

LINEARLY POLARIZED GLUON DISTRIBUTION IN J/ψ PRODUCTION AT EIC

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PLAN OF THE TALK

TMD gluon distributions

Linearly polarized gluon distributions

Probing linearly polarized gluon distribution in J/ψ production

$\cos 2\phi$ asymmetry

Numerical results

Summary and conclusion

GLUON TMDs

Very little known about gluon TMDs, satisfy positivity bound

Unpolarized gluon TMDs have been extracted from LHCb data

[Lansberg, Pisano, Scarpa, Schlegel, PLB 784, 217 \(2018\)](#)

Like quark TMDs, gluon TMDpdfs are process dependent due to the presence of gauge links.
Each gluon TMD contains two gauge links : process dependence more involved than quark TMDs

Simplest possible configurations are ++ or -- and +- or -+

In the literature related to small-x physics, these are known as Weizsacker-Williams (WW) and Dipole distributions, respectively

[Kovchegov and Mueller, Nucl. Phys. B 529, 451 \(1998\)](#)

[McLerran and Venugopalan, PRD 59, 094002 \(1999\)](#)

[Dominguez, Qiu, Xiao, Yuan, PRD 85, 045003 \(2012\)](#)

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LINEARLY POLARIZED GLUON DISTRIBUTIONS

Unpolarized WW gluon distributions can be interpreted as number density of gluons in a nucleon in light cone gauge; can be accessed in processes for example in dijet production in DIS

Dipole distributions are the Fourier transform of color dipole amplitudes and appear in many processes for example in photon-jet and dijet production in pA collision

Operator structure of these two unintegrated gluon distributions are different : studied extensively in the literature

Linearly polarized gluon distributions were first introduced in

[Mulders and Rodrigues, PRD 63, 094021 \(2001\)](#)

Measures an interference between an amplitude when the active gluon is polarized along x (or y) direction and a complex conjugate amplitude with the gluon polarized in y (or x) direction in an unpolarized hadron

Affects unpolarized cross section as well as generates a $\cos 2\phi$ asymmetry

LINEARLY POLARIZED GLUON DISTRIBUTION

T-even; can be WW type or dipole type depending on gauge link

Has not been extracted from data yet, although a lot of theoretical studies has been done

- * Can be probed in dijet imbalance in unpolarized hadronic collision; heavy quark pair production in ep and pp collision [Marquet, Roisnel, Taels \(2018\)](#); [Pisano, Boer, Brodsky, Buffing, Mulders \(2013\)](#); [Efremov, Evanov, Teryaev \(2018\)](#)
- * Quarkonium pair production in pp collision [Lansberg, Pisano, Scarpa, Schlegel \(2018\)](#)
- * Associated production of dilepton and J/ψ [Lansberg, Pisano, Schlegel \(2017\)](#)
- * eA collision: dijet imbalance [Dumitru, Skokov, Ulrich \(2018\)](#)
- * Transverse momentum distribution of Higgs boson and heavy quarkonium in unpolarized pp collision [Sun, Xiao, Yuan \(2011\)](#); [Boer, Dunnen, Pisano, Schlegel, Vogelsang \(2012\)](#); [Boer and Pisano \(2012\)](#); [AM and Rajesh \(2016,2017\)](#)

LINEARLY POLARIZED GLUON DISTRIBUTION IN J/Ψ PRODUCTION

Initial and final state interaction may affect the generalized factorization. Such effects are less complicated in ep collision compared to pp and pA collision

J/Ψ production in ep collision probes h_{1g}^\perp through the LO process $\gamma^* + g \rightarrow c + \bar{c}$

[AM and Rajesh, EPJC 77, 854 \(2017\)](#)

Contributes at $z=1$, where z is the energy fraction of the photon carried by J/Ψ in the proton rest frame

Extended to the region $z < 1$ [Kishore and AM, PRD \(2019\)](#)

Can probe gluon distributions at small x (gluon contribution dominant)

Production mechanism of J/Ψ is calculated in NRQCD. Basic assumption : factorization of amplitude into a hard part with the heavy quark pair produced in the process $\gamma^* + g \rightarrow c + \bar{c} + g$

Heavy quark pair then hadronizes to form J/Ψ. Hadronization is described in terms of long distance matrix elements (LDMEs). These are obtained by fitting data. CS case from lattice calculations and potential models

LINEARLY POLARIZED GLUON DISTRIBUTION IN J/ψ PRODUCTION

LDMEs have scaling property wrt the velocity parameter v (small)

Cross section is expressed as a double expansion in terms of α_s and v . For J/ψ $v \approx 0.3$

In NRQCD, the heavy quark pair can be produced in color singlet (CS) state or in color octet (CO) state

In CS model, heavy quark pair in the hard process is produced in color singlet state with the same quantum numbers as J/ψ

With the assumption of generalized TMD factorization it was shown that low p_T RHIC data can be reasonably described by NRQCD based CS model; although high p_T data needs inclusion of CO states

[D'Alesio, Murgia, Pisano, Taels, PRD 036011 \(2017\)](#)

Both CS and CO states are needed to describe the HERA data

[Rajesh, Kishore, AM, PRD 98, 014007 \(2018\)](#)

J/ψ PRODUCTION IN EP COLLISION

We present a calculation of $\cos 2\phi$ asymmetry in the process $e(l) + p(P) \rightarrow e(l') + J/\psi(P_h) + X$

In the kinematical region $z < 1$ in NRQCD based CS model

The corresponding hard process is $\gamma^* + g \rightarrow c + \bar{c} + g$ Final state gluon not detected

$z = P \cdot P_h / P \cdot q$ energy fraction of J/ψ in rest frame of proton

$$q = l - l'; Q^2 = -q^2; s = (l + P)^2 \quad Q^2 = sx_B y; \quad x_B = \frac{Q^2}{2P \cdot q}; \quad y = P \cdot q / P \cdot l$$

Incoming and outgoing leptons form the lepton plane. Azimuthal angles are measured wrt this plane

We use a framework based on generalized parton model approach with the inclusion of intrinsic transverse momentum effects, and assume TMD factorization.

