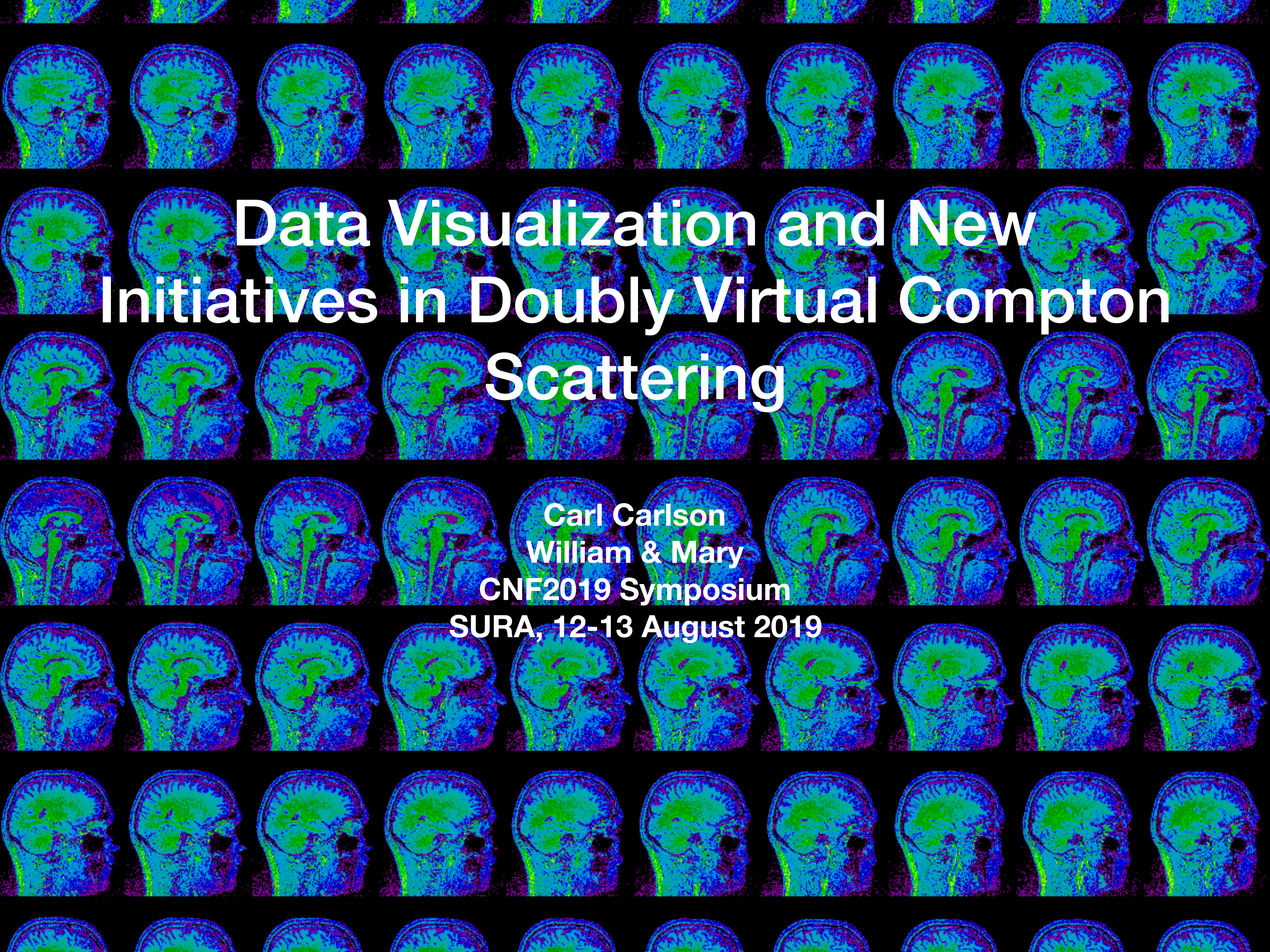

Data Visualization and New Initiatives in Doubly Virtual Compton Scattering

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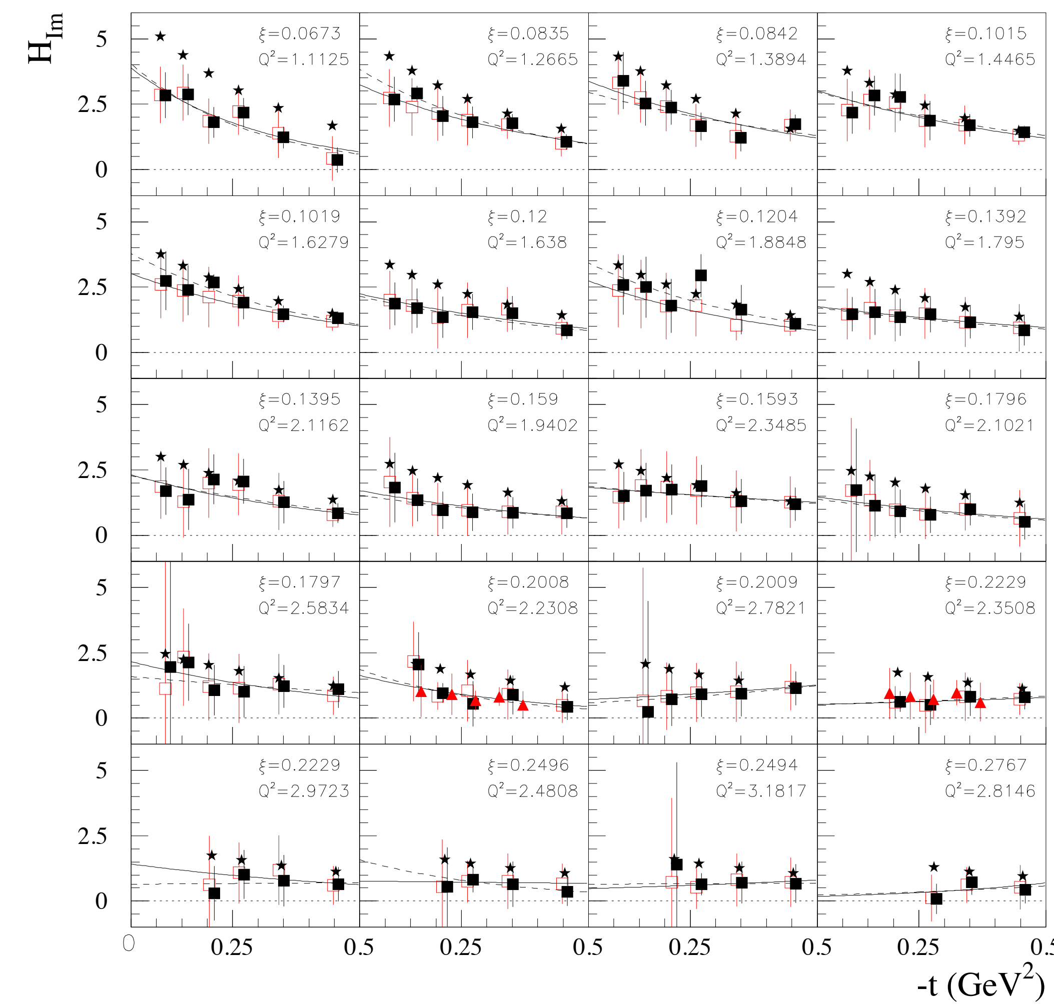
Data Visualization and New Initiatives in Doubly Virtual Compton Scattering

- Two subprojects—discuss then serially

Data Visualization for here

- There is flow of data from JLab and other labs that is already significant and whose increase in the future will also be significant.
- Currently, much data is presented as a many row by many column grid of small two-dimensional plots.
- Maybe very exciting, and presentation as grid of small plots (like postage stamps) may be necessary to get all the data shown, but makes the data hard to see and makes unexpected patterns hard to see



