Parton Distribution Functions from Lattice QCD

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with

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Introduction

- Goal: Compute hadron structure properties from QCD
    - Parton distribution functions (PDFs)
- Operator product: Mellin moments are local matrix elements that can be computed in Lattice QCD
    - Power divergent mixing limits us to few moments
- Few years ago X. Ji suggested an approach for obtaining PDFs from Lattice QCD
- First calculations already available

X. Ji, Phys.Rev.Lett. 110, (2013)

Y.-Q. Ma J.-W. Qiu (2014) 1404.6860

H.-W. Lin, J.-W. Chen, S. D. Cohen, and X. Ji, Phys.Rev. D91, 054510 (2015)

C. Alexandrou, et al, Phys. Rev. D92, 014502 (2015)

- A new approach for obtaining PDFs from LQCD introduced by A. Radyushkin

A. Radyushkin Phys.Lett. B767 (2017)

- Hadronic tensor methods

K-F Liu et al Phys. Rev. Lett. 72 (1994) , Phys. Rev. D62 (2000) 074501
Detmold and Lin 2005
M. T. Hansen et al arXiv:1704.08993.
UKQCD-QCDSF-CSSM Phys. Lett. B714 (2012), arXiv:1703.01153

Ma and Qiu : arXiv:1709.03018

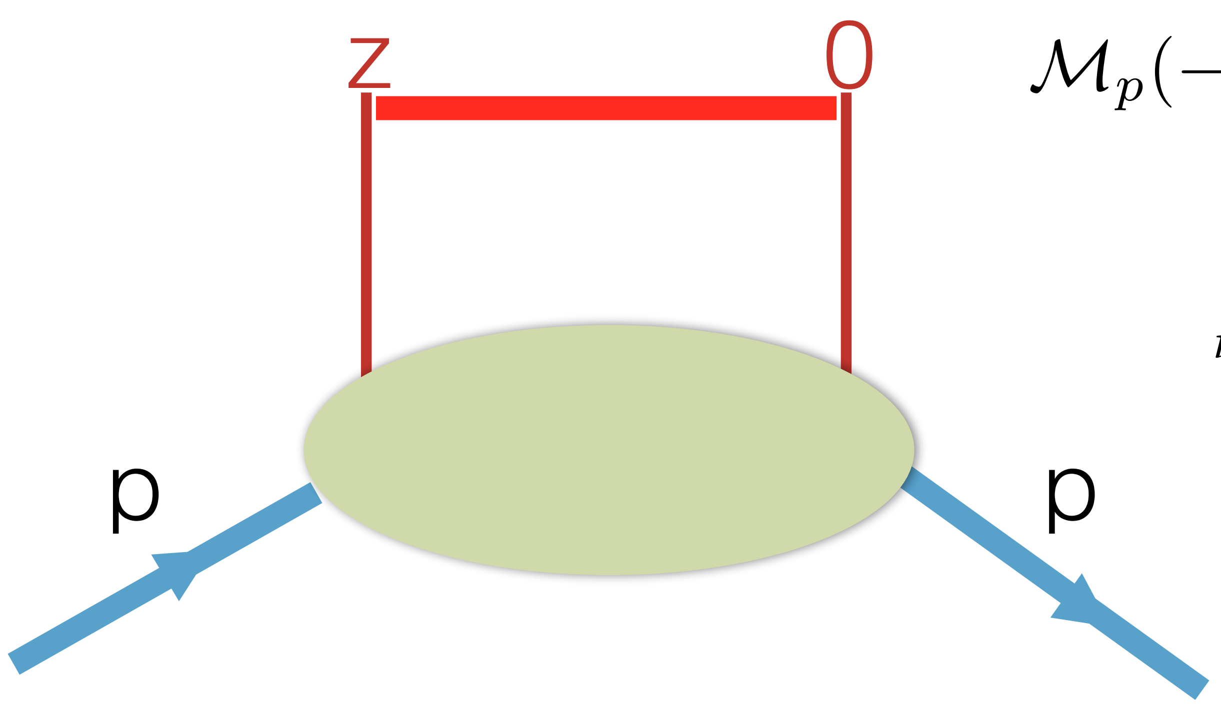
Pseudo-PDFs

Unpolarized PDFs proton:

$$\mathcal{M}^\alpha(z,p) \equiv \langle p|\bar\psi(0)\,\gamma^\alpha\,\hat{E}(0,z;A)\psi(z)|p\rangle \qquad \hat{E}(0,z;A) = \mathcal{P}\exp\left[-ig\int_0^z \mathrm{d}z'_\mu\, A^\mu_\alpha(z')T_\alpha\right]$$

$$\mathcal{M}^\alpha(z,p) = 2p^\alpha \mathcal{M}_p(-(zp),-z^2) + z^\alpha \mathcal{M}_z(-(zp),-z^2)$$

$$\mathcal{M}^+(z,p) = 2p^+\mathcal{M}_p(-p_+z_-,0) \qquad \mathcal{M}_p(-p_+z_-,0) = \int_{-1}^1 dx\, f(x)\, e^{-ixp_+z_-}$$



$\mathcal{M}_p(-pz,-z^2)$ is a Lorentz invariant therefore computable in any frame

$\nu = -zp$ **Ioffe time**

Goal:
Compute PDFs with Lattice QCD using space like separations z

$$\mathcal{M}_p(\nu, z^2) = \int_0^1 d\alpha\, \mathcal{C}(\alpha, z^2\mu^2, \alpha_s(\mu))\, \mathcal{Q}(\alpha\nu, \mu) \; + \mathcal{O}(z^2\Lambda_{qcd}^2)$$