J/ψ PRODUCTION IN EP COLLISION

Diff. cross section

$$d\sigma = \frac{1}{2s} \frac{d^3 l'}{(2\pi)^3 2E_{l'}} \frac{d^3 P_h}{(2\pi)^3 2E_{P_h}} \int \frac{d^3 p_g}{(2\pi)^3 2E_g} \int dx d^2 k_\perp (2\pi)^4 \delta(q + k - P_h - p_g) \\ \times \frac{1}{Q^4} L^{\mu\mu'}(l, q) \Phi^{\nu\nu'}(x, k_\perp) \mathcal{M}_{\mu\nu}^{\gamma^*+g \rightarrow J/\psi+g} \mathcal{M}_{\mu'\nu'}^{*\gamma^*+g \rightarrow J/\psi+g}$$

Leptonic tensor $L^{\mu\mu'}(l, q) = e^2(-g^{\mu\mu'} Q^2 + 2(l^\mu l'^{\mu'} + l'^{\mu'} l^\mu))$

Gluon correlator for unpolarized hadron

$$\phi_g^{\nu\nu'}(x, \mathbf{k}_\perp) = \frac{1}{2x} \left[-g_\perp^{\nu\nu'} f_1^g(x, \mathbf{k}_\perp^2) + \left(\frac{k_\perp^\nu k_\perp^{\nu'}}{M_p^2} + g_\perp^{\nu\nu'} \frac{\mathbf{k}_\perp^2}{2M_p^2} \right) h_1^{\perp g}(x, \mathbf{k}_\perp^2) \right]$$

↑
Unpol. gluon distribution

↑
Linearly polarized gluon distribution

$$g_\perp^{\nu\nu'} = g^{\nu\nu'} - P^\nu n^{\nu'} / P \cdot n - P^{\nu'} n^\nu / P \cdot n$$

