The PDF of the Pion from Lattice Calculation of Current-Current Matrix Elements

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QCD Evolution, May 2019

Introduction

Working definition - calculation of x-dependent PDFs and QDAs (quark distribution amplitudes) from Euclidean-space lattice calculations.

- Quasi-PDF (qPDF) interpreted in LaMET (Large Momentum Effective Theory) was proposed by X.Ji **X. Ji, Phys. Rev. Lett. 110 (2013) 262002**

$$q(x,\mu^2,P^z) = \int \frac{dz}{4\pi} e^{izk^z} \langle P \mid \bar{\psi}(z)\gamma^z e^{-ig\int_0^z dz' A^z(z')}\psi(0) \mid P > + \mathcal{O}((\Lambda^2/(P^z)^2), M^2/(P^z)^2))$$

Quasi distributions approach light-cone distributions in limit of large *P*^{z}

$$q(x,\mu^2,P^z) = \int_x^1 \frac{dy}{y} Z\left(\frac{x}{y}, \frac{\mu}{P^z}\right) q(y,\mu^2) + \mathcal{O}(\Lambda^2/(P^z)^2, M^2/(P^z)^2)$$

- Pseudo-PDF (pPDF) recognizing generalization of PDFs in terms of *Ioffe Time.* $\nu = p \cdot z$ **A. Radyushkin, PLB767 (2017)**

$$\mathcal{M}^\alpha(z,p) = \langle p \mid \bar{\psi}(z)\gamma^\alpha \exp\left(-ig\int_0^z dz' A^z(z')\right)\psi(0) \mid p\rangle$$

Pseudo-PDF vs Quasi-PDF

Relation between qPDF and pPDF approaches

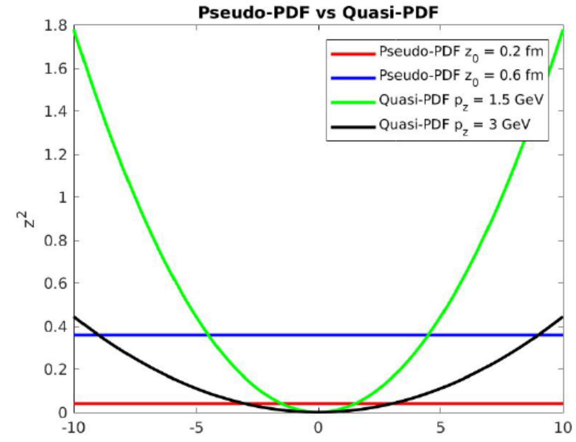
Joe Karpie

- Both integrals of Ioffe-Time Distribution Function
- Computed matrix elements the same
- Should yield same PDF after matching and systematic controls

$$P(x, z_0^2) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\nu\, e^{-i\nu x} M(\nu, z_0^2)$$

$$Q(x, p_z^2) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\nu\, e^{-i\nu x} M(\nu, \frac{\nu^2}{p_z^2})$$

Require sufficiently fine lattice spacings



KF Liu, SJ Dong, PRL72, 1790 (1994)

- Hadronic Tensor (HT)

$$W_{\mu\nu} = \frac{1}{4\pi}\int d^4z\, e^{iq.z}\langle p \mid J_\mu(z)^\dagger J_\nu(0) \mid p\rangle$$

$$C_4(\vec{p}, \vec{q}, \tau) = \sum_{\vec{x}_f} e^{-i\vec{p}.\vec{x}_f} \sum_{\vec{x}_2, \vec{x}_1} e^{-i\vec{q}.(\vec{x}_2 - \vec{x}_1)}\langle N(\vec{x}_f, t_f) J_\mu(\vec{x}_2, t_2) J_\nu(\vec{x}_1, t_1) \bar{N}(\vec{0}, t_0)\rangle$$

This is a ***four-point*** function.

Good "Lattice Cross Sections"

- LCS

Ma and Qiu, Phys. Rev. Lett. 120 022003

$$\sigma_n(\omega, \xi^2, P^2) = \langle P \mid T\{\mathcal{O}_n(\xi)\} \mid P \rangle$$

Expressed in coordinate space

Short distance scale

where

$$\sigma_n(\omega, \xi^2, P^2) = \sum_a \int_{-1}^{1} \frac{dx}{x} f_a(x, \mu^2) K_n^a(x\omega, \xi^2, x^2P^2, \mu^2) + \mathcal{O}(\xi^2 \Lambda_{\rm QCD}^2)$$

Calculated in LQCD

Parton Distribution function

Calculated in perturbation theory ("process dependent")

Factorize in $\omega = P \cdot \xi,\ \xi^2 P^2$ providing $\xi << \frac{1}{\Lambda_{\rm QCD}}$

$P \to \sqrt{s}$ *Collision energy*

$\xi \to \frac{1}{Q}$ *Hard Probe*

LCS - II

Choice of Operators

$$\mathcal{O}(\xi) = \bar{\psi}(0)\Gamma W(0, 0+\xi)\psi(\xi)$$

↑

Wilson line

Gauge-Invariant Currents

$$\begin{aligned} \mathcal{O}_S(\xi) &= \xi^4 Z_S^2 [\bar{\psi}_q \psi_q](\xi)[\bar{\psi}_q \psi](0) \\ \mathcal{O}_{V'}(\xi) &= \xi^2 Z_{V'}^2 [\bar{\psi}_q \xi \cdot \gamma \psi_{q'}](\xi)[\bar{\psi}_{q'} \xi \cdot \gamma \psi](0) \end{aligned}$$

