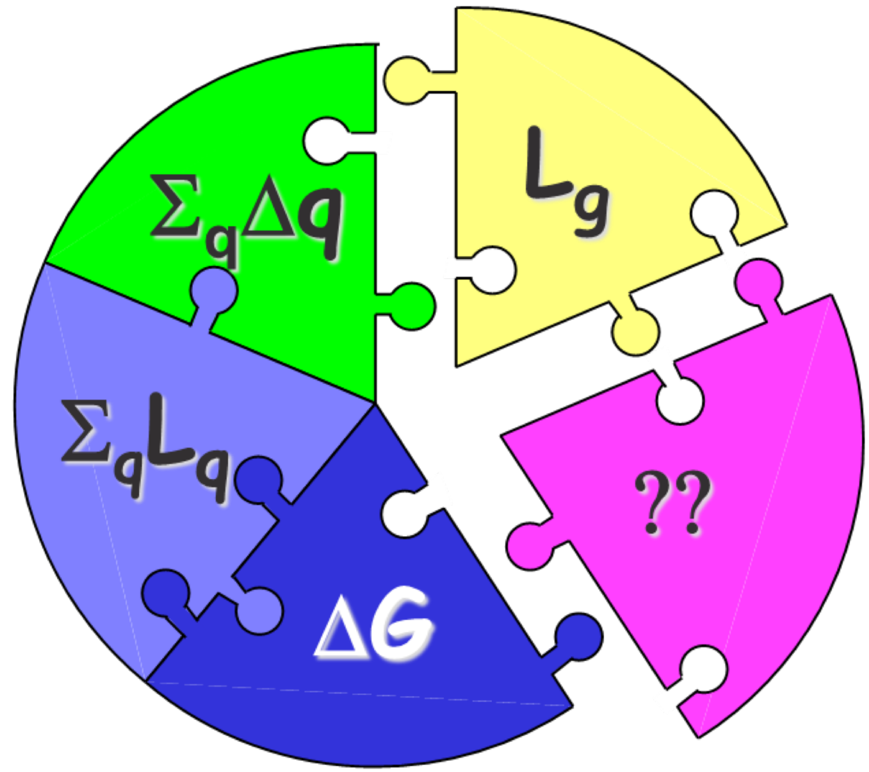
Before-Dinner Discussion

Yuri Kovchegov & Andrea Signori

Proton Spin

Proton Spin Puzzle

- Current experimental status of helicity PDFs
- OAM
- Lattice results
- Small-x
- What is he roadmap to solving the spin puzzle



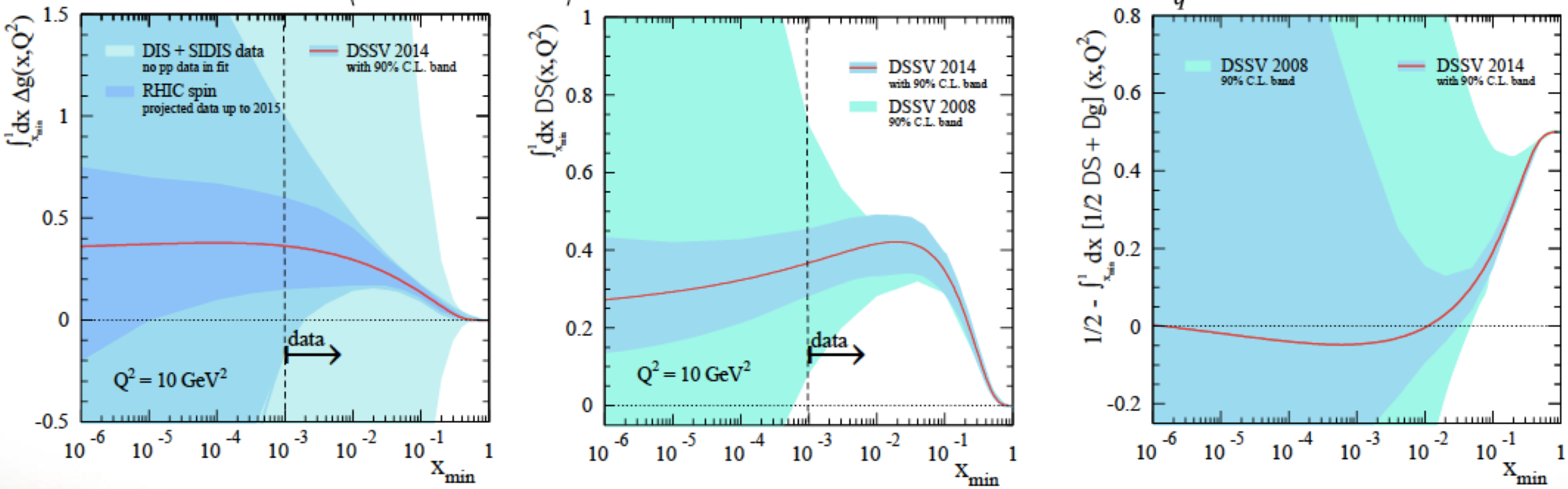
$$S = \frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L$$