FEYNMAN DIAGRAMS CONTRIBUTING

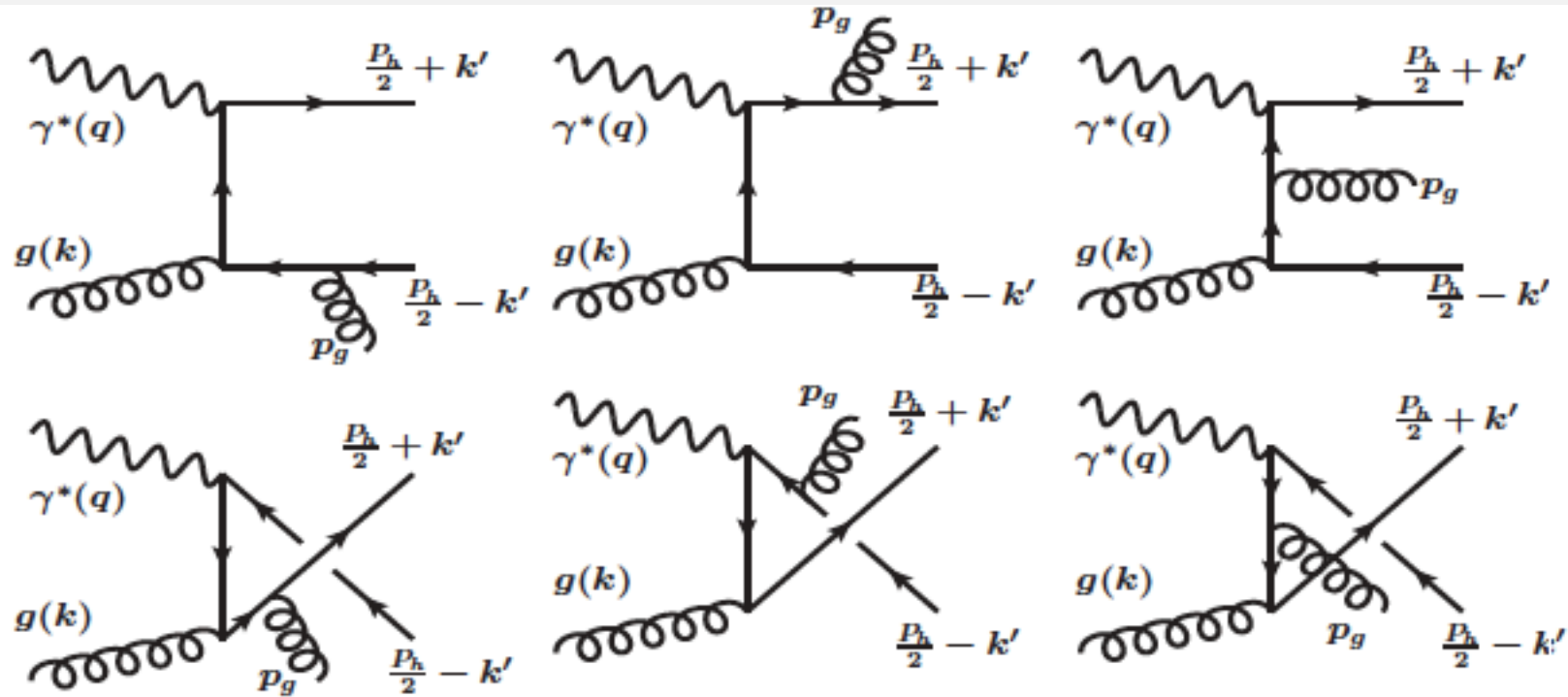


FIG. 1. Feynman diagrams for $\gamma^* + g \rightarrow J/\psi + g$ process

CALCULATION OF THE AMPLITUDE

Virtual diagrams not there, cutoff $z < 0.9$

The amplitude can be written as,

$$\mathcal{M}(\gamma^* g \rightarrow Q\bar{Q}[\psi^{2S+1}L_J^{(1)}](P_h) + g) = \sum_{L_z S_z} \int \frac{d^3 k'}{(2\pi)^3} \Psi_{LL_z}(k') \langle LL_z; SS_z | JJ_z \rangle \times \text{Tr}[O(q, k, P_h, k') \mathcal{P}_{SS_z}(P_h, k')]$$

The operators $O(q, k, P_h, k')$ are calculated from the Feynman diagrams

$$O(q, k, P_h, k') = \sum_{m=1}^6 C_m O_m(q, k, P_h, k').$$

$\mathcal{P}_{SS_z}(P_h, k')$ Is the projection operator that projects the spin triplet and singlet states

$$\begin{aligned} \mathcal{P}_{SS_z}(P_h, k') &= \sum_{s_1 s_2} \langle \frac{1}{2} s_1; \frac{1}{2} s_2 | SS_z \rangle v(\frac{P_h}{2} - k', s_1) \bar{u}(\frac{P_h}{2} + k', s_2) \\ &= \frac{1}{4M^{3/2}} (-\not{P}_h + 2\not{k}' + M) \Pi_{SS_z} (\not{P}_h + 2\not{k}' + M) + \mathcal{O}(k'^2) \end{aligned}$$

CALCULATION OF THE AMPLITUDE

Final expression

$$\mathcal{M}[^3S_1^{(1)}](P_h, k) = \frac{1}{4\sqrt{\pi M}} R_0(0) \frac{\delta_{ab}}{\sqrt{N_c}} \text{Tr} \left[\sum_{m=1}^3 O_m(0) (-\not{P}_h + M) \not{\epsilon}_{s_z} \right],$$

Where

$$\begin{aligned} \sum_{m=1}^3 O_m(0) = & g_s^2 (e e_c) \varepsilon_{\lambda_g}^{\rho*}(p_g) \left[\frac{\gamma_\nu (\not{P}_h - 2\not{q} + M) \gamma_\mu (-\not{P}_h - 2\not{p}_g + M) \gamma_\rho}{(\hat{s} - M^2)(\hat{u} - M^2 + q^2)} \right. \\ & + \frac{\gamma_\rho (\not{P}_h + 2\not{p}_g + M) \gamma_\nu (-\not{P}_h + 2\not{k} + M) \gamma_\mu}{(\hat{s} - M^2)(\hat{t} - M^2)} \\ & \left. + \frac{\gamma_\nu (\not{P}_h - 2\not{q} + M) \gamma_\rho (-\not{P}_h + 2\not{k} + M) \gamma_\mu}{(\hat{t} - M^2)(\hat{u} - M^2 + q^2)} \right]. \end{aligned}$$

CALCULATION OF THE AMPLITUDE

$k' \ll P_h$ Amplitude expanded in Taylor series about $k'=0$

First term in the expansion gives the S wave states ($L=0, J=0, 1$). Terms linear in k' give the P wave states ($L=1, J=0, 1, 2$).

As J/ψ is a 3S_1 state, in CS model we calculate the contribution only from this state

$$\begin{aligned} \mathcal{M}[^{2S+1}S_J^{(1)}](P_h, k) &= \frac{1}{\sqrt{4\pi}} R_0(0) \text{Tr}[O(q, k, P_h, k') \mathcal{P}_{SS_z}(P_h, k')] \Big|_{k'=0} \\ &= \frac{1}{\sqrt{4\pi}} R_0(0) \text{Tr}[O(0) \mathcal{P}_{SS_z}(0)], \end{aligned}$$

$$O(0) = O(q, k, P_h, k') \Big|_{k'=0}, \quad \mathcal{P}_{SS_z}(0) = \mathcal{P}_{SS_z}(P_h, k') \Big|_{k'=0}$$

Symmetry relations :

$$\begin{aligned} \text{Tr}[O_1(0)(-\not{P}_h + M)\not{\epsilon}_{s_z}] &= \text{Tr}[O_4(0)(-\not{P}_h + M)\not{\epsilon}_{s_z}] \\ \text{Tr}[O_2(0)(-\not{P}_h + M)\not{\epsilon}_{s_z}] &= \text{Tr}[O_5(0)(-\not{P}_h + M)\not{\epsilon}_{s_z}] \\ \text{Tr}[O_3(0)(-\not{P}_h + M)\not{\epsilon}_{s_z}] &= \text{Tr}[O_6(0)(-\not{P}_h + M)\not{\epsilon}_{s_z}] \end{aligned}$$

CALCULATION OF THE ASYMMETRY

- * We use a framework based on generalized parton model with the inclusion of intrinsic transverse momentum
- * Consider the region where $P_{h\perp} < M$ M is the mass of J/ψ
- * Impose cutoff $z < 0.9$ to keep outgoing gluon hard ,
Upper limit on $P_{h\perp}$ reduces fragmentation contribution from heavy quark
- * $0.3 < z$ to eliminate fragmentation of hard gluon into J/ψ

Three amplitudes and conjugates, with $i=1,2,3$

$$\mathcal{M}_i[{}^3S_1^{(1)}](P_h, k) = \frac{1}{4\sqrt{\pi M}} R_0(0) \frac{\delta_{ab}}{\sqrt{N_c}} \text{Tr} [O_i(0)(-\not{P}_h + M)\not{\epsilon}_{s_z}],$$