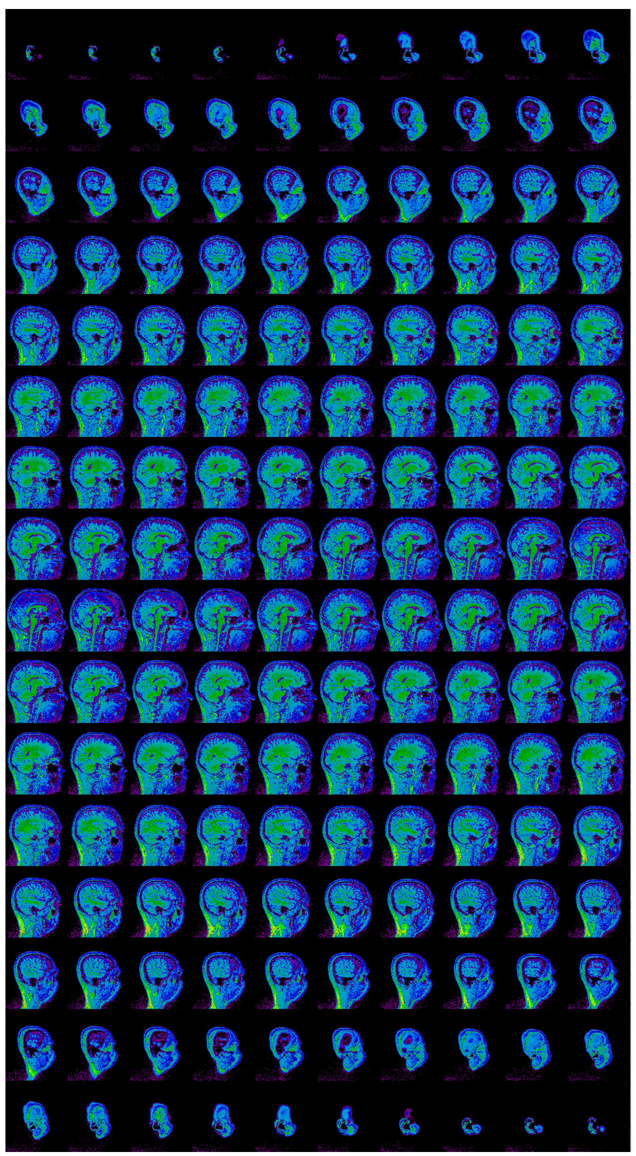
(Dupré et al., 2017, EPJA 53, 171)

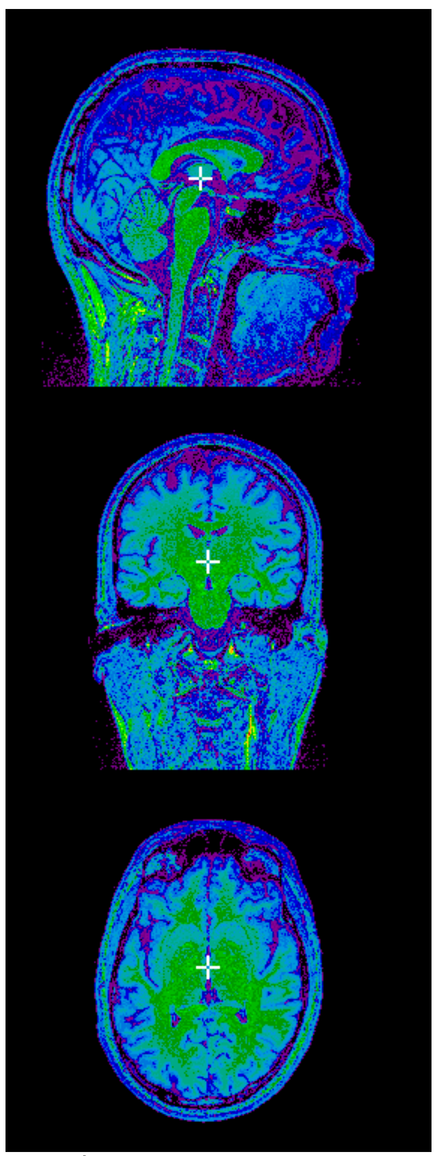
Data Visualization for here

- Related problems are addressed, and to some extent solved, in the medical profession, where one often has a set of scans which can be presented as a set of fixed direction small two-dimensional images, again looking rather like a grid of postage stamps.