$\mathcal{Q}(\nu, \mu)$ is called the Ioffe time PDF

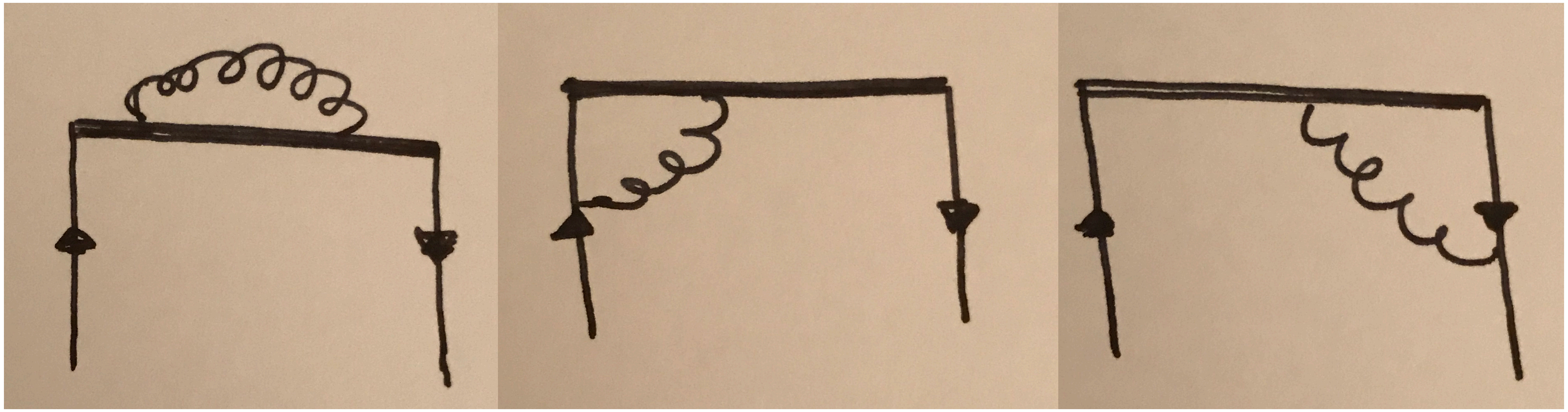
V. Braun, et. al Phys. Rev. D 51, 6036 (1995)

$$\mathcal{Q}(\nu, \mu) = \int_{-1}^1 dx\, e^{-ix\nu} f(x, \mu)$$

Matching to $\overline{MS}$

Radyushkin Phys.Rev. D98 (2018) no.1, 014019
Izubuchi et al. Phys.Rev. D98 (2018) no.5, 056004
Zhang et al. Phys.Rev. D97 (2018) no.7, 074508

UV behaviour



One loop calculation of the UV divergences results in

$$\mathcal{M}^0(z,P,a) \sim e^{-m|z|/a}\left(\frac{a^2}{z^2}\right)^{2\gamma_{end}}$$

after re-summation of one loop result resulting exponentiation

- J.G.M.Gatheral,Phys.Lett.133B,90(1983)
- J.Frenkel, J.C.Taylor,Nucl.Phys.B246,231(1984),
- G.P.Korchemsky, A.V.Radyushkin,Nucl.Phys.B283,342(1987).

Multiplicatively renormalizable

Consider the ratio $$\mathfrak{M}(\nu, z_3^2) \equiv \frac{\mathcal{M}_p(\nu, z_3^2)}{\mathcal{M}_p(0, z_3^2)}$$

UV divergences will cancel in this ratio resulting a renormalization group invariant (RGI) function

The lattice regulator can now be removed

$\mathfrak{M}^{cont}(\nu, z_3^2)$ Universal independent of the lattice

$\mathcal{M}_p(0,0) = 1$ Isovector matrix element

$$\mathfrak{M}(\nu, z^2) = \int_0^1 d\alpha \, \mathfrak{C}(\alpha, z^2\mu^2, \alpha_s(\mu)) \mathcal{Q}(\alpha\nu, \mu) \; + \sum_{k=1}^{\infty} \mathcal{B}_k(\nu)(z^2)^k$$

$$\mathcal{B}_k(\nu)(z^2)^k \sim \mathcal{O}(\Lambda_{qcd}^{2k})$$

Polynomial corrections to the Ioffe time PDF may be suppressed

B. U. Musch, *et al* Phys. Rev. D 83, 094507 (2011)
M. Anselmino et al. 10.1007/JHEP04(2014)005

A. Radyushkin Phys.Lett. B767 (2017)

Polynomial corrections will vanish in the $z^2 = 0$ limit

Factorization:

$$\mathfrak{M}(\nu, z^2) = \int_0^1 dx\, q(x,\mu)\mathcal{K}(x\nu, z^2\mu^2) + \mathcal{O}(z^2\Lambda^2_{qcd})$$

perturbative kernel (known to NLO)

Challenges:

- Compute matrix elements in lattice QCD
  - Optimal interpolating fields to isolate the ground state matrix element requires computation of many correlations functions. This task is dominated by dot-products which are typically not optimally implemented
- Solution of an inverse problem to obtain the PDF from lattice QCD data
  - Controling the polynomial corrections is required for proper estimation of systematic errors. This in turn implies the need for very fine lattice spacings.

Project Goals

- Create pseudo-data using the above model
  - Explore various options for parametrizing the transverse structure of the proton
- Develop methods that allow for reliable extraction of the PDF using pseudo-data
- Understand the systematics of the PDF extraction

Karpie et. al JHEP 1904 (2019) 057

- Optimize the codes for computing the relevant matrix elements from lattice QCD
- Support goes to:
  - 1 graduate student to work on the modeling (J. Karpie)
  - 1 postdoc W&M to work on the LQCD code optimization (E. Romero)

Matrix Element computation

–Eloy Romero

Hadron correlation functions

- One of the LQCD goals is to predict low-energy hadron spectrum of nuclei
- That means measuring the two-point correction functions of field operators with specific quantum numbers

$$C(t', t) = \langle \chi(t')\chi^\dagger(t) \rangle$$

- In Monte-Carlo calculations, the physical relevant signal in correlation functions falls exponentially and is rapidly dominated by statistical fluctuations
- Operators that create low-lying energy eigenstates quickly improve the quality of the data extracted exponentially
- Smearing methods project the operator on the low-energy states and make computationally feasible the evaluation of the correlation functions

$$\tilde{\chi}(t) = \Box(t)\chi(t)\Box(t), \quad \Box(t) = V(t)V^\dagger(t), \quad C(t', t) \approx \langle \tilde{\chi}(t')\tilde{\chi}^\dagger(t) \rangle$$

- The computational cost is dominated by the creation and contraction of tensors with sizes of the smearing projection rank

Higher-order Singular Value Decomposition

$$\tau = (U_1, U_2, \ldots, U_d) \cdot s$$

where U_i are unitary matrices and s is the core tensor.

