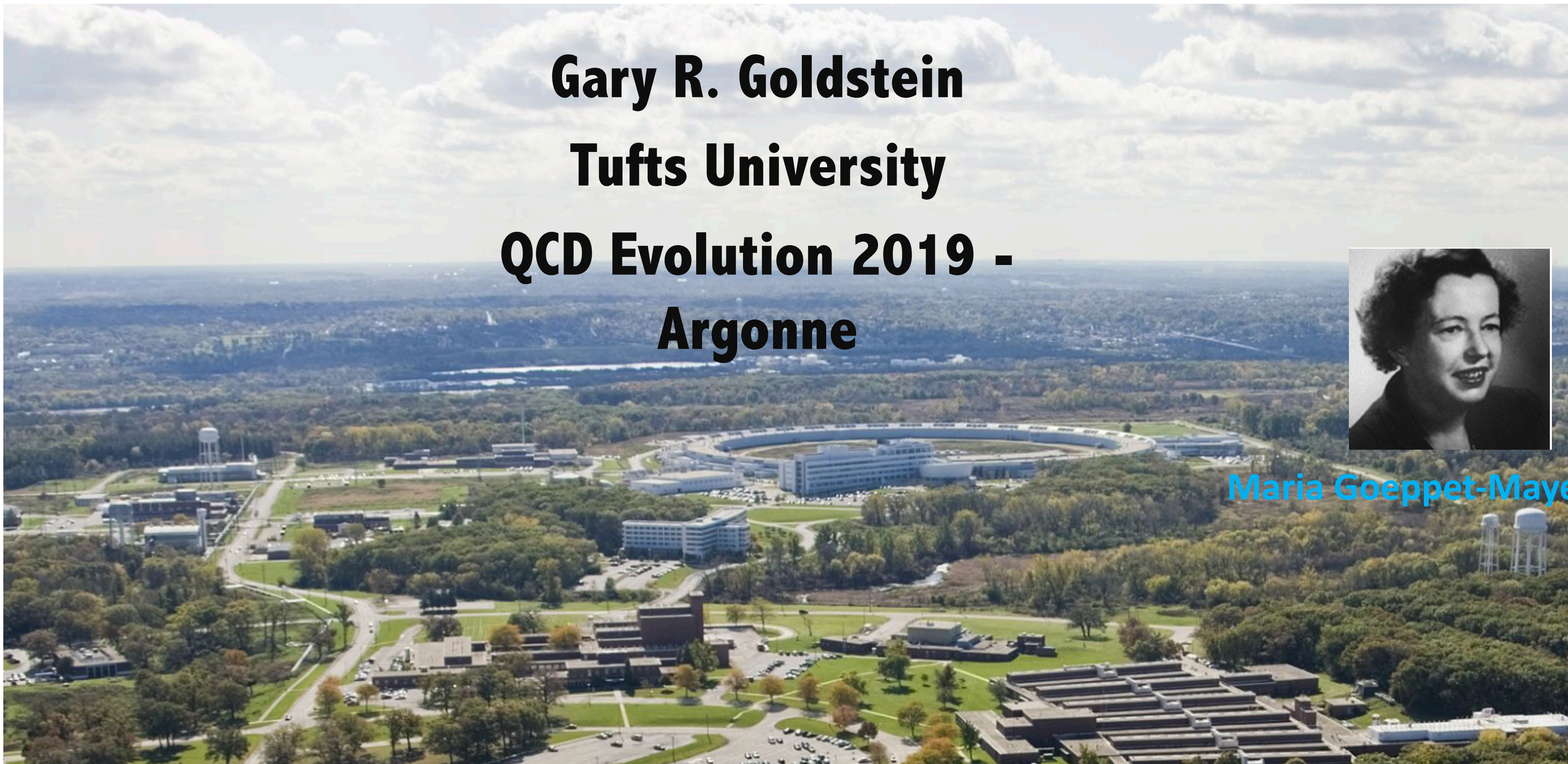
Measuring Polarized Gluon Distributions by Top Pair Production Spin Correlations



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Abstract

Top-antitop pairs are produced prolifically in p+p collisions at the LHC, primarily by gluon fusion. At intermediate values of momentum fraction x for each gluon in g+g to t+tbar, the spin dependences of gluon distributions leave imprints on the momentum and spin correlations of the top pairs. These correlations are distinguishable from the quark-antiquark annihilation mechanism. Decays of such spin entangled top pairs produce a variety of correlations among pairs of the 3-momenta of the decay products - particles and jets. Some different angular correlations will be presented and related to measurable distributions of pairs of jets and/or leptons. Models for spin dependent gluon transverse momentum distributions and generalized transverse momentum distributions will be used to simulate top pair decay product spin correlations, illustrating how to measure the gluon or quark polarizations in the colliding protons.

Gluon Distributions

Transversity

Top quarks

Gluon Transversity → Top Pair Spin Correlations

OUTLINE – GLUONS to TOP quarks to gluons

- GPD's & TMD's in Models– e.g. Reggeized spectator "flexible parameterization"
- Electroproduction
- Gluon GPDs Polarized Gluons?
  Transversity NOW Seen!
- t+t-bar production & decay to measure Gluon polarization in p+p @ LHC. Inclusive → TMDs
- Top spin correlations & Observable quantities

Gluon GPDs

$$\frac{1}{\bar{P}^+}\int \frac{dz^-}{2\pi} e^{ix\bar{P}^+ z^-} \langle P', \Lambda' | G^{+i}(-\frac{1}{2}z) G^{+i}(\frac{1}{2}z) | P, \Lambda \rangle \Big|_{z^+=0, \vec{z}_T=0} =$$
$$\frac{1}{2\bar{P}^+} \bar{U}(P', \Lambda') [H^g(x,\xi,t)\gamma^+ + E^g(x,\xi,t)\frac{i\sigma^{+\alpha}(-\Delta_\alpha)}{2M}] U(P, \Lambda)$$

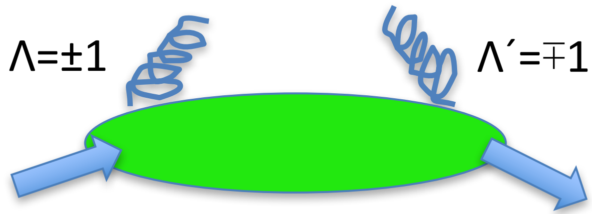
Even t-channel parity & Gluon helicity conserving

$$\frac{-i}{\bar{P}^+}\int \frac{dz^-}{2\pi} e^{ix\bar{P}^+ z^-} \langle P', \Lambda' | G^{+i}(-\frac{1}{2}z) \tilde{G}^{+i}(\frac{1}{2}z) | P, \Lambda \rangle \Big|_{z^+=0, \vec{z}_T=0} =$$
$$\frac{1}{2\bar{P}^+} \bar{U}(P', \Lambda') [\tilde{H}^g(x,\xi,t)\gamma^+\gamma_5 + \tilde{E}^g(x,\xi,t)\frac{\gamma_5(-\Delta^+)}{2M}] U(P, \Lambda)$$

Odd t-channel parity & Gluon helicity conserving

Must have 4 more Gluon helicity NONconserving

Extension to Gluon “Transversity”



$$
\begin{aligned}
&-\frac{1}{P^+}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-}\langle p',\lambda'|\,\mathbf{S}F^{+i}(-\tfrac{1}{2}z)\,F^{+j}(\tfrac{1}{2}z)\,|p,\lambda\rangle\Big|_{z^+=0,\,\mathbf{z}_T=0} \\
&= \mathbf{S}\,\frac{1}{2P^+}\,\frac{P^+\Delta^j-\Delta^+P^j}{2mP^+} \\
&\times\ \bar{u}(p',\lambda')\left[H_T^g\, i\sigma^{+i} + \tilde{H}_T^g\,\frac{P^+\Delta^i-\Delta^+P^i}{m^2}\right. \\
&\qquad \left. + E_T^g\,\frac{\gamma^+\Delta^i-\Delta^+\gamma^i}{2m} + \tilde{E}_T^g\,\frac{\gamma^+P^i-P^+\gamma^i}{m}\right]u(p,\lambda).
\end{aligned}
$$

4 GPDs: see M.Diehl, EPJC19, 485 (2001)

4 Gluon helicity NONconserving Double flip

Gluon "transversity" Double helicity flip *does not mix* with quark distributions

Transversity for on-shell gluons or photons : no |0> helicity

$$|+1)_{trans} = \{|+1> + |-1>\}/2 = |-1)_{trans}$$

$$|0\ )_{trans} = \{|+1> - |-1>\}/\sqrt{2}$$

$$\text{helicity } |\pm 1>= \{-/+\hat{x} - i\hat{y}\}/\sqrt{2}$$

$$\hat{x} = -|0)_{trans} = P_{parallel}$$

Linear polarization in the plane

$$\hat{y} = i\sqrt{2}\,|+1)_{trans} = P_{normal}$$

Linear polarization normal to the plane

GG&M.J.Moravcsik, Ann.Phys.195,213(1989).

Using Reggeized Spectators Model
Many other models & recently

How to Measure? What Processes? Long standing question.

M. Diehl, T. Gousset, B. Pire, and J. P. Ralston, Phys. Lett. B411, 193 (1997).
X. Ji and J. Osborne, UMD PP#98-074, hep-ph/9801260.
P. Kroll, M. Schurmann, and P. A. M. Guichon, Nucl. Phys. A598, 435 (1996).
P. Hoodbhoy & X. Ji, PRD58, 054006 (1998).

