Transverse Hadron Structures from Lattice QCD

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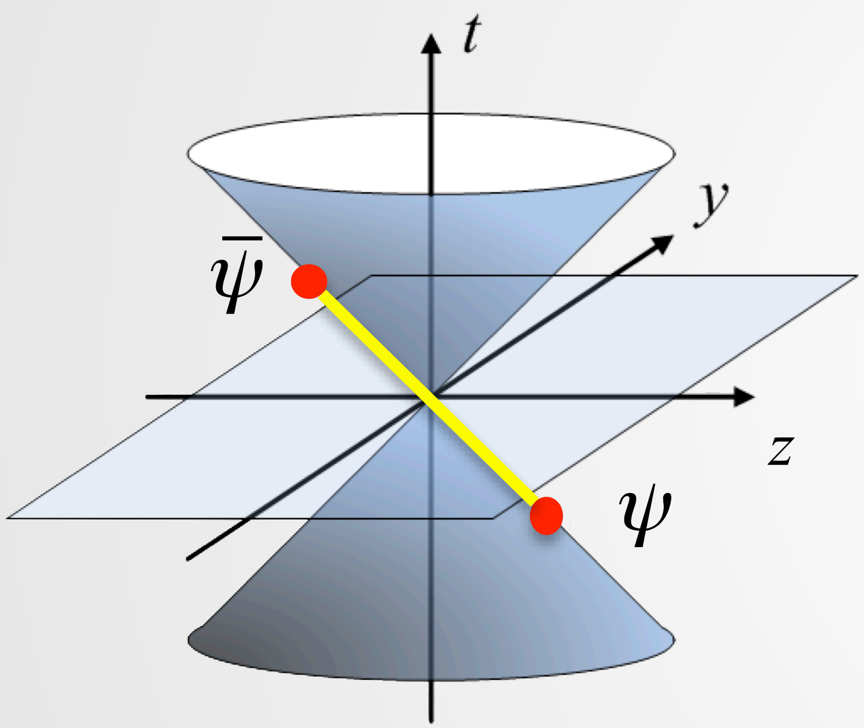
QCD Evolution Workshop
Physics Division, Argonne National Laboratory, IL

Outline

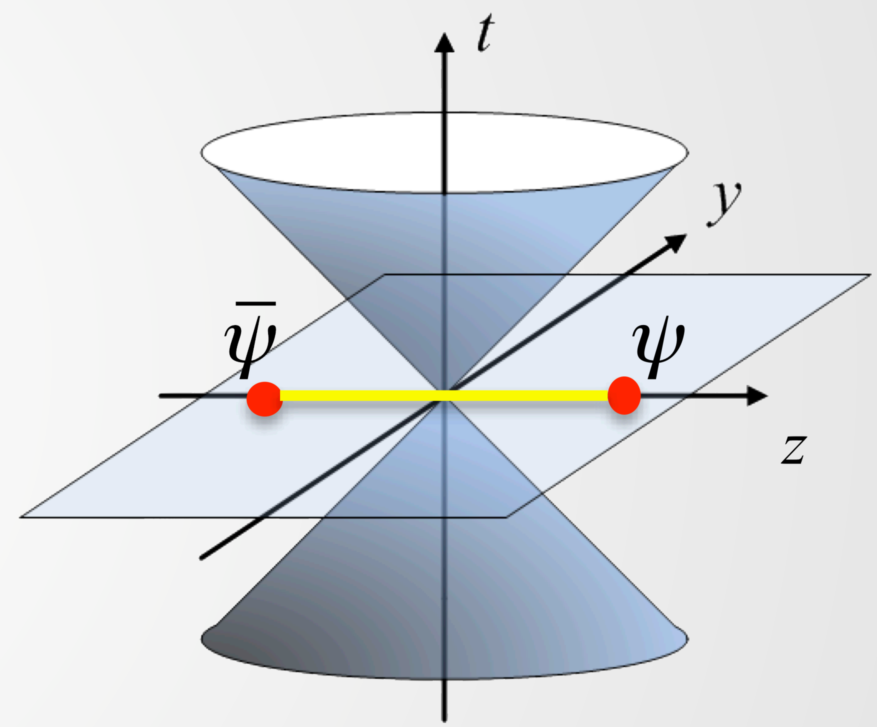
- Large-momentum effective theory
  - Physical picture and factorization formula
  - Systematic approach to extract PDFs from lattice QCD
- Transverse hadron structures from lattice QCD
  - Generalized parton distributions
  - Collins-Soper kernel of TMDPDF from lattice QCD

A novel approach to calculate light-cone PDFs

- Large-Momentum Effective Theory:
  - Ji, PRL110 (2013);
  - Ji, SCPMA57 (2014).



PDF $q(x)$:
Cannot be calculated on the lattice



Quasi-PDF $\tilde{q}(x, P^z)$:
Directly calculable on the lattice

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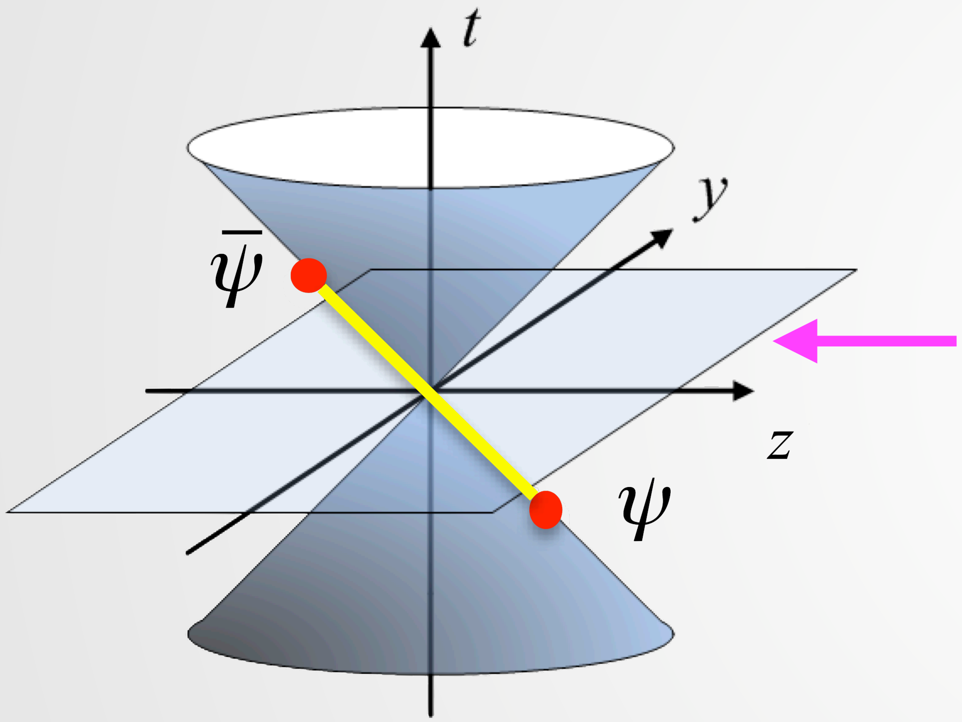
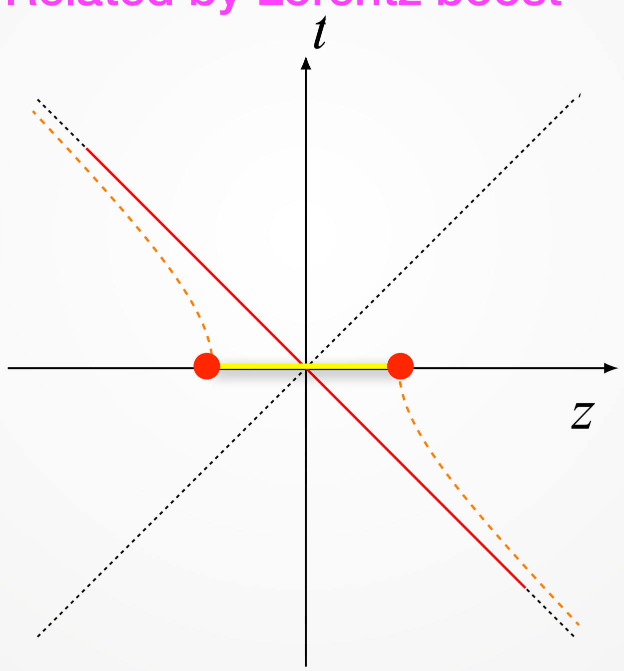
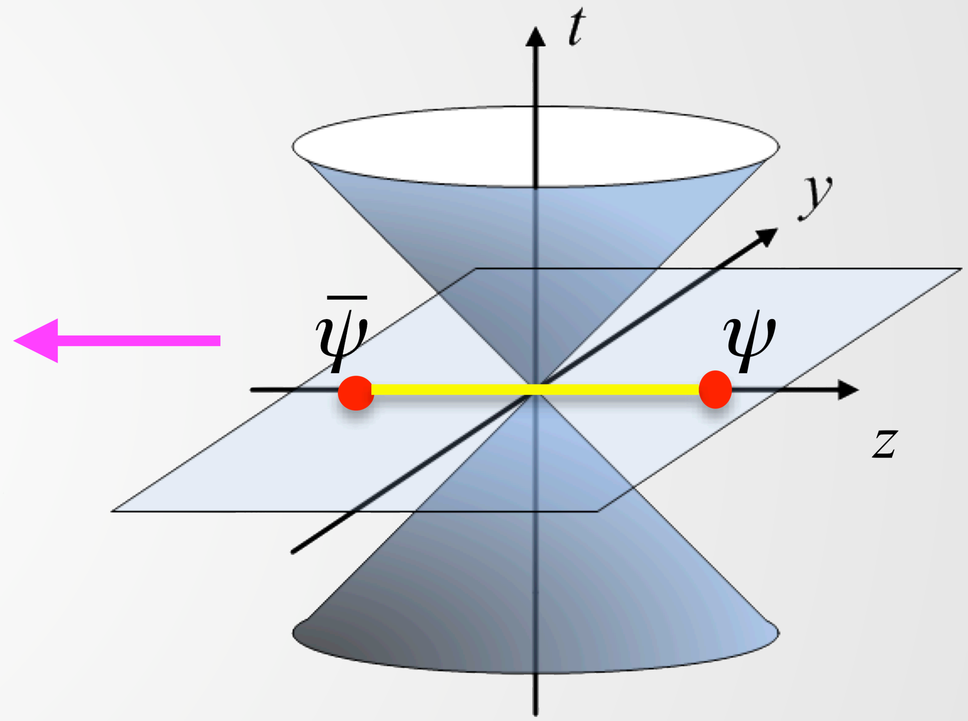