What forms the Spin of the Proton

Spin is more than the number ½ ! It is the interplay between the intrinsic properties and interactions of quarks and gluons

What do we know:

$$\frac{1}{2}\hbar = \left\langle P,\frac{1}{2}\,|\,J^z_{QCD}\,|\,P,\frac{1}{2}\right\rangle = \overbrace{\frac{1}{2}\int_0^1 dx\,\Delta\Sigma(x,Q^2)}^{\text{total quark spin}} + \overbrace{\int_0^1 dx\,\Delta G(x,Q^2)}^{\text{gluon spin}} + \overbrace{\int_0^1 dx\left(\sum_q L_q^z + L_g^z\right)}^{\text{angular momentum}}$$



1/2 - Gluon -- Quarks = orbital angular momentum

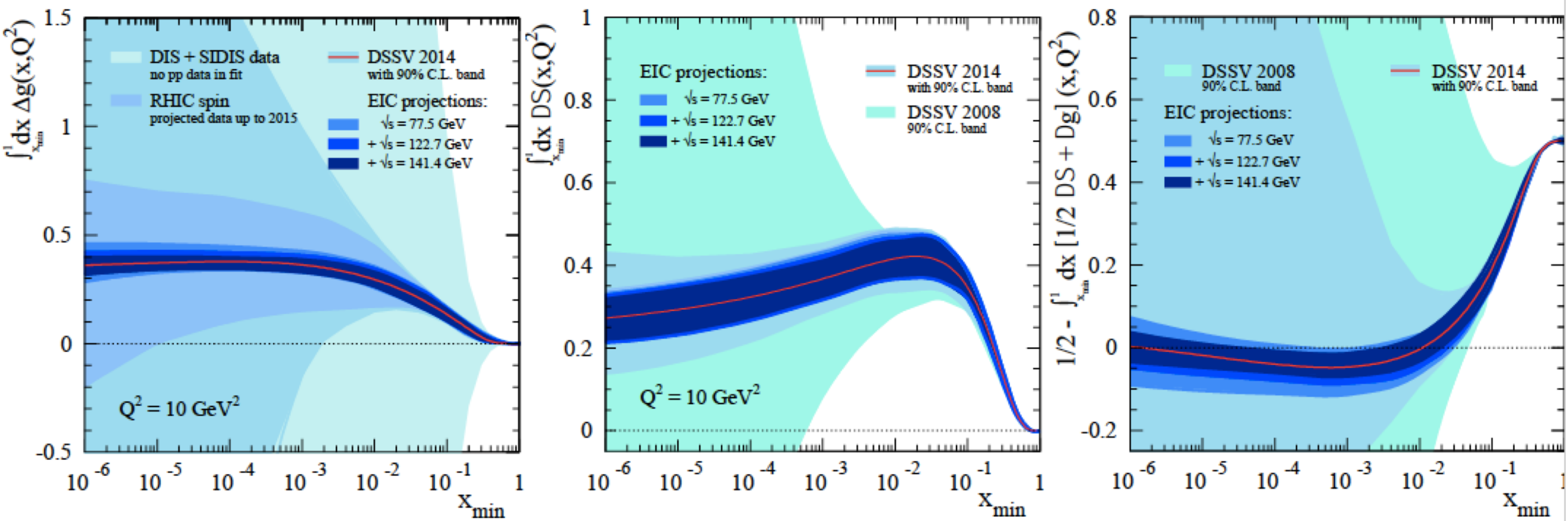
40% 30%

INT EIC program

Where does the Spin of the proton hide

"Helicity sum rule:"

$$\frac{1}{2}\hbar = \left\langle P,\frac{1}{2} \middle| J^z_{QCD} \middle| P,\frac{1}{2}\right\rangle = \overbrace{\sum_q \frac{1}{2} S^z_q}^{\text{total quark spin}} + \overbrace{S^z_g}^{\text{gluon spin}} + \overbrace{\sum_q L^z_q + L^z_g}^{\text{angular momentum}}$$



1/2 - Gluon - Quarks = orbital angular momentum

INT EIC program

`PDF' for OAM?

Take the staple-shaped Wilson line and define

$$L^{q,g} = \int dx \int d^2b_\perp d^2k_\perp (\vec{b}_\perp \times \vec{k}_\perp)_z W^{q,g}(x, \vec{b}_\perp, \vec{k}_\perp)$$

$$L^{q,g}(x) = \int d^2b_\perp d^2k_\perp (\vec{b}_\perp \times \vec{k}_\perp)_z W^{q,g}(x, \vec{b}_\perp, \vec{k}_\perp)$$

Agrees with Harindranath, Kundu (1998); Hagler, Schafer (1998) in the light-cone gauge.