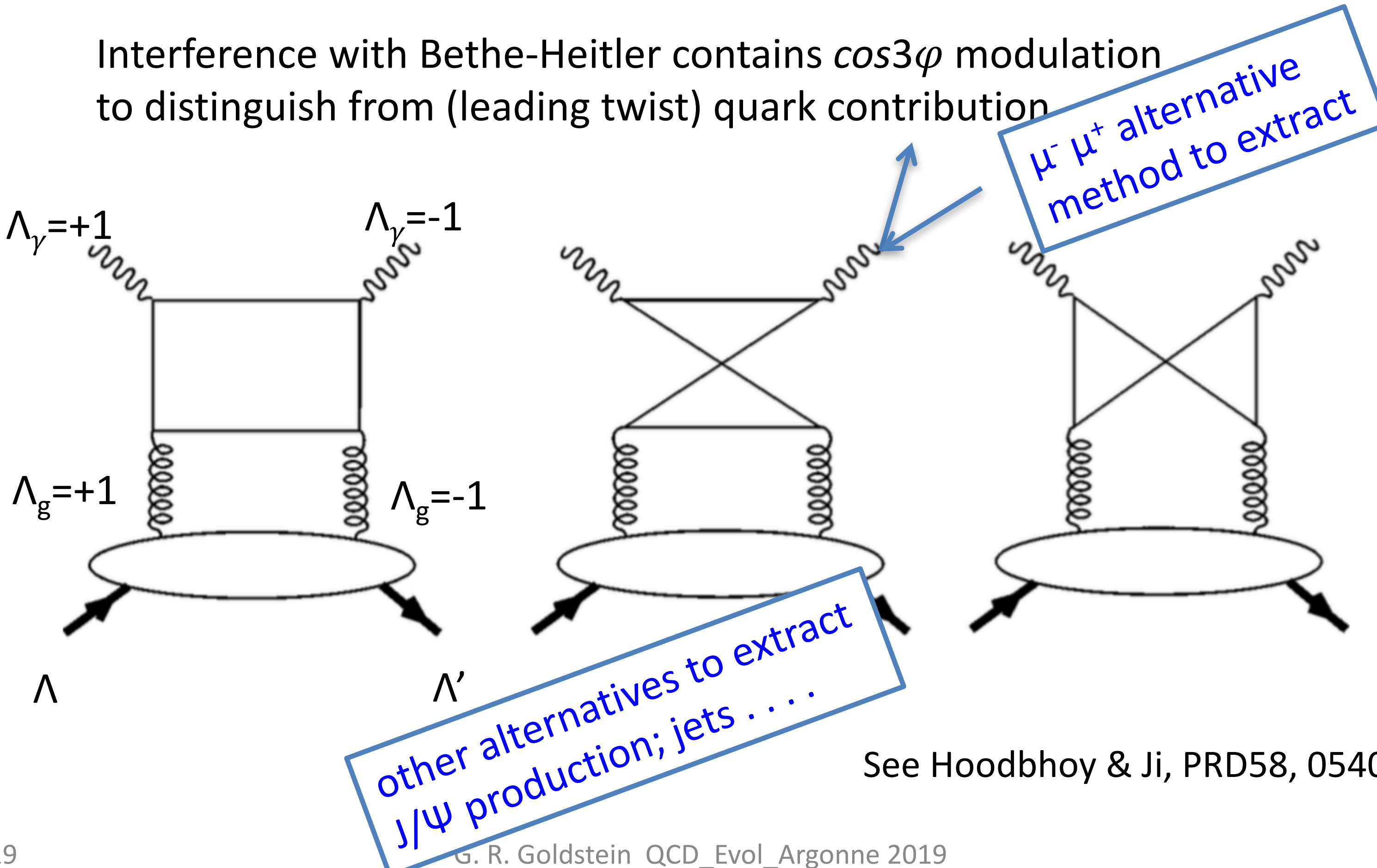
TMDs

P. Mulders, J. Rodrigues,PRD 63, 094021 (2001).
D. Boer, Few-Body Syst. (2017); C. Pisano, et al., JHEP 10, 024 (2013);
D. Boer, et al., PRL 106, 132001 (2011); . . . .

Helicity flip $A_{\Lambda',-1;\Lambda,+1}$ contributes to DVCS ~ α_s

$$M_{\Lambda',\Lambda'\gamma=-1;\Lambda,\Lambda\gamma=+1} = -\frac{\alpha s}{2\pi}\sum_q e_q^2 \int_{-1}^{+1} dx \frac{A_{\Lambda',\Lambda' g=-1;\Lambda,\Lambda g=+1}(x,\xi,t)}{(\xi - x - i\varepsilon)(\xi + x - i\varepsilon)} \; C'(x,\xi,Q^2)$$

Interference with Bethe-Heitler contains *cos*3φ modulation
to distinguish from (leading twist) quark contribution



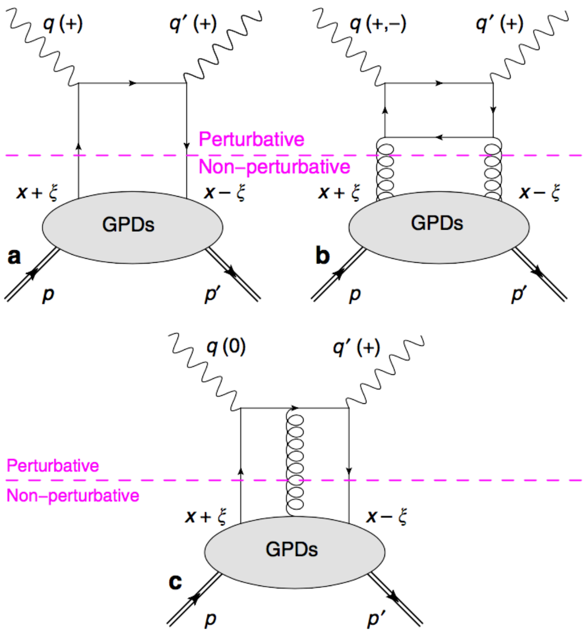
See Hoodbhoy & Ji, PRD58, 054006 (1998)

Gluon GPDs from DVCS – Hall A

Polarized & unpolarized beam measurements

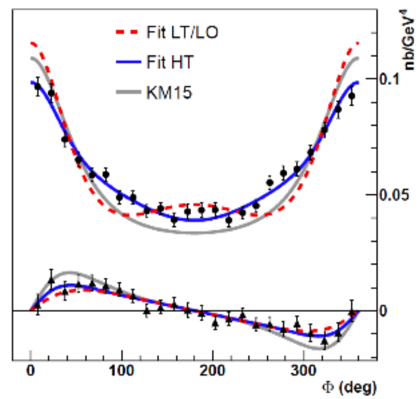
Evidence of gluon transversity

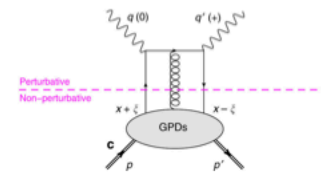
Fitting ϕ distribution requires F_{++} and both F_{+-} gluon transversity and F_{0+} higher twist



$$\frac{d^4\sigma(h)}{dQ^2 dx_B dt d\phi} = \frac{d^2\sigma_0}{dQ^2 dx_B} \times \left[\left|\mathcal{T}^{BH}\right|^2 + \left|\mathcal{T}^{DVCS}(h)\right|^2 - \mathcal{I}(h) \right]$$

A glimpse of gluons through deeply virtual compton scattering on the proton, published in *Nature Communications* 8, 1408 (2017).
doi:10.1038/s41467-017-01819-3





Analysis of 6 GeV Hall A DVCS data on the proton.

Jefferson Lab

See Latifa Elouadrhiri talk at QCD Evolution 2018

LHC - many opportunities for studying gluons
p+p unpolarized → jets, hadrons, leptons
Interactions via g+g→ Q+Qbar +X
gluon TMDs in some kinematics
Extension to Gluon "Transversity"

c.f. TMDs $h_1^{\perp g}(x, p_T^2)$ Mulders & Rodrigues (2001), Gluon Boer-Mulders function
see D. Boer, Frascati talk (Nov.2016) & many references for measurements at EIC, RHIC, LHC

Heuristic for spin TMDs

- Boer-Mulders for Unpolarized nucleon with "Transversly" polarized quark

N/q	U	L	T
U	$\mathbf{f_1}$		$h_1^{\perp}$
L		$\mathbf{g_1}$	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	$\mathbf{h_1}$ $h_{1T}^{\perp}$

Gluon TMDs

$$\begin{pmatrix} G+\Delta G_L & \frac{|\boldsymbol{k}_T|e^{-i\phi}}{M}[\Delta G_T-iG_T] & -e^{-2i\phi}[H^{\perp(1)}+i\Delta H_L^{\perp(1)}] & -i\frac{|\boldsymbol{k}_T|e^{-3i\phi}}{M}\Delta H_T^{\perp(1)} \\ \frac{|\boldsymbol{k}_T|e^{i\phi}}{M}[\Delta G_T+iG_T] & G-\Delta G_L & -i\frac{|\boldsymbol{k}_T|e^{-i\phi}}{M}\Delta H_T & -e^{-2i\phi}[H^{\perp(1)}-i\Delta H_L^{\perp(1)}] \\ -e^{2i\phi}[H^{\perp(1)}-i\Delta H_L^{\perp(1)}] & i\frac{|\boldsymbol{k}_T|e^{i\phi}}{M}\Delta H_T & G-\Delta G_L & -\frac{|\boldsymbol{k}_T|e^{-i\phi}}{M}[\Delta G_T+iG_T] \\ i\frac{|\boldsymbol{k}_T|e^{3i\phi}}{M}\Delta H_T^{\perp(1)} & -e^{2i\phi}[H^{\perp(1)}+i\Delta H_L^{\perp(1)}] & -\frac{|\boldsymbol{k}_T|e^{i\phi}}{M}[\Delta G_T-iG_T] & G+\Delta G_L \end{pmatrix}$$

Mulders & Rodrigues, PRD63, 94021 (2001)

The matrix representation is also convenient to find the physical meaning of the distributions. Well known is *G* which measures the number of gluons with momentum (*x*,***k****T*) in a hadron. The functions *GL* (*GT*) represents the difference of the numbers of gluons with opposite circular polarizations in a longitudinally transversely polarized nucleon. The off- diagonal function *H* also is a difference of densities, but in this case of linearly polarized gluons in an unpolarized hadron. Using the circular polarizations, *H* flips the polarization.

Corresponding GTMDs generalize GPDs & TMDs.
Unintegrated models connect all
Other notation $\Delta H_T^{\perp(1)} = \frac{k_T^2}{2M^2}\Delta H_T^{\perp}$, or $h_1^{g\perp}$, the gluon Boer-Mulders function.
Unpolarized Nucleon → polarized gluon | factorization & evolution

Gluon TMDs

TMD Color gauge invariance

Small x gluons, Kharzeev, Kovchegov, Tuchin, saturation,

2 gluon distributions: WW vs. DP,

Saturation issues: McLerran-Venugopalan model, ColorGlassCondensate, . . ?