Cross section will have contributions of the form

$$M_i M_j = L^{\mu\mu'}(l, q) \Phi^{\nu\nu'}(x, k_\perp) \mathcal{M}_{i\mu\nu}^{\gamma^*+g \rightarrow J/\psi+g} \mathcal{M}_{j\mu'\nu'}^{*\gamma^*+g \rightarrow J/\psi+g}$$

CALCULATION OF THE ASYMMETRY

Final expression of the diff cross section

$$\frac{d\sigma}{dydx_Bdzd^2\mathbf{P}_{h\perp}} = \frac{1}{256\pi^4} \frac{1}{x_B^2 s^3 y^2 z(1-z)} \int k_{\perp} dk_{\perp} |M'|^2$$

Where $|M'|^2 = \int d\phi |M|^2$,

We are interested in small x , we neglect terms with higher powers of x_B , also keep terms only upto $\left(\frac{k_{\perp}^2}{M_p^2}\right)$

Leading terms to the asymmetry come from

$$\frac{d\sigma}{dydx_Bdzd^2\mathbf{P}_{hT}} = \frac{1}{256\pi^4} \frac{1}{x_B^2 s^3 y^2 z(1-z)} \int k_{\perp} dk_{\perp} \left\{ (A_0 + A_1 \cos\phi_h) f_1^g(x, \mathbf{k}_{\perp}^2) \right\} + \frac{k_{\perp}^2}{M_p^2} \left\{ (B_0 + B_1 \cos\phi_h + B_2 \cos 2\phi_h) h_1^{1g}(x, \mathbf{k}_{\perp}^2) \right\}$$

$$\begin{aligned} A_0 = & 64\pi M^4 \left\{ M^2(z-1)(M^2 + P_{h\perp}^2 - s x_B y(z-2)z)(M^2(z-1)(2z-3) \right. \\ & + P_{h\perp}^2(2z(4z-9)+9) - s y(z-1)(x(z+1)(2z-3) \\ & + x_B z(3(y(y+6)-6)z^2 - 6(y(y+4)-3)z + 4y+3) + 2x_B y + x_B)) \\ & + M^4 z(-2M^2(z-1)^2(3z-5) - 4P_{h\perp}^2(z-1)(z(6z-13)+8) \\ & + s y(x(3z-10)(z-1)^2 + x_B z((50-31y(y+2))z^3 \\ & + (y(57y+118)-80)z^2 - y(43y+106)z \\ & + y(9y+46)+78z-56) + 2x_B((y-4)y+10))) \left. \right\} / s \\ & + 2xy z^2 (M^4(z-1)^3 + M^2(P_{h\perp}^2(z-1)(8(z-1)z+3) \\ & + s y(x_B(z((1-6y(y+2))z^2 + 2y(y+4)z - 2y+z+1) - 1) \\ & - x(z-1)^3)) + P_{h\perp}^4(z(z(11z-23)+13)-3) \\ & + P_{h\perp}^2 s y(x(z(-2(z-5)z-9)+3) + x_B(z(-3(y(5y+16)-6)z^2 \\ & + 2(y(y+14)-5)z - 6y+7) - 3)) \\ & + s^2 x x_B y^2 z(z((y-2)y(2z-1)+2(8z-7)+4)) \left. \right\} / \{x^2 y^3 (z-1)^2 z^3 \\ & \times (M^2 + s y(x_B - x))^2 (M^2 + P_{h\perp}^2 - s x_B y(z-2)z)^2 \} \end{aligned}$$

R. Kishore and AM;
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$$\langle \cos(2\phi_h) \rangle = \frac{\int d\phi_h \cos(2\phi_h) d\sigma}{\int d\phi_h d\sigma}$$

$$P_{hT} = P_{h\perp}$$

PARAMETRIZATION OF THE TMDs

Linearly polarized gluon distribution satisfies the positivity bound
 Upper limit of asymmetry obtained when this bound is saturated

$$\frac{k_{\perp}^2}{2M_p^2} \left| h_1^{\perp g}(x, \mathbf{k}_{\perp}^2) \right| \leq f_1^g(x, \mathbf{k}_{\perp}^2)$$

Gaussian parametrization satisfy positivity bound but does not saturate it

$$f_1^g(x, \mathbf{k}_{\perp}^2) = f_1^g(x, \mu) \frac{1}{\pi \langle k_{\perp}^2 \rangle} e^{-k_{\perp}^2 / \langle k_{\perp}^2 \rangle}$$

Boer and Pisano (2012)

We took $r=1/3$

$$\langle k_{\perp}^2 \rangle = 0.25 \text{ GeV}^2.$$

$$h_1^{\perp g}(x, \mathbf{k}_{\perp}^2) = \frac{M_p^2 f_1^g(x, \mu)}{\pi \langle k_{\perp}^2 \rangle^2} \frac{2(1-r)}{r} e^{1 - \frac{k_{\perp}^2}{r \langle k_{\perp}^2 \rangle}}$$

In small x region the WW gluon distributions are calculated in McLerran-Venugopalan (MV) model