- HOSVD accelerates the contractions of tensors if near null rows, columns and fibers can be dropped from the core tensors

Higher-order Singular Value Decomposition

$$\tau = (U_1, U_2, \ldots, U_d) \cdot s$$

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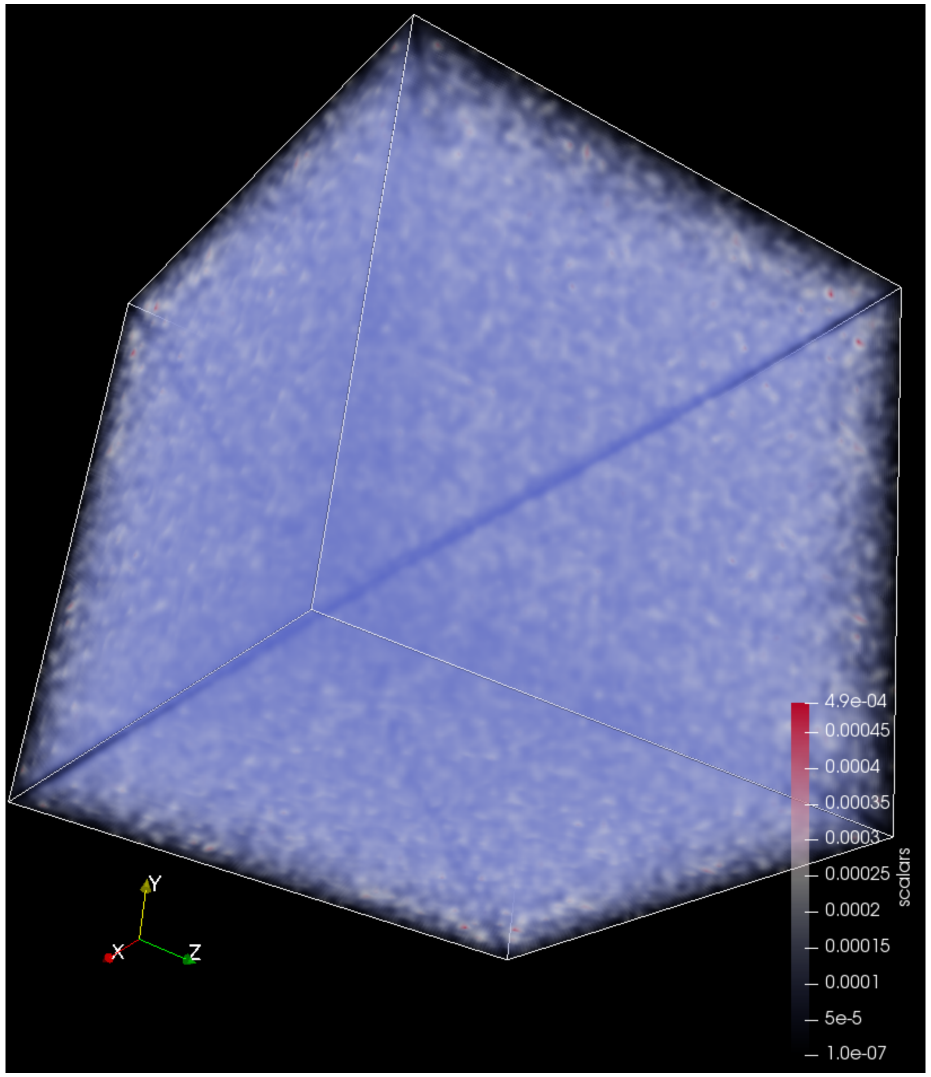
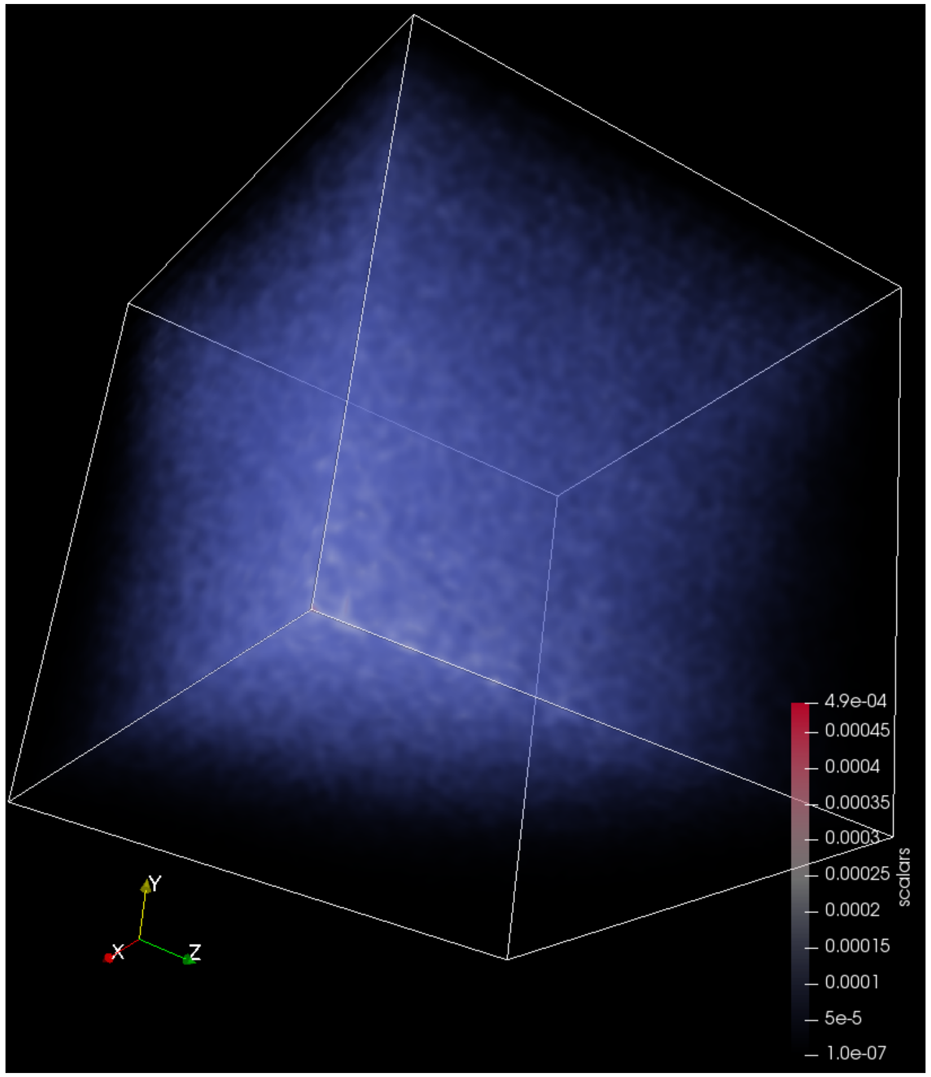
Example from one of the generated tensors:



τ



s

Higher-order Singular Value Decomposition

$$\tau = (U_1, U_2, \ldots, U_d) \cdot s$$

where U_i are unitary matrices and s is the core tensor.