See Mulders, et al.: small x DP is pure gauge link

$$\Gamma^{\mu\nu\,[\mathcal{U},\mathcal{U}']}(x,\boldsymbol{k}_T) \equiv \int \frac{d(\xi\cdot P)\,d^2\xi_T}{(P\cdot n)^2(2\pi)^3} e^{i(xP+k_T)\cdot\xi} \left\langle P \left| \mathrm{Tr}_c \left[F^{n\nu}(0)\,\mathcal{U}_{[0,\xi]}\,F^{n\mu}(\xi)\,\mathcal{U}'_{[\xi,0]} \right] \right| P \right\rangle \Big|_{\xi\cdot n=0} .$$

Gauge link

$\xi=[0^+, \xi^-, \boldsymbol{\xi}_T]$

$$\mathcal{U}_{\mathcal{C}}[0,\xi] = \mathcal{P}\exp\left(-ig \int_{\mathcal{C}[0,\xi]} ds_\mu\, A^\mu(s) \right)$$

Can be forward light front pointing link $+\infty$ FSI. Weizsacker-Williams

Or mixed light front pointing link $-\infty$ ISI Dipole

For *U* and *U'* *have* [+ +] or [+ -] (& parity opposites)

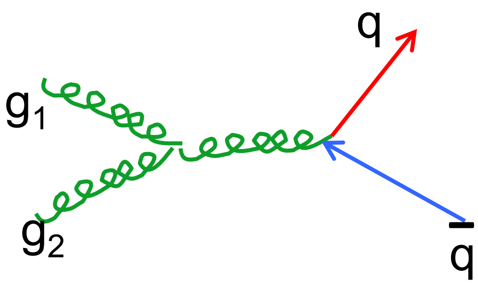
Consider TMD & GPD vs. data at intermediate x

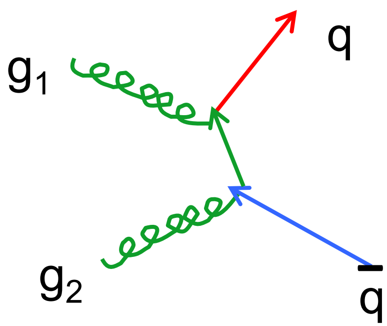
The Model for valence quarks- Reggeized Diquarks

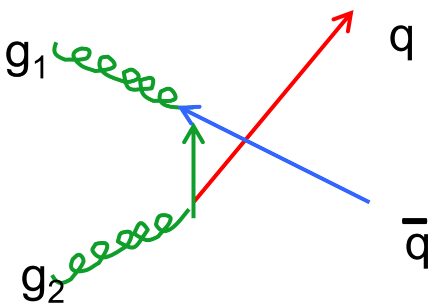
The Model – first for Chiral Even – then Odd
Reggeized Diquark Spectator
Diquark: Color anti-3, scalar & axial vector

Gluon GPDs, TMDs, GTMDs

From p+p to gluon TMDs to quark pairs







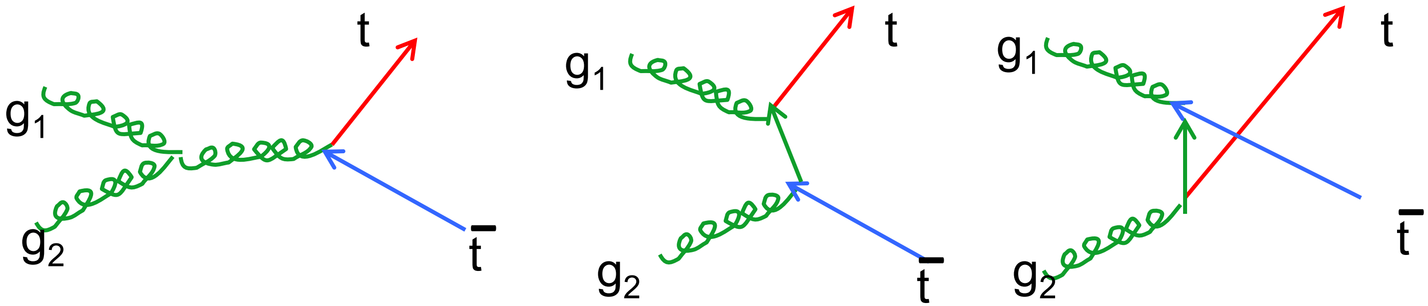
Form quarkonia & different possibilities for *gg*

Complications from f.s.i. & jets – hadronization

See Boer, Brodsky, Pisano, et al., . . .

Factorization and evolution

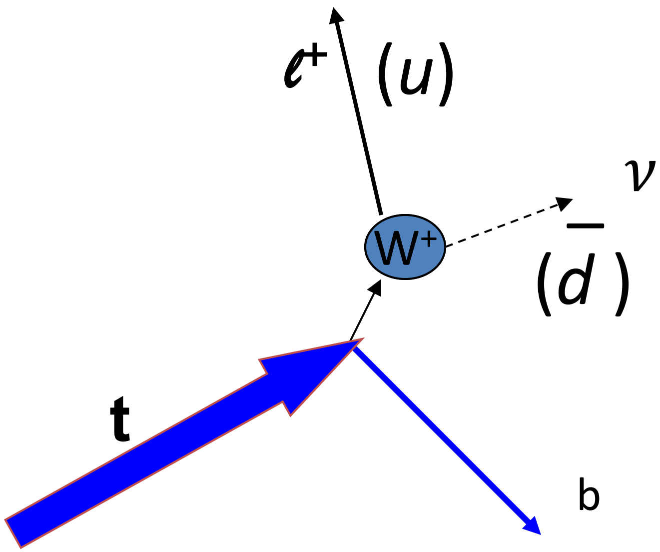
For Gluon fusion top production at LHC



- g_1 & g_2 carry helicity $\Lambda_1\ \Lambda_2 = \pm 1$ & color 1, 8. . . & C=+ or -
- *t* & *t*-bar carry helicity $\lambda_t\ ,\ \lambda_{tbar} = \pm\ 1/2$ & color 1 or 8
- ***t & tbar decay before hadronizing => no toponia & large scale***

How is top polarization determined?

Its decay is good analyzer for transverse polarization.



$$U_{\lambda_t,\lambda_t'} = \sum_{\lambda_b} B^*_{\lambda_b,\lambda_t'} B_{\lambda_b,\lambda_t'}$$

$$\propto (I + \vec{p}_{\bar{l}} \cdot \vec{\sigma}_t / p_{\bar{l}})_{\lambda_t,\lambda_t'} (p_b \cdot p_\nu)$$

Calculated in top rest frame

OR

$$U = (p_t - m_t S_t) \cdot p_{\bar{l}} \, (p_b \cdot p_\nu)$$

$$S_t = \left[\frac{\vec{p} \cdot \vec{P}_t}{m_t}, \vec{P}_t + \frac{(\vec{p} \cdot \vec{P}_t)\vec{P}_t}{m_t(E_t + m_t)} \right]$$

Covariant form in any frame

P_t = strength of top polarization

Dalitz & GRG, PLB287,225(1992); PRD45, 1531(1992)

$(I + \vec{p}_{\bar{l}} \cdot \vec{\sigma}_t / p_{\bar{l}})$ lepton or u-quark moves parallel to transverse polarization

What is known about polarized top production?

Top Single Spin Asymmetry and Double Spin Correlations – Measurements

ATLAS PRD93, 012002 (2016) & ref. PRL114, 142001 (2015)

** SSA: B_1 or A_P =-0.035 +\- 0.040. (syst & stat)
*** Double: $C_{helicity}$ =0.315 +\- 0.07 vs. NLO QCD =0.31
(Bernreuther, et al., PRL 87,242002 (2001) QCD corrections but unpolarized gluons)

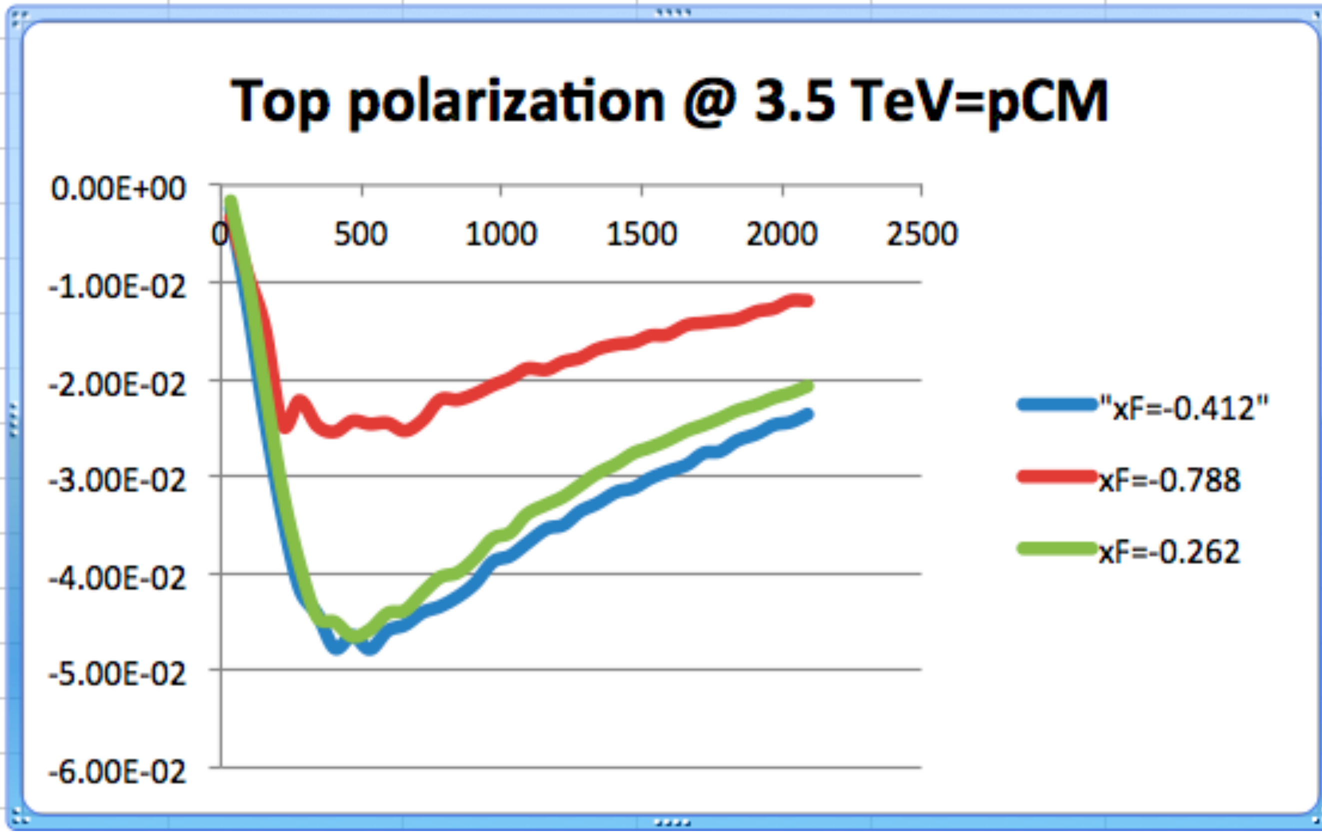
CMS PRL112, 182001 (2014): Different kinematics & selection criteria

** SSA: A_P=0.005+\- 0.01.
*** Double: $A_{\Delta\phi}$ =0.113 +\- 0.01. vs. 0.110 +\- 0.001 (MC & QCD)
A_{c1c2} = -0.021 +\- 0.03 vs -0.078 +\- 0.001

$$\frac{1}{\sigma}\frac{d^2\sigma}{d\cos\theta_1 d\cos\theta_2} = \frac{1}{4}(1 + B_1\cos\theta_1 + B_2\cos\theta_2 - C_{\text{helicity}}\cos\theta_1\cdot\cos\theta_2)$$

