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3D structure and entanglement of QCD

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QCD – entangled states and QIT

- Parton-hadron duality in hard QCD scattering: PDFs x FFs
  - nucleon is pure state → ensemble of partons (good light-front states)
  - hard (short distance) process: partons → partons
  - emerging partons are pure state(s) → ensemble of hadron states
- Entangled (pure) states $|\Phi\rangle$ in bipartite ($\mathcal{H}^A \otimes \mathcal{H}^B$) space, with a density matrix $\rho = |\Phi\rangle\langle\Phi|$, lead to ensembles (non-pure state) in the reduced spaces.
  - EPR bipartite pure state leads to a 50% - 50% ensemble in subspaces
  - Maximal entanglement associated with maximal entropy
  - Tripartite states come in two classes of maximal entangled states.
- Both hadrons and partons might live in a multipartite $\mathcal{H}^A \otimes \mathcal{H}^B \otimes \mathcal{H}^C$ space! This requires careful thinking about 'resolution' or more fundamental about 'local', as well as about 'TMD'
- Multipartite entanglement offers a new perspective to include leptons, quarks, gauge bosons and Higgs field in a more fundamental framework that explains emergence of symmetries in the standard model of particle physics (revival of rishon model).

ENTANGLEMENT OF HADRONS

Entanglement in hard processes

- A nucleon is an (entangled) pure state (or a spin ensemble) $\rho_i = |P\rangle\langle P|$
- Hard processes serve as entanglement witness $\mathrm{Tr}(\rho W) < 0$ (for a hermitean semi-definite operator W)
  - Start with initial state density matrix $\rho_i = |P\rangle\langle P|$
  - Split scattering process into $M(i \to f \to m_f) = M(i \to f)M(f \to m_f)$
  - Define production part and decay/fragmentation part (specify hadrons/spins)

$$R^{prod}_{\alpha\beta;\alpha'\beta'} = M^*_{\beta\to\beta'}(i \to f)M_{\alpha\to\alpha'}(i \to f)$$

