Generalized parton distributions and form factors of the proton

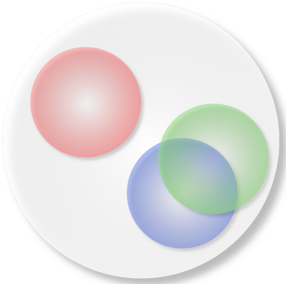
Adam Freese

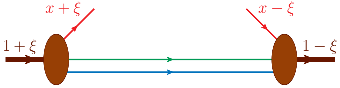
Argonne National Laboratory

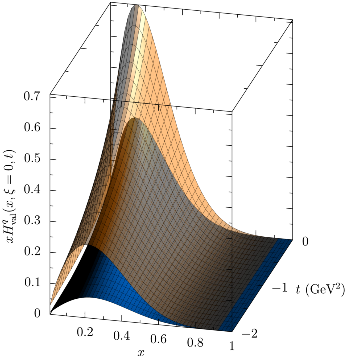
August 5, 2019

Generalized parton distributions and form factors

- **Generalized form factors** (GPDs) are three-dimensional descriptions of partonic structure.
- GPDs contain the elastic form factors.
- Two-dimensional slices of GPDs are seen in **deeply virtual Compton scattering** (DVCS).
- GPDs & DVCS offer additional insight to form factors.
- I will present model calculations/predictions for proton GPDs.



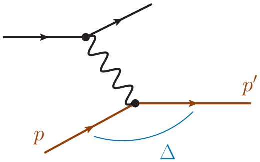




Form factors and proton structure

Point particle

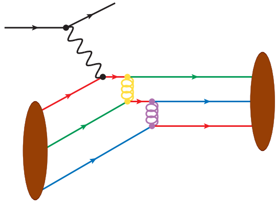
$$\langle p's'|j^\mu(0)|ps\rangle = \bar{u}^{s'}(p')\gamma^\mu u^s(p)$$



$$\Delta = p' - p \qquad t = \Delta^2 = -Q^2$$

General spin-half particle

$$\langle p's'|j^\mu(0)|ps\rangle = \bar{u}^{s'}(p')\left[\gamma^\mu F_1(t) + \frac{i\sigma^{\mu\Delta}}{2m_p}F_2(t)\right]u^s(p)$$



- Form factors appear in the electromagnetic current.
- They tell us about substructure—how is the proton different from a point particle?
- Seen in reactions where proton **gets a momentum kick**.
- How kick is redistributed gives hint at structure—seen in t dependence.
  - Stay together easily with large kick? *Slow t falloff*
  - Trouble staying together? *Fast t falloff*

Electric and magnetic form factors

Sachs form factors

$$G_E(t) = F_1(t) + \frac{t}{4m_p^2} F_2(t)$$
$$G_M(t) = F_1(t) + F_2(t)$$

Sum rules

$$G_E(0) = F_1(0) = 1$$
$$G_M(0) = F_1(0) + F_2(0)$$
$$\mu_p = 1 + \kappa_p$$

- 3D Fourier transforms of $G_E(t)$ and $G_M(t)$ characterize electric and magnetic response of charged objects (electrons) to proton; *e.g.*,

$$\rho_E(\mathbf{r}) = \int \frac{d^3\mathbf{k}}{2E_k(2\pi)^3} G_E(t = -\mathbf{k}^2) e^{-i(\mathbf{k}\cdot\mathbf{r})}$$

- These are **not literal spatial densities** (see *e.g.*, Miller, PRC99 (2019) 035202)
- **Charge and magnetization radii**:

$$\langle r_E^2 \rangle = 6 \frac{\mathrm{d}G_E(t)}{dt}\bigg|_{t=0} \qquad \langle r_M^2 \rangle = 6 \frac{\mathrm{d}G_M(t)}{dt}\bigg|_{t=0}$$

…electrons respond *as if* proton had these radii.

Flavor-separated structure

- Proton & neutron have valence uud and udd structure.
- Assume **charge symmetry**: neutron is a proton with $u \leftrightarrow d$ swap.

$$F_{ip}(t) = \frac{2}{3}F_{ip}^{u}(t) - \frac{1}{3}F_{ip}^{d}(t)$$
$$F_{in}(t) = -\frac{1}{3}F_{ip}^{u}(t) + \frac{2}{3}F_{ip}^{d}(t)$$

(where $i = 1, 2$) and thus

$$F_{ip}^{u}(t) = 2F_{ip}(t) + F_{in}(t)$$
$$F_{ip}^{d}(t) = F_{ip}(t) + 2F_{in}(t)$$

- Neutron measurements can tell us more about the proton.
- Different forms for $F_{ip}^{u}(t)$ and $F_{ip}^{d}(t)$ hint at interesting isospin structure!