θ_1 θ_2 decay product angles w.r.t. t+tbar CM

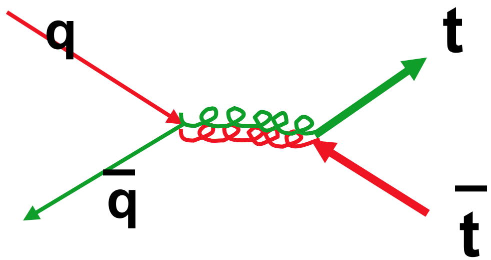
Direct measure of hard process- top polarization
Top decays weakly before hadronizing
⇒ decay "self-analyzing"



Contributions to order α_S Imaginary Part (Dharmaratna & GRG 1990,1996)

Analyze t--> W^+ b

Dilepton events or lepton+hadron jets or all Hadron jets)



Tree-level QCD

$q+\bar{q}$ (Tevatron)
or g+g (LHC)

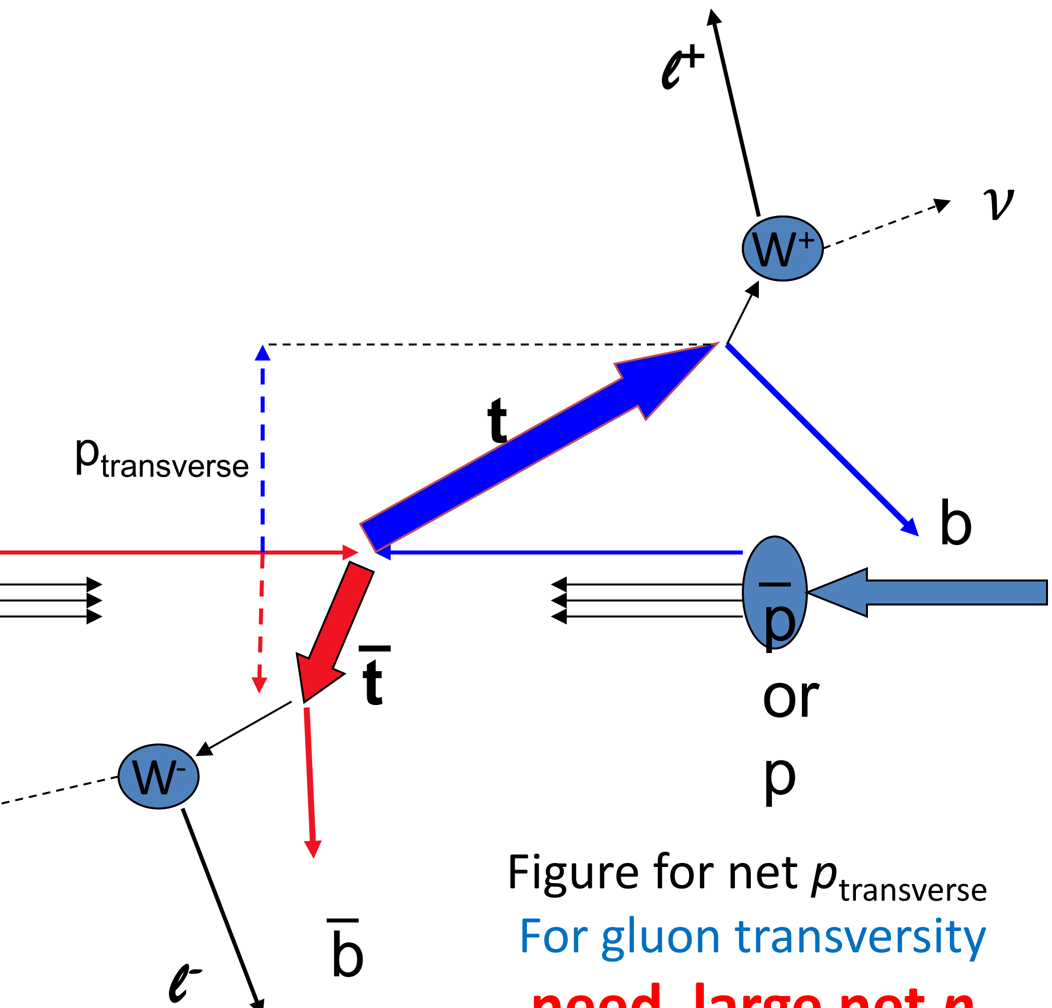
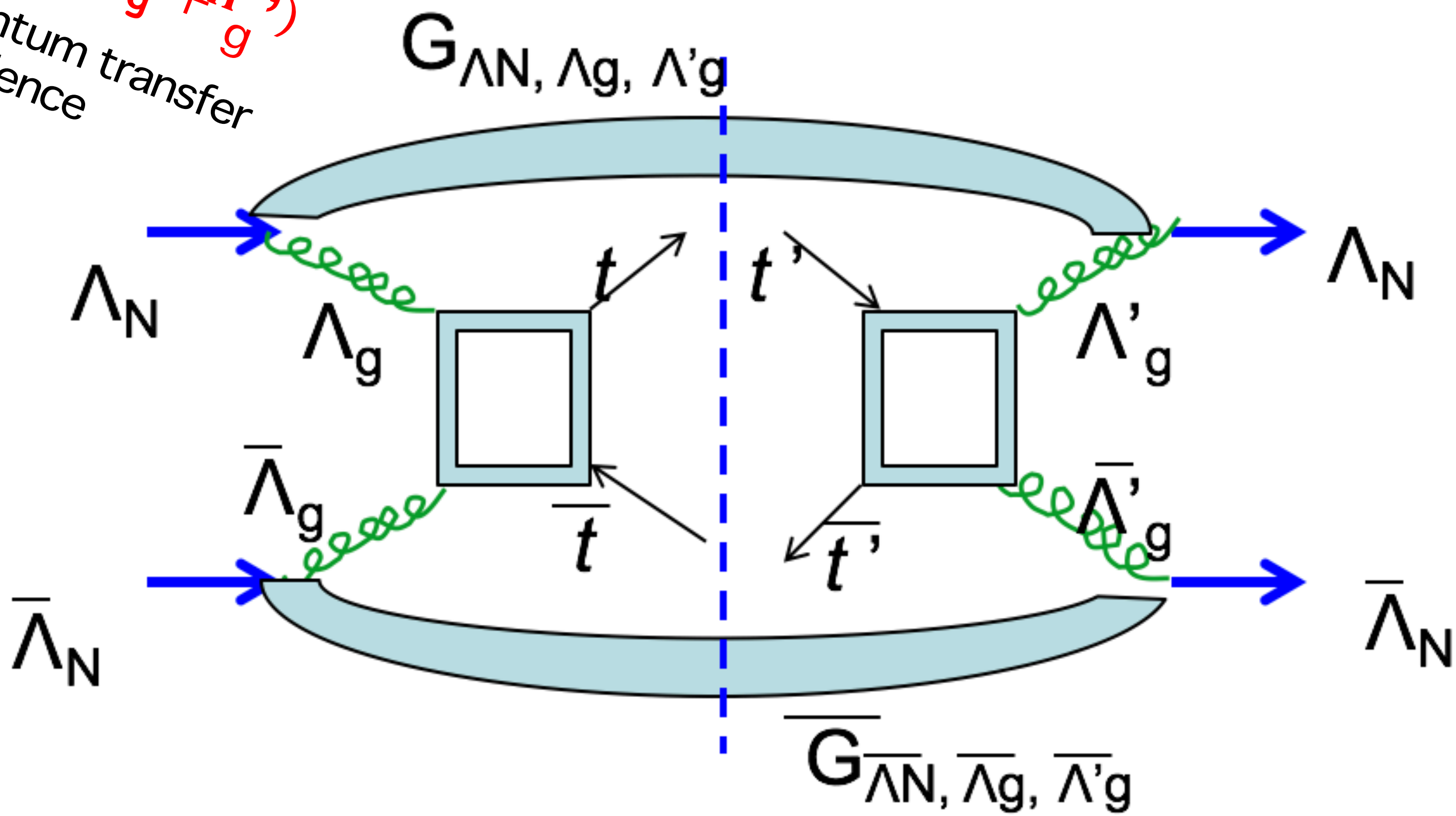


Figure for net $p_{transverse}$
For gluon transversity
need large net $p_{transverse}$
to access transversity

For inclusive p+p→t+tbar+X

Δ(Λ_N-Λ_N'-Λ_g+Λ_g')
Momentum transfer dependence



Gluon linear polarization with like and unlike t-tbar helicities

[work on particular gluon distributions still in progress S.Liuti,GRG,Gonzalez-Hernandez,Poage(thesis)]

$F \sim G_{XX}+G_{YY}$, $H \sim G_{XX}-G_{YY}$ or linear polarization

$\rho_{t',\bar{t}';t,\bar{t}}$	$\bar{F}\,F$	$\bar{H}\,H$	$\bar{F}\,H$	$\bar{H}\,F$
++;++	$\gamma^{-2}(1+\beta^{2}(1+\sin^{4}\theta))$	$\gamma^{-2}(-1+\beta^{2}(1+\sin^{4}\theta))$	$-2\dfrac{\beta^{2}}{\gamma^{2}}\sin^{2}\theta$	$-2\dfrac{\beta^{2}}{\gamma^{2}}\sin^{2}\theta$
+-;+-	$\beta^{2}\sin^{2}\theta(2-\sin^{2}\theta))$	$-\beta^{2}\sin^{4}\theta$	0	0

$$\rho_{t',\bar{t}';t,\bar{t}} \propto \sum_{all-helicities-not-tops} \bar{G}_{\bar{\Lambda}_N \bar{\Lambda}_g \bar{\Lambda}'_g} A^*_{\Lambda'_g \bar{\Lambda}'_g; t',\bar{t}'} A_{\Lambda_g \bar{\Lambda}_g; t,\bar{t}} G_{\Lambda_N \Lambda_g \Lambda'_g}$$

- The gluon spin correlations are transmitted to (determine the spin of) the decay products.
- The correlations between the lepton directions and the parent top spin (in the top rest frame) produce correlations between the lepton directions.
- The gluon fusion mechanism gives rise to a higher order (wrt quark antiquark) angular distribution due to the combination of two spin 1 gluons.

