Multi-Parton Correlations in SIDIS, e^+e^-, and pp collisions

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Outline

- TMD and collinear twist-3 (CT3) functions
- Sivers and Collins effects & A_N in *pp* collisions
- Toward a global analysis of transverse spin observables
- Summary and outlook

TMD and Collinear Twist-3 Functions

TMD PDFs (x, k_T)

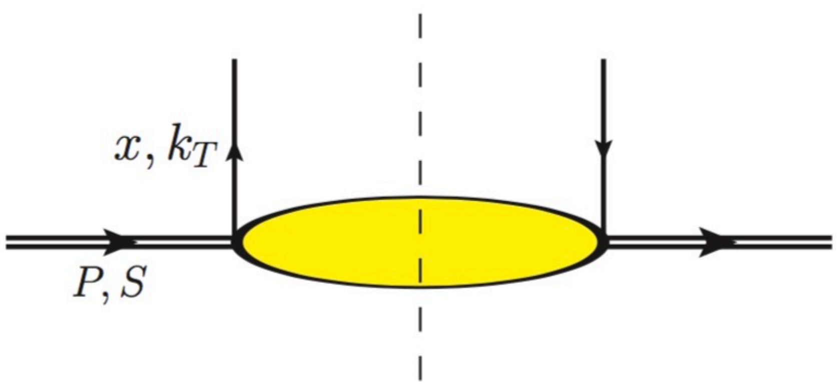
q pol. / H pol.	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_{1T} $h_{1T}^\perp$

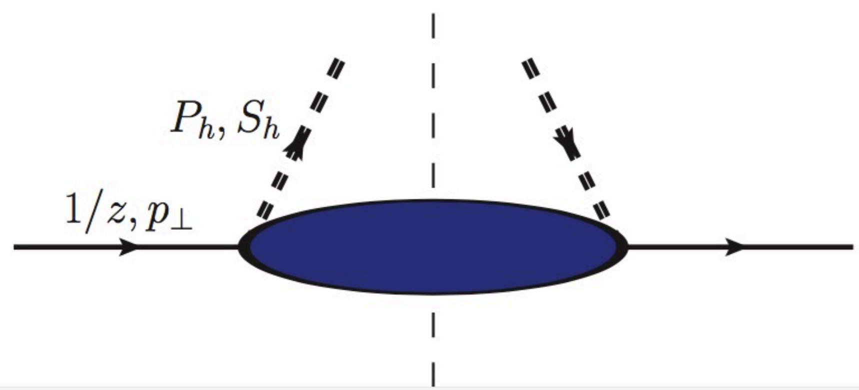
(Mulders, Tangerman (1996); Goeke, Metz, Schlegel (2005))

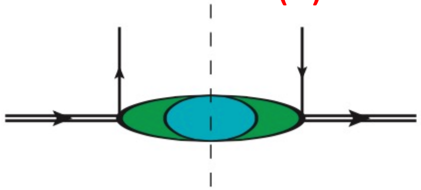
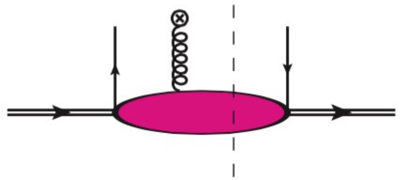
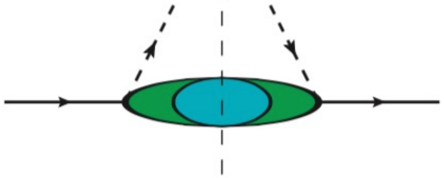
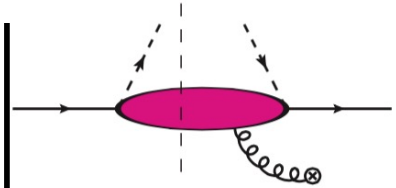
TMD FFs $(z, p_\perp)$

q pol. / H pol.	U	L	T
U	D_1		$H_1^\perp$
L		G_{1L}	$H_{1L}^\perp$
T	$D_{1T}^\perp$	G_{1T}	H_{1T} $H_{1T}^\perp$

(Boer, Jakob, Mulders (1997))





Hadron Pol.	CT3 PDF (x)		CT3 PDF (x, x_1)	CT3 FF (z)		CT3 FF (z, z_1)
						
	intrinsic	kinematical	dynamical	intrinsic	kinematical	dynamical
U	e	$h_1^{\perp(1)}$	H_{FU}	E, H	$H_1^{\perp(1)}$	$\hat{H}_{FU}^{\Re,\Im}$
L	h_L	$h_{1L}^{\perp(1)}$	H_{FL}	H_L, E_L	$H_{1L}^{\perp(1)}$	$\hat{H}_{FL}^{\Re,\Im}$
T	g_T	$f_{1T}^{\perp(1)}$, $g_{1T}^{\perp(1)}$	F_{FT}, G_{FT}	D_T, G_T	$D_{1T}^{\perp(1)}$, $G_{1T}^{\perp(1)}$	$\hat{D}_{FT}^{\Re,\Im}, \hat{G}_{FT}^{\Re,\Im}$

For CT3 observables, QCD equation of motion relations (EoMRs) and Lorentz invariance relations (LIRs) are necessary to guarantee

- EM and color gauge invariance of the cross section
- Frame independence of the cross section

(Kanazawa, Metz, DP, Schlegel, PLB **742** (2015); Kanazawa, Metz, DP, Schlegel, PLB **744** (2015); Koike, DP, Takagi, Yoshida PLB **752** (2016); Koike, DP, Yoshida PLB **759** (2016); Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016); Gamberg, Kang, DP, Schlegel, Yoshida JHEP **1901** (2019))

They are known for both twist-3 PDFs and FFs in the quark sector

EoMRs are known in the gluon sector but LIRs have not been derived

EoMR $$H^q(z) = -2z H_1^{\perp(1),q}(z) + 2z\int_z^\infty \frac{dz_1}{z_1^2}\frac{1}{\frac{1}{z}-\frac{1}{z_1}}\hat{H}_{FU}^{qg,\Im}(z,z_1)$$

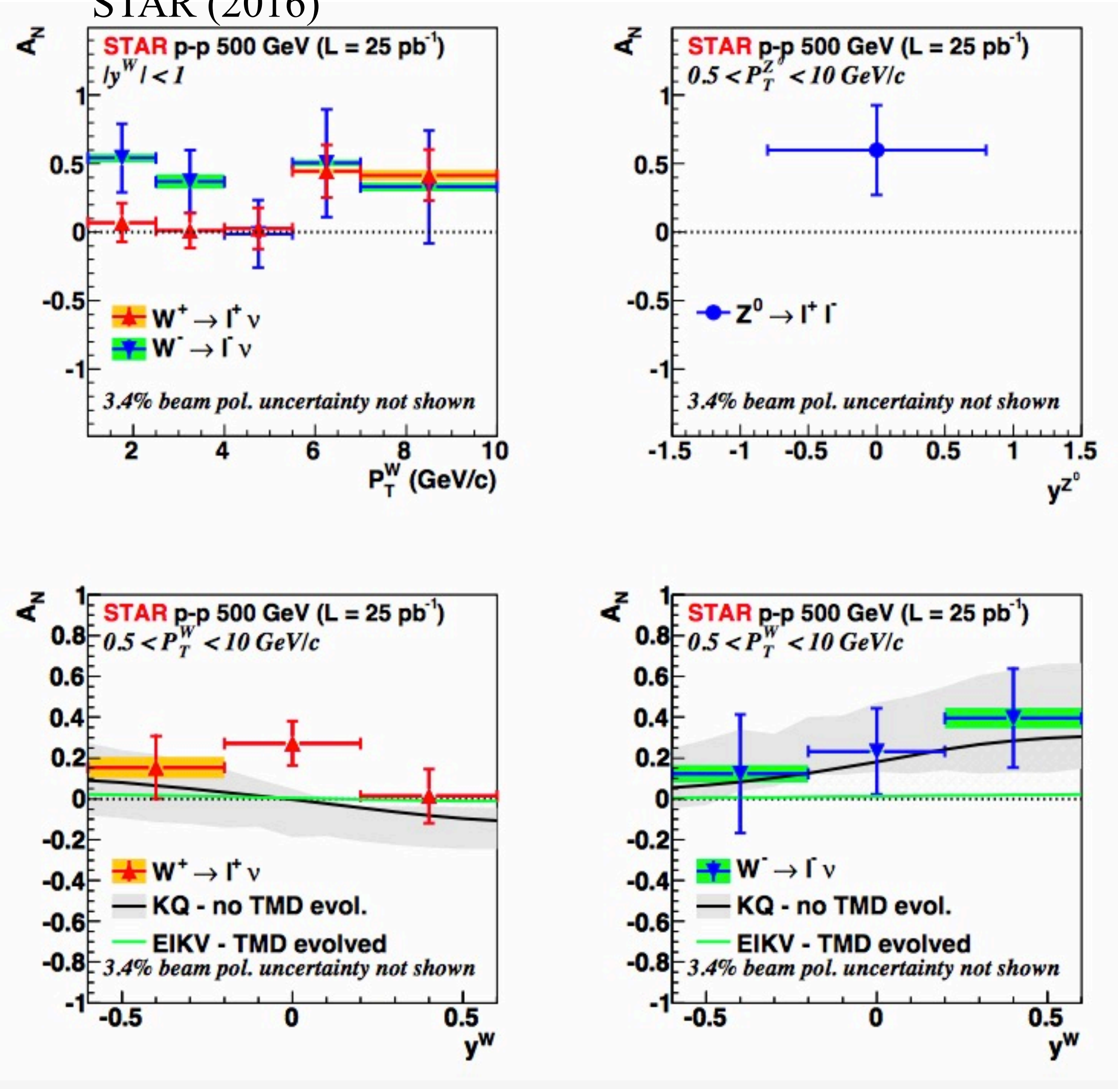
LIR $$\frac{H^q(z)}{z} = -\left(1 - z\frac{d}{dz}\right)H_1^{\perp(1),q}(z) - \frac{2}{z}\int_z^\infty \frac{dz_1}{z_1^2}\frac{\hat{H}_{FU}^{qg,\Im}(z,z_1)}{(1/z-1/z_1)^2}$$

(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))

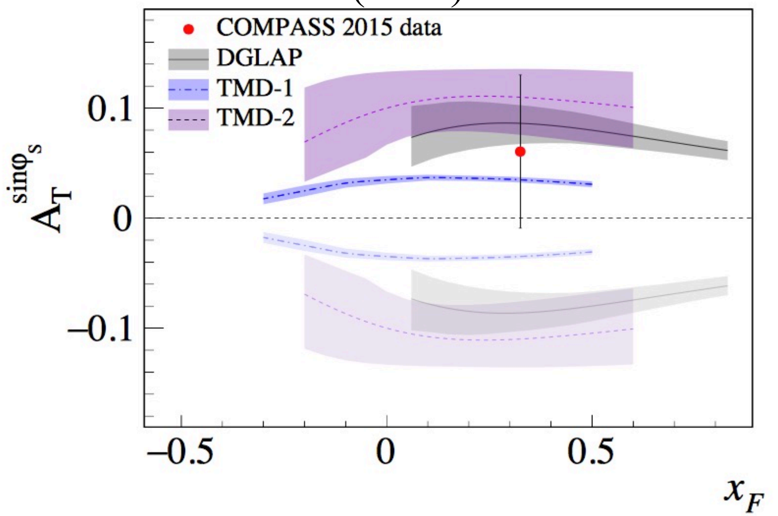
Sivers and Collins Effects & A_N in *pp* Collisions

Drell-Yan Sivers effect

STAR (2016)

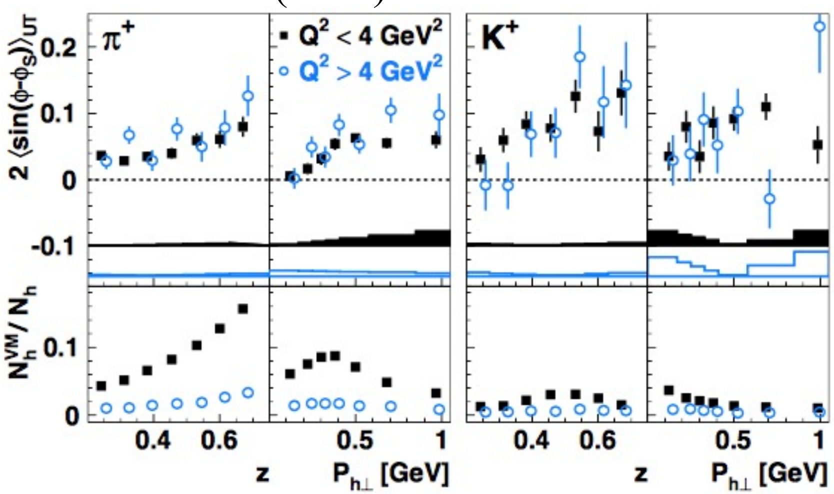


COMPASS (2017)

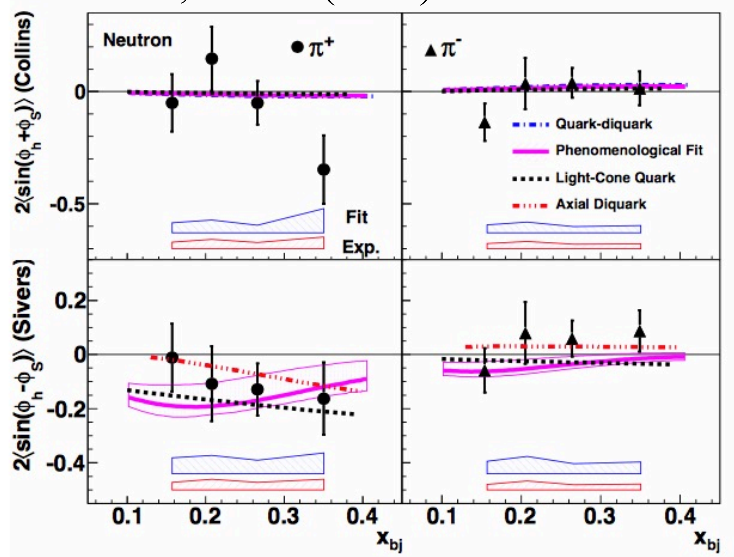


SIDIS Sivers effect ($\sin(\phi_h - \phi_s)$)

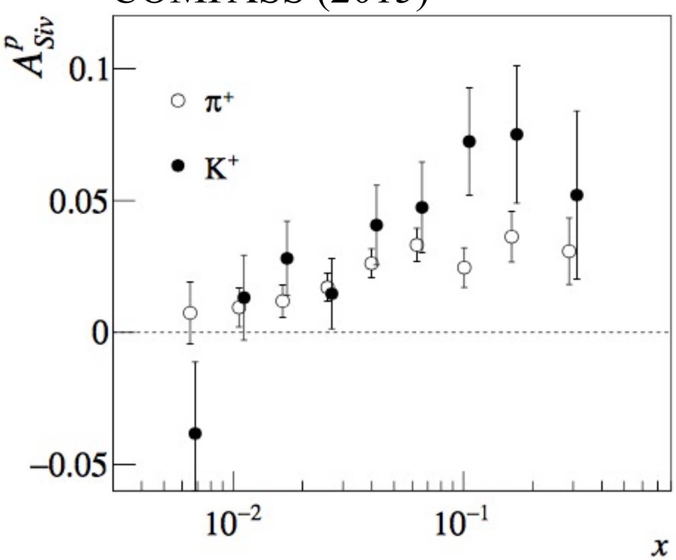
HERMES (2009)



JLab, Hall A (2011)

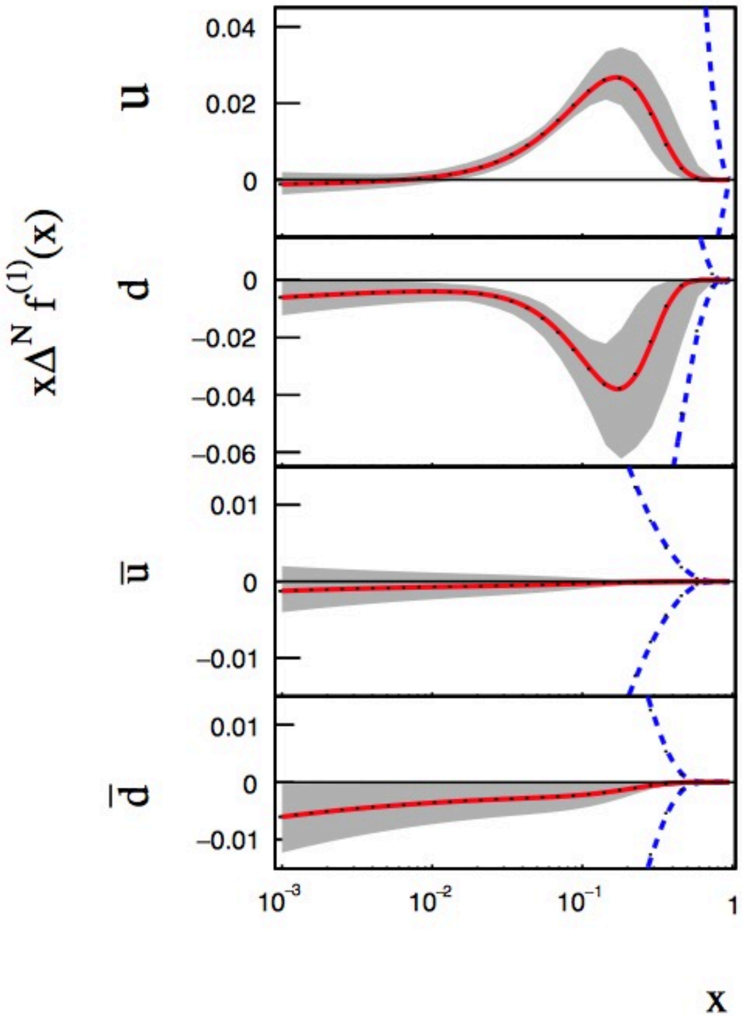


COMPASS (2015)