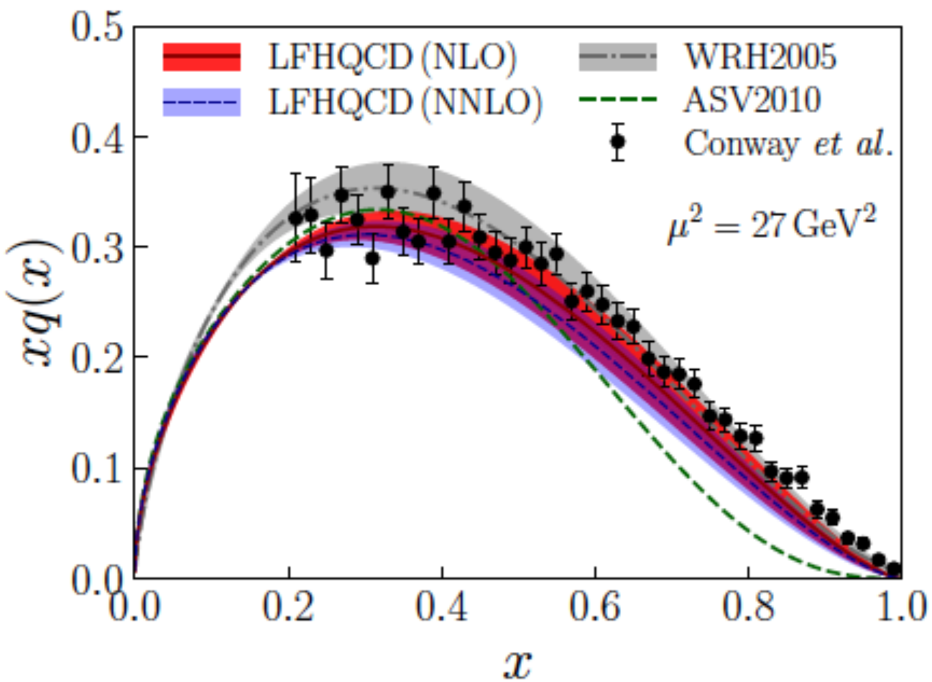
← Flavor-changing

- Straight-forward operator renormalization
- ξ can be off-axis
- Variety of operators
  - Control over systematic uncertainties
  - "Higher-Twist"

First Application - Pion PDF

R. Sufian, J. Karpie, C. Egerer, K. Orginos, J. Qiu, D. Richards, Phys. Rev. D 99, 074507 (2019), arXiv:1901.03921

- u distribution of FNAL E615 to leading order
- C12-15-006 at Hall A will look at structure of pion
- C12-15-006A at Hall A will look at structure of Kaon
- No free pion target



de Teramond, liu, Sufian, Dosch, Brodsky, Deur, PRL (2018)

Discrepancy in large-x behavior of pion distribution

Why the Pion - II?

- Pion less computationally demanding that nucleon
  - *Larger signal-to-noise ratio*

$$C(t,\vec{p}) \equiv \sum_{\vec{x}} \langle 0 \mid \mathcal{O}(t,\vec{x})\mathcal{O}^\dagger(0,0) \mid 0 \rangle e^{-i\vec{p}\cdot\vec{x}} \rightarrow e^{-E(\vec{p})t}$$

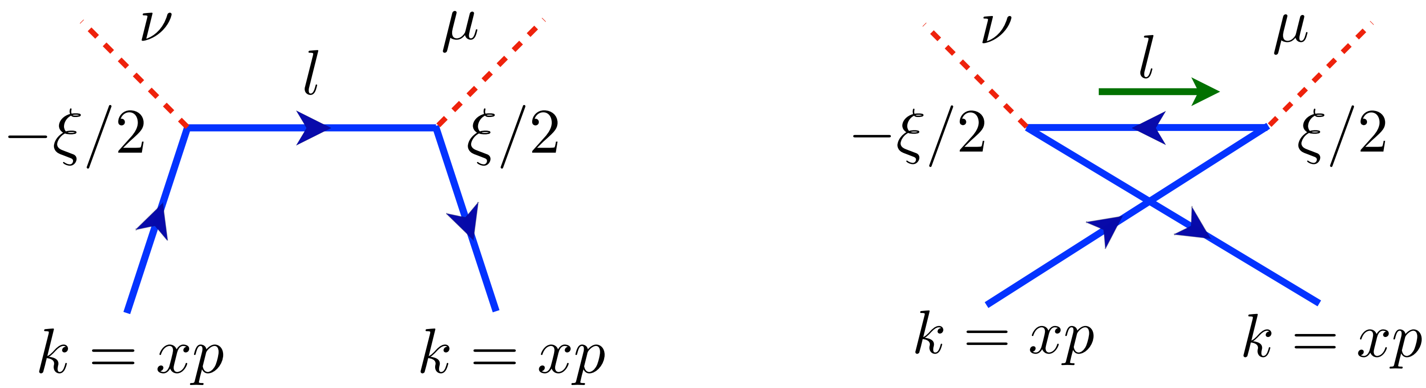
$$C_{\sqrt{\sigma^2}}(t,\vec{p}) \rightarrow \begin{cases} e^{-m_\pi t} & \text{Mesons} \\ e^{-(3m_\pi/2)t} & \text{Baryons} \end{cases}$$

- Important constraint on systematic uncertainty is understanding operator renormalization
  - *Operator renormalization "independent" of external states*