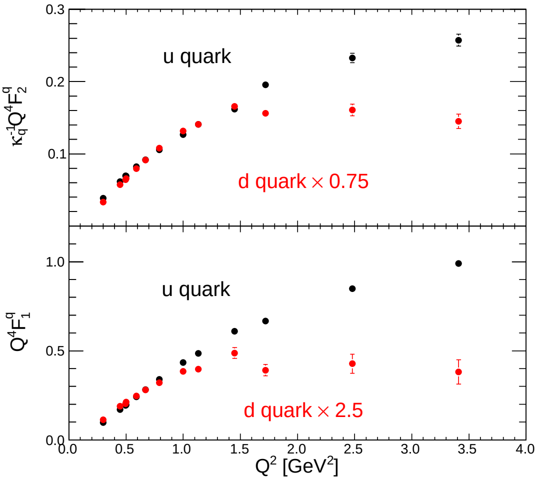
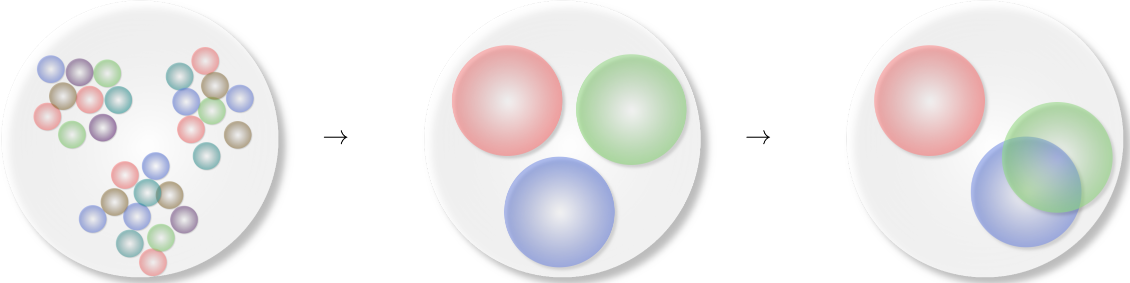


Figure from Cates *et al.*, PRL106 (2011) 252003

Diquark correlations

- One way to get behavior seen in Cates *et al.* is **diquark correlations**.
- Proton is made of three **dressed quarks**.
  - Dressed quarks are quasi-particles.
  - Mass of dressed quark: 300-400 MeV (model-dependent).
  - They are amalgamations of the true (current) quarks.
- Two of the three quarks are often bound together into a diquark.
- Proton then looks like a two-body state: quark and diquark.



Scalar diquarks

- The lightest diquarks are **scalar/isoscalar**.
- Isoscalarity $\Rightarrow$ down quark is in diquark.
- Asymptotic form for two-body form factor $\sim \frac{1}{Q^2}$ at large Q^2.
- Quark outside diquark (up quark): form factor goes as $\frac{1}{Q^2}$.
- Quark within diquark: two layers of two-body behavior, $\frac{1}{Q^4}$
- Cates *et al.* result highly suggestive of isoscalar diquarks!

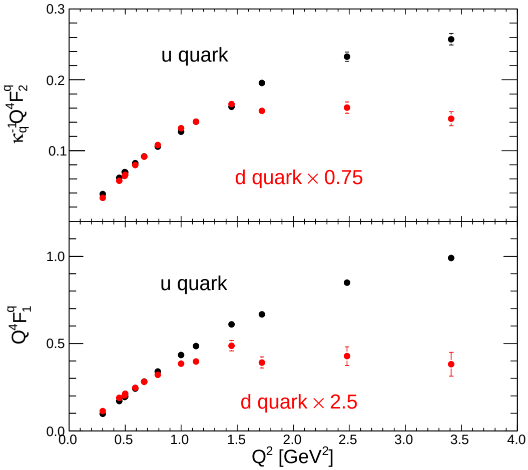
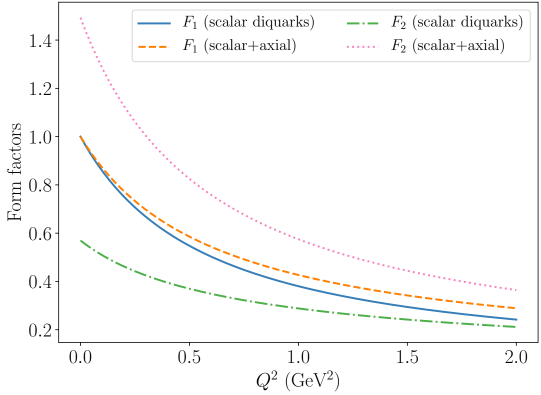