S_{\perp} is the transverse size of the proton

Q_{sg} is the saturation scale

$$f_1^g(x, \mathbf{k}_{\perp}^2) = \frac{S_{\perp} C_F}{\alpha_s \pi^3} \int dr \frac{J_0(k_{\perp} r)}{r} \left(1 - e^{-\frac{r^2}{4} Q_{sg}^2(r)} \right)$$

$$h_1^{\perp g}(x, \mathbf{k}_{\perp}^2) = \frac{S_{\perp} C_F}{\alpha_s \pi^3} \frac{2M_p^2}{k_{\perp}^2} \int dr \frac{J_2(k_{\perp} r)}{r \log\left(\frac{1}{r^2 \Lambda_{QCD}^2}\right)} \left(1 - e^{-\frac{r^2}{4} Q_{sg}^2(r)} \right)$$

$$Q_{sg}^2 = \alpha_s N_c \mu_A \ln \frac{1}{r^2 \Lambda_{QCD}^2} \quad \mu_A S_{\perp} = \alpha_s 2\pi A.$$

For proton $A=1$

McLerran and Venugopalan, PRD (1994)

PARAMETRIZATION OF THE TMDS

MV model calculation for WW gluon distributions in small x region : for large nucleus or energetic proton

We use a regulated version of MV model

Bacchetta, Boer, Pisano, Taelis (2018)

$$\frac{k_{\perp}^2}{2M_p^2} \frac{h_1^{\perp g}(x, k_{\perp}^2)}{f_1^g(x, k_{\perp}^2)} = \frac{\int dr \frac{J_2(k_{\perp} r)}{r \log\left(\frac{1}{r^2 \Lambda_{QCD}^2}\right)} \left(1 - e^{-\frac{r^2}{4} Q_{sg0}^2 \log\left(\frac{1}{r^2 \Lambda_{QCD}^2}\right)}\right)}{\int dr \frac{J_0(k_{\perp} r)}{r} \left(1 - e^{-\frac{r^2}{4} Q_{sg0}^2 \log\left(\frac{1}{r^2 \Lambda_{QCD}^2}\right)}\right)}$$

A regulator is added to the exponent and the log for numerical convergence

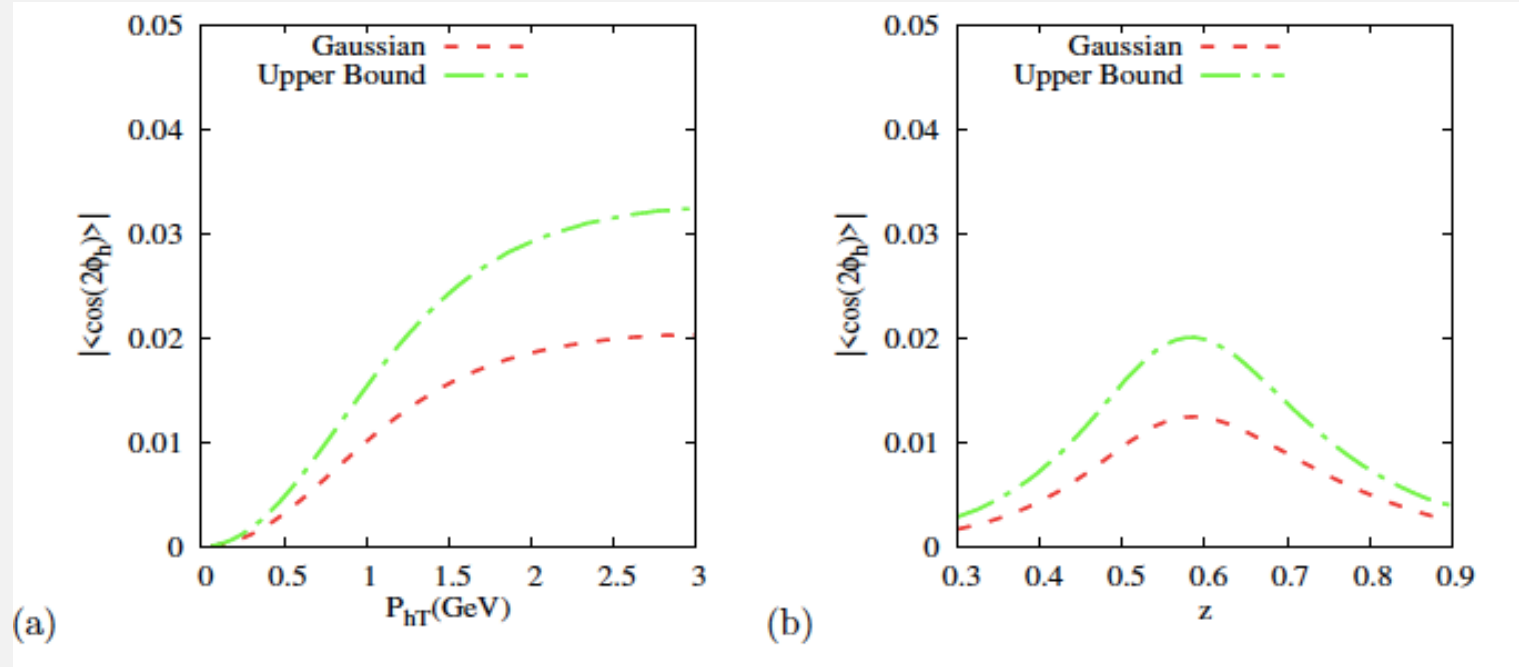
$$Q_{sg0}^2 = (N_c/C_F) \times Q_{s0}^2$$

$$Q_{s0}^2 = 0.35 \text{ GeV}^2 \text{ at } x = 0.01 \text{ and } \Lambda_{QCD} = 0.2 \text{ GeV}$$

From the fits to HERA data

The ratio is less than 1 for all values of transverse momentum

RESULTS



Asymmetry at EIC

$$\sqrt{s} = 45 \text{ GeV}$$

$$x_B = 0.01$$

Integration ranges

$$0 < P_{hT} < 3 \text{ GeV}$$

$$0.3 < z < 0.9$$

$$0.05 < y < 0.4$$

Upper bound of the asymmetry is obtained by saturating the positivity bound, Gaussian indicates using Gaussian parametrization of the TMDs