Perturbative Kernel

$$\sigma_{ij}^{\mu\nu}(\xi,p) = \langle \pi(p) \mid \mathcal{O}_{ij}^{\mu\nu}(\xi) \mid \pi(p)\rangle$$
$$= \xi^4 \langle \pi(p) \mid \mathcal{J}_i^{\mu}(\xi/2)\mathcal{J}^{\nu} + j(-\xi/2) \mid \pi(p)\rangle$$

Calculate *K* at tree-level between quark states



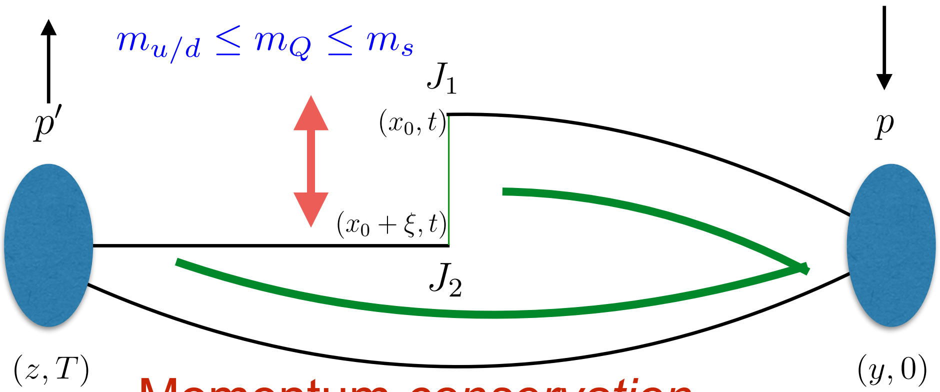
Process, i.e. current, dependent

$$\frac{1}{2}[\sigma_{V,A}^{\mu\nu}(\xi,p) + \sigma_{A,V}^{\mu\nu}(\xi,p)]$$

$$\equiv \epsilon^{\mu\nu\alpha\beta}\xi_\alpha p_\beta T_1(\omega,\xi^2) + (p^\mu\xi^\nu - \xi^\mu p^\nu) T_2(\omega,\xi^2)$$

Computational Setup

Momentum projection



Momentum projection

$C_4(T)$

Momentum *conservation*

$$\langle \Pi(-p')|\mathcal{O}_{J_1}(x_0)\mathcal{O}_{J_2}(\xi)|\Pi(-p')\rangle =$$

$$= \sum_{y,z} e^{i(p'.z-p.y)} \langle \bar{q}\,\Gamma_\Pi\, q(z,T)\, \bar{Q}\, J_2\, Q(x_0+\xi,t)\, \bar{q}\, J_1\, q(x_0,t)\, \bar{q}\,\Gamma_\Pi\, q(y,0)\rangle$$

$$= \sum_{y,z} e^{i(p'.z-p.y)} \mathrm{Tr}[J_2\, G_Q(x_0+\xi,t;x_0,t)\, J_1\, G(x_0,t;y,0)\,\Gamma_\Pi G(y,0;z,T)\Gamma_\Pi\, G(z,T;x_0+\xi,t)]$$

Straightforward computational setup using sequential-source method:

$$D(Z,T;w)H(w;x_0,t) = \sum_y e^{-ip\cdot y}\Gamma_\Pi G(y,0;x_0,t)$$

$$D(s;w)\tilde{H}(w;x_0,t) = \sum_z e^{ip\cdot z}\Gamma_\Pi H(z,T;x_0,t)$$

Computational Details

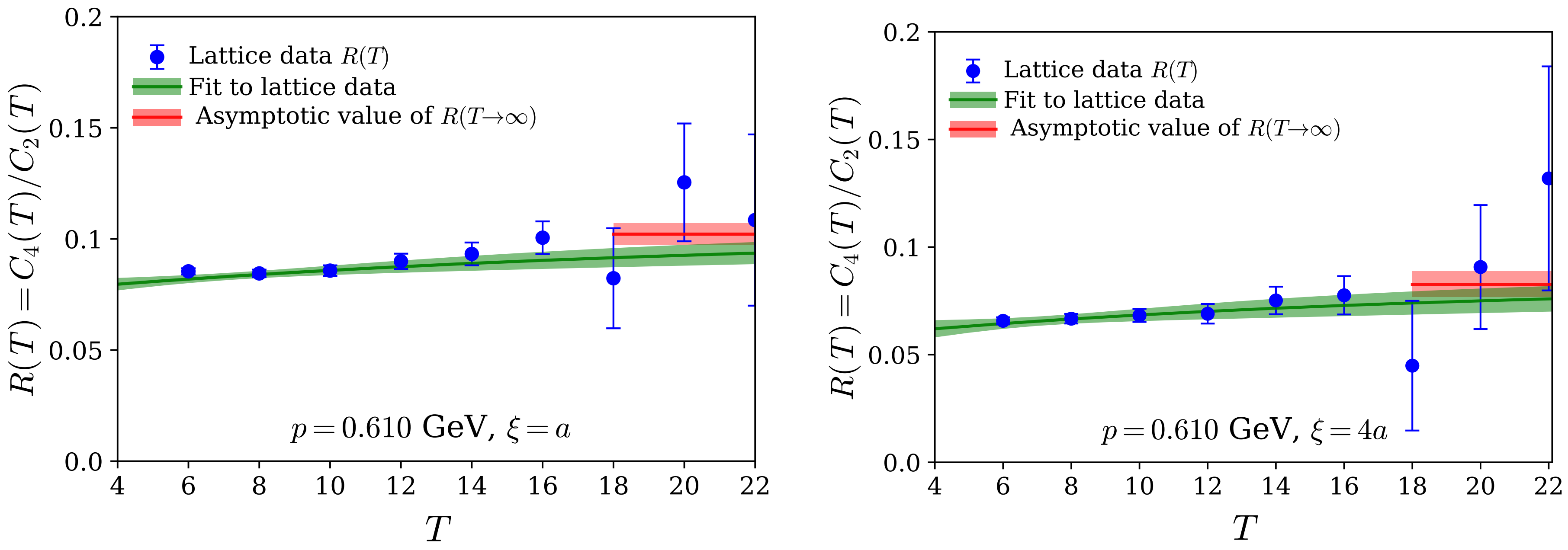
- 2 + 1 Clover-fermion action
  - Lattice spacing a ~ 0.127 fm
  - Pion mass 460 MeV
  - 32^3 x 96 lattice, 490 Configs