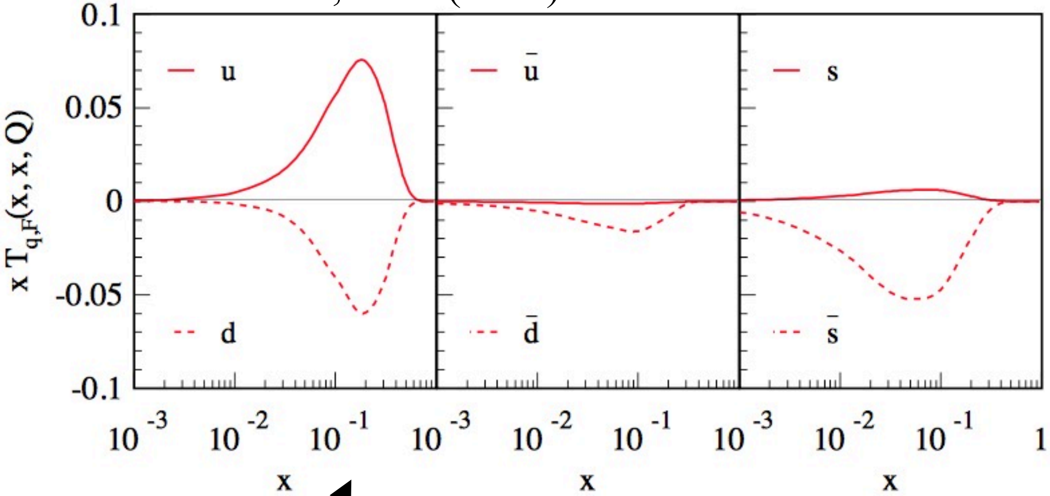


$$F_{UT}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\left[-\frac{\hat{h}\cdot\vec{k}_T}{M} f_{1T}^{\perp} D_1\right]$$

Anselmino, et al. (2017)



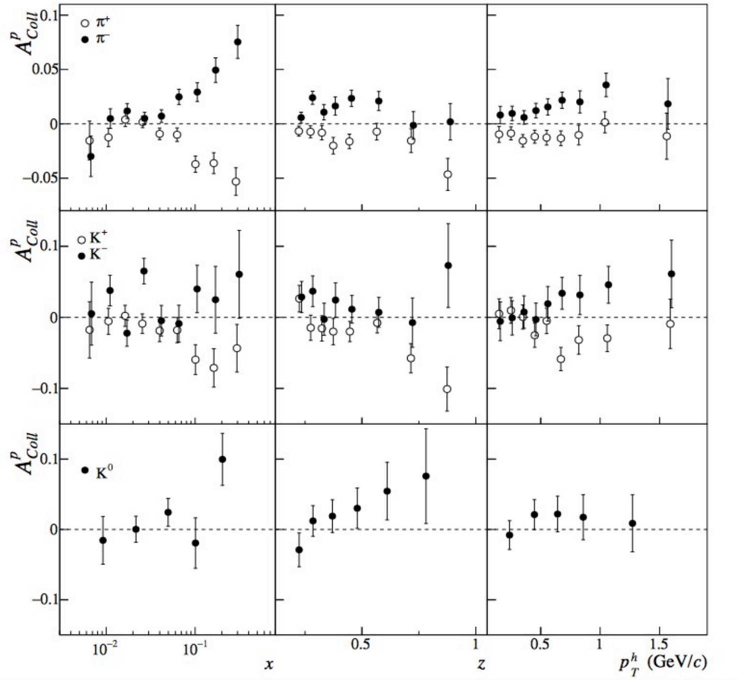
Echevarria, et al. (2014)



TMDs in Collins-Soper-Sterman (CSS) evolution formalism

SIDIS Collins effect ($\sin(\phi_h + \phi_s)$)

COMPASS (2015)

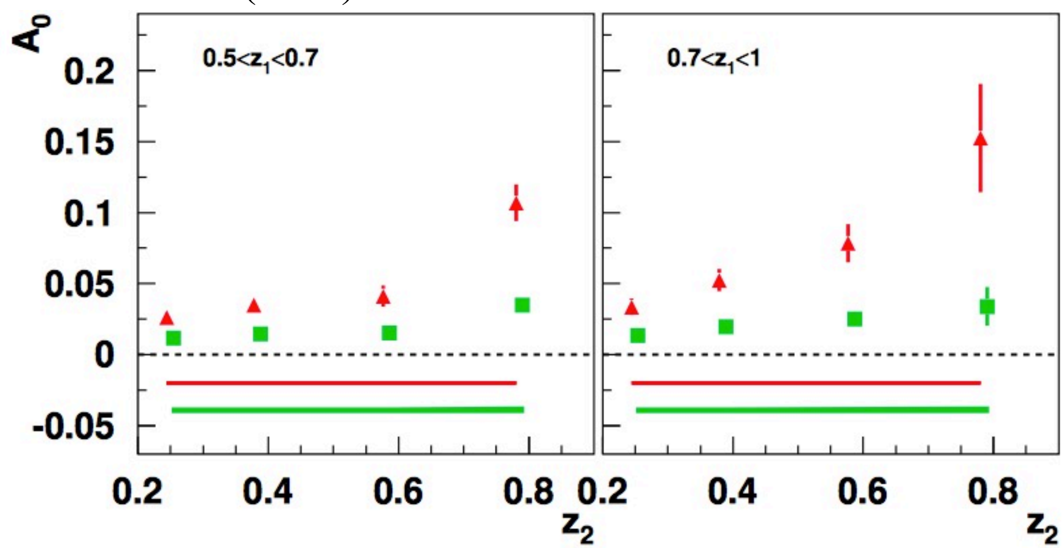


Also data from JLab Hall A (2011, 2014) and HERMES

$$F_{UT}^{\sin(\phi_h+\phi_S)} = \mathcal{C}\left[-\frac{\hat{h}\cdot\vec{p}_\perp}{M_h}\boldsymbol{h_1}\,\boldsymbol{H_1^\perp}\right]$$

e^+e^- Collins effect ($\cos(2\phi_0)$)

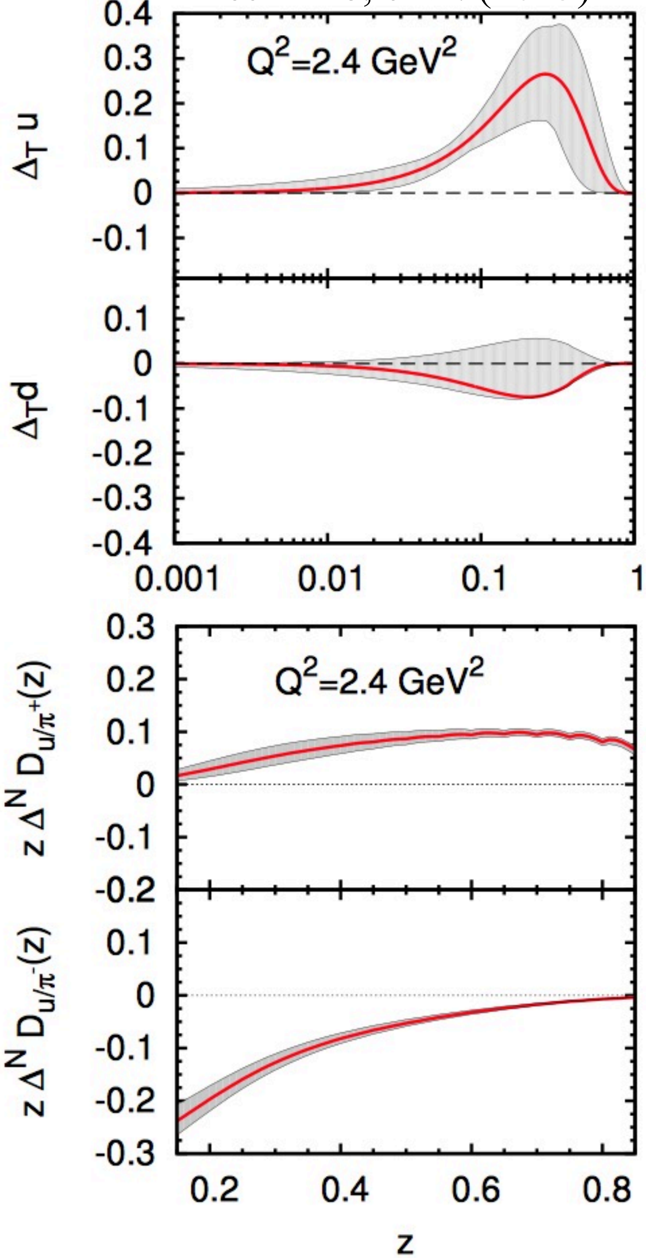
Belle (2008)



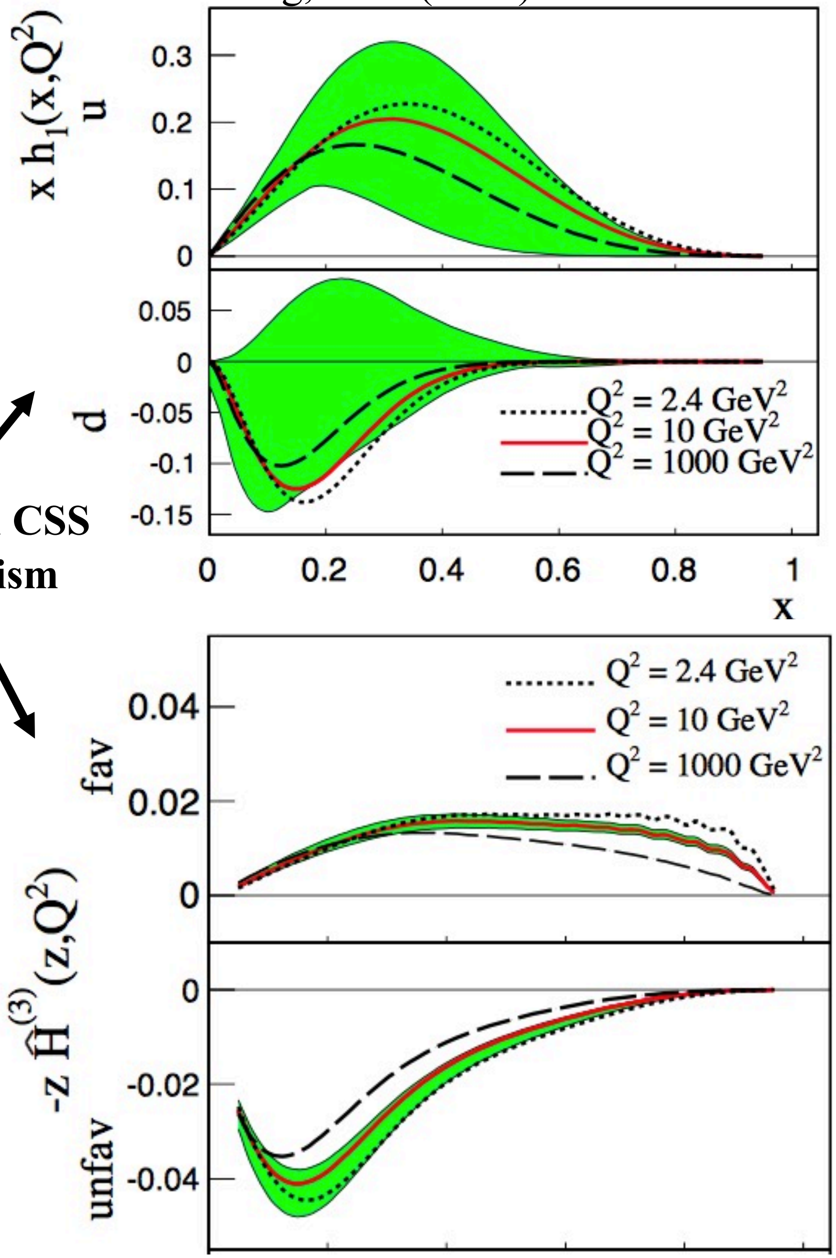
Also data from BaBar (2014) and BESIII (2016)

$$F_{UU}^{\cos(2\phi_0)} = \mathcal{C}\left[\frac{2\hat{h}\cdot\vec{p}_{a\perp}\,\hat{h}\cdot\vec{p}_{b\perp} - \vec{p}_{a\perp}\cdot\vec{p}_{b\perp}}{M_a M_b}\,\boldsymbol{H_1^\perp}\,\boldsymbol{\bar{H}_1^\perp}\right]$$

Anselmino, et al. (2015)



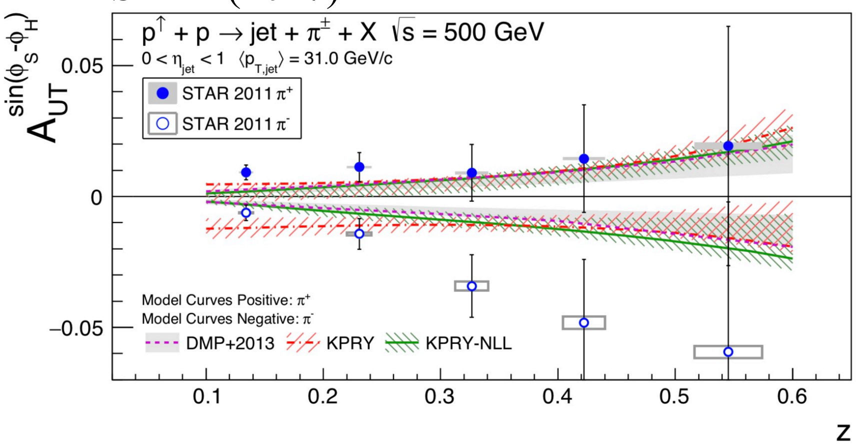
Kang, et al. (2016)



TMDs in CSS formalism

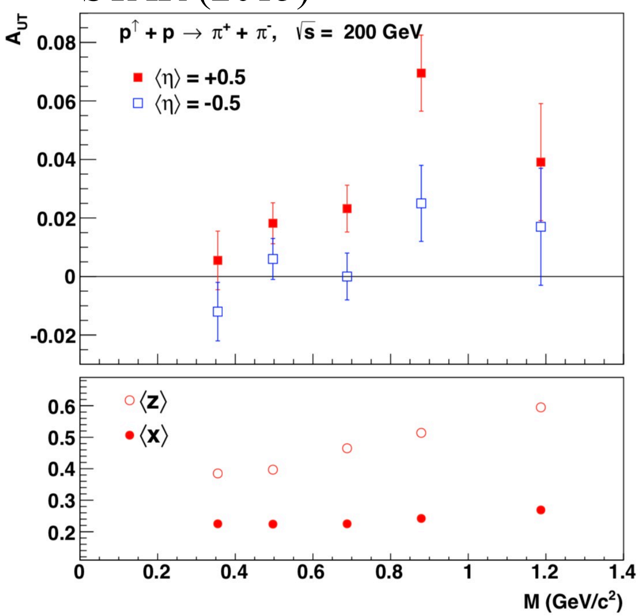
Hadron in a jet Collins effect

STAR (2017)

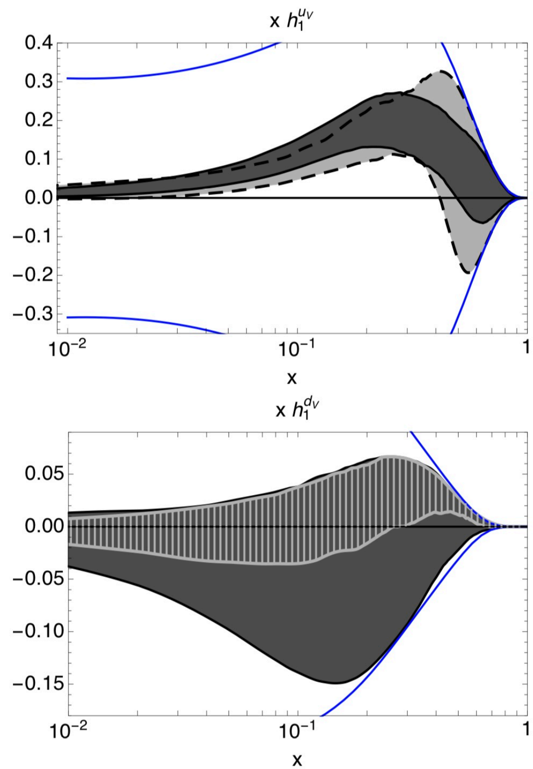


Theory curves from D'Alesio, et al. (2017) & Kang, et al. (2017)

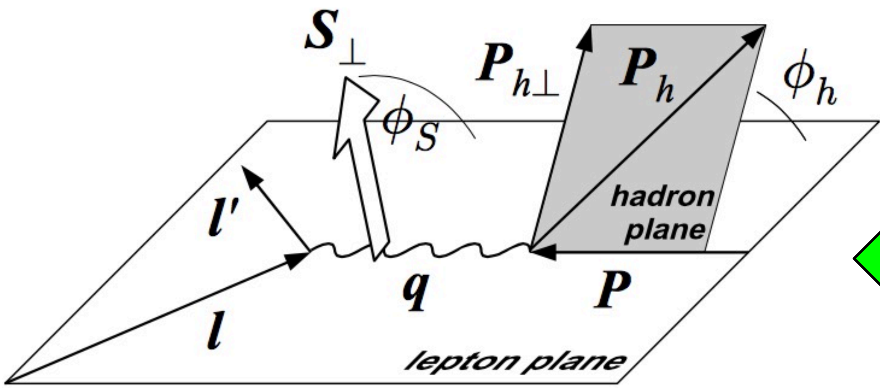
STAR (2015)