- As example see brain scans at https://brainbrowser.cbrain.mcgill.ca/volume-viewer

- Computer processing turns these into interactive images that can be presented with arbitrary centers, with arbitrary scanning planes, and zoomable. We would aim, and be able to do, similar processing of nuclear science data.



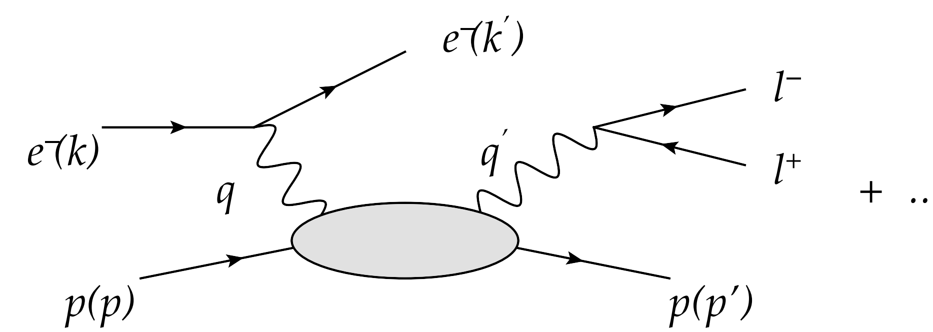


- Original scans present image in *y*-*z* plane (in some coordinate system) for series of values of *x*

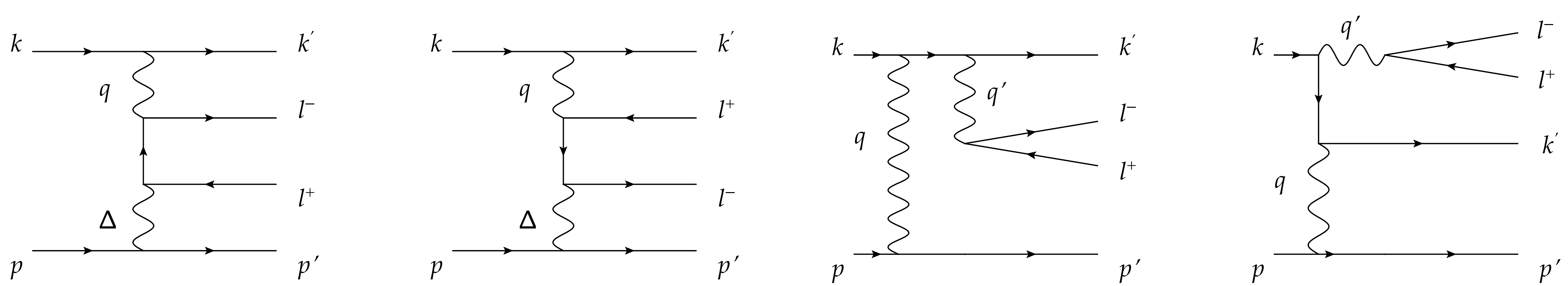
- Processed output allows viewing arbitrary plane for arbitrary value of third coordinate.

Doubly Virtual Compton Scattering

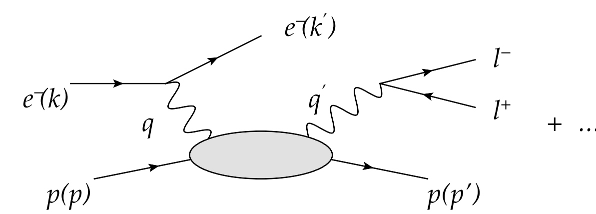
- Describe first: What it is and why we are interested.
- Process: $e + p \to e + p + \ell^+ \ell^-$ or $\gamma^* + p \to \gamma^* + p$ where one photon is spacelike and one is timelike.
- Main interest: Compton amplitude



