

lattice spectroscopy @ JLab

Raúl Briceño - <http://bit.ly/rbricenoPhD>



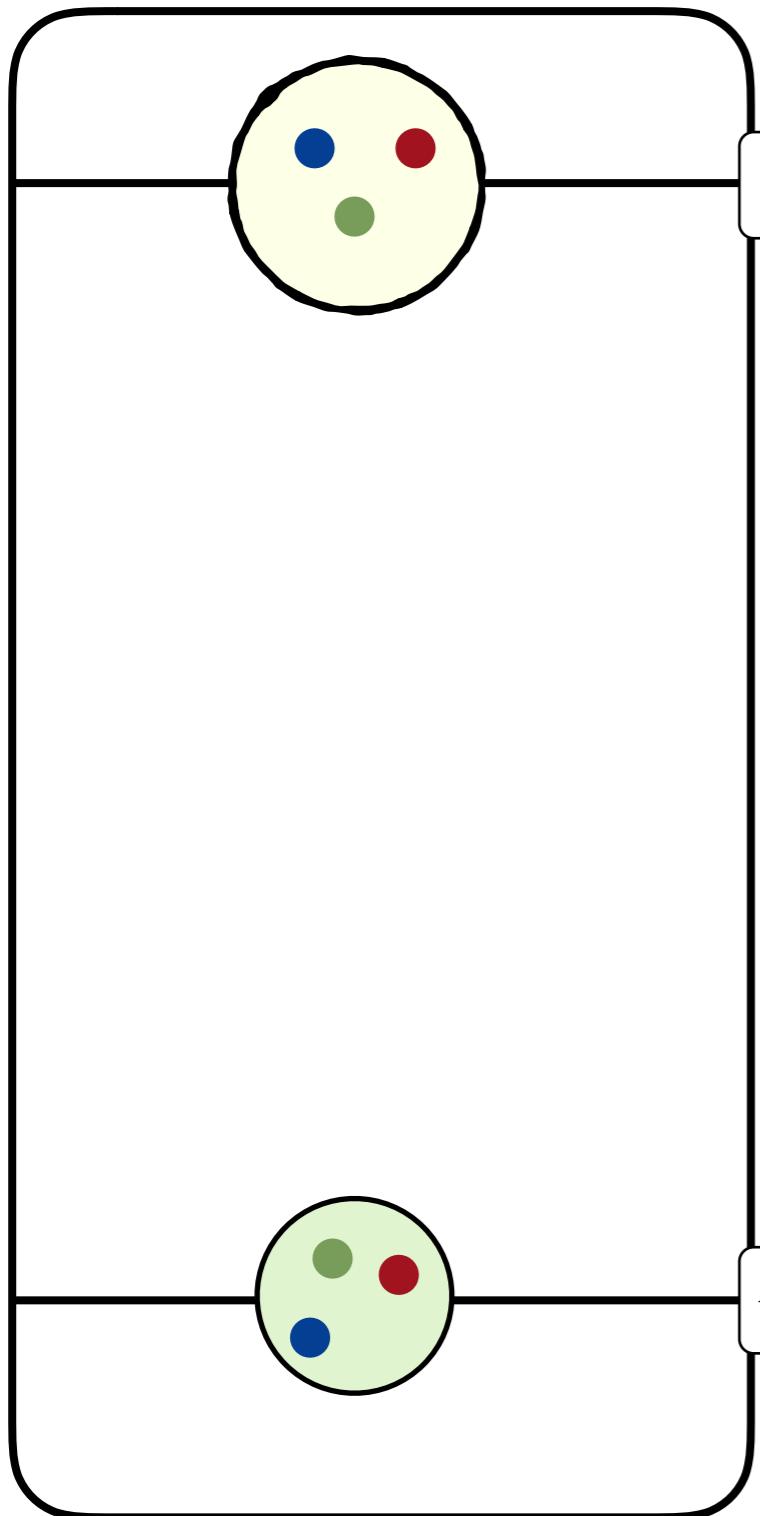
Norfolk, VA [Home to ODU]



JLab, VA



the Roper an outstanding 50yr puzzle

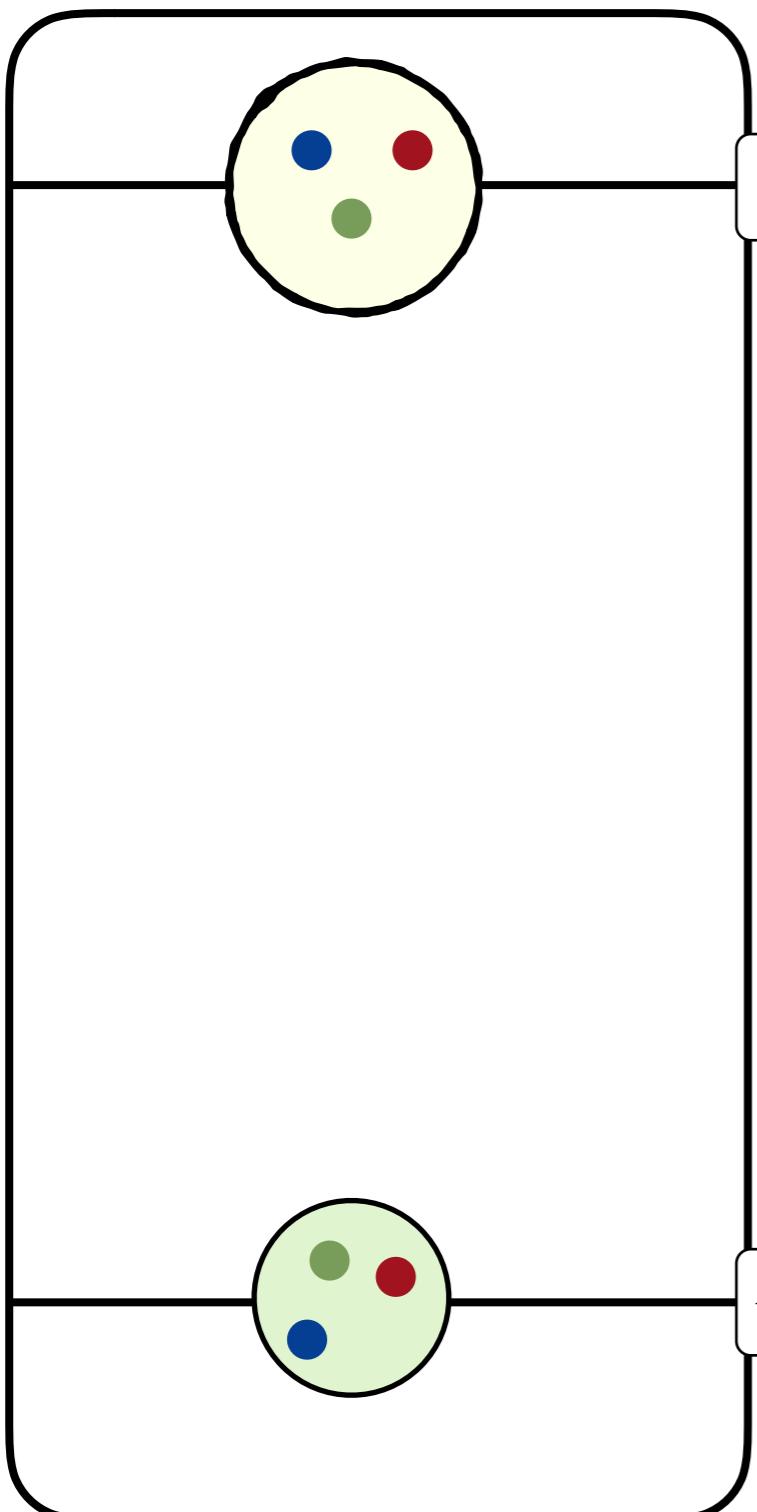


*The Roper is naively simple...
...but understanding its nature, requires us to face
some of the major theoretical challenges in QCD.*

*to claim understanding, we need to convincingly
reproduce experimental observables*

we need a standard of measure

the Roper an outstanding 50yr puzzle



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Volker Burkert

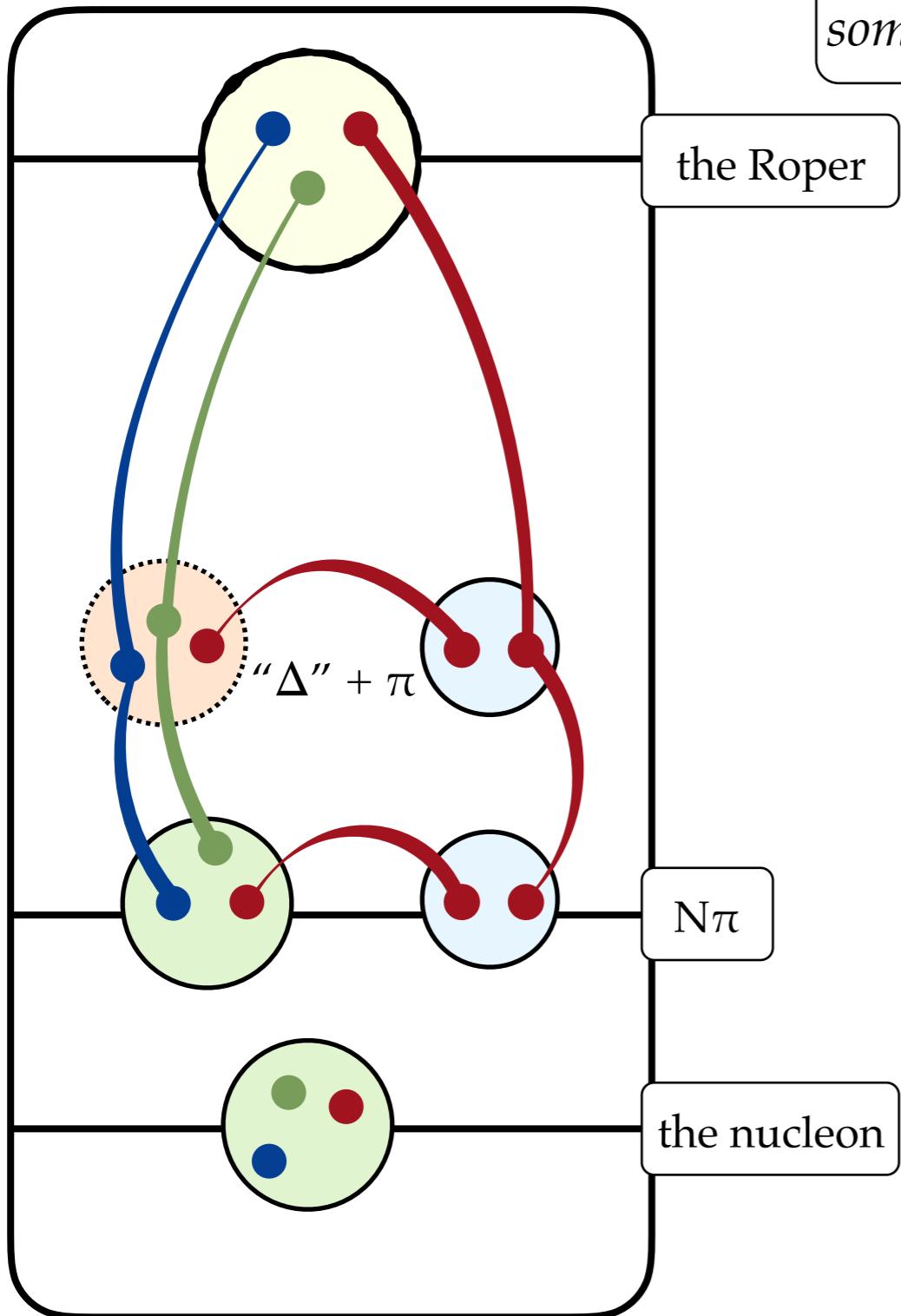
Volker Standard

“Accuracy of predictions should be commensurate
with experiments, i.e. $\mathcal{O}(\text{few MeV})$

Corollary

no theoretical study meets the “Volker standard”
for the Roper!

the Roper an outstanding 50yr puzzle



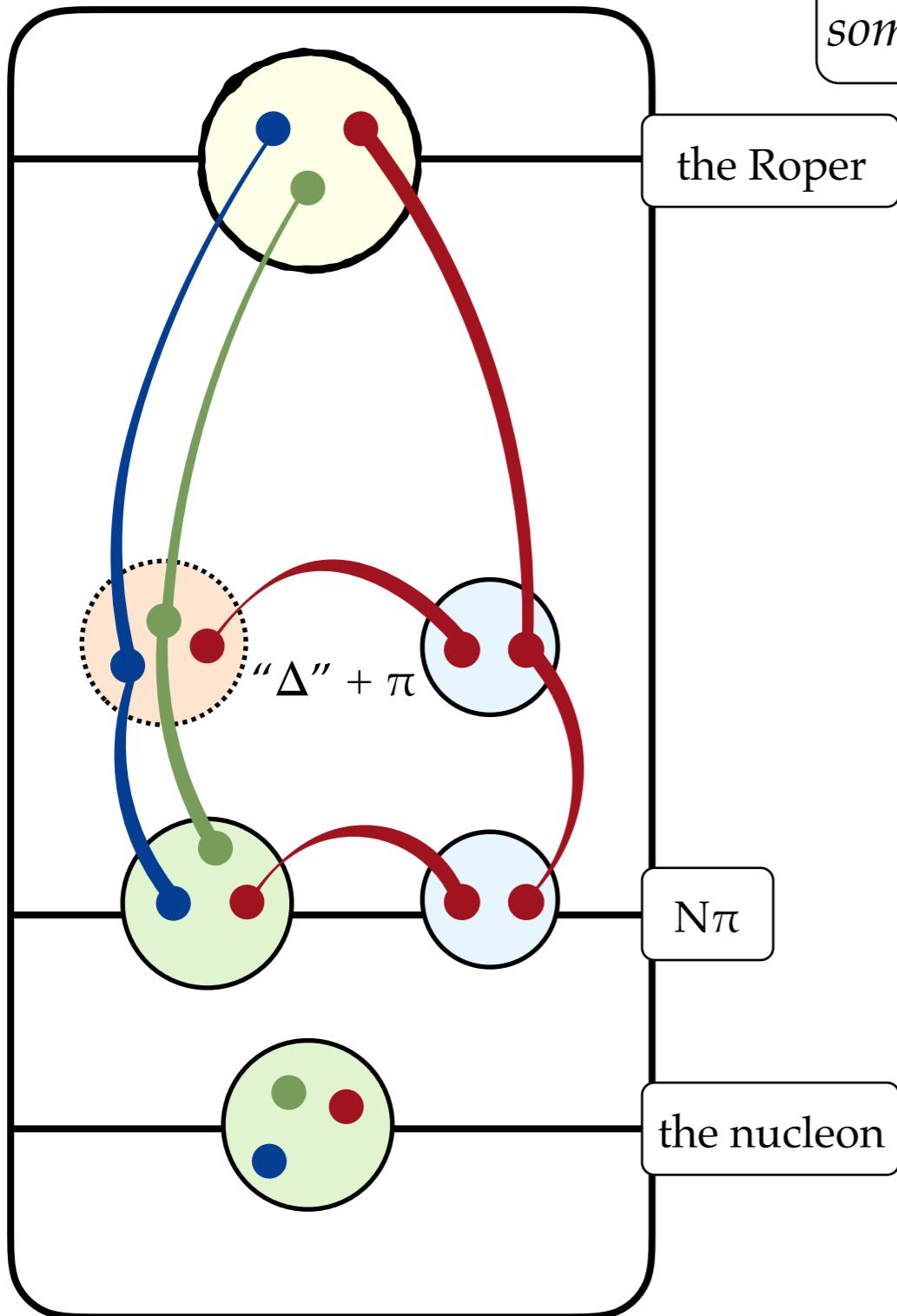
*The Roper is naively simple...
...but understanding its nature, requires us to face
some of the major theoretical challenges in QCD.*

What are the necessary features needed
to be able to claim accuracy?

broad resonance

*this already disqualifies most
if not all theoretical studies!*

the Roper an outstanding 50yr puzzle



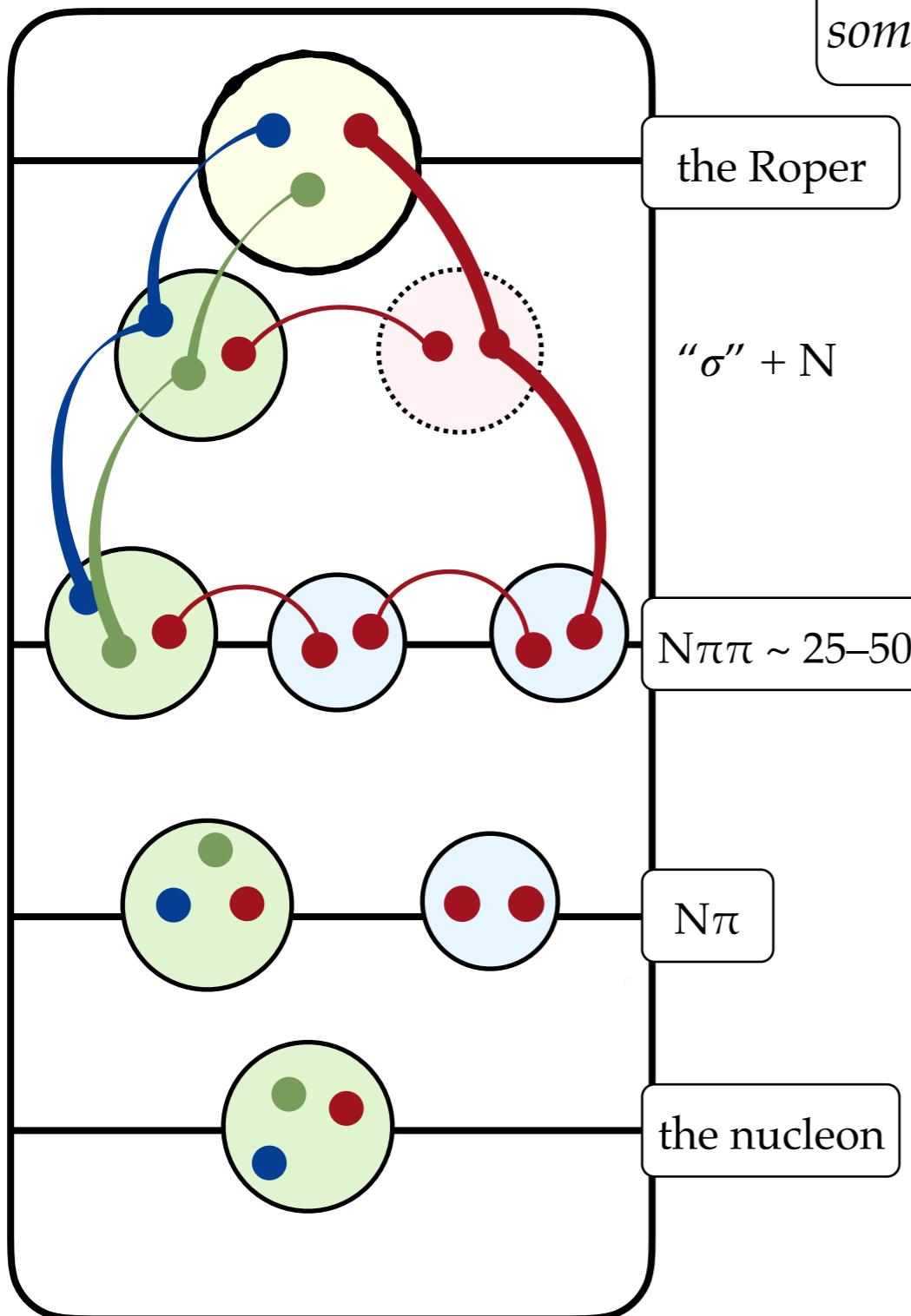
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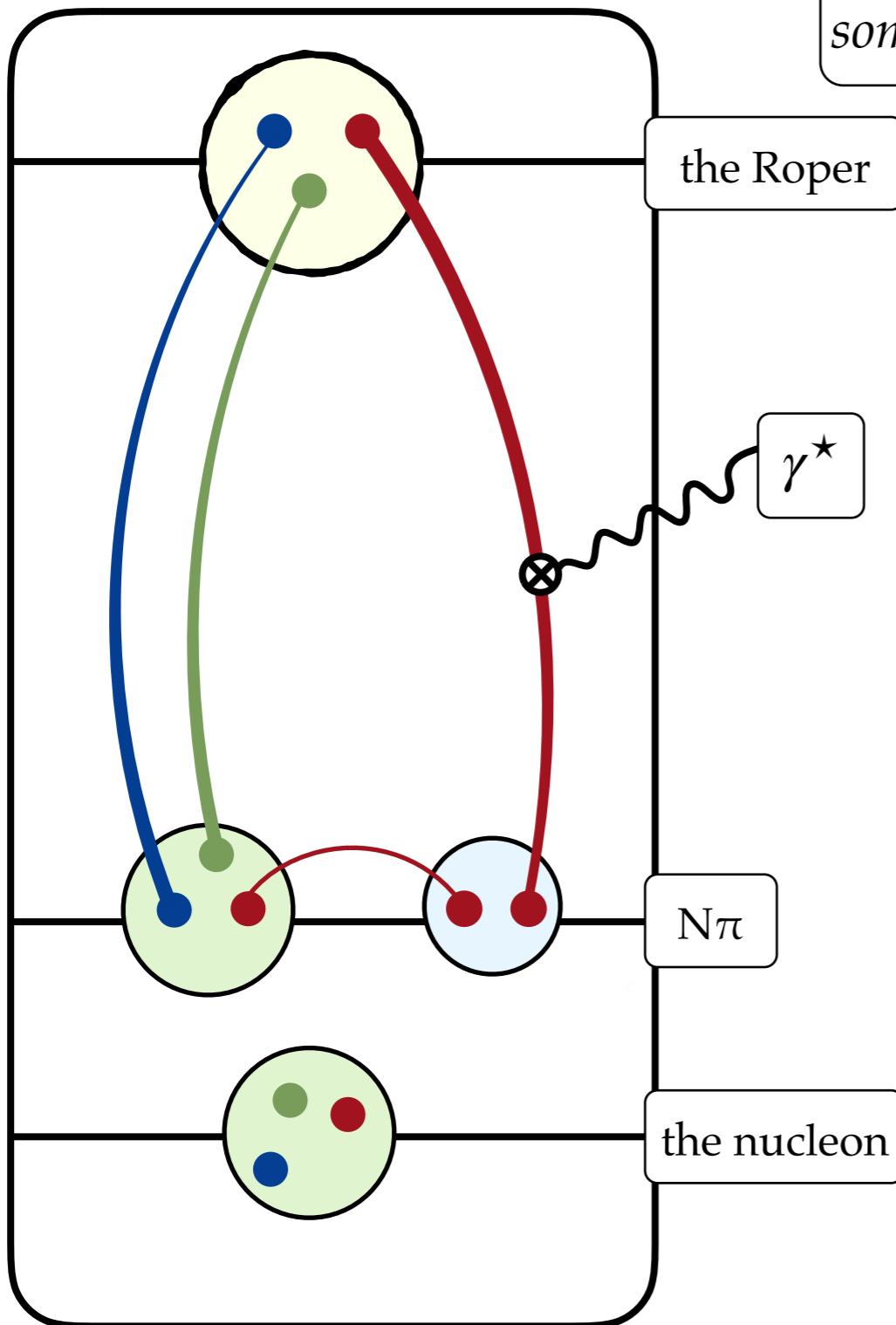
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three-particle system

structure of resonant states

QCD spectroscopy

Amplitude analysis

Experiments

QCD

INT Program : "Accessing and Understanding the QCD Spectrum"

