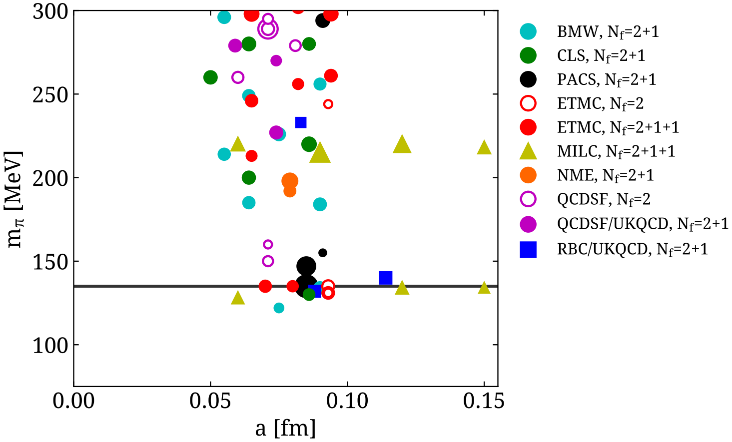
Hadron form factors and moments of parton distribution functions

Constantia Alexandrou
University of Cyprus and The Cyprus Institute

Outline

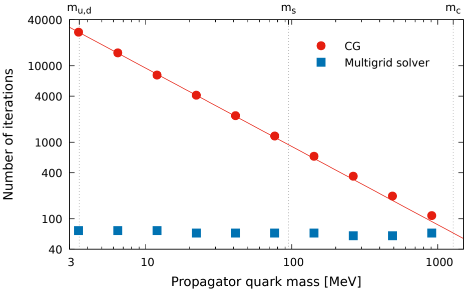
Status of simulations



Size of labels proportional to Lm_π

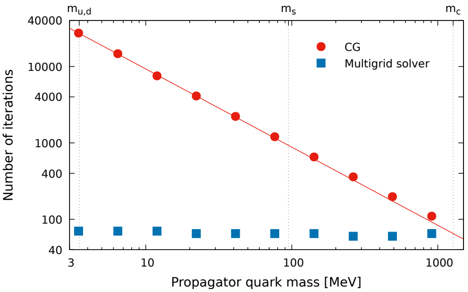
Algorithmic improvements

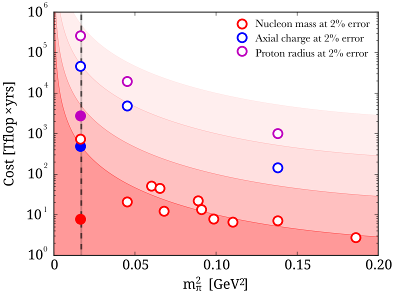
- Utilize leadership computers
- Develop fast scalable codes by exploiting different computer architectures and new algorithms e.g. one bottleneck is critical slow down due to condition number of the Dirac matrix $\rightarrow$ use deflation or multi-grid



Algorithmic improvements

- Utilize leadership computers
- Develop fast scalable codes by exploiting different computer architectures and new algorithms e.g. one bottleneck is critical slow down due to condition number of the Dirac matrix → use deflation or multi-grid

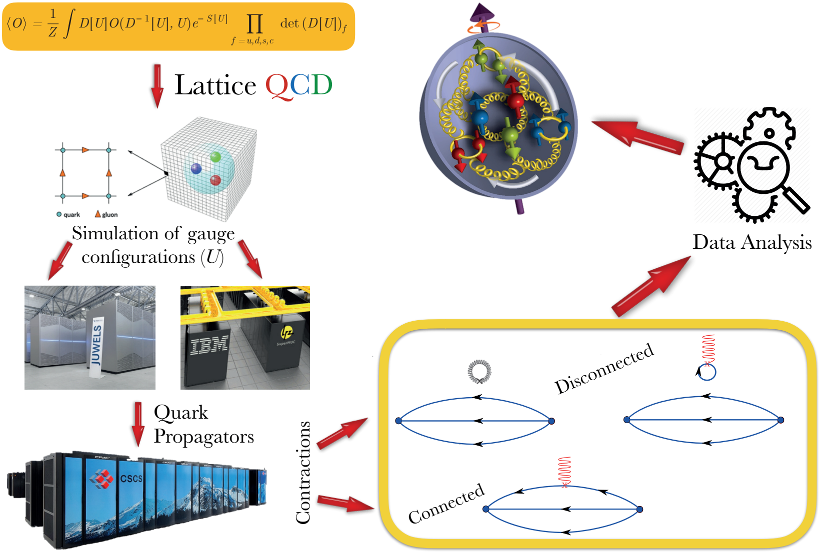




Impact on major experiments world-wide

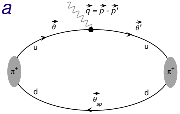
Overview of the lattice QCD computation

$$\langle O\rangle = \frac{1}{Z}\int D[U]\,O(D^{-1}[U], U)\,e^{-S[U]}\prod_{f=u,d,s,c}\det\left(D[U]\right)_f$$