$\vec{p} = [0, 0, p_z]$	ζ	No. of source points (x_0, t)	No. of source-sink separations
$p = 0.610$ GeV	1.75	2	9
$p = 0.915$ GeV	2.50	5	9
$p = 1.220$ GeV	3.75	6	9
$p = 1.525$ GeV	4.50	7	7

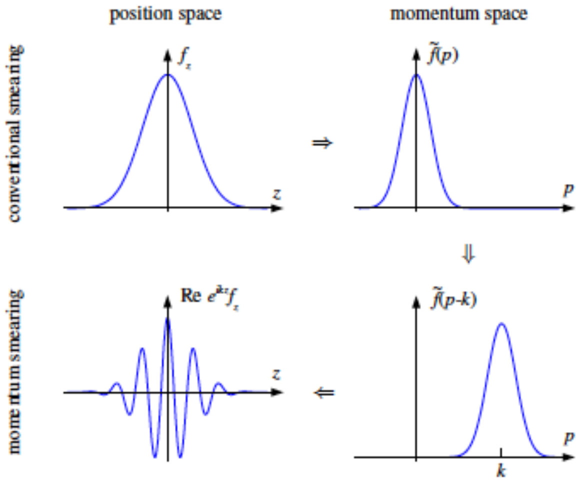
Data degrades at higher momenta

Quality of Data

$$C_{2\mathrm{pt}}(\vec{p}, T) = \langle \Pi_{\vec{p}}(T) \bar{\Pi}_{\vec{p}}(0) \rangle$$