August 17 - September 18, 2020



Pilloni [ECT*]

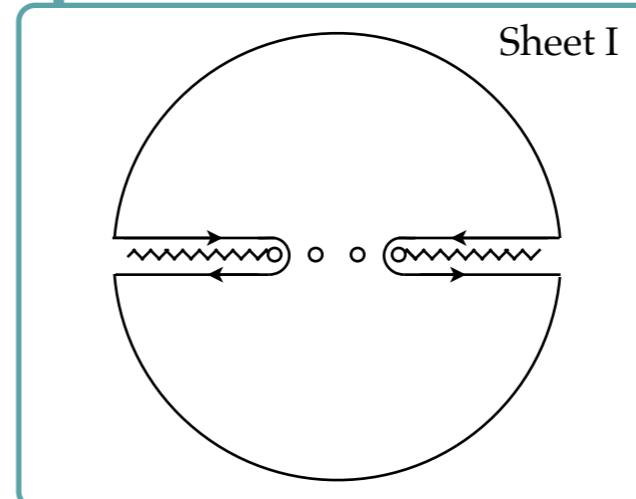


Eichmann [CFTP]

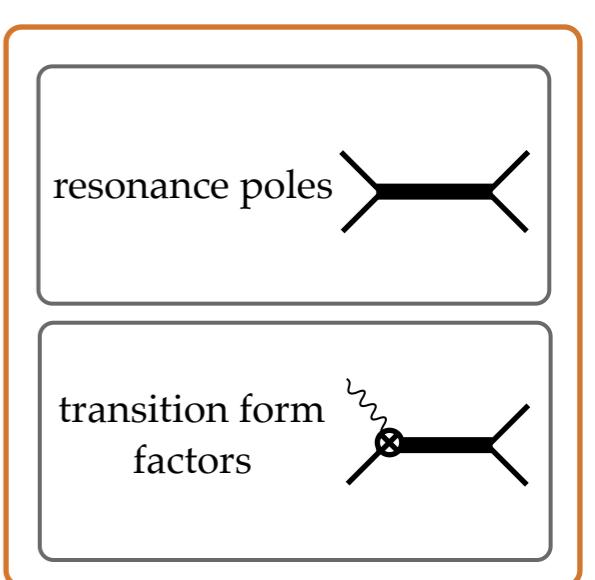
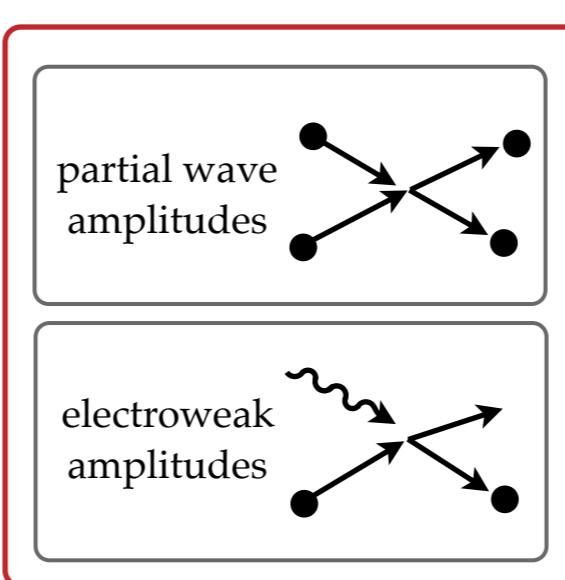
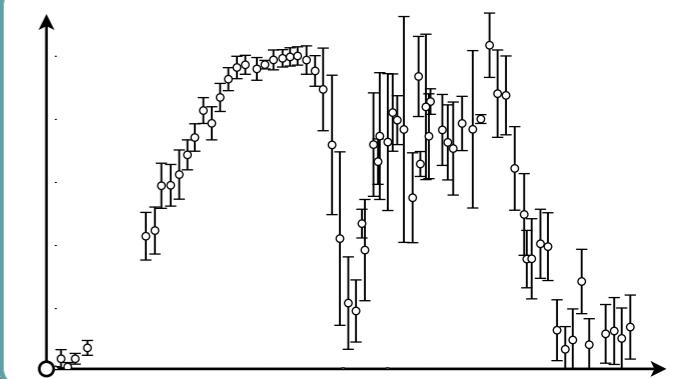
QCD spectroscopy

QCD

Amplitude analysis

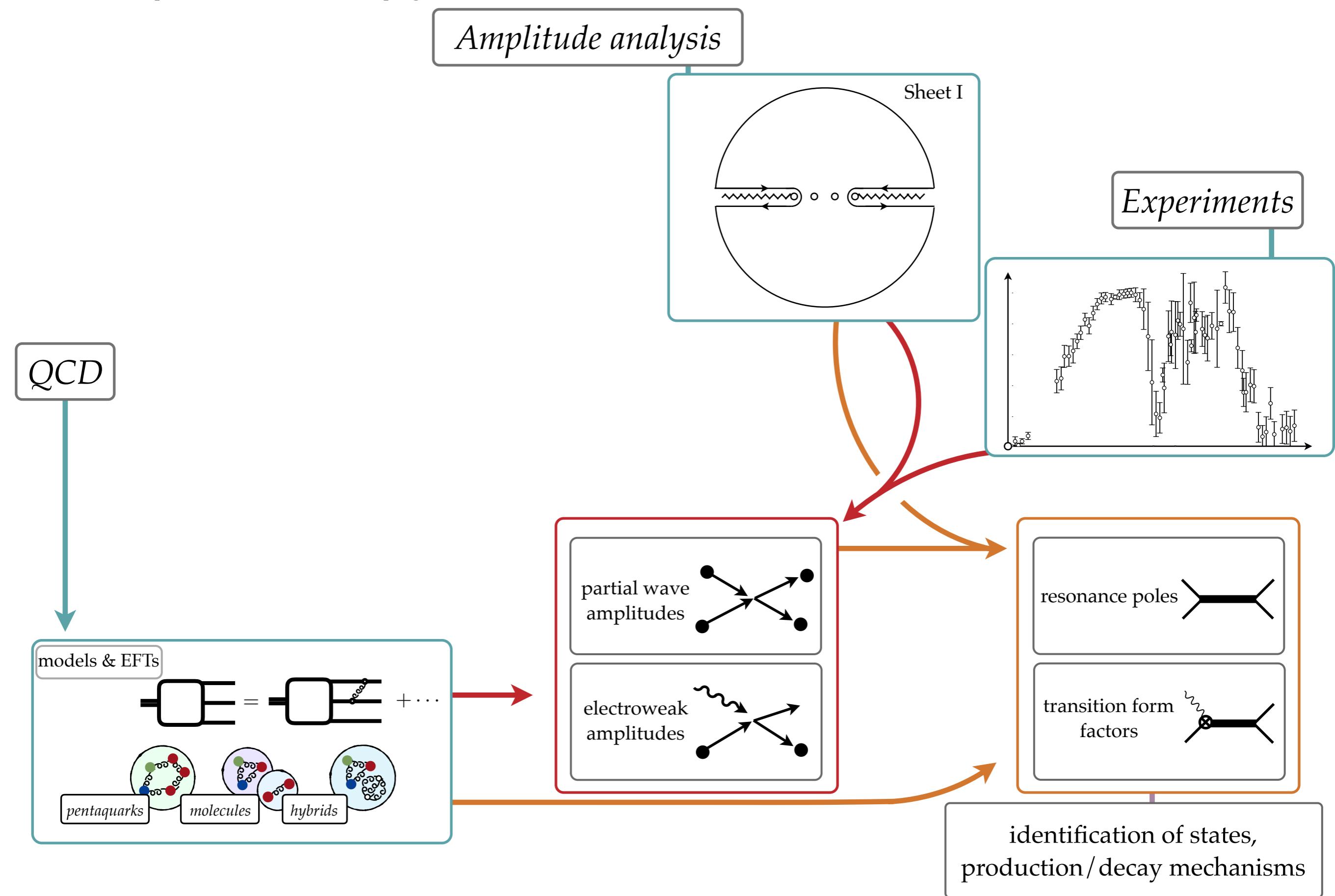


Experiments

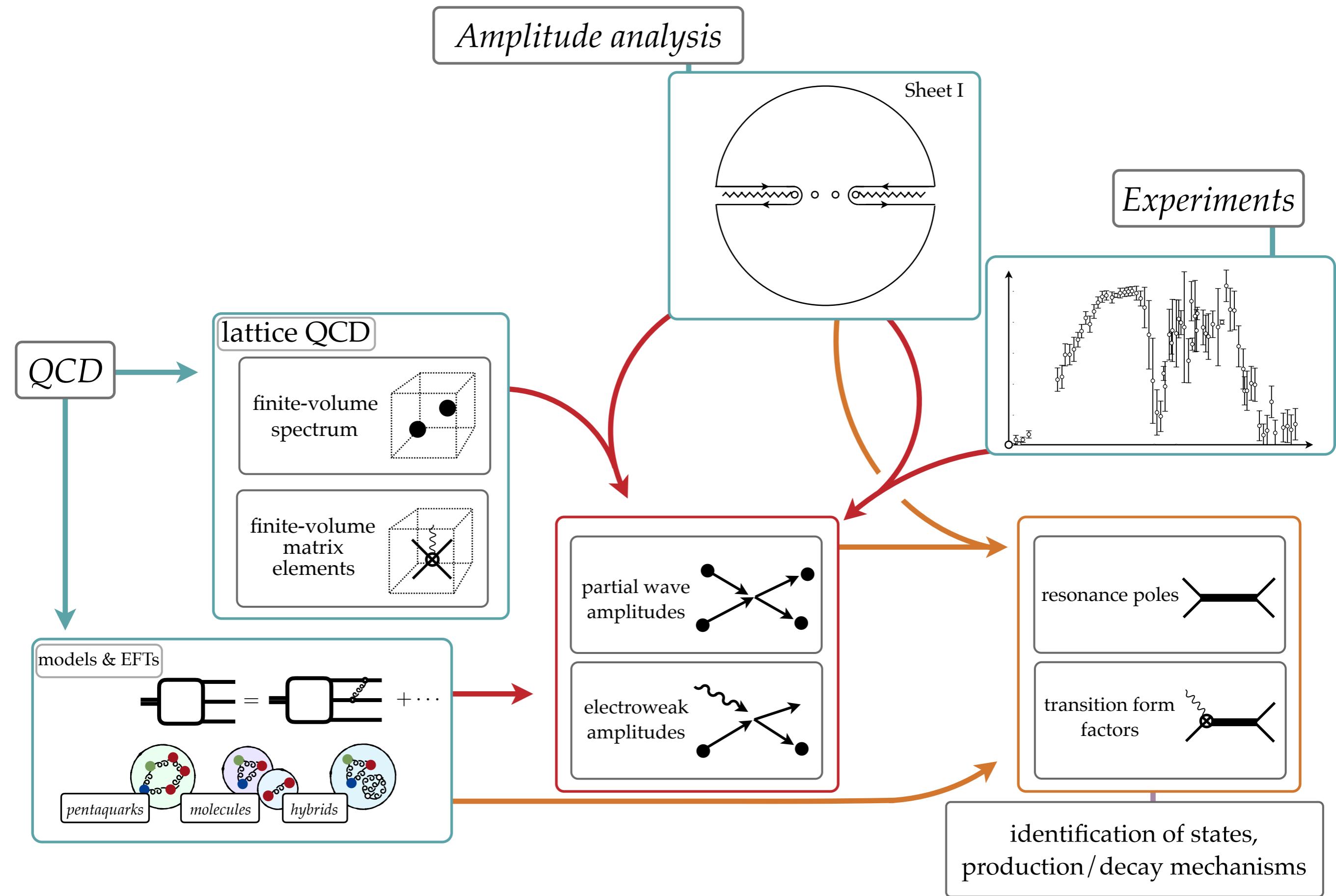


identification of states,
production/decay mechanisms

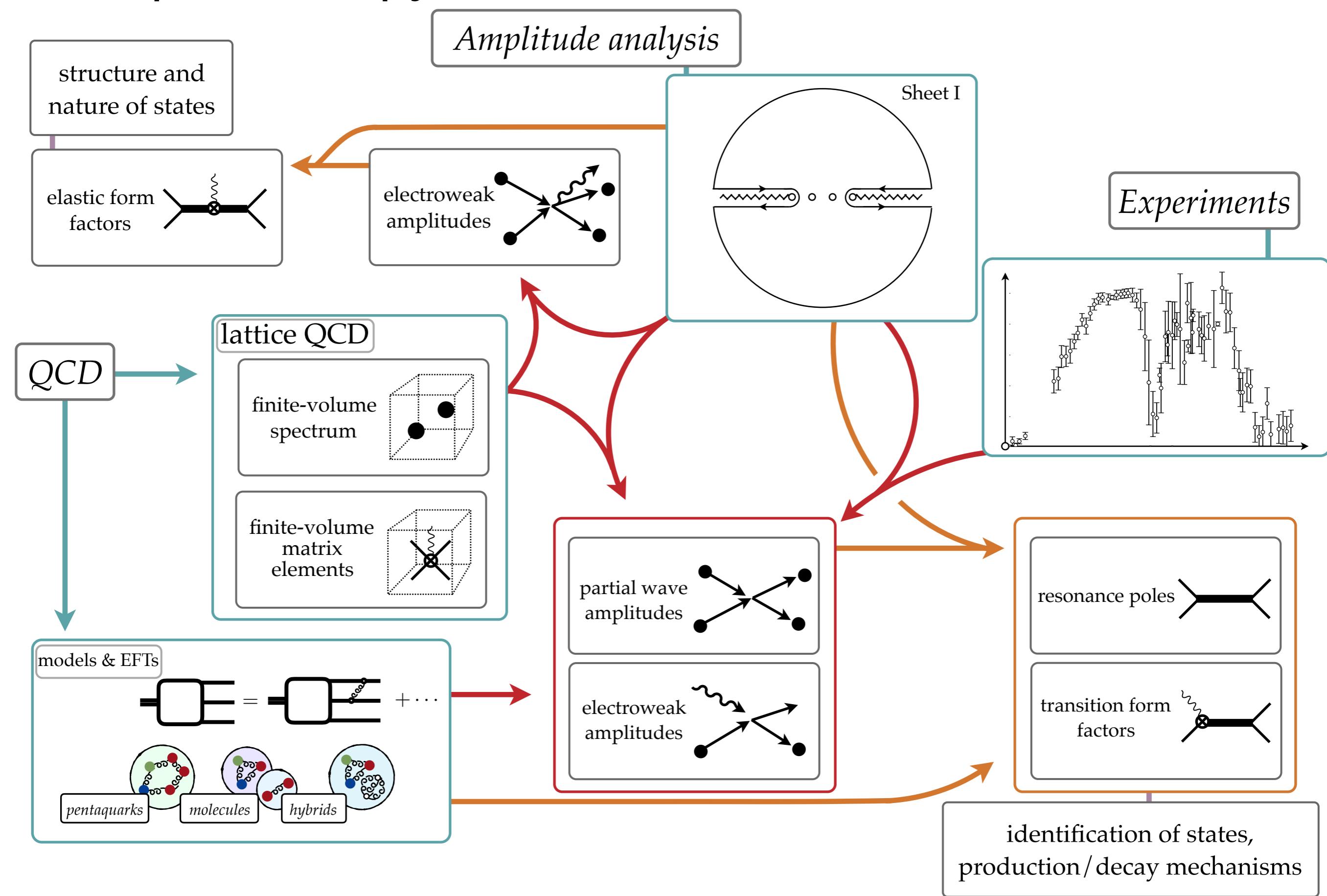
QCD spectroscopy



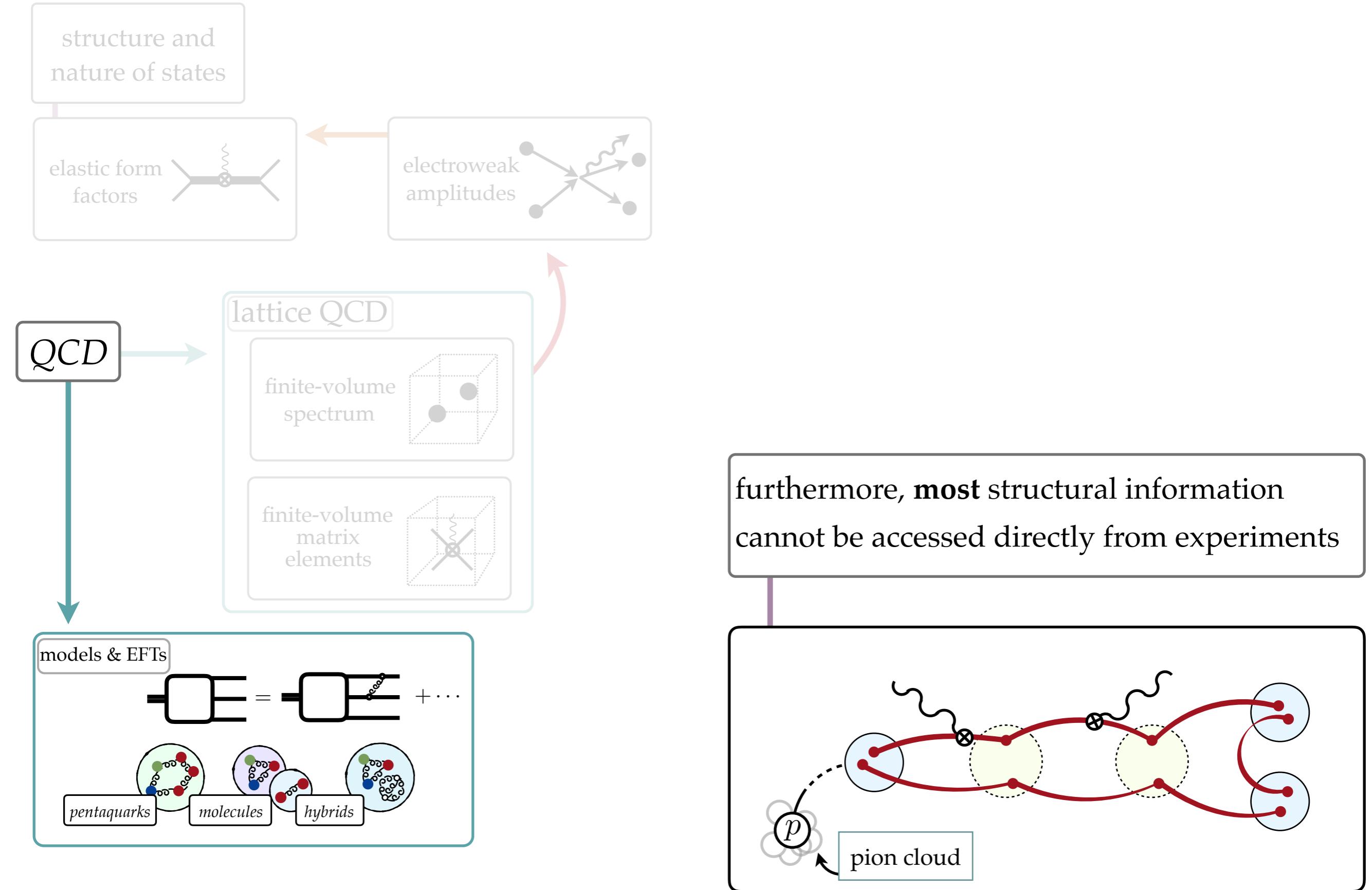
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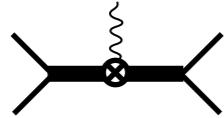
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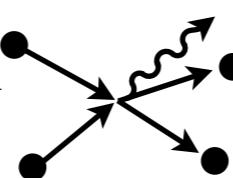
QCD spectroscopy

structure and nature of states

elastic form factors



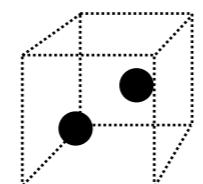
electroweak amplitudes



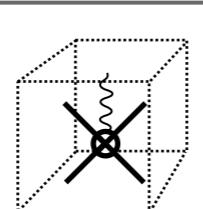
QCD

lattice QCD

finite-volume spectrum



finite-volume matrix elements



Hansen (CERN)



Baroni (USC)



Jackura (IU)



Ortega (W&M)



JLAB-THY-15-2140

Relativistic, model-independent, multichannel
2 → 2 transition amplitudes in a finite volume

Raúl A. Briceño^{1,*} and Maxwell T. Hansen^{2,†}

¹ Thomas Jefferson National Accelerator Facility,
12000 Jefferson Avenue, Newport News, VA 23606, USA

JLAB-THY-18-2878
CERN-TH-2018-263

Form factors of two-hadron states from a covariant finite-volume formalism

Alessandro Baroni,^{1,*} Raúl A. Briceño,^{2,3,†} Maxwell T. Hansen,^{4,‡} and Felipe G. Ortega-Gama^{2,5,§}

¹Department of Physics and Astronomy University of South Carolina,
712 Main Street, Columbia, South Carolina 29208, USA

²Thomas Jefferson National Accelerator Facility,
12000 Jefferson Avenue, Newport News, Virginia 23606, USA

³Department of Physics Old Dominion University, Norfolk, Virginia 23529, USA

JLAB-T
CERN-T

Consistency checks for two-body finite-volume matrix elements:
I. Conserved currents and bound states

Raúl A. Briceño,^{1,2,*} Maxwell T. Hansen,^{3,†} and Andrew W. Jackura^{1,2,‡}

¹Thomas Jefferson National Accelerator Facility, 12000 Jefferson Avenue, Newport News, VA 23606, USA

²Department of Physics, Old Dominion University, Norfolk, Virginia 23529, USA

³Theoretical Physics Department, CERN, 1211 Geneva 23, Switzerland

(Dated: September 24, 2019)