Figure from Cates *et al.*, PRL106 (2011) 252003

Scalar & axial-vector diquarks



	Scalar diquarks	Scalar+axial	Experiment
κ_p	0.57	1.49	1.79

(Pion cloud improves model; Cloët *et al.*, PRC90 (2014) 045202)

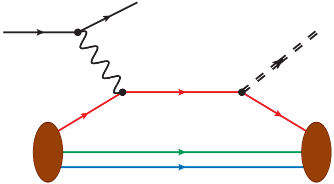
- More than just scalar diquarks may exist.
- **Axial-vector/isovector** diquarks are next-lightest diquark.
- Axial-vector diquarks needed to get large proton magnetic moment.
- **Question**: how could we see evidence of axial diquarks in observables? (a la Cates *et al.* for scalar diquarks).

Model calculations (on left):

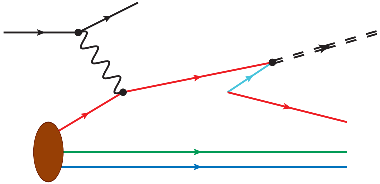
- Nambu–Jona-Lasinio model
- No pion cloud included
- Q^2 falloff is slower than reality
- Introducing dipole with vector meson pole improves Q^2 falloff
- See arXiv:1907.08256 & Cloët &al. (2014) for details

Multidimensional structure

Option 1
Quark re-enters hadron

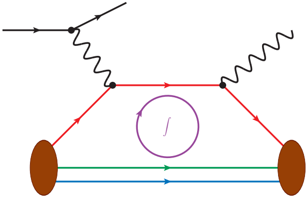


Option 2
Target is destroyed

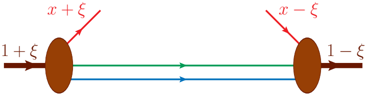


- More proton structure information if a new particle is **created** and **detected** in final state.
- Two possible avenues for exploration:
  1. **Hard exclusive reactions**: proton remains intact.
  2. **Semi-inclusive deeply inelastic scattering** (SIDIS): proton is (generally) destroyed, sum over all final states that include created/detected particle.
- **Option 1** is closest analogue to form factors (elastic scattering).
- **Deeply virtual Compton scattering** (DVCS): the created/detected particle is a photon.

DVCS and GPDs



Deeply virtual Compton scattering
$\mathcal{H}(\xi, t)$

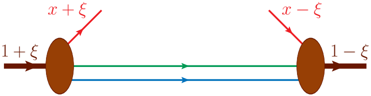


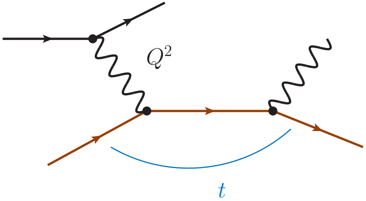
Generalized parton distribution
$H(x, \xi, t)$

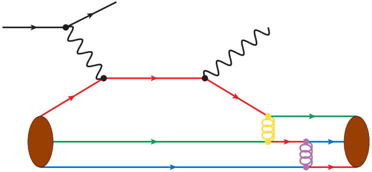
- **Deeply virtual Compton scattering** (DVCS) is the hard electro-production of a photon.
- **Generalized parton distributions** (GPDs) encode the QCD structure seen in DVCS.
- Presence of a loop means one GPD variable is integrated over.
- Integrated quantities seen in experiment: **Compton form factors**

$$\mathcal{H}(\xi, t) = \int \mathrm{d}x \left[\frac{1}{\xi - x - i0} \mp \frac{1}{\xi + x - i0} \right] H(x, \xi, t)$$

The GPD/DVCS variables







- $\frac{x+\xi}{1+\xi}$ and $\frac{x-\xi}{1-\xi}$ are initial and final **light cone momentum fractions** of struck quark.
- t is the invariant momentum transfer **to the target**.
- Q^2 is the invariant momentum transfer **from the electron**.
- $t \neq -Q^2$, in contrast to elastic scattering.
- Q^2 acts as a **resolution scale**, like in deeply inelastic scattering (DIS).
  (I will use $Q^2 = 4\ \mathrm{GeV}^2$ for remainder of talk.)
- t tells us about structure seen from redistribution of momentum kick.
- t is the "form factor variable" rather than Q^2.