G.R.Goldstein,``Spin Correlations in Top Quark Production and the Top Quark Mass” in Proc. 12th Intl Symp. High Energy Spin Physics, Amsterdam, ed.C.W. deJager, et al., World Sci., Singapore (1997) p. 328

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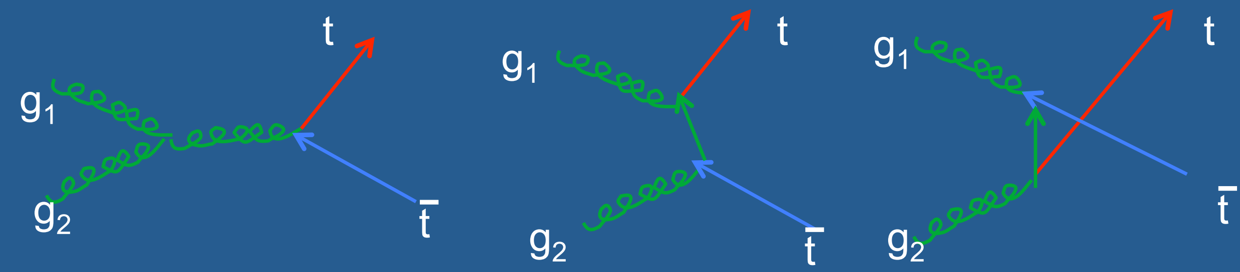
At LHC:

Gluon fusion tree level mechanism
(Color gauge invariance)



g_1, g_2 carry helicity $\Lambda_1\ \Lambda_2$ = ± 1 **OR transversity 1 or 0**

t, *t*-bar carry helicity $\lambda_{t}\ \lambda_{tbar}$ = ± ½ OR transversity $\pm 1/2$

Introduced in:
G.R.Goldstein,``Spin Correlations in Top Quark Production and the Top Quark Mass" in Proc. 12th Intl Symp. High Energy Spin Physics, Amsterdam, ed.C.W. deJager, et al., World Sci., Singapore (1997) p. 328.
R.H. Dalitz, G.R. Goldstein and R. Marshall, "Heavy Quark Spin Correlations in e+e- annihilations", Phys. Lett. B215, 783 (1988);
R.H. Dalitz, G.R. Goldstein and R. Marshall, "On the Helicity of Charm Jets", Zeits.f. Phys. C42, 441 (1989).

$q+q$-bar → t + t-bar
Dilepton channel

- The light (polarized) quark-antiquark annihilation mechanism gives rise to the **angular distribution between opposite charge lepton pairs**, more information than C_{helicity} or $A_{c1\ c2}$

$$
\begin{aligned}
W(\theta, p, p_{\bar{l}}, p_l) = \frac{1}{4}\{1 &+ [\sin^2\theta([p^2+m^2](\hat{p}_{\bar{l}})_x(\hat{p}_l)_{\bar{x}} + [p^2-m^2](\hat{p}_{\bar{l}})_y(\hat{p}_l)_{\bar{y}}) \\
&- 2mp\cos\theta\sin\theta((\hat{p}_{\bar{l}})_x(\hat{p}_l)_{\bar{z}} + (\hat{p}_{\bar{l}})_z(\hat{p}_l)_{\bar{x}}) + ([p^2-m^2] \\
&+[p^2+m^2]\cos^2\theta)(\hat{p}_{\bar{l}})_z(\hat{p}_l)_{\bar{z}}]/[(p^2+m^2)+(p^2-m^2)cos^2\theta]\} \\
= \frac{1}{4} + \frac{1}{4}\{&(2-\beta^2)\sin^2\theta(\hat{p}_{\bar{l}})_x(\hat{p}_l)_{\bar{x}} + \beta^2(\hat{p}_{\bar{l}})_y(\hat{p}_l)_{\bar{y}} \\
&+[\beta^2+(2-\beta^2)\cos^2\theta](\hat{p}_{\bar{l}})_z(\hat{p}_l)_{\bar{z}} \\
&-\frac{2}{\gamma}\cos\theta\sin\theta((\hat{p}_{\bar{l}})_x(\hat{p}_l)_{\bar{z}} + (\hat{p}_{\bar{l}})_z(\hat{p}_l)_{\bar{x}})\Big\}/[(2-\beta^2)+\beta^2\cos^2\theta]
\end{aligned}
$$

m =top mass, θ = t production angle in q+q-bar CM
p= light quark 3-momentum in CM
Unit vectors p-hat are anti-lepton$^+$ and lepton$^-$ 3-momenta directions in the top and anti-top rest frames.

See G.R.Goldstein,``Spin Correlations in Top Quark Production and the Top Quark Mass'' in Proc. 12th Intl Symp. High Energy Spin Physics, Amsterdam, ed.C.W. deJager, et al., World Sci., Singapore (1997) p. 328.

$g_1+g_2 \rightarrow t + t\text{-bar}$
Spin correlations – dilepton channel

Correlations expressed as a weighting factor first ***for unpolarized gluons***.

- The gluon fusion mechanism gives rise to a higher order angular distribution ($\sin^4\theta$) due to the combination of two spin 1 gluons.

$$\begin{aligned} W(\theta,p,p_{\bar{l}},p_l) = \frac{1}{4} - \frac{1}{4}\{&[p^4\sin^4\theta + m^4](\hat{p}_{\bar{l}})_x(\hat{p}_l)_{\bar{x}} + [p^2(p^2-2m^2)\sin^4\theta - m^4](\hat{p}_{\bar{l}})_y(\hat{p}_l)_{\bar{y}} \\ &+ [p^4\sin^4\theta - 2p^2(p^2-m^2)\sin^2\theta + m^2(2p^2-m^2)](\hat{p}_{\bar{l}})_z(\hat{p}_l)_{\bar{z}} \\ &+ 2mp^2\sqrt{p^2-m^2}\cos\theta\sin^3\theta[(\hat{p}_{\bar{l}})_x(\hat{p}_l)_{\bar{z}} - (\hat{p}_{\bar{l}})_z(\hat{p}_l)_{\bar{x}}]\} \\ &/\left[p^2(2m^2-p^2)\sin^4\theta + 2p^2(p^2-m^2)\sin^2\theta + m^2(2p^2-m^2)\right] \qquad (20) \end{aligned}$$

$$\begin{aligned} = \frac{1}{4} - \frac{1}{4}\{&[(1-\beta^2)^2 + \sin^4\theta)](\hat{p}_{\bar{l}})_x(\hat{p}_l)_{\bar{x}} \\ &+[-(1-\beta^2)^2 - (1-2\beta^2)\sin^4\theta](\hat{p}_{\bar{l}})_y(\hat{p}_l)_{\bar{y}} \\ &+[(1-\beta^4) - 2\beta^2\sin^2\theta + \sin^4\theta](\hat{p}_{\bar{l}})_z(\hat{p}_l)_{\bar{z}} \\ &+2\frac{\beta}{\gamma}\sin^3\theta\cos\theta[(\hat{p}_{\bar{l}})_x(\hat{p}_l)_{\bar{z}} - (\hat{p}_{\bar{l}})_z(\hat{p}_l)_{\bar{x}}]\Big\} \\ &/\left[(1-\beta^4) + 2\beta^2\sin^2\theta + (1-2\beta^2)\sin^4\theta\right] \qquad (21) \end{aligned}$$

m =top mass, θ = t production angle in g+g CM; p= gluon 3-momentum in CM
p-hat's are lepton$^-$ 3-momenta directions in the top and anti-top rest frames.

Use these to test SM vs. BSM – Integrated version agrees – with big errors -- GRG *in process* – see also Mahlon & Parke

See GG& Liuti, 1710.01683; 2024742 (APS-DPF 2017)

$g_1+g_2 \rightarrow t + t\text{-bar}$
Spin correlations

Correlations expressed as a weighting factor ***for polarized gluons***.

- The gluon fusion mechanism gives rise to a higher order angular distribution ($\sin^4\theta$) due to the combination of two spin 1 gluons.

$$W^{(LP,LP)}(\theta, p, p_{\bar{l}}, p_l) = -\frac{1}{4} + \frac{1}{4}\{[(1-\beta^4) + \beta^2\sin^2\theta(-2+(2-\beta^2)\sin^2\theta)](\hat{p}_{\bar{l}})_x(\hat{p}_l)_{\bar{x}}$$
$$+[(1-\beta^4)+\beta^2\sin^2\theta(2-\beta^2\sin^2\theta)](\hat{p}_{\bar{l}})_y(\hat{p}_l)_{\bar{y}}$$
$$+[-(1-\beta^2)^2+\beta^2(2-\beta^2)\sin^4\theta](\hat{p}_{\bar{l}})_z(\hat{p}_l)_{\bar{z}}$$
$$-4\frac{\beta^2}{\gamma}\sin^3\theta\cos\theta[(\hat{p}_{\bar{l}})_x(\hat{p}_l)_{\bar{z}} - (\hat{p}_{\bar{l}})_z(\hat{p}_l)_{\bar{x}}]\Big\}$$
$$/\left[(1-\beta^2)^2+\beta^4\sin^4\theta\right]$$

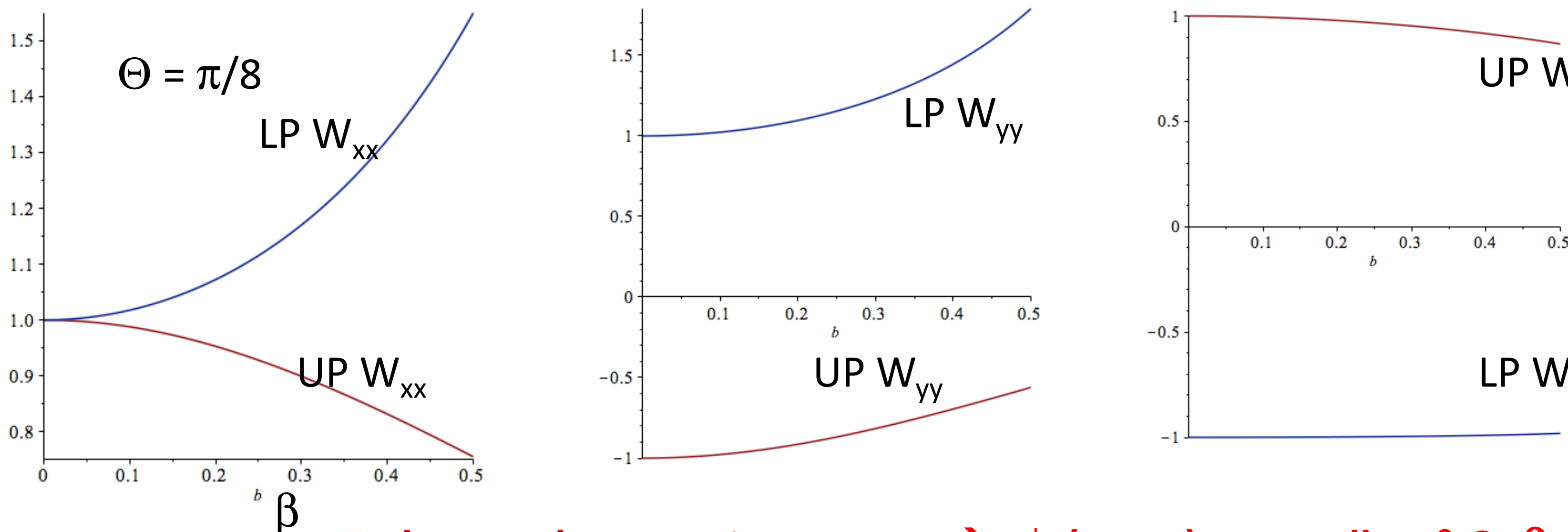
Crucial measurements $(\hat{p}_{\bar{l}})_x(\hat{p}_l)_{\bar{x}} = W_{xx}$, $(\hat{p}_{\bar{l}})_x(\hat{p}_l)_{\bar{z}} = W_{xz}$, ... **Weighting tensor**

