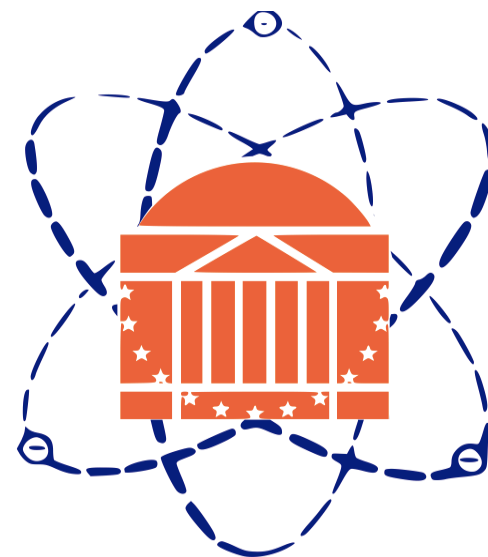


NUCLEAR FEMTOGRAPHY AS A BRIDGE FROM THE NUCLEON TO NEUTRON STARS

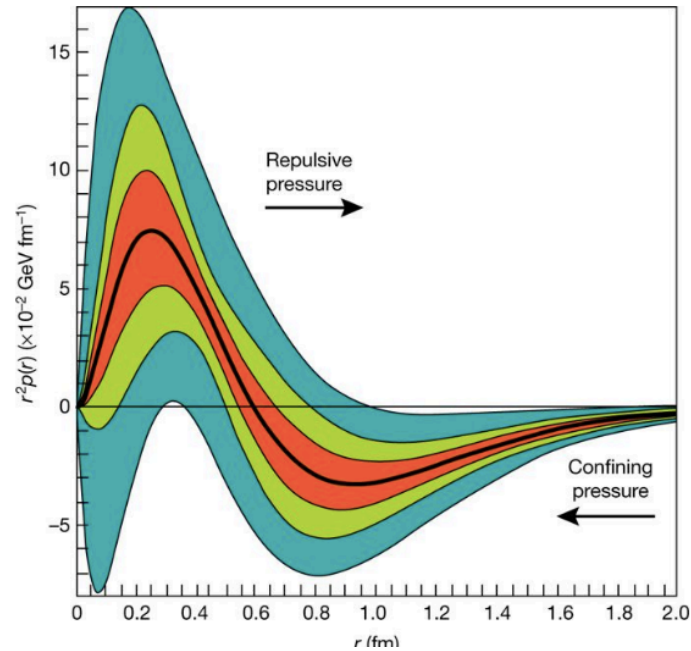
QCD EVOLUTION 2019

ARGONNE NATIONAL LAB, MAY 13-17, 2019

Simonetta Liuti
University of Virginia



Burkert, Elouadrhiri, Girod, Nature 557, 396 (2018)



“The average peak pressure near the center is about 10^{35} pascals, which exceeds the pressure estimated for the most densely packed known objects in the Universe, neutron stars”

How is the pressure radial distribution extracted from data? (How does the proton/neutron get its mass and spin?)

$$\mathcal{L}_{QCD} = \bar{\psi} (i\gamma_{\mu} D^{\mu} - m) \psi - \frac{1}{4} F_{a,\mu\nu} F_a^{\mu\nu}$$

Invariance of \mathcal{L}_{QCD} under **translations** and **rotations**

Energy Momentum Tensor

from **translation** inv. \rightarrow
$$T_{QCD}^{\mu\nu} = \frac{1}{4} \bar{\psi} \gamma^{(\mu} D^{\nu)} \psi + Tr \left\{ F^{\mu\alpha} F_{\alpha}^{\nu} - \frac{1}{2} g^{\mu\nu} F^2 \right\}$$

Angular Momentum Tensor

from **rotation** inv. \rightarrow
$$M_{QCD}^{\mu\nu\lambda} = x^{\nu} T_{QCD}^{\mu\lambda} - x^{\lambda} T_{QCD}^{\mu\nu}$$

S=1/2



Parametrization of QCD EMT matrix element between proton states (X. Ji, 1997)

$$\begin{aligned}
 \langle p', \Lambda | T_{\mathbf{q}, \mathbf{g}}^{\mu\nu} | p, \Lambda \rangle = & A(t) \bar{U}(p', \Lambda') [\gamma^\mu P^\nu + \gamma^\nu P^\mu] U(p, \Lambda) + B(t) \bar{U}(p', \Lambda') i \frac{\sigma^{\mu(\nu} \Delta^{\nu)} }{2M} U(p, \Lambda) \\
 & + C(t) [\Delta^2 g^{\mu\nu} - \Delta^{\mu\nu}] \bar{U}(p', \Lambda') U(p, \Lambda) + \tilde{C}(t) g^{\mu\nu} \bar{U}(p', \Lambda') U(p, \Lambda)
 \end{aligned}$$

forward

off-forward

q and g not separately conserved

$$\left\{ \begin{aligned}
 P &= \frac{p + p'}{2} \\
 \Delta &= p' - p = q - q' \\
 t &= (p - p')^2 = \Delta^2
 \end{aligned} \right.$$

Direct calculation of EMT form factors

Donoghue et al. PLB529 (2002)

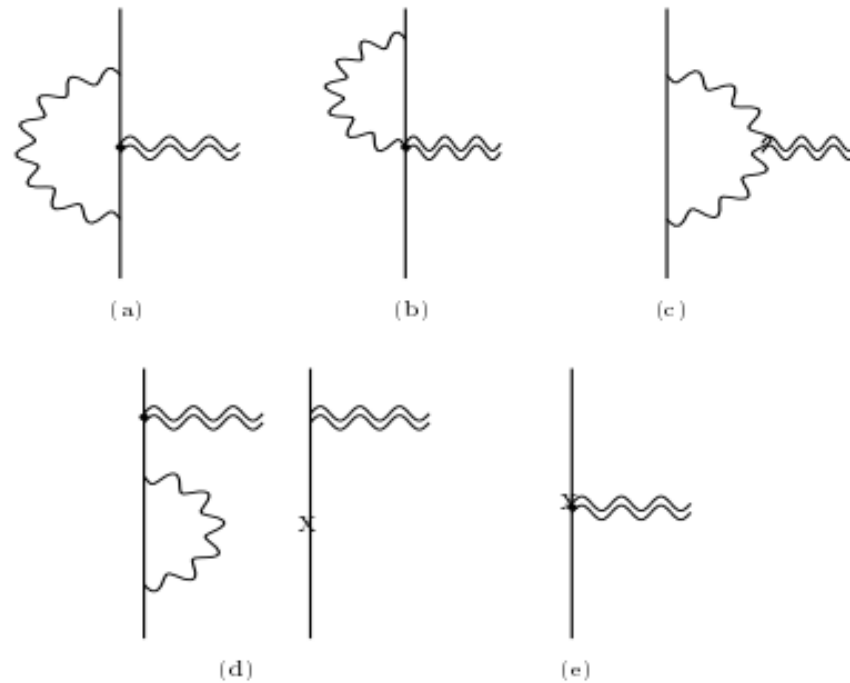
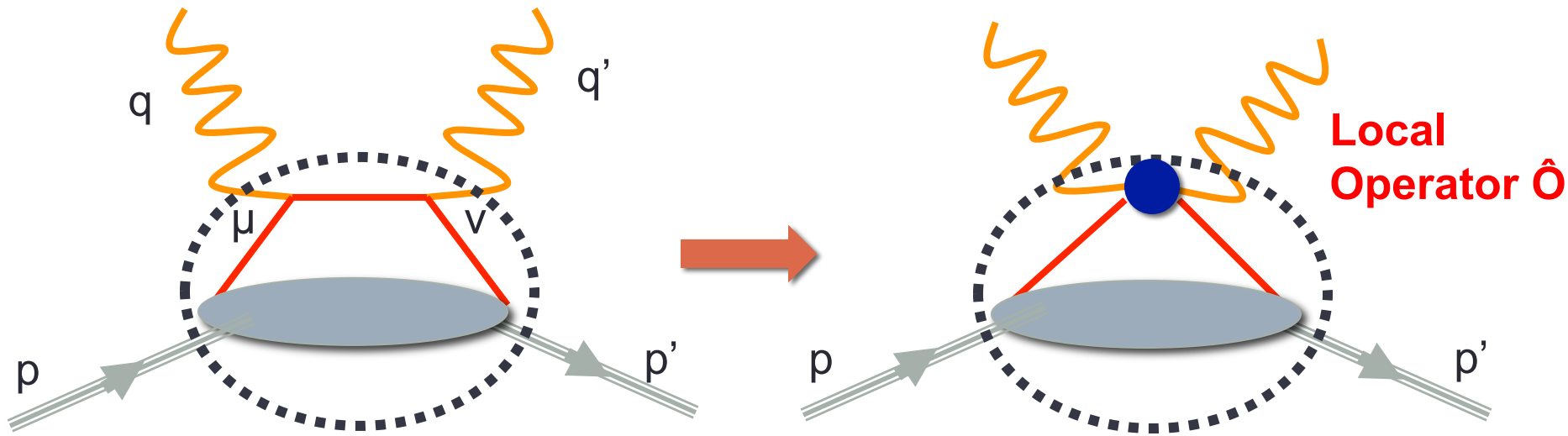


Figure 2: Feynman diagrams for spin 1/2 radiative corrections to $T_{\mu\nu}$.

GPDs and EMT matrix elements



$Q^2 \gg M^2 \rightarrow$ “deep”

$W^2 \gg M^2 \rightarrow$ equivalent to an “inelastic”
process but not directly accessible

2nd Mellin moments

Nucleon

From OPE  From EMT

$$\int dx x H(x, \xi, t) = A_{20}(t) + \xi^2 C_{20}(t) \equiv A(t) + \xi^2 C(t) \leftarrow \text{D-term}$$

$$\int dx x E(x, \xi, t) = B_{20}(t) - \xi^2 C_{20}(t) \equiv B(t) - \xi^2 C(t)$$

Physical interpretation of EMT form factors

$$\frac{1}{2} (A_q + B_q) = \boxed{J_q} = \frac{1}{2} (A_{20} + B_{20}) \Rightarrow J_q^i = \int d^3r \epsilon^{ijk} r_j T_{0k}$$

$$A_q = \boxed{\langle x_q \rangle} = A_{20}$$

$$C_q = \boxed{\text{Internal Forces}} = C_{20} \Rightarrow \int d^3r (r^i r^j - \delta^{ij} r^2) T_{ij}$$

C (D-term) is related to pressure

Static approximation

$$T_{ij} = \left(\frac{r_i r_j}{r^2} - \frac{1}{3} \delta_{ij} \right) s(r) + \delta_{ij} p(r)$$

Landau&Lifshitz, Vol.7

M. Polyakov, hep-ph/0210165

M. Polyakov, P. Schweitzer, arXiv:1805.06596

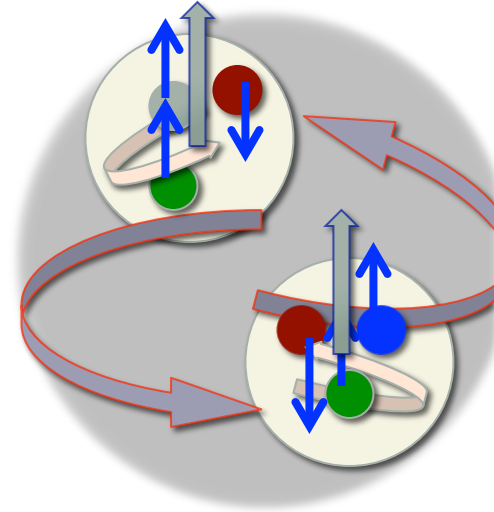
C is a measure of the **Elastic Free Energy** in the proton

Energy Momentum Tensor in a spin 1 system

Angular momentum sum rule for spin one hadronic systems

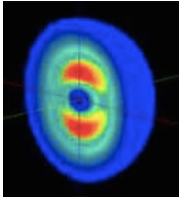
Swadhin K. Taneja,^{1,*} Kunal Kathuria,^{2,†} Simonetta Liuti,^{2,‡} and Gary R. C. S. ^{3,§}

PRD86(2012)



7 conserved form factors!