GPDs and form factors

- Helicity-independent GPDs defined using a non-local operator

$$\frac{1}{2}\int \frac{dz}{2\pi} e^{-iP\cdot nzx} \langle p'|\bar{q}\left(\frac{nz}{2}\right) \not{n} q \left(-\frac{nz}{2}\right)|p\rangle = \bar{u}(p')\left[\not{n} H^q(x,\xi,t) + \frac{i\sigma^{n\Delta}}{2m_N} E^q(x,\xi,t)\right] u(p)$$

...in the light cone gauge. Other gauges require a Wilson line.

- Breakdown analogous to form factors:

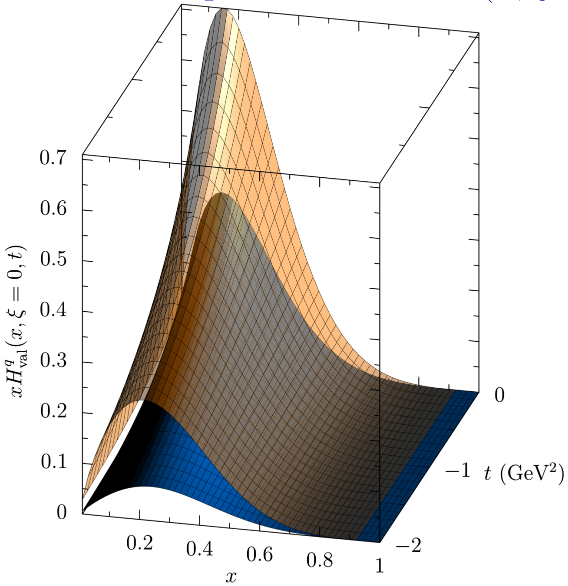
$$\langle p's'|j^\mu(0)|ps\rangle = \bar{u}^{s'}(p')\left[\gamma^\mu F_1(t) + \frac{i\sigma^{\mu\Delta}}{2m_p} F_2(t)\right] u^s(p)$$

- These are even related through integration

$$\int \mathrm{d}x\, H^q(x,\xi,t;Q^2) = F_1^q(t) \qquad \int \mathrm{d}x\, E^q(x,\xi,t;Q^2) = F_2^q(t)$$

- GPDs are, in a sense, extensions of form factors

Non-skewed proton GPD: $H^q(x, \xi = 0, t)$



Orange is up; blue is down.

- **Forward limit**: $H^q(x, 0, 0) = q(x)$
- Related to (flavor-separated) **Dirac form factor**:

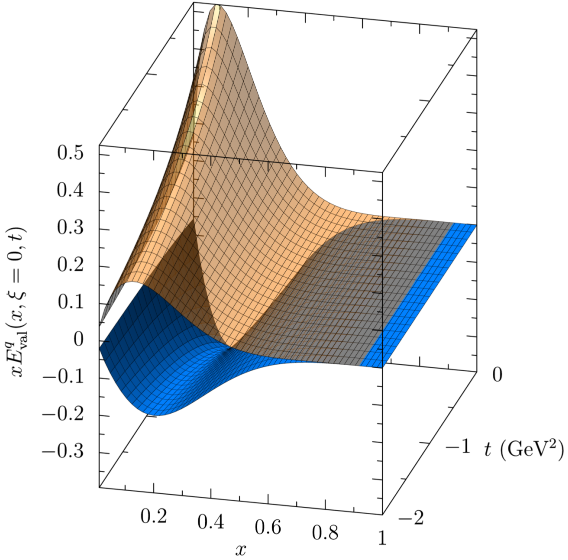
$$\int \mathrm{d}x\, H^q(x, 0, t) = F_1^q(t)$$

- Additionally related to **gravitational form factor**:

$$\int \mathrm{d}x\, xH^q(x, 0, t) = A^q(t)$$

  describing distribution of energy
- Show hybrid of behaviors of form factors and parton distribution functions (PDFs)

Non-skewed proton GPD: $E^q(x, \xi = 0, t)$



Orange is up; blue is down.