$$R^{decay}_{\beta'\alpha'} = M^*_{\beta'\to m_f}(f)M_{\alpha'\to m_f}(f)$$

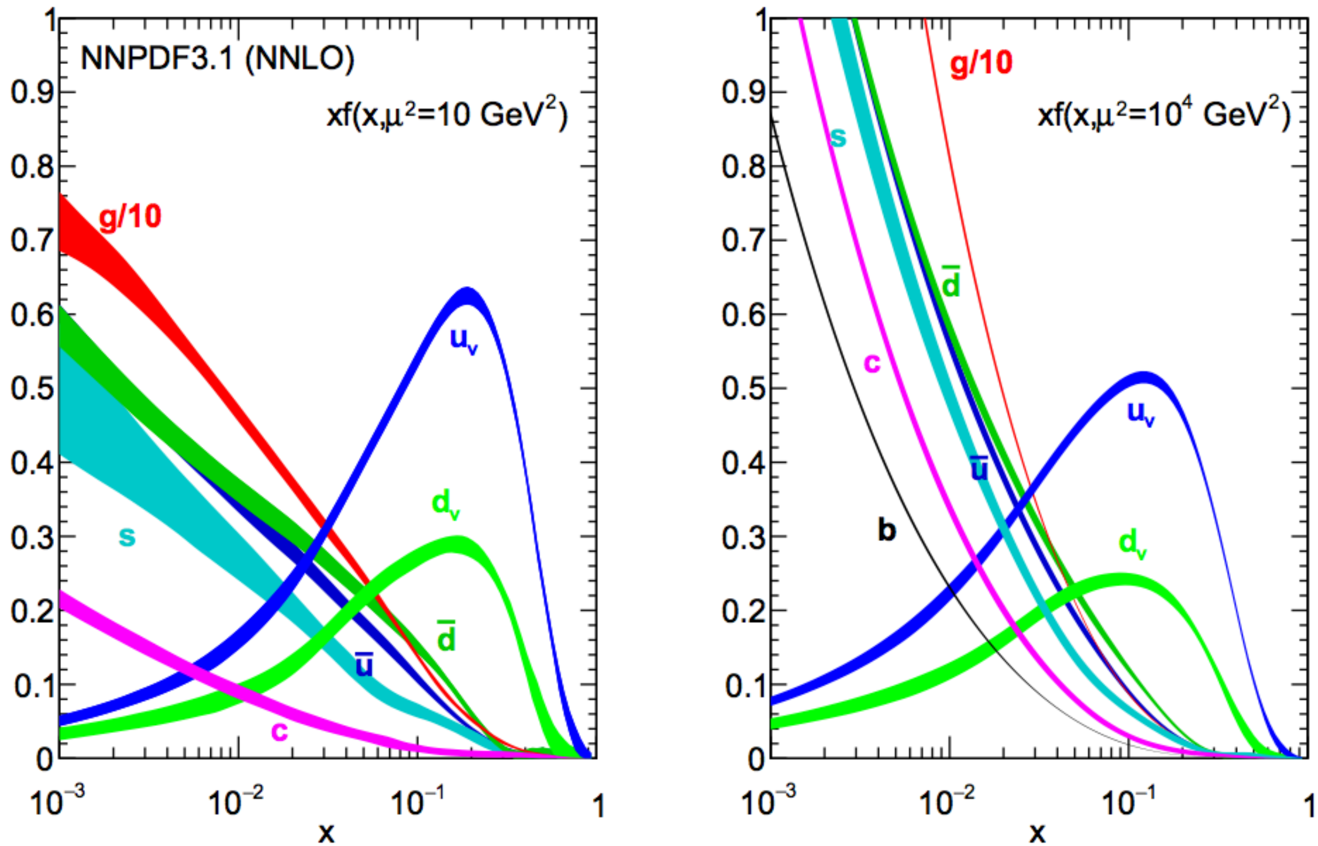
- Final state distribution is $\rho_f = \dfrac{M|P\rangle\langle P|M^\dagger}{\mathrm{Tr}(\rho_i\, M^\dagger M)} = \dfrac{\rho_i\, R^{prod}}{\mathrm{Tr}(\rho_i\, R^{prod})}$
  - Entanglement studies via fragmenting (specific hadrons or spin analyzers)

$$\mathrm{Tr}(\rho_f R^{decay}) = \frac{\mathrm{Tr}(\rho_i R^{prod} R^{decay})}{\mathrm{Tr}(\rho_i R^{prod})}$$

  - 'Maximal entanglement' and 'scattering enhanced entanglement' (ask me later)
    Cervera-Lierta, Latorre, Rojo & Rottoli (1703.02989)

Entanglement and distribution functions

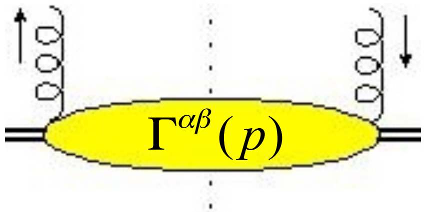
- PDFs are naturally, the 'momentum ensembles' upon selecting $x = p^+/P^+$ via hard interaction $\delta(x-x_B)$