S=1



$$\begin{aligned}
 \langle p', \Lambda' | p, \Lambda \rangle = & -\frac{1}{2} P^\mu P^\nu (\epsilon'^* \epsilon) \mathcal{G}_1(t) - \frac{1}{4} P^\mu P^\nu \frac{(\epsilon P)(\epsilon'^* P)}{M^2} \mathcal{G}_2(t) \\
 & -\frac{1}{2} [\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2] (\epsilon'^* \epsilon) \mathcal{G}_3(t) - \frac{1}{4} [\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2] \frac{(\epsilon P)(\epsilon'^* P)}{M^2} \mathcal{G}_4(t) \\
 & +\frac{1}{4} [(\epsilon'^{* \mu} (\epsilon P) + \epsilon^\mu (\epsilon'^* P)) P^\nu + \mu \leftrightarrow \nu] \mathcal{G}_5(t) \\
 & +\frac{1}{4} [(\epsilon'^{* \mu} (\epsilon P) - \epsilon^\mu (\epsilon'^* P)) \Delta^\nu + \mu \leftrightarrow \nu + 2g_{\mu\nu} (\epsilon P)(\epsilon'^* P) - (\epsilon'^{* \mu} \epsilon^\nu + \epsilon'^{* \nu} \epsilon^\mu) \Delta^2] \mathcal{G}_6(t) \\
 & +\frac{1}{2} [\epsilon'^{* \mu} \epsilon^\nu + \epsilon'^{* \nu} \epsilon^\mu] \mathcal{G}_7(t) + g^{\mu\nu} (\epsilon'^* \epsilon) M^2 \mathcal{G}_8(t)
 \end{aligned}$$

General rule to count form factors: t-channel J^{PC} q. numbers

n	$J^{PC}(S; L)$				
0	0^{+-}	$1^{--}(1; 0, 2)$			
1	$0^{++}(1; 1)$	1^{-+}	$2^{++}(1; 1, 3)$		
2	0^{+-}	$1^{--}(1; 0, 2)$	2^{+-}	$3^{--}(1; 2, 4)$	
3	$0^{++}(1; 1)$	1^{-+}	$2^{++}(1; 1, 3)$	3^{-+}	$4^{++}(1; 3, 5)$
...			...		

Haegler, PLB(2004)
Z.Chen&Ji, PRD(2005)

TABLE III: J^{PC} of the vector operators with $(S; L, L')$ for the corresponding $N\bar{N}$ state. Where there are no $(S; L, L')$ values there are no matching quantum numbers for the $N\bar{N}$ system.

Nucleon

	$L = 0$	1	2	3	4	...
$S = 0$	$J^{PC} 0^{-+}$	1^{+-}	2^{-+}	3^{+-}	4^{-+}	
$S = 1$	1^{--}	0^{++}	1^{--}	2^{++}	3^{--}	
		1^{++}	2^{--}	3^{++}	4^{--}	
		2^{++}	3^{--}	4^{++}	5^{--}	

TABLE I: J^{PC} of the $N\bar{N}$ states.

Deuteron

	$L = 0$	1	2	3	4	...
$S = 0$	$J^{PC} 0^{++}$	1^{--}	2^{++}	3^{--}	4^{++}	
$S = 1$	1^{+-}	0^{-+}	1^{+-}	2^{-+}	3^{+-}	
		1^{-+}	2^{+-}	3^{-+}	4^{+-}	
		2^{-+}	3^{+-}	4^{-+}	5^{+-}	
$S = 2$	2^{++}	1^{--}	0^{++}	1^{--}	2^{++}	
		2^{--}	1^{++}	2^{--}	3^{++}	
		3^{--}	2^{++}	3^{--}	4^{++}	
			3^{++}	4^{--}	5^{++}	
			4^{++}	5^{--}	6^{++}	

TABLE II: J^{PC} of the $d\bar{d}$ states.

Taneja et al., PRD86(2012)

Deuteron

From OPE \longleftrightarrow From EMT

$$2 \int dxx [H_1(x, \xi, t) - \frac{1}{3} H_5(x, \xi, t)] = \mathcal{G}_1(t) + \xi^2 \mathcal{G}_3(t)$$

Momentum

$$2 \int dxx H_2(x, \xi, t) = \mathcal{G}_5(t)$$

Angular Momentum

$$2 \int dxx H_3(x, \xi, t) = \mathcal{G}_2(t) + \xi^2 \mathcal{G}_4(t)$$

Quadrupole

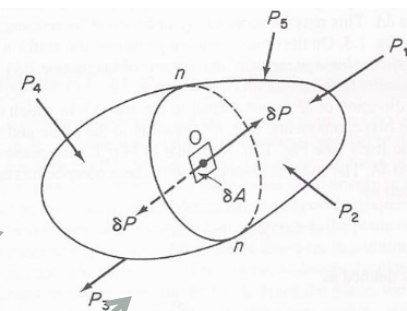
$$-4 \int dxx H_4(x, \xi, t) = \xi \mathcal{G}_6(t)$$

 ξ -odd

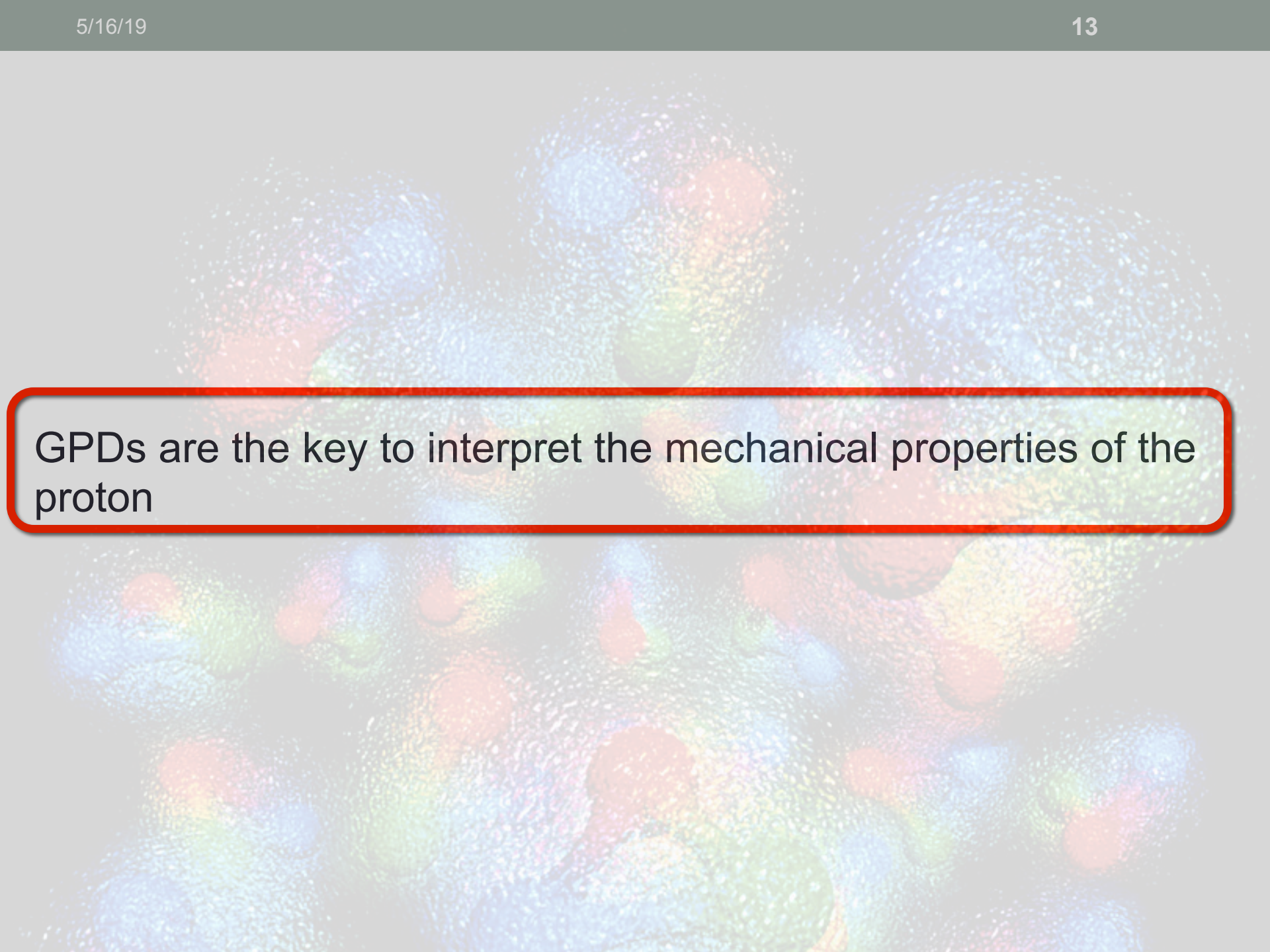
$$\int dxx H_5(x, \xi, t) = -\frac{t}{8M_D^2} \mathcal{G}_6(t) + \frac{1}{2} \mathcal{G}_7(t)$$

Connected to b_1 SR

Double flip
D-term
dependent on
polarization

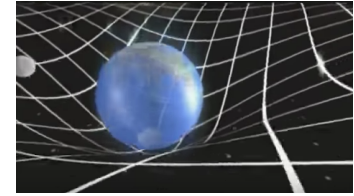


Connecting with observables: work in progress with Brandon Kriesten and Adam Freese



GPDs are the key to interpret the mechanical properties of the proton

EMT and the source of the gravitational field



action $S = \int \sqrt{-g} \left[\frac{1}{16\pi G} \mathcal{R} - \mathcal{L}_m \right] d^4x$

Ricci scalar \rightarrow curvature
Flat space

Matter and energy in the universe $\rightarrow T_{\mu\nu} = \frac{\delta \mathcal{L}_m}{\delta g^{\mu\nu}} + g_{\mu\nu} \mathcal{L}_m$

Spacetime geometry $\rightarrow G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$

Ricci tensor

Einstein's Eqs. $\rightarrow G_{\mu\nu} = 8\pi G T_{\mu\nu}$

Using the metric, $g_{\mu\nu}$, one can calculate:

- the curvature tensor $R_{\mu\nu}$
- the Einstein tensor $G_{\mu\nu}$
- the stress energy tensor for a perfect fluid $T_{\mu\nu} = (p + \epsilon)u_\mu u_\nu + pg_{\mu\nu}$

TOV Equations

energy density pressure matter density

$$\frac{dp}{dr} = - \frac{G(\epsilon(r) + 4\pi r^3 p(r)/c^2)(p(r)/c^2 + \rho)}{r(r - 2G\epsilon(r)/c^2)}$$