- No forward limit.
- Related to (flavor-separated) **Pauli form factor**:

$$\int \mathrm{d}x\, E^q(x, 0, t) = F_2^q(t)$$

- Up and down quarks have opposite signs—contribute constructively to κ_p
- Additionally related to **gravitational form factor**:

$$\frac{1}{2}\int \mathrm{d}x\, x\Big(H^q(x, 0, t) + E^q(x, 0, t)\Big) = J^q(t)$$

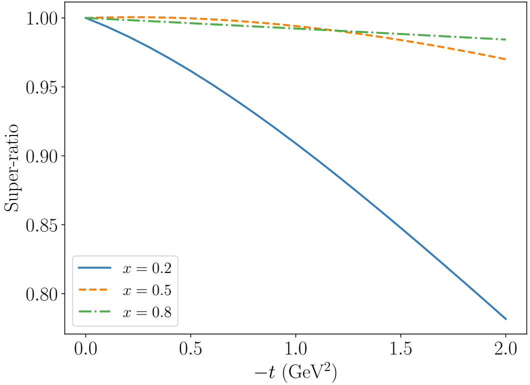
describing distribution of *total* angular momentum

A normalized, flavor-separated super-ratio

- How much faster is down quark falloff **for different x slices**?
- Normalize out the $t = 0$ value:

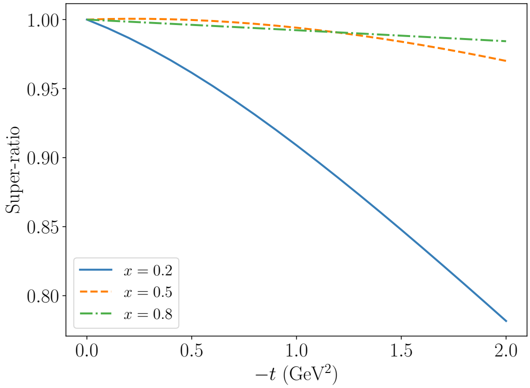
$$S = \frac{H^d(x,0,t)}{H^d(x,0,0)} \Big/ \frac{H^u(x,0,t)}{H^u(x,0,0)}$$

- In **scalar diquark** model, down quarks are within diquark.
  - The d/u ratio is small to begin with at large x
  - At small x, d/u ratio decreases as diquark dies off with t
- We see new behavior offered by this multidimensional description!