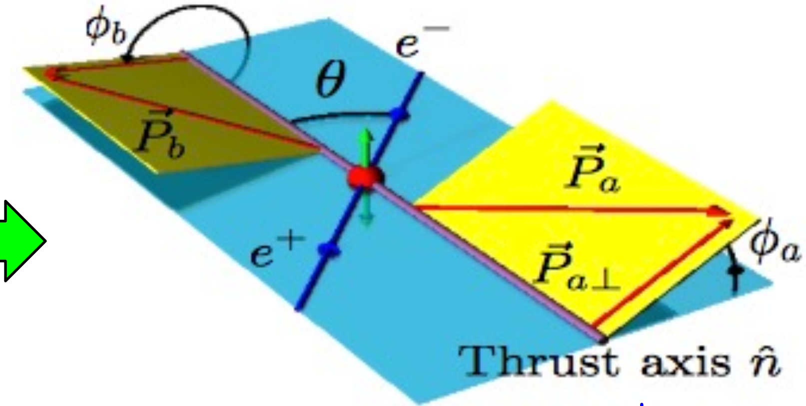


Transversity from dihadron FF



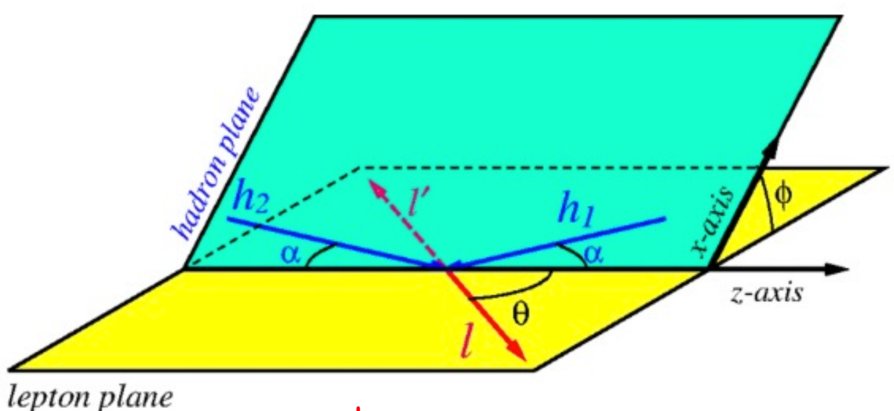
Bacchetta and Radici (2017)





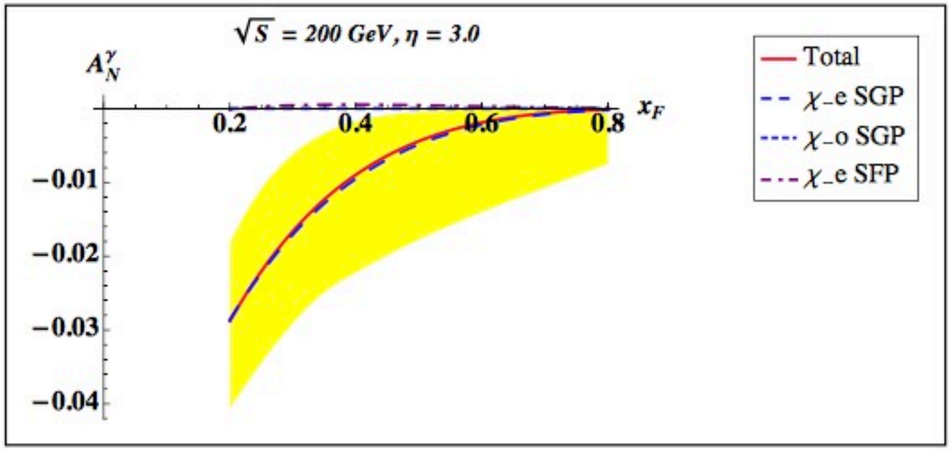
h_1, $f_{1T}^\perp$, $H_1^\perp$

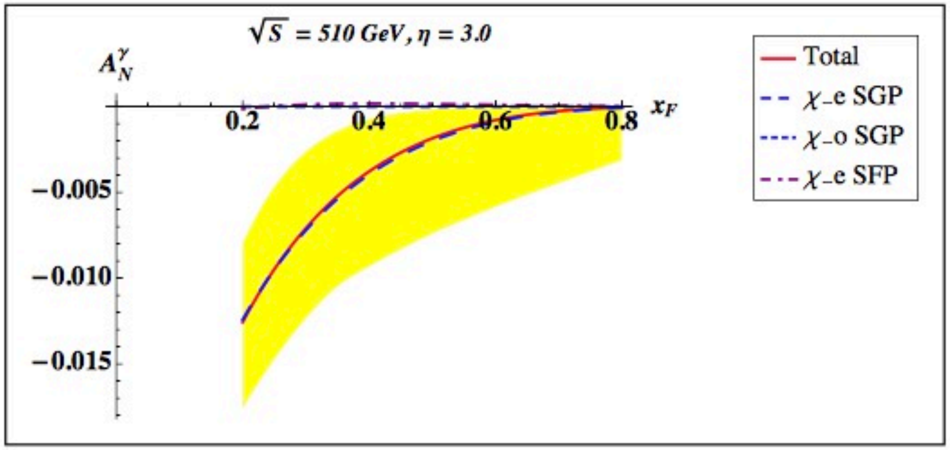
$H_1^\perp$



$f_{1T}^\perp$

A_N in $pp \to \gamma X$





(Kanazawa, Koike, Metz, DP – PRD **91** (2015))
(See also Gamberg, Kang, Prokudin (2013))

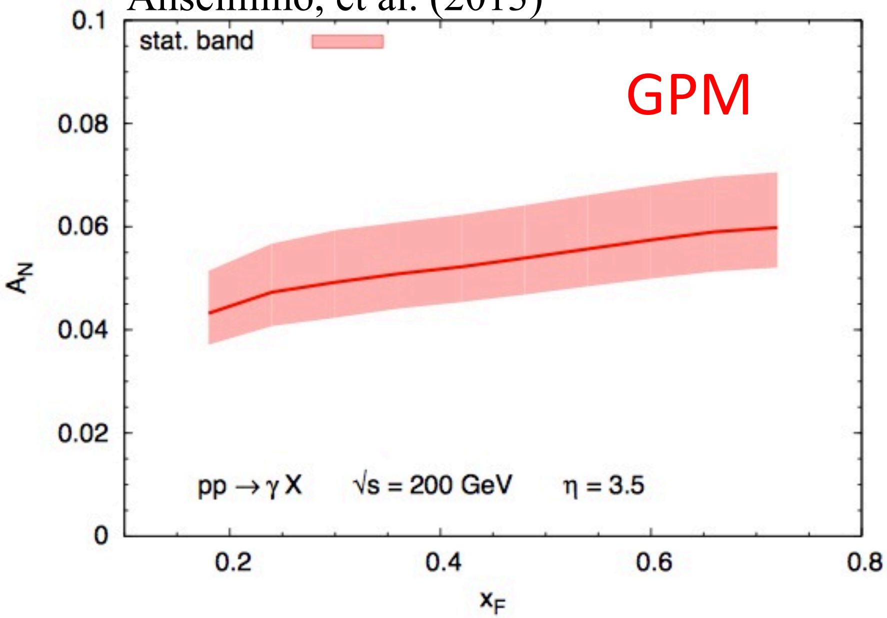
Qiu-Sterman term is the main cause of A_N in $pp \to \gamma X$ → Test of the process dependence of the Sivers function

$$d\Delta\sigma^\pi \sim H \otimes f_1 \otimes \boldsymbol{F_{FT}(x,x)}$$

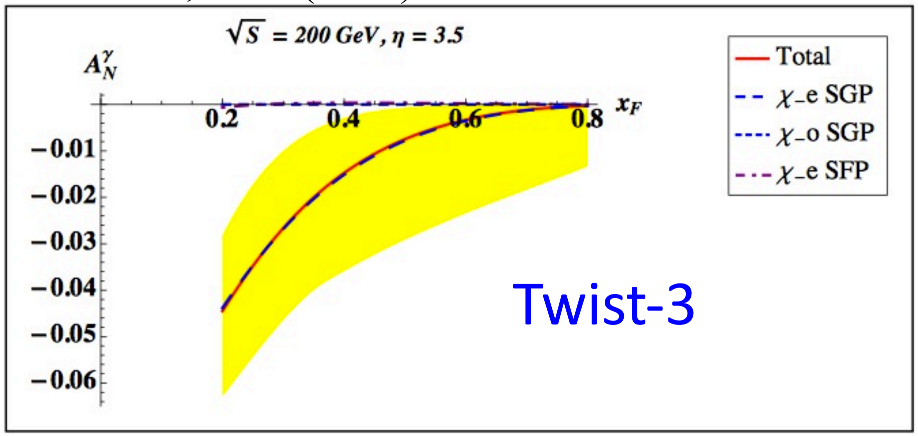
↳ Qiu-Sterman function

A_N in $pp \to \gamma X$

Anselmino, et al. (2013)

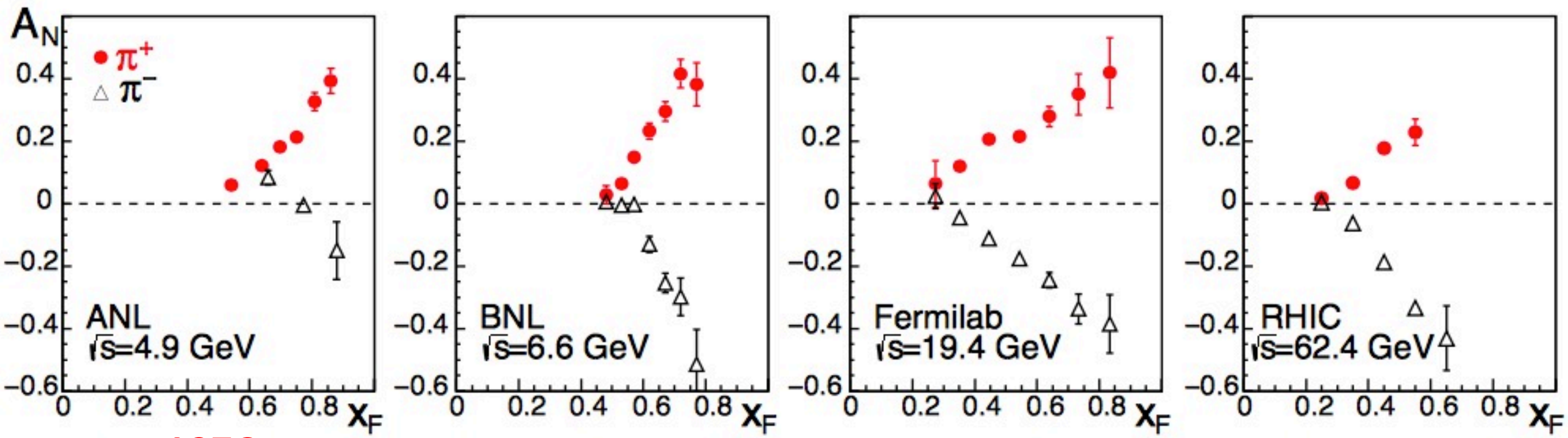


Kanazawa, et al. (2015)



GPM predicts a positive asymmetry while twist-3 predicts a negative one

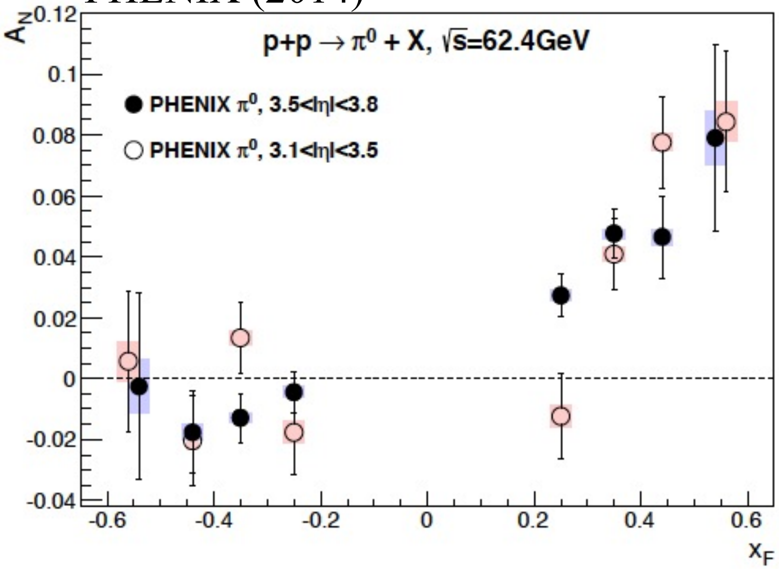
A_N in $pp \rightarrow \pi X$ – PUZZLE FOR 40+ YEARS!



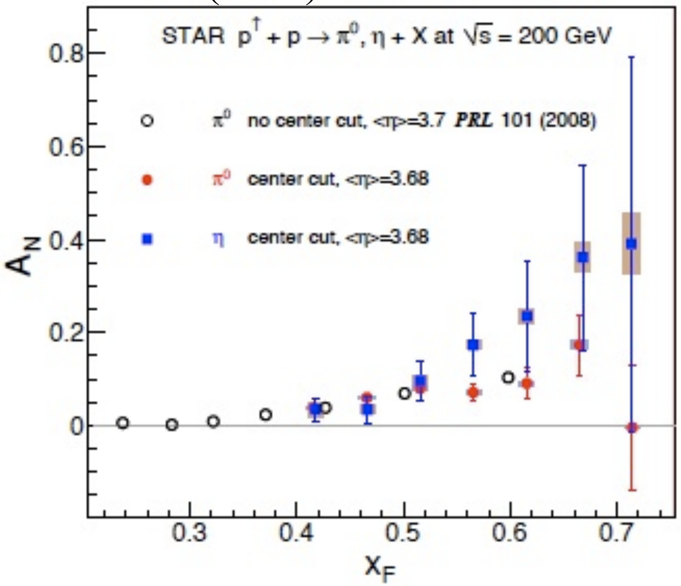
1976 →

A_N in $pp \to \pi X$ – PUZZLE FOR 40+ YEARS!

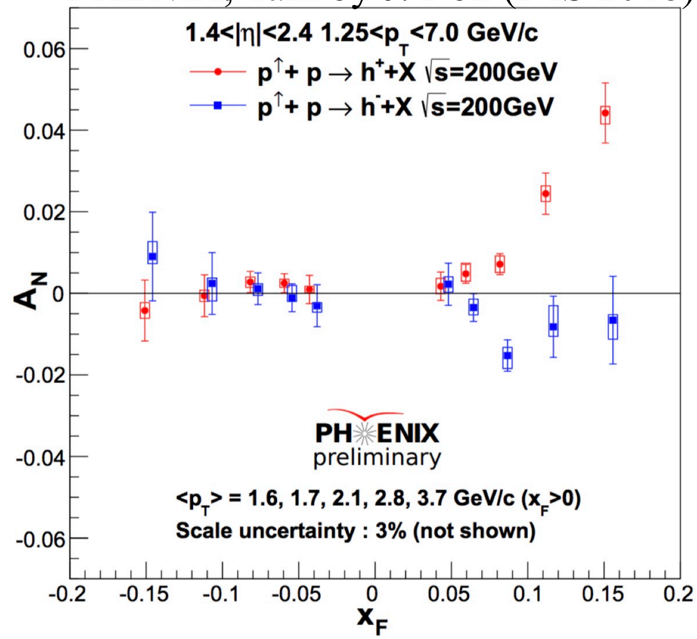
PHENIX (2014)



STAR (2012)



PHENIX, Talk by J. Bok (DIS 2018)



$$d\Delta\sigma^{\pi} \sim H \otimes f_1 \otimes \boldsymbol{F_{FT}(x,x)}$$

$$E_\ell \frac{d^3\Delta\sigma(\vec{s}_T)}{d^3\ell} = \frac{\alpha_s^2}{S} \sum_{a,b,c} \int_{z_{\min}}^1 \frac{dz}{z^2} D_{c\to h}(z) \int_{x'_{\min}}^1 \frac{dx'}{x'} \frac{1}{x'S + T/z} \phi_{b/B}(x')$$
$$\times \sqrt{4\pi\alpha_s} \left(\frac{\epsilon^{\ell s_T n \bar{n}}}{z\hat{u}}\right) \frac{1}{x} \left[T_{a,F}(x,x) - x\left(\frac{d}{dx}T_{a,F}(x,x)\right)\right] H_{ab\to c}(\hat{s},\hat{t},\hat{u})$$

$$\boxed{F_{FT} \sim T_F}$$

(Qiu & Sterman (1999), Kouvaris, et al. (2006))

For many years the Qiu-Sterman/Sivers-type contribution was thought to be the dominant source of TSSAs in $p^{\uparrow}p \longrightarrow \pi\, X$

$$\sout{d\Delta\sigma^{\pi} \sim H \otimes f_1 \otimes \boldsymbol{F_{FT}(x,x)}}$$

(Kang, Qiu, Vogelsang, Yuan (2011); Kang and Prokudin (2012); Metz, DP, Schäfer, Schlegel, Vogelsang, Zhou (2012))

$$\sout{d\Delta\sigma^{\pi} \sim H \otimes f_1 \otimes \boldsymbol{F_{FT}(x,x)}}$$

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes S \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{H}, \int \frac{dz_1}{z_1^2} \frac{\boldsymbol{\hat{H}_{FU}^{\Im}}}{(1/z - 1/z_1)^2} \right)$$

$$E_h \frac{d\Delta\sigma^{Frag}(S_T)}{d^3\vec{P}_h} = -\frac{4\alpha_s^2 M_h}{S}\,\epsilon^{P'PP_hS_T} \sum_i \sum_{a,b,c} \int_0^1 \frac{dz}{z^3} \int_0^1 dx' \int_0^1 dx\; \delta(\hat{s}+\hat{t}+\hat{u}) \frac{1}{\hat{s}\,(-x'\hat{t}-x\hat{u})}$$
$$\times\; h_1^a(x)\, f_1^b(x') \left\{ \left[H_1^{\perp(1),c}(z) - z\frac{dH_1^{\perp(1),c}(z)}{dz} \right] S_{H_1^\perp}^i + \frac{1}{z} H^c(z)\, S_H^i \right.$$
$$\left. + \frac{2}{z} \int_z^\infty \frac{dz_1}{z_1^2} \frac{1}{\left(\frac{1}{z}-\frac{1}{z_1}\right)^2} \hat{H}_{FU}^{c,\Im}(z,z_1)\, S_{\hat{H}_{FU}}^i \right\}$$

Non-pole matrix element!