Warning: This is NOT the usual (twist-two) PDF.

Y. Hatta, INT EIC program, 2018

Observables for OAM

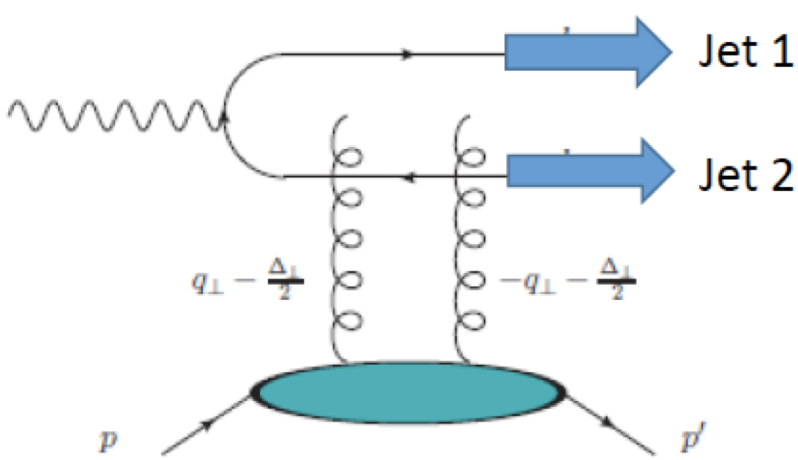
Y. Hatta, INT EIC program, 2018

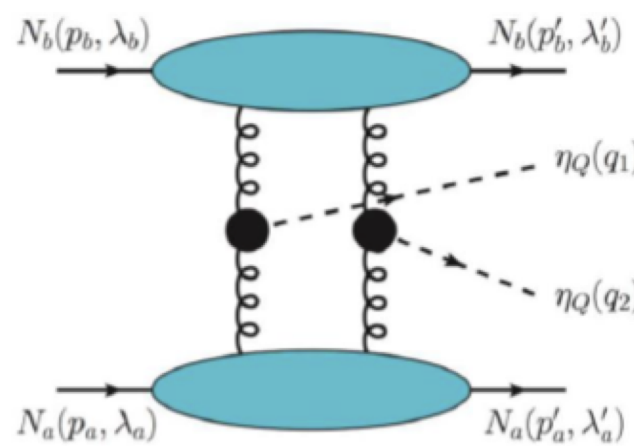
Essentially the measurement of Wigner/GTMD .
Tag two hadrons (jets) in the final state, together with the recoiling proton