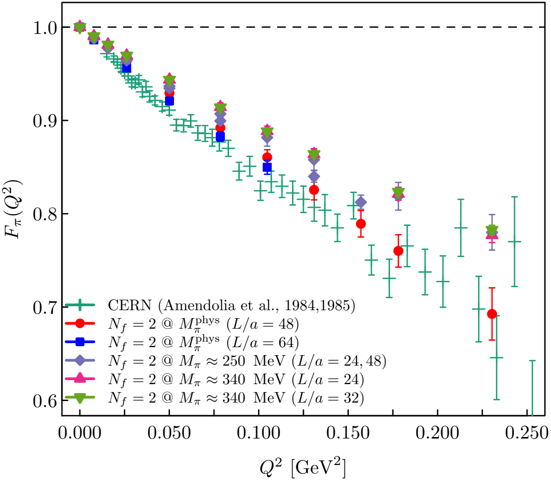
Pion electric form factor

A lot of progress has been achieved in recent years.
E.g. the Extended Twisted Mass Collaboration (ETMC) used several $N_f = 2$ ensembles including one at physical pion mass to extrapolate to infinity volume at a given value of the lattice spacing a



$$\left\langle \pi^+(\vec{p}')|V_\mu(0)|\pi^+(\vec{p})\right\rangle = (p'_\mu + p_\mu)\, F_\pi(Q^2)$$

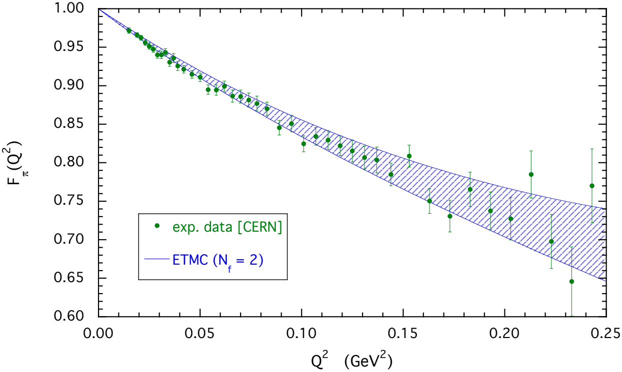
Twisted b.c. allows to reach small Q^2 values



$F(Q^2) \sim \frac{1}{1+(Q/m_\pi)^2}$ well satisfied in accord with the VMD hypothesis.

C. A., S. Bacchio, P. Dimopoulos, J. Finkenrath, R. Frezzotti, K. Jansen, B. Kostrzewa, M. Mangin-Brinet, F. Sanfilippo, S. Simula, C. Urbach, U. Wenger (ETMC), Phys. Rev. D97 (2018) 014508

Pion charge radius



$\langle r^2 \rangle_\pi = 0.443\,(21)_{stat}\,(7)_{ratio}\,(1)_{fit-range}\,(7)_{M_\pi}\,(6)_{ChPT}\,(15)_{FVE}\,(6)_{Q^2-range}$ fm^2

$\langle r^2 \rangle_\pi = 0.443\,(21)_{stat}\,(20)_{syst}$ fm^2 in agreement with $\langle r^2 \rangle_\pi^{exp.} = 0.452\,(11)$ fm^2

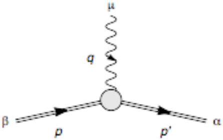
C. A., S. Bacchio, P. Dimopoulos, J. Finkenrath, R. Frezzotti, K. Jansen, B. Kostrzewa, M. Mangin-Brinet, F. Sanfilippo, S. Simula, C. Urbach, U. Wenger (ETMC), Phys. Rev. D97 (2018) 014508

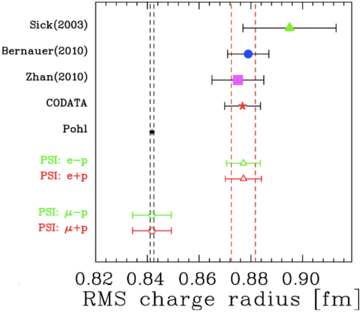
Electromagnetic form factors

$$\langle N(p',s')|j^\mu(0)|N(p,s)\rangle = \bar{u}_N(p',s')\left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2)\right]u_N(p,s)$$

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4m_N^2}F_2(q^2)$$

$$G_M(q^2) = F_1(q^2) + F_2(q^2)$$





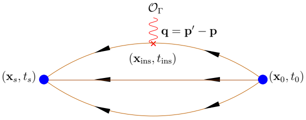
E. J. Downie, EPJ Conf. 113 (2016) 05021

- Proton radius extracted from muonic hydrogen is 7.9 σ different from the one extracted from electron scattering, R. Pohl *et al.*, Nature 466 (2010) 213
- Muonic measurement is ten times more accurate and a reanalysis of electron scattering data may give agreement with muonic measurement

Evaluation of matrix elements in lattice QCD

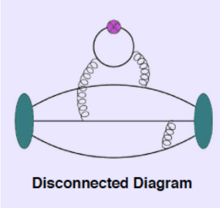
Three-point functions:

$G^{\mu\nu}(\Gamma, \vec{q}, t_s, t_{\rm ins}) = \sum_{\vec{x}_s, \vec{x}_{\rm ins}} e^{i\vec{x}_{\rm ins}\cdot\vec{q}}\, \Gamma_{\beta\alpha}\, \langle J_\alpha(\vec{x}_s, t_s) \mathcal{O}^{\mu\nu}_\Gamma(\vec{x}_{\rm ins}, t_{\rm ins}) \bar{J}_\beta(\vec{x}_0, t_0)\rangle$



connected contribution

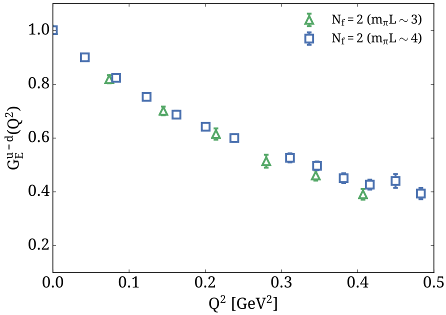
disconnected contribution



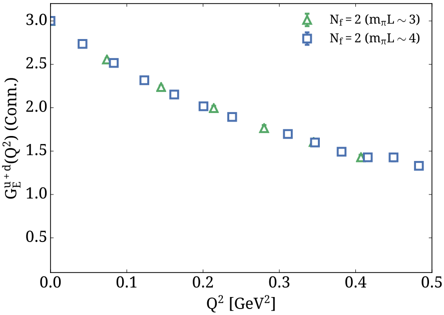
Various methods to ensure ground state dominance: plateau, two-state and summation methods $\rightarrow$ all should agree