Recently, a framework has been developed to study form factors of two-hadron states probed by an external current. The method is based on relating finite-volume matrix elements, computed using numerical lattice QCD, to the corresponding infinite-volume observables. As the formalism is complicated, it is important to provide non-trivial checks on the final results and also to explore its validity. In this paper, we apply this method to the two-body finite-volume matrix elements of the

v1 [hep-lat] 28 Sep 2015

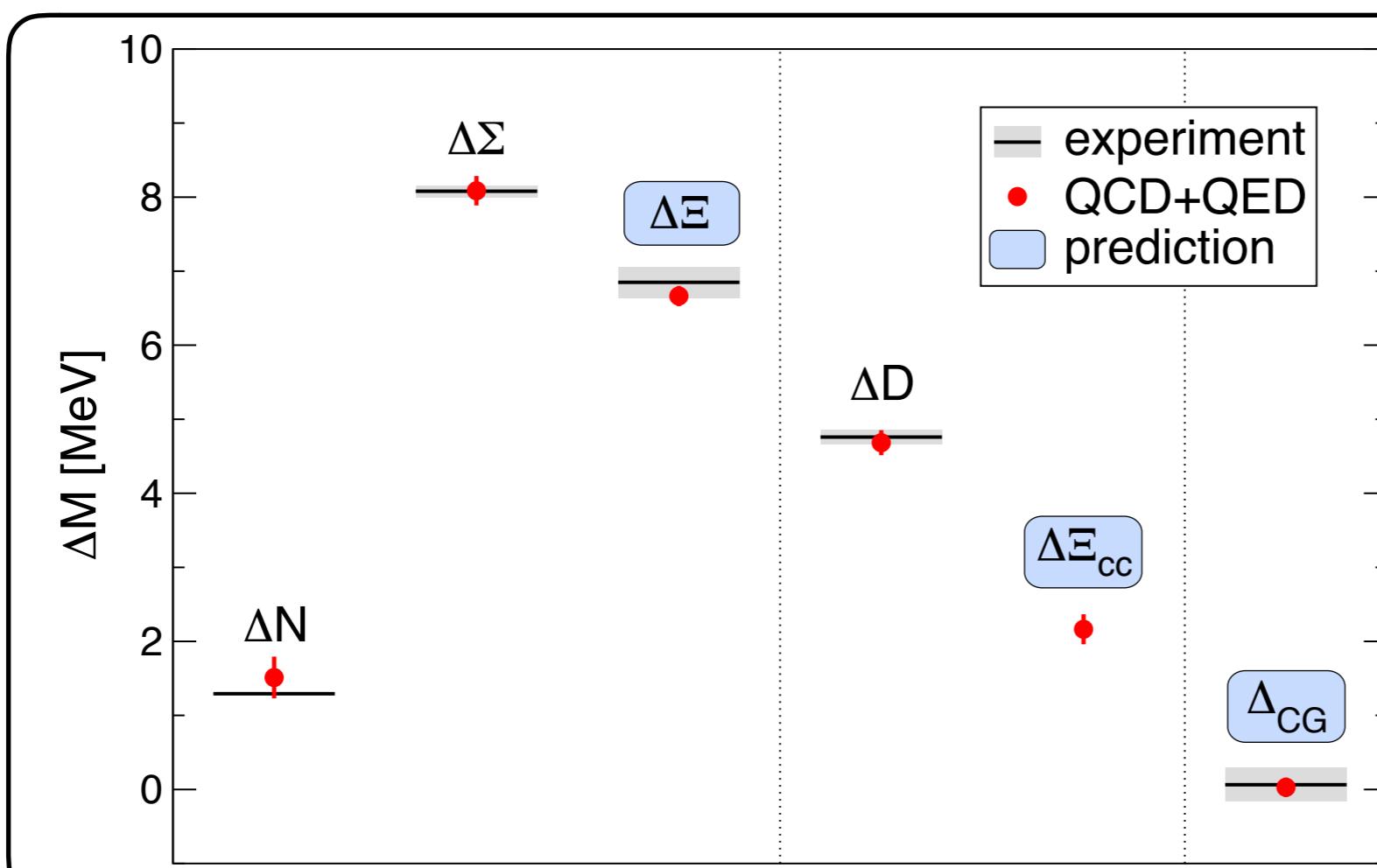
] 26 Dec 2018

Scattering observables from lattice QCD

- Wick rotation [Euclidean spacetime]: $t_M \rightarrow -it_E$
- Monte Carlo sampling
- quark masses: $m_q \rightarrow m_q^{\text{phys.}}$
- lattice spacing
- finite volume

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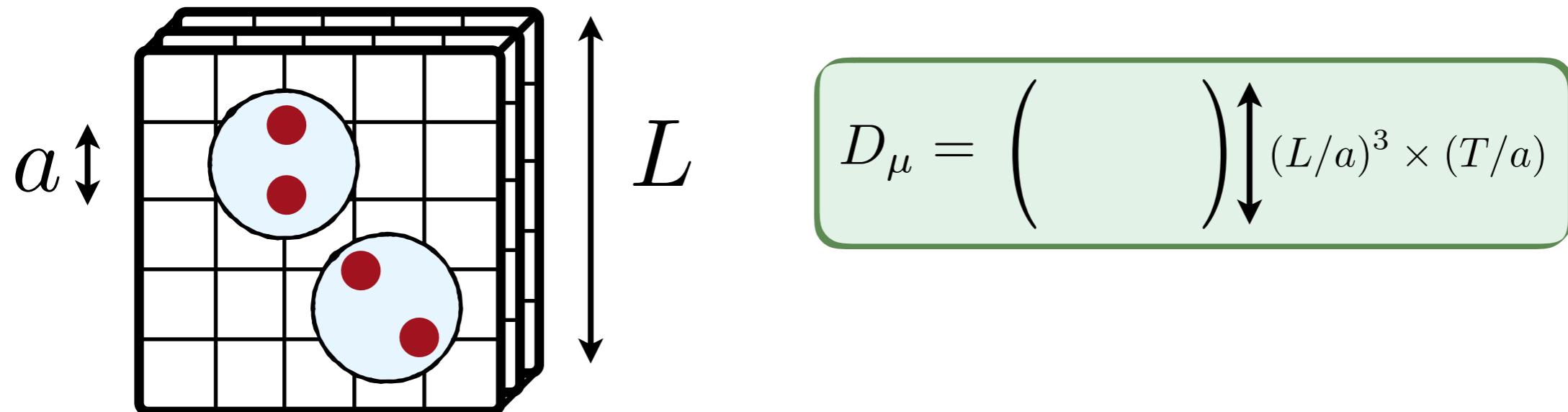
satisfies the
Volker Standard



BMW Collaboration (2015)

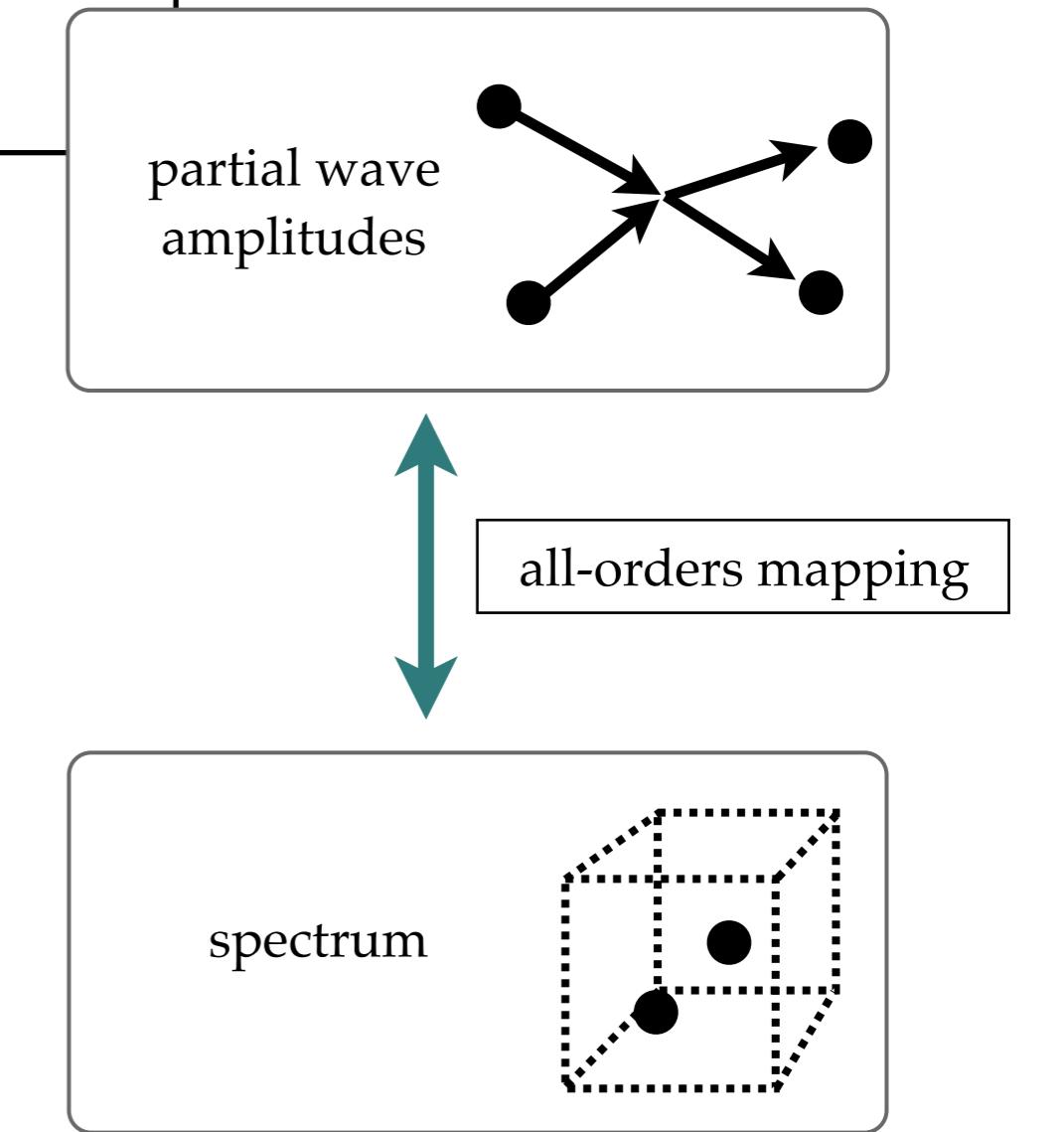
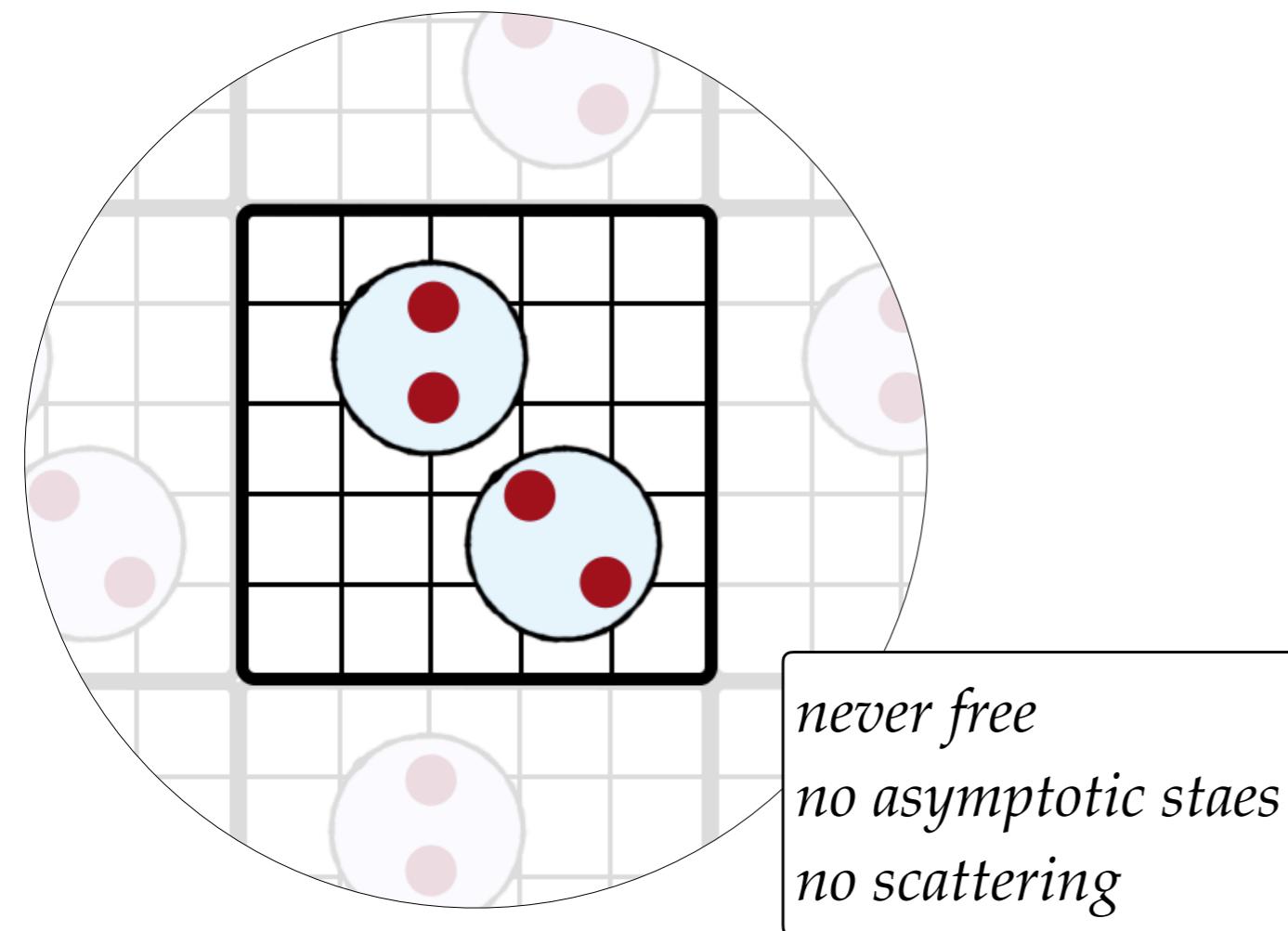
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the light sector



had spec

Wilson

Dudek

Edwards

Thomas

Woss

PRL 118, 022002 (2017)

PHYSICAL REVIEW LETTERS

week ending
13 JANUARY 2017

Isoscalar $\pi\pi$ Scattering and the σ Meson Resonance from QCD

Raul A. Briceño,^{1,*} Jozef J. Dudek,^{1,2,†} Robert G. Edwards,^{1,‡} and David J. Wilson^{3,§}

JLAB-THY-17-2534

¹Thomas

Isoscalar $\pi\pi, K\bar{K}, \eta\eta$ scattering and the σ, f_0, f_2 mesons from QCD

Raul A. Briceño,^{1, 2, *} Jozef J. Dudek,^{1, 3, †} Robert G. Edwards,^{1, ‡} and David J. Wilson^{4, §}
(for the Hadron Spectrum Collaboration)

¹Thomas Jefferson National Accelerator Facility, 12000 Jefferson Avenue, Newport News, VA 22606, USA

PHYSICAL REVIEW LETTERS 123, 042002 (2019)

Quark-Mass Dependence of Elastic πK Scattering from QCD

David J. Wilson,^{1,*} Raúl A. Briceño,^{2,3,†} Jozef J. Dudek,^{2,4,‡} Robert G. Edwards,^{2,§} and Christopher E. Thomas^{5,||}

PHYSICAL REVIEW D 100, 054506 (2019)

²Thomas

b_1 resonance in coupled $\pi\omega, \pi\phi$ scattering from lattice QCD

Antoni J. Woss,^{1,*} Christopher E. Thomas,^{1,†} Jozef J. Dudek,^{2,3,‡} Robert G. Edwards,^{2,‡} and David J. Wilson^{4,||}

Introduction
as a tool to in-
interactions, q
scalar channel
to 0, is domi-
despite experi-
place for many

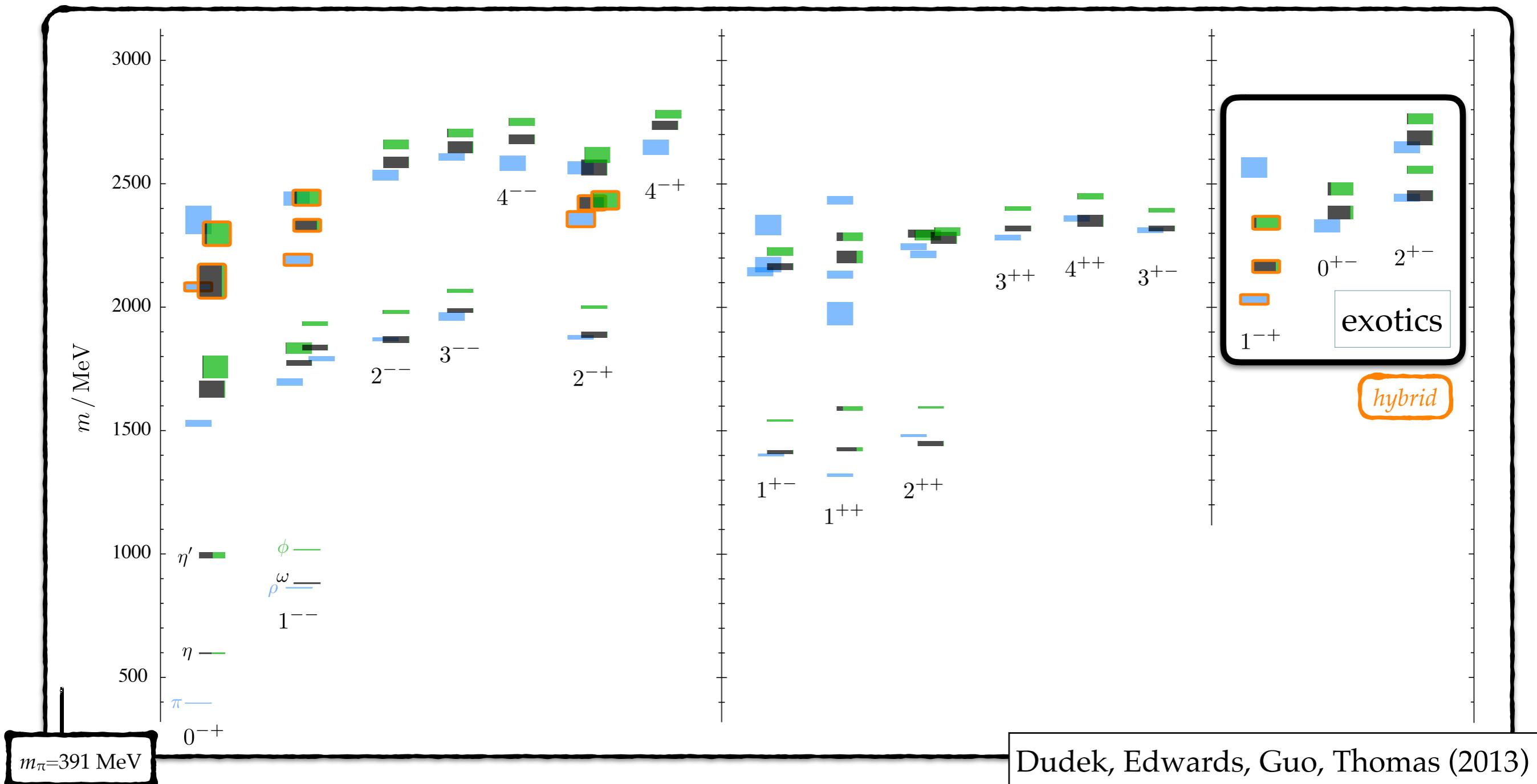
[0-lat] 20 Aug 2017

The co-
($J^P = 0^+$)
one view
thus hav-

Interesting but "old-school spectroscopy"

Evaluate: $C_{ab}^{2pt.}(t, \mathbf{P}) \equiv \langle 0 | \mathcal{O}_b(t, \mathbf{P}) \mathcal{O}_a^\dagger(0, \mathbf{P}) | 0 \rangle = \sum_n Z_{b,n} Z_{a,n}^* e^{-E_n t}$

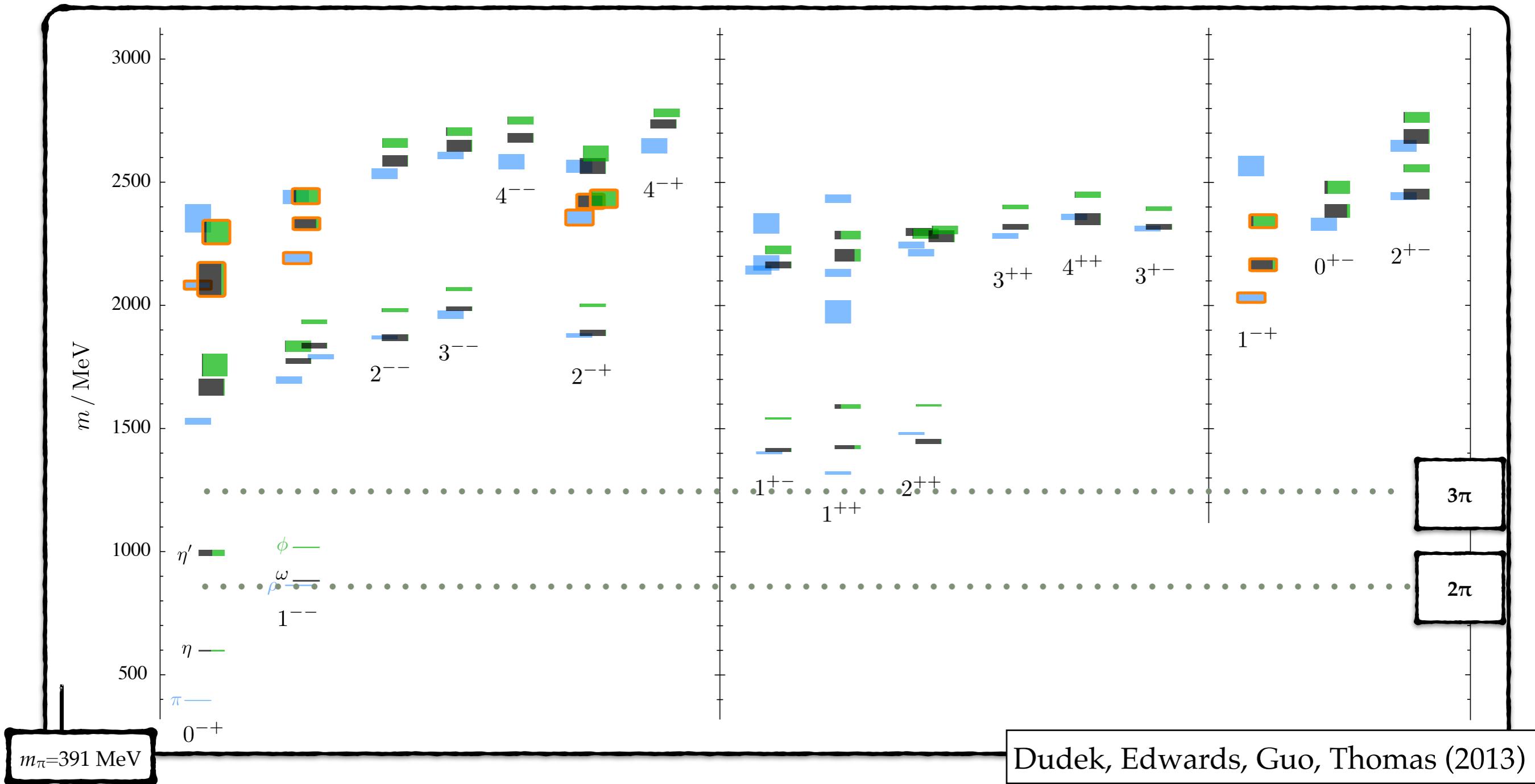
...using distillation and a large number [10-30] of local ops, $\mathcal{O}_b \sim \bar{q} \Gamma_b q$