- Also have Bethe-Heitler amplitudes



Doubly Virtual Compton Scattering

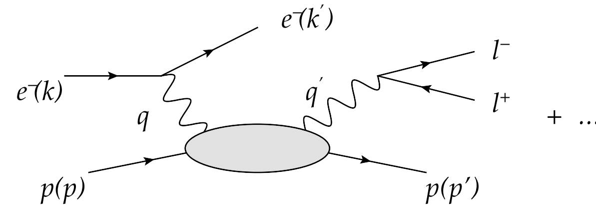
()

- Natural sequence from Real Compton Scattering (RCS, both photons real) and DVCS (Deeply Virtual Compton Scattering, initial photon from electron scattering, final photon real)

- Allows determining Compton Form Factors (related to Generalized Parton Distributions) as functions of three variables, e.g., $\mathscr{H}(\xi', \xi, t)$

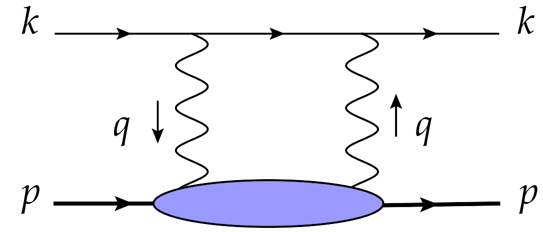
- Accesses terms in Compton amplitude that are crucial in calculating (*viz.*, fixing subtraction terms) in two-photon exchange contributions (more to come on this)

- Shengying Zhao, 1904.09335v1, argues this experiment, DDVCS or VVCS, is feasible at 12 GeV JLab

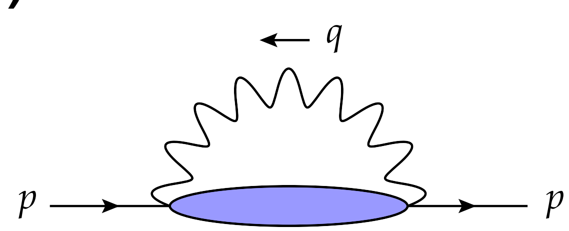


- About the subtraction term interests:

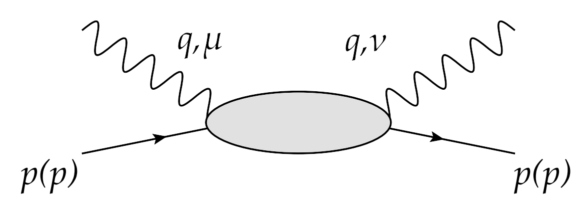
- The TPE part of the Lamb shift in muonic (or other!) atoms



- and the e.m. part of the proton-neutron mass difference (Cottingham formula)



- both depend on the Compton amplitudes $T_{1,2}(\nu, Q^2)$



(both photons equally off shell for these applications)

Doubly Virtual Compton Scattering

- The $T_{1,2}$ are gotten from dispersion relations, but the dispersion relation for T_1 requires a subtraction.
- Bottom line meaning: need to know $T_1(0,Q^2)$ for $Q^2 > 0$.
- Ugh: Not experimentally measurable.

(or maybe one can dream of positronium-proton scattering with both electron and positron interacting: but …)

Doubly Virtual Compton Scattering

- But something is known: the leading term in a low momentum expansion is

$$\lim_{Q^2 \to 0} T_1(0,Q^2) = \frac{Q^2}{e^2}\beta_M$$

where β_M is the nucleon's magnetic polarizability (known).

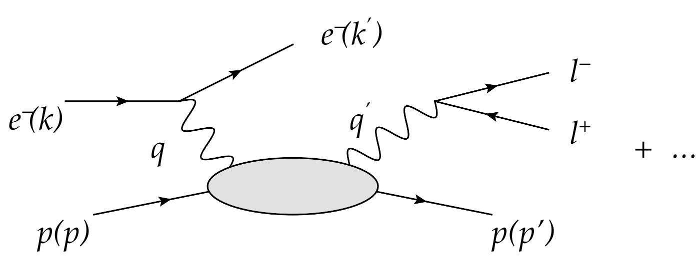
- Further term available theoretically for the proton, from Birse and McGovern and chiral perturbation theory,

$$T_1(0,Q^2) = \frac{Q^2}{e^2}\beta_M\left(1 - \frac{Q^2}{M_\beta^2} + \dots\right)$$