Related by Lorentz boost





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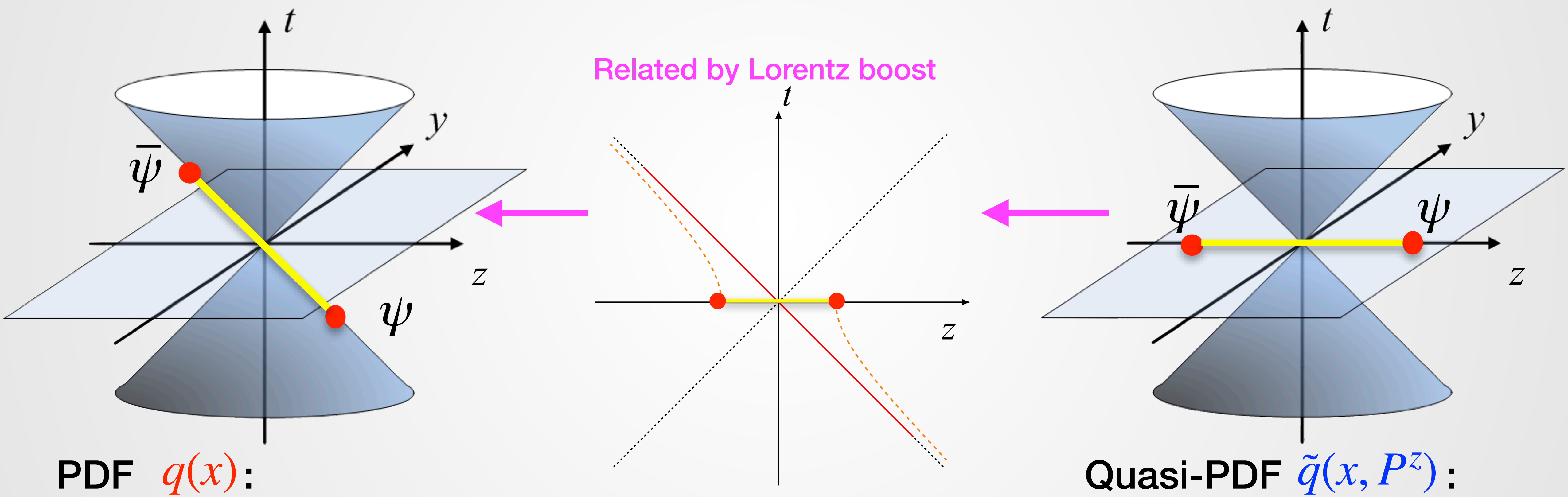


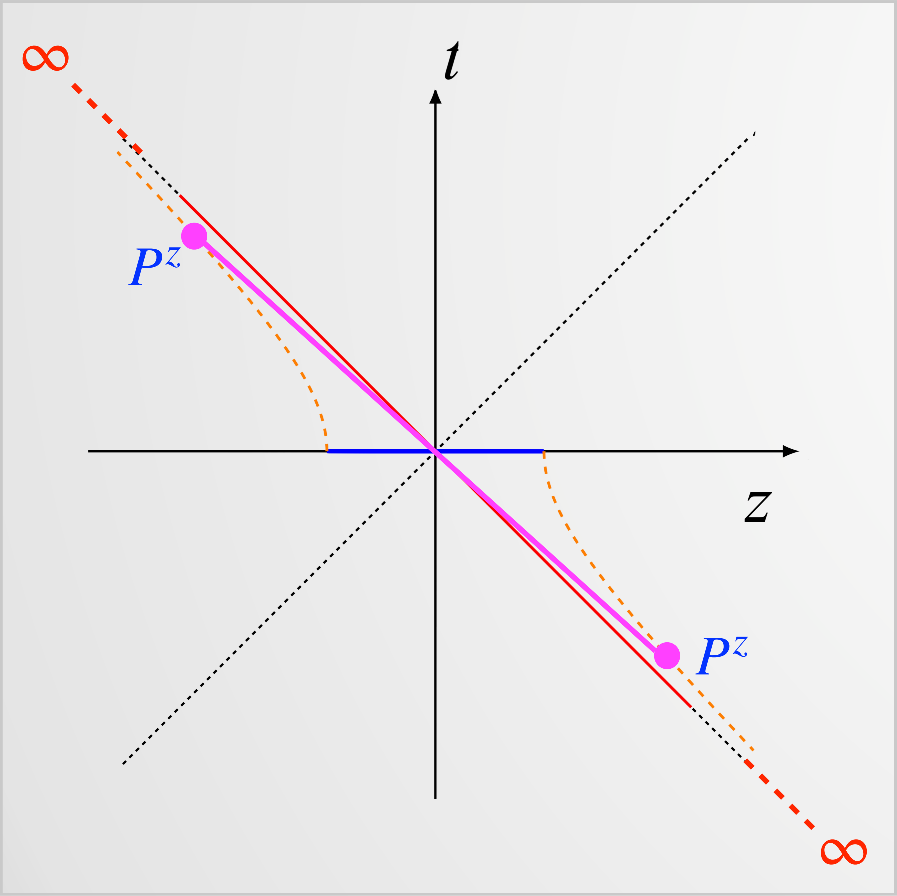
PDF $q(x)$:
Cannot be calculated on the lattice

Calculating the quasi-PDF at hadron momentum P^z is equivalent to boosting it.

Quasi-PDF $\tilde{q}(x, P^z)$:
Directly calculable on the lattice

A novel approach to calculate light-cone PDFs

$$\lim_{P^z\to\infty} \tilde{q}(x,P^z) = ?$$ ✗



Instead of taking $P^z\to\infty$ limit, one can perform an expansion for large but finite P^z:

$$\tilde{q}(x,P^z) = C\left(x,P^z\right) \otimes q(x) + O\left(1/(P^z)^2\right)$$

- $\tilde{q}(x,P^z)$ and $q(x)$ have the same infrared physics (nonperturbative), but different ultraviolet (UV) physics (perturbative);
- Therefore, the matching coefficient $C(x, P^z)$ is perturbative, which controls the logarithmic dependences on P^z.

Systematic procedure of calculating the PDFs

$$\tilde{q}(x, P^z, \mu) = \int \frac{dy}{|y|} C\left(\frac{x}{y}, \boxed{\frac{\mu}{yP^z}}\right) q(y, \mu) + O\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{x^2 P_z^2}\right)$$

- X. Xiong, X. Ji, J.-H. Zhang and Y.Z., PRD90 (2014);
- Y.-Q. Ma and J. Qiu, PRD98 (2018), PRL 120 (2018);
- T. Izubuchi, X. Ji, L. Jin, I. Stewart, and Y.Z., PRD98 (2018).