Interesting but "old-school spectroscopy"

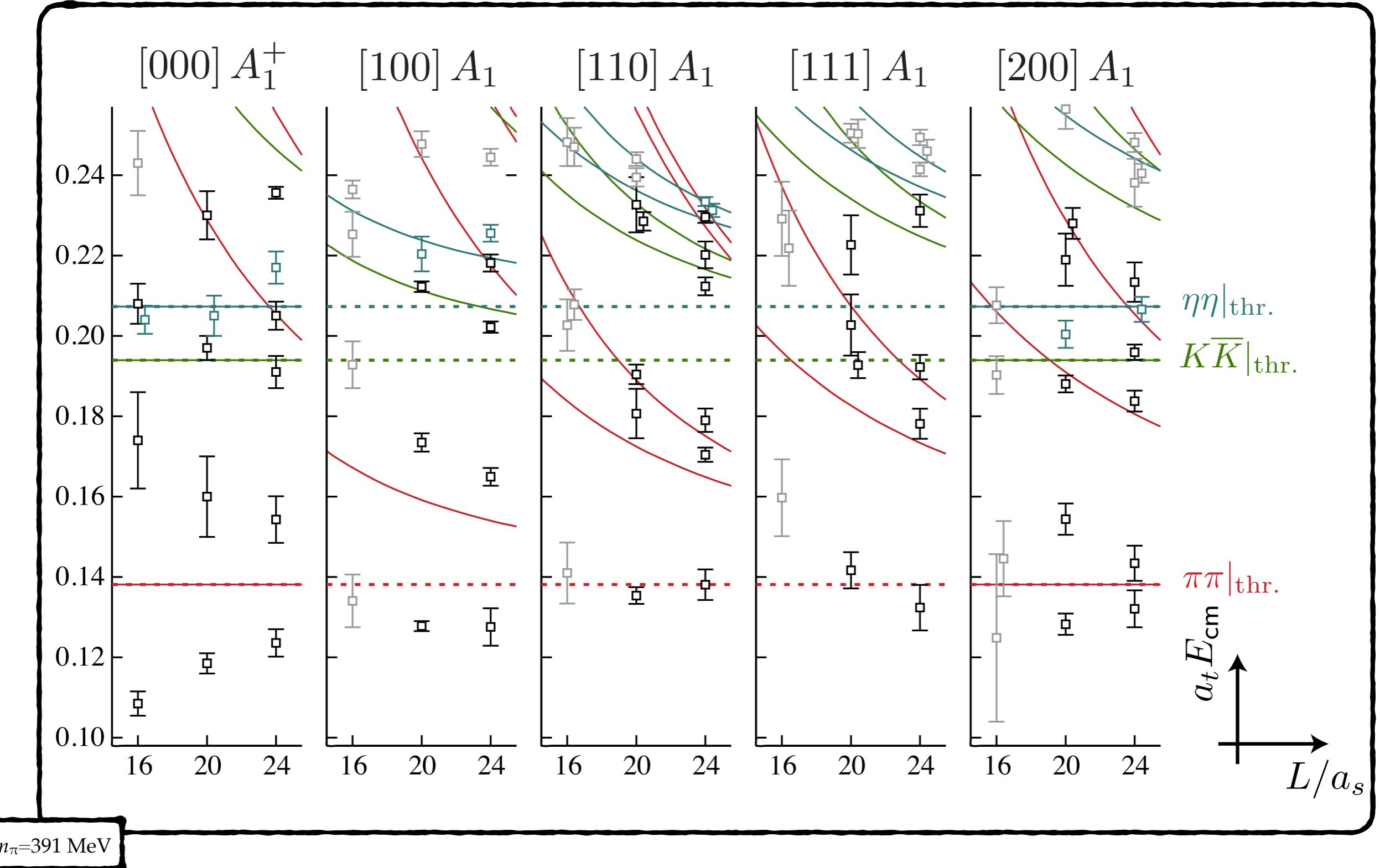
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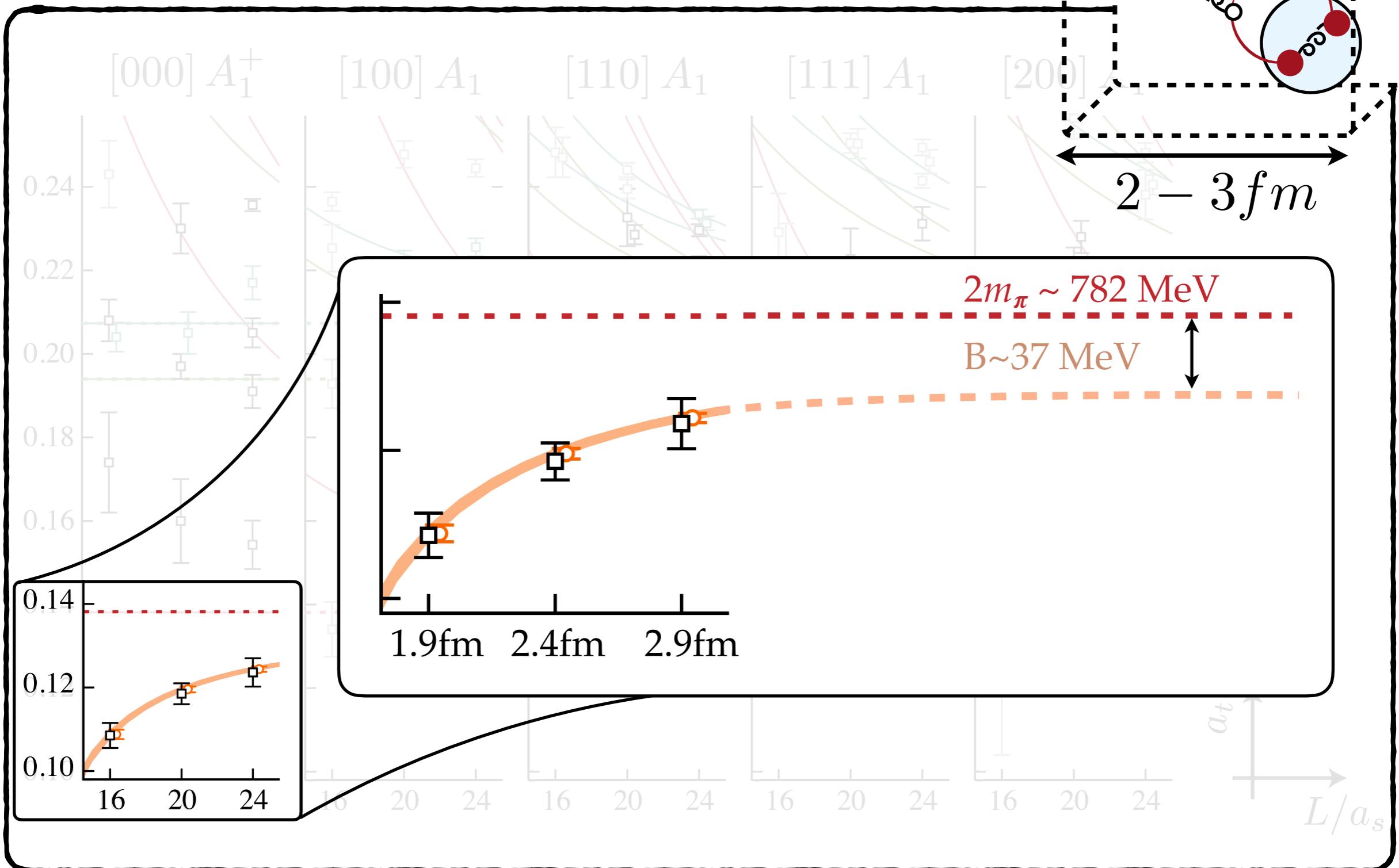
Isoscalar spectra: S-wave dominant

📌 Spectrum including a large basis: $\{\pi\pi, K\bar{K}, \eta\eta, \ell\bar{\ell}, s\bar{s}\}$



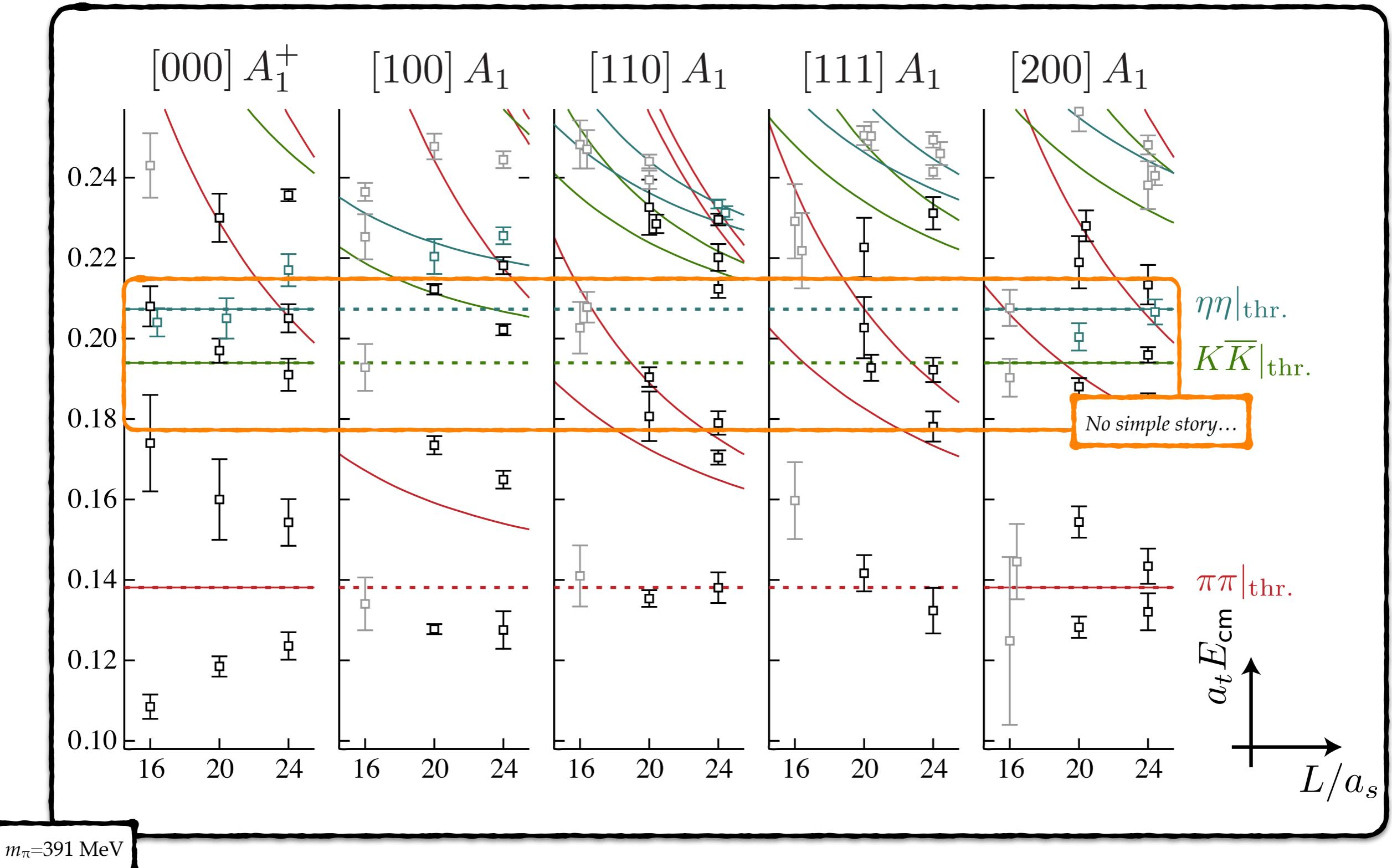
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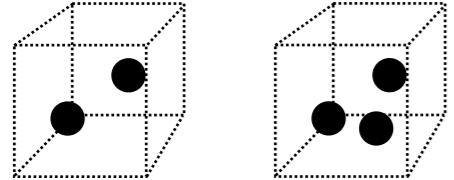
lattice spectroscopy

lattice QCD

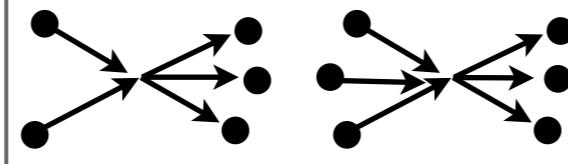
identification of

- states [masses & widths],
- production/decay mechanisms

finite-volume spectrum



PW amplitudes



analytic continuation

resonance poles



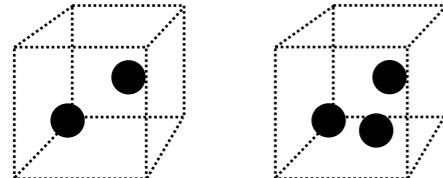
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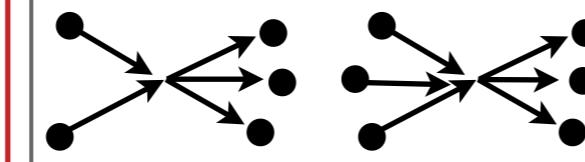
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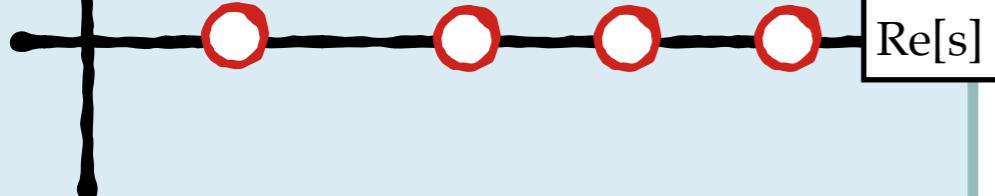
analytic continuation

resonance poles



$\text{Im}[s]$

no continuum of states:
- no cuts
- no sheet structure
- no resonances



bound state

$\text{Im}[s]$

Sheet II

$\text{Re}[s]$

$$s_R = (E_R - \frac{i}{2} \Gamma_R)^2$$

resonance

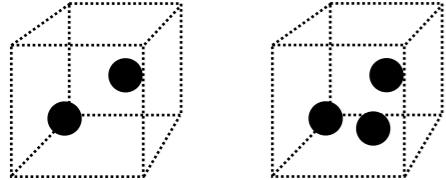
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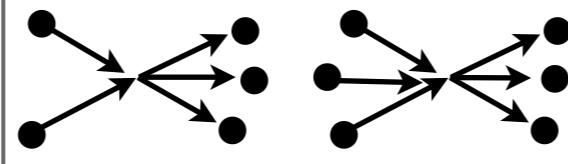
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two-particle
spectrum satisfies: $\det[F^{-1}(P, L) + \mathcal{M}(P)] = 0$

- Lüscher (1986, 1991)
- Rummukainen & Gottlieb (1995)
- Kim, Sachrajda, & Sharpe / Christ, Kim & Yamazaki (2005)
- Feng, Li, & Liu (2004); Hansen & Sharpe / RB & Davoudi (2012)
- RB (2014)

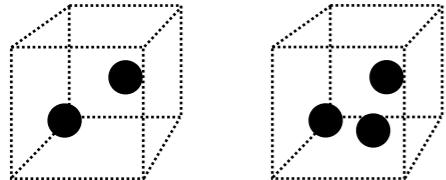
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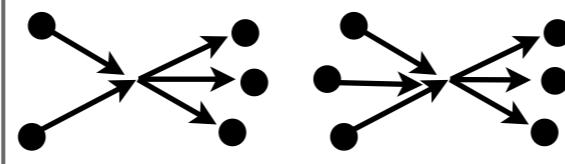
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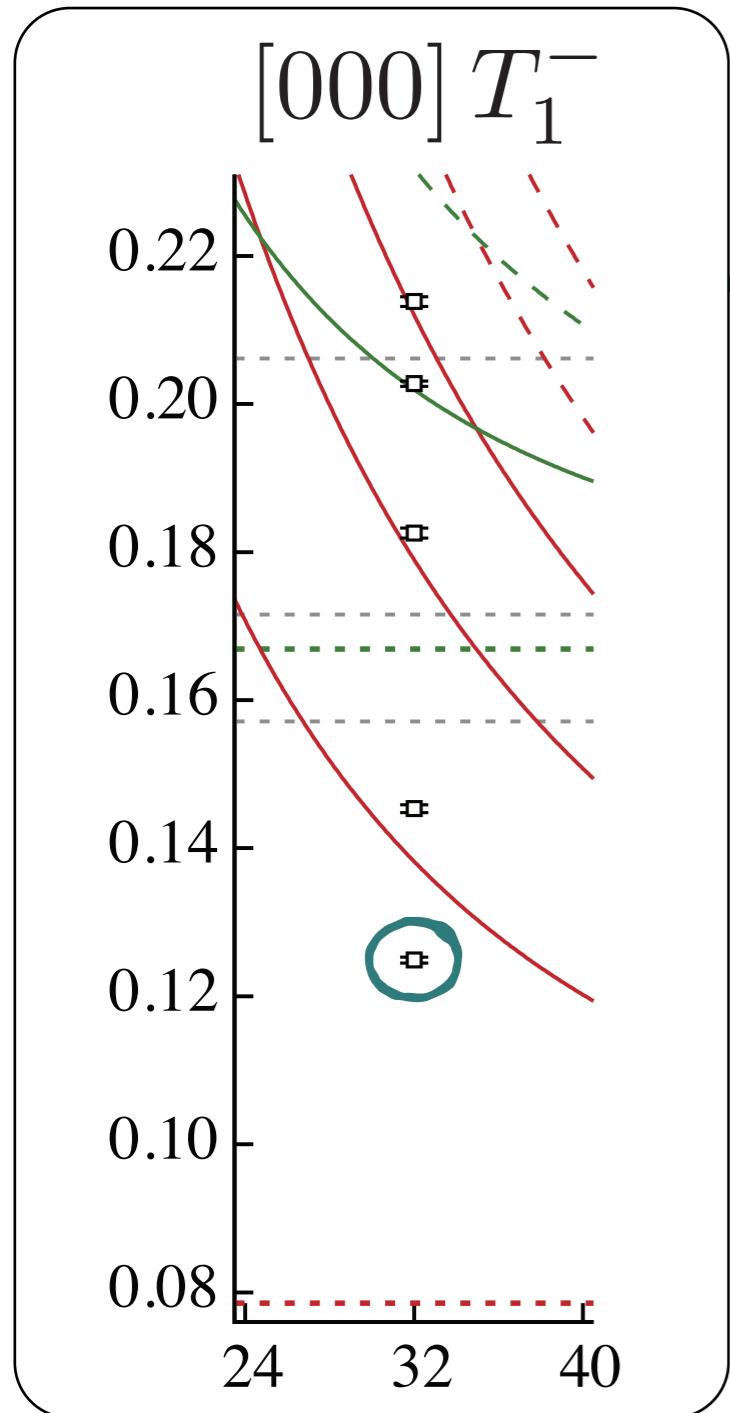
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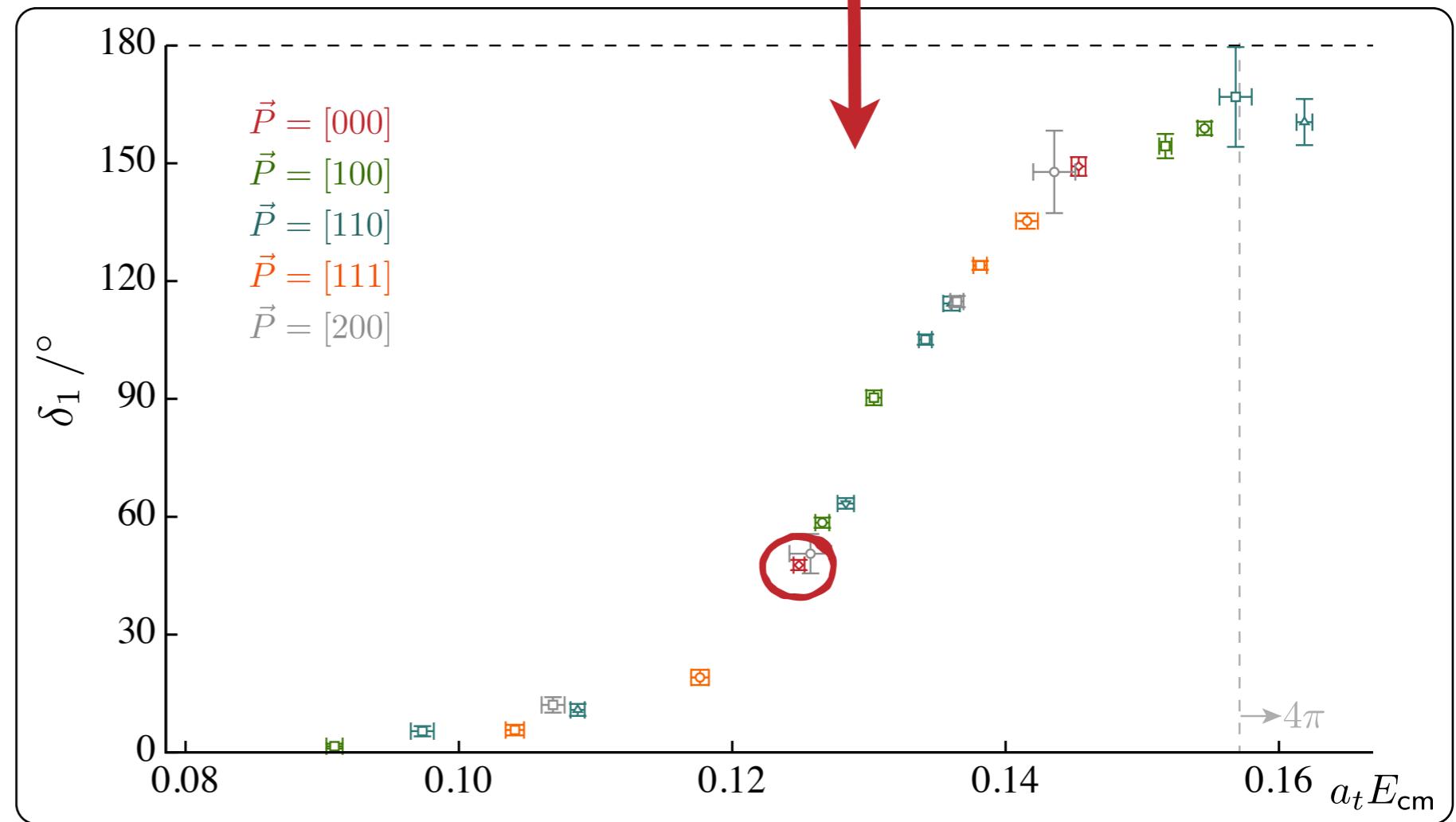
• RB (2014)

single channel:
“easy”