- Gluons in entangled proton state: partonic entropy $S(x) = \ln(xg(x))$

Kharzeev & Levin, 1702.03489; see also 1903.07133 and 1904.11974

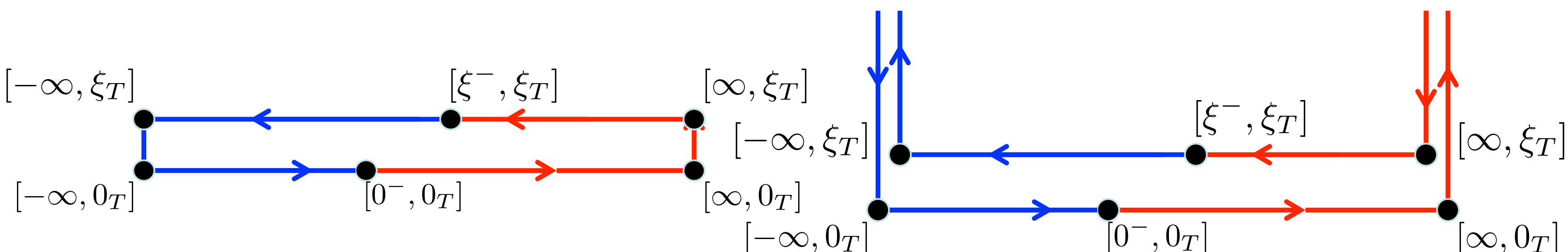
- Selection/probing through the quark and gluon operators in PDFs and FFs including nonlocal extensions and chiral or directional projections
  - $\overline{\psi}\gamma^+\psi = \psi_+^\dagger\psi_+$
  - $F^{+\alpha}F^{+\beta}$



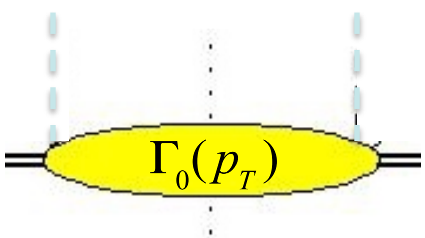
$$\epsilon^\alpha(k)\epsilon^{\beta *}(k) \Longrightarrow$$

$$\Gamma^{[U,U']\,\mu\nu}(x,p_T;n) = \int \frac{d\,\xi{\cdot}P\,d^2\xi_T}{(2\pi)^3}\,e^{ip\cdot\xi}\,\langle P|\,F^{n\mu}(0)\,U_{[0,\xi]}\,F^{n\nu}(\xi)\,U'_{[\xi,0]}\,|P\rangle\big|_{\xi\cdot n=0}$$

- Interesting is probing with something containing color: Wilson loops



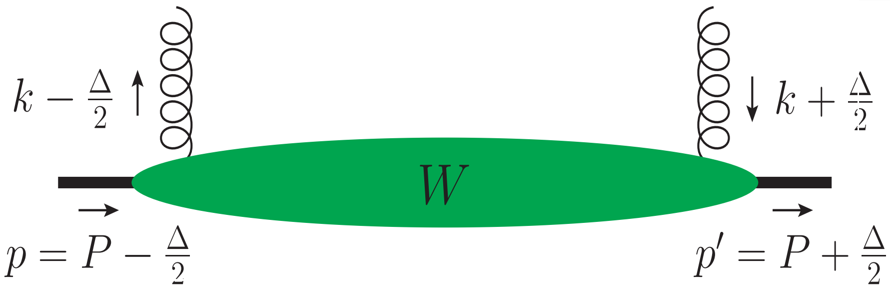
$$\Gamma_0^{[U,U']}(k_T;n) = \int \frac{d^2\xi_T}{(2\pi)^2}\,e^{ik\cdot\xi}\,\langle P|U_{[0,\xi]}\,U'_{[\xi,0]}\,|P\rangle\big|_{\xi=\xi_T}$$



- Diffractive probing

see also Tarasov & Venugopalan, 1903.11624

Gluon TMDs and Wilson loops via GTMDs



$x, y \longleftrightarrow z, b$

$$G^{[+,-]\alpha\beta}(x, k_T, \xi, \Delta_T) = 4 \int \frac{d^3z\, d^3b}{(2\pi)^3}\, e^{ik\cdot z - i\Delta\cdot b}\, \frac{\langle p'| F^{n\beta}(x) U^{[-]}_{[x,y]} F^{n\alpha}(y) U^{[+]}_{[y,x]} |p\rangle \Big|_{LF}}{\langle P|P\rangle}$$

x=ξ=0

$$G^{[+,-]\alpha\beta}(k_T, \Delta_T) = 16 \int \frac{d^2z\, d^2b}{(2\pi)^3}\, e^{ik_T\cdot z_T - i\Delta_T\cdot b_T}\, \frac{\langle p'| G^{\beta}_T(x) U^{[-]}_{[x,y]} G^{\alpha}_T(y) U^{[+]}_{[y,x]} |p\rangle \Big|_{LF}}{\langle P|P\rangle}$$