- **Use these to compare with unpolarized to extract the Gluon transversity**
- **or linear polarizations G_{xx}-G_{yy}**
- **Careful about Frames:**
- **Collider LAB, $t+\bar{t}$ pair CM, separate *t* & *t*-bar rest, $W^{+/-}$ rest frames**

Comparing lepton directional correlations

Weighting tensor for lepton$^+$ lepton$^-$ when $\theta=\pi/8$

or lepton$^+$ d-quark *or* u-quark lepton$^-$



Each event has μ^- μ^+ momenta → $p^\pm$ (x, y, z) as well as θ & β

Probability for given event configuration is given by

G(UP) W(θ,p,$\bar{p}$l,pl) +G(LP) W^{LP} (θ,p,$\bar{p}$l,pl)

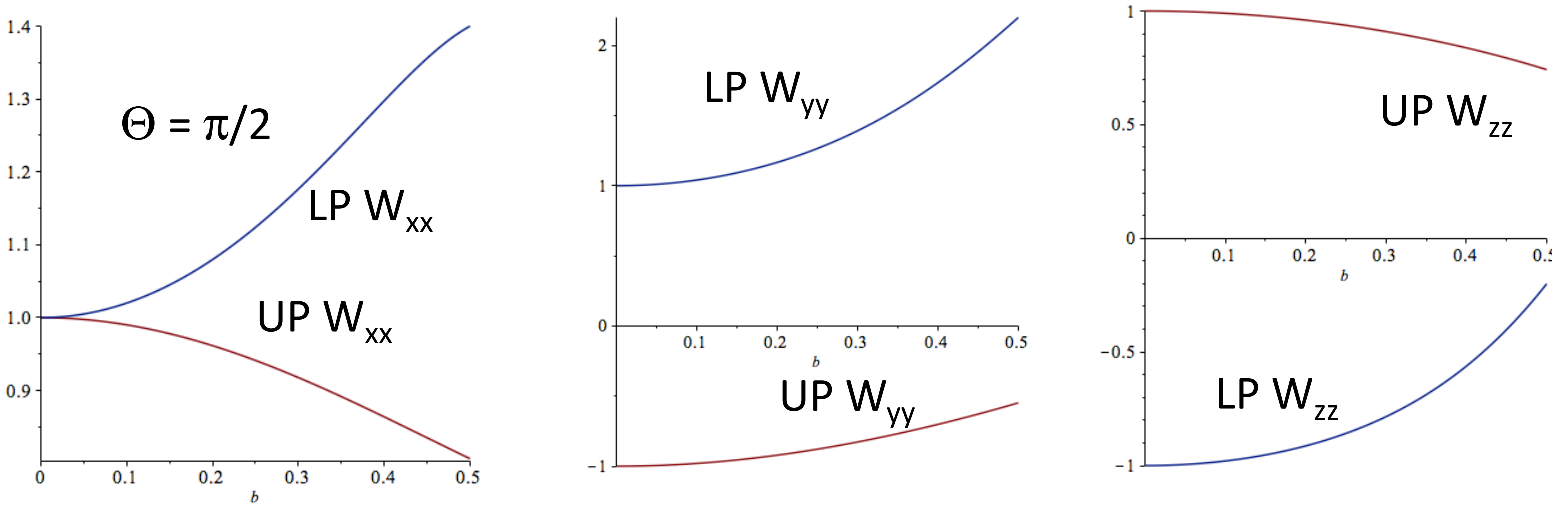
Quite distinct! x & y components are aligned for LP, anti-aligned for UP

Can Diagonalize (with $W_{XY,\ YX}$) to obtain positive ellipsoidal weighting

Comparing lepton directional correlations

Weighting factors for lepton$^+$ lepton$^-$ when $\theta=\pi/2$

$W_{xz}=0$ for the off-diagonal



β

Each event has μ^- μ^+ momenta → $p^\pm$ (x, y, z) as well as θ & β

Probability for given event configuration is given by

$G(UP)\,W(\theta,p,\bar{p}l,pl)$ $+G(LP)\,W^{LP}(\theta,p,\bar{p}l,pl)$

Quite distinct! x & y components are aligned for LP, anti-aligned for UP

Diagonalize (with $W_{XY,\,YX}$) to obtain positive ellipsoidal weighting

Separating polarized gluons

* Each event has μ^- μ^+ momenta → $p^{\pm}$ (x, y, z)
in t & tbar rest frame

* t+tbar CM determines θ direction as well as β for t & tbar

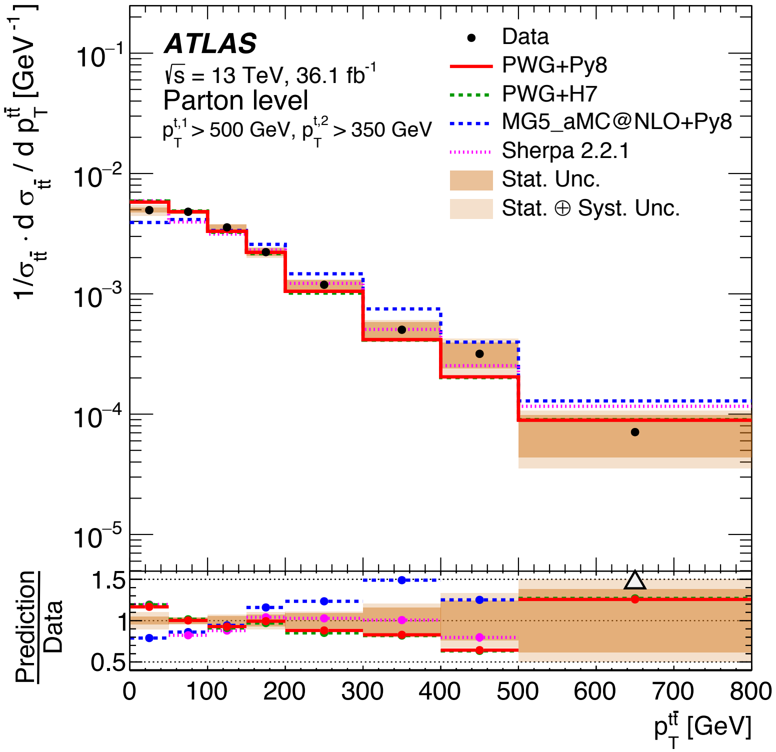
* Probability for given event configuration is given by

$G(UP)\, W(\theta,p,\bar{p}l,pl) + G(LP)\, W^{LP}(\theta,p,\bar{p}l,pl)$ (ignoring light quarks)

- Quite distinct! x & y components are
- aligned for LP, anti-aligned for UP
- G's convoluted with W's all gluon k_T & $\bar{k}_T$ satisfying
- measured $p_t+p_{anti\text{-}t}$ <-> large transverse momenta : transversity

Large transverse momentum

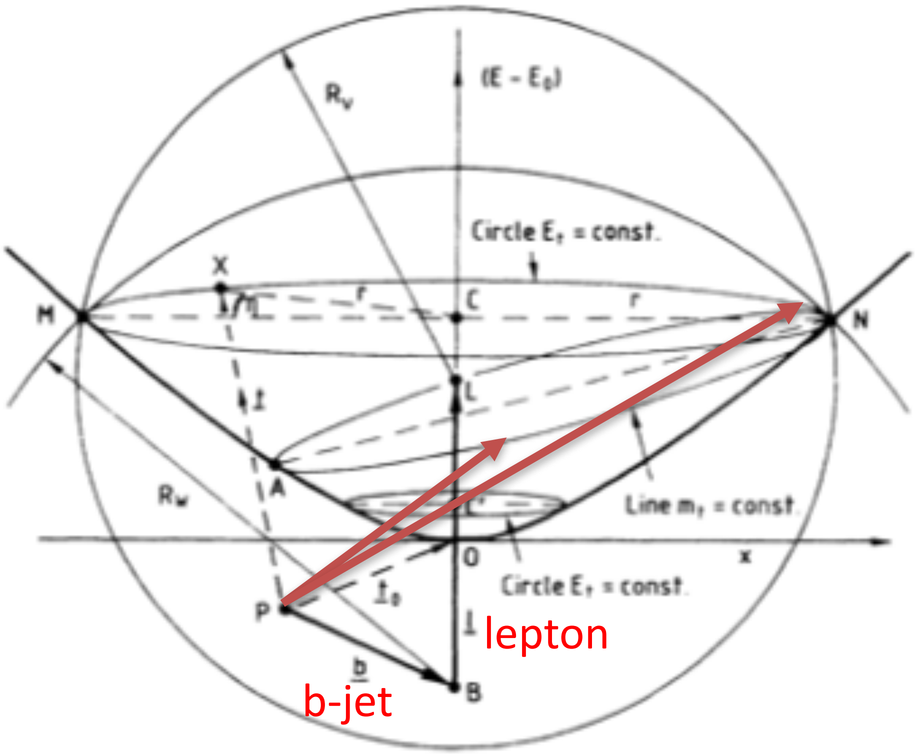
- t-tbar inclusive at 13 TeV



How are top pair polarizations measured at LHC?

- Purely hadronic events ⊃ 6 particles/jets (b,u,d +b̄,ū,d). Combinatorics!
- Dilepton events leave unknown ν & $\bar{\nu}$ momenta (b,e^+ ν +b̄,ū,d). Clean, lower dσ
- Single lepton events ⊃ one ν missing
  - Most promising: with H.Beauchemin(ATLAS), M. Yampolskaya, T. Lachance
- What is t or t̄ momentum?
- Measuring e or μ and b-jet fixes t to an ellipse

Finding top momentum



- top leptonic decay in lab
- → momenta in lab
- b-jet & lepton measured ⇒
- Ellipse of t-vectors determined
- Boost to rest frame
- helicity preserved
- Lepton direction correlated

FIG. 5. Momentum vectors **b** and $\bar{l}$ observed in the laboratory frame for bottom quark and lepton, and the construction for locating all top-quark momenta **t** such that these three vectors can correspond to the decay sequence $t \rightarrow bW^+$, $W^+ \rightarrow \bar{l}^+ \nu_l$ for a given top-quark mass m_t.