$\pi\pi$ Spectrum - ($l=1$ channel)



→
$$\det[F^{-1}(P, L) + \mathcal{M}(P)] = 0$$

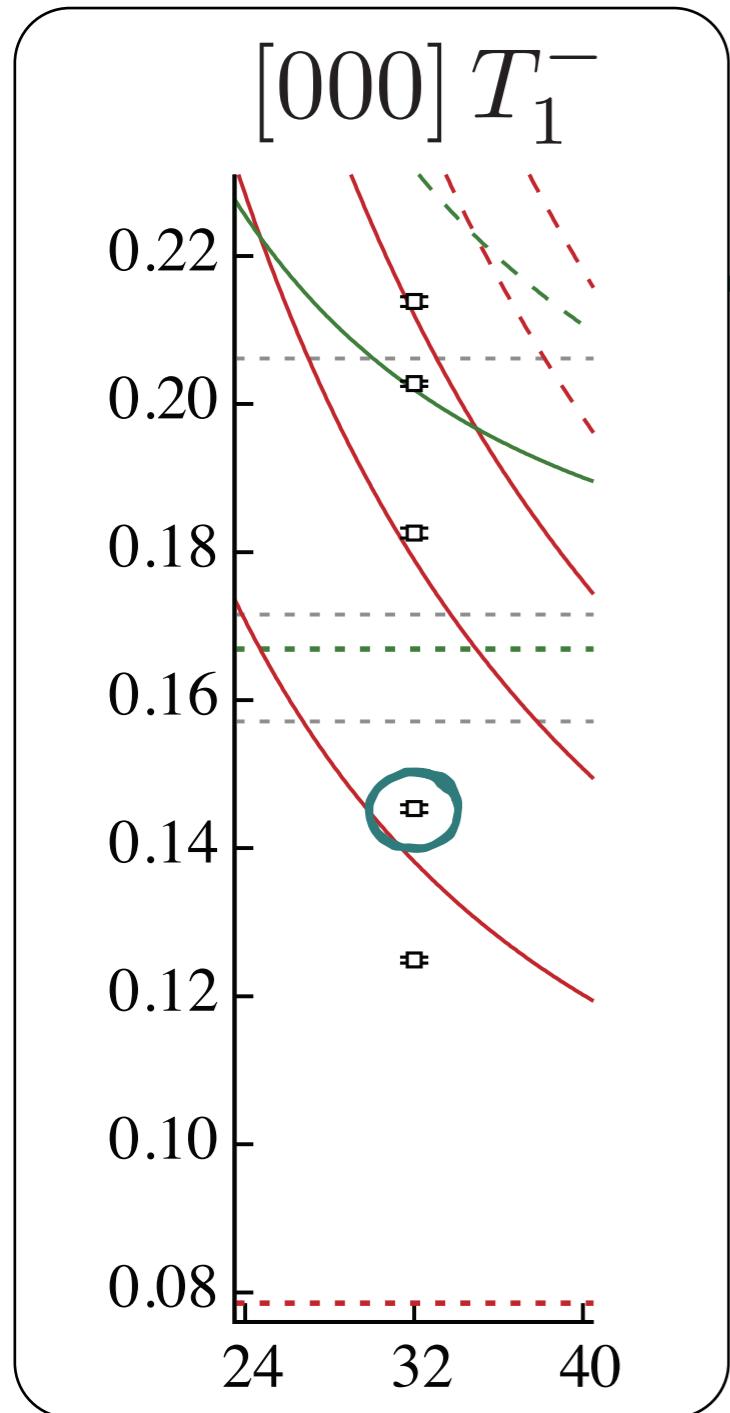


$$\mathcal{M} \propto \frac{1}{\cot \delta_1 - i}$$

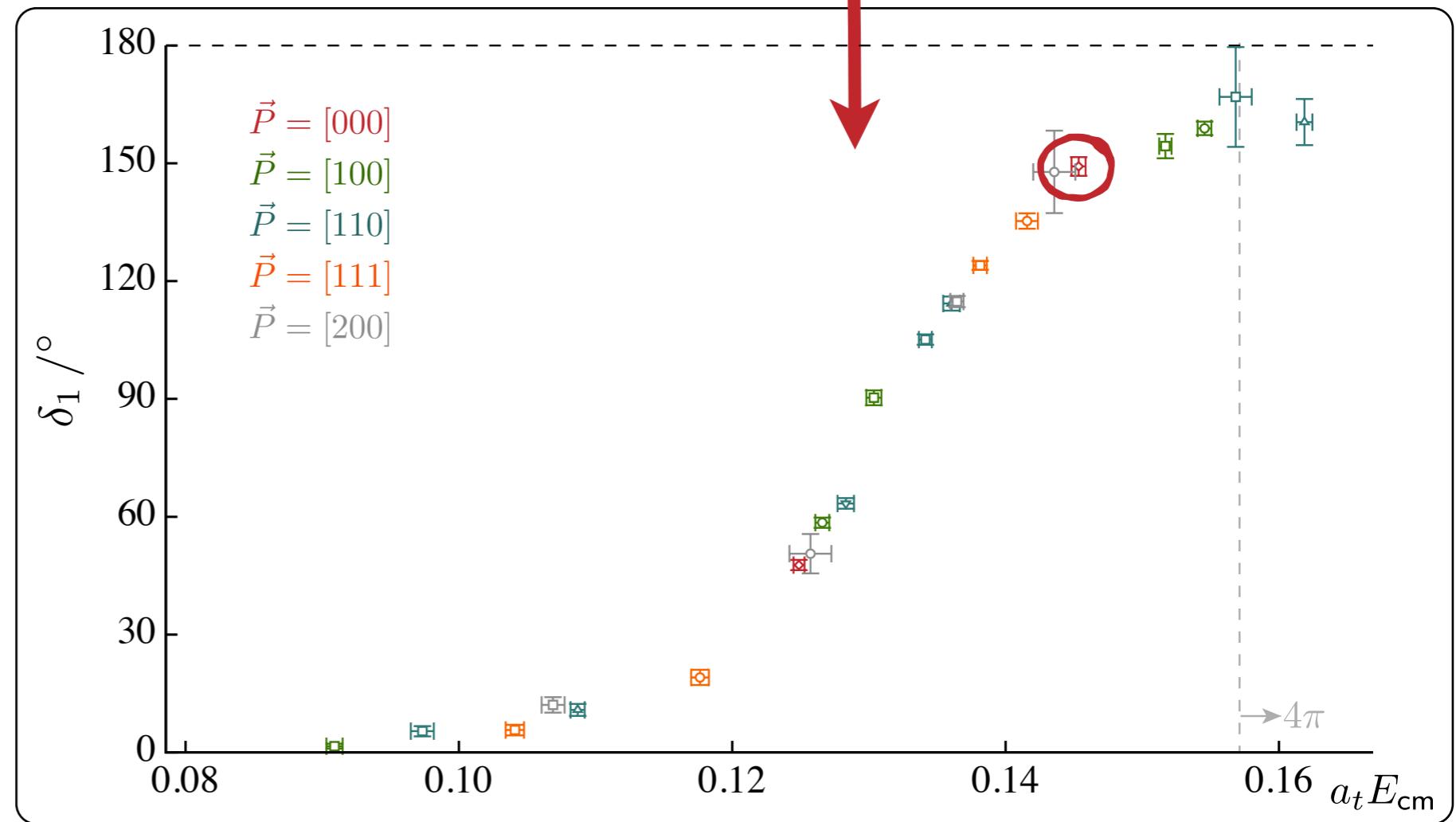
$m_\pi \sim 240$ MeV

Wilson, RB, Dudek, Edwards & Thomas (2015)

$\pi\pi$ Spectrum - ($l=1$ channel)



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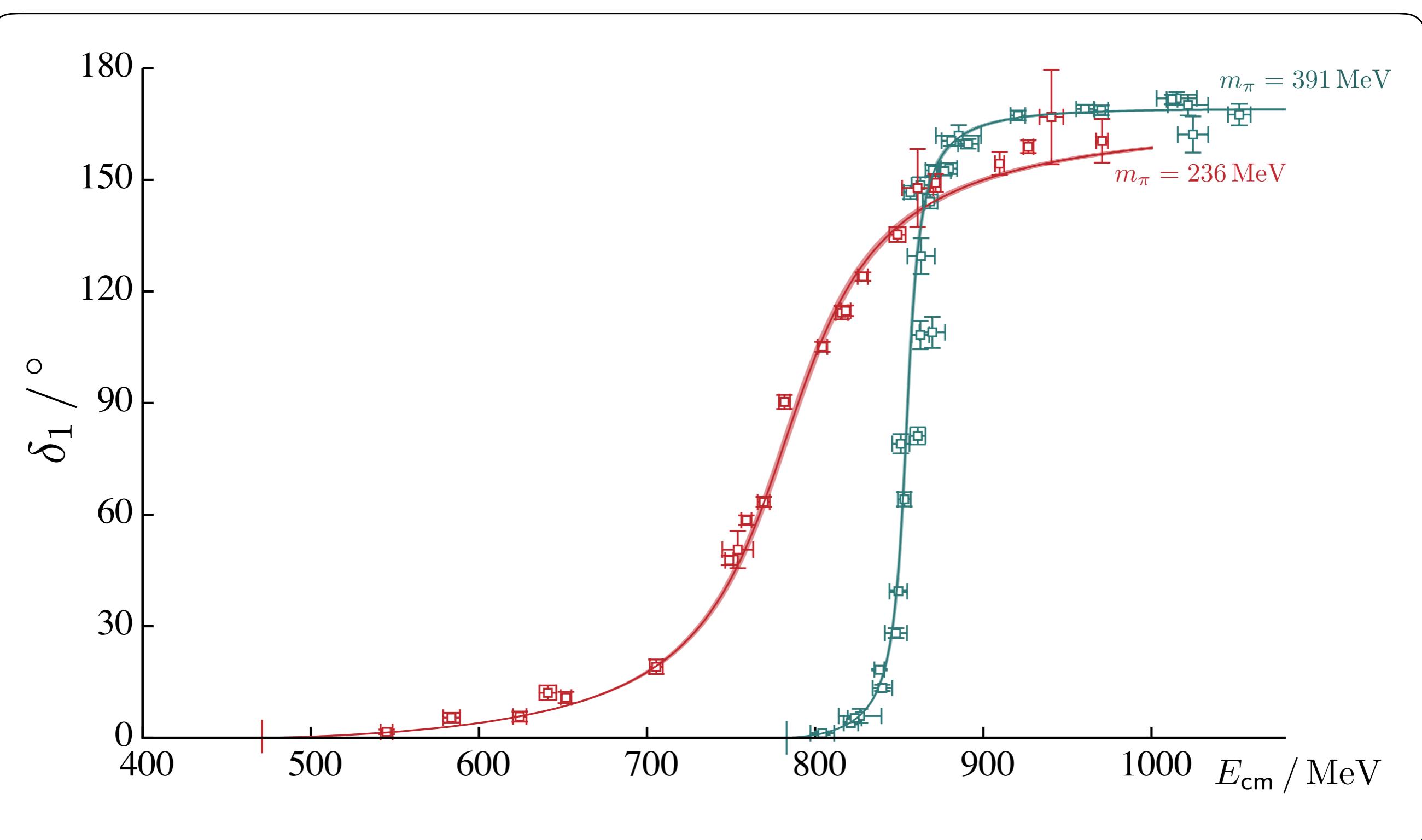


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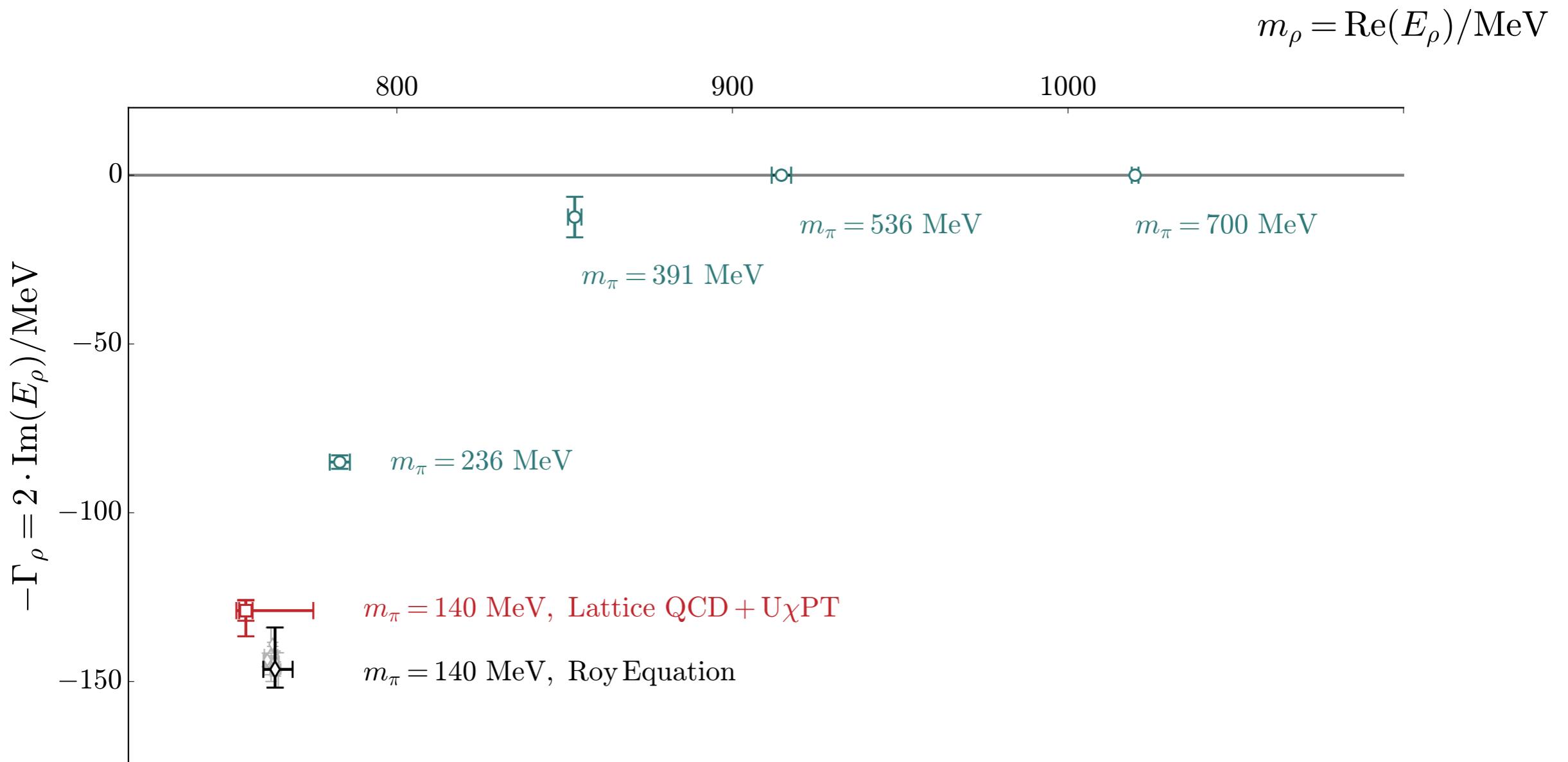
Wilson, RB, Dudek, Edwards & Thomas (2015)

$\pi\pi$ scattering - ($I=1$ channel)



Dudek, Edwards & Thomas (2012)
Wilson, RB, Dudek, Edwards & Thomas (2015)

The ρ vs m_π



Lin *et al.* (2009)

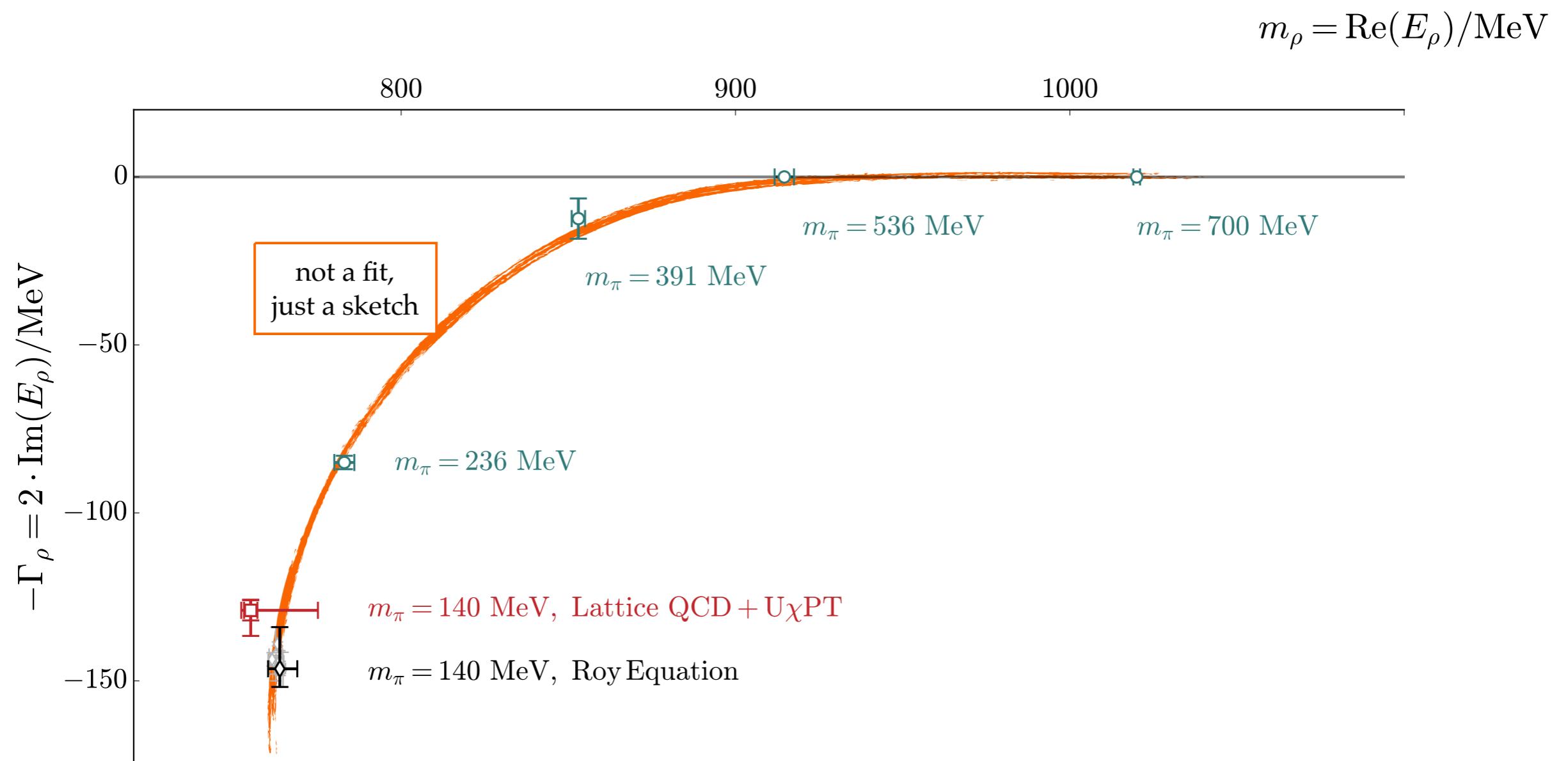
Dudek, Edwards, Guo & Thomas (2013)

Dudek, Edwards & Thomas (2012)

Wilson, RB, Dudek, Edwards & Thomas (2015)