Systematic procedure of calculating the PDFs

1. Simulation of the quasi PDF in lattice QCD

$$\tilde{q}(x, P^z, \mu) = \int \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP^z}\right) q(y, \mu) + O\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{x^2 P_z^2}\right)$$

Systematic procedure of calculating the PDFs

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2. Renormalization of the lattice quasi PDF, and then taking the continuum limit

Proof of renormalizability:

- X. Ji, J.-H. Zhang, and Y.Z., PRL120 (2018);
- J. Green et al., PRL121 (2018);
- T. Ishikawa, Y.-Q. Ma, J. Qiu, S. Yoshida, PRD96 (2017).

Nonperturbative renormalization on the lattice:

- I. Stewart and Y.Z., PRD97 (2018);
- J.-W. Chen, Y.Z. et al., LP3 Collaboration, PRD97 (2018).
- Constantinou and Panagopoulos, PRD96 (2017);
- C. Alexandrou et al., ETM Collaboration, NPB923 (2017).

Systematic procedure of calculating the PDFs

3. Subtraction of power corrections

- O Nachtmann, NPB63 (1973);
- J.W. Chen et al. (LP3), NPB911 (2016).

$$\tilde{q}(x, P^z, \mu) = \int \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP^z}\right) q(y, \mu) + O\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{x^2 P_z^2}\right)$$

$$q(x) \cdot O\left(\frac{\Lambda_{\rm QCD}^2}{x^2(1-x)P_z^2}\right)$$

Renormalon contribution to the power correction:
Braun, Vladimirov, and Zhang, PRD99 (2019).

Systematic procedure of calculating the PDFs

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- Matching for the quasi-PDF:
  - X. Xiong, X. Ji, J.-H. Zhang and Y.Z., PRD90 (2014);
  - I. Stewart and Y.Z., PRD97 (2018);
  - Y.-S. Liu, Y.Z. et al. (LP3), arXiv:1807.06566;
  - T. Izubuchi, X. Ji, L. Jin, I. Stewart, and Y.Z., PRD98 (2018);
  - Y.-S. Liu, Y.Z. et al., arXiv:1810.10879;
  - Y.Z., Int.J.Mod.Phys. A33 (2019);
  - C. Alexandrou et al. (ETMC), arXiv: 1902.00587.

See F. Steffens' talk.

4. Matching to the PDF.

For recent progress on the lattice calculation of quark iso-vector PDFs, see M. Constantinou's talk.

Systematic procedure of calculating the PDFs

5. Extract $q(y)$

$$\tilde{q}(x, P^z, \mu) = \int \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP^z}\right) q(y, \mu) + O\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{x^2 P_z^2}\right)$$

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  - T. Izubuchi, X. Ji, L. Jin, I. Stewart, and Y.Z., PRD98 (2018);
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PDFs for transverse hadron structures

- Generalized Parton Distribution (GPD)

  $F_i(x, \xi = 0, \vec{b}_T)$

  - $\vec{b}_T$: transverse position of the parton.

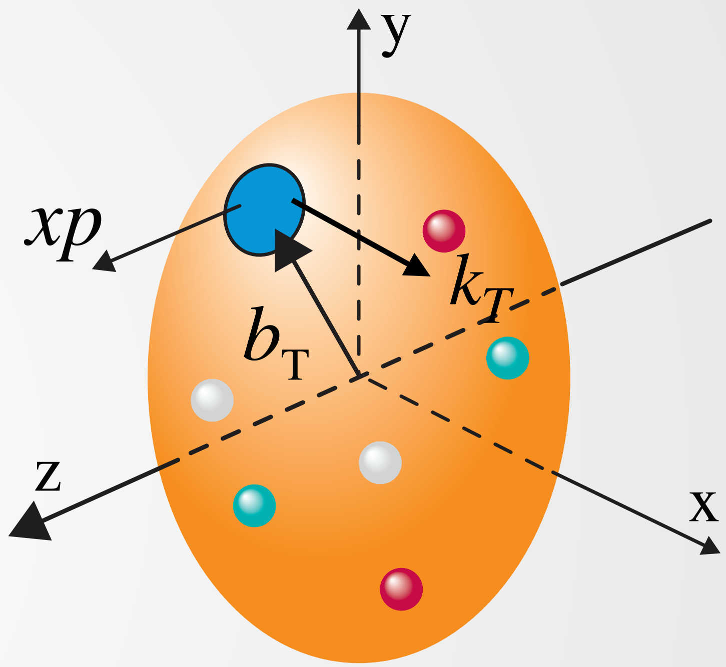
- Transverse momentum dependent (TMD) PDF

  $q_i(x, \vec{k}_T)$

  - $\vec{k}_T$: transverse momentum of the parton.

- Wigner distribution or generalized transverse momentum dependent distribution

  $W_i(x, \xi = 0, \vec{k}_T, \vec{b}_T)$



The longitudinal and transverse PDFs provide complete 3D structural information of the proton.

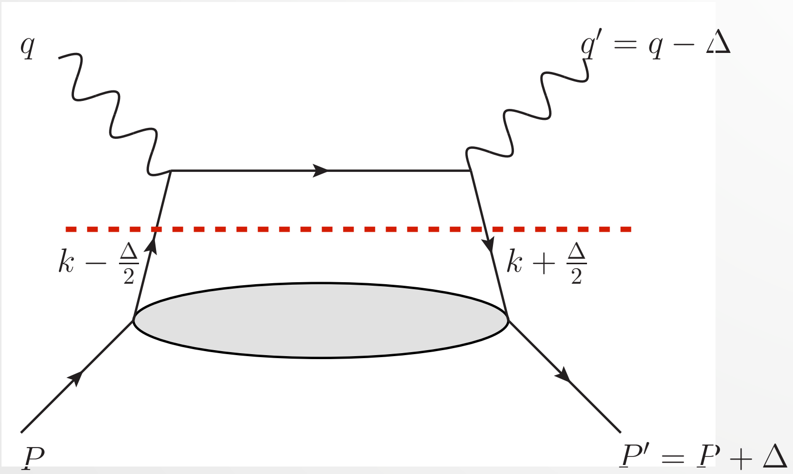
GPD

- Light-cone GPD:

$$\xi = \frac{P^+ - P'^+}{P^+ + P'^+}, \quad t = (P' - P)^2 \equiv \Delta^2$$

$$F_\Gamma(x,\xi,t,\mu) = \int \frac{d\zeta^-}{4\pi} e^{-ix\bar{P}^+\zeta^-} \langle P', S' | \bar{\psi}(\frac{\zeta^-}{2}) \Gamma U(\frac{\zeta^-}{2}, -\frac{\zeta^-}{2}) \psi(-\frac{\zeta^-}{2}) | P, S \rangle$$

- Measurable in hard exclusive processes such as deeply virtual Compton scattering:



$$\sim \int dx\ C(x,\xi) F(x,\xi,t)$$

- Efforts to extract GPDs with global analysis have begun.

"Status and prospects of GPD extraction from DVCS", K. Kumericki, @INT Workshop INT-18-3, 2018.

Quasi-GPD

$$\tilde{\xi} = \frac{P^z - P'^z}{P^z + P'^z} = \xi + O\left(\frac{M^2}{P_z^2}\right)$$

- **Definition:**

$$\tilde{F}_{\tilde{\Gamma}}(x, \tilde{\xi}, t, \mu) = \int \frac{dz}{4\pi} e^{-ixP^z z} \langle P', S' | \bar{\psi}\left(\frac{z}{2}\right) \tilde{\Gamma} U\left(\frac{z}{2}, -\frac{z}{2}\right) \psi\left(-\frac{z}{2}\right) | P, S \rangle$$

- **Renormalization:**
  - Same operator as the quasi-PDF, can be renormalized the same way!