(Metz & DP - PLB **723** (2013))

~~$d\Delta\sigma^{\pi} \sim H \otimes f_1 \otimes F_{FT}(x,x)$~~

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes S \otimes \left(H_1^{\perp(1)}, H, \int \frac{dz_1}{z_1^2} \frac{\hat{H}_{FU}^{\Im}}{(1/z - 1/z_1)^2} \right)$$

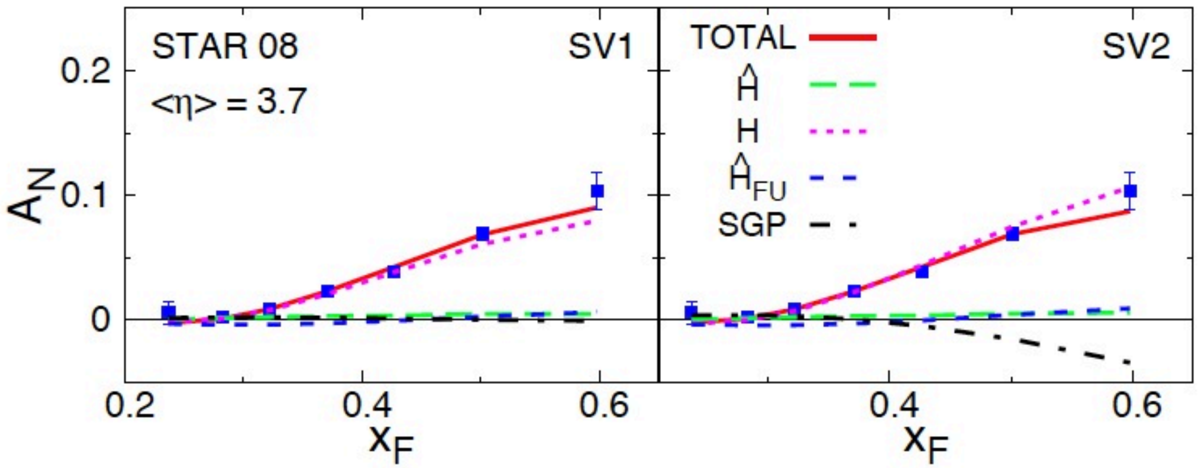
$$E_h \frac{d\Delta\sigma^{Frag}(S_T)}{d^3\vec{P}_h} = -\frac{4\alpha_s^2 M_h}{S}\, \epsilon^{P'PP_hS_T} \sum_i \sum_{a,b,c} \int_0^1 \frac{dz}{z^3} \int_0^1 dx' \int_0^1 dx\, \delta(\hat{s}+\hat{t}+\hat{u}) \frac{1}{\hat{s}\,(-x'\hat{t} - x\hat{u})}$$
$$\times\; h_1^a(x)\, f_1^b(x') \left\{ \left[H_1^{\perp(1),c}(z) - z\frac{dH_1^{\perp(1),c}(z)}{dz} \right] S^i_{H_1^\perp} + \frac{1}{z} H^c(z)\, S^i_H \right.$$
$$\left. + \frac{2}{z} \int_z^\infty \frac{dz_1}{z_1^2} \frac{1}{\left(\frac{1}{z} - \frac{1}{z_1}\right)^2} \hat{H}_{FU}^{c,\Im}(z,z_1)\, S^i_{\hat{H}_{FU}} \right\}$$

(Metz & DP - PLB **723** (2013))

We now believe the TSSAs in $p^{\uparrow}p \to \pi\, X$ are due to fragmentation effects as the partons form pions in the final state

(Kanazawa, Koike, Metz, DP, PRD **89**(RC) (2014); Gamberg, Kang, DP, Prokudin, PLB **770** (2017))

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes S \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{H}, \int \frac{dz_1}{z_1^2} \frac{\hat{\boldsymbol{H}}_{\boldsymbol{FU}}^{\Im}}{(1/z - 1/z_1)^2} \right)$$



(Kanazawa, Koike, Metz, DP, PRD **89**(RC) (2014))

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes S \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{H}, \int \frac{dz_1}{z_1^2} \frac{\boldsymbol{\hat{H}_{FU}^{\Im}}}{(1/z - 1/z_1)^2} \right)$$

EoMR & LIR

$$2z \int_z^{\infty} \frac{dz_1}{z_1^2} \frac{1}{\frac{1}{z} - \frac{1}{z_1}} \hat{H}_{FU}^{qg,\Im}(z, z_1)$$

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes \tilde{S} \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{\tilde{H}} \right)$$

$$E_h \frac{d\Delta\sigma^{Frag}(S_T)}{d^3\vec{P}_h} = -\frac{4\alpha_s^2 M_h}{S} \epsilon^{P'PP_hS_T} \sum_i \sum_{a,b,c} \int_0^1 \frac{dz}{z^3} \int_0^1 dx' \int_0^1 dx\, \delta(\hat{s}+\hat{t}+\hat{u}) \frac{1}{\hat{s}}$$
$$\times\ h_1^a(x)\, f_1^b(x') \left\{ \left[H_1^{\perp(1),c}(z) - z\frac{dH_1^{\perp(1),c}(z)}{dz} \right] \tilde{S}_{H_1^{\perp}}^i + \left[-2H_1^{\perp(1),c}(z) + \frac{1}{z}\tilde{H}^c(z) \right] \tilde{S}_H^i \right\}$$

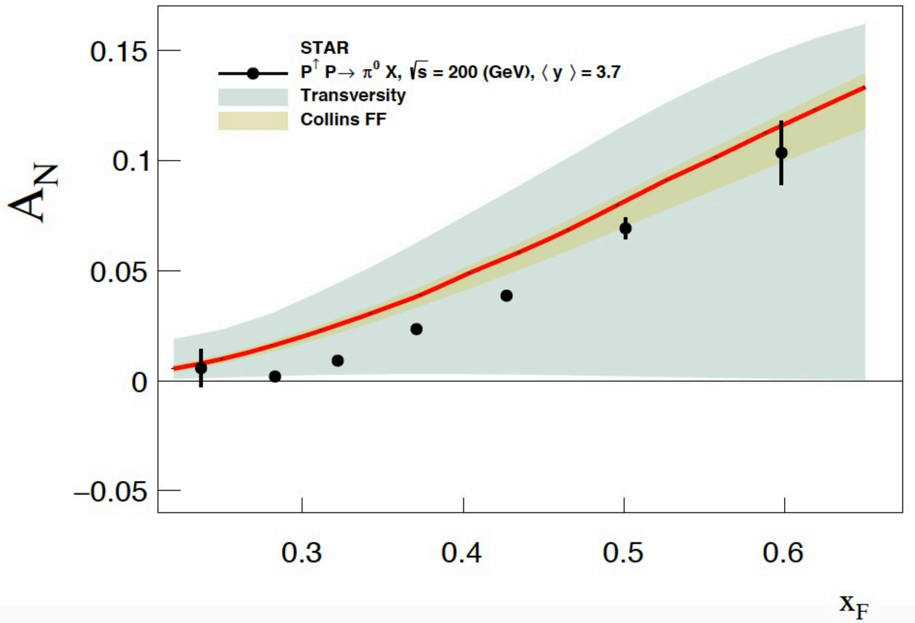
where $\tilde{S}_{H_1^{\perp}}^i \equiv \dfrac{S_{H_1^{\perp}}^i - S_{H_{FU}}^i}{-x'\hat{t} - x\hat{u}}$ and $\tilde{S}_H^i \equiv \dfrac{S_H^i - S_{H_{FU}}^i}{-x'\hat{t} - x\hat{u}}$

(Gamberg, Kang, DP, Prokudin, PLB **770** (2017))

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes S \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{H}, \int \frac{dz_1}{z_1^2} \frac{\boldsymbol{\hat{H}_{FU}^{\Im}}}{(1/z - 1/z_1)^2} \right)$$

EoMR & LIR

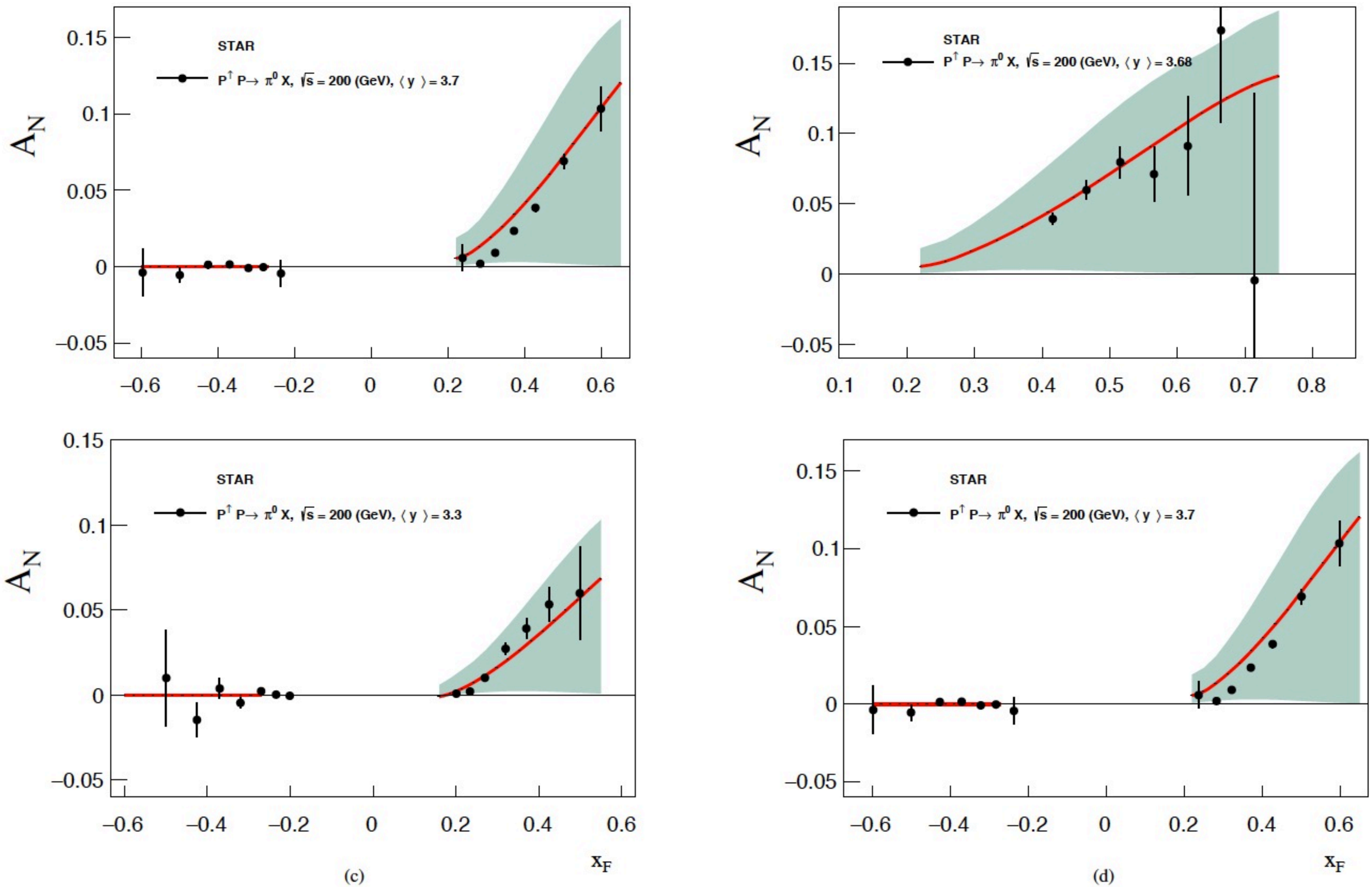
$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes \tilde{S} \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{\tilde{H}} \right)$$



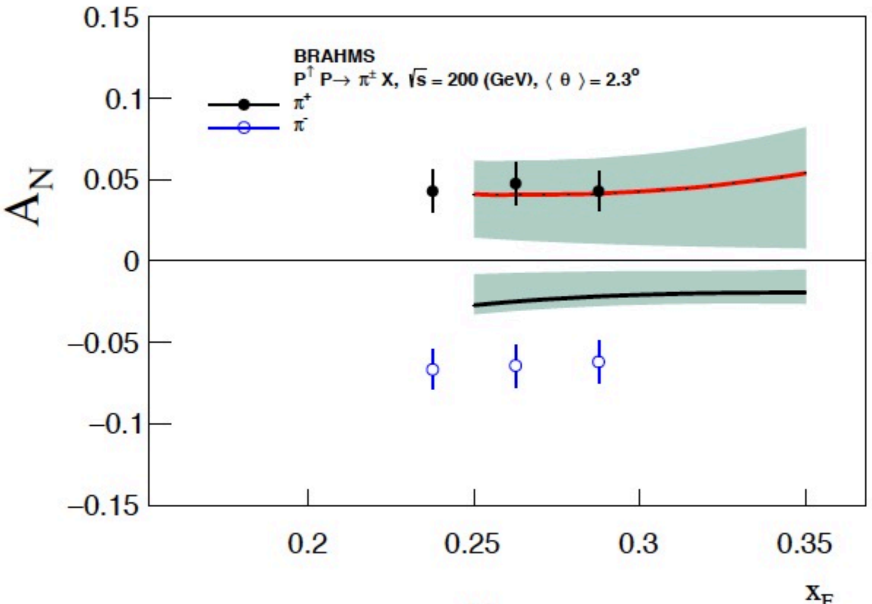
Fragmentation term is the main cause of A_N in *pp -> π X*

The ***A_N data from RHIC*** can be used to constrain ***transversity at large x!***

(Gamberg, Kang, DP, Prokudin, PLB **770** (2017))



(Gamberg, Kang, DP, Prokudin, PLB **770** (2017))



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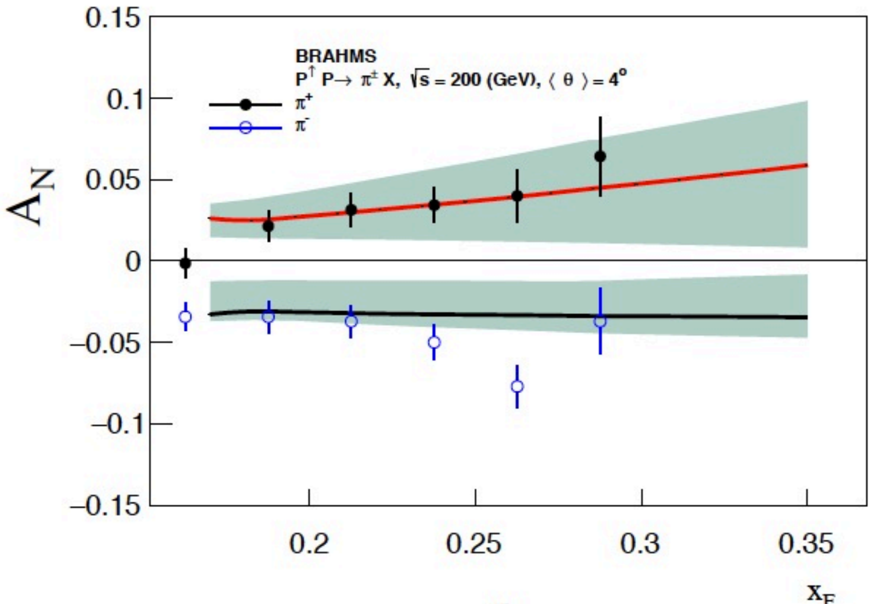
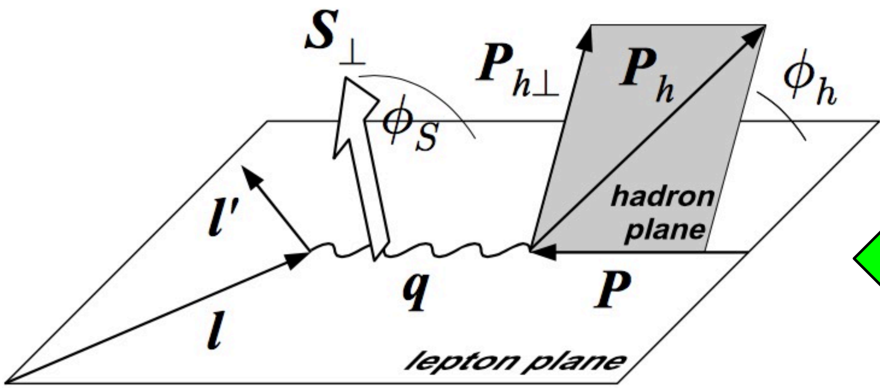
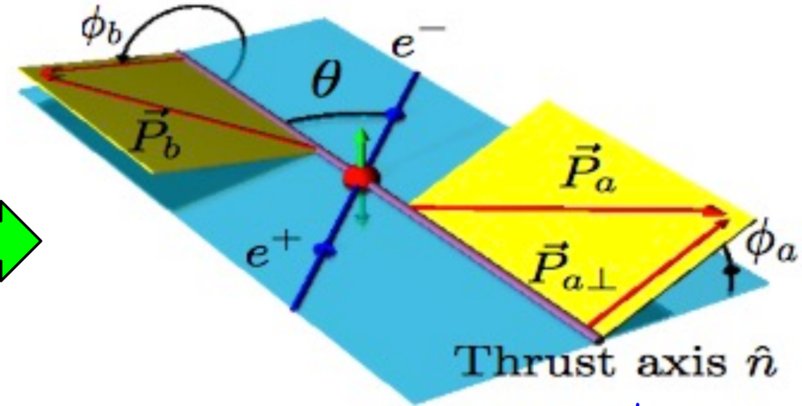