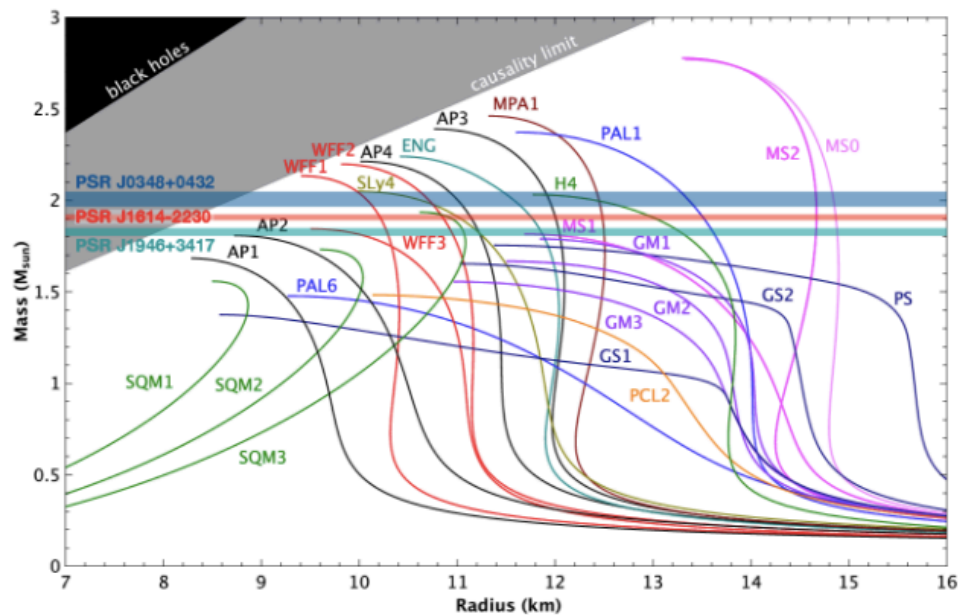
$$\frac{dM}{dr} = 4\pi\rho r^2$$

Knowing the EoS

EoS $\rho(r)$ - $\epsilon(r)$ relation

One can solve TOV to find the mass radius relation

Constraints from Pulsar Masses

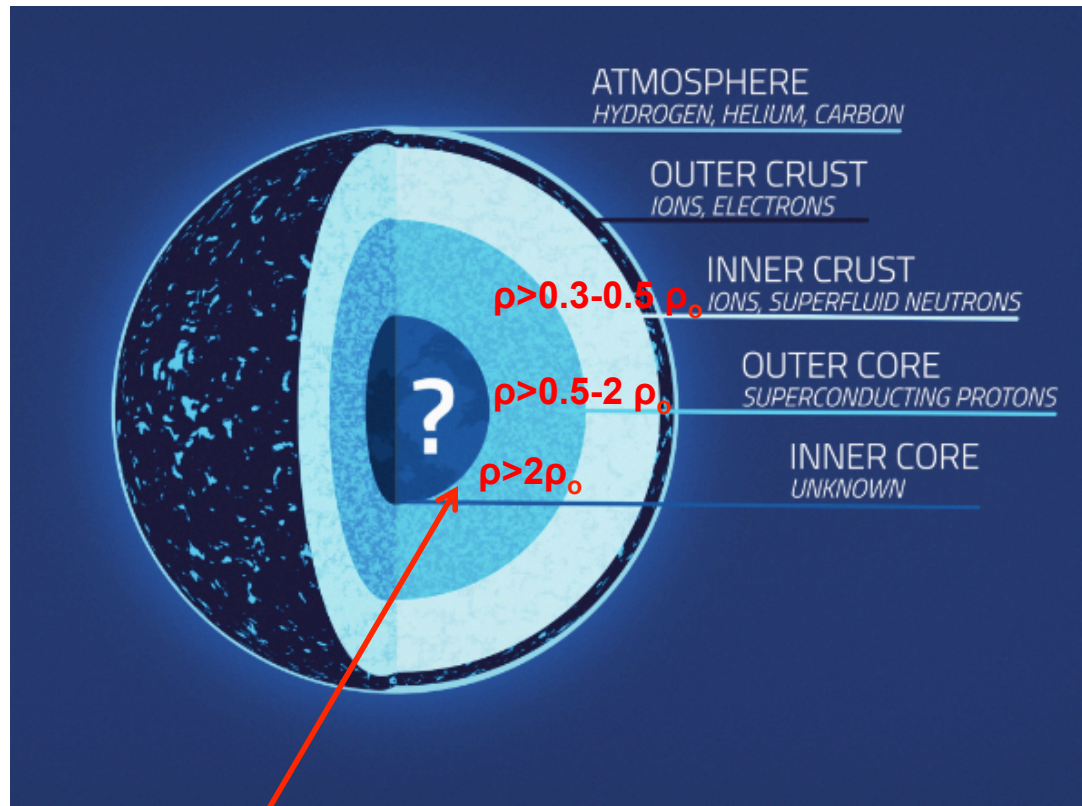


Özel and Freire, <http://xtreme.as.arizona.edu/NeutronStars>

J. Antoniadis et al., *Science* 340, 6131 (2013)
 P. B. Demorest et al., *Nature* 467 (2010)

Constraints require the EoS to be stiff → consistent with predictions for ordinary nuclear matter composed of mostly neutrons and few protons including three body interactions

What governs the EoS of neutron stars?

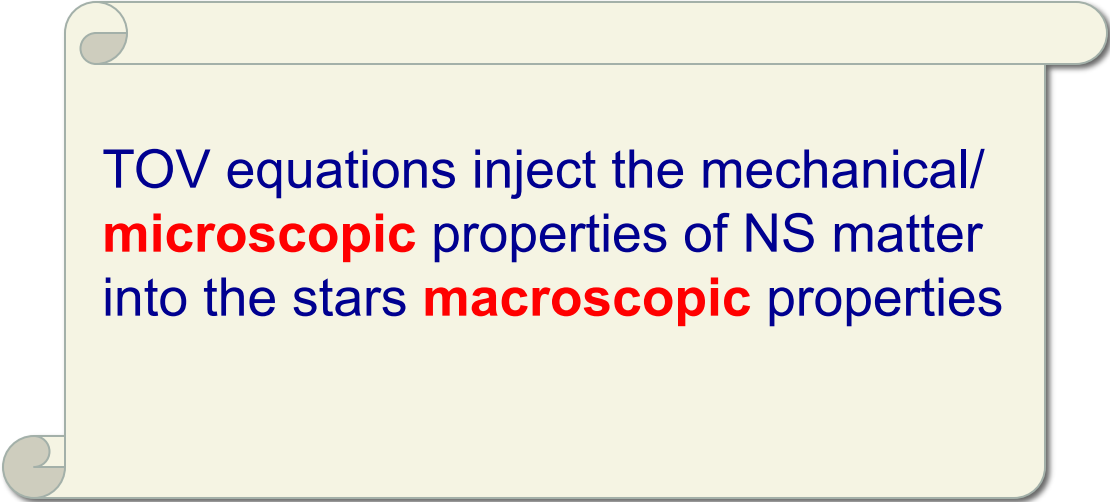


<https://svs.gsfc.nasa.gov/20267>

Due to their extreme compactness, the central density of neutron stars exceeds the nuclear saturation density, $\rho_0 = 0.16 \text{ fm}^{-3}$

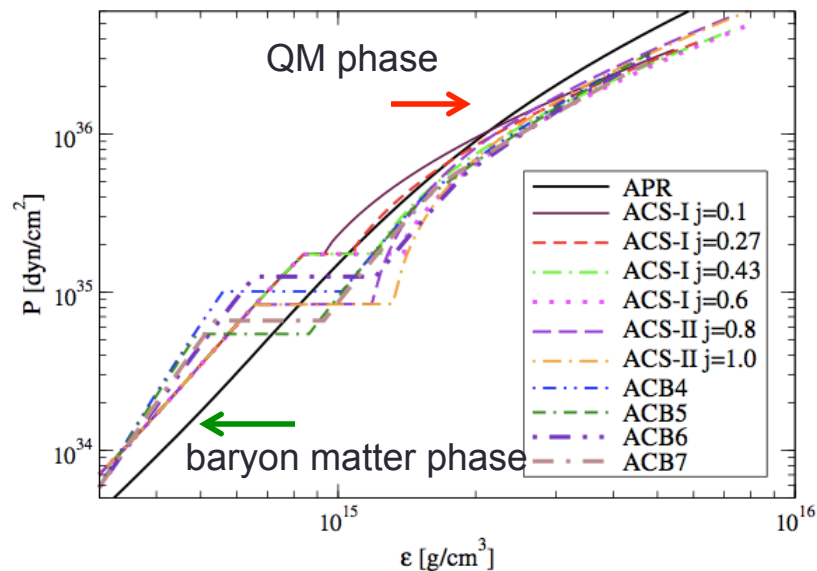
Most observed neutron star masses are $> 1.3M_{\odot}$

Gravitational collapse is countered by pressure originating from nuclear forces

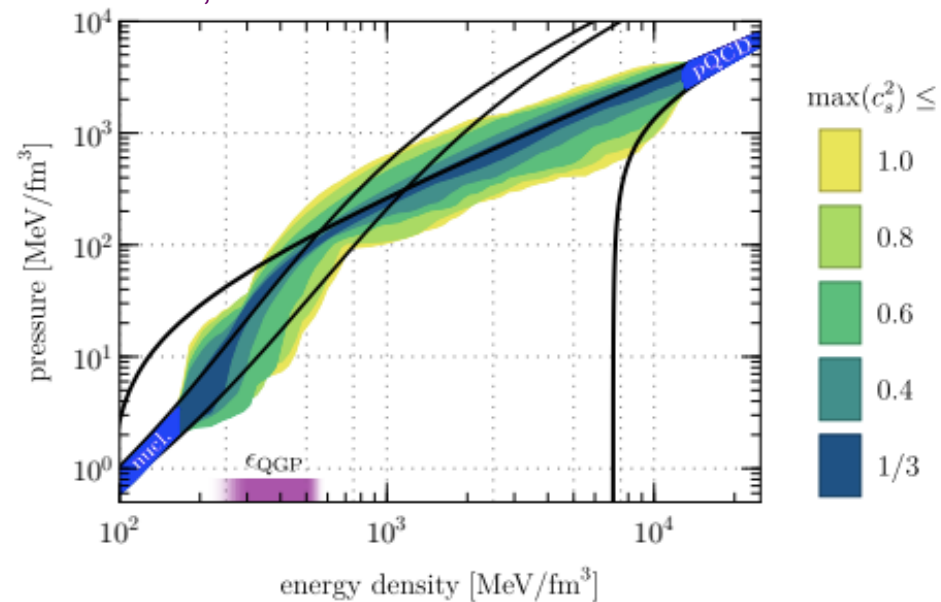


TOV equations inject the mechanical/
microscopic properties of NS matter
into the stars **macroscopic** properties

Pascalidhis et al., arXiv:1712.00451



Annala, E., Gorda, T., Kurkela, A., Nattila, J., & Vuorinen, A. arXiv:1903.09121



“...the existence of quark-matter cores inside very massive NSs should be considered the standard scenario, not an exotic alternative. QM is altogether absent in NS cores only under very specific conditions, ...”