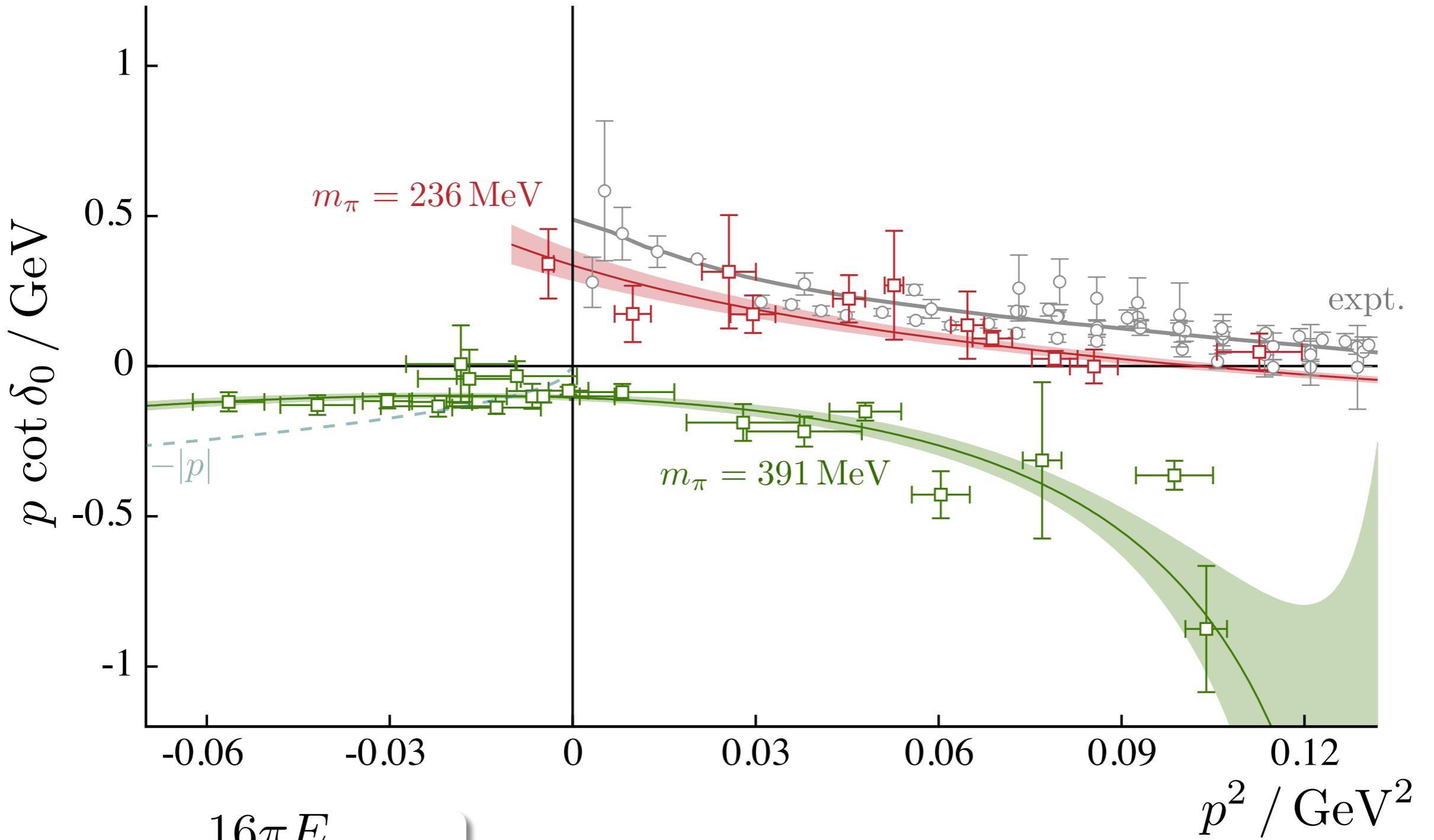
Bolton, RB & Wilson (2015)

The ρ vs m_π



- Lin *et al.* (2009)
Dudek, Edwards, Guo & Thomas (2013)
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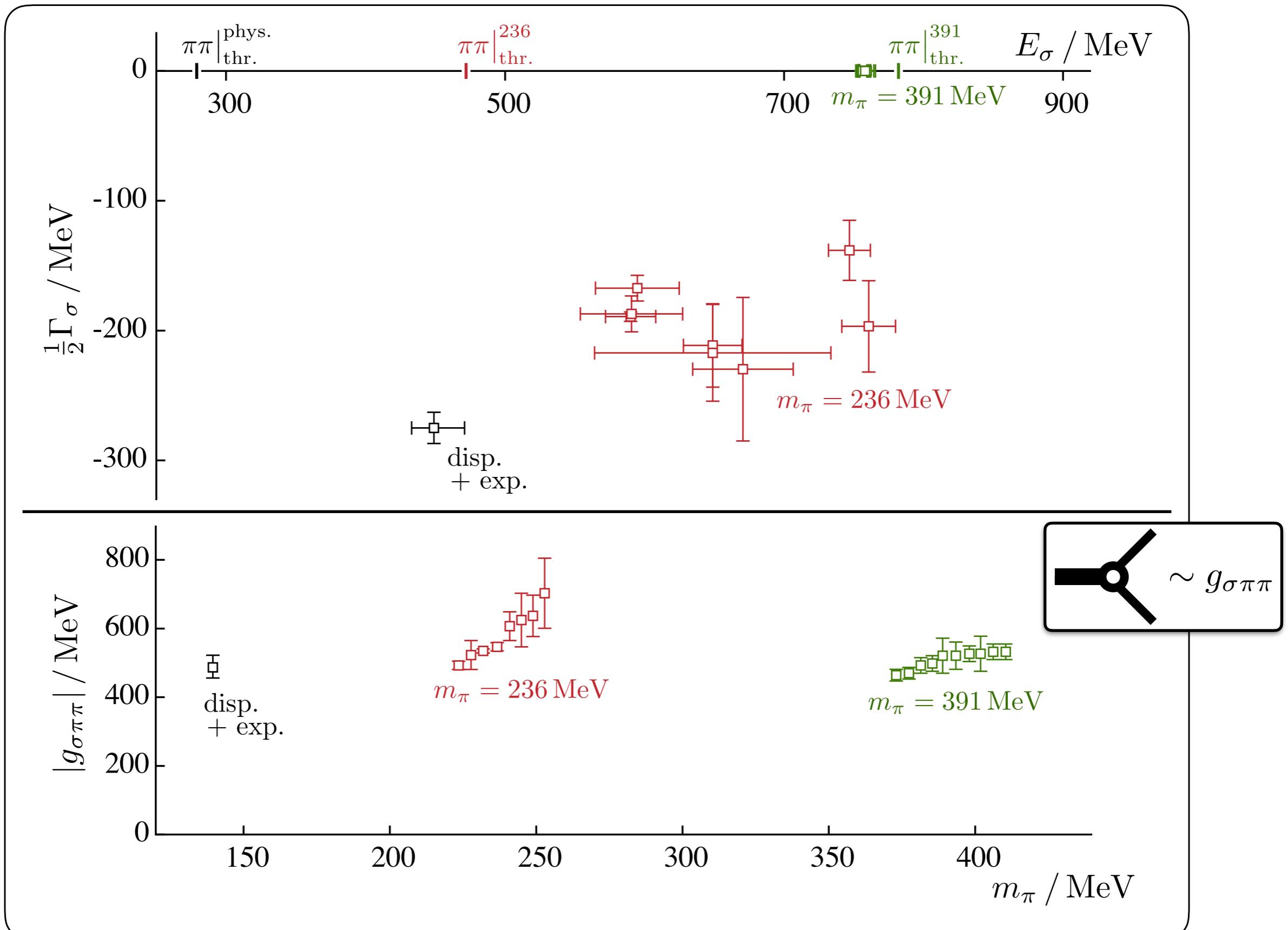
$\pi\pi$ scattering - ($I=0$ channel)



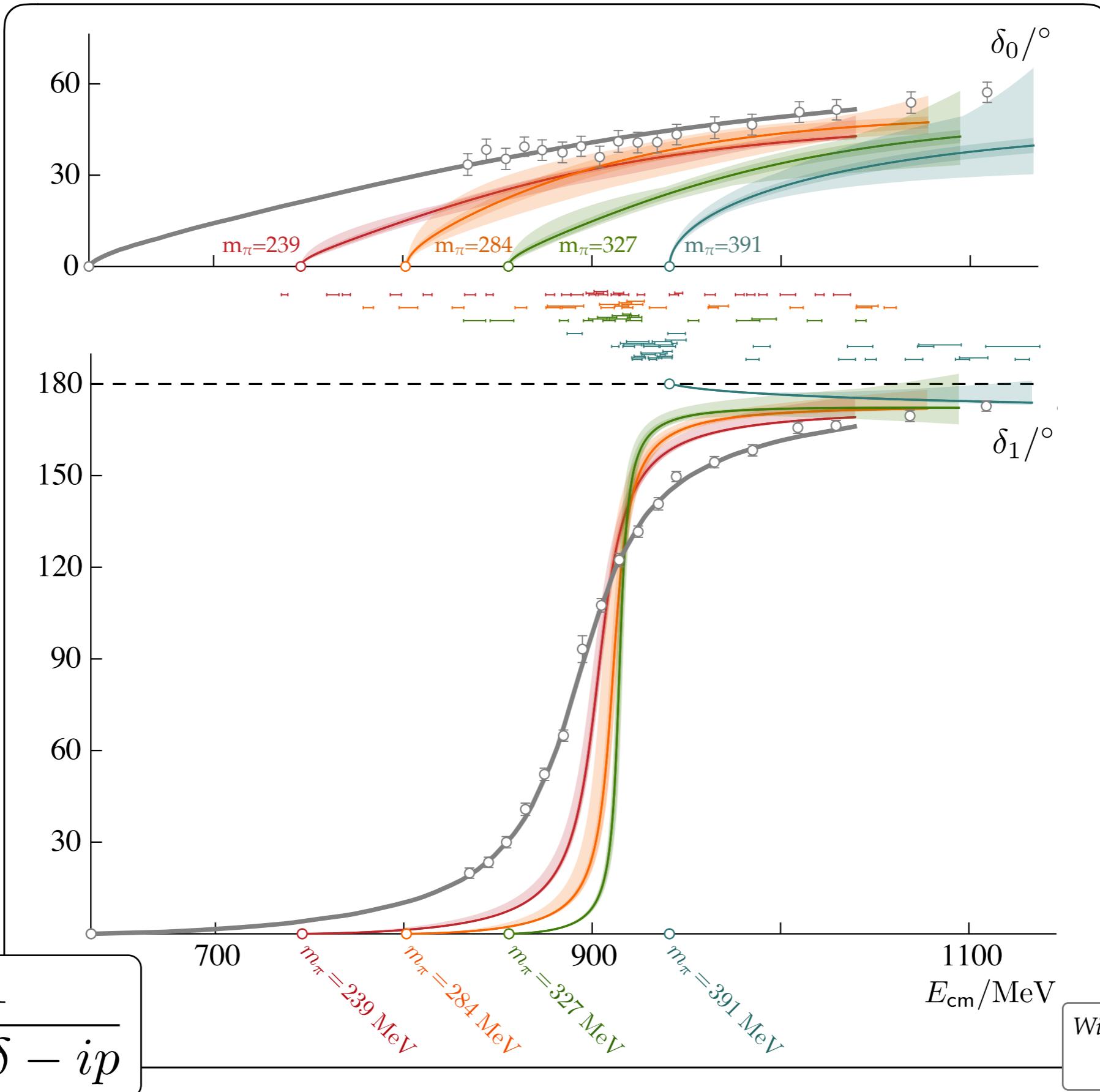
$$\mathcal{M}_0 = \frac{16\pi E_{\text{cm}}}{p \cot \delta_0 - ip}$$

RB, Dudek, Edwards, Wilson - PRL (2017)

The σ vs m_π

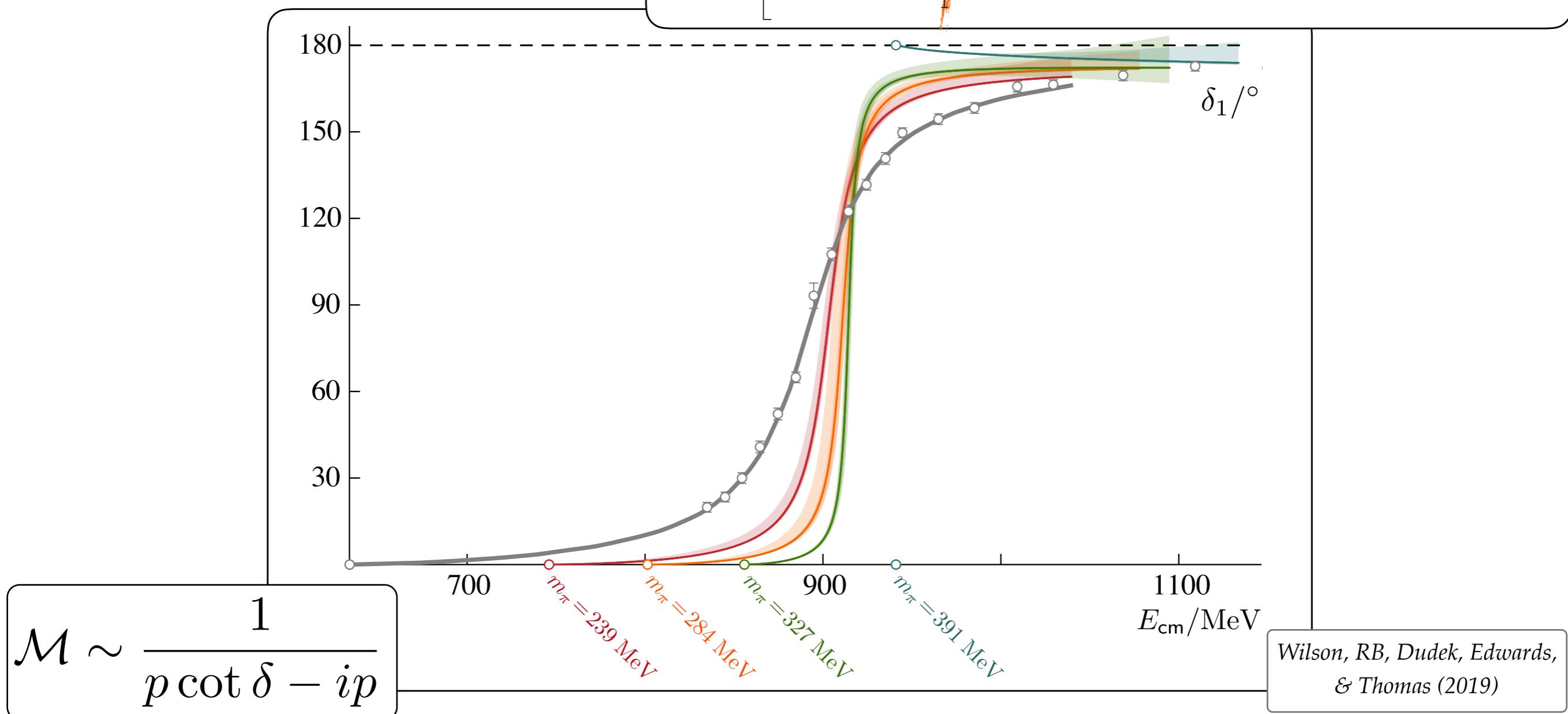


πK scattering - ($|l|=1/2$ channel)



Wilson, RB, Dudek, Edwards,
& Thomas (2019)

πK scattering - ($|l|=1/2$ channel)



multi-channel systems - the cutting edge!

- ✿ the *necessary* formalism for doing coupled-channel scattering of

Feng, Li, & Liu (2004) [inelastic scalar bosons]

Hansen & Sharpe / RB & Davoudi (2012) [moving inelastic scalar bosons]

RB (2014) [general 2-body result]

- ✿ to date, the *Hadron Spectrum collaboration* is the only one to have extracted coupled-channel scattering amplitude information from QCD

$\pi\pi, KK, \eta\eta$ [isoscalar]:

RB, Dudek, Edwards, Wilson - PRL (2017)

RB, Dudek, Edwards, Wilson - PRD (2018)

$K\pi, K\eta$:

Dudek, Edwards, Thomas, Wilson - PRL (2015)

Wilson, Dudek, Edwards, Thomas - PRD (2015)

$\pi\eta, KK$:

Dudek, Edwards, Wilson - PRD (2016)

$D\pi, D\eta, D_sK$:

Moir, Peardon, Ryan, Thomas, Wilson - JHEP (2016)

$\pi\pi, KK$ [isovector]:

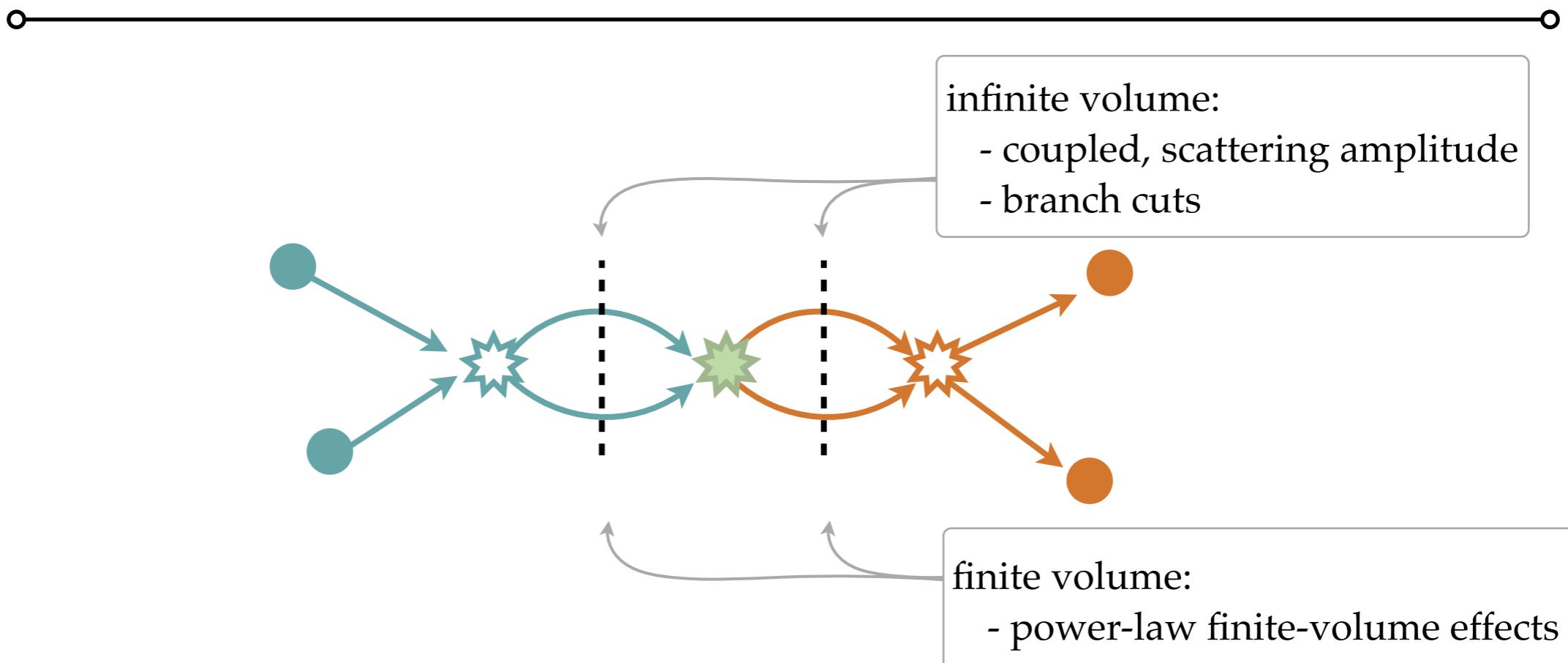
Wilson, RB, Dudek, Edwards, Thomas - PRD (2015)

multi-channel systems - the cutting edge!

📌 Above $2m_K$, there is not a one-to-one correspondence

$$\det \begin{bmatrix} F_{\pi\pi}^{-1} + \mathcal{M}_{\pi\pi,\pi\pi} & \mathcal{M}_{\pi\pi,K\bar{K}} \\ \mathcal{M}_{\pi\pi,K\bar{K}} & F_{K\bar{K}}^{-1} + \mathcal{M}_{K\bar{K},K\bar{K}} \end{bmatrix} = 0$$

Feng, Li, & Liu (2004),
Hansen & Sharpe / RB & Davoudi (2012)



multi-channel systems - the cutting edge!

- ✿ Above $2m_K$, there is not a one-to-one correspondence

$$\det \begin{bmatrix} F_{\pi\pi}^{-1} + \mathcal{M}_{\pi\pi,\pi\pi} & \mathcal{M}_{\pi\pi,K\bar{K}} \\ \mathcal{M}_{\pi\pi,K\bar{K}} & F_{K\bar{K}}^{-1} + \mathcal{M}_{K\bar{K},K\bar{K}} \end{bmatrix} = 0$$

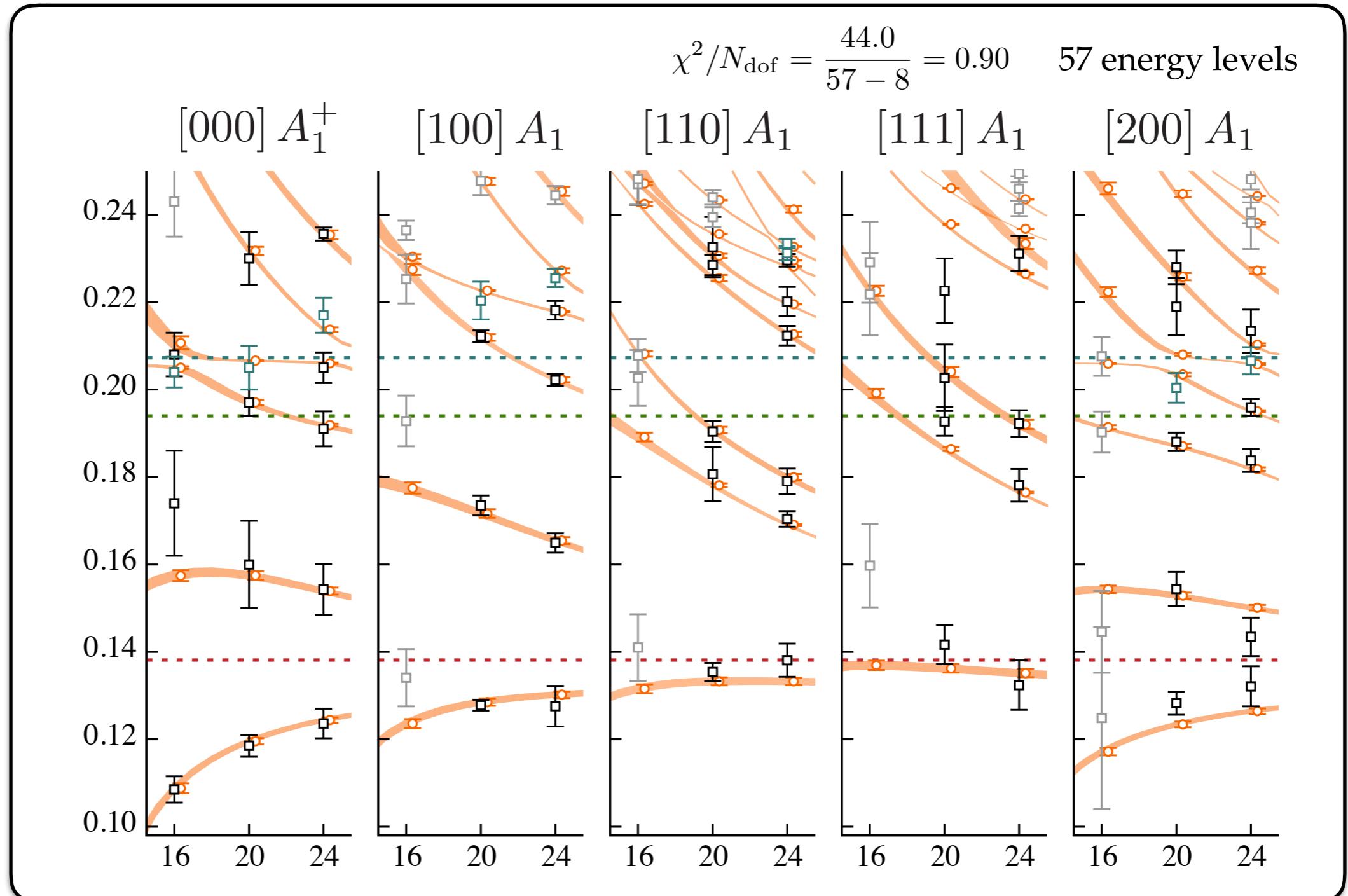
Feng, Li, & Liu (2004),
Hansen & Sharpe / RB & Davoudi (2012)