(f)

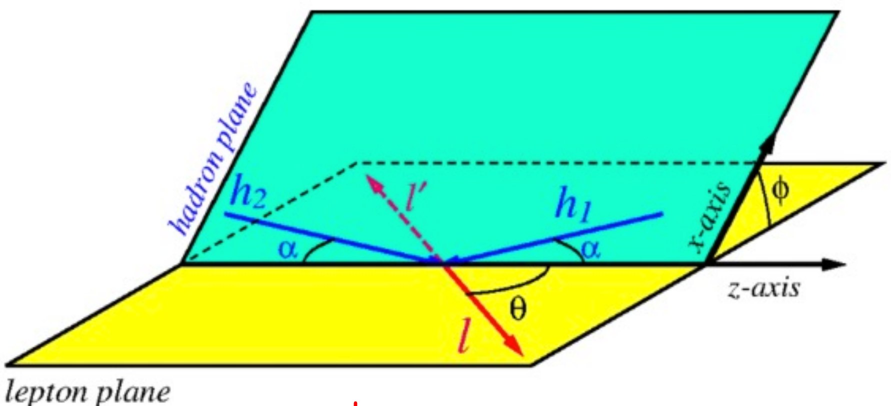
(Gamberg, Kang, DP, Prokudin, PLB **770** (2017))



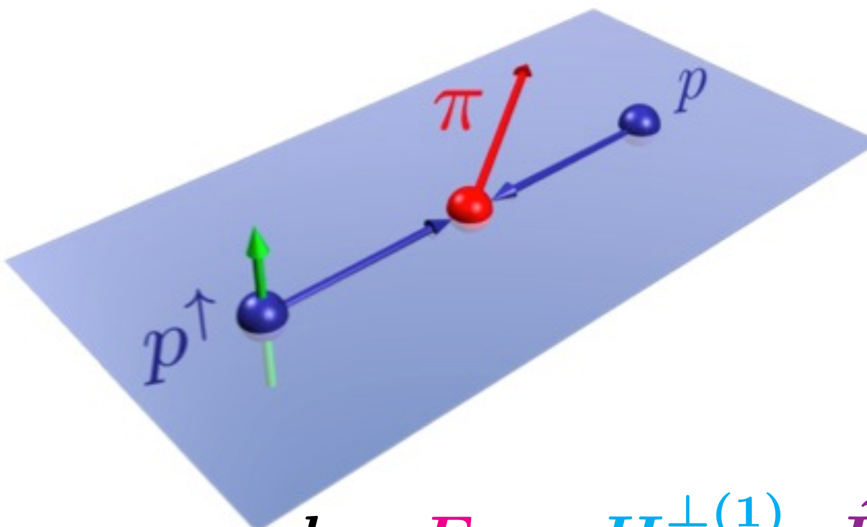
h_1, $f_{1T}^{\perp}$, $H_1^{\perp}$



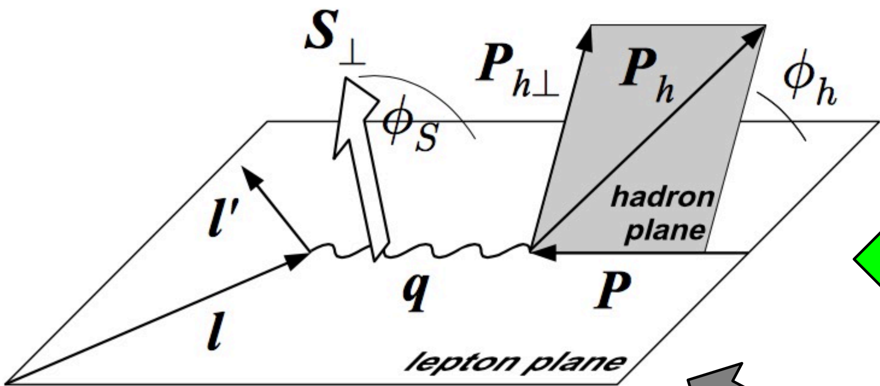
$H_1^{\perp}$

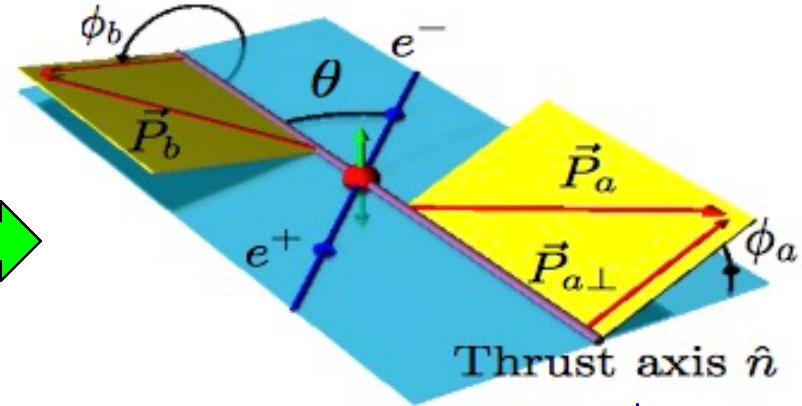


$f_{1T}^{\perp}$



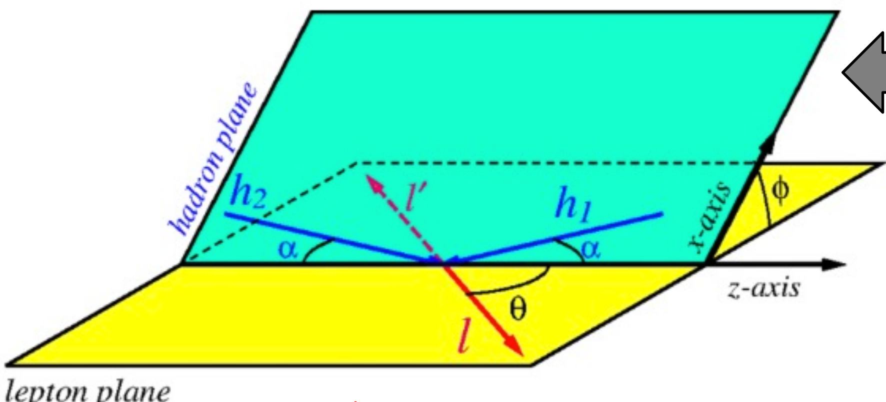
h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$

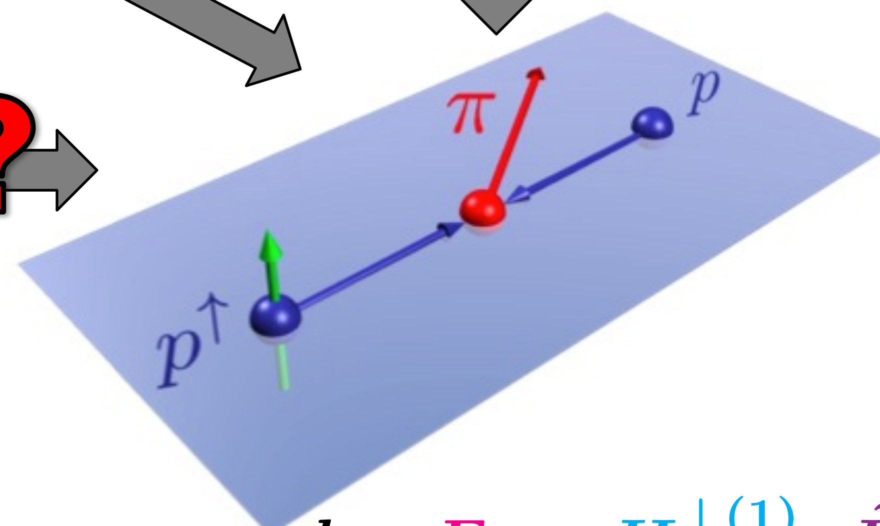




h_1, $f_{1T}^{\perp}$, $H_1^{\perp}$

$H_1^{\perp}$





$f_{1T}^{\perp}$

h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$

Toward a Global Analysis of Transverse Spin Observables

Recall the current phenomenology of TMD observables...

$$\tilde{f}_{1T}^{\perp(1)}(x, b_T; Q^2, \mu_Q) \sim F_{FT}(x, x; \mu_{b_*}) \exp\left[-S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q) - S_{NP}^{f_{1T}^\perp}(b_T, Q)\right]$$

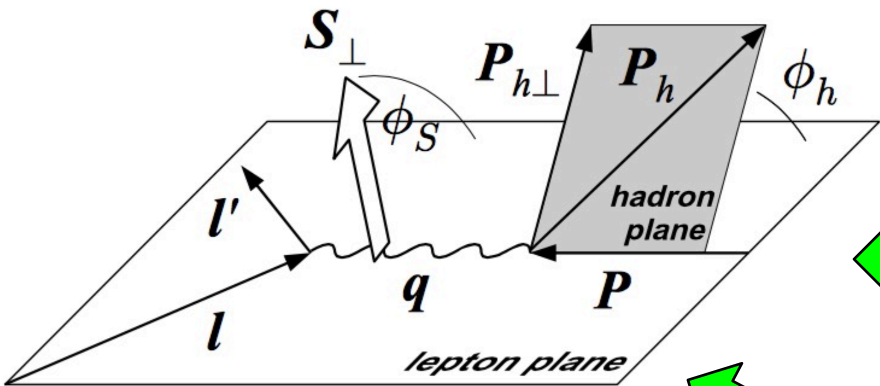
$$S_{NP}^{f_{1T}^\perp}(b_T, Q) \to g_{f_{1T}^\perp}(x, b_T) + g_K(b_T)\ln(Q/Q_0)$$

$$\tilde{H}_1^{\perp(1)}(z, b_T; Q^2, \mu_Q) \sim H_1^{\perp(1)}(z; \mu_{b_*}) \exp\left[-S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q) - S_{NP}^{H_1^\perp}(b_T, Q)\right]$$

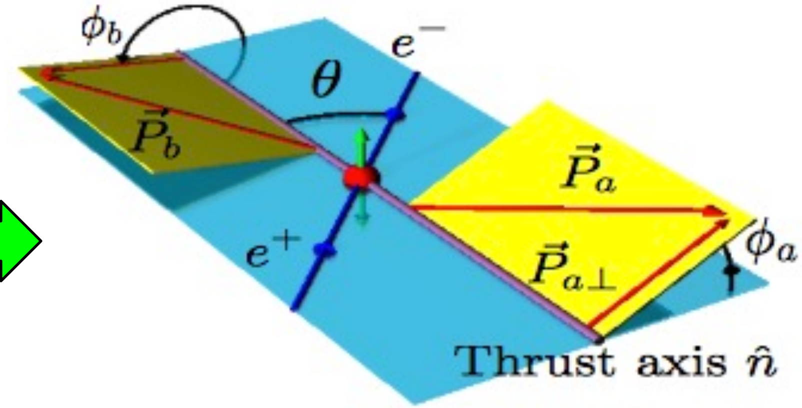
$$S_{NP}^{H_1^\perp}(b_T, Q) \to g_{H_1^\perp}(z, b_T) + g_K(b_T)\ln(Q/Q_0)$$

The **CT3 functions** (along with the NP *g*-functions) are what get extracted in analyses of TSSAs in ***TMD processes*** that use CSS evolution!

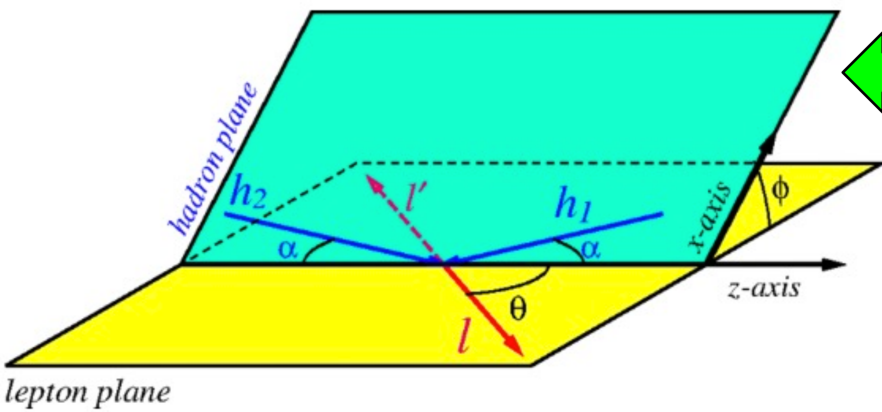
(Echevarria, Idilbi, Kang, Vitev (2014); Kang, Prokudin, Sun, Yuan (2016))



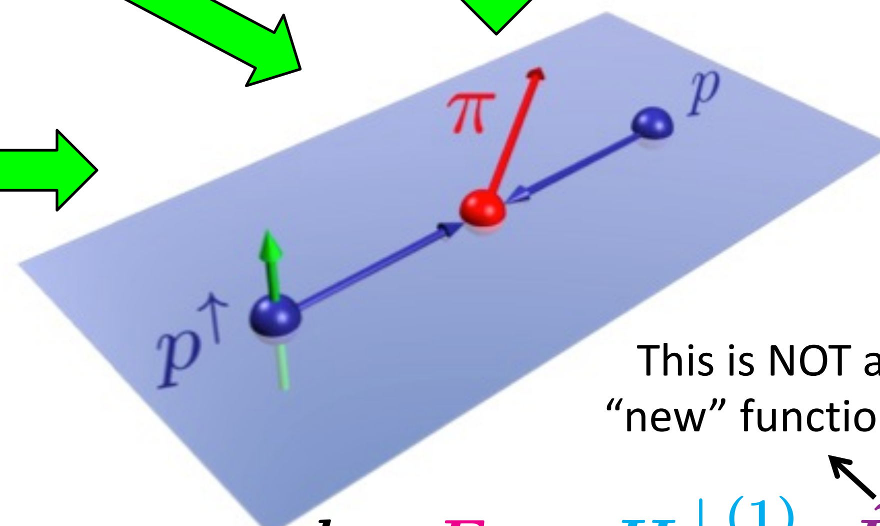
h_1, F_{FT}, $H_1^{\perp(1)}$



$H_1^{\perp(1)}$



F_{FT}



This is NOT a "new" function!

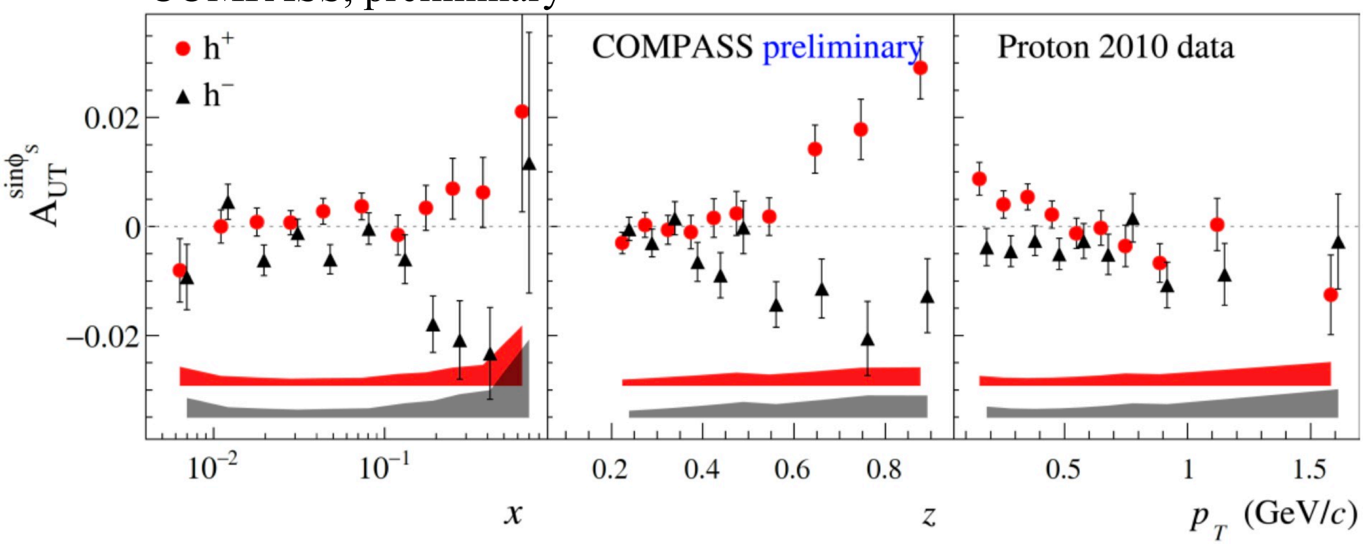
h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$

$A_{UT}^{\sin\phi_S}$ in SIDIS integrated over P_T

$$F_{UT}^{\sin\phi_S} \propto \sum_a e_a^2 \, \frac{2M_h}{Q} \, h_1^a(x) \, \frac{\tilde{H}^a(z)}{z}$$

(Mulders, Tangerman (1996); Bacchetta, et al. (2007); Wang & Lu (2016))