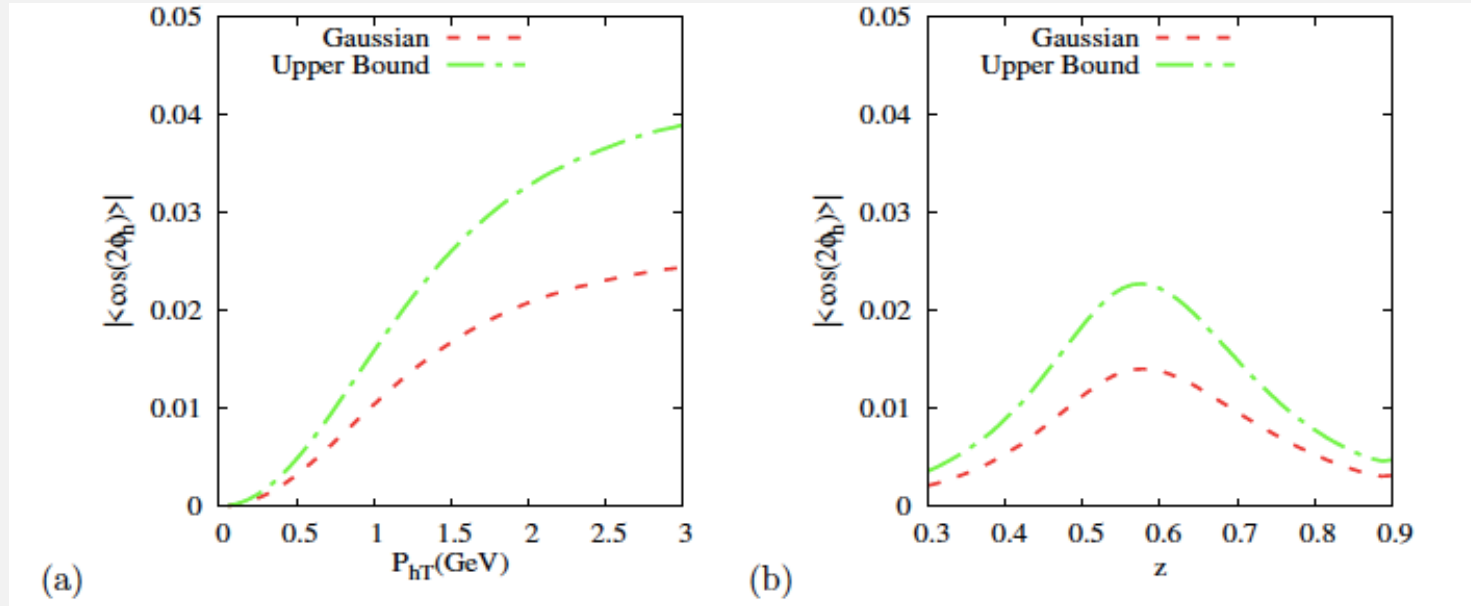
We have used MSTW2008 pdfs, TMD evolution not used Small x : affects x_B and Q^2

y is constrained by Q^2 and x_B

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$$1 < Q^2 < 9 \text{ GeV}^2$$

RESULTS



Asymmetry at EIC

$$\sqrt{s} = 150 \text{ GeV}$$

$$x_B = 0.01$$

$$0 < P_{hT} < 3 \text{ GeV}$$

$$0.3 < z < 0.9$$

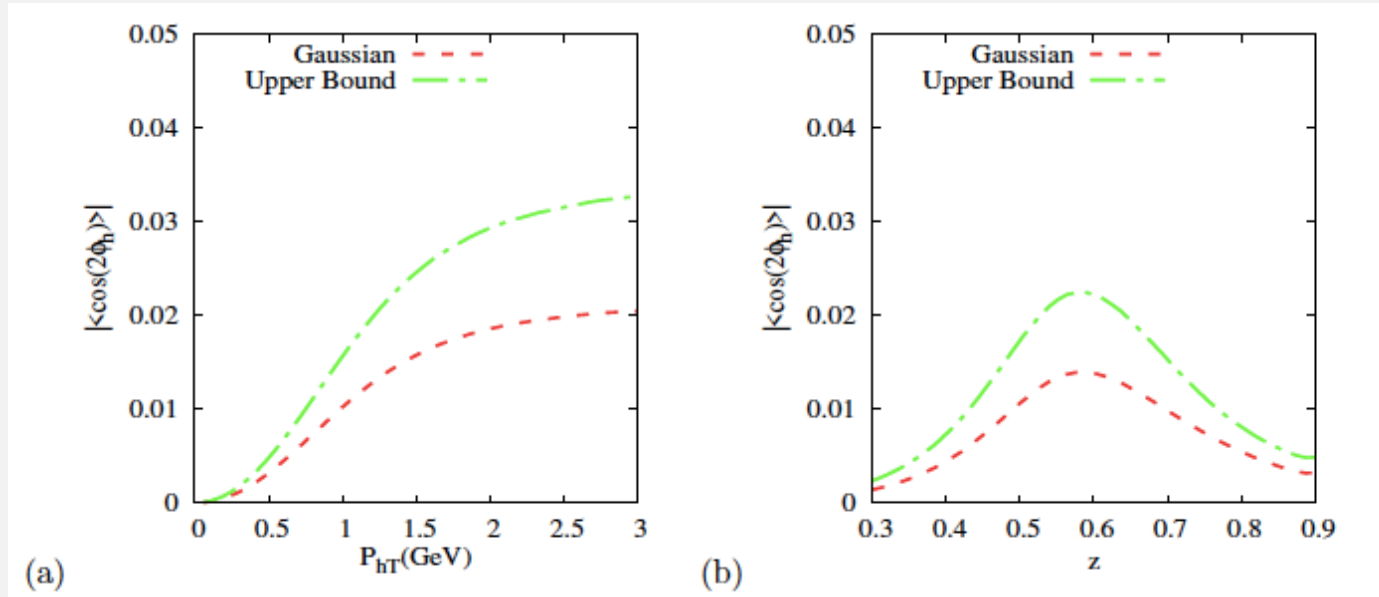
$$0.005 < y < 0.04$$

Asymmetry is negative, consistent with LO estimate. Plot shows magnitude of the asymmetry