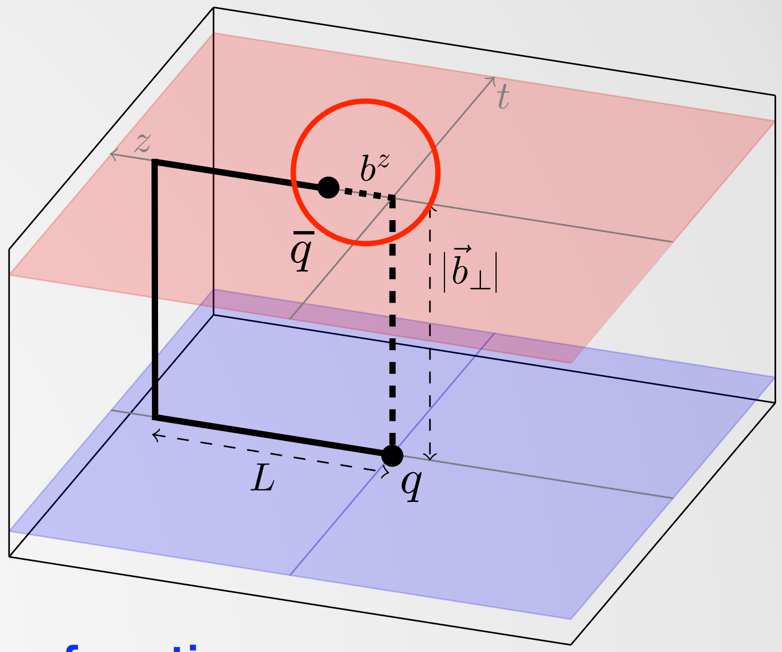
- **Factorization formula:**
  - Y.-S. Liu, Y.Z. et al., arXiv:1902.00307.

$$\tilde{F}_{\tilde{\gamma}^z}(x, \xi, t, \mu) = \int_{-1}^{1} \frac{dy}{|\xi|} C\left(\frac{x}{\xi}, \frac{y}{\xi}, \frac{\mu}{\xi P^z}\right) F_{\gamma^+}(y, \xi, t, \mu) + O\left(\frac{M^2}{P_z^2}, \frac{t}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{x^2 P_z^2}\right)$$

$$= \int_{-1}^{1} \frac{dy}{|y|} \bar{C}\left(\frac{x}{y}, \frac{\xi}{y}, \frac{\mu}{y P^z}\right) F_{\gamma^+}(y, \xi, t, \mu) + O\left(\frac{M^2}{P_z^2}, \frac{t}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{x^2 P_z^2}\right)$$

- First lattice calculation of pion GPD, Chen, Lin and Zhang, arXiv: 1904.12376.
- Preliminary results for quasi-GPDs (ETMC), see M. Constantinou's talk.

Collins-Soper kernel of TMDPDF from lattice QCD



For introduction see
Markus Ebert's talk.

- M. Ebert, I. Stewart, Y.Z., PRD99 (2019);
- M. Ebert, I. Stewart, Y.Z., arXiv:1901.03685;
- M. Ebert, I. Stewart, Y.Z., in progress.

Physical limit:

$$b^z \sim \frac{1}{P^z} \ll b_T \ll L$$

Quasi-beam function
(or un-subtracted quasi-TMD)

$$\gamma_\zeta^q(\mu, b_T) = \frac{1}{\ln(P_1^z/P_2^z)}$$

$$\times \ln \frac{C_{\rm ns}^{\rm TMD}(\mu, xP_2^z) \int db^z\ e^{ib^z xP_1^z}\ \tilde{Z}'(b^z, \mu, \tilde{\mu})\tilde{Z}_{\rm UV}(b^z, \tilde{\mu}, a)\tilde{B}_{\rm ns}(b^z, \vec{b}_T, a, L, P_1^z)}{C_{\rm ns}^{\rm TMD}(\mu, xP_1^z) \int db^z\ e^{ib^z xP_2^z}\ \tilde{Z}'(b^z, \mu, \tilde{\mu})\tilde{Z}_{\rm UV}(b^z, \tilde{\mu}, a)\tilde{B}_{\rm ns}(b^z, \vec{b}_T, a, L, P_2^z)}$$

The idea of forming ratios to cancel the soft function has been used in the calculation of *x*-moments of TMDPDFs by
Hagler, Musch, Engelhardt, Yoon, et al., EPL88 (2009), PRD83 (2011), PRD85 (2012), PRD93 (2016), arXiv:1601.05717, PRD96 (2017)

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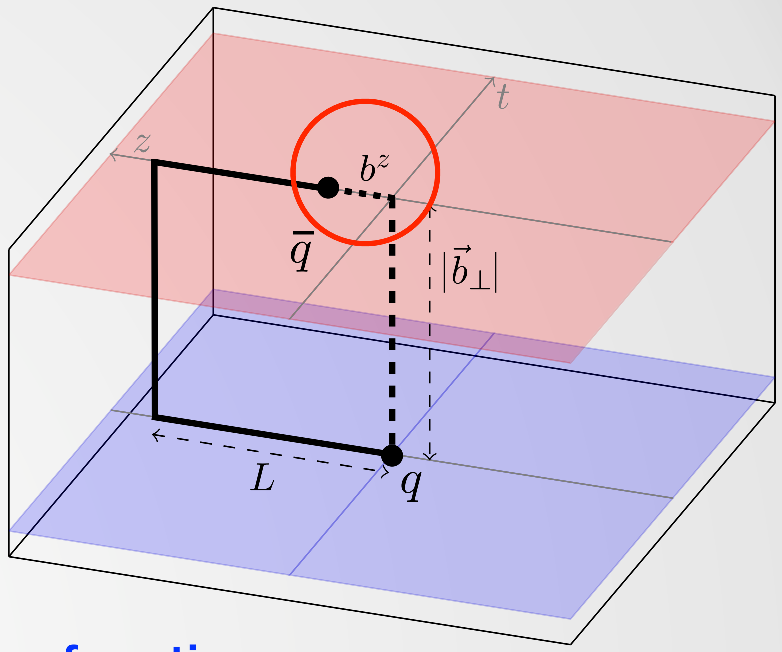

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Physical Collins-Soper (CS) kernel does not depend on the external hadron state, which means that one can just calculate it with a pion state including heavier than physical valence quarks.

work in progress with

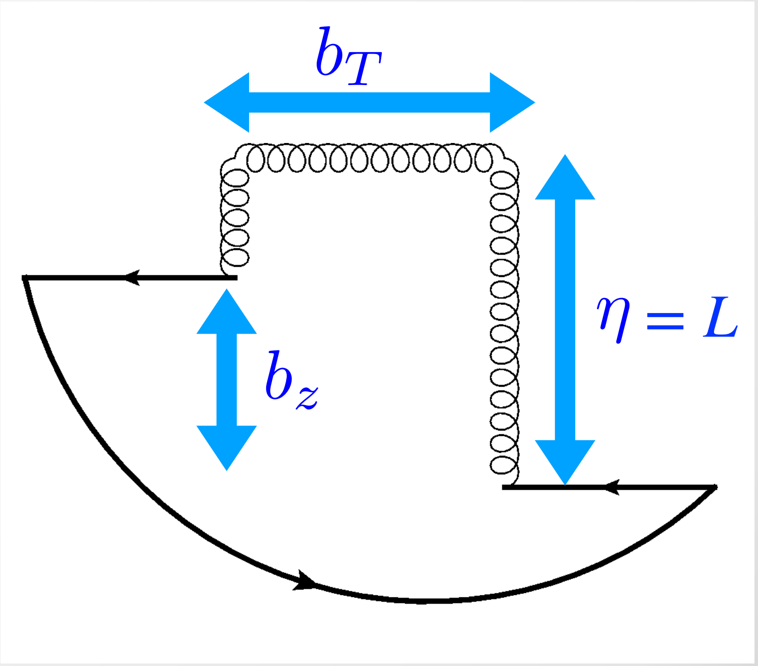
Phiala Shanahan and Michael Wagman.

Procedure of lattice calculation

- 1. Lattice simulation of the bare quasi-beam function

$$\gamma_\zeta^q(\mu, b_T) = \frac{1}{\ln(P_1^z/P_2^z)} \times \ln \frac{C_{\rm ns}^{\rm TMD}(\mu, xP_2^z) \int db^z\, e^{ib^z x P_1^z} Z'(b^z, \mu, \tilde\mu) \tilde Z_{\rm UV}(b^z, \tilde\mu, a) \tilde B_{\rm ns}(b^z, \vec b_T, a, L, P_1^z)}{C_{\rm ns}^{\rm TMD}(\mu, xP_1^z) \int db^z\, e^{ib^z x P_2^z} Z'(b^z, \mu, \tilde\mu) \tilde Z_{\rm UV}(b^z, \tilde\mu, a) \tilde B_{\rm ns}(b^z, \vec b_T, a, L, P_2^z)}$$

$$b^z \sim \frac{1}{P^z} \ll b_T \ll \eta < \frac{L_{\rm Lat}}{2}$$



Choice of γ matrix: to choose γ^t or γ^z depending on seriousness of operator mixing.

- M. Constantinou et al., PRD99 (2019)

Procedure of lattice calculation

- 2. Renormalization and matching to the MSbar scheme

$$\gamma_\zeta^q(\mu, b_T) = \frac{1}{\ln(P_1^z/P_2^z)} \times \ln \frac{C_{\rm ns}^{\rm TMD}(\mu, xP_2^z) \int db^z\, e^{ib^z xP_2^z}\, \tilde{Z}'(b^z, \mu, \tilde{\mu})\tilde{Z}_{\rm UV}(b^z, \tilde{\mu}, a)\tilde{B}_{\rm ns}(b^z, \vec{b}_T, a, L, P_1^z)}{C_{\rm ns}^{\rm TMD}(\mu, xP_1^z) \int db^z\, e^{ib^z xP_1^z}\, \tilde{Z}'(b^z, \mu, \tilde{\mu})\tilde{Z}_{\rm UV}(b^z, \tilde{\mu}, a)\tilde{B}_{\rm ns}(b^z, \vec{b}_T, a, L, P_2^z)}$$

Multiplicative renormalizability of the Wilson line operator assumed to be provable using the auxiliary field formalism.