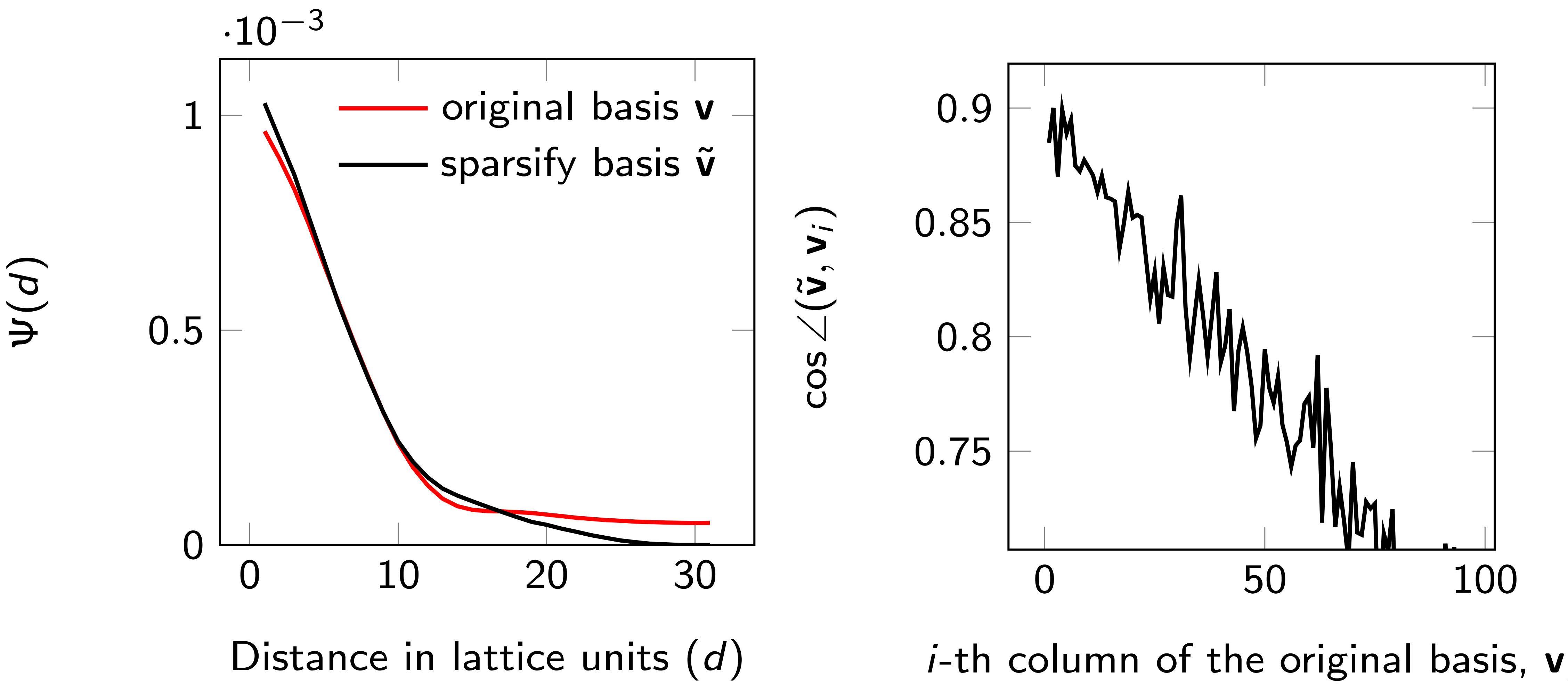
- HOSVD accelerates the contractions of tensors if near null rows, columns and fibers can be dropped from the core tensors
- In the studied cases, the core tensors can be reduced 20% in dimension
- But to keep the symmetries of the original tensors requires contracting the core tensors six times per original contraction
- At the end, we expect modest reduction of time, about 2-3 times

Smearing basis sparsification

- Instead of sparsifying the tensors, we propose to sparsify the smearing basis $V(t)$ that creates the tensors
- The smearing basis consist of the eigenvectors of the lattice Laplacian with the smallest energies
- On the local scale those eigenvectors look similar (local coherence)
- Given a regular partition of a lattice with fully connected components, the eigenvectors can be approximated as the linear combinations of vectors with support on one of the components

Smearing basis sparsification

- We are able to generate a new basis $\tilde{V}(t)$ that has similar spatial distribution to the original one, and
- lower energy eigenvectors are better represented



$$\Psi(d) = \sqrt{\sum_{|\mathbf{x}-\mathbf{y}|=d,\ \alpha\in C,\ i\leq K} |\mathbf{v}_{\mathbf{x}\alpha i}\mathbf{v}^{\dagger}_{\mathbf{y}\alpha i}|^2}.$$

- This basis will accelerate the contractions 16-64 times!