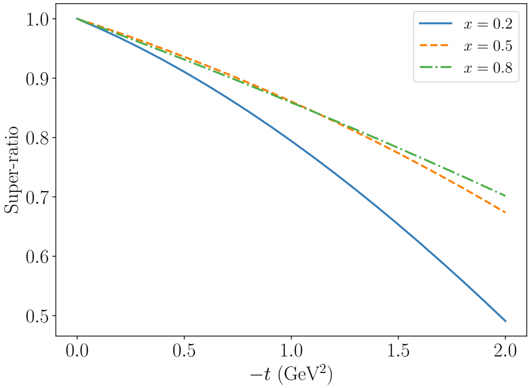


Scalar diquarks only

Super-ratio: a model comparison



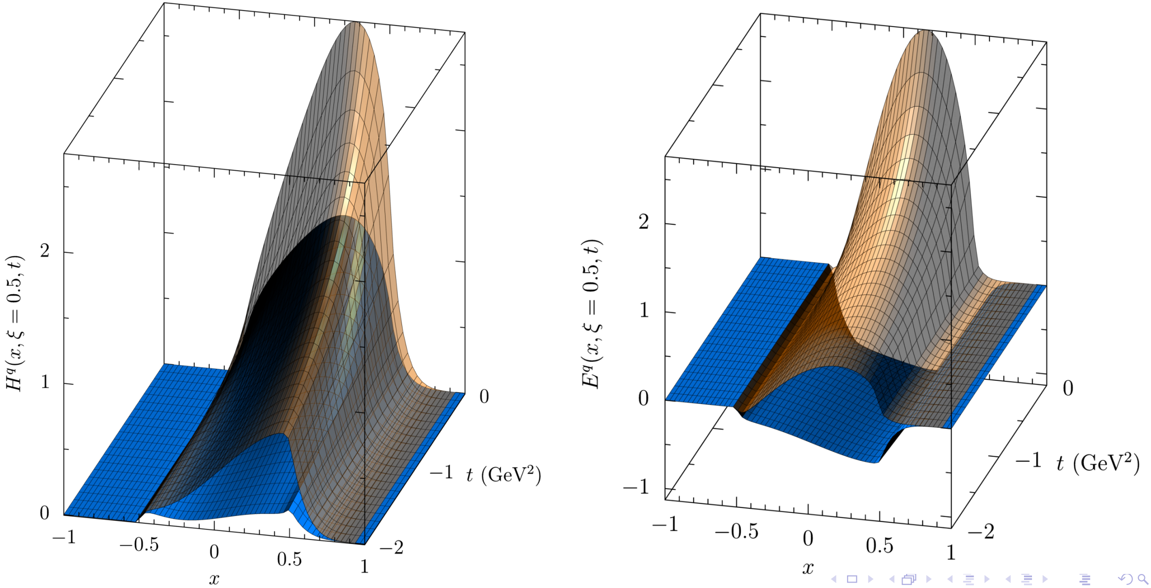
Scalar diquarks only



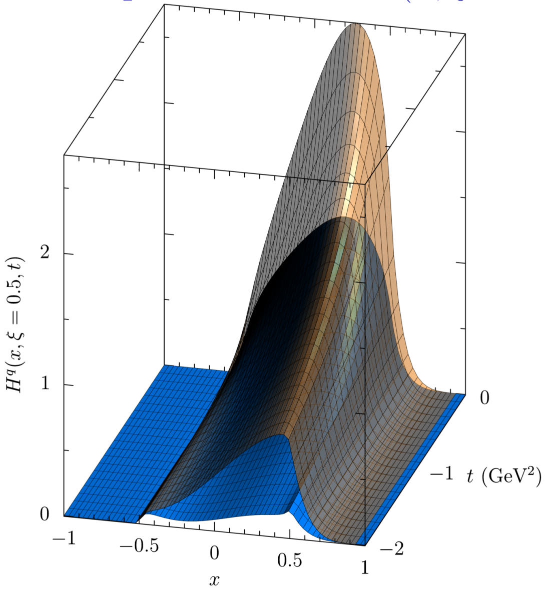
Scalar and axial diquarks

- Noticeable *qualitative* difference!
- This is because down quark can be *outside* an axial diquark.

Skewed GPDs


$H^q(x, \xi = 0.5, t)$
0
1
2
−1
−0.5
0
0.5
1
x
0
−1
−2
t (GeV^2)
$E^q(x, \xi = 0.5, t)$
−1
0
1
2
−1
−0.5
0
0.5
1
x
0
−1
−2
t (GeV^2)

Skewed proton GPD: $H^q(x, \xi = 0.5, t)$



Orange is up; blue is down.

- Still related to (flavor-separated) **Dirac form factor**:

$$\int \mathrm{d}x\, H^q(x, \xi, t) = F_1^q(t)$$

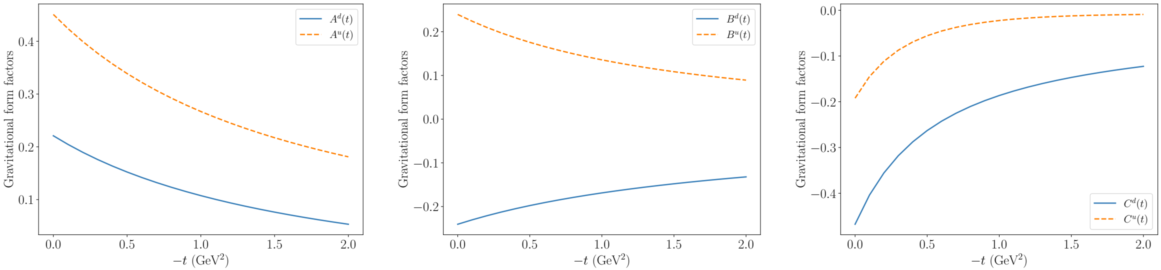
(integration kills ξ dependence!)

- Related to *two* **gravitational form factors**:

$$\int \mathrm{d}x\, x H^q(x, \xi, t) = A^q(t) + \xi^2 C^q(t)$$

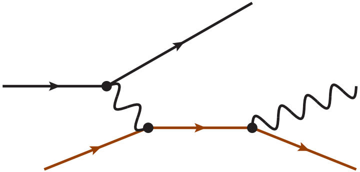
- $C^q(t)$ describes how **shear and pressure** are distributed within the proton (see Polyakov & Schweitzer).
- Being able to study ξ dependence offers an exciting new window into probing forces!

Flavor-separated gravitational form factors?