where $M_\beta^2 = \left(460 \pm 100\,\text{MeV}\right)^2$

Doubly Virtual Compton Scattering

- Should want direct experimental measurement
- Possible from VVCS, but not just from RCS or DVCS.
- The whole story: Compton scattering (unpolarized) expands generally in terms of 5 Compton amplitudes,



$$\leftrightarrow \quad \bar{u}(p')\, M_{\mu\nu}(p,q,q')\, u(p) = i \int d^4x\, e^{-iqx} \langle p' \,|\, T j_\mu(x) j_\nu(0) \,|\, p \rangle$$

with $M_{\mu\nu}$ given by functions times tensors with Lorentz indices

Doubly Virtual Compton Scattering

$$M^{\mu\nu} = \sum_{i=\{1,2,3,4,19\}} B_i(q^2, q'^2, q\cdot q', q\cdot P) \quad T_i^{\mu\nu}$$

$$T_1^{\mu\nu} = -\, q\cdot q'\, \hat{g}^{\mu\nu}$$

$$T_2^{\mu\nu} = 4\left[-(q\cdot P)^2\, \hat{g}^{\mu\nu} - q\cdot q'\, \hat{P}^{\mu}\hat{P}^{\nu}\right]$$

$$T_3^{\mu\nu} = q^2 q'^2\, \hat{g}^{\mu\nu} + q\cdot q'\, \hat{q}^{\mu}\hat{q}^{'\nu}$$

$$T_4^{\mu\nu} = 2\left[q\cdot P(q^2 + q'^2)\, \hat{g}^{\mu\nu} + q\cdot q' \left(\hat{P}^{\mu}\hat{q}^{'\nu} + \hat{q}^{\mu}\hat{P}^{\nu}\right)\right]$$

$$T_{19}^{\mu\nu} = 4\left[q^2 q'^2\, \hat{P}^{\mu}\hat{P}^{\nu} - q\cdot P\, q^2\, \hat{P}^{\mu}\hat{q}^{'\nu} - q\cdot P\, q'^2\, \hat{q}^{\mu}\hat{P}^{\nu} + (q\cdot P)^2\, \hat{q}^{\mu}\hat{q}^{'\nu}\right]$$

$$\hat{g}^{\mu\nu} = g^{\mu\nu} - \frac{q'^{\mu} q^{\nu}}{q\cdot q'} \qquad \hat{P}^{\mu} = P^{\mu} - \frac{P\cdot q}{q\cdot q'} q'^{\mu} \qquad \hat{P}^{\nu} = P^{\nu} - \frac{P\cdot q}{q\cdot q'} q^{\nu}$$

$$\hat{q}^{\mu} = q^{\mu} - \frac{q^2}{q\cdot q'} q'^{\mu} \qquad \hat{q}^{'\nu} = q'^{\nu} - \frac{q'^2}{q\cdot q'} q^{\nu}$$

Doubly Virtual Compton Scattering

- For RCS, only #1 and #2 contribute. (Check it out.)
- For DVCS, also #4 contributes. (Ditto.)
- Need VVCS to involve #3 and #19.
- Importance seen by looking at expansion

Doubly Virtual Compton Scattering

- Can low-momentum-expand the functions B_i, e.g.,

$$B_i(q^2, q'^2, q \cdot q', q \cdot P) = b_{i0} + b_{iq}(q^2 + q'^2) + b_{iX} q \cdot q' + b_{i\nu^2}(2M\nu)^2 + \ldots$$

- The amplitudes we want are forward limits of the B_i

- So work out the (confusingly named) amplitude T_1,

$$T_1(\nu, Q^2)/2M = -Q^2 B_1 + 4M^2\nu^2 B_2 - Q^4 B_3 + 4M\nu Q^2 B_4$$

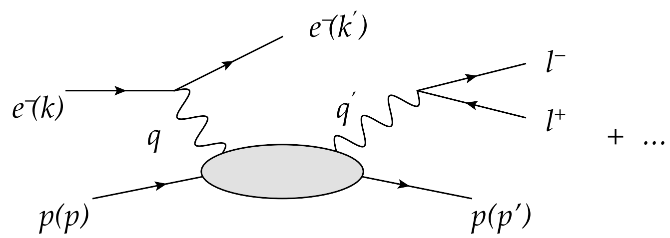
and

$$T_1^{\text{NB}}(0,Q^2) = \frac{2MQ^2}{\alpha_{\text{em}}}\beta_M + Q^4\left[2b_{1q} + b_{1X} - b_{30}\right] + \mathcal{O}(Q^6)$$