- X. Ji, J.-H. Zhang, and Y.Z., PRL120 (2018);
- J. Green et al., PRL121 (2018);

Linear power divergence: $\sim \frac{L - b_z}{a} + \frac{b_T}{a} + \frac{L}{a}$

Nonperturbative Renormalization: $\tilde{Z}_{\rm UV}(b^z, \tilde{\mu}, a)$

Perturbative matching to MSbar scheme: $\tilde{Z}'(b^z, \mu, \tilde{\mu})$

Procedure of lattice calculation

- 3. Fourier transform and calculate the ratio at different P^z

$$\gamma_\zeta^q(\mu, b_T) = \frac{1}{\ln(P_1^z/P_2^z)} \times \ln \frac{C_{\rm ns}^{\rm TMD}(\mu, xP_2^z) \int db^z\ e^{ib^z xP_1^z}\ \tilde{Z}'(b^z,\mu,\tilde{\mu})\tilde{Z}_{\rm UV}(b^z,\tilde{\mu},a)\tilde{B}_{\rm ns}(b^z,\vec{b}_T,a,L,P_1^z)}{C_{\rm ns}^{\rm TMD}(\mu, xP_1^z) \int db^z\ e^{ib^z xP_2^z}\ \tilde{Z}'(b^z,\mu,\tilde{\mu})\tilde{Z}_{\rm UV}(b^z,\tilde{\mu},a)\tilde{B}_{\rm ns}(b^z,\vec{b}_T,a,L,P_2^z)}$$

  - Independent of the choice of *x*!
  - Independent of P^z!
  - Fourier transform has truncation errors, but for given P^z there is always a region of *x* that is insensitive to such truncation effects;
  - One may still seek alternatives to Fourier transforms that can be done directly in coordinate space.

Lattice calculation

- Lattice setup:

  generated by Michael Endres

  - Quenched Wilson gauge configurations;
  - β=6.30168, a=0.06(1) fm, $32^3 \times 64$;
  - Probe valence pion with $m_\pi \sim 1.2$ GeV
  - Each momentum uses 2 gauge fixed plane wave ("wall") quark sources;
  - **A first look at N_{cfg}=7.**

$$0 \leq b^z, b_T \leq \eta, \quad \eta = \{7,8,9,10\}a \quad p^z = \{2,\ 3,\ 4\}\frac{2\pi}{L}, p^z_{\max} = 2.6\ \mathrm{GeV}$$

Bare matrix elements

- 3pt/2pt for matrix element fit:

$$\left(1+\sum_{n>0}\frac{Z_n}{Z_0}e^{-[E_n(p)-E_0(p)]t}\right)\frac{G^{3pt}(b^z,b_T,\eta,P^z,t,\tau)}{G^{2pt}(P^z,t)}=$$

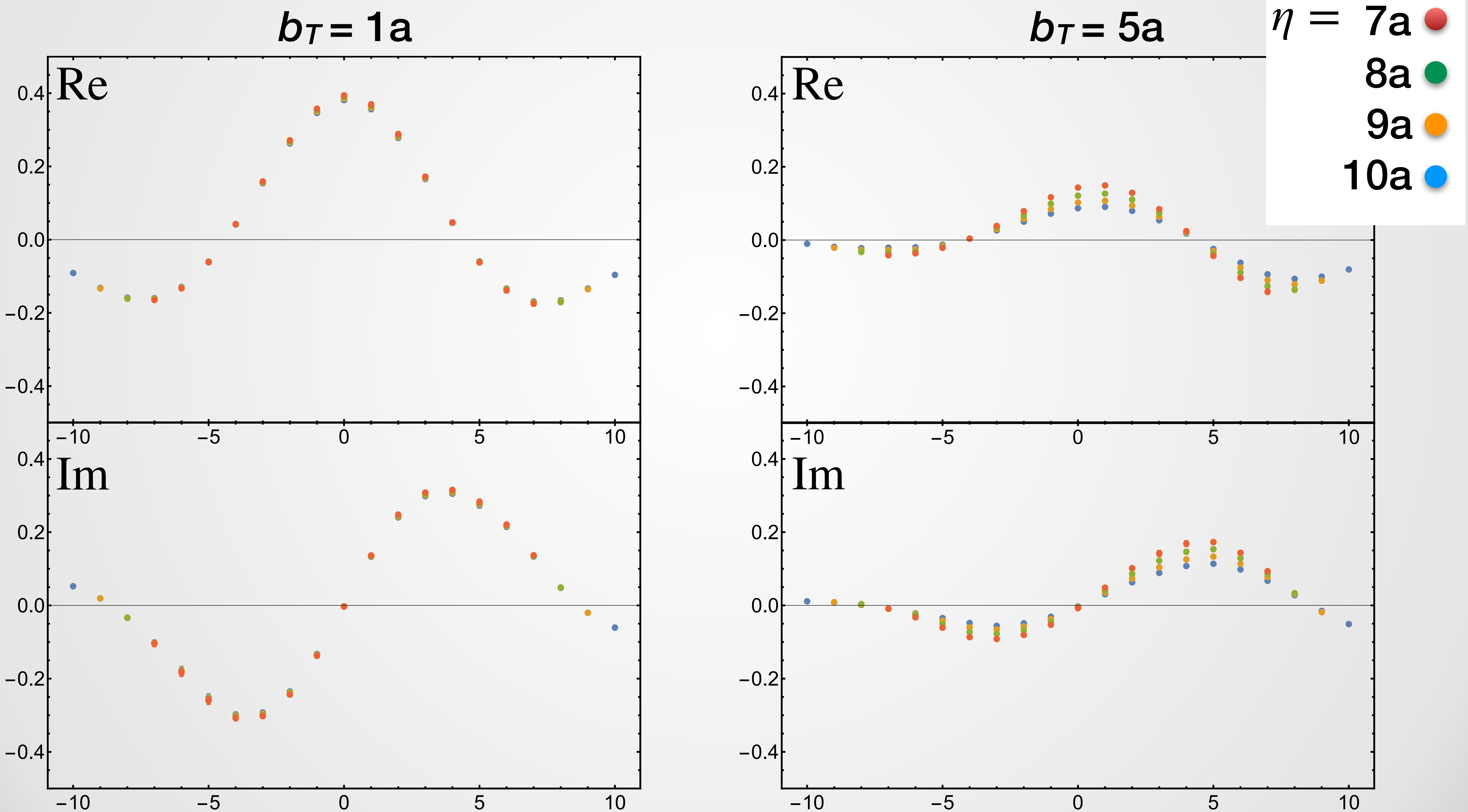
$$\langle 0|\mathcal{O}|0\rangle+\sum_{n,m>0}\langle n|\mathcal{O}|m\rangle e^{-[E_n(p)-E_0(p)]\tau}e^{-[E_m(p)-E_0(p)](t-\tau)}$$

- 6 sources-sink separations $t=8a,\ 10a,\ 12a,\ 14a,\ 16a,\ 18a$
- Simultaneous fit to all *t* and *τ*.

Caveat N_{cfg}=7

Bare matrix elements

P^z=2.6 GeV



- Error bars smaller than point size.
- Asymmetric real and imaginary parts in b_z due to stapled shape.

Lattice renormalization in the RI/MOM Scheme

Green's function: $G(b,p) = \sum_x \left\langle \gamma_5 S^\dagger(p, b+x)\gamma_5 U(b+x,x)\frac{\Gamma}{2}S(p,x)\right\rangle$

Amputated Green's function (or vertex function): $\Lambda(b,p) = \left(\gamma_5\left[S^{-1}(p)\right]^\dagger\right)G(b,p)S^{-1}(p)$

Momentum subtraction condition:

$$Z_{\mathcal{O}}^{-1}(b,p_\mu^R,\mu_R)Z_q(\mu_R)\ G(b,p)\Big|_{p_\mu=p_\mu^R} = G^{\text{tree}}(b,p_R)\,,$$

$$Z_q(\mu_R) = \frac{1}{12}\text{Tr}\left[S^{-1}(p)S^{\text{tree}}(p)\right]\Big|_{p^2=\mu_R^2}$$

- I. Stewart and Y.Z., PRD97 (2018);
- Constantinou and Panagopoulos, PRD96 (2017);
- M. Constantinou et al., PRD99 (2019).