Magnitude increases with increase of cm energy

Asymmetry independent of specific choice of LDME in CS model as only one state contributes. However cross section depends on the LDME set

RESULTS



$$\sqrt{s} = 190 \text{ GeV}$$

$$x_B = 0.005$$

$$0 < P_{hT} < 3 \text{ GeV}$$

$$0.3 < z < 0.9$$

$$0.006 < y < 0.05$$

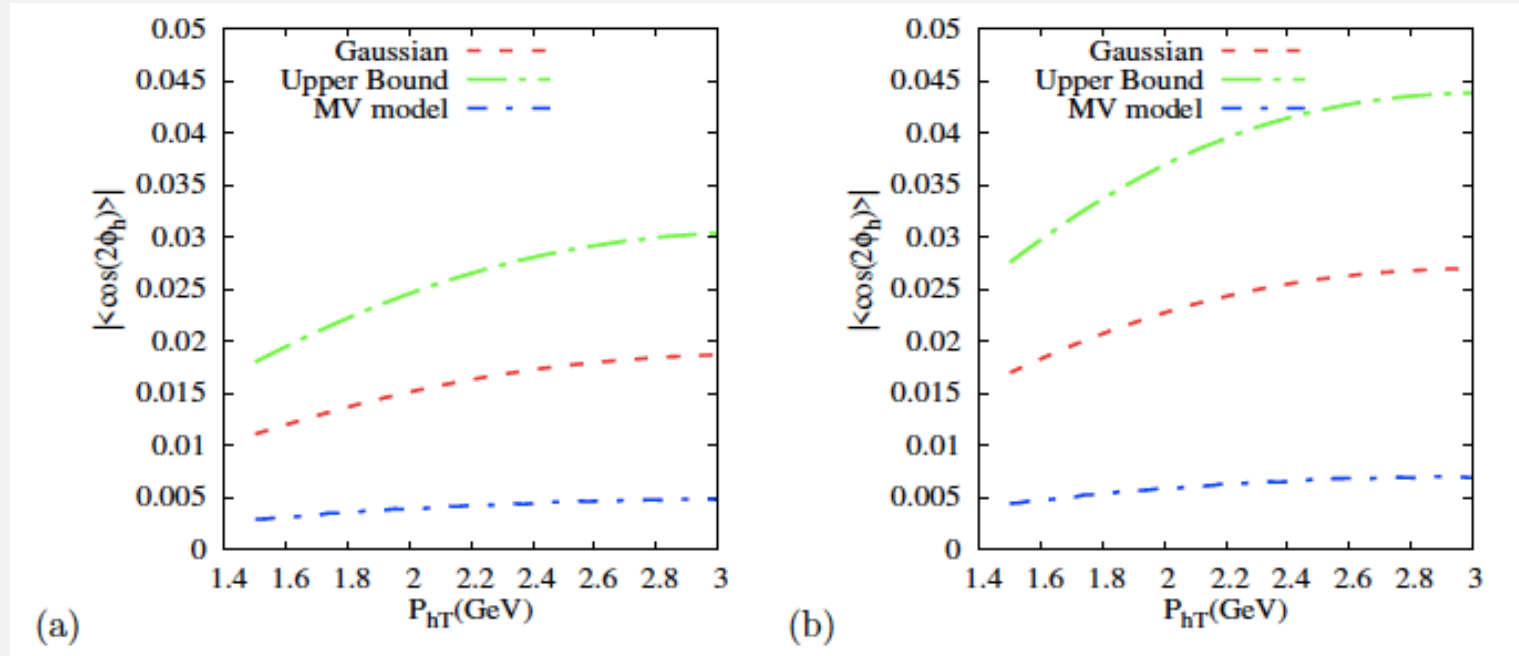
$$1 < Q^2 < 9 \text{ GeV}^2$$

Asymmetry decreases for smaller x_B

y is constrained by Q^2 and x_B

Reaches a peak around $z = 0.6$

RESULTS



$$Q^2 = 9 \text{ GeV}^2$$

$$(a) x = 0.01, z = 0.5$$

$$(b) x = 0.01, z = 0.7$$

$$0.2 < y < 1$$

Asymmetry for fixed Q^2

For (a) \sqrt{s} is in the range 61 to 181 GeV, and for (b) \sqrt{s} is in the range 58 to 182 GeV.