Inverse Problem

with

Joe Karpie
Alexander Rothkopf
Savas Zafeiropoulos

Factorization:

$$\mathfrak{M}(\nu, z^2) = \int_0^1 dx\, q(x,\mu)\mathcal{K}(x\nu, z^2\mu^2) + \mathcal{O}(z^2\Lambda^2_{qcd})$$

perturbative kernel (known to NLO)

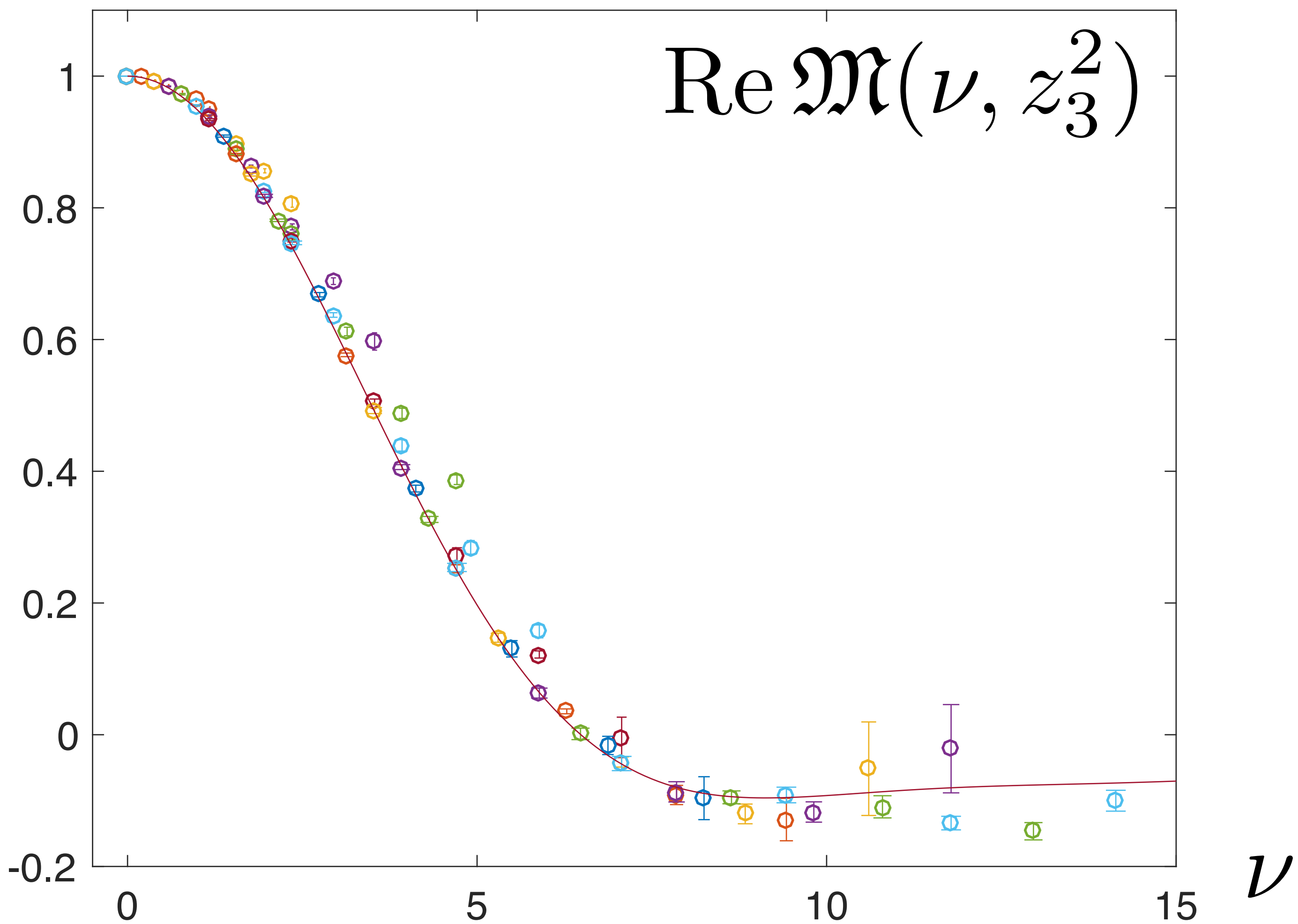
Model for lattice matrix elements

$$\mathfrak{M}(\nu, z^2) = N\int_0^1 dx\, q(x,\mu)\mathcal{K}(x\nu, z^2\mu^2)e^{-z^2\Lambda^2 x(1-x)}$$

Priomordial TMD

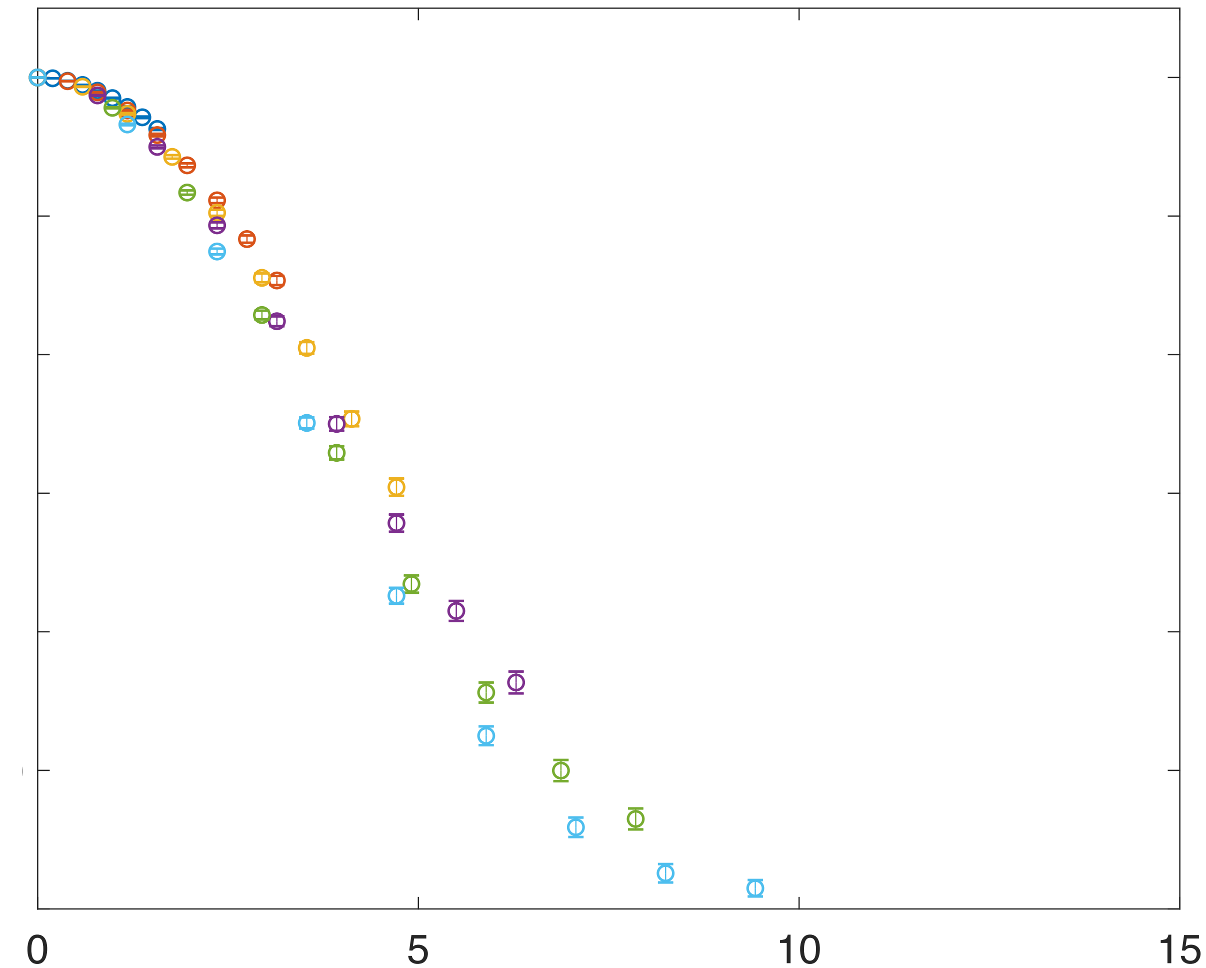
$$\mathfrak{M}(\nu, z^2) = N \int_0^1 dx\, q(x,\mu) \mathcal{K}(x\nu, z^2\mu^2) e^{-z^2\Lambda^2 x(1-x)}$$