Parametrization of amputated Green's functions:

$$\Lambda_{\gamma^t}(z,p) = \tilde{F}_t\gamma^t + \underline{\tilde{F}_z\gamma^z + \tilde{F}_T\frac{b\!\!\!/_T}{b_T} + \tilde{F}_{p\!\!\!/}\frac{p^t p\!\!\!/}{p^2}}$$

Equation of motion

$$+ \underline{\tilde{F}_{\sigma_{tz}}\sigma^{tz} + \tilde{F}_{\sigma_{tT}}\sigma^{tT}}$$

Chiral symmetry breaking

Mixing

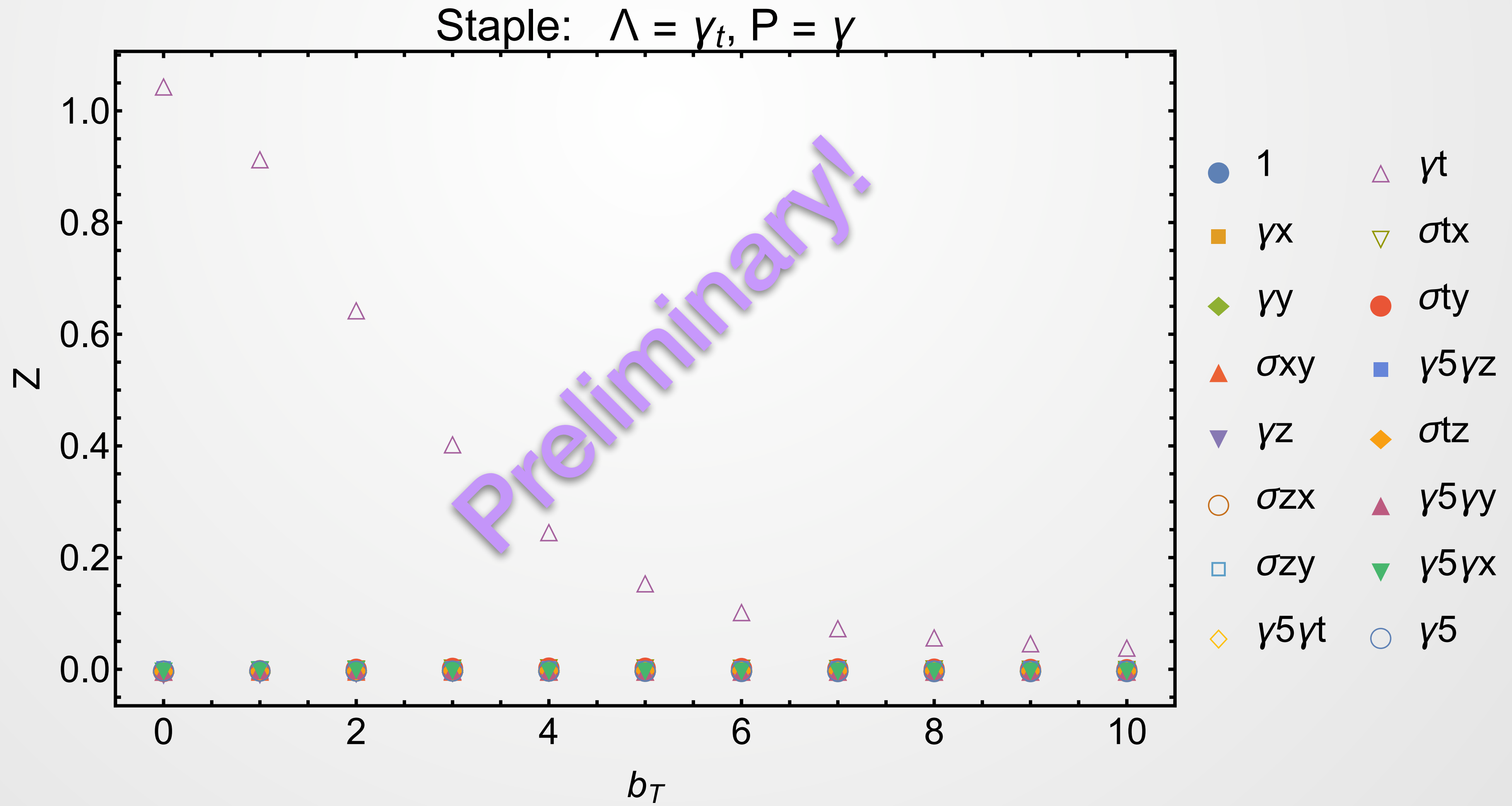
- Tracing with a projection operator to define the renormalization factors.

Tr $\left[\Lambda_{\gamma^t}(z,p)\mathscr{P}\right]$

- For simplicity, we choose $\mathscr{P} = \gamma^t$

- To study the mixing effects, we also choose all the other 15 Gamma matrices.

$b_z = 0$,
eta = 10a.



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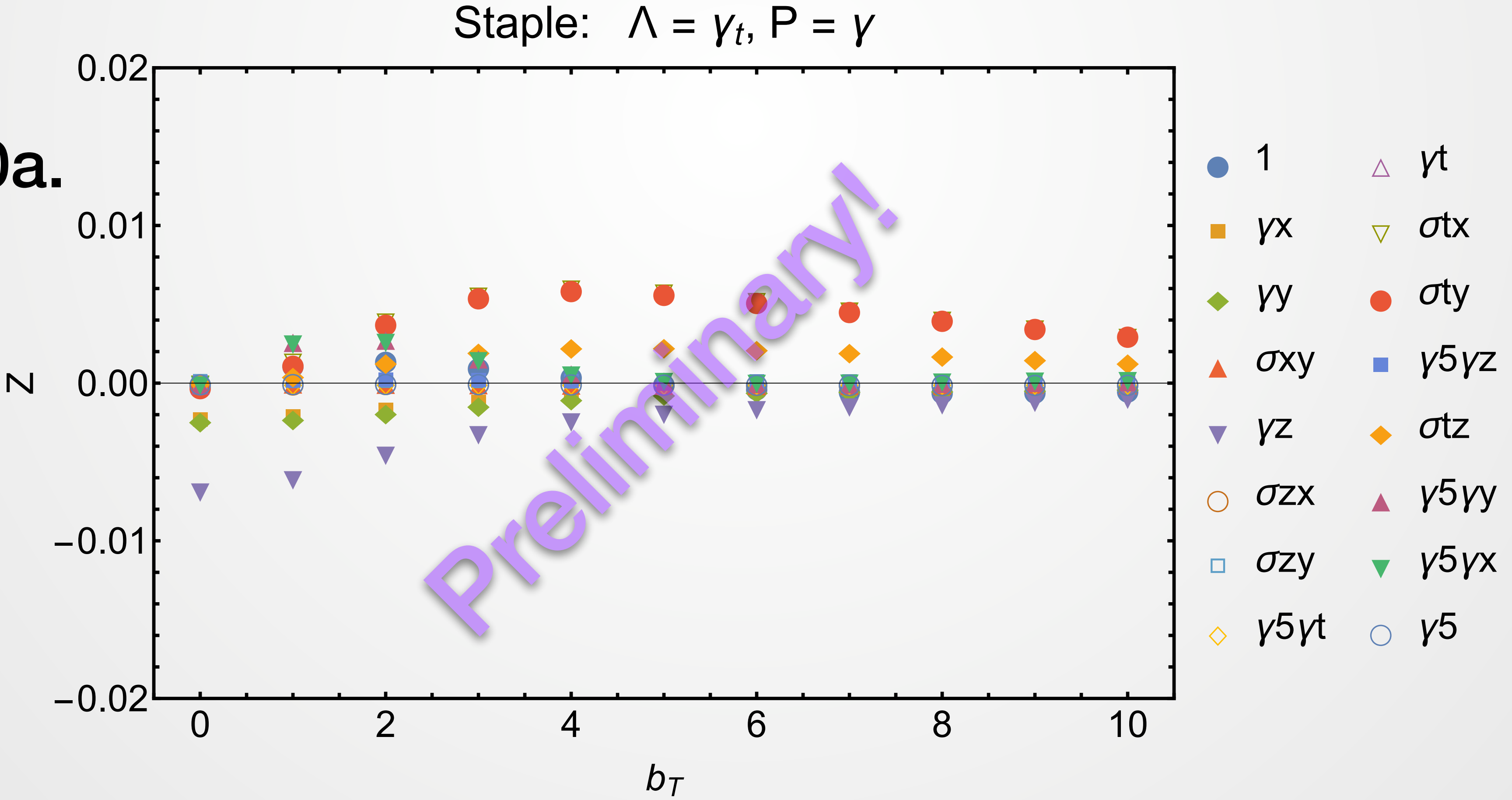
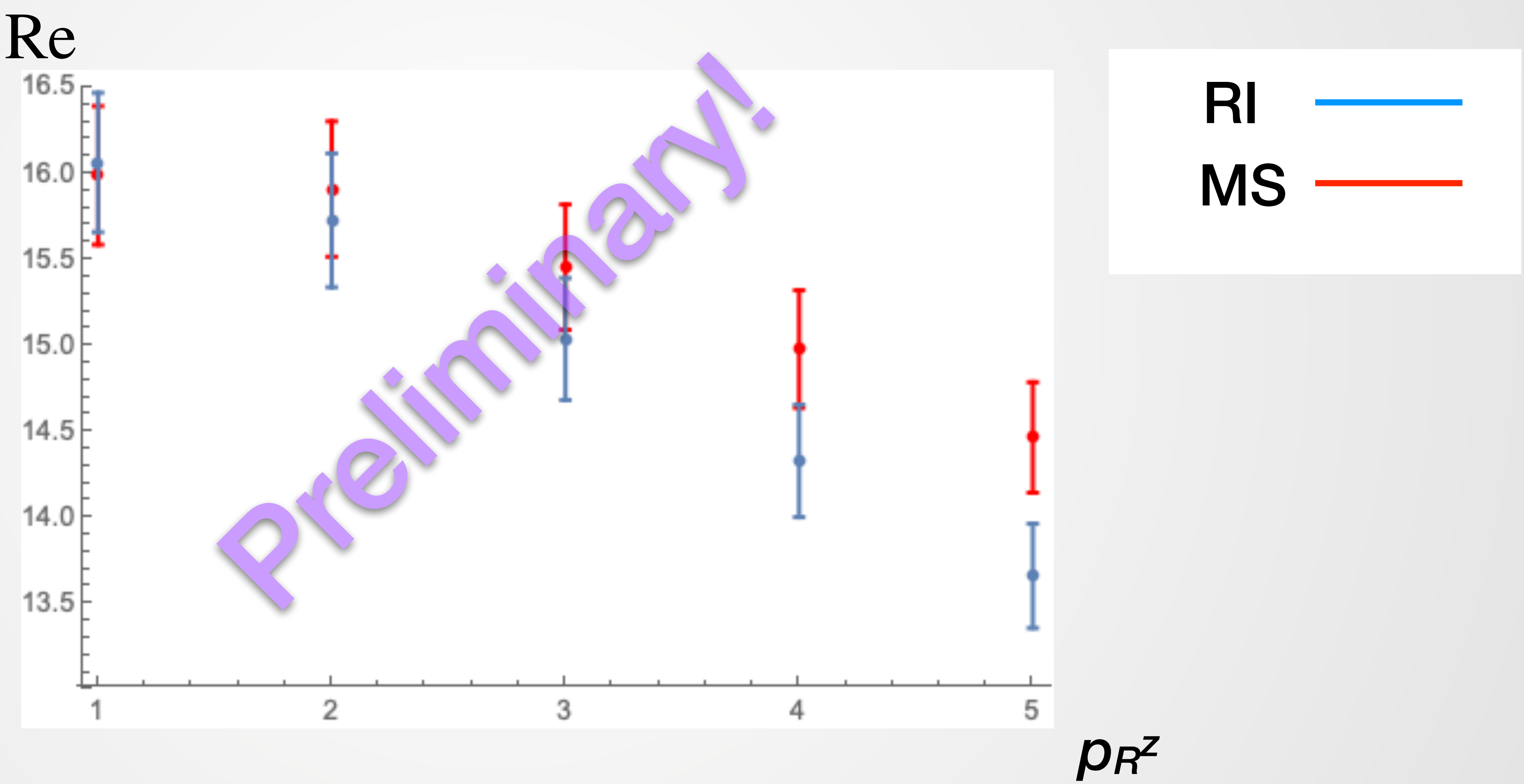