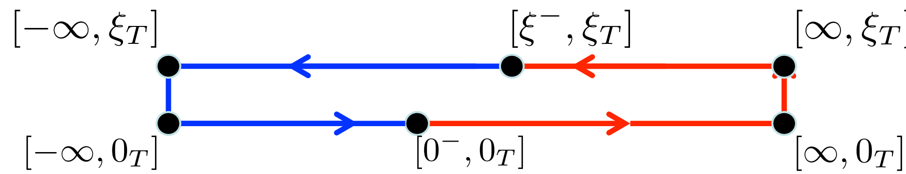
gluonic pole: $G^{\alpha}_T(x_T) = \int_{-\infty}^{\infty} dz^-\; U^{[-]}_{[0,x]} F^{n\alpha}(z^-, x_T) U^{[+]}_{[0,x]}$

$$\left[i\partial^{\alpha}_x, U^{[\pm]}_{[a,x]} \right] = \pm g U^{[\pm]}_{[a,x]} G^{\alpha}_T(x)$$

- Basis: Wilson loop GTMD

$$G^{[\Box]}(k_T, \Delta_T) = \int \frac{d^2z\, d^2b}{(2\pi)^4}\, e^{ik_T\cdot z_T - i\Delta_T\cdot b_T}\, \frac{\langle p'| \frac{1}{N_c} \mathrm{Tr}\left(U^{[\Box]}(x,y)\right) |p\rangle \Big|_{LF}}{\langle P|P\rangle}$$

Gluon TMDs and Wilson loops via GTMDs



$$F^{\alpha\beta} = \frac{\delta W[C]}{\delta \sigma_{\alpha\beta}}$$

- Thus we can use the (off-forward) Wilson loop GTMD,

$$G^{[\Box]}(k_T, \Delta_T) = \int \frac{d^2z\, d^2b}{(2\pi)^4}\, e^{ik_T\cdot z_T - i\Delta_T\cdot b_T}\, \frac{\langle p'|\, \frac{1}{N_c}\mathrm{Tr}\left(U^{[\Box]}(x,y)\right)|p\rangle\Big|_{LF}}{\langle P|P\rangle}$$

$$= \frac{\alpha_s}{2N_c\, M^2}\mathcal{E}(k_T^2, \Delta_T^2, k_T\cdot\Delta_T)$$

- ... to obtain the gluon GTMD at x = 0 as well as the gluon TMD at x = 0

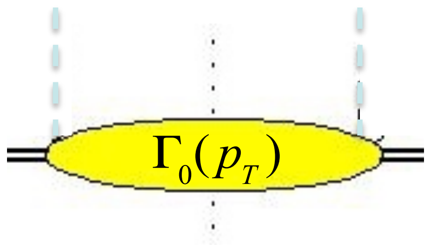
$$G^{[+,-]\alpha\beta}(k_T, \Delta_T) = \left[\frac{k_T^\alpha k_T^\beta}{M^2} - \frac{\Delta_T^\alpha \Delta_T^\beta}{4M^2} - \frac{k_T^{[\alpha}\Delta_T^{\beta]}}{2M^2}\right]\mathcal{E}(k_T^2, \Delta_T^2, k_T\cdot\Delta_T)$$

$$\stackrel{\Delta\to 0}{\Longrightarrow} \quad \frac{k_T^\alpha k_T^\beta}{M^2}\, e(k_T^2)$$

Gluon TMDs in unpolarized hadrons

- Wilson loop correlator

$$\Gamma_0^{[+,-]}(k_T) = \frac{1}{2M^2}\, e^{[+,-]}(k_T^2)$$

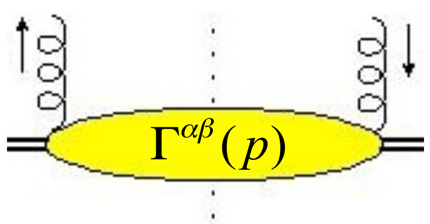


- Gluon (G)TMD at x=ξ=0

$$G^{[+,-]\alpha\beta}(k_T,\Delta_T) \stackrel{\Delta\to 0}{\Longrightarrow} \frac{k_T^\alpha k_T^\beta}{M^2}\, e^{[+,-]}(k_T^2)$$

