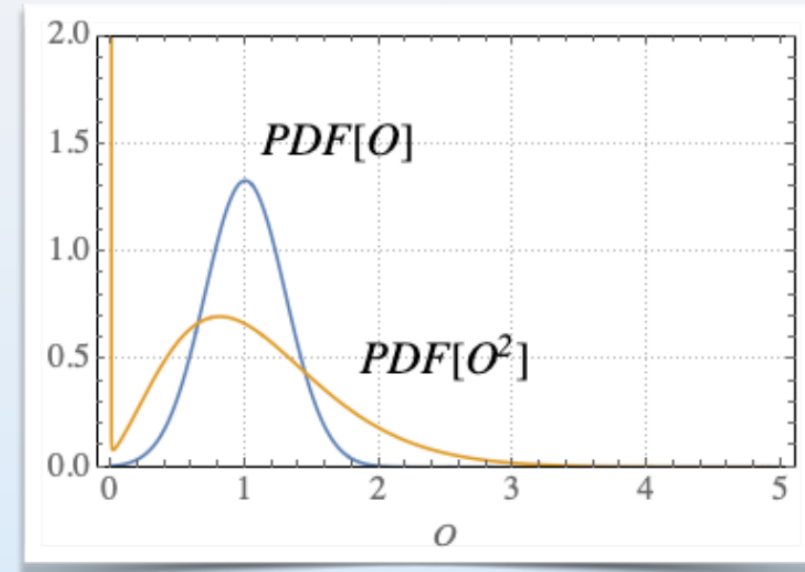
B. Non-linearity

$$\mathcal{O} \sim \mathcal{N}[\mu, \sigma^2] \implies \mathcal{Y} = \mathcal{O}^2 \sim NC_{\chi^2}$$

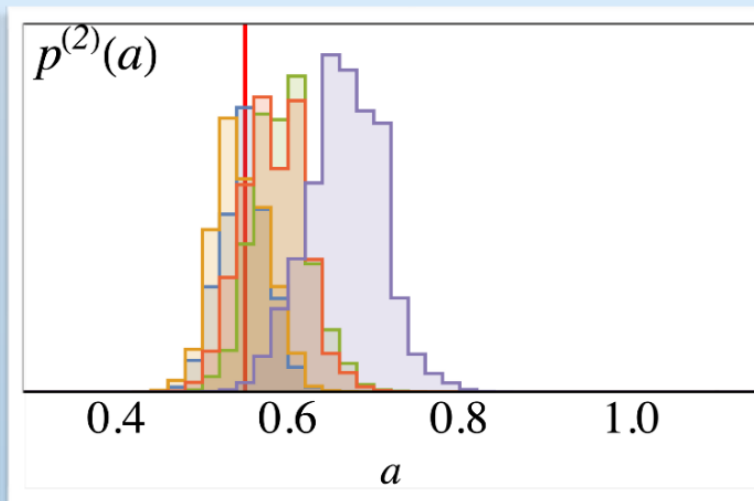
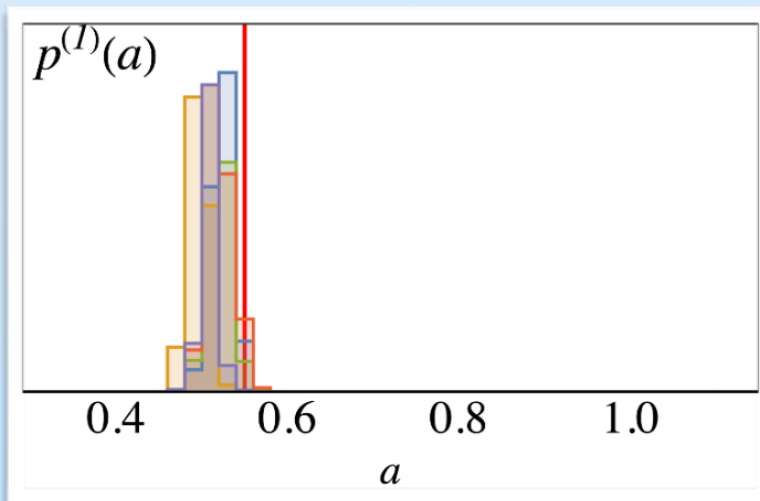
non-central chi squared distribution

$$\mu_{\mathcal{Y}} = \mu_{\mathcal{O}}^2 + \sigma_{\mathcal{O}}^2, \quad \sigma_{\mathcal{Y}}^2 = 2\sigma_{\mathcal{O}}^2(2\mu_{\mathcal{O}}^2 + \sigma_{\mathcal{O}}^2)$$

\implies Expectation value of Fierz identity $\neq 1$



Ultimately – blind test on synthetic data



re-sampling test of both Fierz identities:

- 300 kin. points
- 200 000 samples
- **$a_{test} = 0.55$**