COMPASS, preliminary

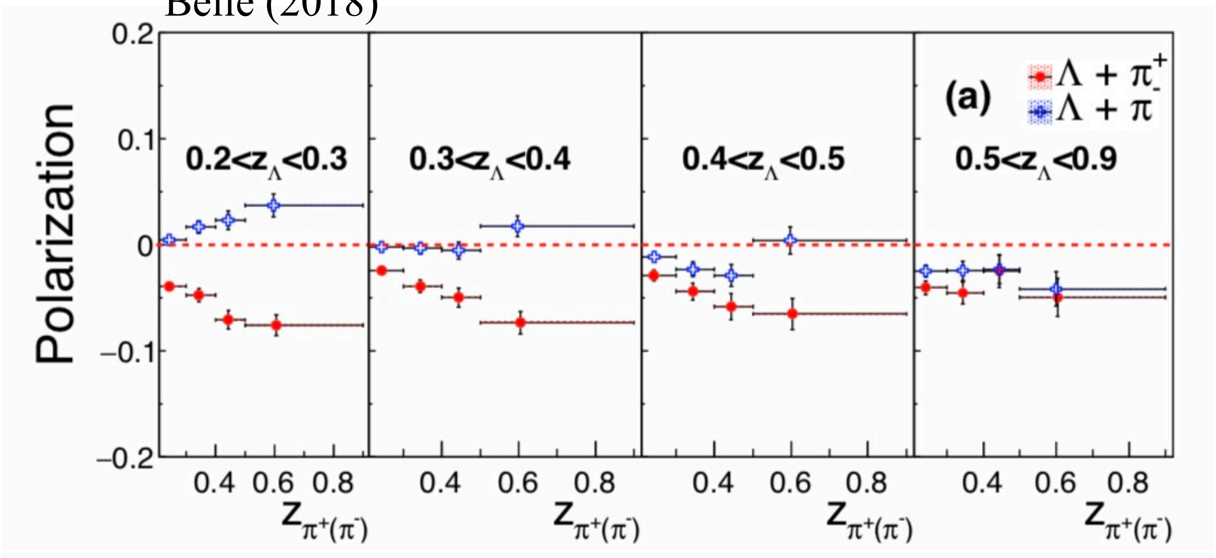


$A_{UT}^{\sin\phi_S}$ in $e^+e^- \rightarrow h_1 h_2 X$ integrated over q_T

$$F_{UT}^{\sin\phi_S} \propto \sum_{a,\bar{a}} e_a^2 \left(\frac{2M_2}{Q} D_1^a(z_1) \frac{D_T^{\bar{a}}(z_2)}{z_2} + \frac{2M_1}{Q} \frac{\tilde{H}^a(z_1)}{z_1} H_1^{\bar{a}}(z_2) \right)$$

(Boer, Jakob, Mulders (1997))

Belle (2018)



A_N in $e^+e^- \rightarrow h\,X$

$$\frac{E_h\, d\sigma(S_h)}{d^3\vec{P}_h} = \sigma_0\,(1-2v)\,\frac{8M_h}{z_h\,s^2}\,\epsilon^{ll'P_h S_h}\sum_{f=q,\bar{q}} e_f^2\,\frac{D_T^f(z_h)}{z_h}$$

(Boer, Jakob, Mulders (1997); Gamberg, Kang, DP, Schlegel, Yoshida JHEP **1901** (2019))

NLO calculation is available => evolution of D_T

(Gamberg, Kang, DP, Schlegel, Yoshida JHEP **1901** (2019))

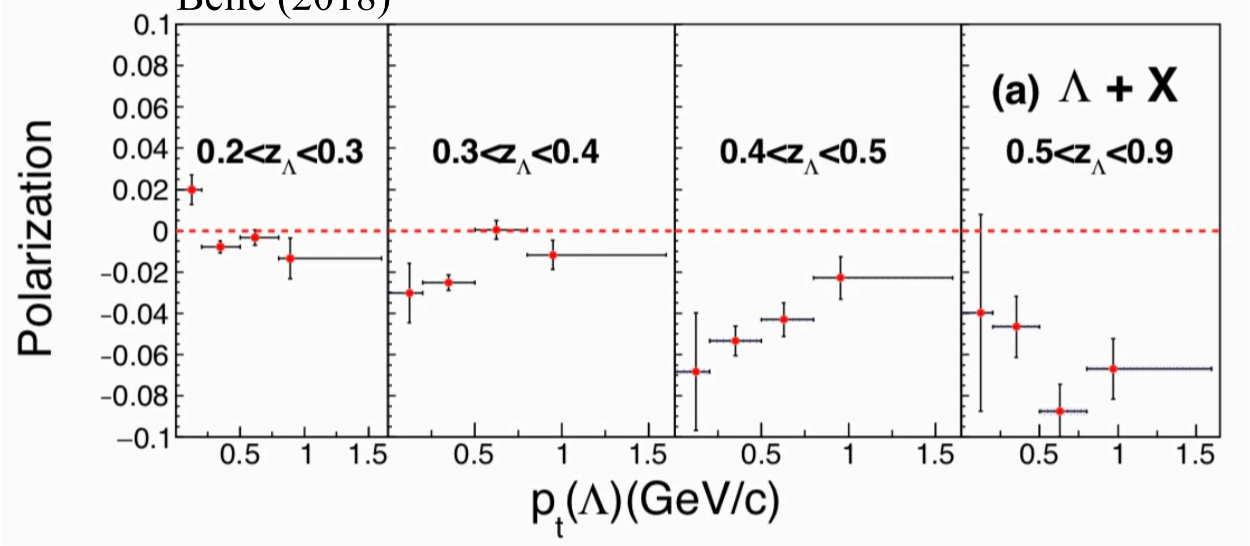
Note that this observable probes the *intrinsic FF* D_T and NOT the polarizing FF $D_{1T}^{\perp}$

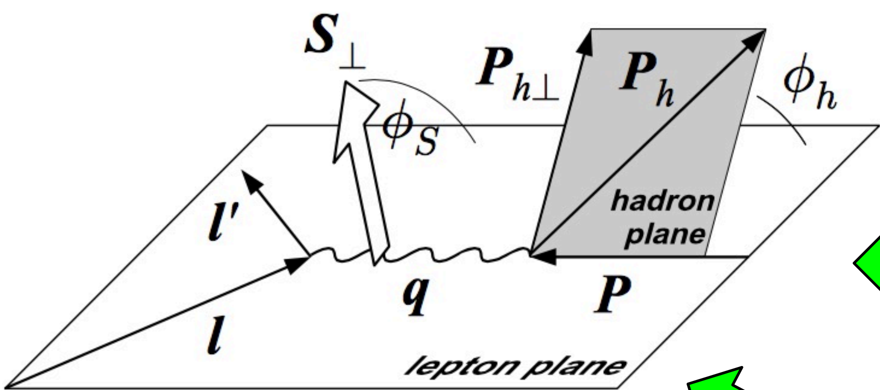
A_N in $e^+e^- \to h\, X$

$$\frac{E_h\, d\sigma(S_h)}{d^3\vec{P}_h} = \sigma_0\,(1-2v)\,\frac{8M_h}{z_h\, s^2}\,\epsilon^{ll' P_h S_h}\sum_{f=q,\bar{q}} e_f^2\,\frac{D_T^f(z_h)}{z_h}$$

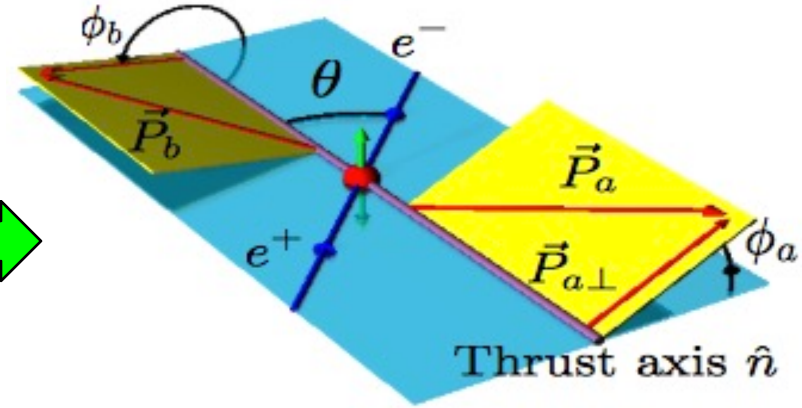
(Boer, Jakob, Mulders (1997); Gamberg, Kang, DP, Schlegel, Yoshida JHEP **1901** (2019))

Belle (2018)

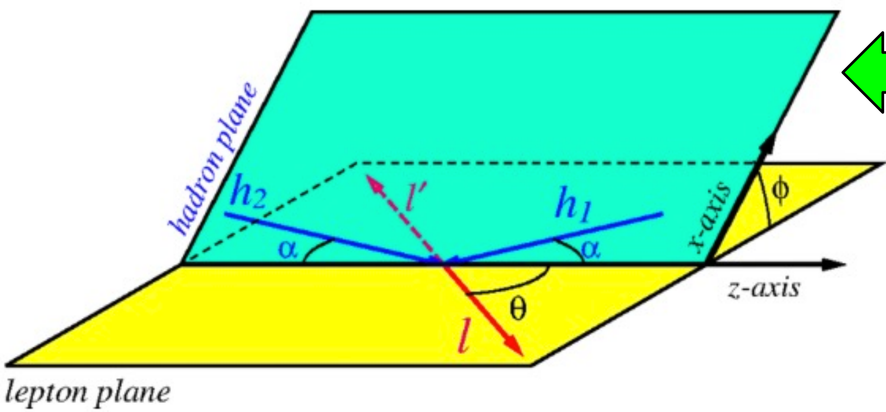




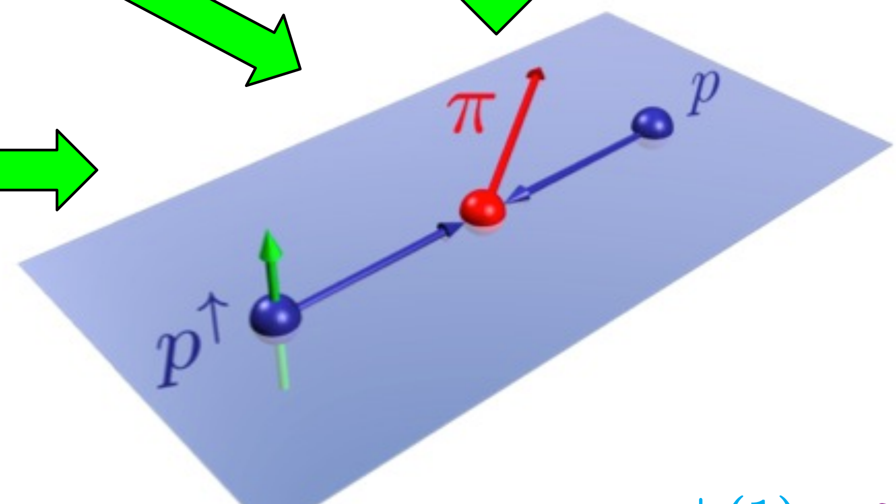
h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$



$H_1^{\perp(1)}$, $\tilde{H}$



F_{FT}



h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$

EOMR

$$H^q(z) \,=\, -2z\, H_1^{\perp(1),q}(z) + 2z \int_z^\infty \frac{dz_1}{z_1^2} \, \frac{1}{\frac{1}{z} - \frac{1}{z_1}} \hat{H}_{FU}^{q,\Im}(z, z_1)$$

LIR

$$\frac{H^q(z)}{z} = -\left(1 - z\frac{d}{dz}\right) H_1^{\perp(1),q}(z) - \frac{2}{z} \int_z^\infty \frac{dz_1}{z_1^2} \frac{\hat{H}_{FU}^{q,\Im}(z, z_1)}{(1/z - 1/z_1)^2}$$

(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))

EOMR

$$H^q(z) = -2z\, H_1^{\perp(1),q}(z) + 2z \int_z^\infty \frac{dz_1}{z_1^2} \frac{1}{\frac{1}{z} - \frac{1}{z_1}} \hat{H}_{FU}^{q,\Im}(z, z_1)$$

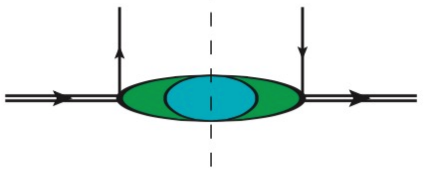
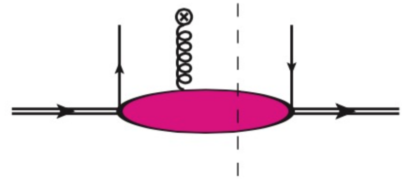
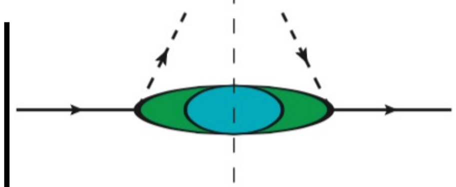
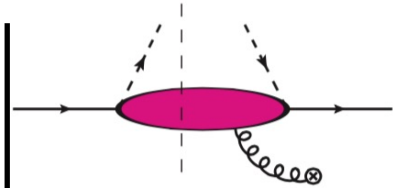
LIR

$$\frac{H^q(z)}{z} = -\left(1 - z\frac{d}{dz}\right) H_1^{\perp(1),q}(z) - \frac{2}{z}\int_z^\infty \frac{dz_1}{z_1^2} \frac{\hat{H}_{FU}^{q,\Im}(z, z_1)}{(1/z - 1/z_1)^2}$$

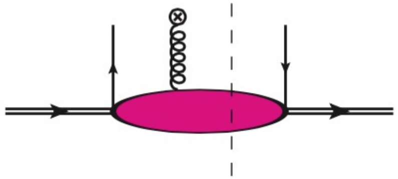
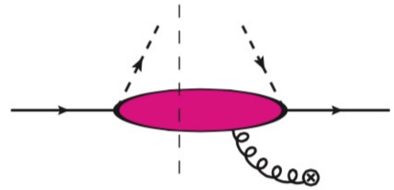
$$\boldsymbol{H(z)} = \int_z^1 dz_1 \int_{z_1}^\infty \frac{dz_2}{z_2^2}\, 2\left[\frac{\left(2(\frac{2}{z_1} - \frac{1}{z_2}) + \frac{1}{z_1}\left(\frac{1}{z_1} - \frac{1}{z_2}\right)\delta\left(\frac{1}{z_1} - \frac{1}{z}\right)\right)}{\left(\frac{1}{z_1} - \frac{1}{z_2}\right)^2} \boldsymbol{\hat{H}_{FU}^{\Im}(z_1, z_2)}\right]$$

$$\boldsymbol{H_1^{\perp(1)}(z)} = -\frac{2}{z}\int_z^1 dz_1 \int_{z_1}^\infty \frac{dz_2}{z_2^2} \frac{\left(\frac{2}{z_1} - \frac{1}{z_2}\right)}{\left(\frac{1}{z_1} - \frac{1}{z_2}\right)^2} \boldsymbol{\hat{H}_{FU}^{\Im}(z_1, z_2)}$$

(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))

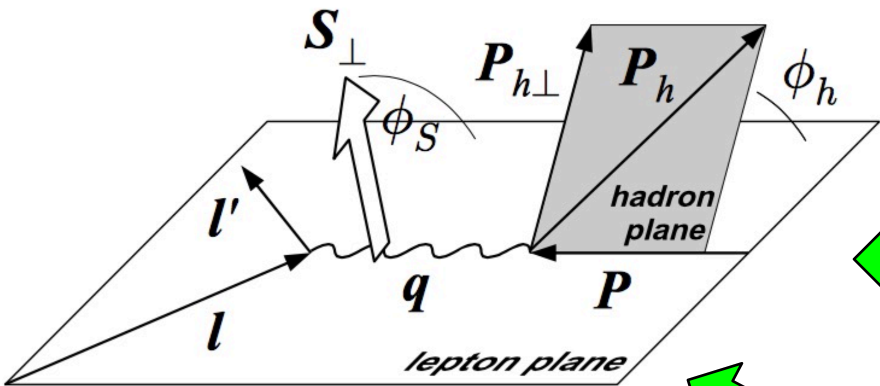
Hadron Pol.	PDF (x)		PDF (x, x_1)	FF (z)		FF (z, z_1)
						
	intrinsic	kinematical	dynamical	intrinsic	kinematical	dynamical
U	✗	✗ $h_1^{\perp(1)}$	H_{FU}	✗ E, ✗ H	✗ $H_1^{\perp(1)}$	$\hat{H}_{FU}^{\Re,\Im}$
L	✗ h_L	✗ $h_{1L}^{\perp(1)}$	H_{FL}	✗ H_L, ✗ E_L	✗ $H_{1L}^{\perp(1)}$	$\hat{H}_{FL}^{\Re,\Im}$
T	✗ g_T	✗ $f_{1T}^{\perp(1)}$, ✗ $g_{1T}^{(1)}$	F_{FT}, G_{FT}	✗ D_T, ✗ G_T	✗ $D_{1T}^{\perp(1)}$, ✗ $G_{1T}^{(1)}$	$\hat{D}_{FT}^{\Re,\Im}, \hat{G}_{FT}^{\Re,\Im}$

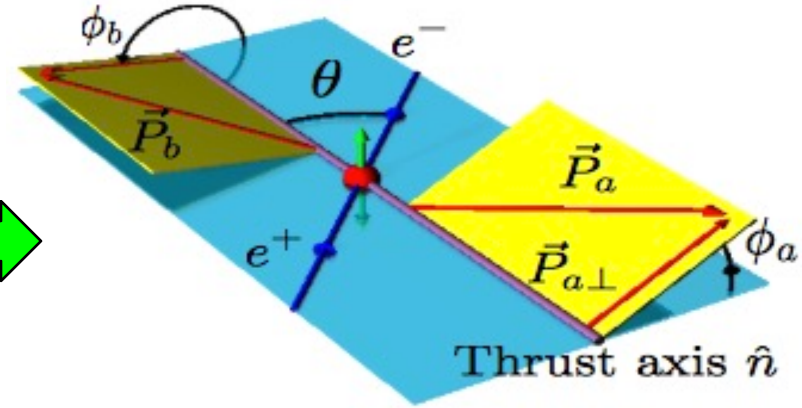
(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))