Dalitz & GG, PRD45,1531(1992)

Summary

- Gluon GPDs & TMDs (from spectator & Regge R✖Dq)
- *Helicity* conserving & Helicity flip →gluon ***Transversity***
- Electroproduction & DVCS → gluon transversity GPDs
- pp→gluons→t+tbar + X => gluon TMDs
- Measurements? Single Top polarization
- **t+tbar spin correlations via lepton decays or hadron jets**

  – To Do List

- **More phenomenology to come**
- **Parton showers & jets**
- **Care about evolution, factorization, power counting, . . .**

Collaborators: Gluons

Simonetta Liuti[2], Osvaldo Gonzalez Hernandez[3], Jon Poage[1]

- GRG, Gonzalez, Liuti, PRD91, 114013 (2015)
- GRG, Gonzalez Hernandez, Liuti, J. Phys. G: Nucl. Part. Phys. **39** 115001 (2012)
- GRG, Liuti, IJMP: Conf. 37, 1560038 (2015); arXiv: 1710.01683 [hep-ph]
- J.Poage, Tufts U. dissertation (2016)
- GRG & Liuti, Hernandez, PoS QCDEV2017, 037 (2017)

Collaborators: Tops

Richard Dalitz,

Discussions: Krzysztof Sliwa, Hugo Beauchemin Tufts and Atlas

- Dalitz, R.H., and GRG, Phys. Rev. D45, 1531 (1992); Phys.Lett.B287, 225 (1992);
- GRG, Sliwa, K., Dalitz, R.H., Phys. Rev. D47, 967 (1993).

Collaborators: Transversity

Michael J. Moravcsik,

- GRG & M.J. Moravcsik, Ann. Phys. 98, 128 (1976); ibid. 142, 219 (1982);
- Ibid. 195, 213 (1989).

See also K. Chen, GRG, R.L. Jaffe, X.-D. Ji, Nucl Phys B 445 (1995) 380-396;
GRG, R.L. Jaffe, X.-D. Ji, Phys. Rev. D52 (1995) 506.

Thank you!

Backup Slides

Construct helicity flip amps Spectator Model, then GPDs

$$A_{++,+-} = \sqrt{1-\xi^2}\,\frac{t_0-t}{4M^2}\left(\tilde{H}_T^g + (1-\xi)\frac{E_T^g+\tilde{E}_T^g}{2}\right)$$

$$A_{-+,--} = \sqrt{1-\xi^2}\,\frac{t_0-t}{4M^2}\left(\tilde{H}_T^g + (1+\xi)\frac{E_T^g-\tilde{E}_T^g}{2}\right)$$

$$A_{++,--} = +e^{-i\phi}(1-\xi^2)\frac{\sqrt{t_0-t}}{2M}\left(H_T^g + \frac{t_0-t}{M^2}\tilde{H}_T^g - \frac{\xi^2}{1-\xi^2}E_T^g + \frac{\xi}{1-\xi^2}\tilde{E}_T^g\right)$$

$$A_{-+,+-} = -e^{i\phi}(1-\xi^2)\frac{\sqrt{t_0-t}^3}{8M^3}\tilde{H}_T^g,$$

Compare to spectator model results

$$\tilde{H}_T^g = 0$$

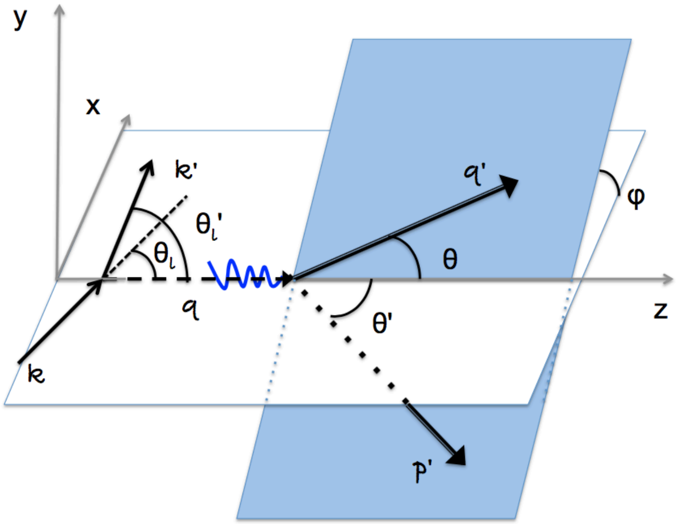
$$(1-X)A^0_{-+.--} = (1-X')A^0_{++,+-}$$

$$\tilde{E}_T^g = 0.$$

As in Hoodbhoy & Ji, PRD58, 054006 (1998)

Measuring Gluon GPDs in Nucleons

DVCS



$$\frac{d^5\sigma}{dx_{Bj}dQ^2d|t|d\phi d\phi_S} = \frac{\alpha^3}{16\pi^2(s-M^2)^2\sqrt{1+\gamma^2}}|T|^2$$

$$T(k,p,k',q',p') = T_{DVCS}(k,p,k',q',p') + T_{BH}(k,p,k',q',p'),$$

$$|T|^2 = |T_{\rm BH} + T_{\rm DVCS}|^2 = |T_{\rm BH}|^2 + |T_{\rm DVCS}|^2 + \mathcal{I}\,.$$

$$\mathcal{I} = T^*_{BH}T_{DVCS} + T^*_{DVCS}T_{BH}.$$

For unpolarized e+p→e'+γ+p' cross section depends on azimuthal angle ϕ.
cos3ϕ modulation in interference dσ measures gluon transversity GPDs (CFF's)

$$\frac{\sqrt{t_0-t}^3}{8M^3}\left[H_T^gF_2 - E_T^gF_1 - 2\tilde{H}_T^g\left(F_1 + \frac{t}{4M^2}F_2\right)\right]\cos 3\phi$$

$\mathcal{H}_T^g \sim \int dx\, H^g_T/(x-\xi)(x+\xi)$ CFF's

But $\mathcal{H}_T^g \sim$ may need EIC

See Diehl, *et al*. PLB411, 193 (1997);
Diehl, EPJC25, 223 (2002);
Belitsky, Mueller, PLB486, 369 (2000).

A glimpse of gluons through deeply virtual compton scattering on the proton, published in *Nature Communications* 8, 1408 (2017).
doi:10.1038/s41467-017-01819-3

Evidence of gluon transversity

Fitting ϕ distribution requires
F_{++} and both F_{+-} gluon transversity
and F_{0+} higher twist

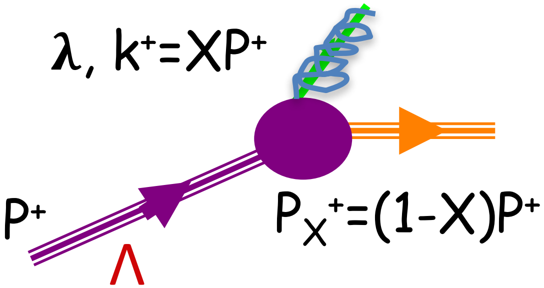
Table 2 Results of the cross-section fits

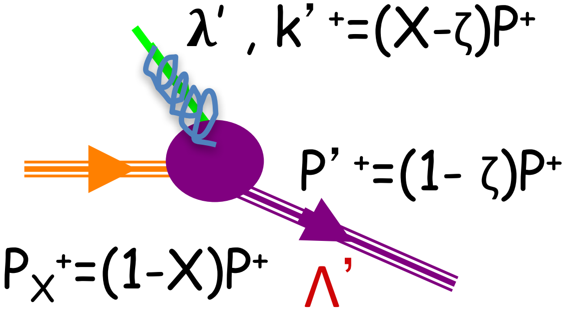
Fit description	**LO/LT**	**Higher twist**	**NLO**
Helicity states	**++**	**++/0+**	**++/−+**
$t = -0.18$ GeV^2	250	204	206
$t = -0.24$ GeV^2	367	206	208
$t = -0.30$ GeV^2	415	189	190

Values of χ^2 (ndf = 208) obtained in the leading-order, leading-twist (++); higher-twist (++/0+); and next-to-leading-order (++/−+) scenarios. The fit is not performed at the highest value of $-t$ because of the lack of full acceptance in ϕ, resulting in a large statistical uncertainty. The fits include statistical and point-to-point systematic uncertainties

Constructing gluon GPDs

Gluon 'vertex functions' $G_{\Lambda x\,;\,\Lambda g,\,\Lambda}$





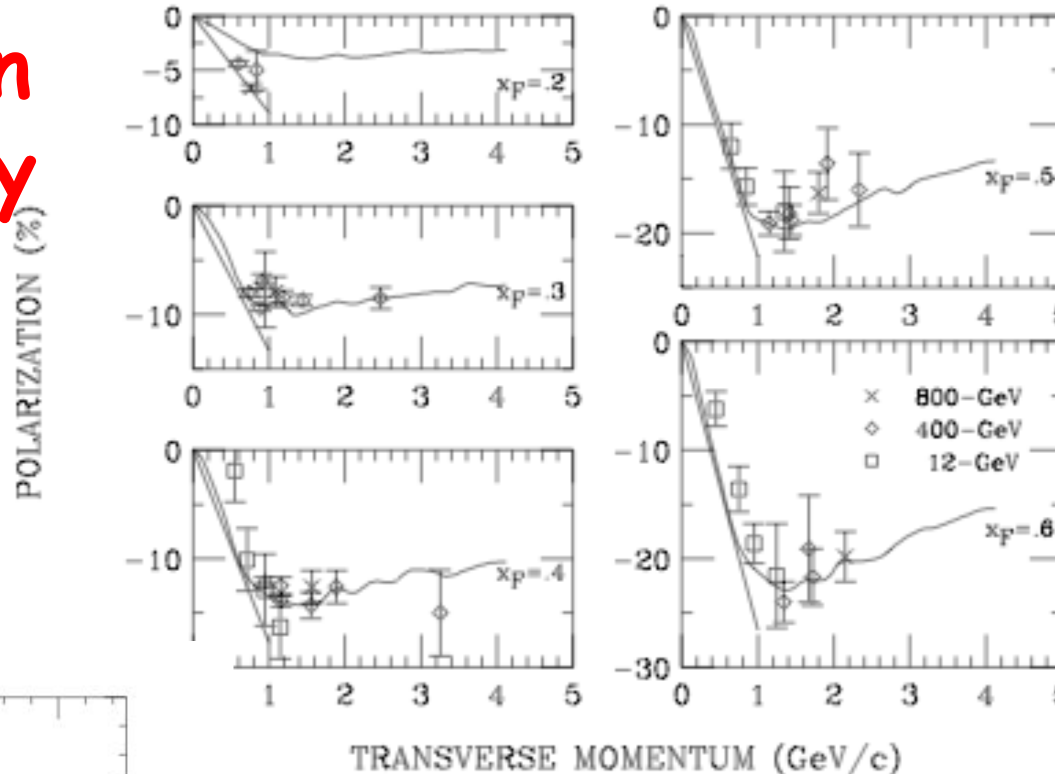
$\mathcal{G}_{+++}(x,\vec{k}_T^2)$	$-\frac{2}{\sqrt{2(1-X)}}\frac{(k_x-ik_y)}{X}$
$\mathcal{G}_{-++}(x,\vec{k}_T^2)$	$-\frac{2}{\sqrt{2(1-X)}}(M(1-X)-M_x)$
$\mathcal{G}_{++-}(x,\vec{k}_T^2)$	0
$\mathcal{G}_{-+-}(x,\vec{k}_T^2)$	$-\frac{2}{\sqrt{2(1-X)}}(1-X)\frac{(k_x-ik_y)}{X}$
$\mathcal{G}^*_{+++}(x,\vec{k}_T^{\prime 2})$	$-\frac{2}{\sqrt{2(1-X')}}\frac{(\tilde{k}_x+i\tilde{k}_y)}{X'}$
$\mathcal{G}^*_{-++}(x,\vec{k}_T^{\prime 2})$	$-\frac{2}{\sqrt{2(1-X')}}(M(1-X')-M_x)$
$\mathcal{G}^*_{++-}(x,\vec{k}_T^{\prime 2})$	0
$\mathcal{G}^*_{-+-}(x,\vec{k}_T^{\prime 2}))$	$-\frac{2}{\sqrt{2(1-X')}}(1-X')\frac{(\tilde{k}_x+i\tilde{k}_y))}{X'}$