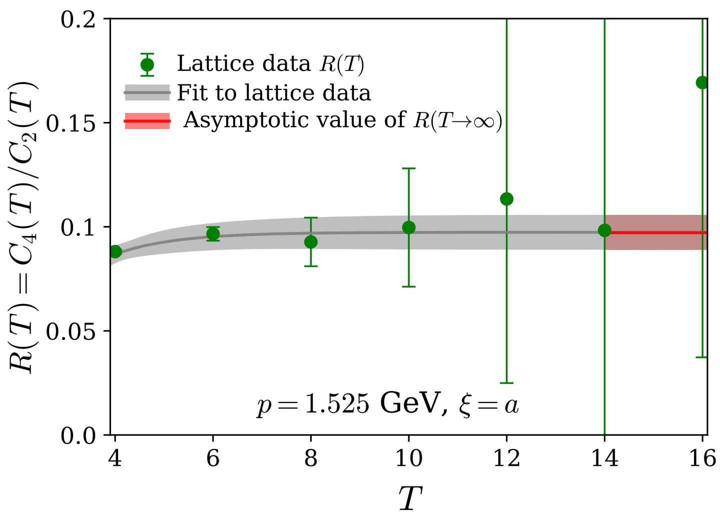


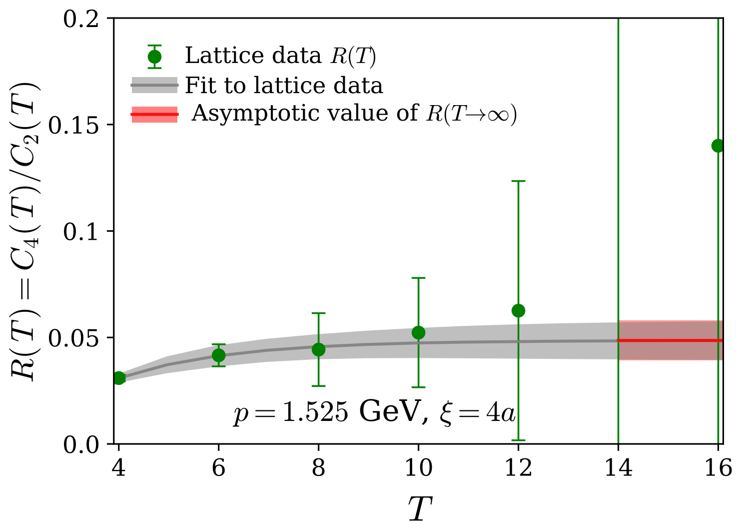
Challenges of Higher Momenta



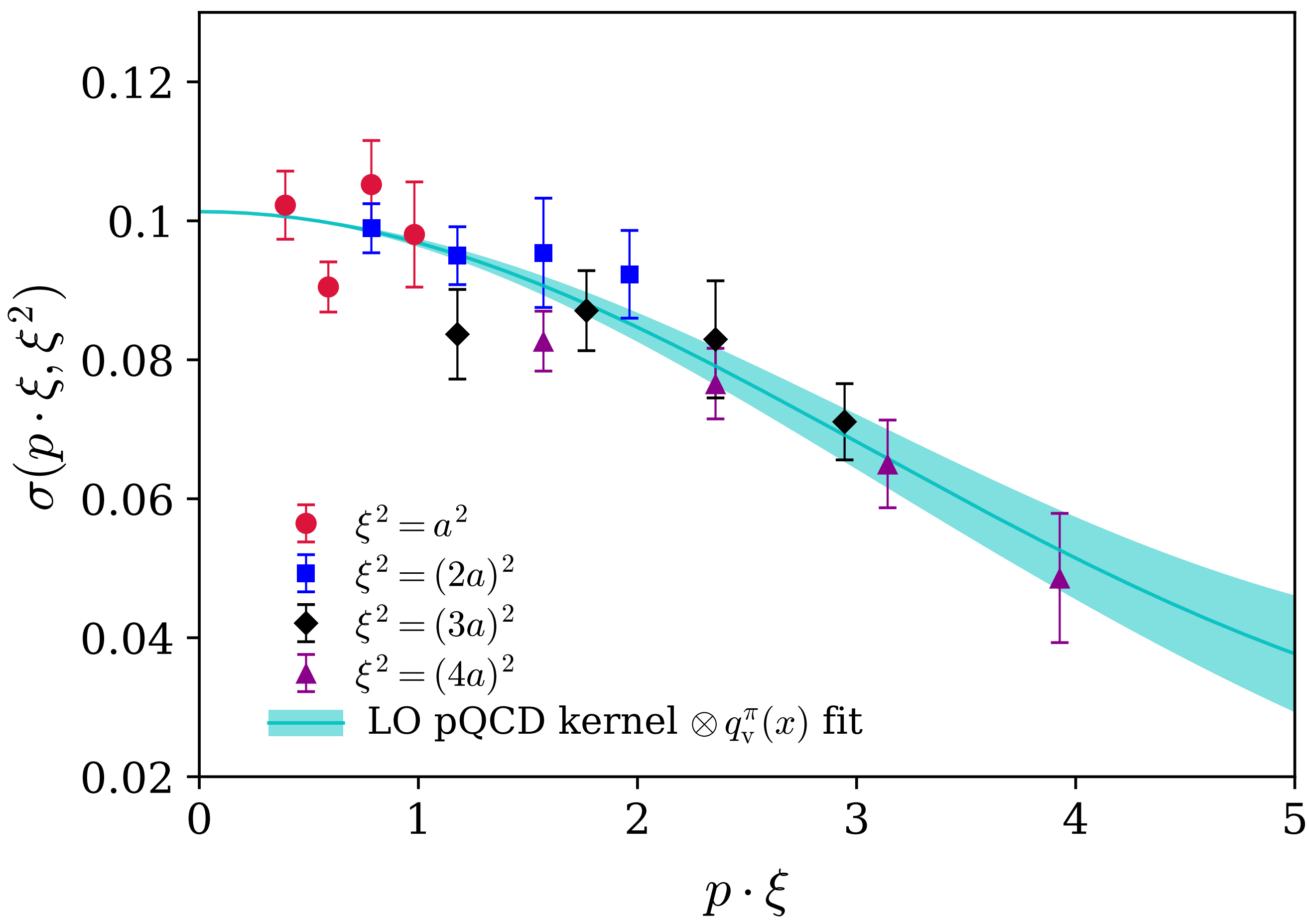
Boosted interpolating operators

Bali *et al.*, Phys. Rev. D 93, 094515 (2016)





Our "Good Cross Section"



Extraction of PDF

"Inverse Problem" - ill-posed inverse Fourier transform.

$$\sigma_n(\omega, \xi^2, P^2) = \sum_a \int_{-1}^{1} \frac{dx}{x} f_a(x, \mu^2) K_n^a(x\omega, \xi^2, x^2P^2, \mu^2) + \mathcal{O}(\xi^2 \Lambda_{\rm QCD}^2)$$