Hadron Pol.	PDF (x, x_1) 	FF (z, z_1) 
U	dynamical H_{FU}	dynamical $\hat{H}_{FU}^{\Re,\Im}$
L	H_{FL}	$\hat{H}_{FL}^{\Re,\Im}$
T	F_{FT}, G_{FT}	$\hat{D}_{FT}^{\Re,\Im}, \hat{G}_{FT}^{\Re,\Im}$

(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))

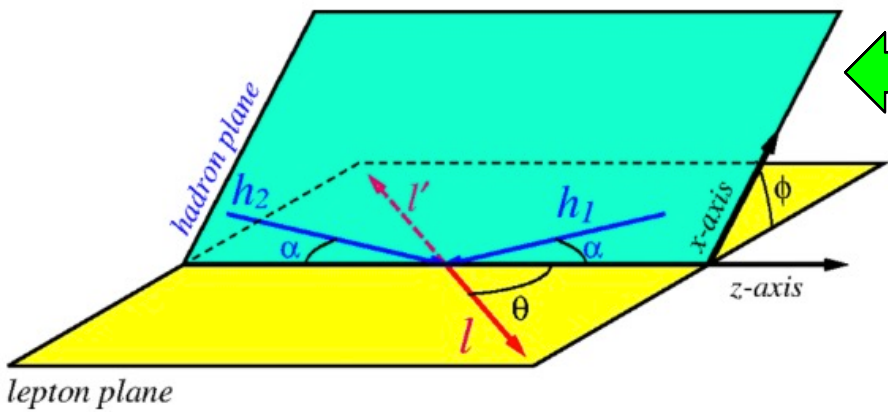
***ALL* transverse-spin observables are driven by**
***multi-parton correlations*!**

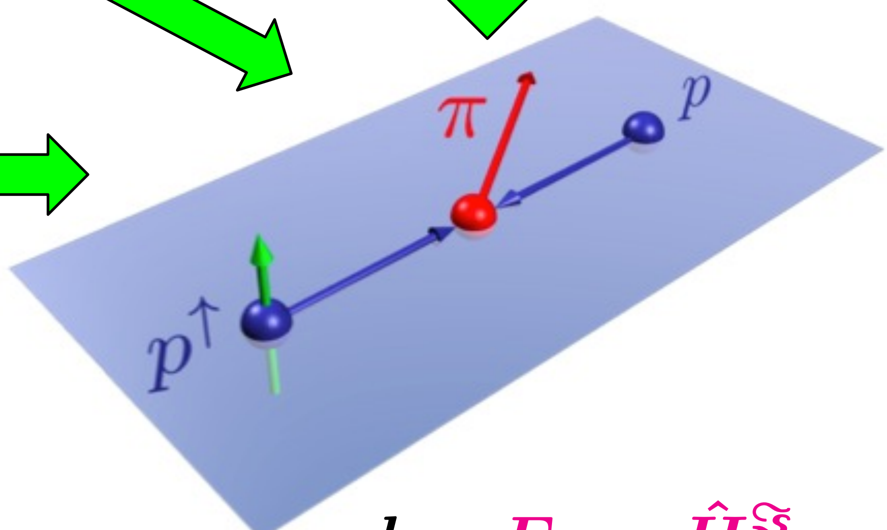




h_1, F_{FT}, $\hat{H}_{FU}^{\mathfrak{S}}$

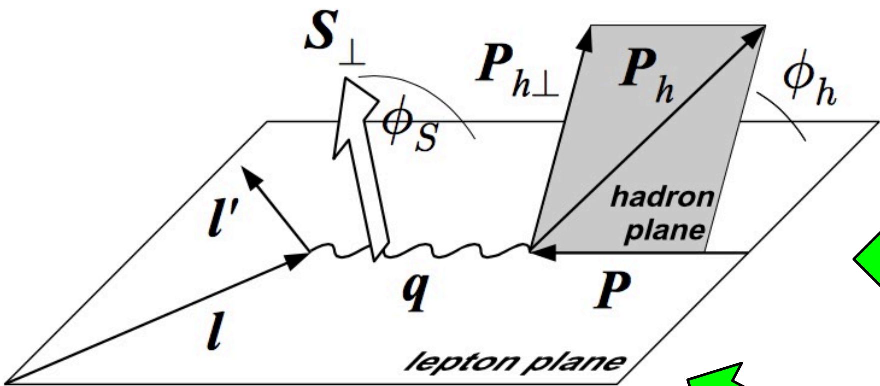
$\hat{H}_{FU}^{\mathfrak{S}}$



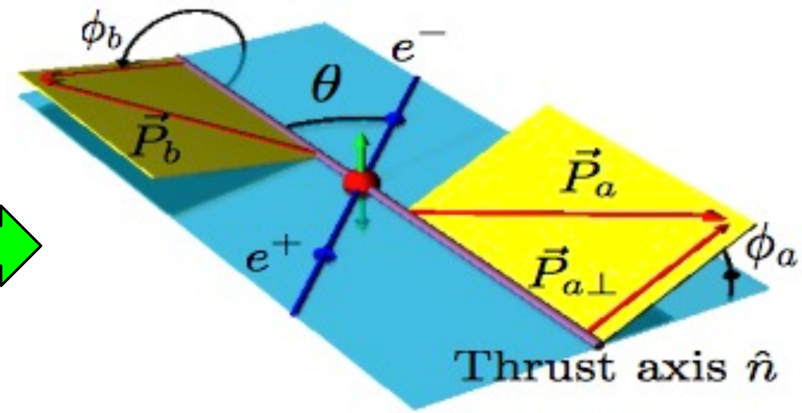


F_{FT}

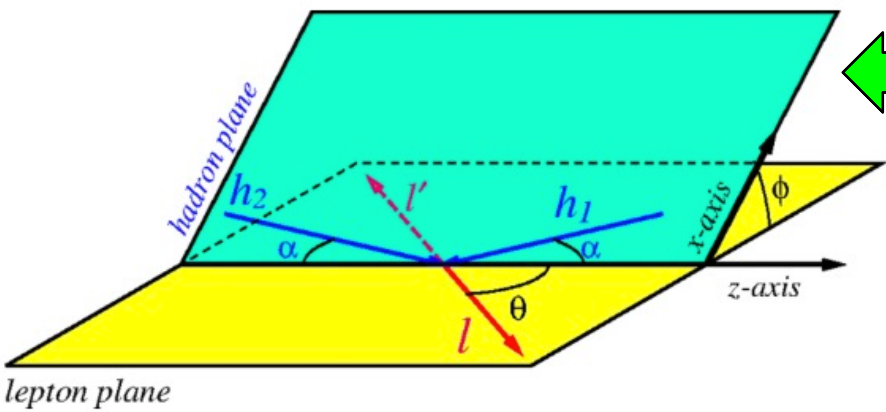
h_1, F_{FT}, $\hat{H}_{FU}^{\mathfrak{S}}$



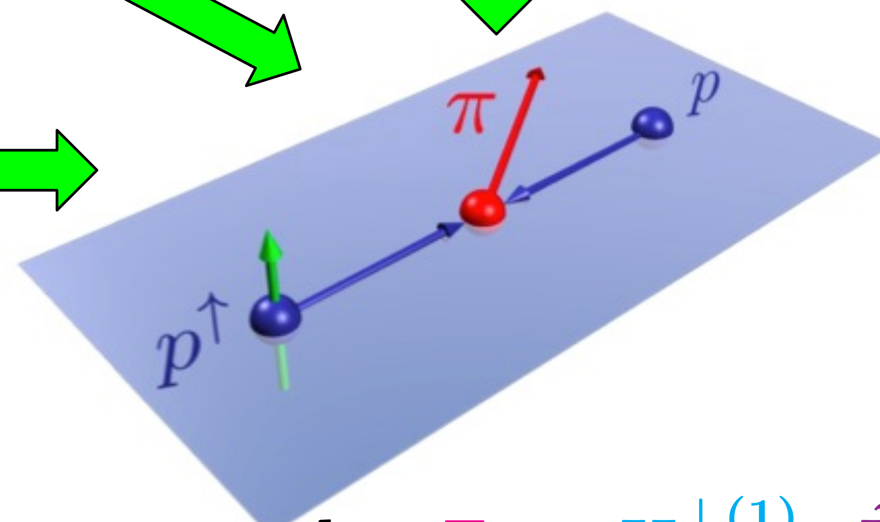
h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$



$H_1^{\perp(1)}$, $\tilde{H}$



F_{FT}



h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$

- What follows are *very preliminary* results of a global fit of
  1) Collins effects in SIDIS and e^+e^-
  2) (Integrated) $A_{UT}^{\sin\phi_s}$ in SIDIS
  3) Sivers effect in SIDIS
  4) A_N in proton-proton collisions (fragmentation + Qiu-Sterman terms)

- The plots only show the results of a single max likelihood fit. The final analysis will eventually include Monte Carlo sampling to determine the best parameters and the error bands. For now, we use a simple parameterization:

  TMDs: $Nx^{\alpha}(1-x)^{\beta}e^{-k_T^2/\langle k_T^2\rangle}$ (or with z and p_T for FF)

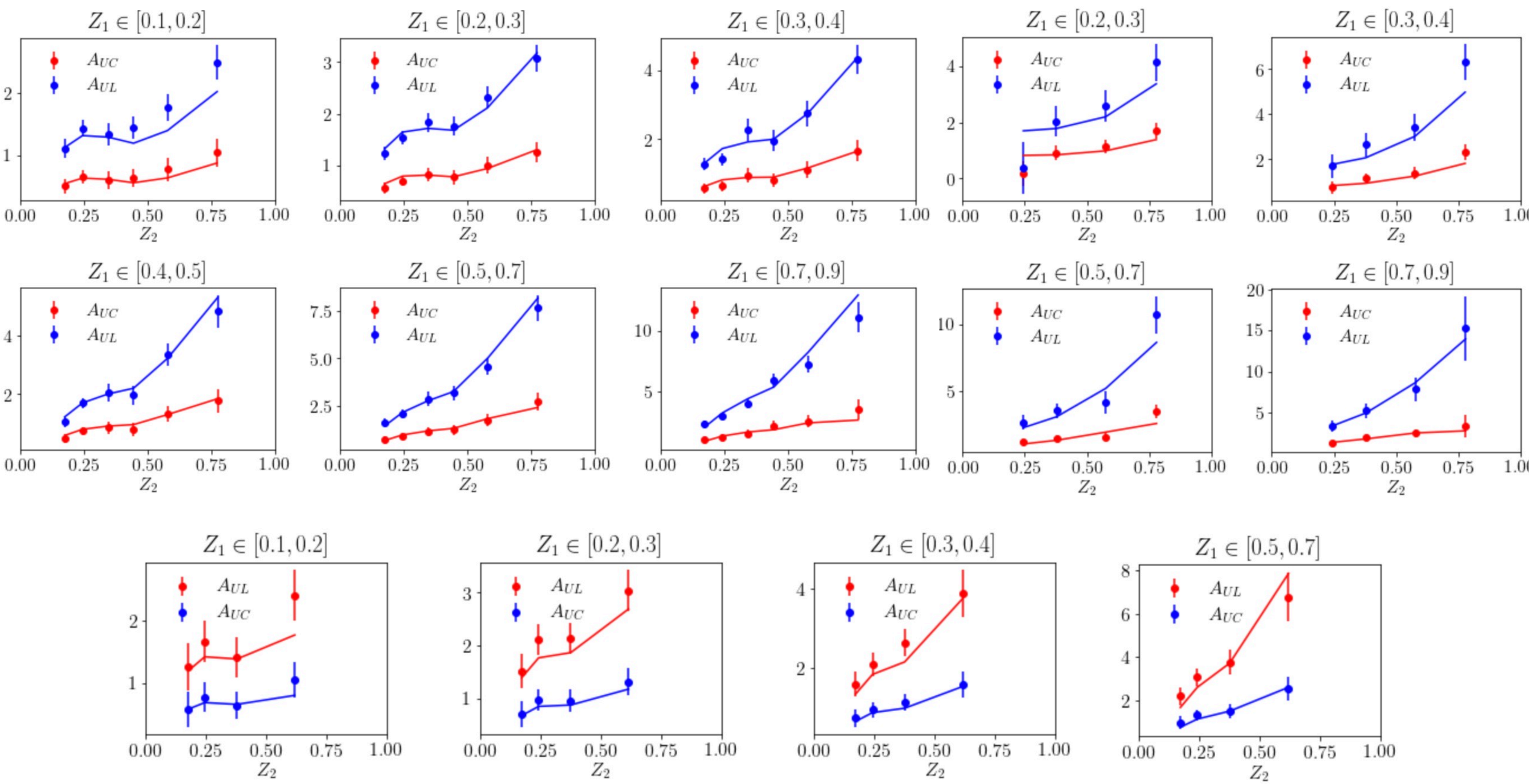
  Collinear: $Nx^{\alpha}(1-x)^{\beta}$ (or with z for FF)

- We have found solutions for the relevant non-perturbative functions that describe simultaneously a non-trivial amount of observables.

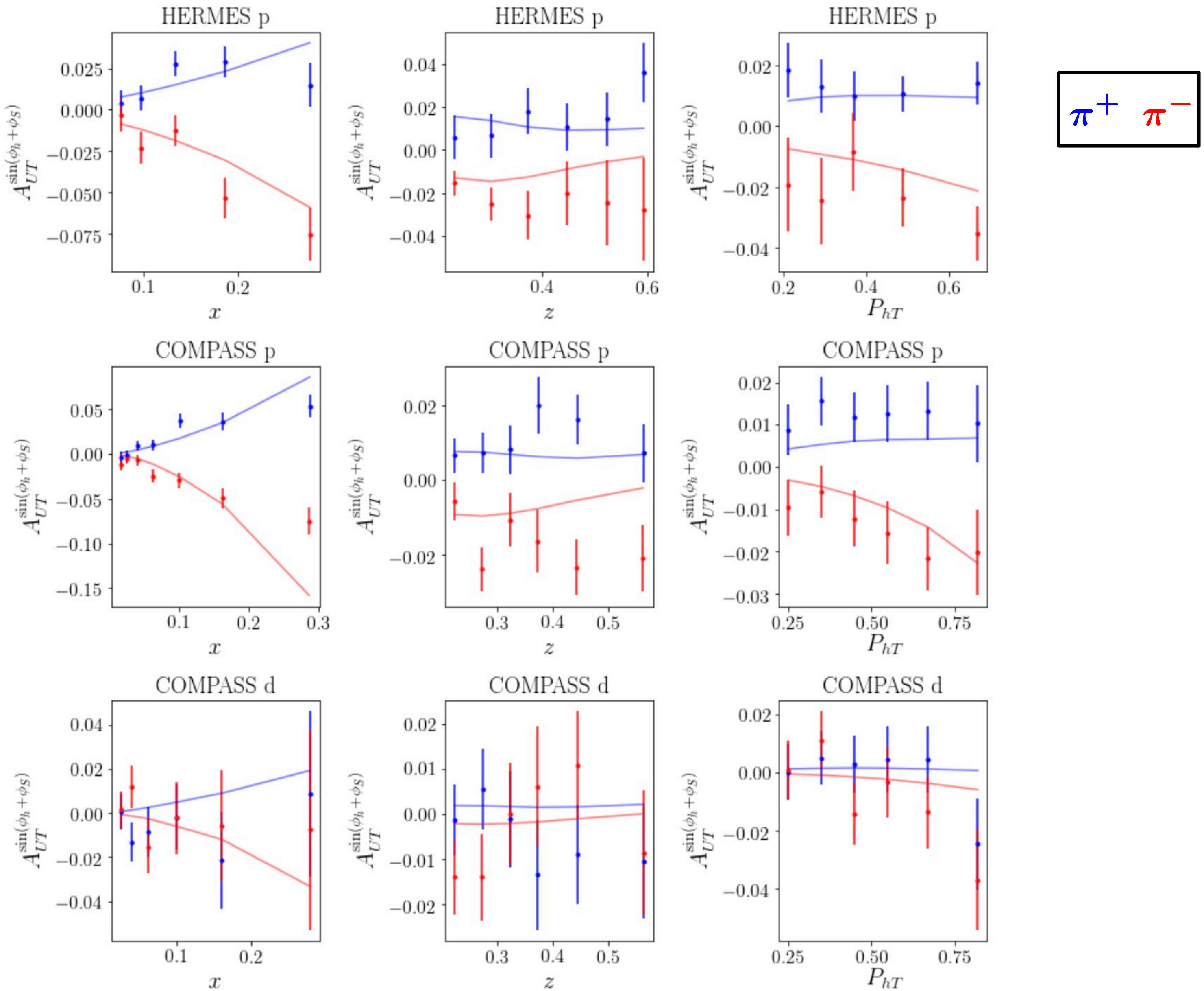
(Gamberg, Kang, DP, Prokudin, Sato, …, on-going work)