- Need B_3

VVCS—practical matters

- Have



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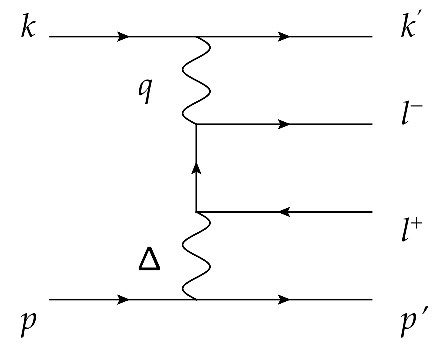
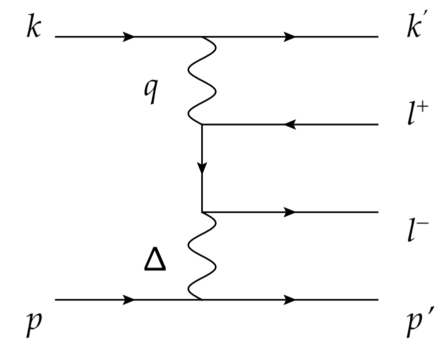


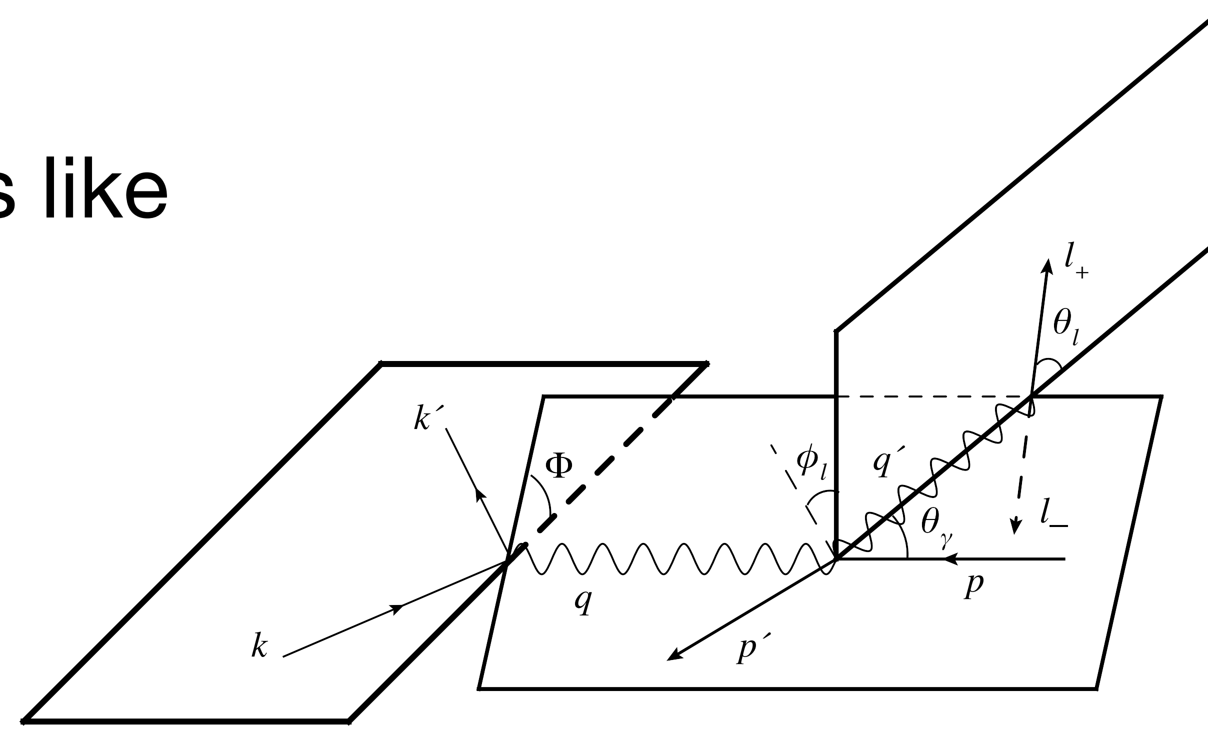


+ two more

- The BH amplitudes are not trivial. Pure BH must be calculated, as well as BH-Compton interference terms.
- Well under way.

VVCS—practical matters

- $e + p \to e + p + \ell^+ \ell^-$ looks like

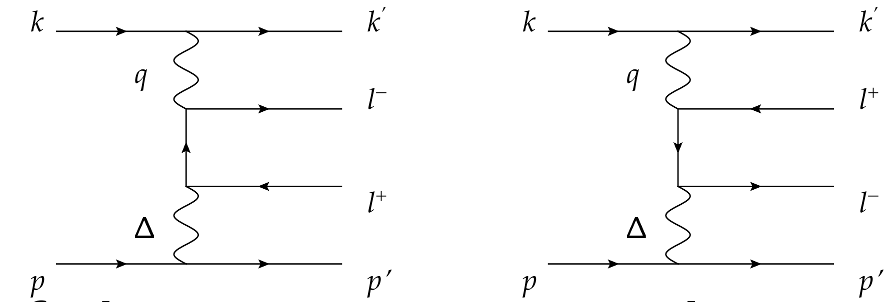


- Chosen 8 independent variables are

$$s = (k+p)^2, \quad Q^2 \equiv -(k-k')^2, \quad W^2 = (q+p)^2,$$
$$t = \Delta^2, \quad M^2_{\ell\ell} = q'^2, \quad \Phi, \quad \theta_\ell, \quad \varphi_\ell$$

- Observe only proton, so integrate over lepton angles θ_ℓ, φ_ℓ .

VVCS—practical matters



- Regarding amplitudes and squares of them, as example take spacelike BH

- Electron side squared has lepton tensor
$$L^{\mu\nu} = 4k^\mu k^\nu - Q^2 g^{\mu\nu}$$

- Hadron side has hadron tensor