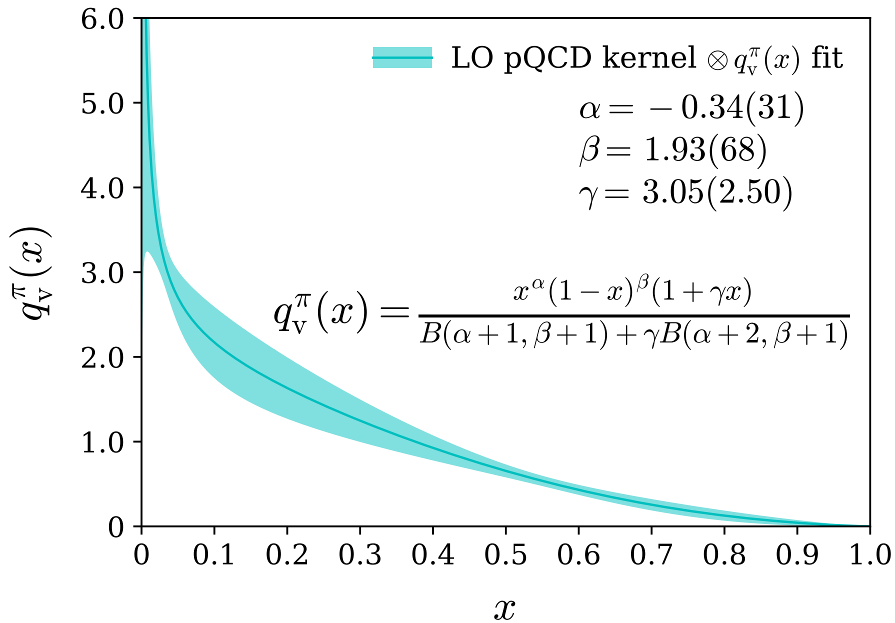
Calculate in PQCD

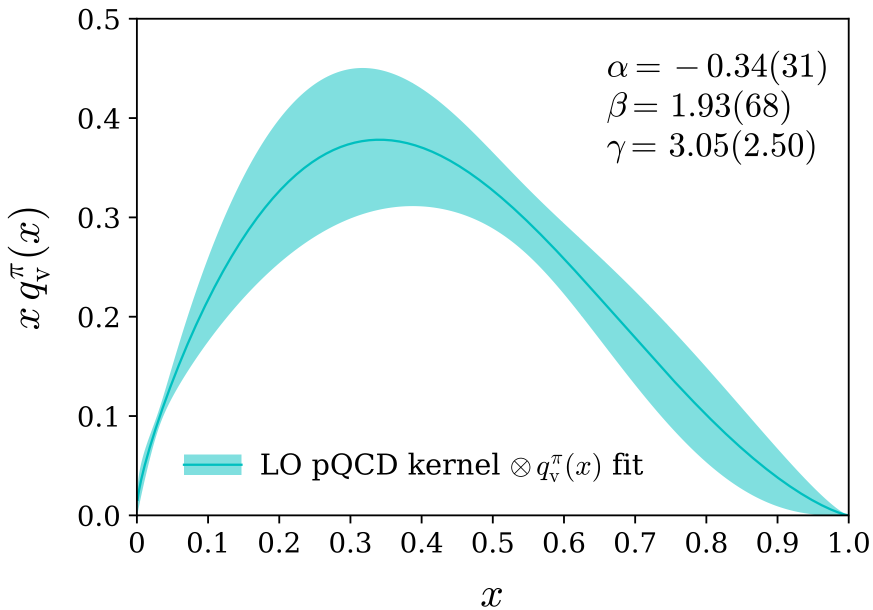
Calculate on Lattice

Extract PDF?

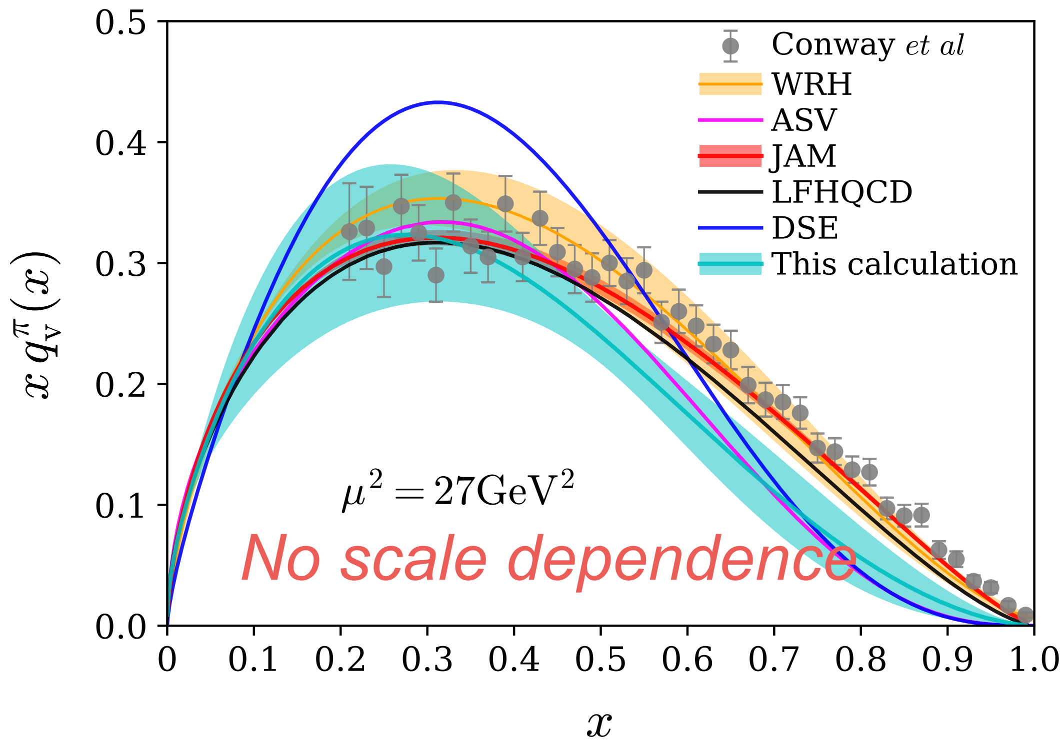
Similar challenge to global fitting community!