- We propose a new, **model independent** way of evaluating the EoS in the quark matter phase by inferring it directly from the matrix elements of the QCD Energy Momentum Tensor (EMT) between nucleon states.
- **Model independent means relating measurement to measurement**
- The ingredients of our calculation are **GPDs**
- This allows us to introduce **spatial coordinates/distance** in the picture in a novel way

Based on

Bounds on the Equation of State of Neutron Stars from High Energy Deeply Virtual Exclusive Experiments

Simonetta Liuti,^{1,*} Abha Rajan,^{2,†} and Kent Yagi^{1,‡}

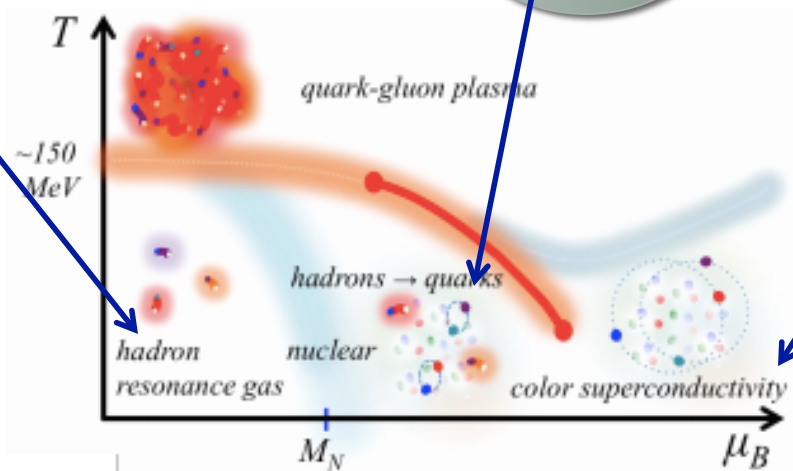
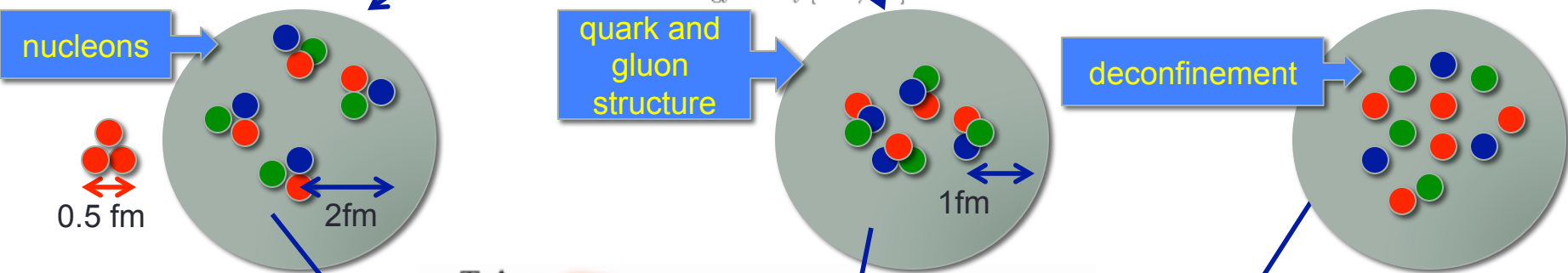
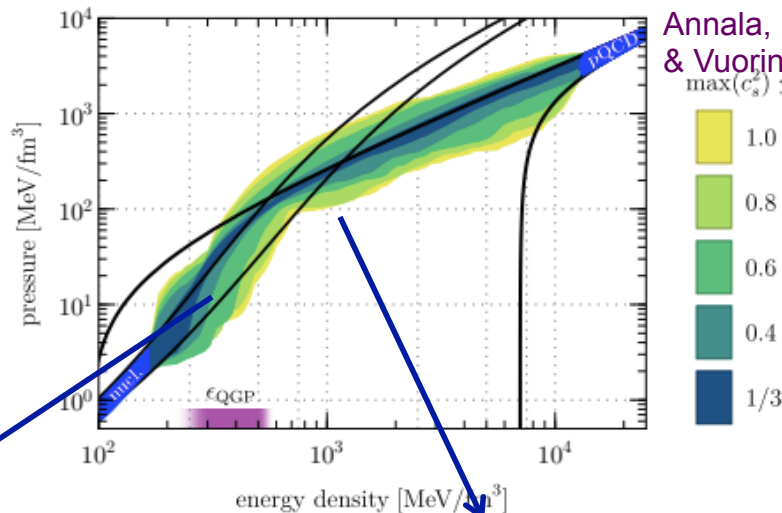
¹Department of Physics, University of Virginia, Charlottesville, VA 22904, USA.
²Physics Department, Brookhaven National Laboratory, Upton, New York 11973, USA.

The recent detection of gravitational waves from merging neutron star events has opened a new window on the many unknown aspects of their internal dynamics. A key role in this context is played by the transition from baryon to quark matter described in the neutron star equation of state (EoS). In particular, the binary pulsar observation of heavy neutron stars requires appropriately stiff dense matter in order to counter gravitational collapse, at variance with the predictions of many phenomenological quark models. On the other side, the LIGO observations favor a softer EoS therefore providing a lower bound to the equation stiffness. We introduce a quantum chromodynamics (QCD) description of the neutron star's high baryon density regime where the pressure and energy density distributions are directly obtained from the matrix elements of the QCD energy momentum tensor. Recent ab initio calculations allow us to evaluate the energy-momentum tensor in a model independent way including both quark and gluon degrees of freedom. Our approach is a first effort to replace quark models and effective gluon interactions with a first principles, fully QCD-based description. Most importantly, the QCD energy momentum tensor matrix elements are connected to the Mellin moments of the generalized parton distributions which can be measured in deeply virtual exclusive scattering experiments. As a consequence, we establish a connection between observables from high energy experiments and from the analysis of gravitational wave events. Both can be used to mutually constrain the respective sets of data. In particular, the emerging QCD-based picture is consistent with the GW170817 neutron star merger event once we allow a first-order phase transition from a low-density nuclear matter EoS to the newly-constructed high-density quark-gluon one.

14 Jan 2019 6107

Densities and distance scales

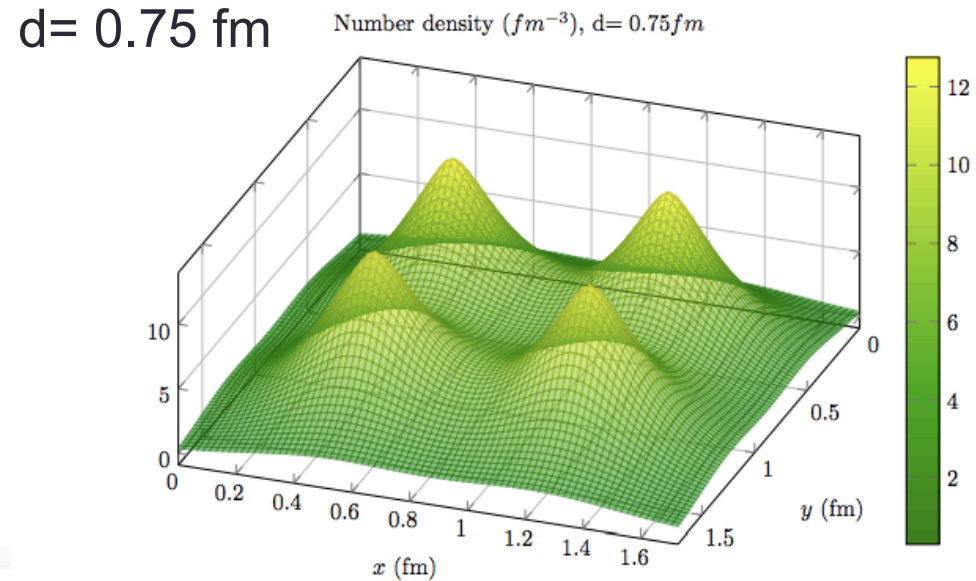
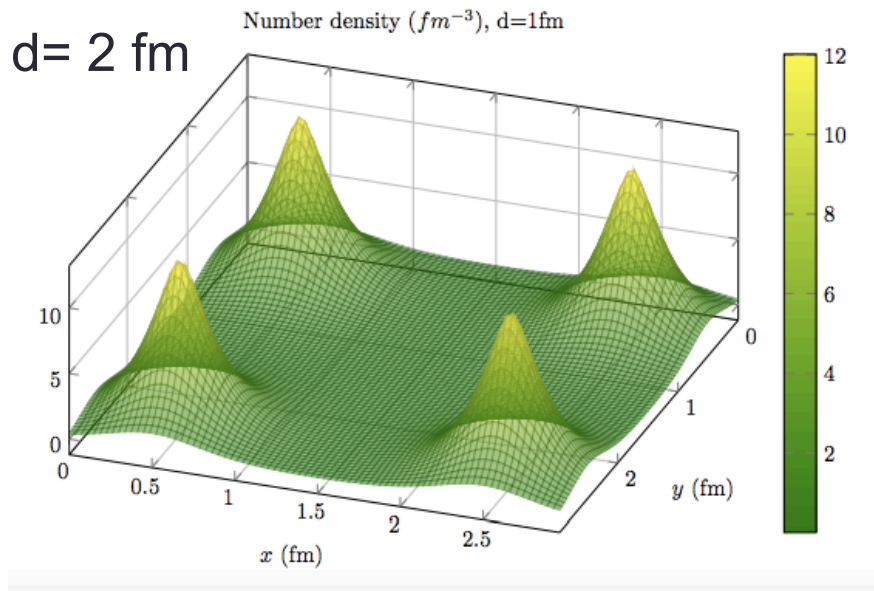
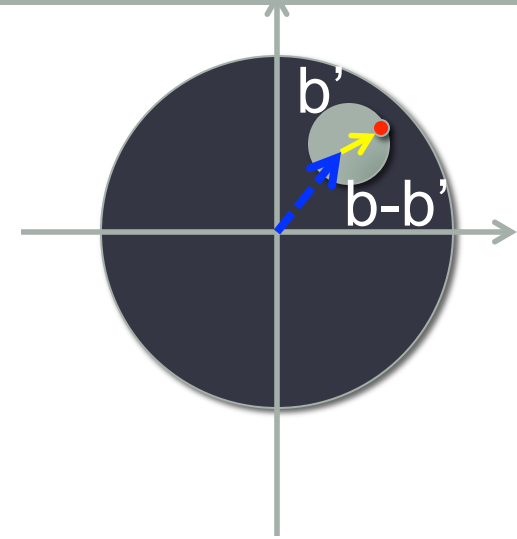
Annala, E., Gorda, T., Kurkela, A., Nattila, J., & Vuorinen, A. arXiv:1903.09121



G. Baym et al. arXiv:1707.04966

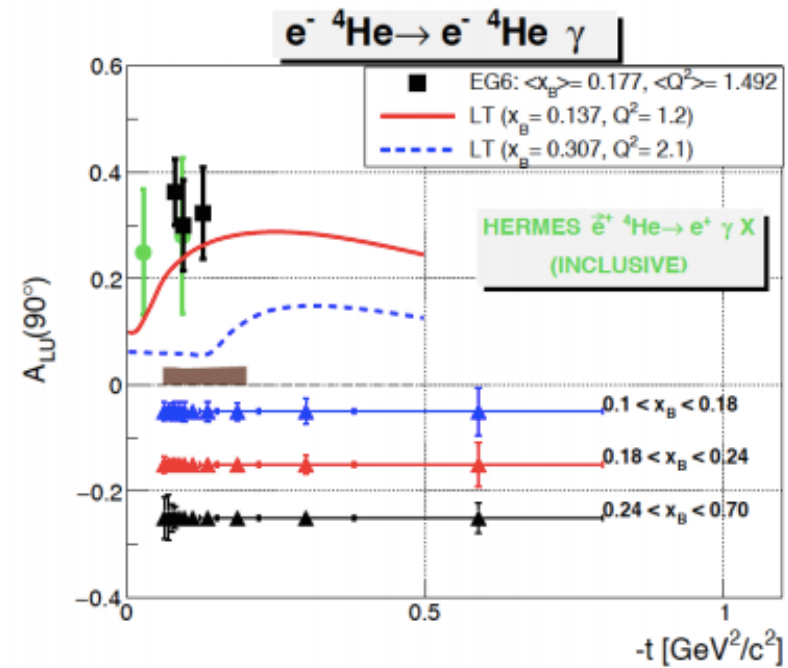
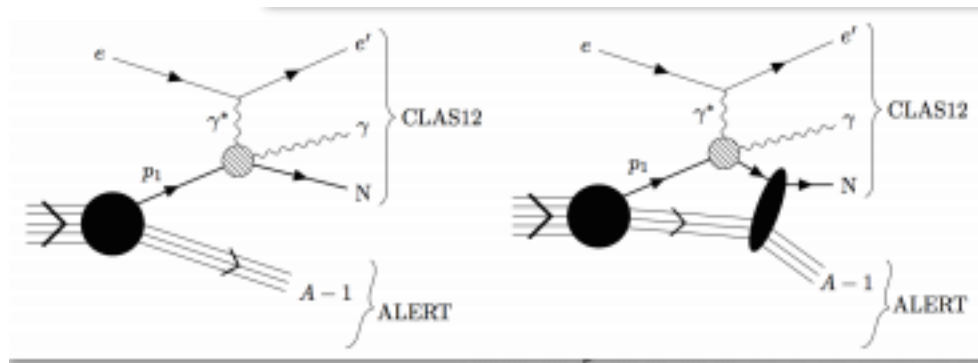
$$q_A(b) = \int d^2 b' \rho_A(|\vec{b} - \vec{b}'|) q_N(b')$$

$$\approx k_F^3 \int d^2 \beta q_N(|\vec{b} - \vec{\beta}|), \quad \vec{\beta} = \vec{b} - \vec{b}'$$