Volume effects on the electric form factor

Electric form factor using twisted mass clover improved fermion simulations at the physical point



Isovector

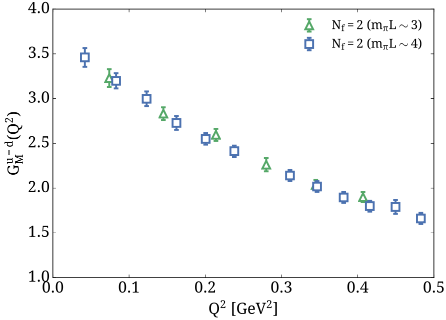


Isoscalar connected

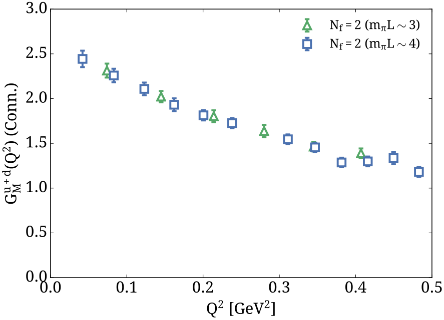
N_f	$N_L^3 \times N_T$	a	$Lm_\pi =$	L [fm]
2	$48^3 \times 96$	0.0938(4)(1)	2.98	4.52
2	$64^3 \times 128$	0.0938(4)(1)	3.97	6.02
2+1+1	$64^3 \times 128$	0.0801(4)	3.62	5.13

Volume effects on the magnetic form factor

Magnetic form factor using twisted mass clover improved fermion simulations at the physical point



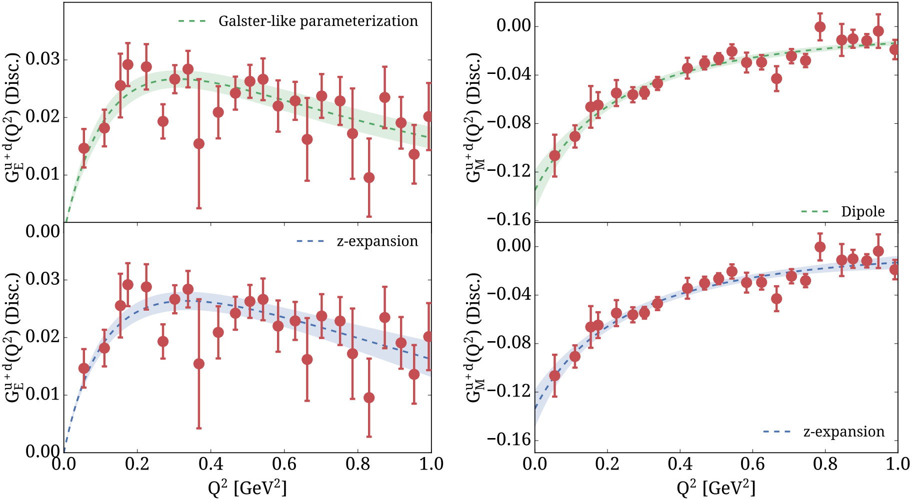
Isovector



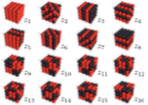
Isoscalar connected

N_f	$N_L^3 \times N_T$	a	$Lm_\pi =$	L [fm]
2	$48^3 \times 96$	0.0938(4)(1)	2.98	4.52
2	$64^3 \times 128$	0.0938(4)(1)	3.97	6.02
2+1+1	$64^3 \times 128$	0.0801(4)	3.62	5.13

Disconnected contributions to the electric and magnetic form factors



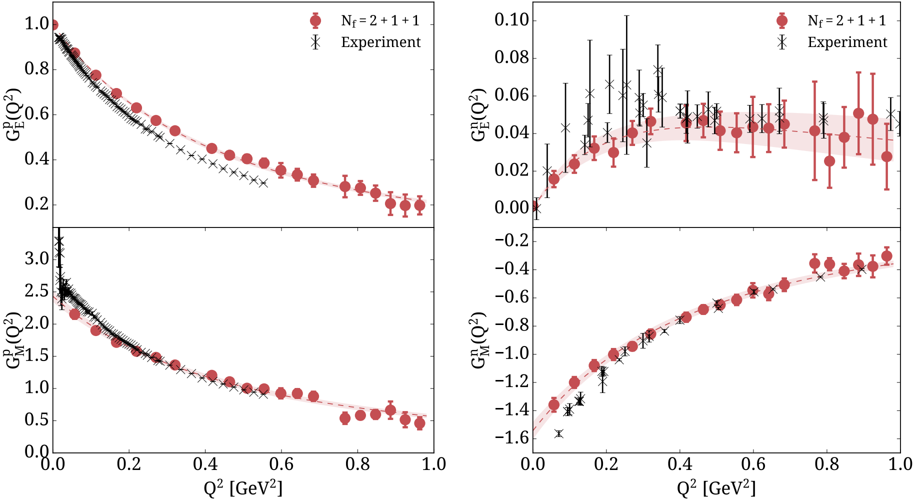
Sampling of the fermion propagator using site colouring schemes, A. Stathopoulos, J. Laeuchli, K. Orginos, arXiv:1302.4018



- C. A., M. Constantinou, K. Hadjiyiannakou, K. Jansen, C. Kallidonis, G. Koutsou and A. Vaquero Aviles-Casco. Phys. Rev. D96 (2017) 034503, arXiv:1706.00469;
- C. A., S. Bacchio, M. Constantinou, J. Finkenrath, K. Hadjiyiannakou, K. Jansen, G. Koutsou and A. Vaquero Aviles-Casco, arXiv:1812.10311.