see F. Steffens

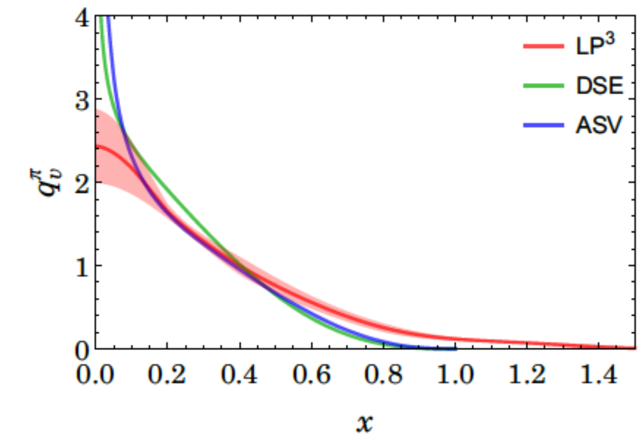


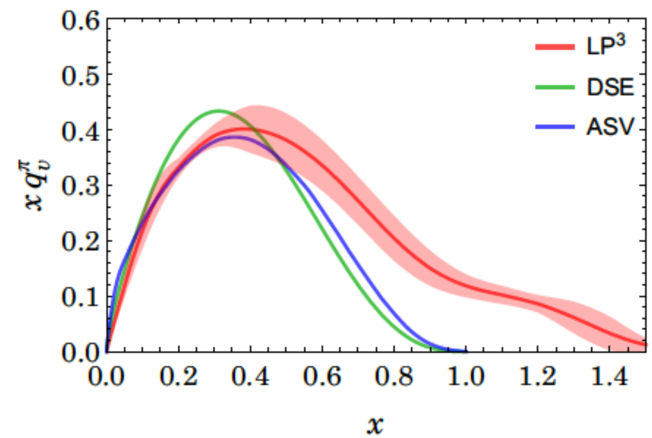


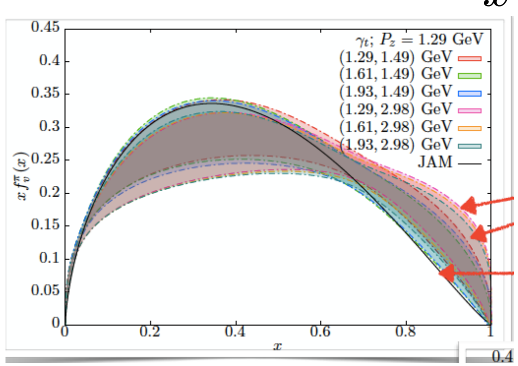
Comparison with Fits/QCD



LP3, arXiv:1804.01483





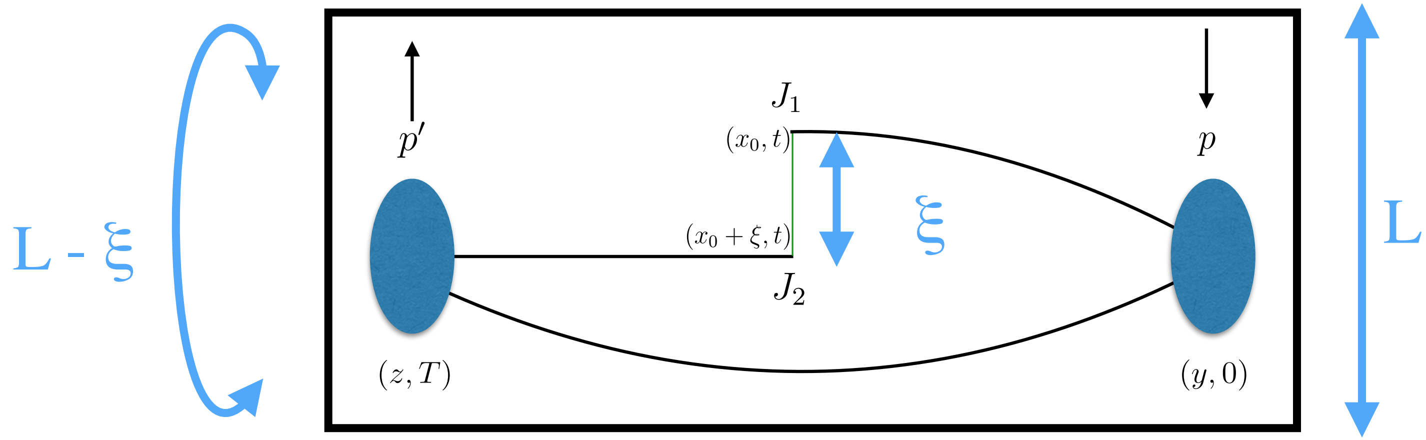


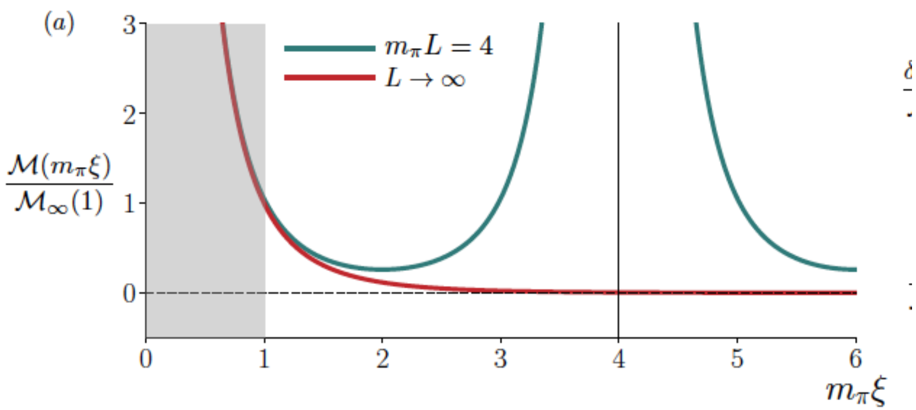
Preliminary

N. Karthik, APS 2019; Lattice 2018

Finite Volume Effects

Briceno, Guerrero, Hansen and Monahan, arXiv:1805.01304





Typically $m_\pi L \simeq 4$

Future? $\begin{cases} \xi \text{ shortdistance} \\ m_\pi \to m_\pi^{\text{phys}} \end{cases}$