ALERT Proposal at Jefferson Lab: Nuclear Exclusive and Semi-inclusive Measurements with A New CLAS12 Low Energy Recoil Tracker

W. Armstrong et al.

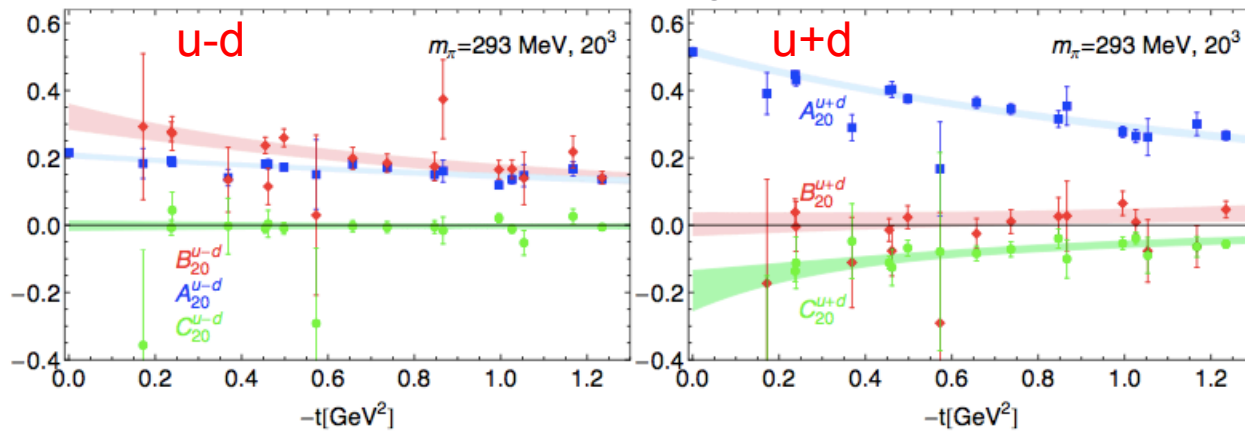


$S=0$

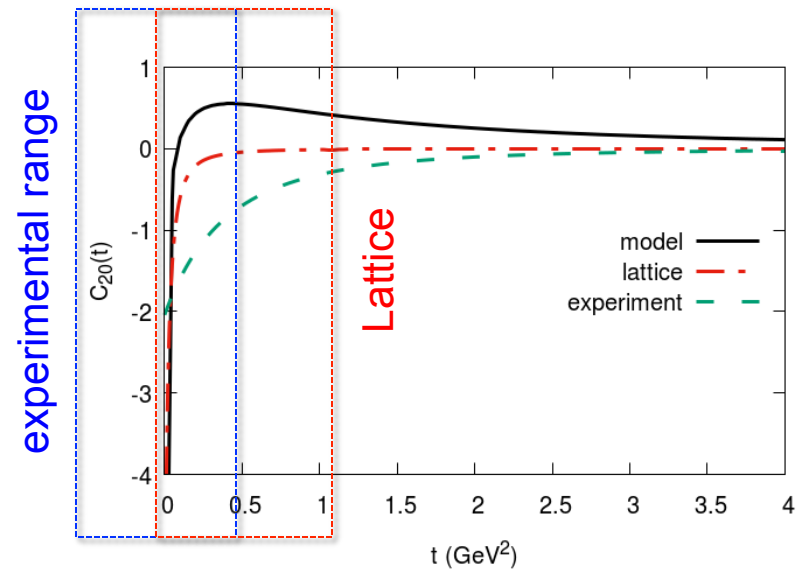
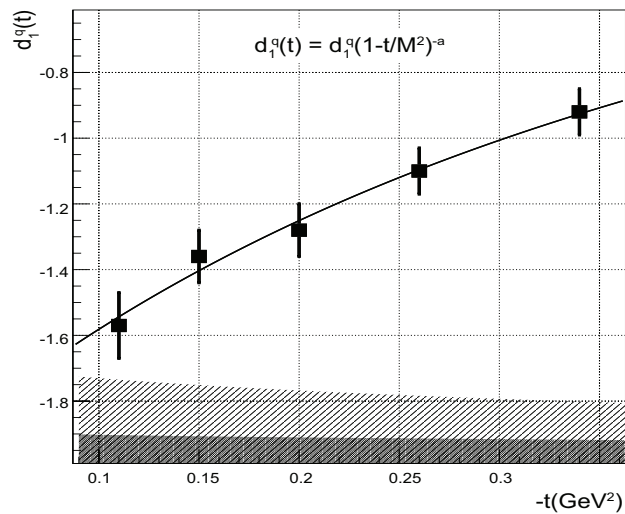
$$\langle p' | T^{\mu\nu} | p \rangle = 2 \left[A(t) P^\mu P^\nu + C(t) (\Delta^2 g^{\mu\nu} - \Delta^\mu \Delta^\nu) \right] + \tilde{C}(t) g^{\mu\nu}$$



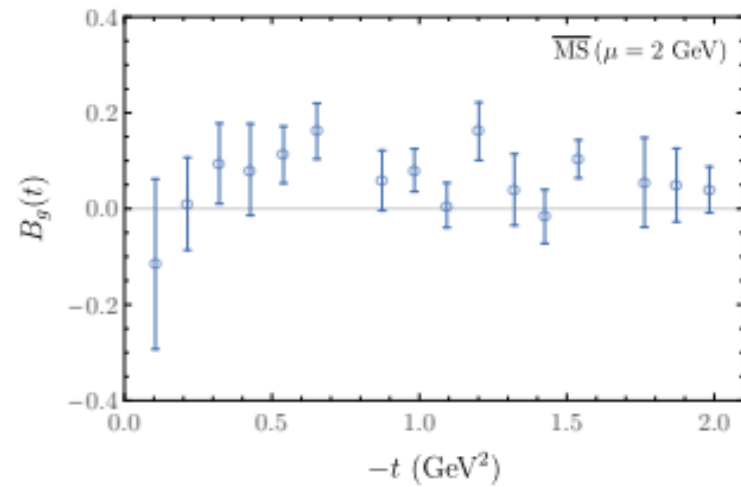
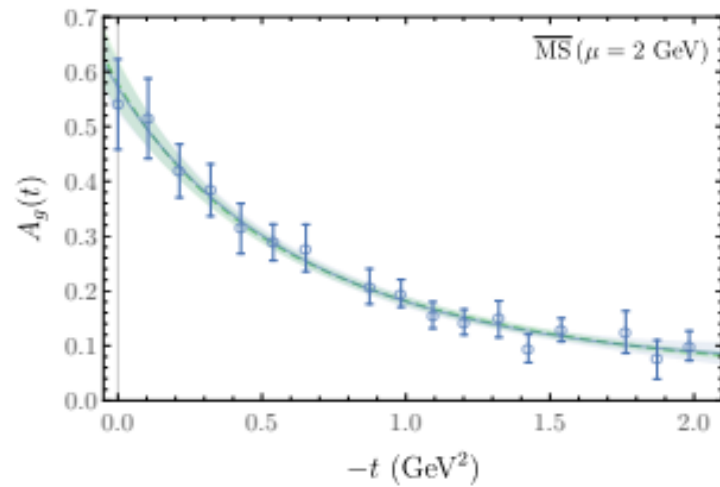
Nucleon Gravitomagnetic Form Factors

C₂₀Ph. Haegler, JoP: **295** (2011) 012009

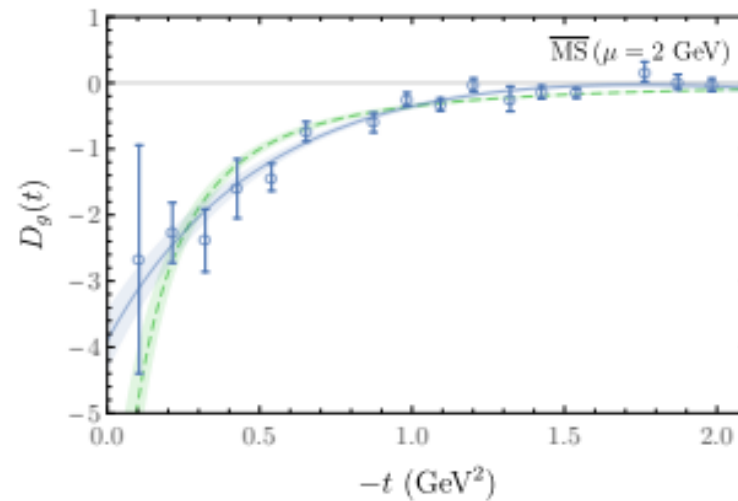
Jlab Hall B, Burkert Elouadrhiri, Girod, Nature (2018)