$$X'=\frac{X-\zeta}{1-\zeta},\ \tilde{k}_{i=1,2}=k_i-\frac{1-X}{1-\zeta}\Delta_i$$

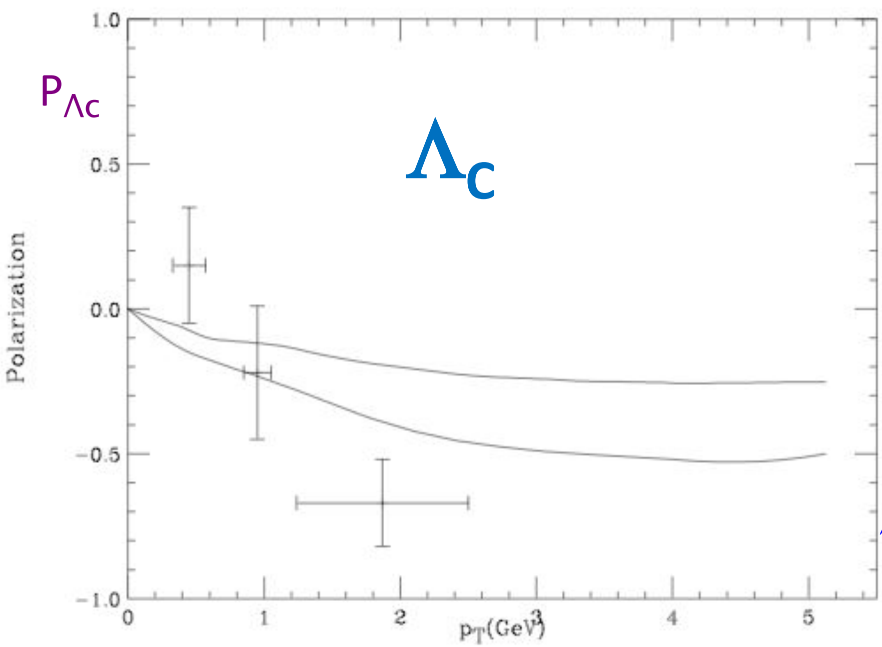
GRG & S. Liuti, QCD Evolution 2014, IJMP: Conf. 37, 1560038 (2015); arXiv: 1710.01683 [hep-ph]
GRG, Gonzalez Hernandez, Liuti, Poage, in progress

Single Spin Asymmetry

$$pp \rightarrow \Lambda^{\uparrow}(\Lambda_c^{\uparrow})X$$

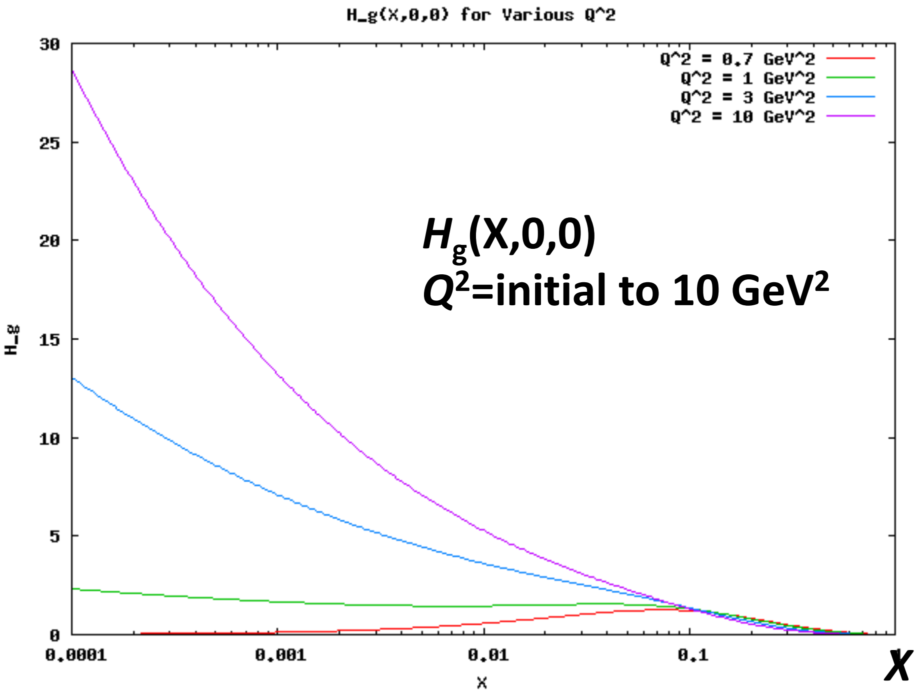


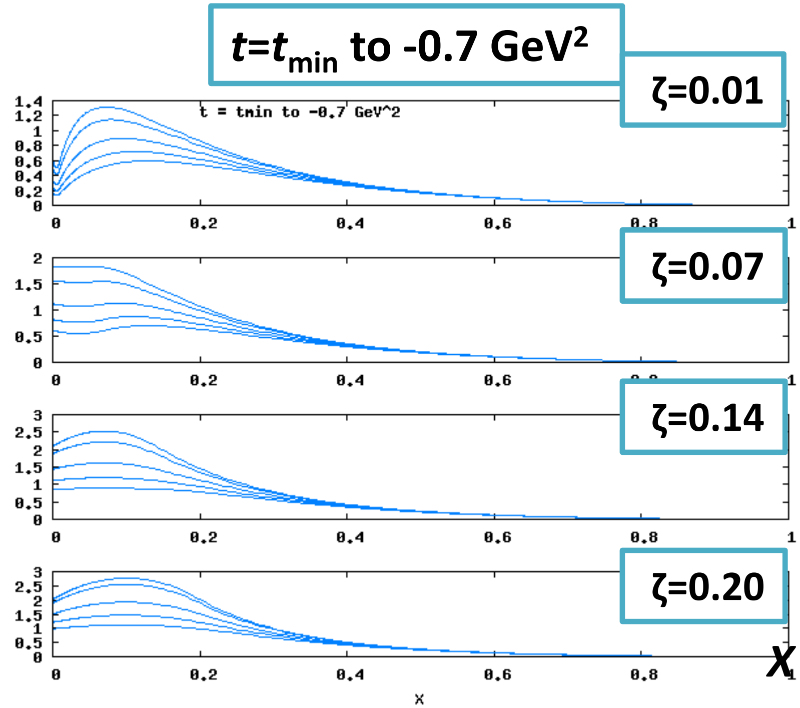
K. Heller, PRD1997
curves from model of
Dharmaratna & GRG PRD '90 & '97



E791, PLB 471, 449 (2000)
$\pi^- + p \rightarrow \Lambda_C + X$
curves from GRG hep-ph/9907573

After pdf's vs. Q^2 → fix x dependence
Regge behavior determines t dependence
Spectator determines ζ dependence





$H_g(X,\zeta,t)$
Q^2=initial

from J. Poage

Top spin correlations & gluon polarizations

$\rho_{t',\bar{t}';t,\bar{t}}$	UP,UP	LP,LP	UP,LP + LP,UP
$++,++$	$\gamma^{-2}(1+\beta^2(1+sin^4\theta))$	$\gamma^{-2}(-1+\beta^2(1+sin^4\theta))$	$-4\gamma^{-2}\beta^2 sin^2\theta$
$+-,+-$	$\beta^2 sin^2\theta(2-sin^2\theta))$	$-\beta^2 sin^4\theta$	0
$++,--$	$\gamma^{-2}(-1+\beta^2(1+sin^4\theta))$	$\gamma^{-2}(+1+\beta^2(1+sin^4\theta))$	$+4\gamma^{-2}\beta^2 sin^2\theta$
$+-,-+$	$\beta^2 sin^4\theta$	$-\beta^2 sin^2\theta(2-sin^2\theta))$	0
$++,+-$	$-2\gamma^{-1}\beta^2 sin^3\theta cos\theta$	$-2\gamma^{-1}\beta^2 sin^3\theta cos\theta$	$-4\gamma^{-1}\beta^2 sin\theta cos\theta$
$++,-+$	$2\gamma^{-1}\beta^2 sin^3\theta cos\theta$	$2\gamma^{-1}\beta^2 sin^3\theta cos\theta$	$4\gamma^{-1}\beta^2 sin\theta cos\theta$

11 using values of

$$G^{(1)}_{\Lambda_{N1},R,R} + G^{(1)}_{\Lambda_{N1},L,L} = G^{(1)}_{\Lambda_{N1},XX} + G^{(1)}_{\Lambda_{N1},YY} = G^{(1)}_{\Lambda_{N1},UP}$$

$$G^{(1)}_{\Lambda_{N1},R,L} + G^{(1)}_{\Lambda_{N1},L,R} = G^{(1)}_{\Lambda_{N1},YY} - G^{(1)}_{\Lambda_{N1},XX} = G^{(1)}_{\Lambda_{N1},LP}$$

UP = unpolarized, LP = Linearly polarized gluon distributions
assuming g+g→ t + t-bar in single plane CM
γ & β for top & antitop in CM.
θ = top production angle in CM relative to (t+tbar) momentum direction in lab
Taking X-Z plane for p+p→$(t+tbar)_{CM}$+X gives ϕ dependence to t+tbar plane for opposite helicities: $Re(e^{\pm(1or2)i\phi} \cdot e^{\pm(-i(1or2)\phi)})$ leading to cos2ϕ for UP,LP and LP,UP and cos4ϕ modulations for LP,LP.