- There are several gluon TMDs for unpolarized hadrons:

$$\Gamma^{\alpha\beta[U]}(x,k_T) = \frac{x}{2}\left\{-g_T^{\alpha\beta}\, f_1^{[U]}(x,k_T^2) + \frac{k_T^{\alpha\beta}}{M^2}\, h_1^{\perp\,[U]}(x,k_T^2)\right\}$$



- Small x behavior of (dipole) gluon TMDs

$$x\, f_1^{[+,-]}(x,k_T^2) \stackrel{x\to 0}{\Longrightarrow} \frac{k_T^2}{2M^2}\, e^{[+,-]}(k_T^2)$$

$$x\, h_1^{\perp[+,-]}(x,k_T^2) \stackrel{x\to 0}{\Longrightarrow} e^{[+,-]}(k_T^2)$$

- Naively, TMDs behave as 1/x, diverge for x → 0 (or less naively: small x behavior x^α with $1 < \alpha < 2$)

Gluon TMDs in polarized nucleon

- Polarized target (vector polarization)

$$\Gamma_L^{ij[U]}(x,k_T) = \frac{x}{2}\left\{ i\epsilon_T^{ij}\, S_L g_1^{[U]}(x,k_T^2) + \frac{\epsilon_{T\,\alpha}^{\{i}k_T^{j\}\alpha}}{M^2}\, S_L h_{1L}^{\perp\,[U]}(x,k_T^2)\right\}$$

$$\Gamma_T^{ij[U]}(x,k_T) = \frac{x}{2}\left\{ \frac{g_T^{ij}\epsilon_T^{kS_T}}{M}\, f_{1T}^{\perp[U]}(x,k_T^2) - \frac{i\epsilon_T^{ij}\,k_T\cdot S_T}{M}\, g_{1T}^{[U]}(x,k_T^2) - \frac{\epsilon_T^{k\{i}S_T^{j\}} + \epsilon_T^{S_T\{i}k_T^{j\}}}{4M}\, h_1(x,k_T^2) - \frac{\epsilon_{T\,\alpha}^{\{i}k_T^{j\}\alpha S_T}}{2M^3}\, h_{1T}^{\perp}(x,k_T^2)\right\}$$

- Cf. Wilson loop TMDs in polarized nucleon (no TMD for L polarization)

$$\Gamma_0(k_T) = \frac{1}{2M^2}\left\{ e(k_T^2) - \frac{\epsilon^{kS_T}}{M}\, e_T(k_T^2)\right\}$$

'pomeron' 'odderon'

Note on bounds:

$$\frac{|k_T|}{M}\,|e_T(k_T^2)| \le e(k_T^2)$$

Dominguez, Xiao, Yuan 2011; Hatta, Xiao, Yuan 2016

D Boer, MG Echevarria, PJM, J Zhou, PRL 116 (2016) 122001, ArXiv 1511.03485

D Boer, S Cotogno, T van Daal, PJM, A. Signori, Y Zhou, JHEP 1610 (2016) 013, ArXiv 1607.01654

S. Cotogno, T. van Daal, PJM, JHEP 1711 (2017) 185, ArXiv 1709.07827

Small x physics in terms of TMDs

- Dipole gluon TMDs: at small x only two structures for unpolarized and transversely polarized nucleons: pomeron & odderon structure

$$x\, f_1^{[+,-]}(x,k_T^2) \longrightarrow \frac{k_T^2}{2M^2}\, e^{[+,-]}(k_T^2)$$

$$x\, h_1^{\perp[+,-]}(x,k_T^2) \longrightarrow e^{[+,-]}(k_T^2)$$

$$x\, f_{1T}^{\perp[+,-]}(x,k_T^2) \longrightarrow \frac{k_T^2}{2M^2}\, e_T^{[+,-]}(k_T^2)$$