Gluons

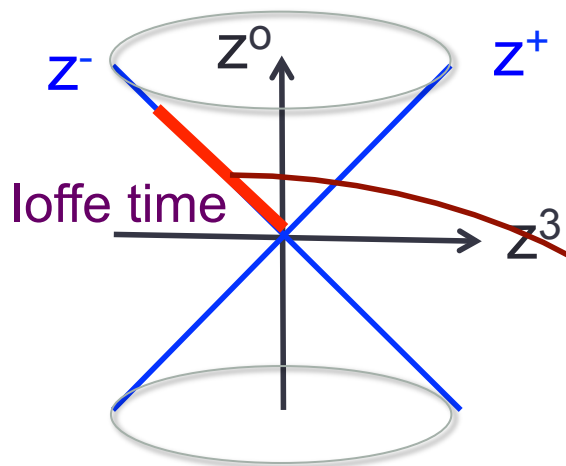


$m_\pi = 450 \text{ MeV}$

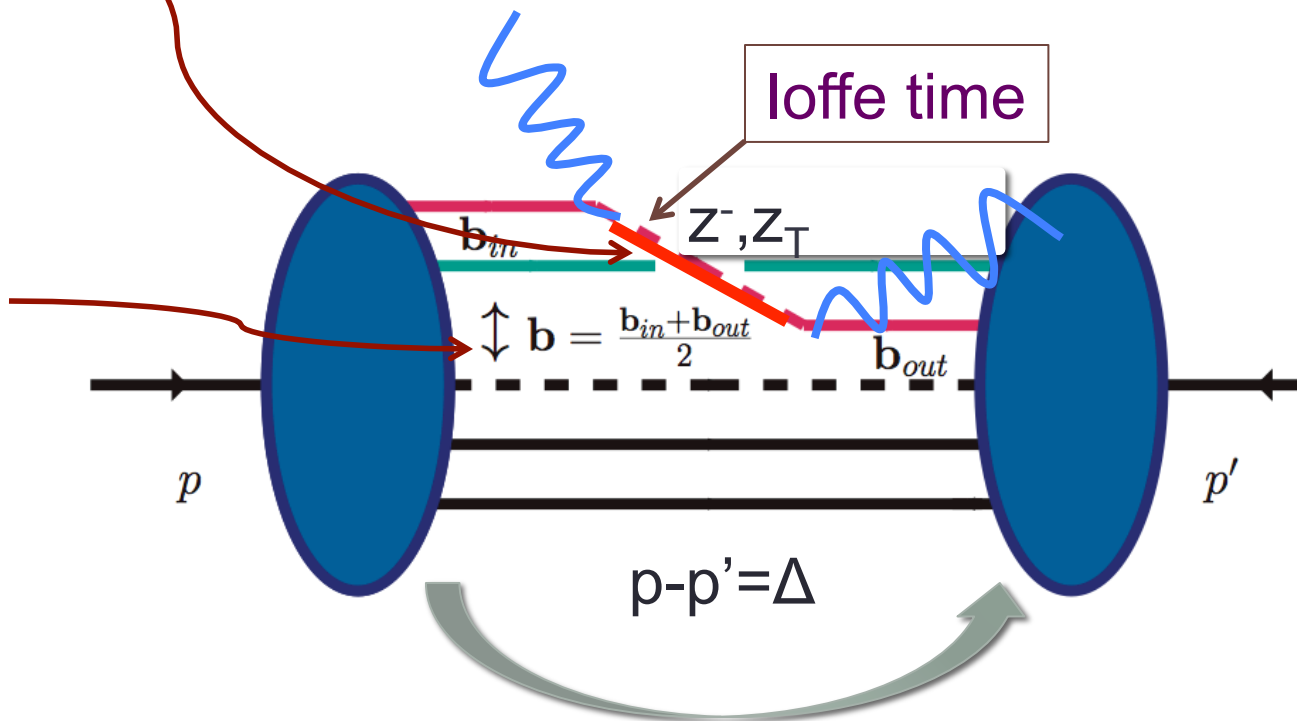


Two distinct distance scales

$$q(x, \vec{b}) = \frac{dn}{dx d^2 \vec{b}}$$



partons location

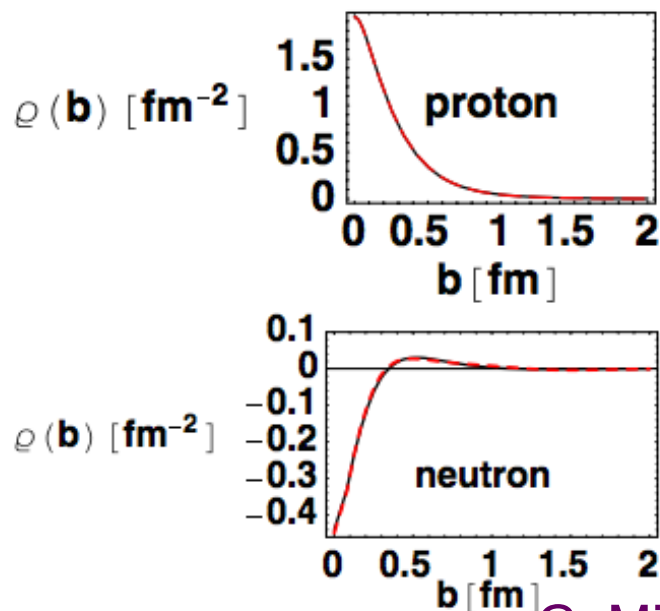


We can map out **faithfully** the spatial quark distributions in the transverse plane (no modeling/approximation)

$$q(x, \vec{b}) = \frac{dn}{dx d^2\vec{b}}$$

Soper (1977), Burkardt (2001)

Already a surprise: re-evaluation of nucleon charge distribution

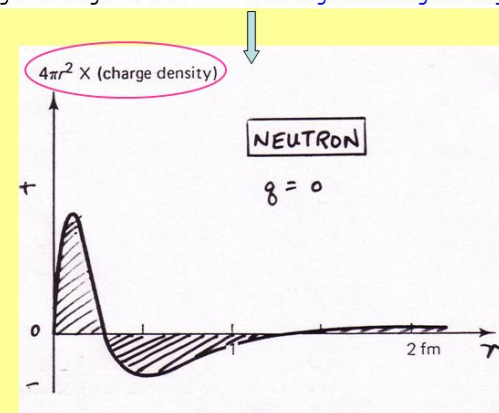


G. Miller(2007)

What does negative $\langle r^2 \rangle$ mean? Neutron “textbook” density

$$\langle r^2 \rangle \equiv \int r^2 \rho(r) d^3r = \int r^2 (4\pi r^2 \rho(r)) dr$$

- charge density must have both -ve and +ve regions, since net charge = 0
- integral is weighted with $r^2 \rightarrow$ more negative charge at large radius



Including all polarization configurations:

$$\rho_{\Lambda\lambda}^q(\mathbf{b}) = H_q(\mathbf{b}^2) + \frac{b^i}{M} \epsilon_{ij} S_T^j \frac{\partial}{\partial b} E_q(\mathbf{b}^2) + \Lambda\lambda \tilde{H}_q(\mathbf{b}^2),$$

1st Mellin M. $H_q(\mathbf{b}^2) = \int \frac{d^2 \Delta_T}{(2\pi)^2} e^{i\Delta_T \cdot \mathbf{b}} A_1^q(t);$ number/mass density

2nd Mellin M. $\epsilon_{q,g}(r) = \int \frac{d^2 \Delta_T}{(2\pi)^2} e^{i\Delta_T \cdot \mathbf{b}} A_2^{q,g}(t)$ Energy density

$$r = |\vec{\mathbf{b}}|$$

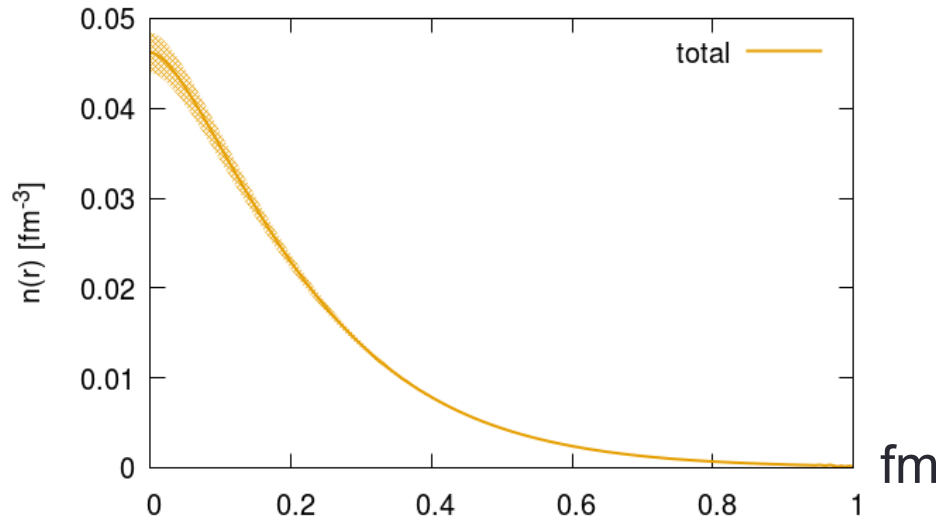
$p_{q,g}(r) = \int \frac{d^2 \Delta_T}{(2\pi)^2} e^{i\Delta_T \cdot \mathbf{b}} 2t C_2^{q,g}(t)$ Pressure

From transverse distance to z_3 using Lorentz invariance

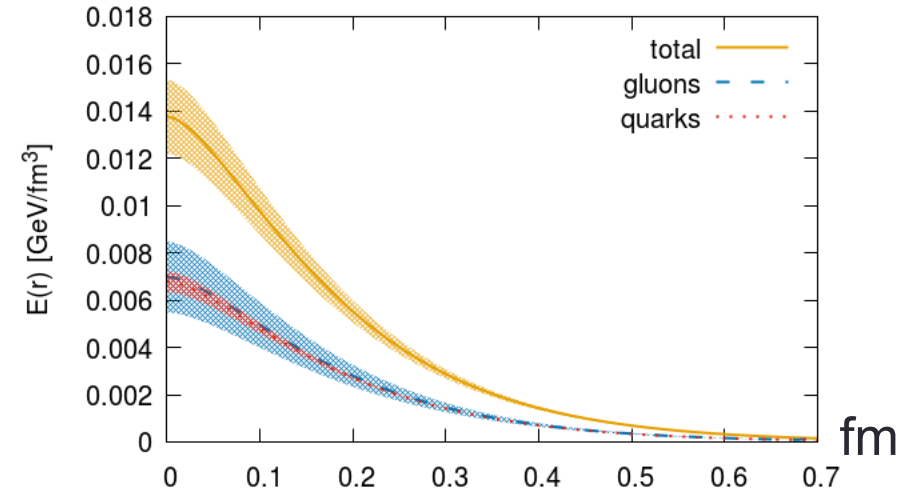
Radyushkin and Orginos, arXiv:1706.05373