Relative momentum between the two hadrons

$$W(x, k_\perp, \Delta_\perp)$$

Recoiling proton momentum

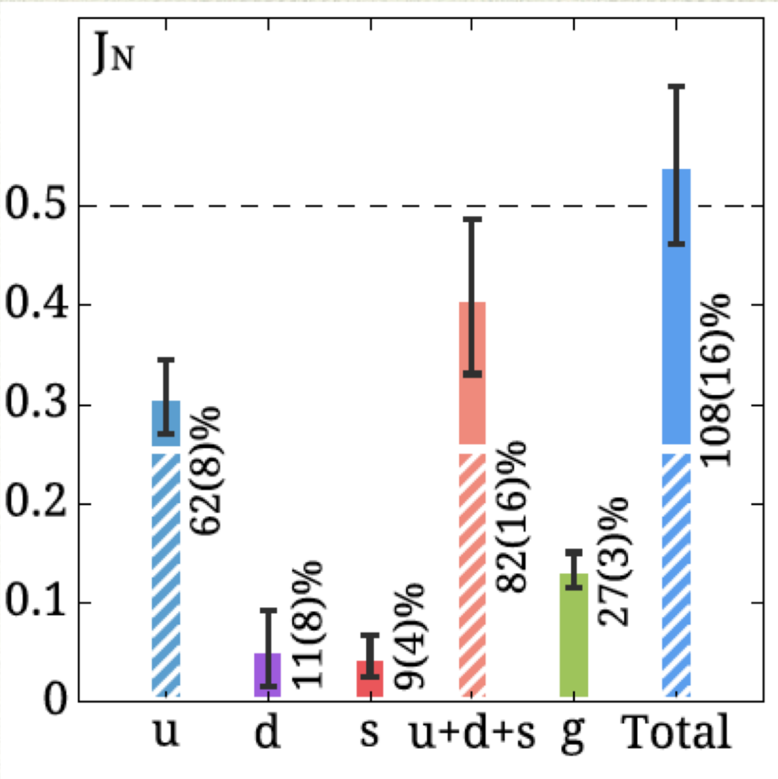




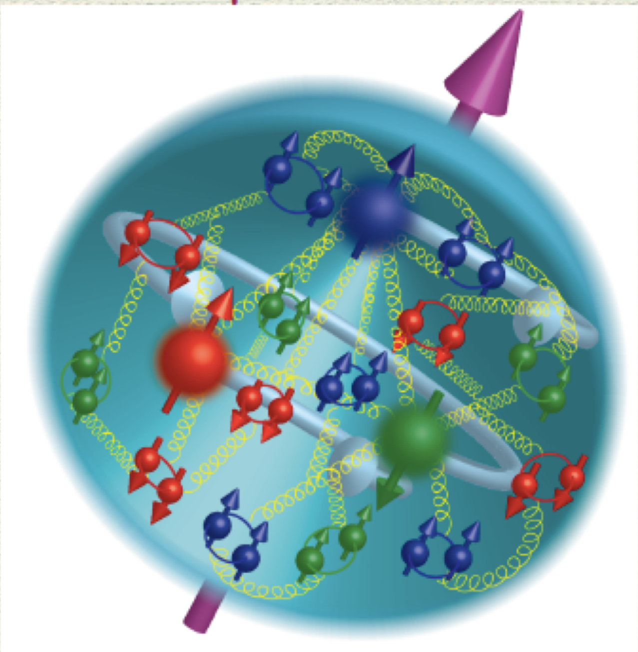
Ji, Yuan, Zhao (2017); YH, Nakagawa, Xiao, Yuan, Zhao (2017),
Bhattacharya, Metz, Zhou (2017); Bhattacharya, Metz, Ojha, Tsai, Zhou (2018)

The proton spin from LQCD

[C. Alexandrou et al., Phys. Rev. Lett. 119, 142002 (2017), [arXiv:1706.02973]]



Better understanding of the spin distribution



Designed by Z.-E. Meziani

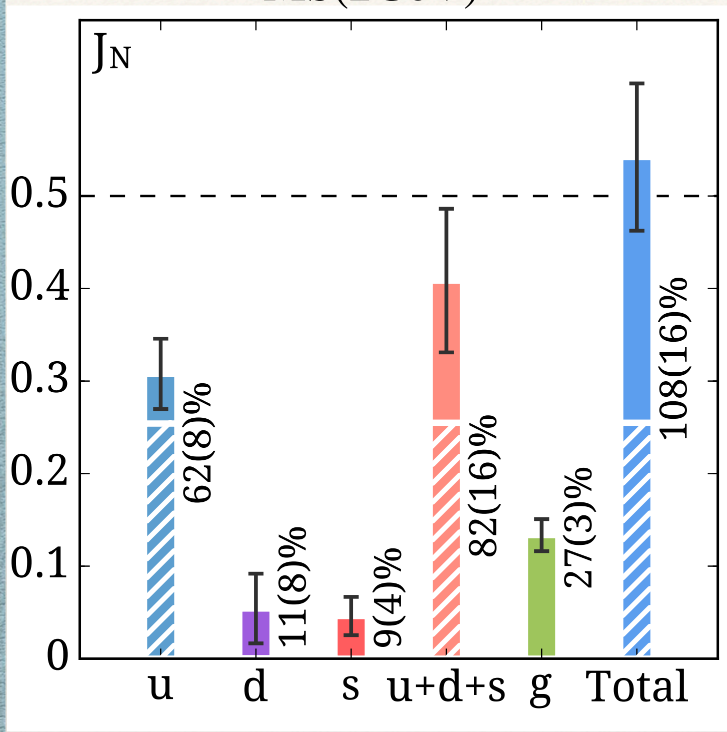
Striped segments: valence quark contributions (connected)
Solid segments: sea quark & gluon contributions (disconnected)

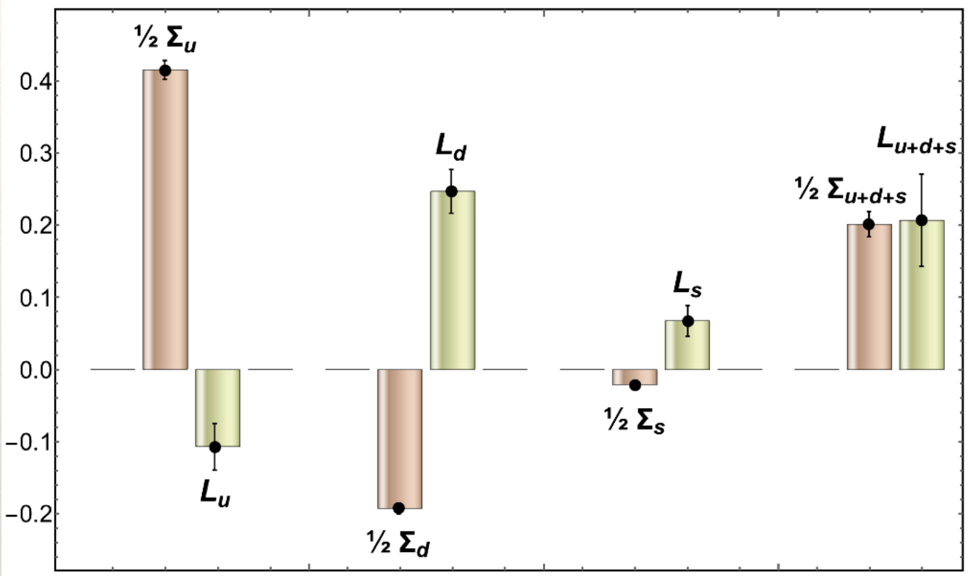
- Satisfaction of spin and momentum sum rule is not forced