Proton and neutron electric and magnetic form factors

Statistically accurate results are emerging



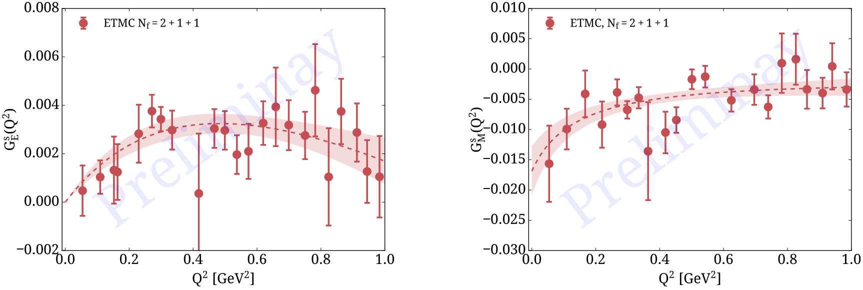
Deviation from experimental results under investigation, arising from e.g. finite volume and/or excited states

C. A., S. Bacchio, M. Constantinou, J. Finkenrath, K. Hadjiyiannakou, K. Jansen, G. Koutsou and A. Vaquero Aviles-Casco, arXiv:1812.10311.

Strange Electromagnetic form factors

Experimental determination: Parity violating $e - N$ scattering
HAPPEX experiment finds $G_M^s(0.62) = -0.070(67)$

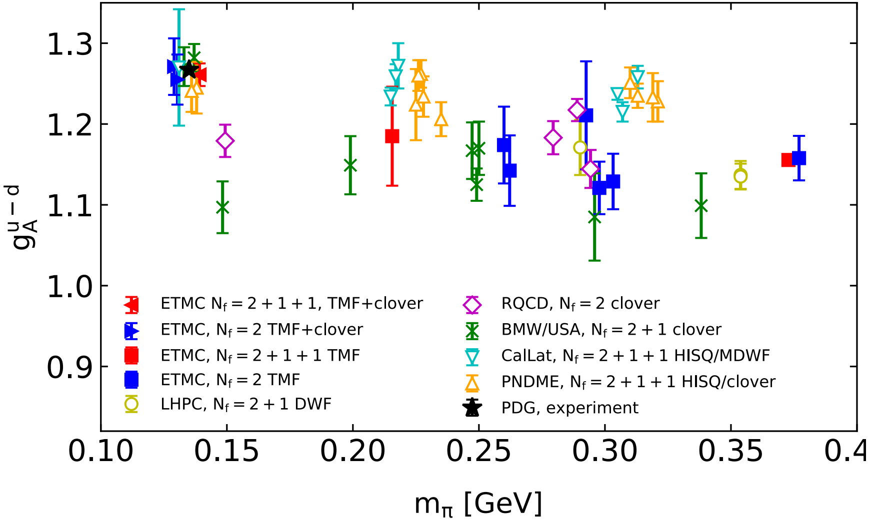
PRELIMINARY



- $N_f = 2+1+1$, $m_\pi = 139(1)$ MeV, $64^3 \times 128$, $a = 0.08$, ETM collaboration
- Overlap valence on $N_f = 2+1$ domain wall fermions, $24^3 \times 64$, $a = 0.11$ fm, $m_\pi = 330$ MeV; $32^3 \times 64$, $a = 0.083$ fm, $m_\pi = 300$ MeV and 48^3, a=0.11 fm, $m_\pi = 139$ MeV, R. S. Sufian *et al.* (χQCD Collaboration) 1606.07075
- $N_f = 2+1$ clover fermions, $m_\pi \sim 320$ MeV, , J. Green et al., Phys.Rev. D92 (2015) 31501

Nucleon axial charge

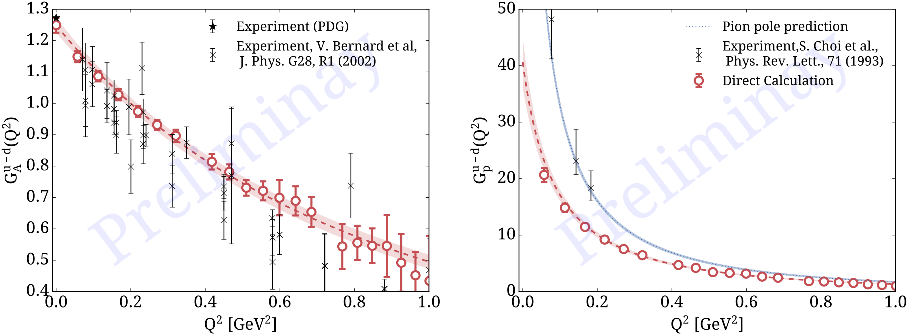
After a long-term effort g_A is emerging from lattice QCD.