O(1%) effects, negligible for exploratory study.

Matching to MSbar scheme @ 2GeV

$$Z_{\overline{\rm MS}}(\eta, b_z, b_T, \mu, a) = Z_{\mathcal{O}}^{-1}(\eta, b^z, b_T, p_\mu^R, \mu^R, a) \cdot C(\eta, b^z, b_T, p_\mu^R, \mu^R, \mu)$$

$b_z = 0$,
$b_T = 8a$,
eta = 10a,

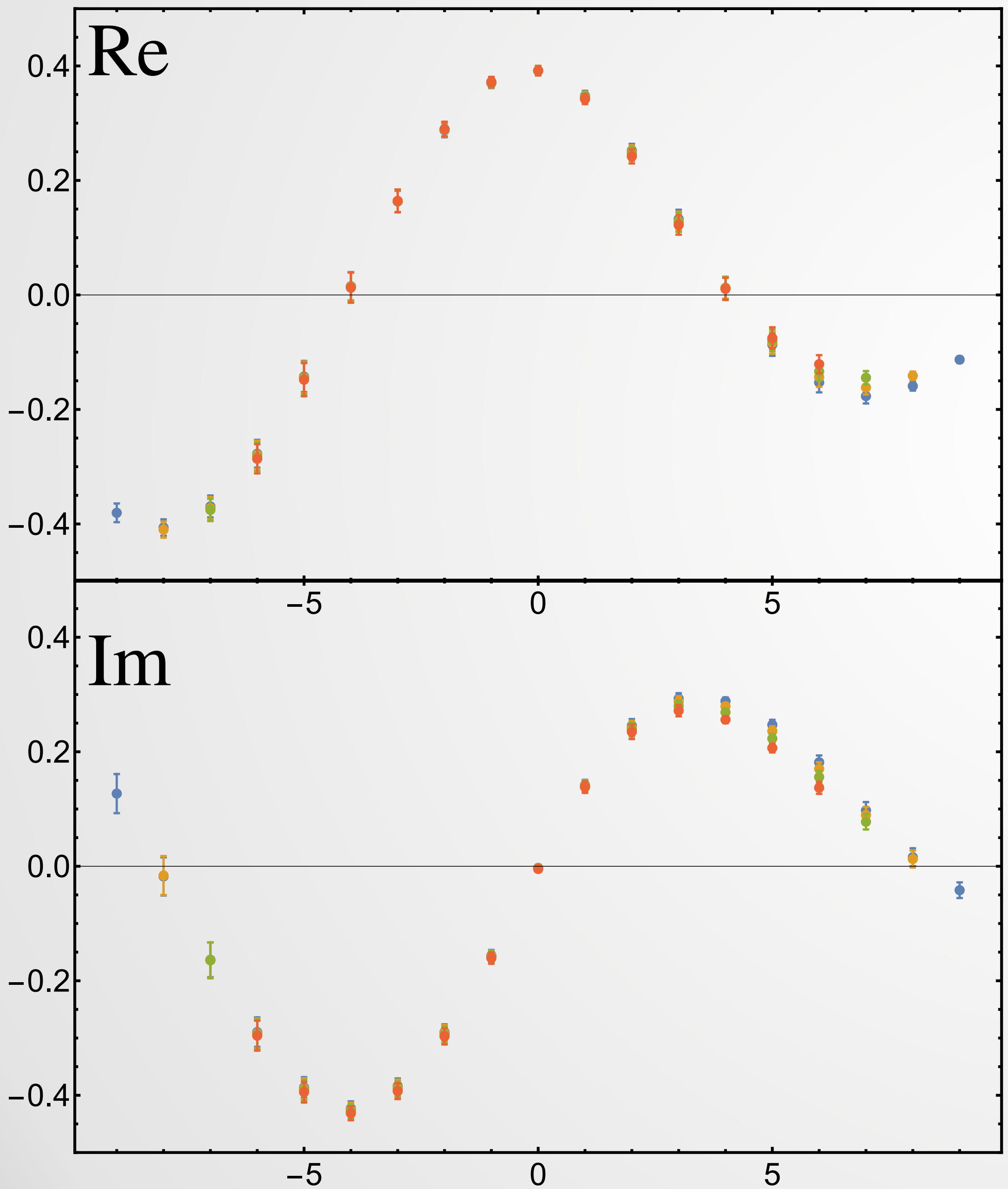


- The RIMOM renormalization factor is most sensitive to p_R^z;
- Perturbative matching is a small correction, but it compensates for the p_R^z dependence in the matching factor;
- Matched result can be fitted with a constant within the uncertainties.