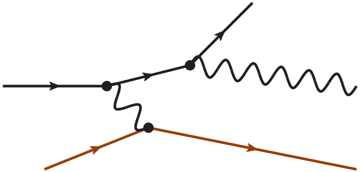


- **Sum rules**: $A(0) = 1$, $B(0) = 0$, $C(0) < 0$, *after* sum over quarks & gluons
  ... hence why $A^u(0) + A^d(0) < 1$ in plot above
- $A^u(t) > A^d(t)$ since there are two up quarks, but ...
- ... down quarks contribute more to shear/pressure??? (We need to understand this better within the model)
- Flavor-separated gravitational form factors could tell us so much more about the proton.
- But GPDs & GFFs are not *directly* measured—long term project.
- **What could we do more directly?**

DVCS and Bethe-Heitler processes



DVCS



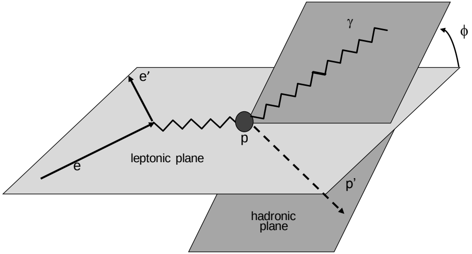
Bethe-Heitler process

- DVCS interferes with **Bethe-Heitler** process: photon emission by electron
- This is a boon rather than a bane: **interference between diagrams seen in beam-spin asymmetry**!
- Interference term seen in $\sin\phi$ modulation of **beam spin asymmetry**:

$$A_{LU}(\phi) = \frac{\sigma^+(\phi) - \sigma^-(\phi)}{\sigma^+(\phi) + \sigma^-(\phi)} \propto \mathrm{Im}\left[F_1\mathcal{H} - \frac{t}{4m_p^2}F_2\mathcal{E} + \xi(F_1 + F_2)\tilde{\mathcal{H}}\right] \sin\phi + \cos\phi \text{ modulations} + \ldots$$

Compton form factors

$$A_{LU}(\phi) \propto \mathrm{Im}\left[F_1\mathcal{H} - \frac{t}{4m_p^2}F_2\mathcal{E} + \xi(F_1+F_2)\tilde{\mathcal{H}}\right]\sin\phi + \dots$$



(diagram taken from Pisano *et al.*, PRD91 (2015) 052014)

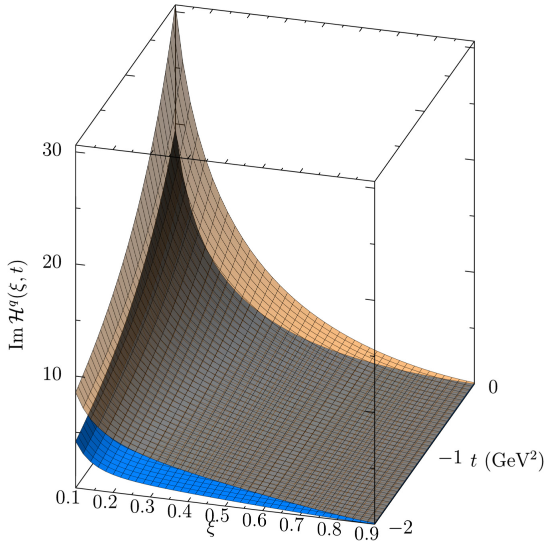
- $\tilde{\mathcal{H}}$ is a helicity-dependent Compton form factor.
- The imaginary parts of Compton form factors appear as:

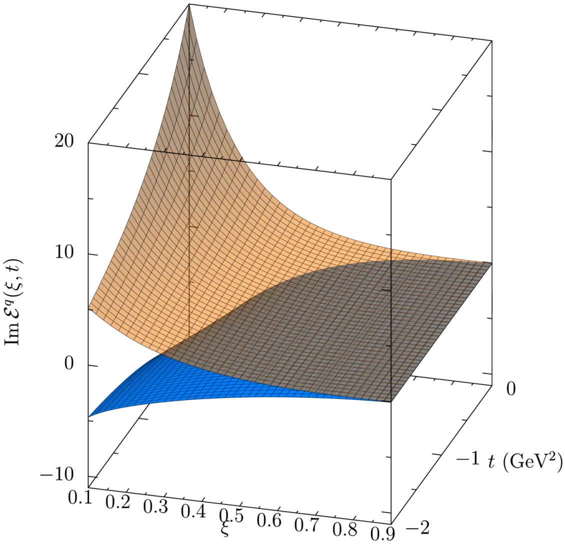
$$\mathcal{H}(\xi,t) = -\pi\sum_q e_q[H^q(\xi,\xi,t) - H^q(-\xi,\xi,t)] \qquad \mathcal{E}(\xi,t) = -\pi\sum_q e_q[E^q(\xi,\xi,t) - E^q(-\xi,\xi,t)]$$