Results on the axial form factors

$$\langle N(p',s')|A_\mu|N(p,s)\rangle = i\sqrt{\frac{m_N^2}{E_N(\vec{p}')E_N(\vec{p})}}\bar{u}_N(p',s')\left(\gamma_\mu G_A(Q^2) - i\frac{Q_\mu}{2m_N}G_p(Q^2)\right)\gamma_5 u_N(p,s)$$

Isovector

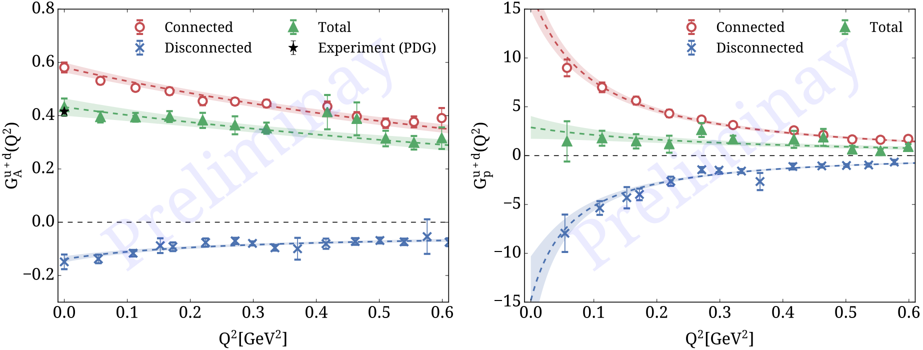


- ETMC using $N_f = 2+1+1$ twisted mass fermions, $a = 0.08$ fm, $64^3 \times 128$

Recent results on the isoscalar axial form factors

$$\langle N(p',s')|A_\mu|N(p,s)\rangle = i\sqrt{\frac{m_N^2}{E_N(\vec{p}')E_N(\vec{p})}}\bar{u}_N(p',s')\left(\gamma_\mu G_A(Q^2) - i\frac{Q_\mu}{2m_N}G_p(Q^2)\right)\gamma_5 u_N(p,s)$$

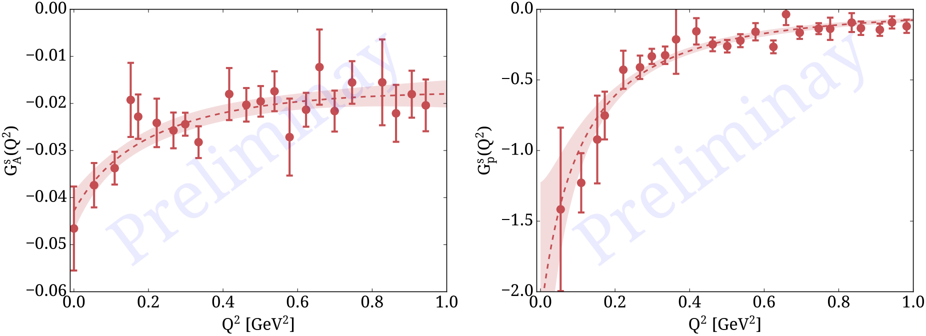
Isoscalar



- ETMC using $N_f = 2+1+1$ twisted mass fermions, $a = 0.08$ fm, $64^3 \times 128$

Strange axial form factors

$$\langle N(p',s')|A_\mu|N(p,s)\rangle = i\sqrt{\frac{m_N^2}{E_N(\vec{p}\,')E_N(\vec{p})}}\,\bar{u}_N(p',s')\left(\gamma_\mu G_A(Q^2) - i\frac{Q_\mu}{2m_N}G_p(Q^2)\right)\gamma_5 u_N(p,s)$$



- ETMC using $N_f = 2+1+1$ twisted mass fermions, $a = 0.08$ fm, $64^3 \times 128$

Moments of PDFs

High energy scattering: Formulate in terms of light-cone correlation functions, M. Diehl, Phys. Rep. 388 (2003)

Consider one-particle states p' and $p \to$ GPDs, X. Ji, J. Phys. G24 (1998) 1181

$$F_\Gamma(x,\xi,q^2) = \frac{1}{2}\int \frac{d\lambda}{2\pi} e^{ix\lambda} \langle p'|\bar{\psi}(-\lambda n/2)\Gamma \mathcal{P} e^{ig\int_{-\lambda/2}^{\lambda/2} d\alpha\, n\cdot A(n\alpha)} \psi(\lambda n/2)|p\rangle$$

where $q = p' - p$, $\bar{P} = (p'+p)/2$, n is a light-cone vector with and $\bar{P}.n = 1$

Expansion of the light cone operator leads to a tower of local operators $\mathcal{O}^{\mu\mu_1\cdots\mu_n}$

$\longrightarrow$ Entails computing nucleon matrix elements of quark bilinears: $\langle N(p',s')|\mathcal{O}_\Gamma^{\mu_1\cdots\mu_n}|N(p,s)\rangle$

- Unpolarized:

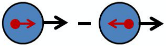
$$\mathcal{O}_V^{\mu\mu_1\cdots\mu_n} = \bar{\psi}(x)\gamma^{\{\mu} i \overleftrightarrow{D}^{\mu_1} \ldots i \overleftrightarrow{D}^{\mu_n\}} \psi(x)$$

$n=0: \to \langle 1\rangle_q = g_V^q, \qquad n=1: \to J^q = \frac{1}{2}\left[A_{20}^q(0) + B_{20}^q(0)\right]$ and