$$H^{\alpha\beta} = 4\frac{G_E^2(t) + \tau G_M^2(t)}{1+\tau} p^\alpha p^\beta - Q^2\, G_M^2(t) g^{\alpha\beta}$$

- Remains part from photons connected to final leptons, which we will call $\mathscr{M}$

VVCS—practical matters

- One term is

$$\frac{1}{4\pi}\int_{-1}^{+1} d\cos\theta_l \int_0^{2\pi} d\phi_l \, g^{\mu\nu}\, g^{\alpha\beta}\, \mathcal{M}_{\mu\alpha,\nu\beta} = C_a^{(1)} + C_a^{(2)} \frac{1}{\beta\beta_Q} \log\left(\frac{1+\beta\beta_Q}{1-\beta\beta_Q}\right)$$

with

$$\beta^2 \equiv 1 - \frac{4m_l^2}{q'^2}, \qquad \beta_Q^2 \equiv 1 + \frac{4Q^2 t}{(q'^2 + Q^2 - t)^2}$$

- and further

$$C_a^{(1)} = 16\left\{-1 + \frac{4(Q^2 - 2m_l^2)(t + 2m_l^2)}{(q'^2 + Q^2 - t)^2\,(1 - \beta^2\beta_Q^2)}\right\}$$

$$C_a^{(2)} = \frac{16}{(q'^2 + Q^2 - t)^2}\left\{q'^4 + (Q^2 - t)^2 + 4m_l^2(q'^2 + Q^2 - t) - 8m_l^4\right\}$$

VVCS—practical matters

- Have other terms a well for the pure BH
- To go: interference terms and exploring good kinematic regions to isolate B_3 and/or b_{30} .

Most recent proton radius plot