- Use the fenomenological PDFs from CJ15, NNPDF, etc. for q(x,μ)
- Explore models for the “TMD” that produce data that look similar to the lattice QCD results
- Reconstruct the PDF from mock matrix element data and assess the fidelity of the reconstruction by comparing with the known input PDF



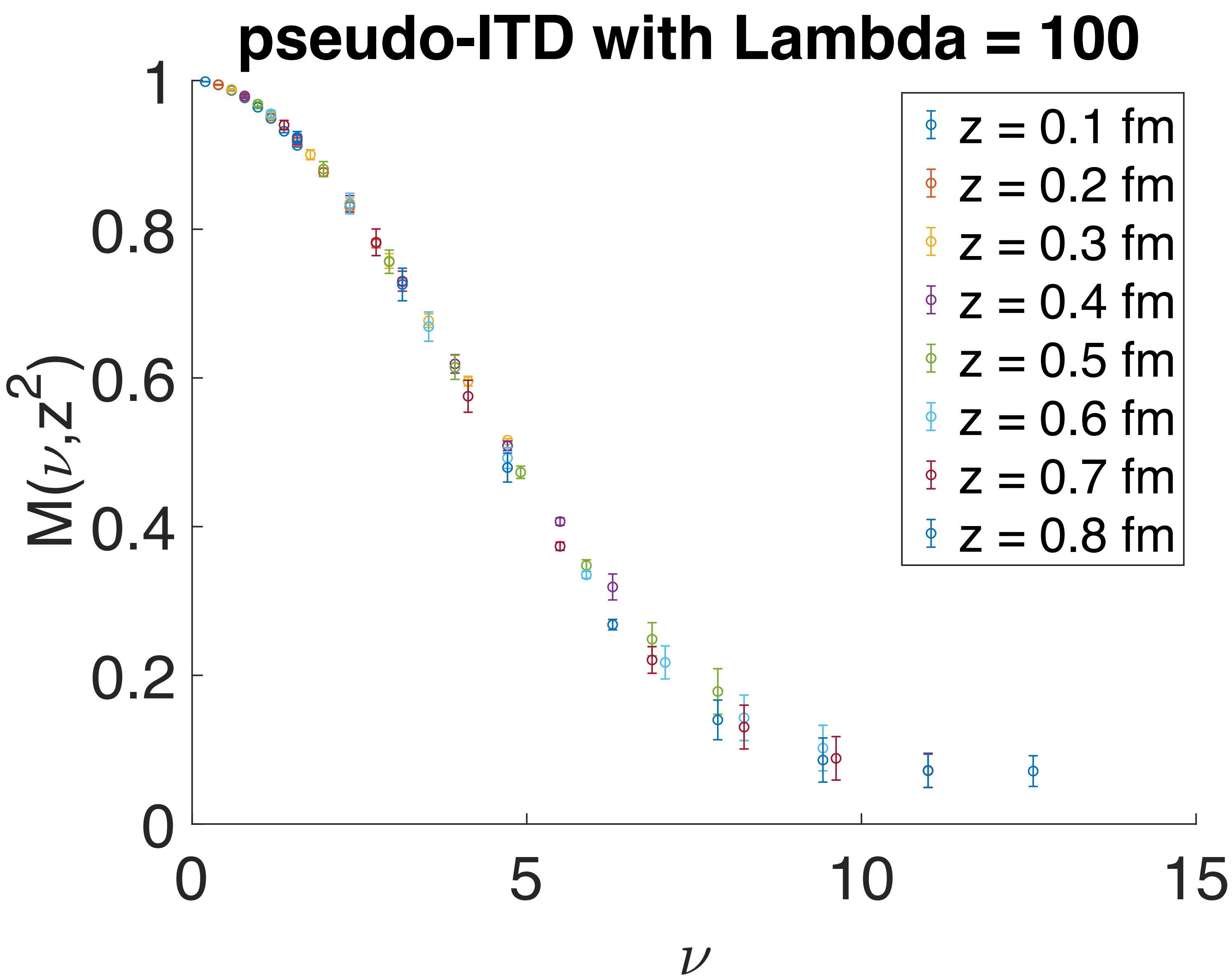
Lattice QCD result

Orginos et al. (Phys.Rev. D96 (2017) no.9, 094503)



Mock data produced by model using CJ15 pdfs (μ=2 GeV)

LO kernel



NLO kernel with CJ15 PDFs

Methods to be studied

- Backus-Gilbert
- Bayesian reconstruction
- Neural network parametrization
- Simple functional form parameterizations
- Understand how to properly handle and remove the polynomial corrections.