$\langle x\rangle_q = A_{20}^q(0)$

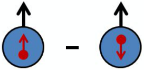
- Helicity:

$$\mathcal{O}_A^{\mu\mu_1\cdots\mu_n} = \bar{\psi}(x)\gamma^{\{\mu} i \overleftrightarrow{D}^{\mu_1} \ldots i \overleftrightarrow{D}^{\mu_n\}} \gamma_5 \psi(x)$$



$n=0: \to \langle 1\rangle_{\Delta q} = \Delta\Sigma^q = g_A^q, \qquad n=1: \to \langle x\rangle_{\Delta q} = \tilde{A}_{20}^q(0)$

- Transversity:



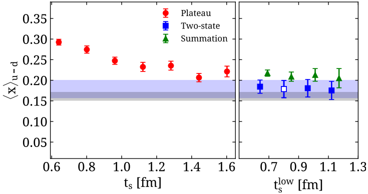
$$\mathcal{O}_T^{\nu\mu\mu_1\cdots\mu_n} = \bar{\psi}(x)\sigma^{\{\nu,\mu} i \overleftrightarrow{D}^{\mu_1} \ldots i \overleftrightarrow{D}^{\mu_n\}} \frac{\tau^a}{2}\psi(x)$$

$n=0: \to \langle 1\rangle_{\delta q} = g_T^q, \qquad n=1: \to \langle x\rangle_{\delta q} = \tilde{\tilde{A}}_{20}^q(0)$

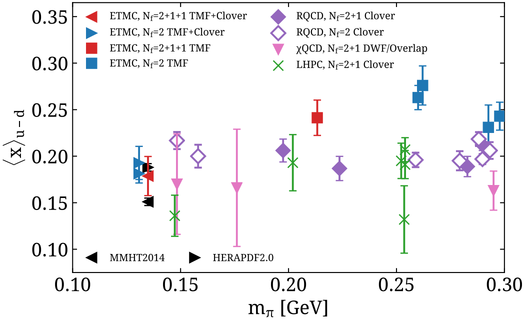
Momentum fraction $\langle \mathbf{x} \rangle_{u-d}$

Preliminary

- $N_f = 2+1+1$ twisted mass fermions with a clover term at a physical value of the pion mass, $64^3 \times 128$ and $a = 0.080$ fm



Results for the isovector in the $\overline{\text{MS}}$ at 2 GeV



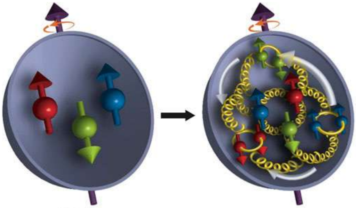
At the physical point we find in the $\overline{\text{MS}}$ at 2 GeV:

$N_f = 2$ $\quad \langle x \rangle_{u-d} = 0.194(9)(10)$

$N_f = 2+1+1$ $\quad \langle x \rangle_{u-d} = 0.179(22)$

Proton spin puzzle

European Muon Collaboration (EMC) experiment at CERN: Deep Inelastic Scattering (DIS) of high energy polarized muons on polarized protons , J. Ashman *et al.* (EMC) Phys. Lett. B206 (1988) 364 and Nucl. Phys. B328 (1989) 1.



Naive quark model: Only valence quarks $\frac{1}{2} = \frac{1}{2}(\Delta u_v + \Delta d_v)$ where $\Delta u_v = \frac{4}{3}$ and $\Delta d_v = -\frac{1}{3}$

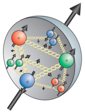
EMC result: $\frac{1}{2}\sum_q \Delta\Sigma_q \sim \frac{1}{4}$ $\rightarrow$ Spin puzzle

How does the spin of the nucleon arise?

Gluons and sea quarks are important $\rightarrow \Delta G$ and $\Delta q_{\rm sea}$

But also orbital angular momentum of quarks and gluons.

Spin of the nucleon



Ji decomposition: $\frac{1}{2} = \underbrace{\sum_q \frac{1}{2}\Delta\Sigma^q}_{\text{quark spin}} + \underbrace{\sum_q L^q + J^g}_{\text{dark spin}}$

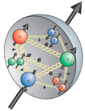
$\Delta\Sigma_q \equiv \Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s} + \cdots$
Total quark angular momentum $J^q = \frac{1}{2}\Delta\Sigma^q + L^q$ and total gluon angular momentum J^g.

The total quark angular momenta J^q can be extracted from generalized form factors at zero momentum transfer $Q^2 = 0$ (unpolarized and helicity PFDs):

$J^q = \frac{1}{2}\left(A_{20}^q(0) + B_{20}^q(0)\right)$ while $\Delta\Sigma^q = \tilde{A}_{10}^q(0)$.

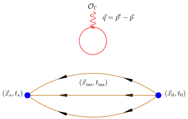
Need to compute nucleon matrix elements of local operators.

Quark intrinsic spin



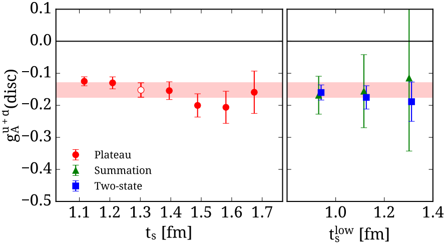
Spin sum: $\frac{1}{2} = \sum_q \underbrace{\left(\frac{1}{2}\Delta\Sigma^q + L^q\right)}_{J^q} + J^g$

$J^q = \frac{1}{2}\left(A^q_{20}(0) + B^q_{20}(0)\right)$ and $\Delta\Sigma^q = g^q_A$