- ✿ In general, must constrain $(1/2) [N^2 + N]$ functions of energy
- ✿ Need that many energy levels at the same energy
- ✿ Alternatively, parametrize scattering amplitude and do a global fit

coupled-channels analysis

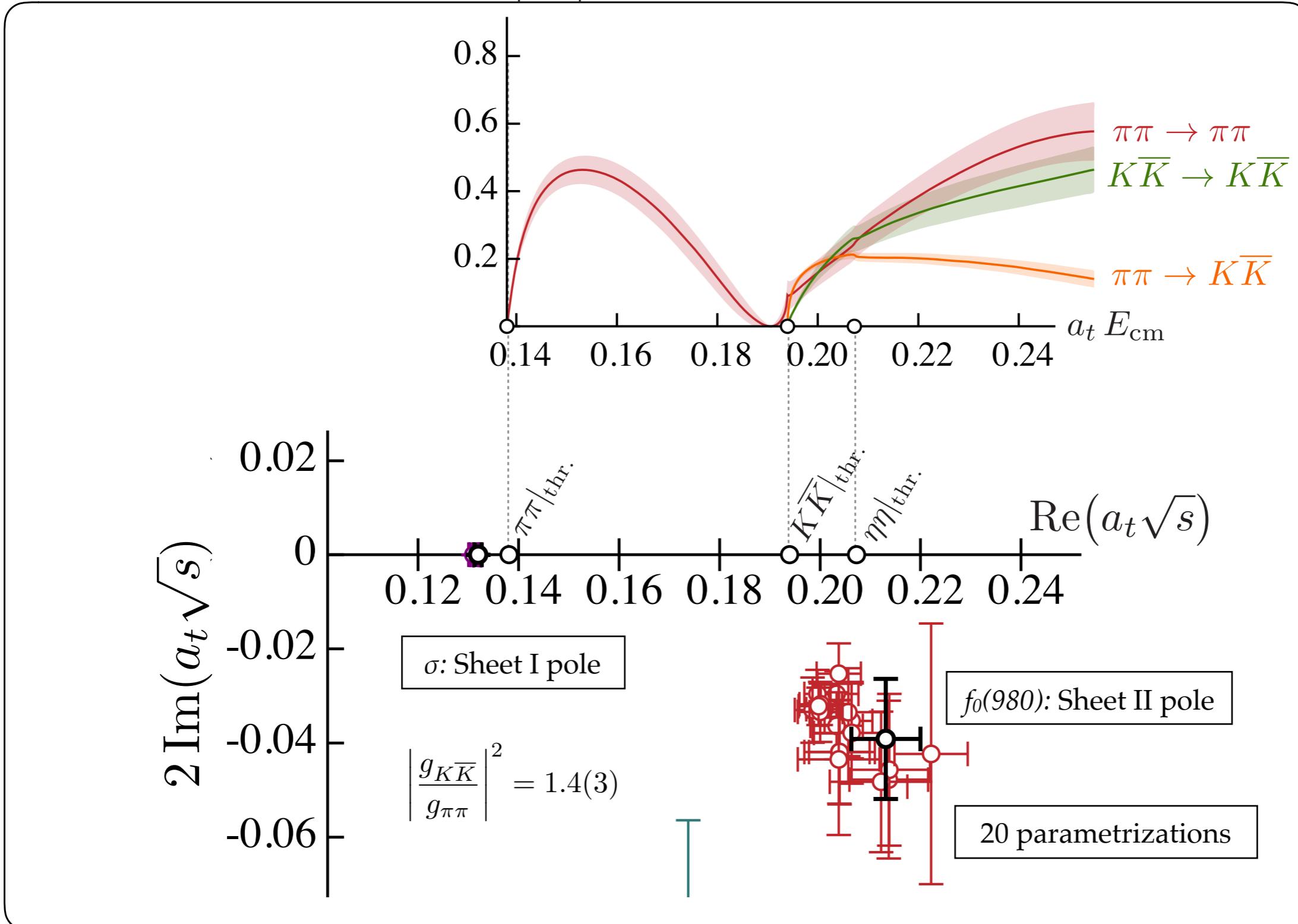
⌚ S-wave above $2m_\pi, 2m_K$, and $2m_\eta$

⌚ Ansatz $\mathbf{K}^{-1}(s) = \begin{pmatrix} a + bs & c + ds & e \\ c + ds & f & g \\ e & g & h \end{pmatrix}$



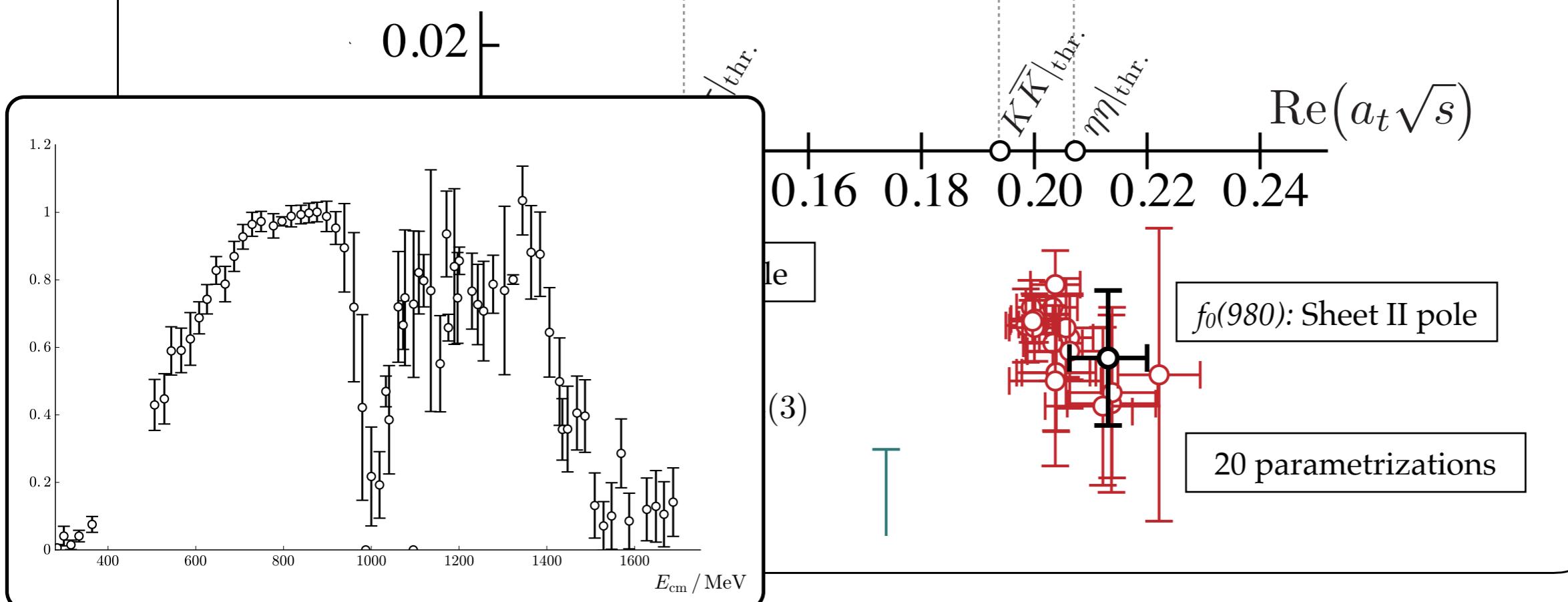
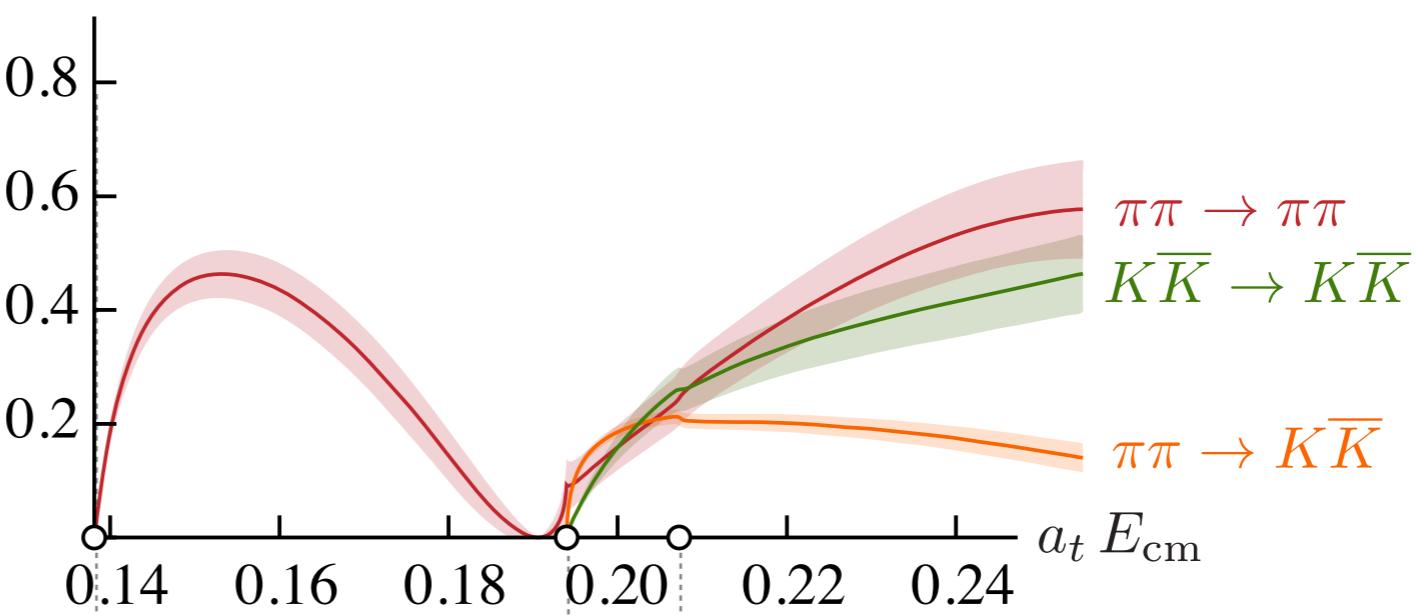
scalar $\pi\pi$ - KK

📌 Near poles: $\mathcal{M} \sim \frac{g^2}{s_0 - s}$

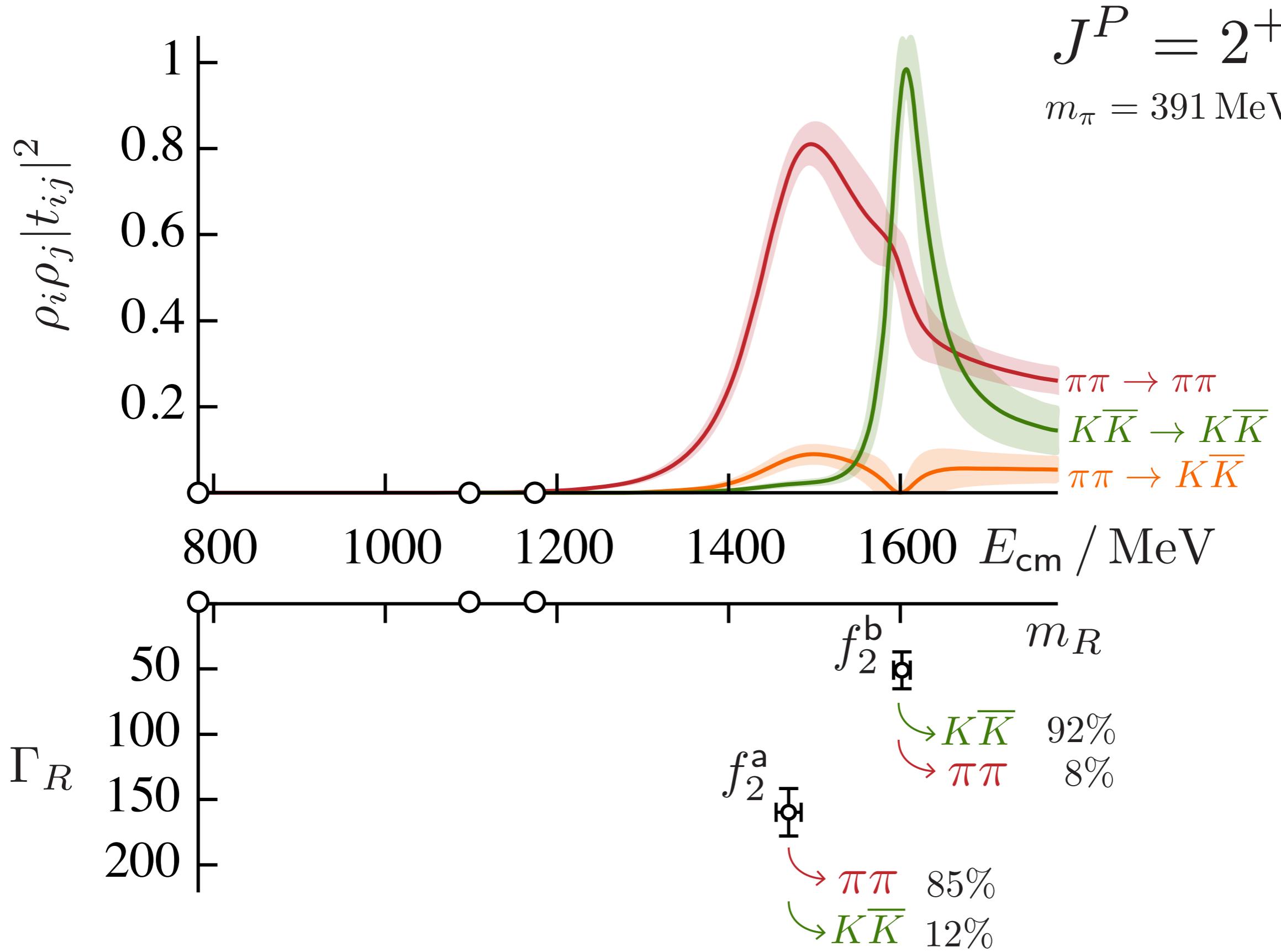


scalar $\pi\pi$ - KK

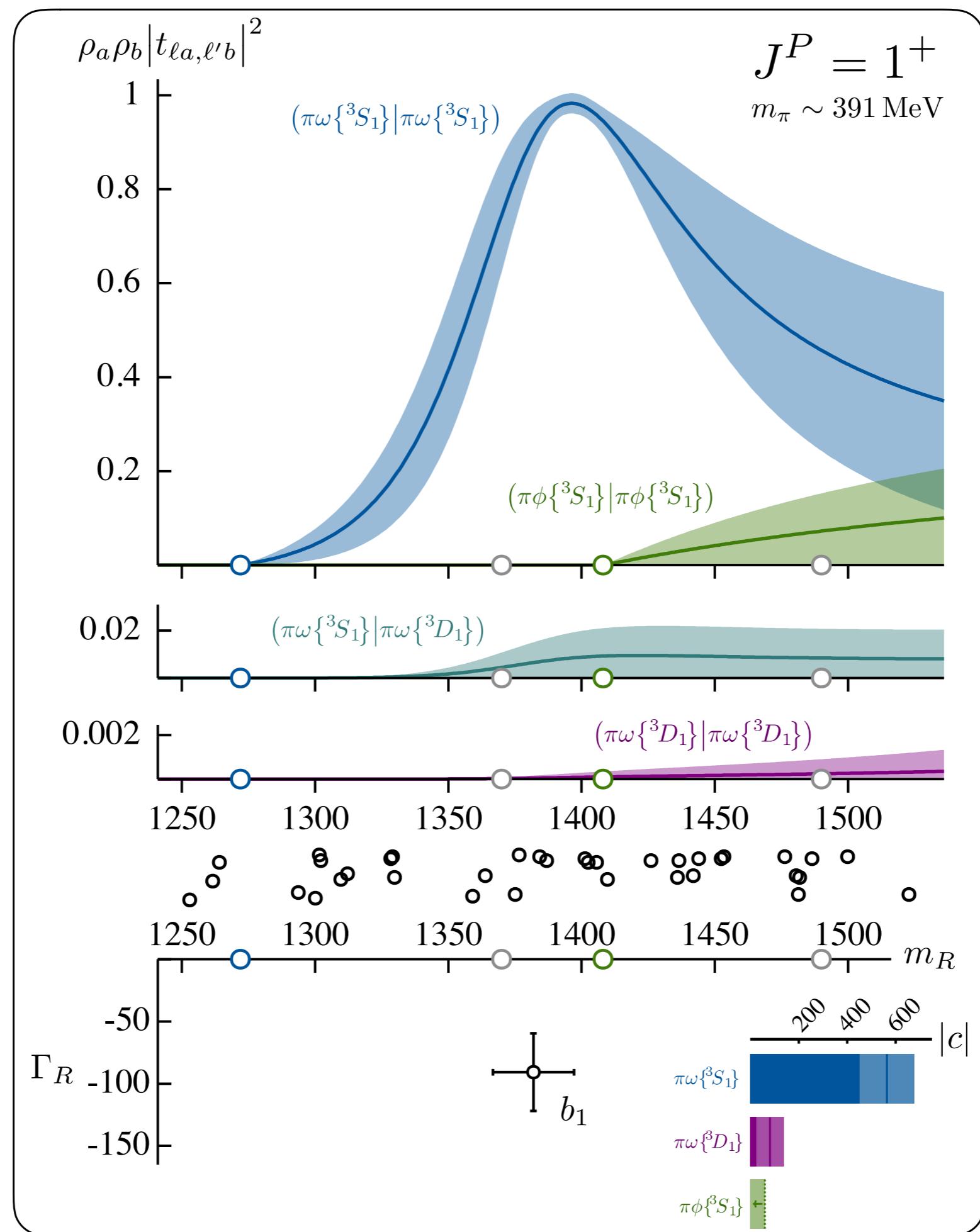
📌 Near poles: $\mathcal{M} \sim \frac{g^2}{s_0 - s}$



tensor $\pi\pi$ - KK



$\pi\omega$ - $\pi\phi$
and the b_1



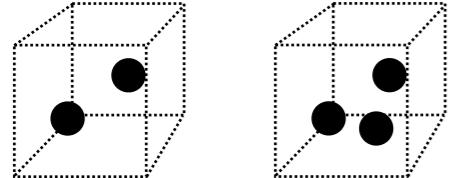
lattice spectroscopy

lattice QCD

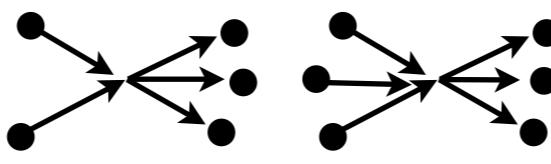
identification of

- states [masses & widths],
- production/decay mechanisms

finite-volume spectrum



PW amplitudes

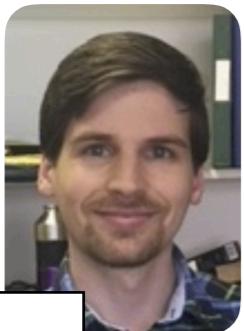


analytic continuation

resonance poles



inside the box



Hansen

Blanton

Romero-López

outside the box



Szczepaniak

three-particle spectrum satisfies:

$$\det[F_3^{-1} + \mathcal{K}_{\text{df},3}] = 0$$

- Hansen & Sharpe (2014, 2015)
- RB, Hansen & Sharpe (2017, 2018)
- Romero-López, Sharpe, Blanton, RB, & Hansen (2019)
- RB, Hansen, Sharpe, & Szczepaniak (2019)

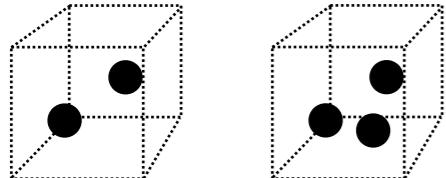
lattice spectroscopy

lattice QCD

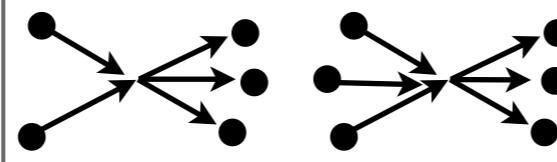
identification of

- states [masses & widths],
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finite-volume spectrum