Summary

- First calculation of PDFs from gauge-invariant current-current correlators using VA currents.
- Calculation for variety of lattice currents *including Wilson line* in progress
  - Higher-order kernel - scale dependence
  - Off-axis separations and momenta
  - "Higher-twist" contributions
- Important to understand finite-volume effects -
  $24^3 \times 96, a \simeq 0.127\,\mathrm{fm}, m_\pi \simeq 430\,\mathrm{MeV}$

Projected calculations with

$$m_\pi \approx 380\ \mathrm{MeV},\ a \approx 0.09\ \mathrm{fm}\ (32^3 \times 64$$

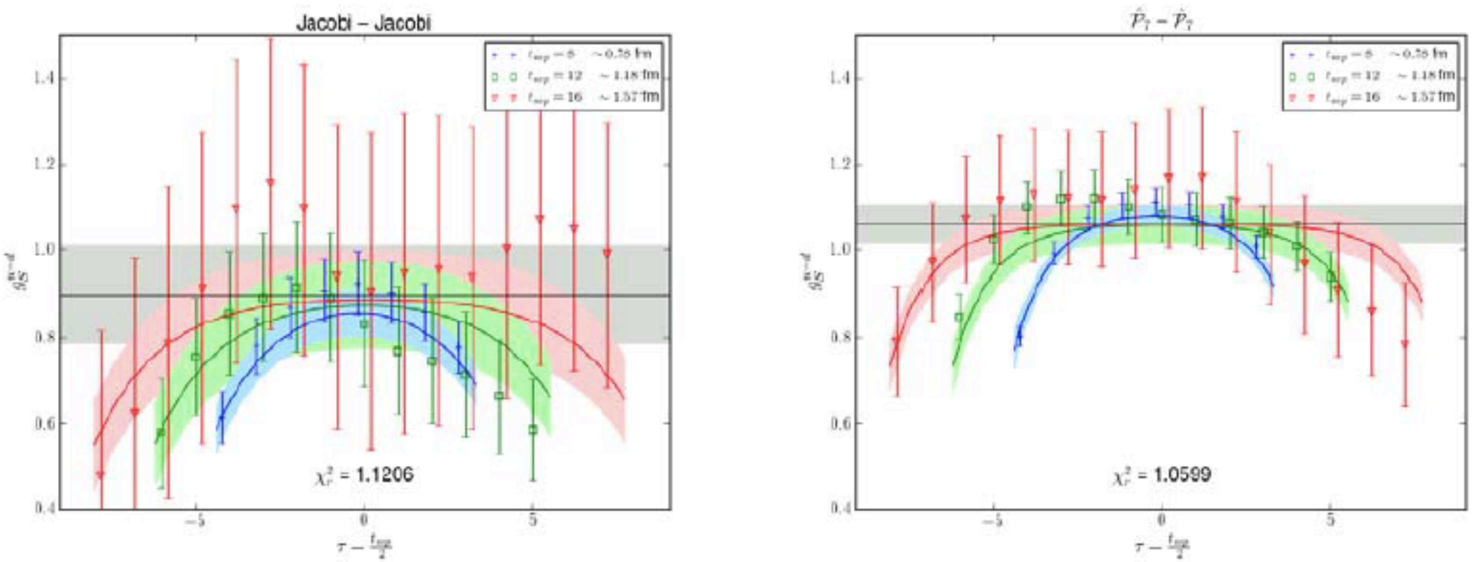
$$m_\pi \approx 170\ \mathrm{MeV}\ (64^3 \times 128)$$

Nucleon?

- Perturbative Calculation Proceeds in same way
- Does not admit straightforward computational method
- Exploit ideas from Spectroscopy?

Nucleon Scalar Charge

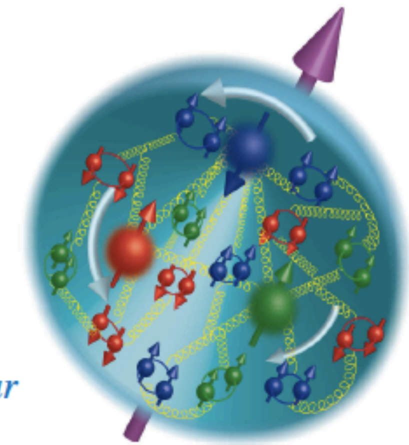
C.Egerer, D.Richards, F.Winter, Phys. Rev. D 99, 034506 (2019)



Improved sampling of lattice through distillation

And a word from Ian and Me….

Benefits of GHP Membership



- The *Topical Group on Hadronic Physics (GHP)* is the dedicated organization that advocates for the science of QCD within the APS; and therefore to the broader physics community, funding agencies, and general public [www.aps.org/units/ghp/]
- Effectiveness of this advocacy and its impact is strongly coupled to the number of GHP members. Importantly, membership determines:
  - Number of APS Fellows the GHP can nominate — *250 members ≃ 1 APS Fellow per-year*
  - Number of invited parallel talks and our own sorting categories at the APS April Meeting

- Hadron Physics is a vibrant field, with upgrades at Jefferson Lab and RHIC, and the proposed $1.5 billion EIC — this growth should also be apparent in the GHP
  - GHP helps reward and highlight the world-class research in our field through, e.g., the GHP Dissertation award and APS Fellows — very important for hires, grants, and promotions
- Please consider joining the GHP — $10/yr with APS membership