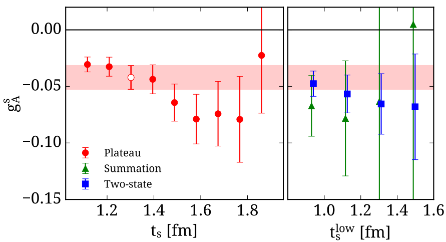


Need isoscalar g_A, which has disconnected contributions

- $N_f = 2$ twisted mass fermions with a clover term at a physical value of the pion mass, $48^3 \times 96$ and $a = 0.093(1)$ fm
- Intrinsic quark spin: $\Delta\Sigma^q = g^q_A$
- $N_f = 2 + 1 + 1$ twisted mass clover-improved fermions under analysis



Isoscalar disconnected

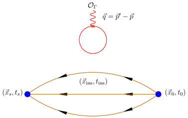


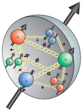
Strange

Quark intrinsic spin

Spin sum: $\frac{1}{2} = \sum_q \underbrace{\left(\frac{1}{2}\Delta\Sigma^q + L^q\right)}_{J^q} + J^g$

$J^q = \frac{1}{2}\left(A_{20}^q(0) + B_{20}^q(0)\right)$ and $\Delta\Sigma^q = g_A^q$





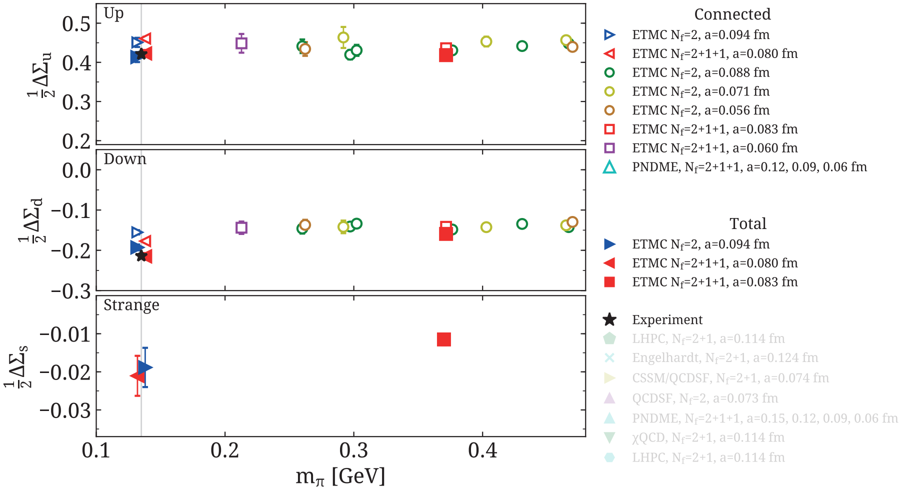
Need isoscalar g_A, which has disconnected contributions

We find from the plateau method:

- $g_A^{u+d} = -0.15(2)$ (disconnected only) with 854,400 statistics
- Combining with the isovector we find: $g_A^u = 0.828(21)$, $g_A^d = -0.387(21)$
- $g_A^s = -0.042(10)$ with 861,200 statistics

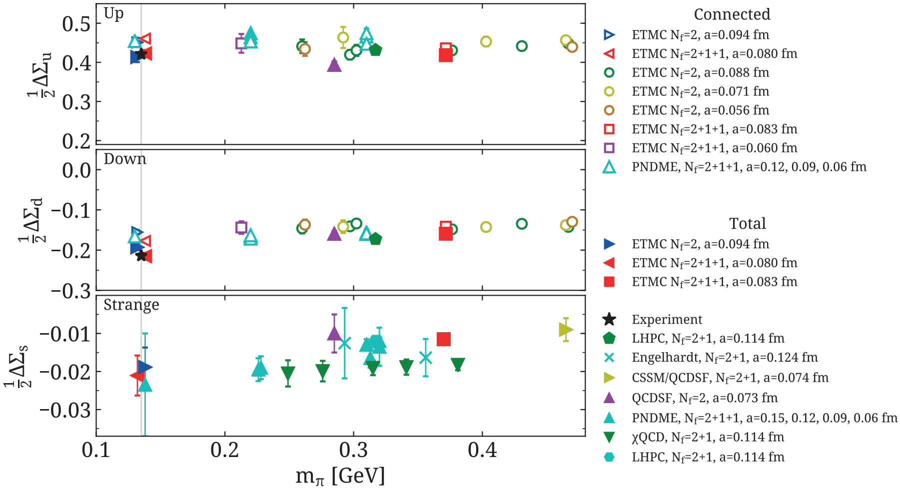
Quark intrinsic spin

- Volume and finite-*a* effects smaller than statistical errors at heavier that physical pion masses
- Disconnected contributions non-zero. Our result agrees with recent analysis by COMPASS that found $0.13 < \frac{1}{2}\Delta\Sigma < 0.18$ C. Adolph et al., Phys. Lett. B753, 18 (2016), 1503.08935



Quark intrinsic spin

- Volume and finite-*a* effects smaller than statistical errors at heavier that physical pion masses
- Disconnected contributions non-zero. Our result agrees with recent analysis by COMPASS that found $0.13 < \frac{1}{2}\Delta\Sigma < 0.18$ C. Adolph et al., Phys. Lett. B753, 18 (2016), 1503.08935
- Good agreement with other lattice QCD results