PW amplitudes



analytic continuation

resonance poles



Blanton

Romero-López

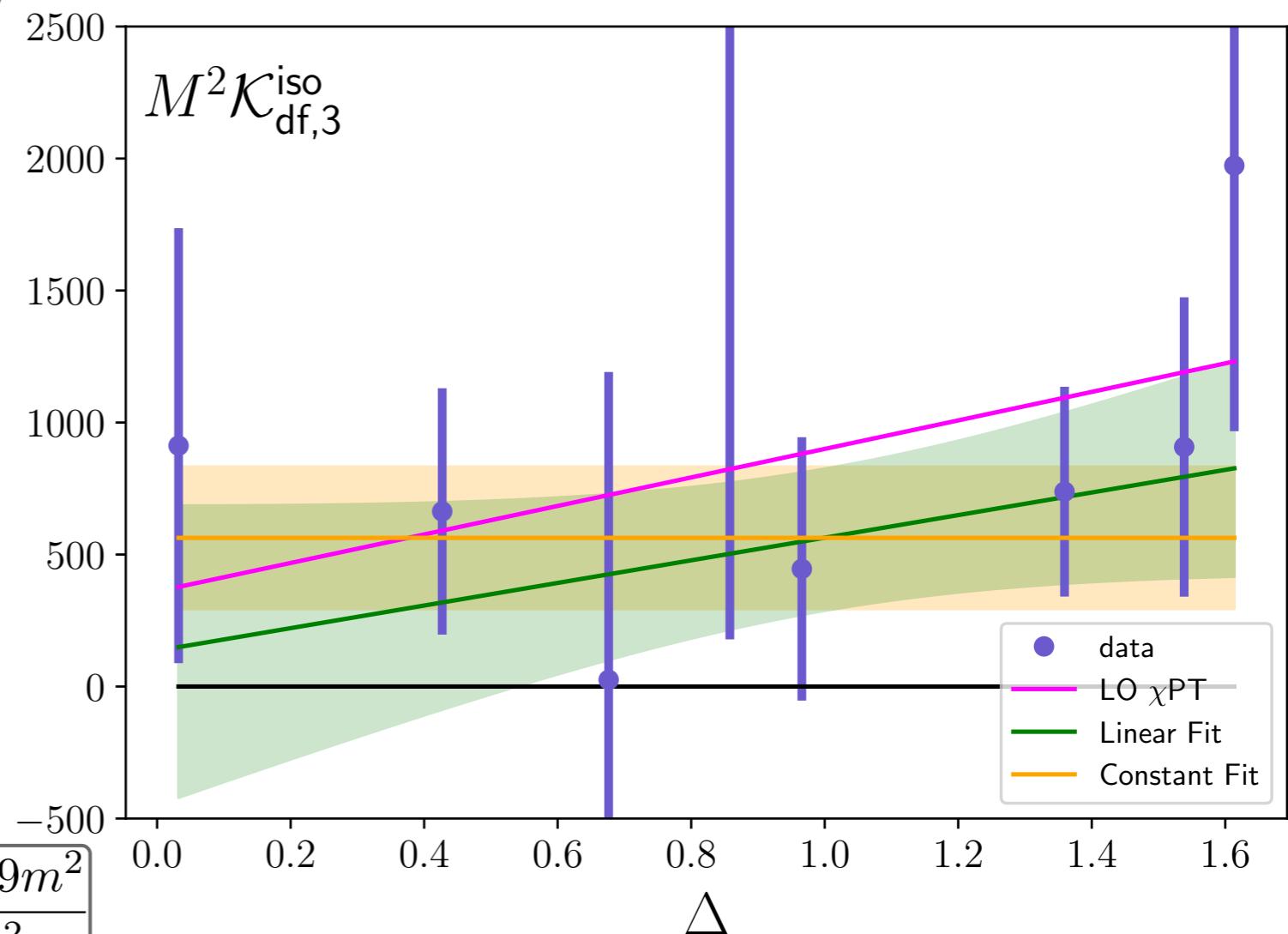
three-particle spectrum satisfies:

$$\det[F_3^{-1} + \mathcal{K}_{\text{df},3}] = 0$$

Hörz and Hanlon (2019)

Romero-López, Blaton, & Sharpe (2019)

$$\Delta = \frac{E^2 - 9m^2}{9m^2}$$



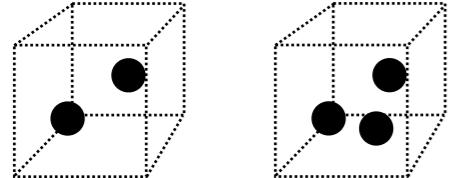
lattice spectroscopy

lattice QCD

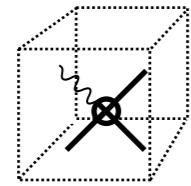
identification of

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- production/decay mechanisms

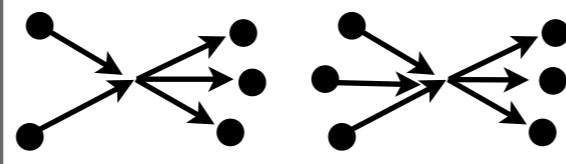
finite-volume spectrum



0/1-to-2
FV matrix
elements

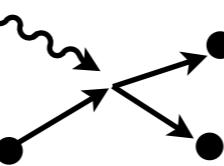


PW amplitudes



analytic
continuation

electroweak
amplitudes



resonance poles



transition
form factors



Lellouch & Lüscher (2000)

Kim, Sachrajda, & Sharpe

Christ, Kim & Yamazaki (2005)

...

Hansen & Sharpe (2012)

RB, Hansen Walker-Loud (2014)

RB & Hansen (2015)

$$|\langle 2 | \mathcal{J} | 1 \rangle_L| = \sqrt{\mathcal{A} \mathcal{R} \mathcal{A}}$$

Lellouch-Lüscher matrix:

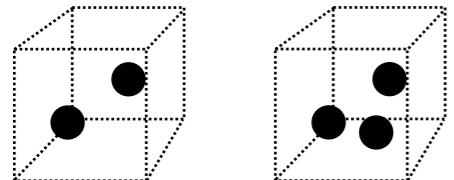
$$\mathcal{R}(E_n, \mathbf{P}) \equiv \lim_{E \rightarrow E_n} \left[\frac{(E - E_n)}{F^{-1}(P, L) + \mathcal{M}(P)} \right]$$

lattice spectroscopy

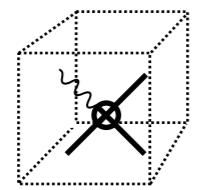
lattice QCD

identification of
 • states [masses & widths],
 • production/decay mechanisms

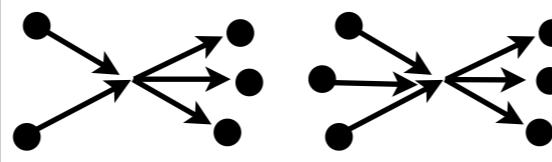
finite-volume spectrum



0/1-to-2
FV matrix
elements



PW amplitudes

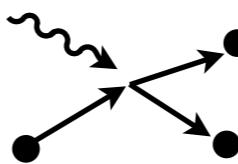


analytic
continuation

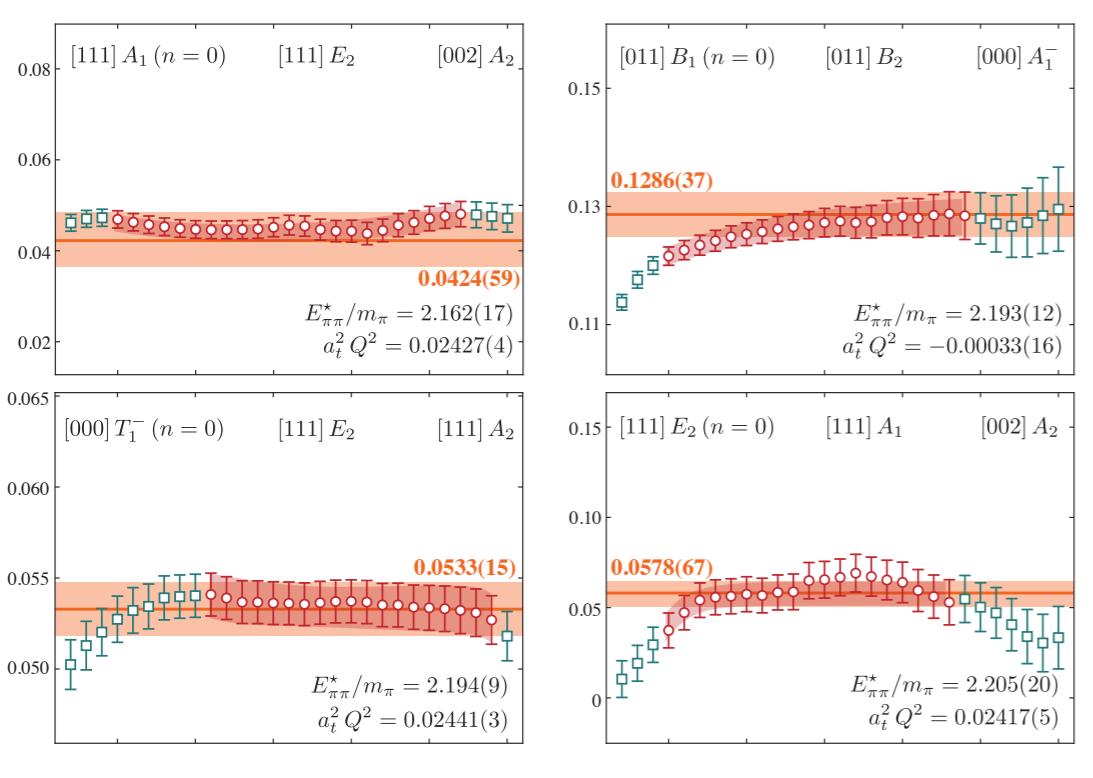
resonance poles



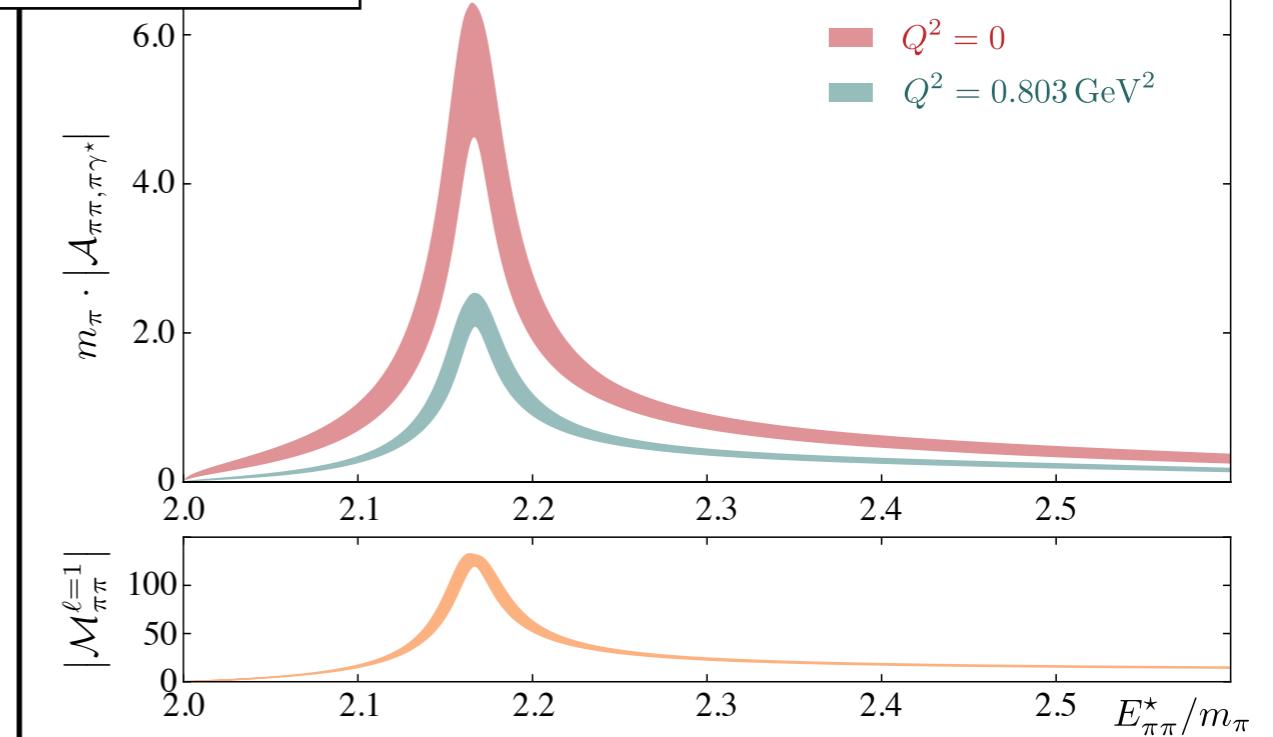
electroweak
amplitudes



transition
form factors



$\pi\gamma^*$ -to- $\pi\pi$



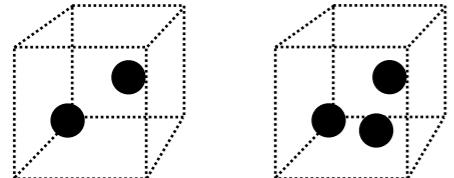
lattice spectroscopy

lattice QCD

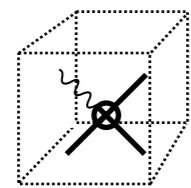
identification of

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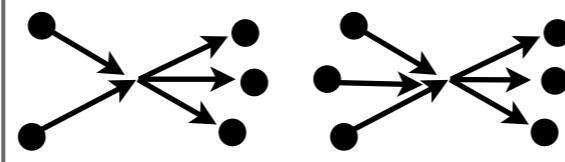
finite-volume spectrum



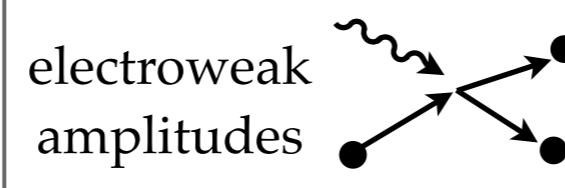
0/1-to-2
FV matrix
elements



PW amplitudes



analytic
continuation



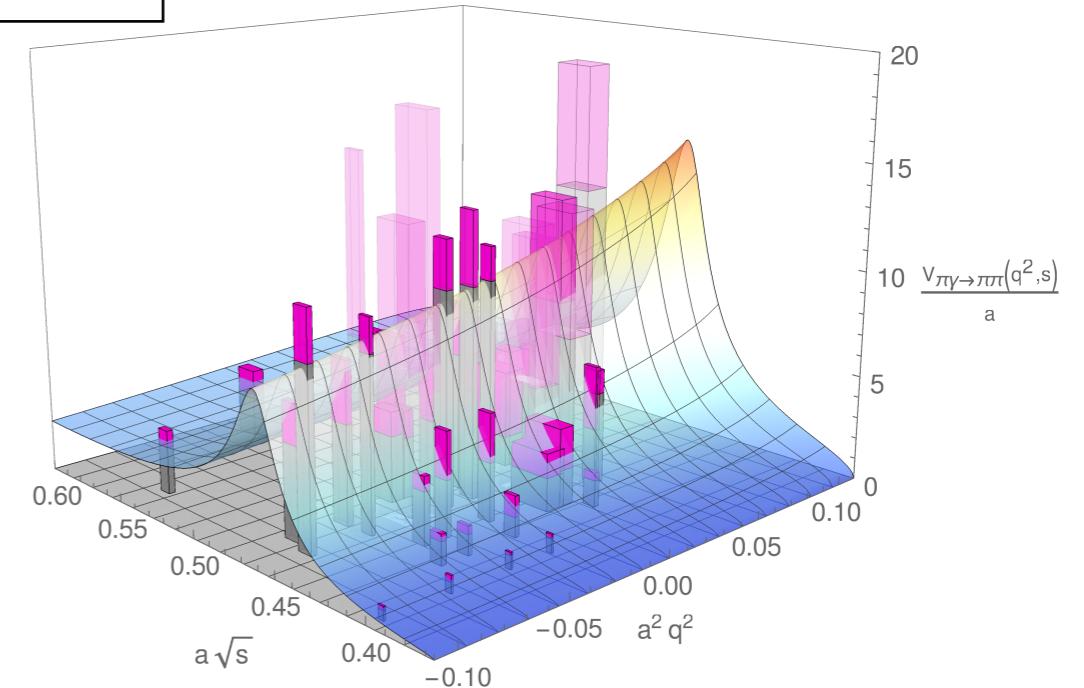
resonance poles



transition
form factors



$\pi\gamma^*\text{-to-}\pi\pi$

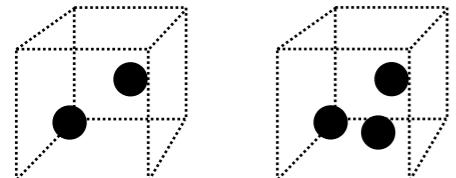


lattice spectroscopy

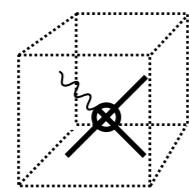
lattice QCD

identification of
• states [masses & widths],
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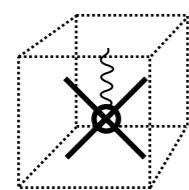
finite-volume spectrum



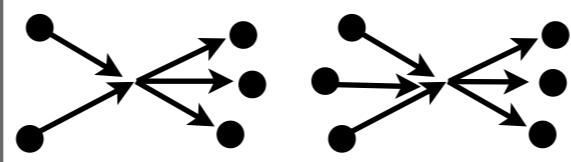
0/1-to-2
FV matrix
elements



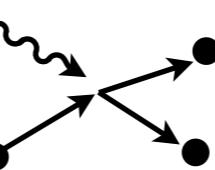
2-to-2
FV matrix
elements



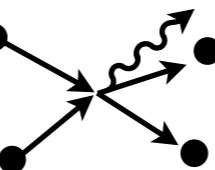
PW amplitudes



electroweak
amplitudes



electroweak
amplitudes

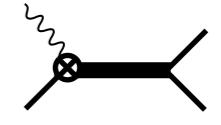


analytic
continuation

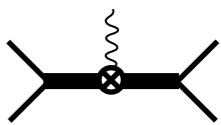
resonance poles



transition
form factors



elastic form
factors



structure and nature of states

Baroni

Hansen

Ortega



• RB & Hansen (2015)

• Baroni, RB, Hansen, Ortega (2018)

lattice spectroscopy

lattice QCD

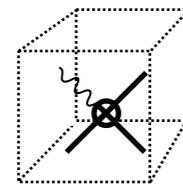
identification of

- states [masses & widths],
- production/decay mechanisms

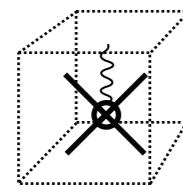
finite-volume spectrum



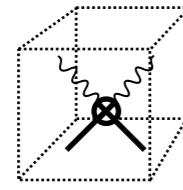
0/1-to-2
FV matrix
elements



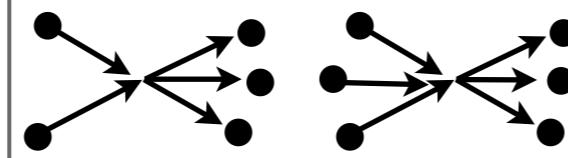
2-to-2
FV matrix
elements



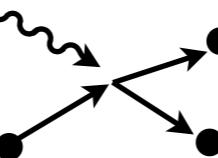
0/1-to-0/1
with two
currents



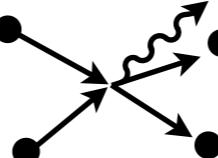
PW amplitudes



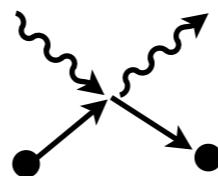
electroweak
amplitudes



electroweak
amplitudes



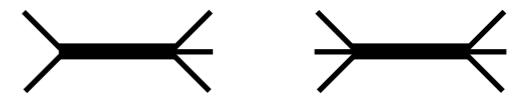
electroweak
amplitudes



analytic
continuation



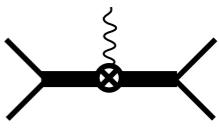
resonance poles



transition
form factors



elastic form
factors



Baroni



Hansen



Schindler



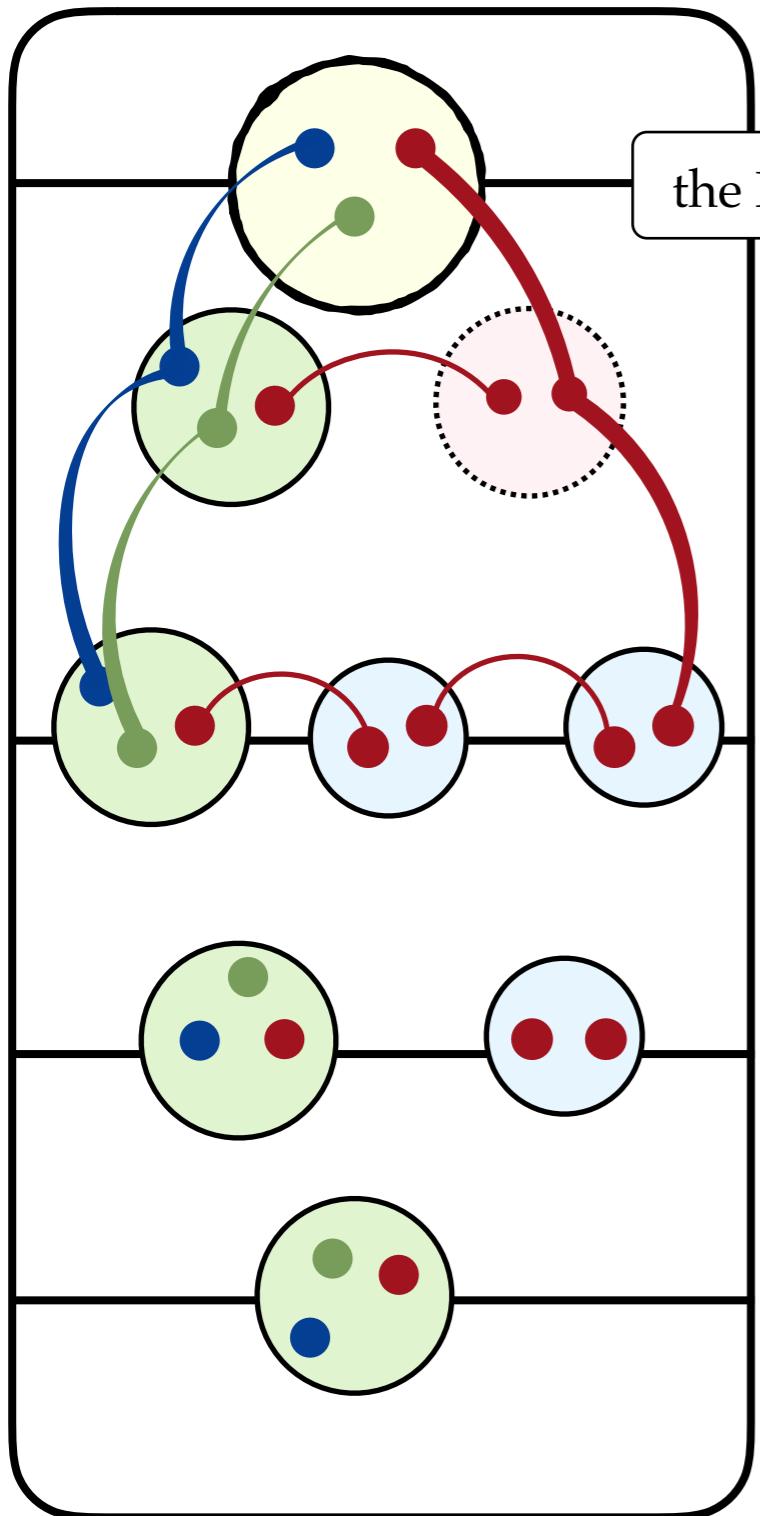
Christ, Feng, Martinelli, & Sachrajda (2017)

Baroni, RB, Davoudi, Hansen, Schindler (to appear)

Davoudi



the Roper an outstanding 50yr puzzle



What are the necessary features needed
to be able to claim accuracy?

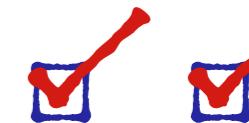
exploratory study

formalism

broad resonance



non-trivial spin structure



strongly coupled



three-particle system



structure of resonant states