MV model gives much smaller asymmetry than the Gaussian for fixed values of x and z , asymmetry increases as z increases

LO CALCULATION IN NRQCD BASED CO MODEL

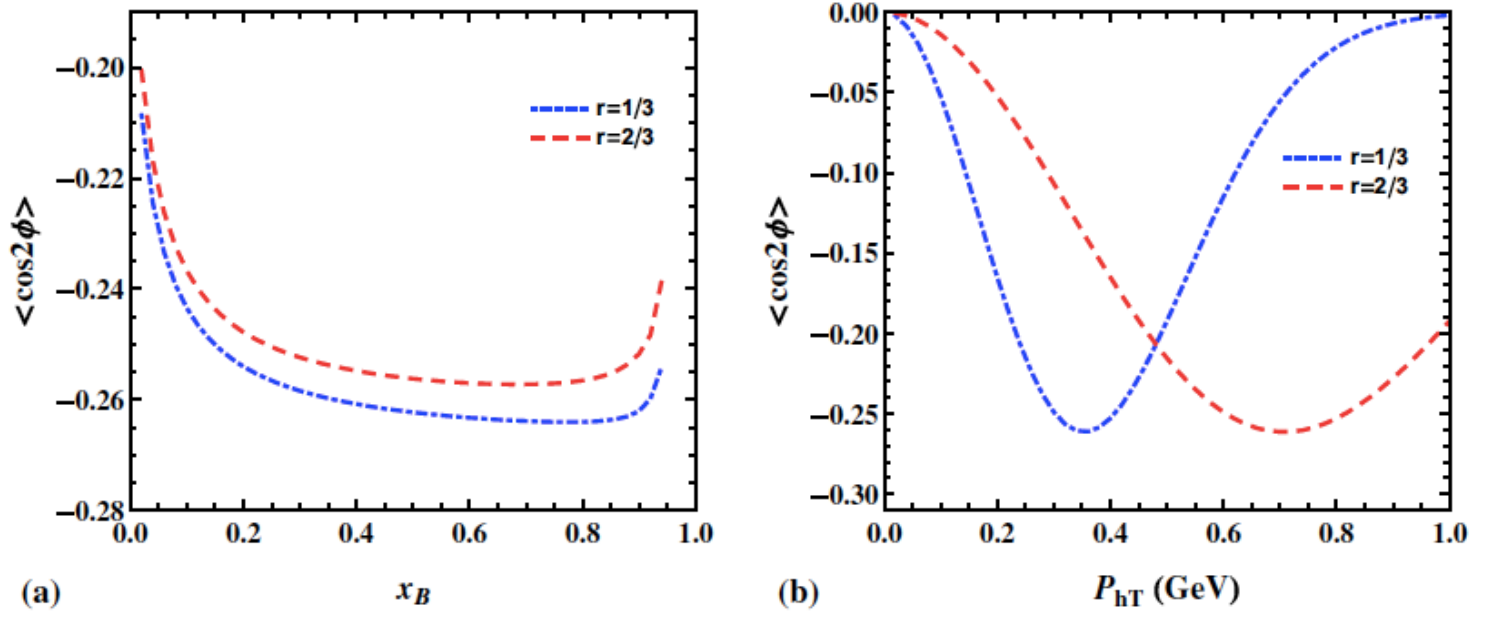
AM, Rajesh, EPJC 77, 854 (2017)

Production of J/ψ through LO process

$$\gamma^* + g \rightarrow c + \bar{c}$$

Contributes at $z = 1$

NRQCD based CO model



The integration ranges are $0 < P_{hT} < 1.0$ GeV, $0.1 < y < 0.9$ and $0.0001 < x_B < 0.9$

Contributions from $^1S_0, ^3P_0, ^3P_1, ^3P_2$ states have been considered

Gaussian parametrization of TMDs are used

26 % negative asymmetry at EIC energy

SUMMARY AND CONCLUSION

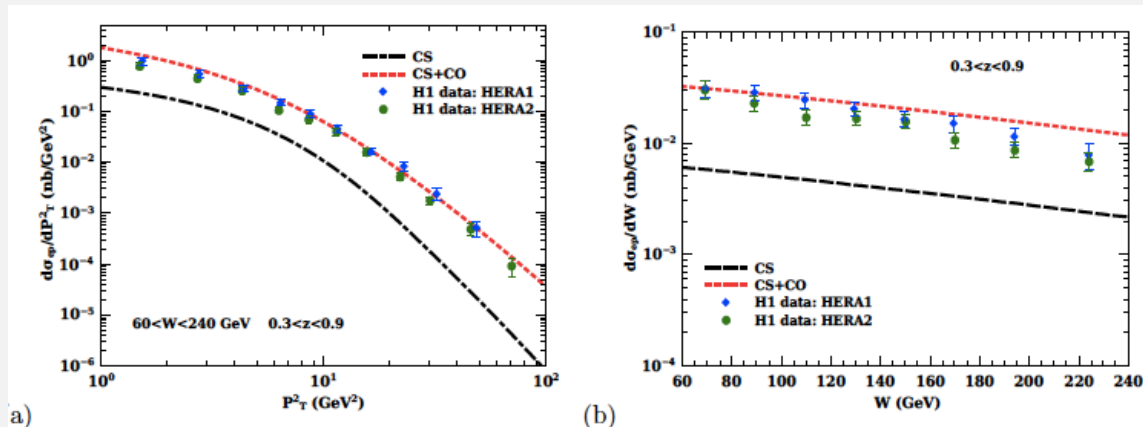
Presented a calculation of $\cos 2\phi$ asymmetry in the electroproduction of J/ψ production at the EIC

In the kinematical region $z < 1$ and small x , where contribution from the process $\gamma^* + g \rightarrow c + \bar{c} + g$ dominates

In this process the WW gluon distributions are probed; asymmetry probes the WW linearly polarized gluon distribution

We calculated the asymmetry in NRQCD based CS model

Presented the upper limit of the asymmetry as well as predictions using a Gaussian model and MV model for the TMDs



Asymmetry small but sizable

Future work would involve calculation of the contributions of CO states to the asymmetry

These play an important role in explaining the data for unpolarized cross section from HERA



QCD with Electron-Ion Collider (QEIC)

<https://indico.cern.ch/event/797767/>

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