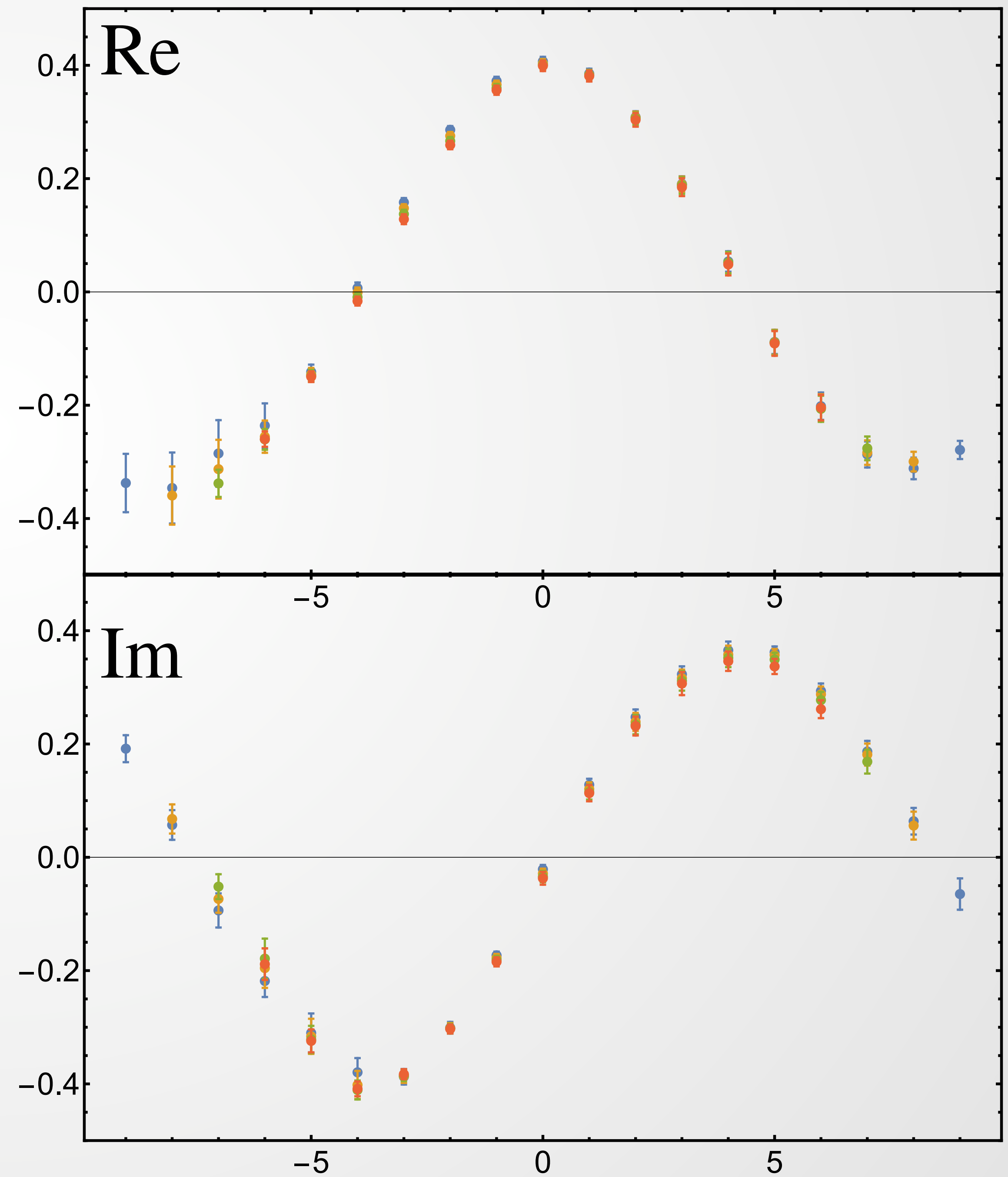
Renormalized matrix element in the MSbar scheme @ 2 GeV

Caveat N_{cfg}=7

$b_T = 1a$



$b_T = 5a$



Renormalization renders real and imaginary parts more symmetric in b_z

Caveat N_{cfg}=7

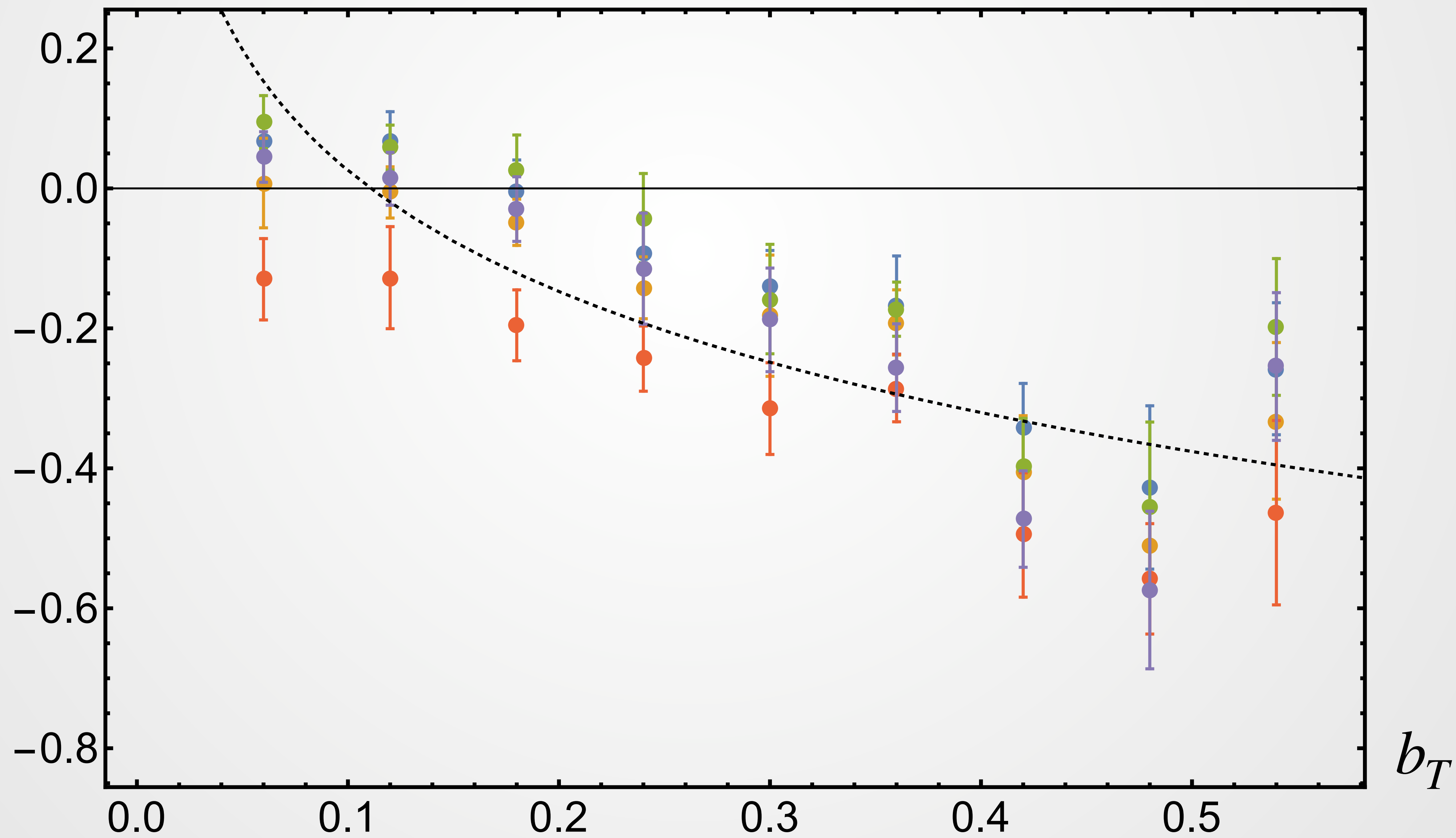
Extraction of the CS kernel with Naive Fourier transform and without matching

Comparison to perturbative results at one-loop (dashed line):

$$\gamma_\mu^q\left[\alpha_s(\mu)\right] = -\frac{\alpha_s(\mu)C_F}{\pi}\ln\frac{b_T^2\mu^2}{4e^{-2\gamma_E}} + O(\alpha_s^2)$$

- M. Ebert, I. Stewart, Y.Z., PRD99 (2019).

$\gamma_\zeta(b_T, \mu = 2\text{ GeV})$



Different colored points correspond to CS kernel calculated at *x* = 0.4, 0.45, 0.5, 0.55, 0.6.

Conclusion

- The LaMET approach can be readily applied to the lattice calculation of GPDs;
- The Collins-Soper kernel can be calculated with LaMET by forming the ratios of quasi-beam functions;
- The operator mixing for the staple-shaped Wilson line operator is negligible on the current lattice setup;
- Encouraging results that LQCD calculations of the CS kernel might be achieved with present-day resources;
- Future work will include (much) larger statistics, different lattice spacings (for taking the continuum limit), and more systematic treatment than the naive Fourier transform.