Collins effect e^+e^-

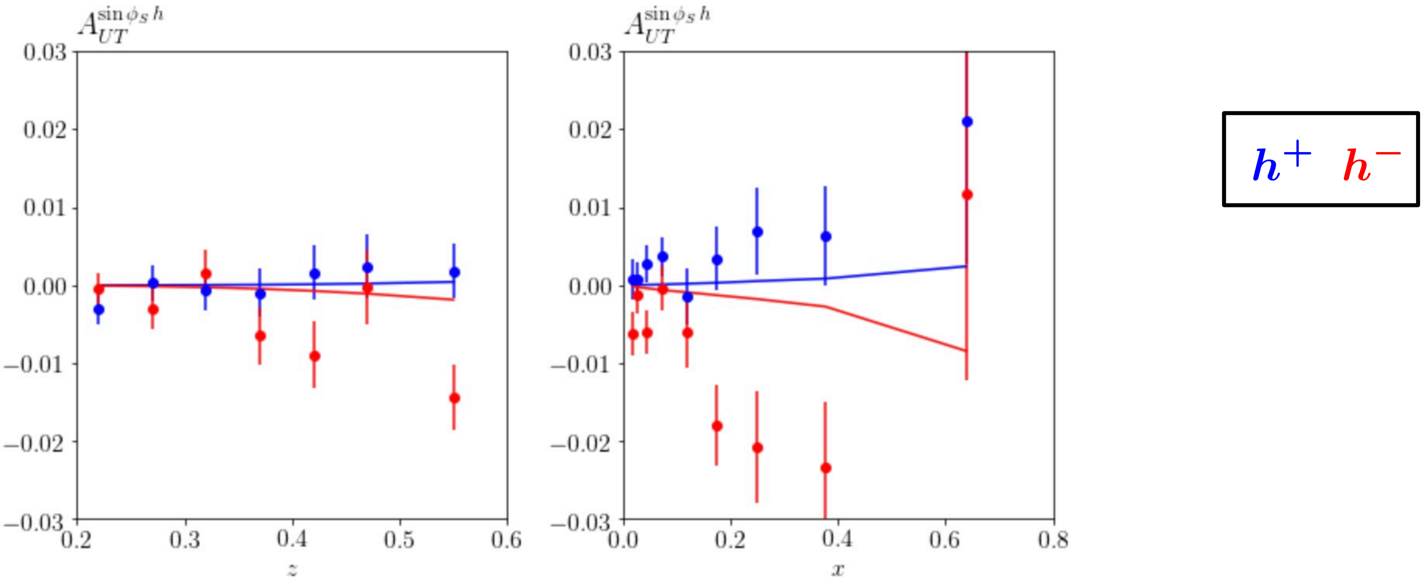
A_{UC} A_{UL}



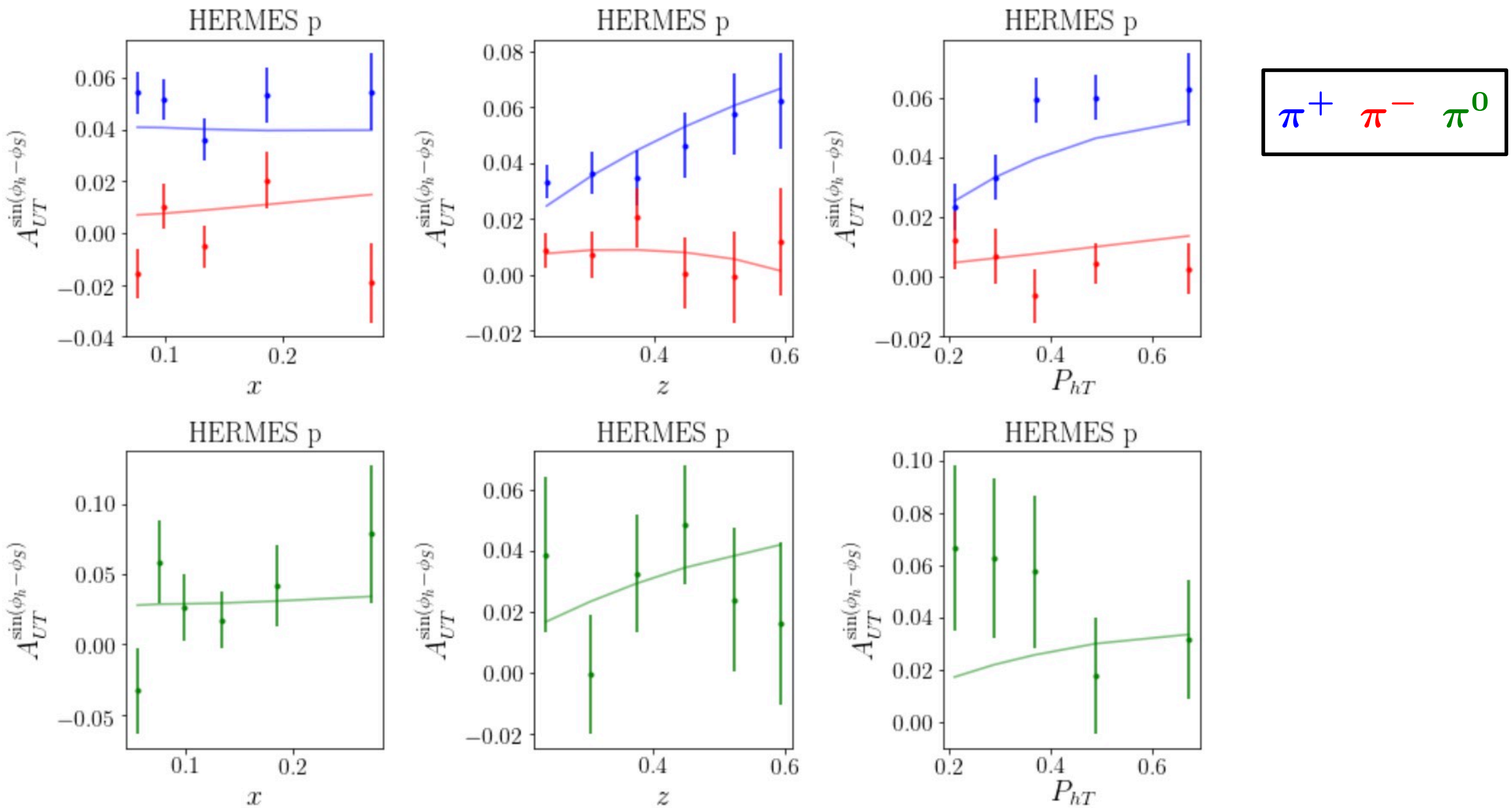
Collins effect SIDIS



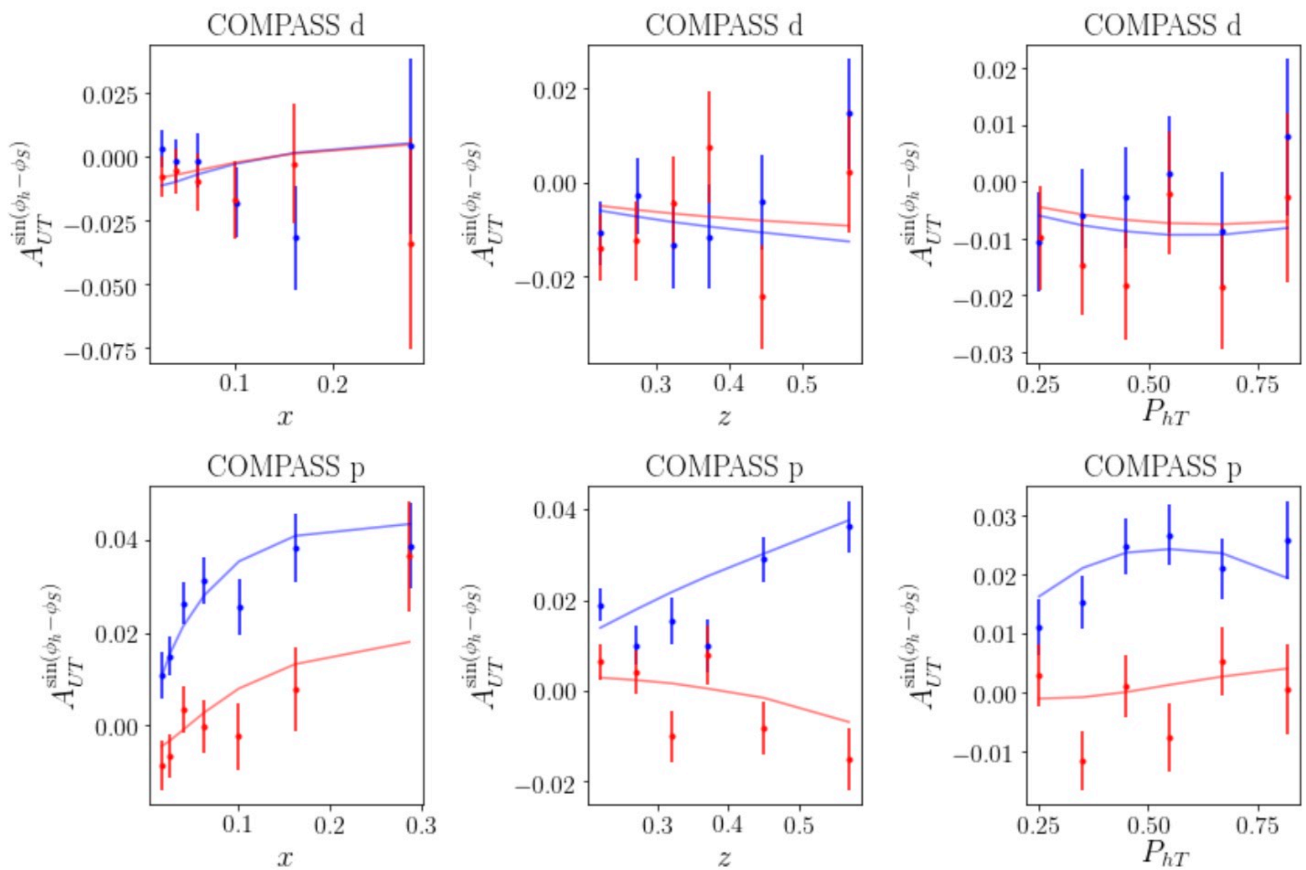
$A_{UT}^{\sin\phi_S}$ in SIDIS



Sivers effect SIDIS



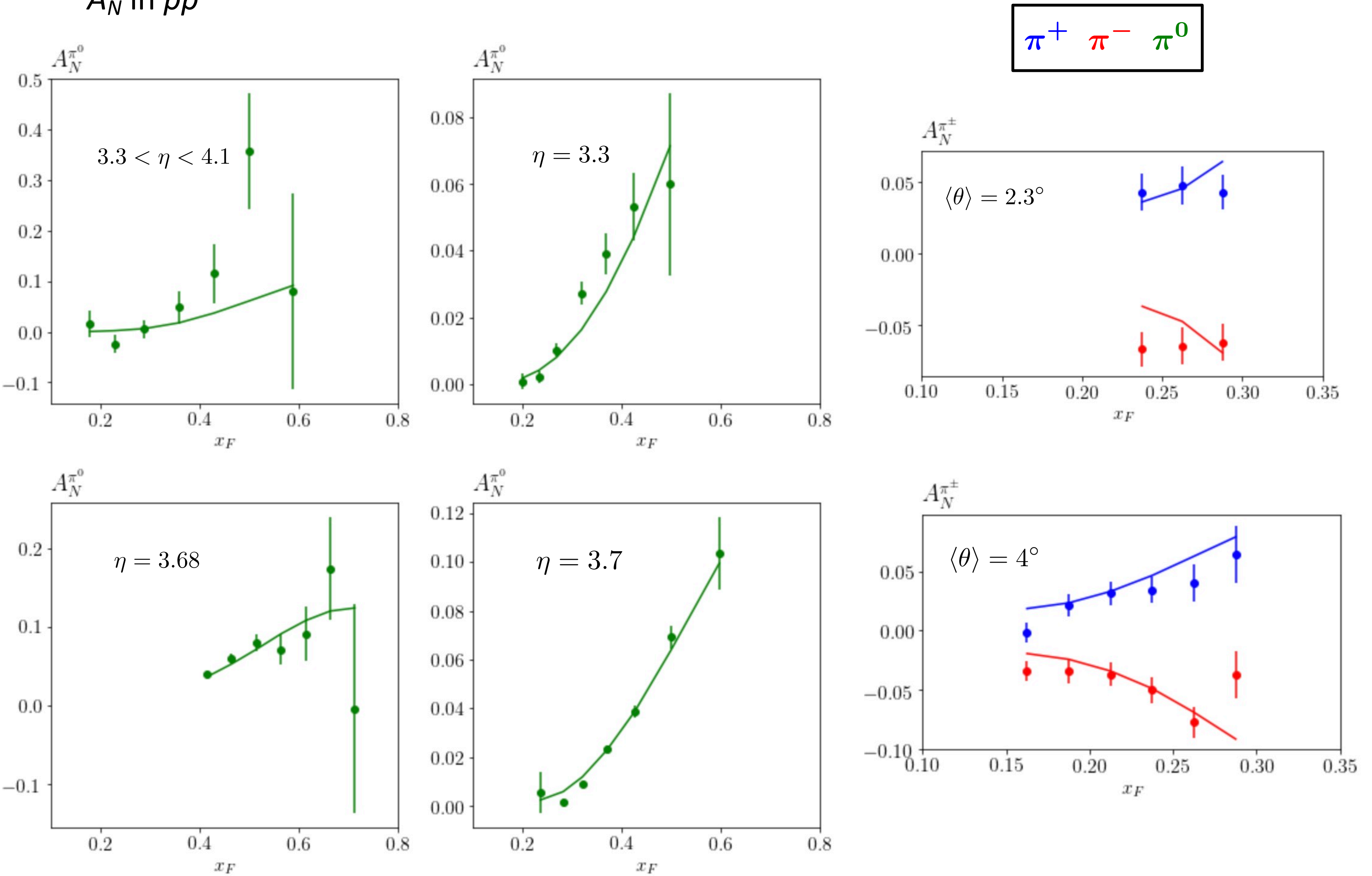
Sivers effect SIDIS

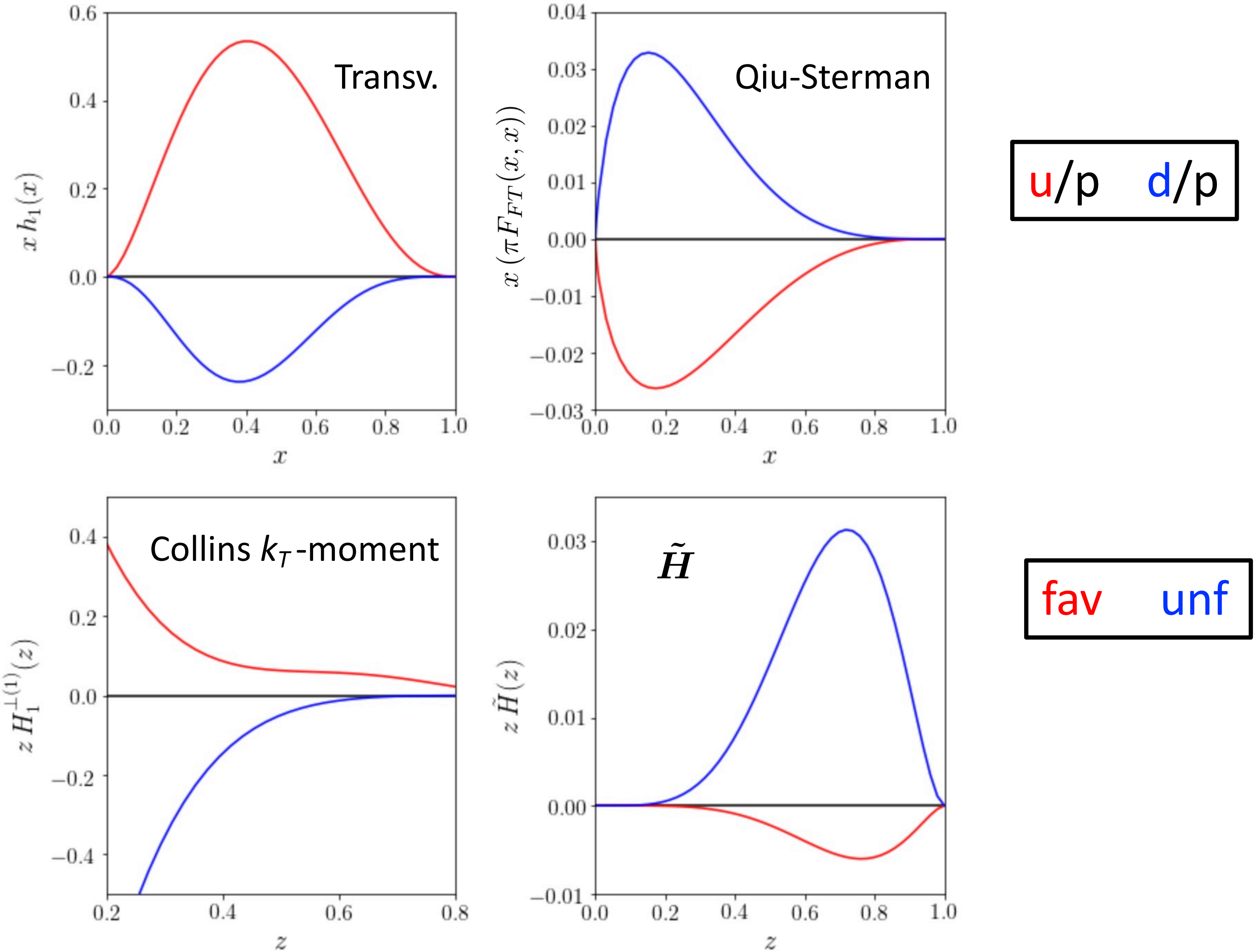


π^+ π^-

h^+ h^-

A_N in pp


π+ π− π0
$A_N^{\pi^0}$
$3.3 < \eta < 4.1$
$\eta = 3.3$
$\eta = 3.68$
$\eta = 3.7$
$A_N^{\pi^\pm}$
$\langle\theta\rangle = 2.3^\circ$
$\langle\theta\rangle = 4^\circ$
x_F


Transv.
$x\,h_1(x)$
x
Qiu-Sterman
$x\,(\pi F_{FT}(x,x))$
x
Collins k_T-moment
$z\,H_1^{\perp(1)}(z)$
z
$\tilde{H}$
$z\,\tilde{H}(z)$
z
u/p d/p
fav unf

Summary and Outlook

- TMD and collinear functions are highly interconnected, especially for reactions involving transverse spin, and we should treat both types of observables on the same footing.

- A global analysis can be performed of TMD (Sivers and Collins effects) *AND* collinear twist-3 (A_N in *pp,* $A_{UT}^{\sin\phi_s}$ in SIDIS) transverse-spin observables.

- In addition to the Sivers and Collins effects that will be measured at a future EIC (with improved statistics needed for deuterium), we must also include measurements of A_N in electron-nucleon collisions.