Spin decomposition

[C. Alexandrou et al., Phys. Rev. Lett. 119, 142002 (2017), [arXiv:1706.02973]]

$\overline{\text{MS}}$(2GeV)





Quark orbital angular momentum extracted indirectly ($L_q = J_q - \Sigma_q$)

- ★ Striped segments: connected contributions (connected)
- ★ Solid segments: disconnected contributions (quark & gluon)

Satisfaction of spin and momentum sum rule is not forced

Helicity PDFs at Small-x

- Theoretical calculations by Bartels, Ermolaev and Ryskin (BER, 1996) and by YK, Pitonyak and Sievert (2015-17, KPS).
- KPS results (large-N_C):

$$\Delta q(x, Q^2) \sim \left(\frac{1}{x}\right)^{\alpha_h^q} \quad \text{with} \quad \alpha_h^q = \frac{4}{\sqrt{3}} \sqrt{\frac{\alpha_s \, N_c}{2\pi}} \approx 2.31 \sqrt{\frac{\alpha_s \, N_c}{2\pi}}$$

$$\Delta G(x, Q^2) \sim \left(\frac{1}{x}\right)^{\alpha_h^G} \quad \text{with} \quad \alpha_h^G = \frac{13}{4\sqrt{3}} \sqrt{\frac{\alpha_s \, N_c}{2\pi}} \approx 1.88 \sqrt{\frac{\alpha_s \, N_c}{2\pi}}$$

- BER results at large-N_C (for both quark and gluon helicity distributions):

$$\alpha_h^{BER} = \sqrt{\frac{17+\sqrt{97}}{2}} \sqrt{\frac{\alpha_s N_c}{2\pi}} \approx 3.66 \sqrt{\frac{\alpha_s N_c}{2\pi}}$$

For finite N_C and N_f=4 BER have 3.66 → 3.45.

Helicity PDFs at Small-x

$$PDF(x) \sim \left(\frac{1}{x}\right)^{\alpha}$$

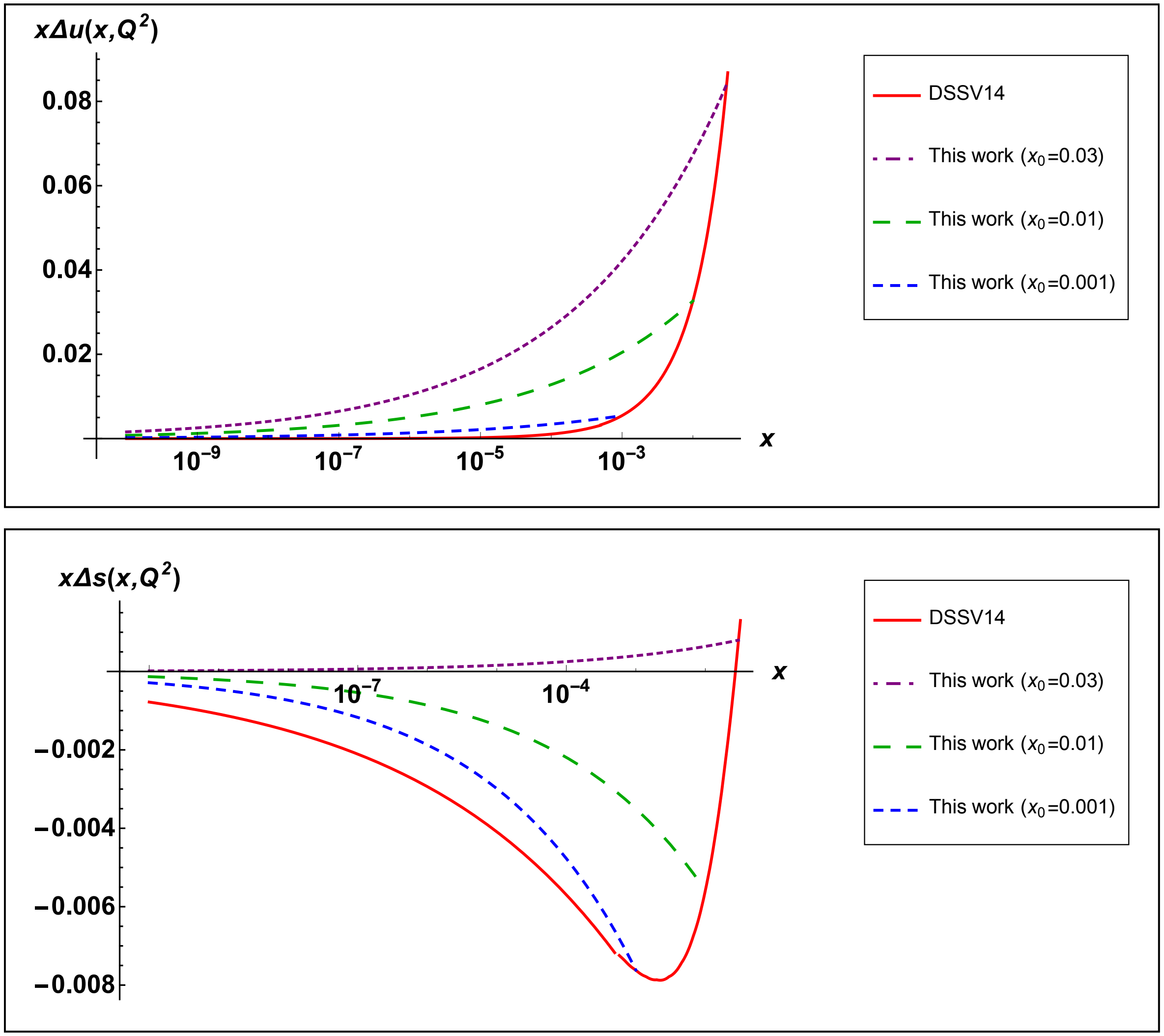
- The summary of the existing powers of x is