A₁₀

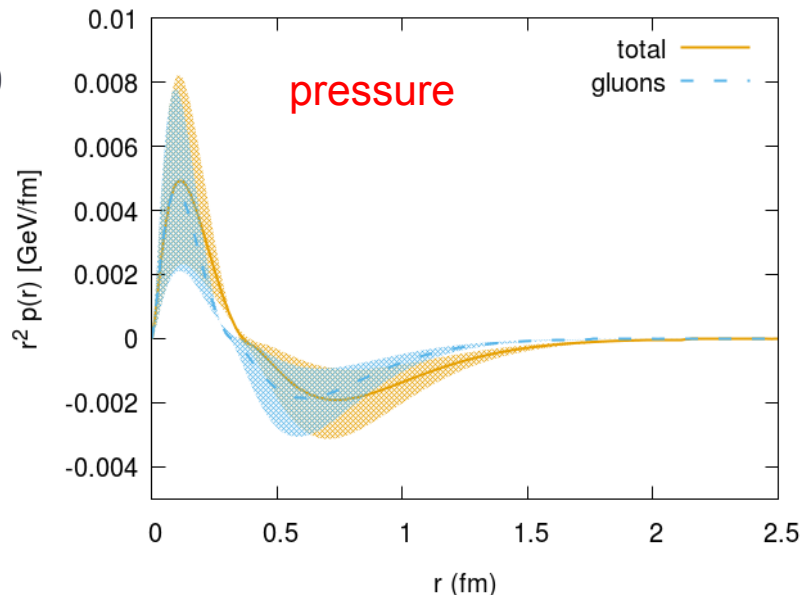
Number density vs radius

A₂₀

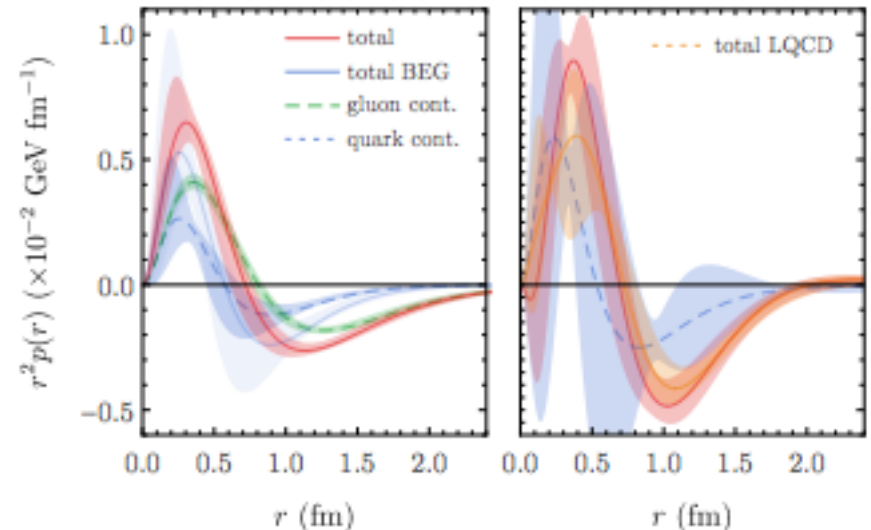
Energy density vs radius

C₂₀

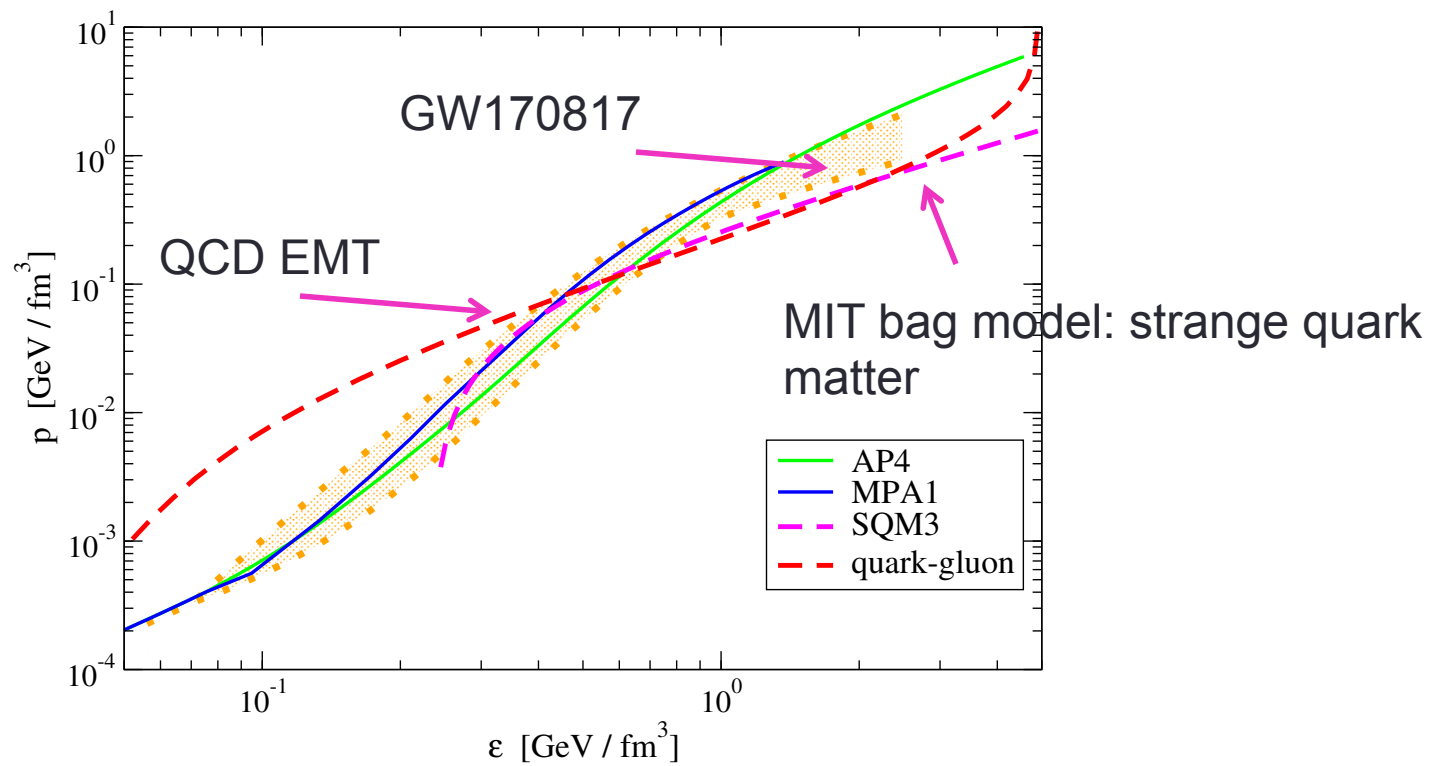
pressure



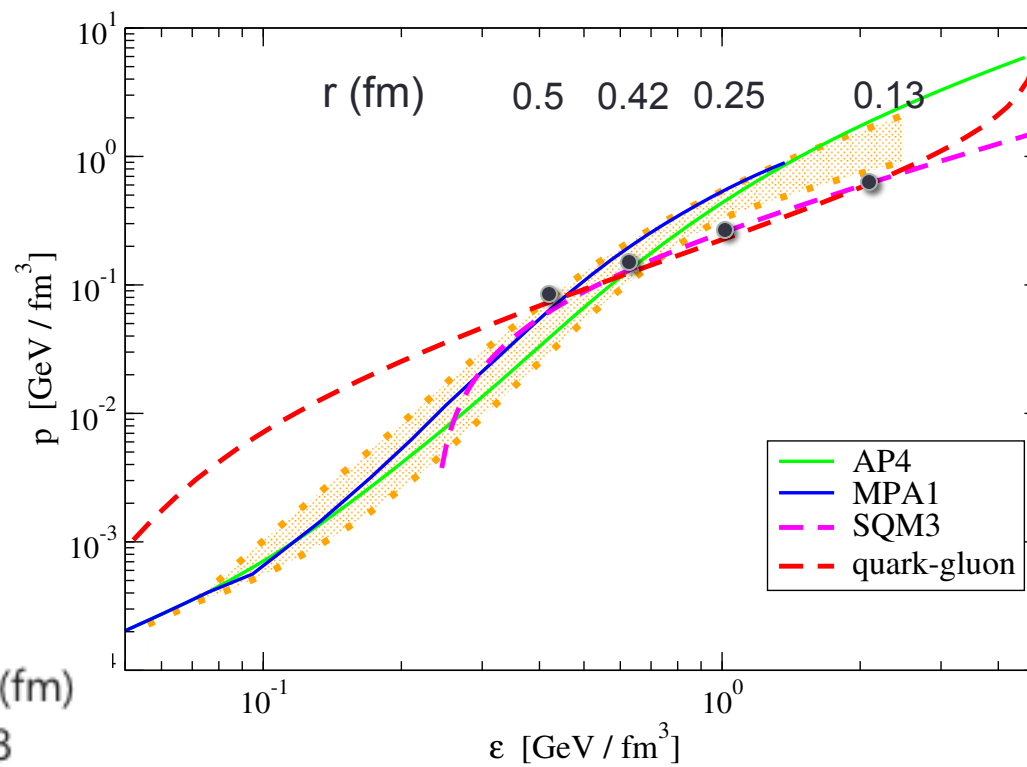
pressure



Detmold, Shahanan, Phys.Rev.Lett. 122 (2019)