Some tests

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Ignoring (z^2) corrections

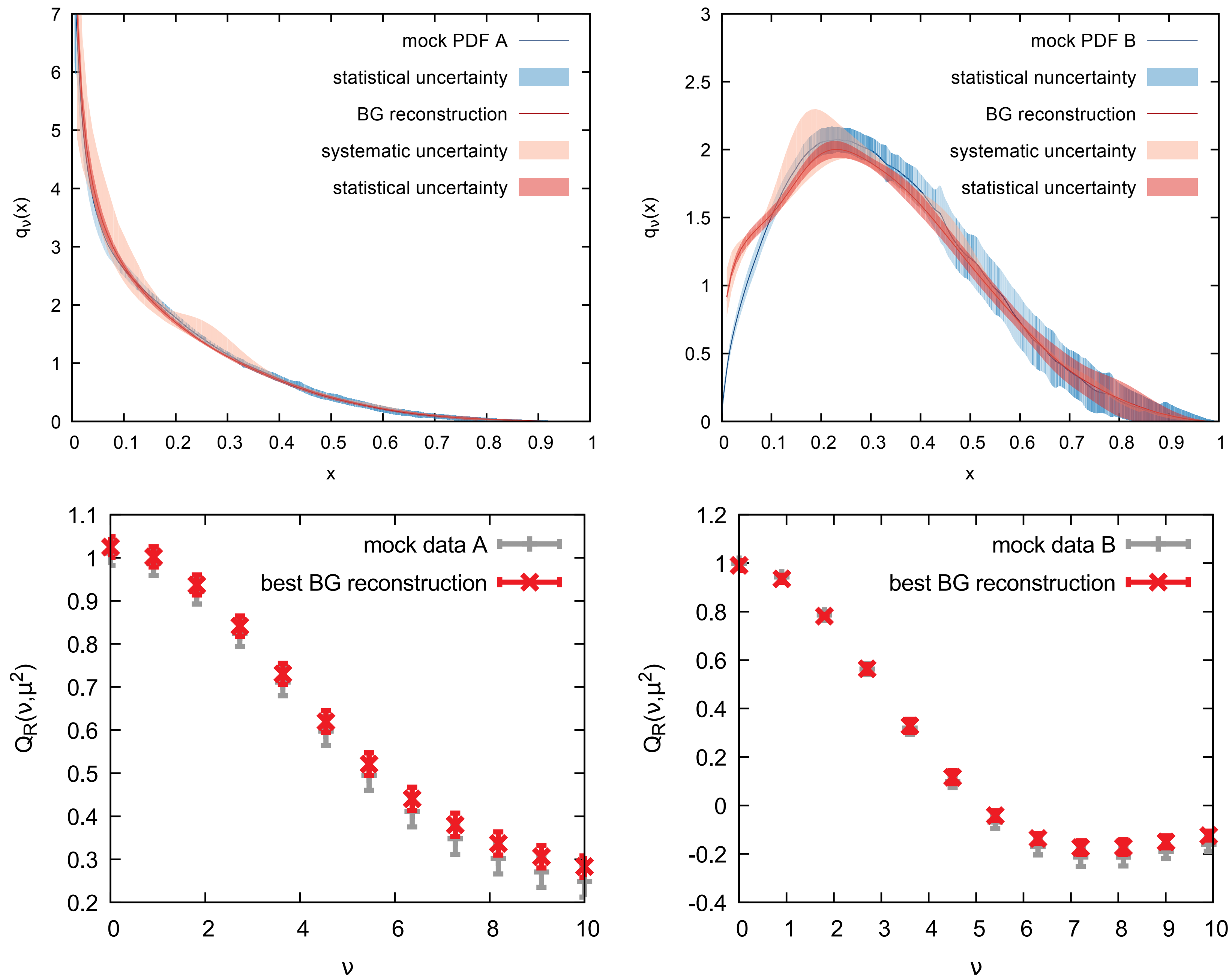
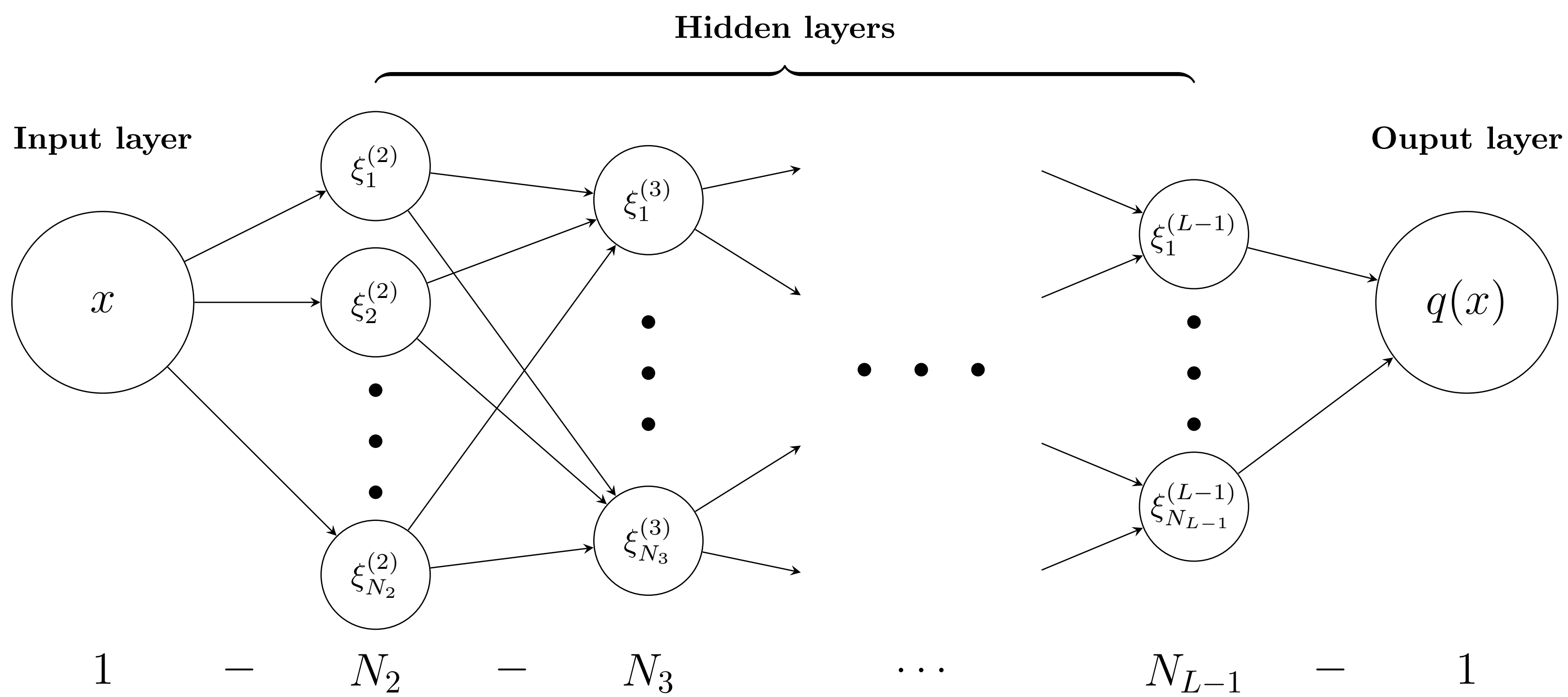