Observable	Evolution	Intercept	$Q^2 = 3$ GeV^2 $\alpha_s = 0.343$	$Q^2 = 10$ GeV^2 $\alpha_s = 0.249$	$Q^2 = 87$ GeV^2 $\alpha_s = 0.18$
Unpolarized flavor singlet structure function F_2	LO BFKL Pomeron	$1 + \frac{\alpha_s N_c}{\pi} 4 \ln 2$	1.908	1.659	1.477
Unpolarized flavor non-singlet structure function F_2	Reggeon	$\sqrt{\frac{2\,\alpha_s C_F}{\pi}}$	0.540	0.460	0.391
Flavor singlet structure function g_1^S	us (Pure Glue)	$2.31 \sqrt{\frac{\alpha_s N_c}{2\pi}}$	0.936	0.797	0.678
	BER (Pure Glue)	$3.66 \sqrt{\frac{\alpha_s N_c}{2\pi}}$	1.481	1.262	1.073
	BER ($N_f = 4$)	$3.45 \sqrt{\frac{\alpha_s N_c}{2\pi}}$	1.400	1.190	1.011
Flavor non-singlet structure function g_1^{NS}	BER and us (large-N_c)	[illegible]	0.572	0.488	0.415

- Similar disagreement exists for OAM distributions at small x.
- However, even the smaller KPS powers lead to potentially important contributions to the proton spin coming from small x. (next)