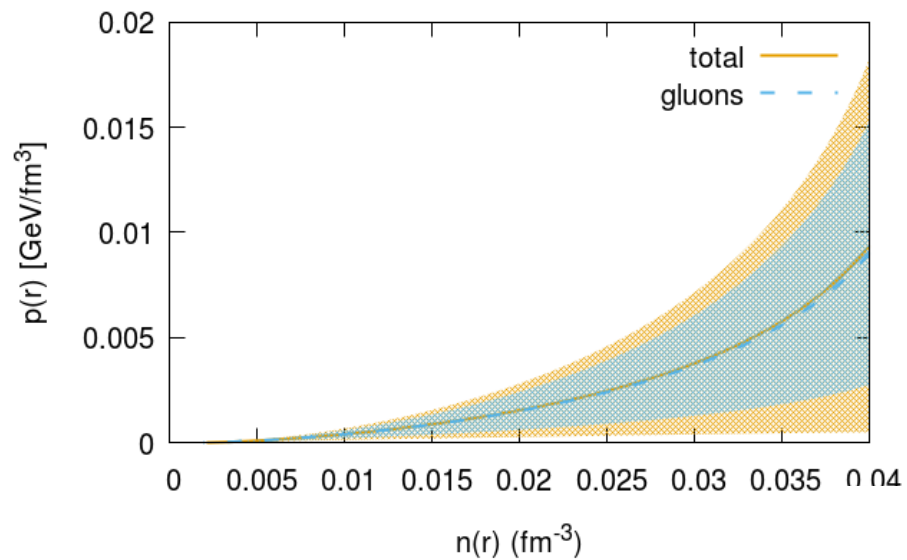


SL, Rajan, Yagi, arXiv:1812.01479

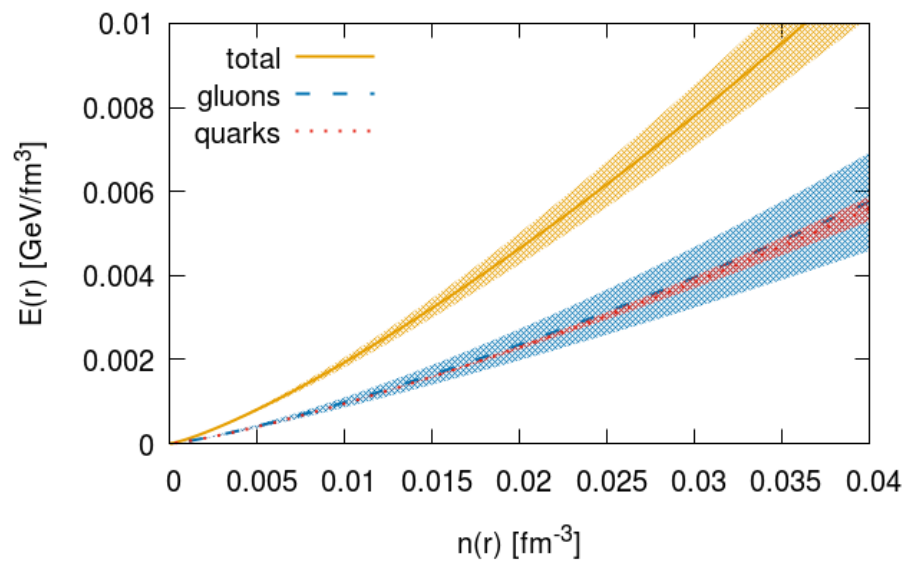


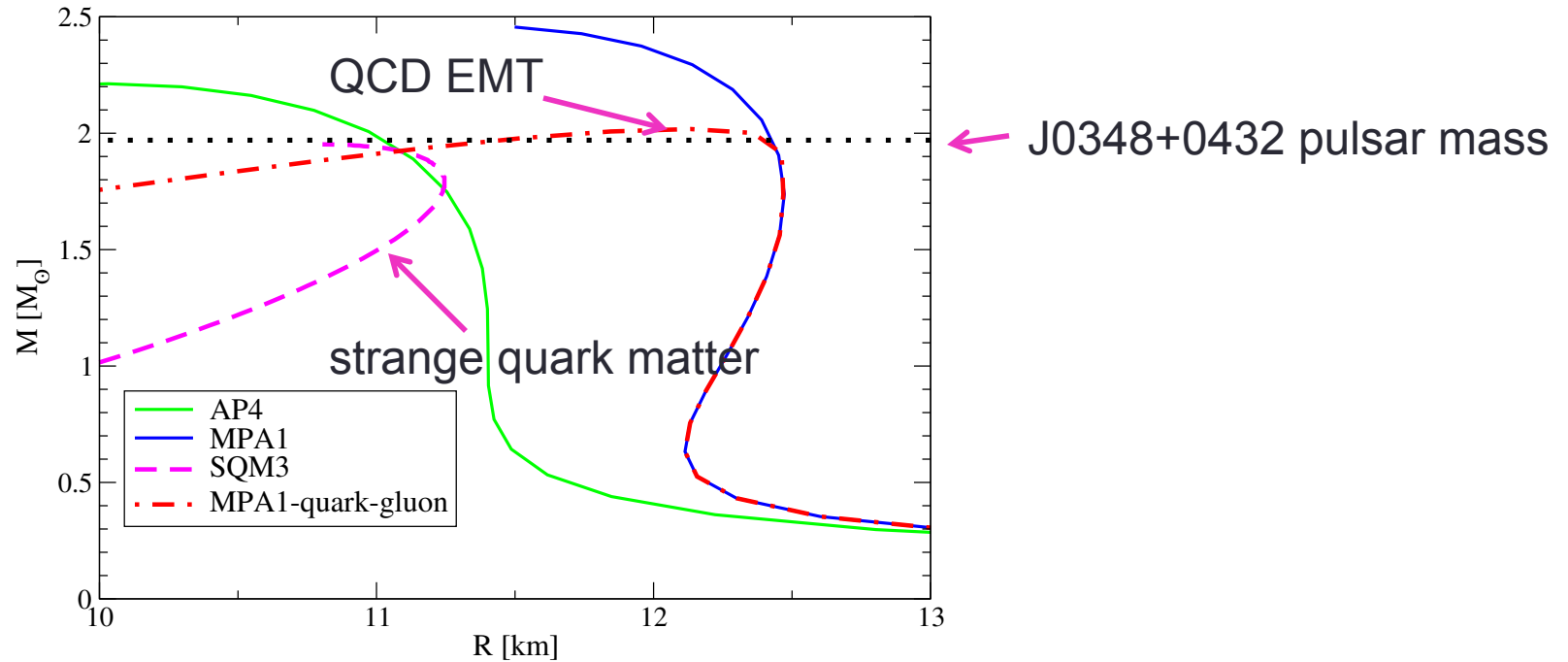
E (GeV/fm ³)	r (fm)
0.2	0.48
0.3	0.426
0.6	0.325
1	0.25
2	0.134
3	0.05

Pressure vs number density



Energy density vs number density





“stitching” at transition pressure $0.15 \text{ GeV}/\text{fm}^3$

Jefferson Lab's measurement on the pressure inside the nucleon/hadronic matter needs to be corroborated by an independent set of measurements

Neutron stars mergers/multimessenger astronomy provide an independent constraint

WHAT'S NEXT...

Key question: how to extract accurate information on the mechanical properties of the nucleon from data

B. Kriesten's talk

Extraction of Generalized Parton Distribution Observables from Deeply Virtual Electron Proton Scattering Experiments

Brandon Kriesten,^{*} Simonetta Liuti,[†] Liliet Calero Diaz,[‡] Dustin Keller,[§] and Andrew Meyer[¶]
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Gary R. Goldstein^{**}

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J. Osvaldo Gonzalez-Hernandez^{††}

INFN, Torino

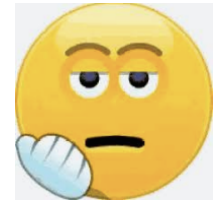
(Dated: April 6, 2019)

We provide the general expression of the cross section for exclusive deeply virtual photon electroproduction from a spin 1/2 target using current parameterizations of the off-forward correlation function in a nucleon for different beam and target polarization configurations up to twist three accuracy. All contributions to the cross section including deeply virtual Compton scattering, the Bethe-Heitler process, and their interference, are described within a helicity amplitude based framework which is also relativistically covariant and readily applicable to both the laboratory frame and in a collider kinematic setting. Our formalism renders a clear physical interpretation of the various components of the cross section by making a connection with the known characteristic structure of the electron scattering coincidence reactions. In particular, we focus on the total angular momentum, J_z , and on the orbital angular momentum, L_z . On one side, we uncover an avenue to a precise extraction of J_z , given by the combination of generalized parton distributions, $H + E$, through a generalization of the Rosenbluth separation method used in elastic electron proton scattering. On the other, we single out for the first time, the twist three angular modulations of the cross section that are sensitive to L_z . The proposed generalized Rosenbluth technique adds constraints and can be extended to additional observables relevant to the mapping of the 3D structure of the nucleon.

arXiv:1903.05742

New formalism for deeply virtual exclusive processes

- 1) No harmonics, please.....this is just a coincidence experiment
(write the cross section a la Donnelly...)
- 2) This formalism has **not** been developed in previous work for GPDs

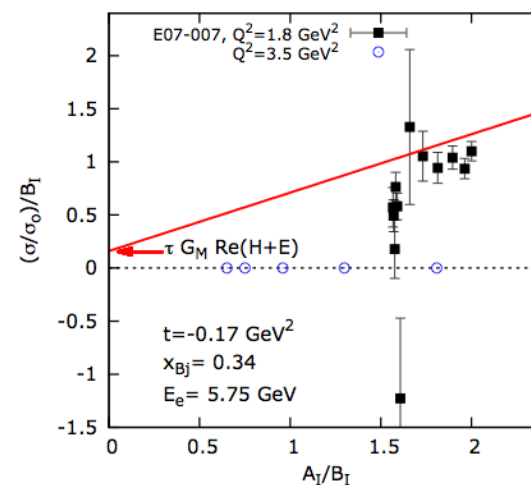
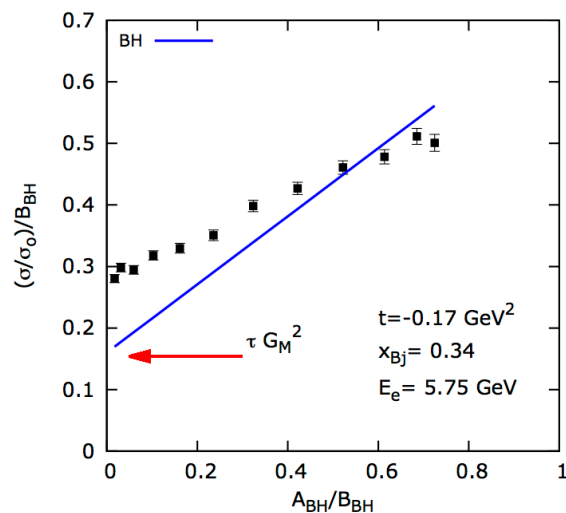


$$\frac{d^5 \sigma_{unpol}^{BH}}{dx_{Bj} dQ^2 d|t| d\phi d\phi_S} = \frac{\Gamma}{t^2} [A_{BH} (F_1^2 + \tau F_2^2) + B_{BH} \tau G_M^2(t)]$$

BH

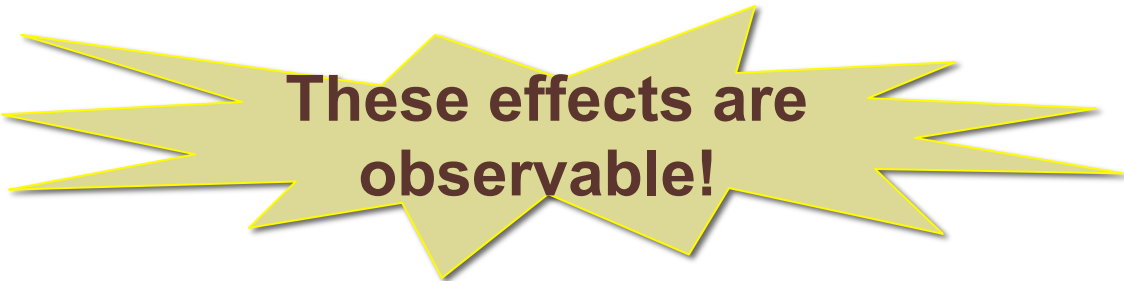
$$\frac{d^5 \sigma_{unpol}^I}{dx_{Bj} dQ^2 d|t| d\phi d\phi_S} = \frac{\Gamma}{Q^2(-t)} [A_I (F_1 \text{Re}\mathcal{H} + \tau F_2 \text{Re}\mathcal{E}) + B_I G_M \text{Re}(\mathcal{H} + \mathcal{E}) + C_I G_M \text{Re}\tilde{\mathcal{H}}]$$

DVCS



Conclusions and Outlook

- The EoS of dense matter in QCD can be obtained from first principles, using **ab initio calculations for both quark and gluon d.o.f.**
- **Gluons** are found to dominate the EoS providing a trend in the high density regime which is consistent with the constraint from LIGO.



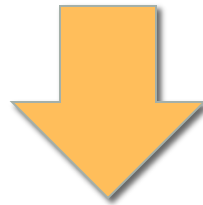
**These effects are
observable!**

- We can connect the **pressure and energy density** in neutron stars with collider observables: the **GPDs**.
- The proposed line of research opens up a new framework for understanding the properties of **hybrid stars**. In the future we hope to set more stringent constraints on the nature of the **hadron to quark matter transition** at zero temperature.

...To observe, evaluate and interpret

Wigner distributions at the subatomic level

requires stepping up data analyses from the standard methods → developing new numerical/analytic/quantum computing methods



Center for Nuclear Femtography at Jefferson Lab

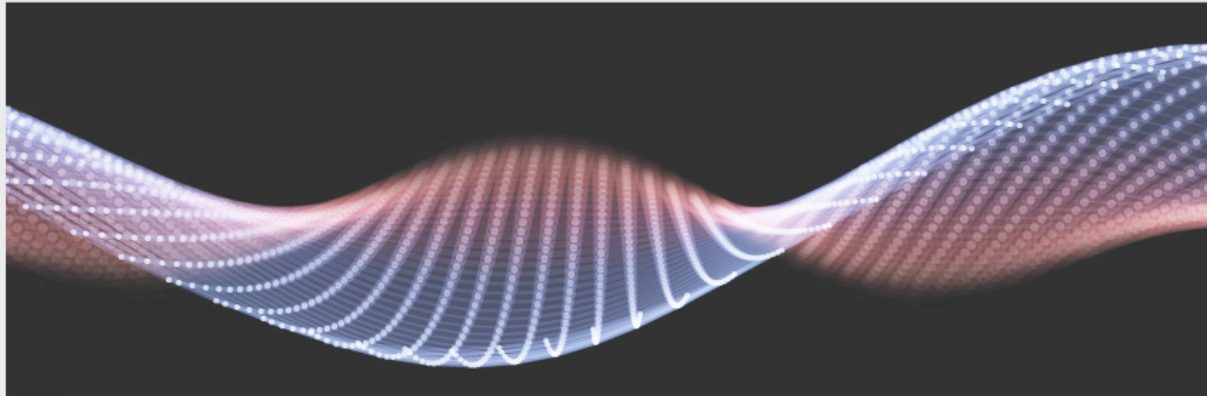
After the

Virginia Symposium on Imaging and Visualization in Science

from Sunday, 9 December 2018 at 08:00 to Tuesday, 11 December 2018 at 18:00 (America/New_York)

University of Virginia

Description



Symposium on Imaging and Visualization in Science

December 10-11, 2018, University of Virginia

**Summer Institute on Wigner Imaging and Femtography,
SIWIF@UVA: May-August 2019**

Organizers

P. Alonzi, M. Burkardt, D. Keller, S. Liuti, O. Pfister, P. Reinke