Figure 6. x-space PDF's reconstructed using the Backus-Gilbert (BG) method from $N_\nu = 12$ Ioffe-time data points on the interval $\nu = [0, 10]$ (top) as well as the input data (gray crosses) compared to the data arising from the reconstructed PDF (red crosses) in the bottom panels. The plots in the left column show the results for mock data based on a phenomenological PDF, while the right column from the modified scenario where the PDF vanishes at the origin. The reconstruction was performed with preconditioning exponents $a = -0.35$ and $b = 2$ for scenario A and $a = 0.3$ and $b = 2$ for scenario B.

Neural-Network reconstruction



Flexible parametrization of q(x)

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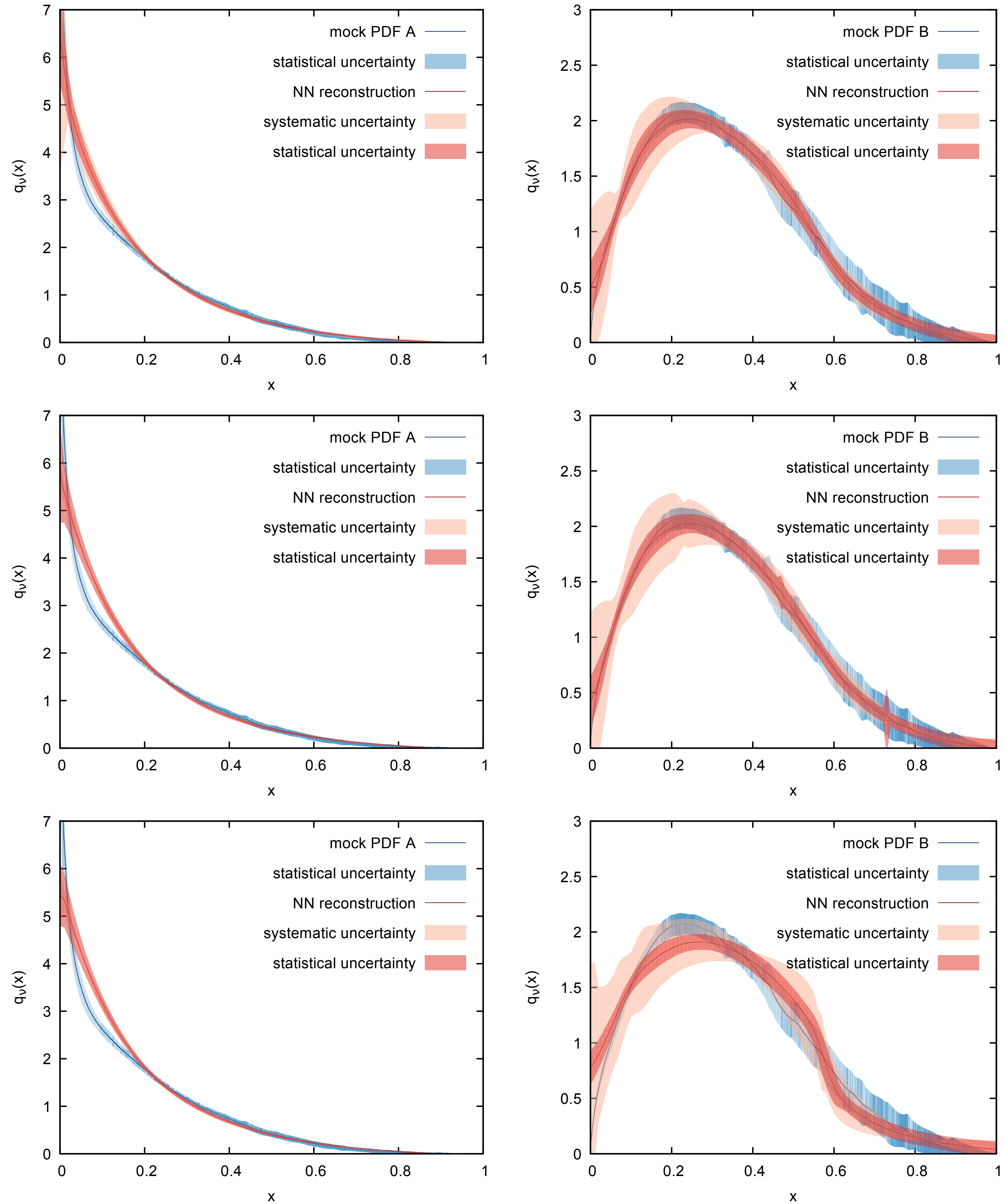

Figure 9. The genetically trained neural nets. The blue band is the original data. The red band is the reconstructed PDF with statistical and systematic errors. The left column is with NNPDF data. The right column is with modified data. The first row has a network geometry of 1-3-1. The second row has a geometry of 1-4-1. The third row has a geometry of 1-2-2-1.

Summary

- Ideas for improving the efficiency of the matrix element computation have been developed
  - Study of their performance is under way
- We have constructed models for producing mock data to study the effectiveness of various methods for solving the inverse problem at hand
  - Study the fidelity of the reconstruction is under way