Results for the gluon content

- 2094 gauge configurations with 100 different source positions each $\rightarrow$ more than 200 000 measurements
- Due to mixing with the quark singlet operator, the renormalization and mixing coefficients had to be extracted from a one-loop perturbative lattice calculation, M. Constantinou and H. Panagopoulos
- $\langle x \rangle_{g,\text{bare}} = 0.318(24) \xrightarrow{\text{Renormalization}}$ $< x >^R_g = Z_{gg} < x >_g + Z_{gq} < x >_{u+d+s} = 0.267(12)_{\text{stat}}(10)_{\text{syst}}$. The renormalization is perturbatively done using two-levels of stout smearing. The systematic error is the difference between using one- and two-levels of stout smearing.
- Momentum sum is satisfied:
  $\sum_q \langle x \rangle_q + \langle x \rangle_g = \langle x \rangle_{u+d}|_{\text{conn.}} + \langle x \rangle_{u+d+s}|_{\text{disconn.}} + \langle x \rangle_g = 1.07(12)_{\text{stat}}(10)_{\text{syst}}$

Nucleon spin

Spin sum: $\frac{1}{2} = \sum_q \underbrace{\left(\frac{1}{2}\Delta\Sigma^q + L^q\right)}_{J^q} + J^g$

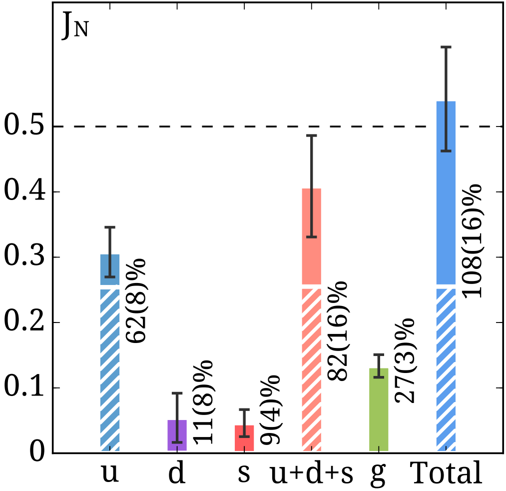
$$\frac{1}{2}\Delta\Sigma^u = 0.415(13)(2), \quad \tfrac{1}{2}\Delta\Sigma^d = -0.193(8)(3), \quad \frac{1}{2}\Delta\Sigma^s = -0.021(5)(1) \tag{1}$$
$$J^u = 0.308(30)(24), \quad J^d = 0.054(29)(24), \quad J^s = 0.046(21)$$
$$L^u = -0.107(32)(24), \quad L^d = 0.247(30)(24), \quad L^s = 0.067(21)(1)$$

We find that $B^q_{20}(0) \sim 0 \longrightarrow$ taking $B_{20}(0)^g \sim 0$ we can directly check the nucleon spin sum:

$$J_N = (0.308)_u + (0.054)_d + (0.046)_s + (0.133)_g = 0.54(6)(5)$$

The proton spin puzzle

1987: the European Muon Collaboration showed that only a fraction of the proton spin is carried by the quarks $\Longrightarrow$ ETMC has now provided the solution



Recent results from lattice QCD at the physical point

C.A. *et al.*, Phys. Rev. Lett. 119 (2017) arXiv:1706.02973



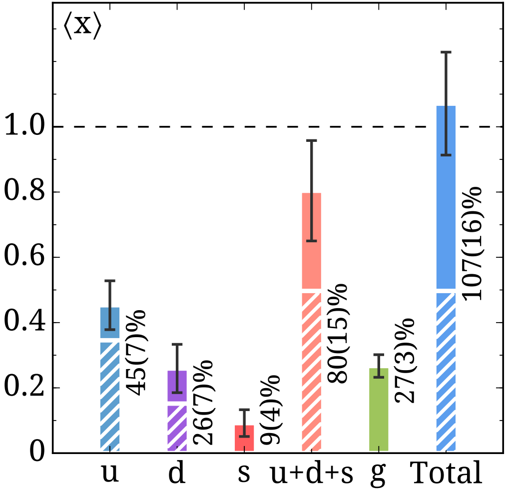




The proton momentum sum

$\Longrightarrow$ Momentum sum also satisfied

$\sum_q \langle x \rangle_q + \langle x \rangle_g = 0.497(12)(5)|_{\text{conn.}} + 0.307(121)(95)|_{\text{disc.}} + 0.267(12)(10)|_{\text{gluon}} = 1.07(12)(10)$









Recent results from lattice QCD at the physical point

C.A. *et al.*, Phys. Rev. Lett. 119 (2017) arXiv:1706.02973

Conclusions

- Precision nucleon structure is now possible with all contributions taken into account and performing the simulation at the physical point
  $\Longrightarrow$ no chiral extrapolation needed
- A number of collaborations are computing of g_A, $\langle x \rangle_{u-d}$, etc, at the physical point allowing cross-check of approaches

On-going studies:

- Continuum limit $\longrightarrow$ need at least three lattice spacings
- Assessment volume effects
- Investigation of the proton radius using new methods e.g. position methods
- Computation of gluonic observables
- Study of excited states and resonances
- Study of scattering lengths and interactions
- etc.

Extended Twisted Mass Collaboration

E

Cyprus (Univ. of Cyprus, Cyprus Inst.), France (Orsay, Grenoble), Germany (Berlin/Zeuthen, Bonn, Frankfurt, Hamburg, Münster), Italy (Rome I, II, III, Trento), Netherlands (Groningen), Poland (Poznan), Spain (Valencia), Switzerland (Bern), UK (Liverpool)

Collaborators:
S. Bacchio, K. Cichy, M. Constantinou, J. Finkenrath, K. Hadjiyiannakou, K.Jansen, Ch. kallidonis, G. Koutsou, A. Scapellato, F. Steffens, A. Vaquero

Thank you for your attention