Impact on proton spin

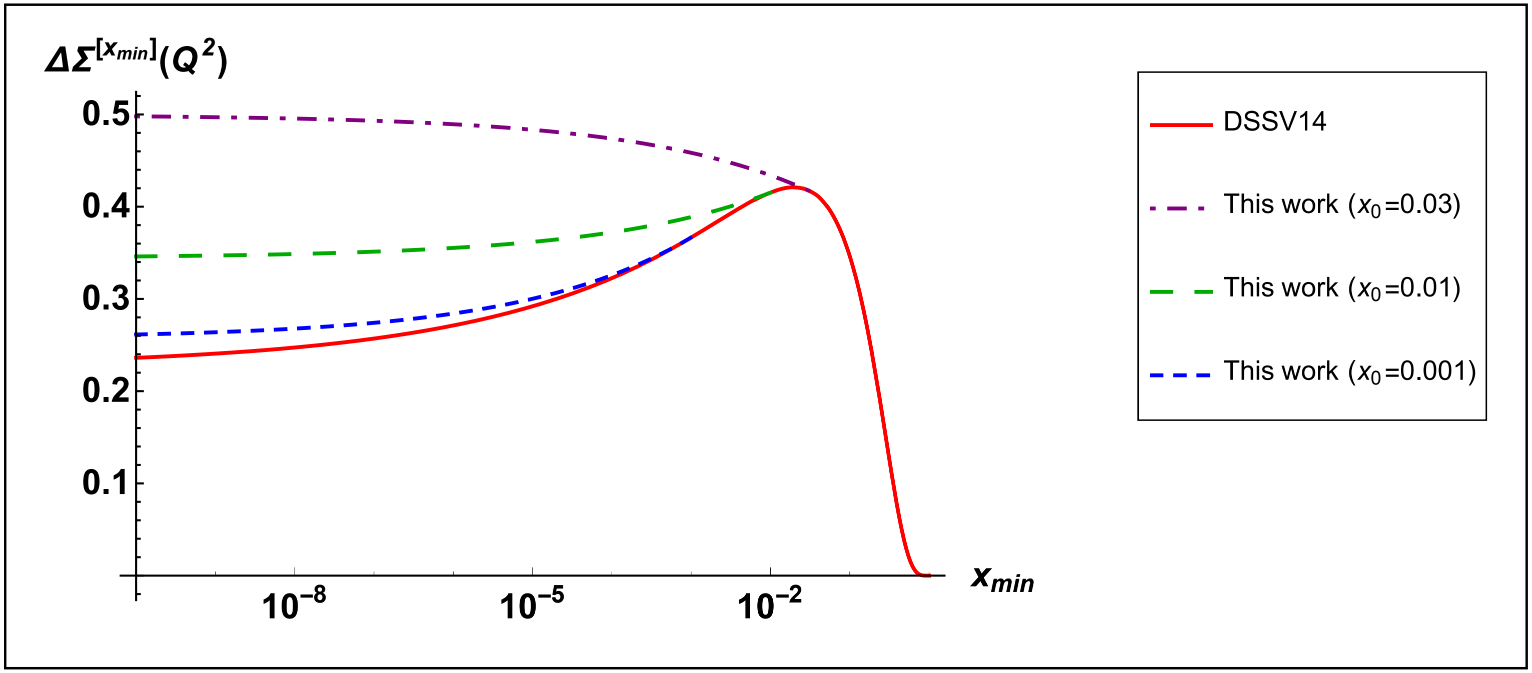
- We have attached a $\Delta\tilde{\Sigma}(x,Q^2) = N\,x^{-\alpha_h}$ curve to the existing hPDF's fits at some ad hoc small value of x labeled x_0 :



"ballpark" phenomenology

Impact on proton spin

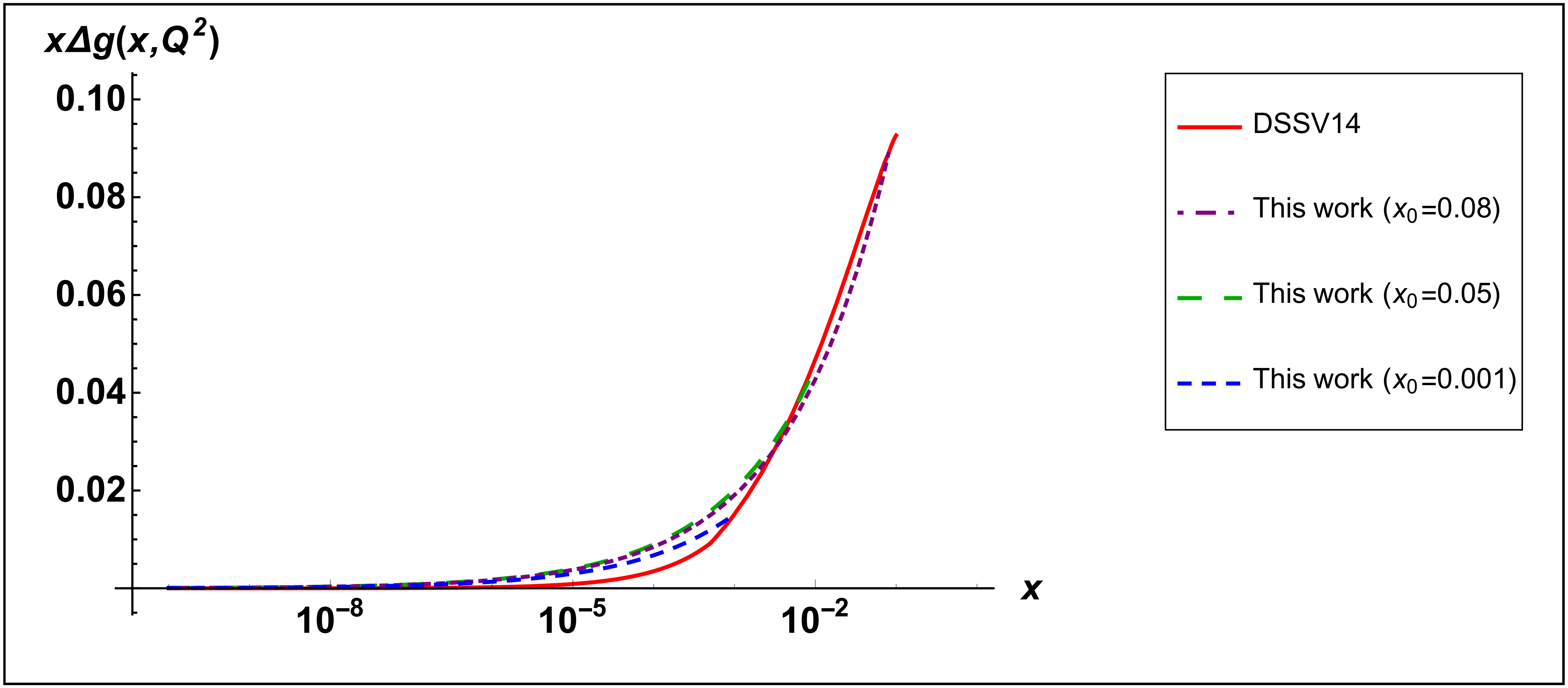
- Defining $\Delta\Sigma^{[x_{min}]}(Q^2) \equiv \int_{x_{min}}^{1} dx\, \Delta\Sigma(x, Q^2)$ we plot it for x_0=0.03, 0.01, 0.001:



- We observe a moderate to significant enhancement of quark spin.
- More detailed phenomenology is needed in the future.

Impact of our ΔG on the proton spin

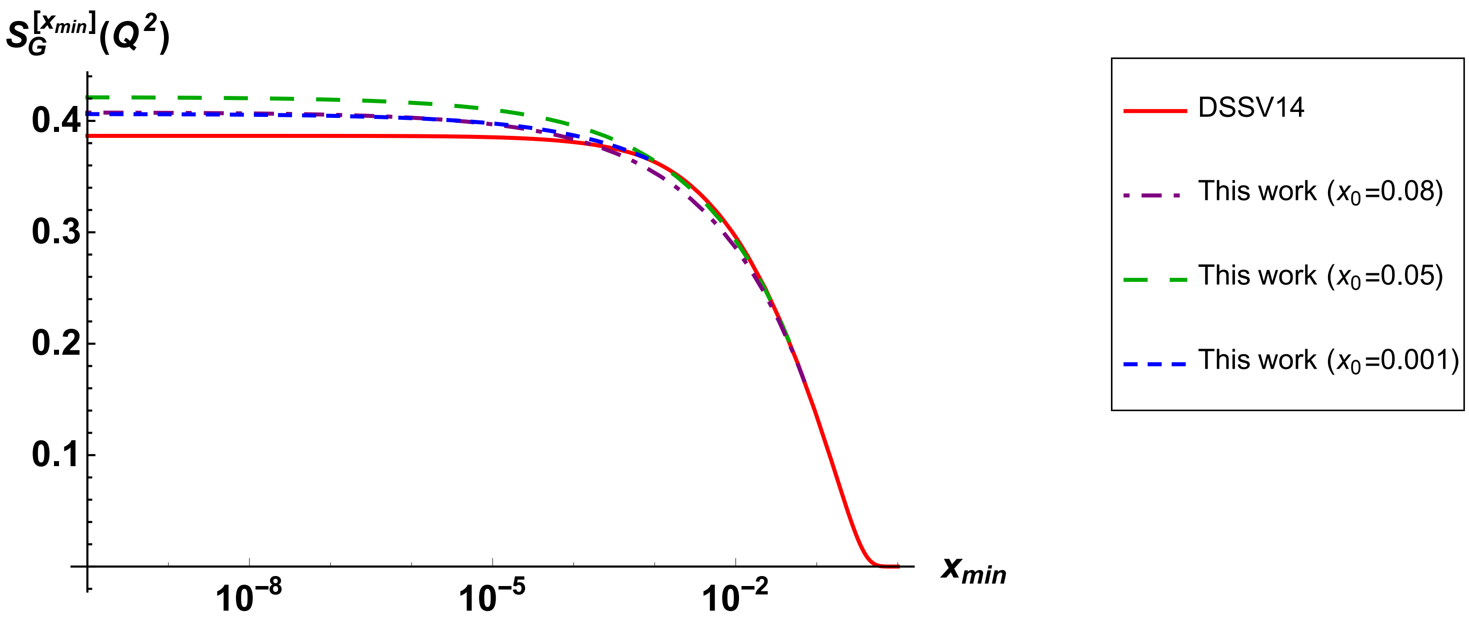
- We have attached a $\Delta\tilde{G}(x,Q^2) = N\,x^{-\alpha_h^G}$ curve to the existing hPDF's fits at some ad hoc small value of x labeled x_0 :



"ballpark"
phenomenology

Impact of our ΔG on the proton spin

- Defining $S_G^{[x_{min}]}(Q^2) \equiv \int_{x_{min}}^{1} dx\, \Delta G(x, Q^2)$ we plot it for x_0=0.08, 0.05, 0.001:



- We observe a moderate enhancement of gluon spin.
- More detailed phenomenology is needed in the future.