$$\tilde{\mathcal{H}}(\xi,t) = -\pi\sum_q e_q[\tilde{H}^q(\xi,\xi,t) + \tilde{H}^q(-\xi,\xi,t)] \qquad \tilde{\mathcal{E}}(\xi,t) = -\pi\sum_q e_q[\tilde{E}^q(\xi,\xi,t) + \tilde{E}^q(-\xi,\xi,t)]$$

- CFFs and beam spin asymmetry give **two-dimensional slices** of GPDs.
- Need **precision determinations of form factors** in order to extract CFFs.
- **Question**: is there anything about diquarks we can see in *just* imaginary CFFs?

Model calculation: flavor-separated Compton form factors



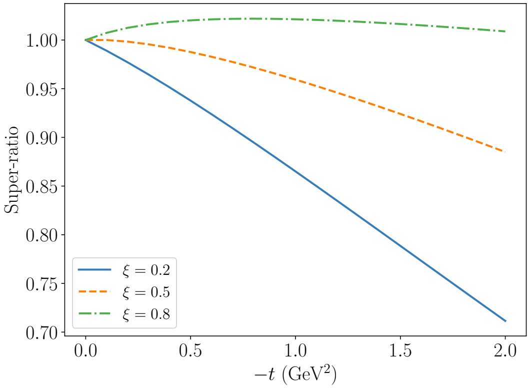


Super-ratio for Compton form factors

- Let's see if this works for CFFs, which are measured.
- How much faster is down quark falloff **for different ξ slices**?
- This is a new lever-arm offered by virtual Compton scattering!
- Normalize out the $t = 0$ value:

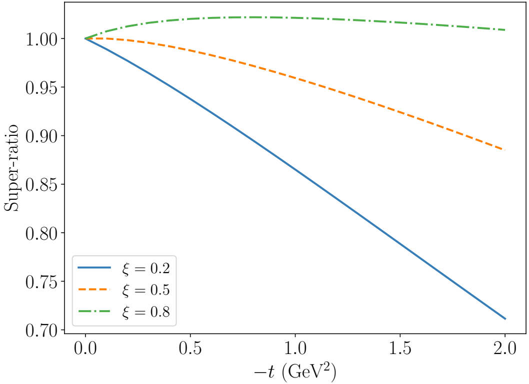
$$S = \frac{\mathcal{H}^d(\xi,t)}{\mathcal{H}^d(\xi,0)} \Bigg/ \frac{\mathcal{H}^u(\xi,t)}{\mathcal{H}^u(\xi,0)}$$

- High $x \Rightarrow$ high ξ in imaginary CFFs
- In **scalar diquark** model, down quarks are within diquark.
  - The d/u ratio is small to begin with at large ξ
  - At small ξ, d/u ratio decreases as diquark dies off with t

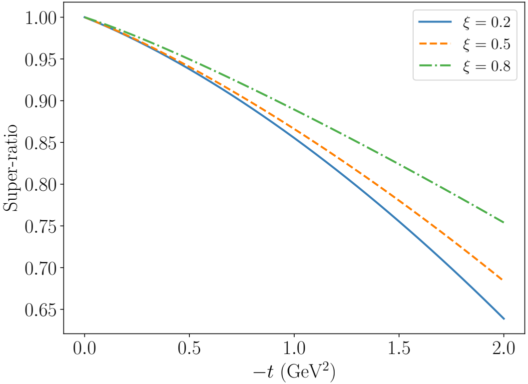


Scalar diquarks only

Super-ratio: Compton form factor case



Scalar diquarks only



Scalar and axial diquarks

- Noticeable *qualitative* difference, still!
- Again because down quark can be *outside* an axial diquark.

Conclusions and summary

- **Generalized parton distributions** (GPDs) are multi-dimensional descriptions of proton structure which contain the elastic form factors.
- GPDs can also be used to determine **gravitational form factors**.
- **Compton form factors** are two-dimensional slices of GPDs, which are measured in **deeply virtual Compton scattering** (DVCS).
- Compton form factors contain the potential to elaborate upon the quark-diquark structure of the proton.

Thank you for your time and attention!