$$x\, h_1^{[+,-]}(x,k_T^2) \longrightarrow \frac{k_T^2}{2M^2}\, e_T^{[+,-]}(k_T^2)$$

$$x\, h_{1T}^{\perp[+,-]}(x,k_T^2) \longrightarrow e_T^{[+,-]}(k_T^2)$$

- Circularly polarized gluons in transversely polarized nucleons and TMDs in longitudinally polarized nucleons: Naively: TMDs tend to zero for x → 0 (less naively: small x behavior x^α with $\alpha < 1$)

D Boer, S Cotogno, T van Daal, PJM, A. Signori, Y Zhou, JHEP 1610 (2016) 013, ArXiv 1607.01654

Nuclear measurements of GTMDs

- Back to unpolarized GTMDs (at x=ξ=0)

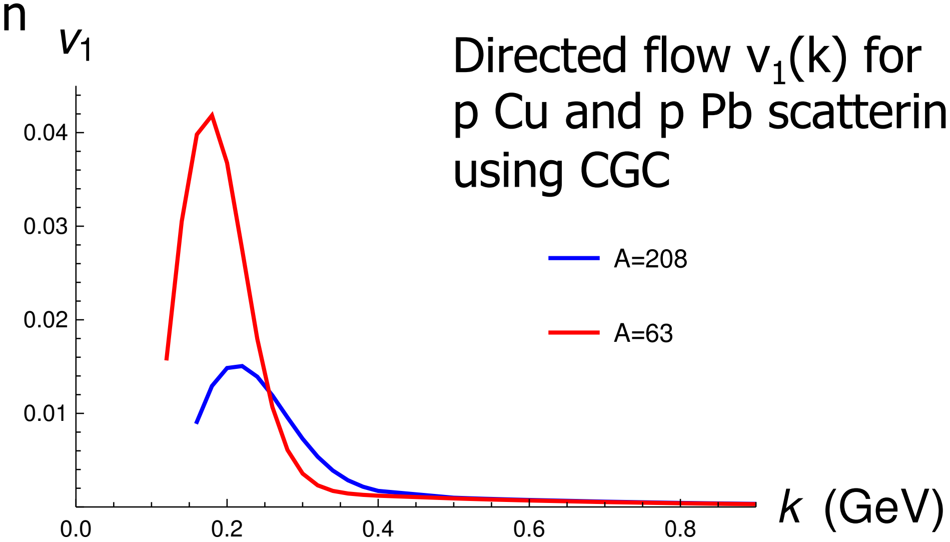
$$G^{[+,-]\alpha\beta}(k_T,\Delta_T) = \frac{2N_c}{\alpha_s}\left[k_T^\alpha k_T^\beta - \frac{1}{4}\Delta_T^\alpha\Delta_T^\beta - \frac{1}{2}k_T^{[\alpha}\Delta_T^{\beta]}\right] G^{[\Box]}(k_T,\Delta_T)$$

- Can be complex

$$\text{Hermiticity: } G^{[\Box]*}(k_T,\Delta_T) = G^{[\Box]}(k_T,-\Delta_T)$$

$$\text{Time reversal: } G^{[\Box]*}(k_T,\Delta_T) = G^{[\Box^\dagger]}(-k_T,-\Delta_T)$$

- Odderon (C-odd) contribution in $G^{[\Box]}(k_T,\Delta_T)$ only has odd powers in $k_T.\Delta_T$ and vanishes in the forward limit
- As b_T (FT of Δ_T) and z_T (FT of k_T) involve different scales in a nucleus (nuclear for b_T and nucleonic for z_T) the odderon part could show up, e.g. in ultra - peripheral pA scattering via azimuthal asymmetries (directed flow)
  - Needs cubic term in CGC action
  - Leads to $1/A^{1/3}$ effect



Directed flow $v_1(k)$ for p Cu and p Pb scattering using CGC

D Boer, PJM, E. Petreska, JHEP 1807 (2018) 140, ArXiv 1805.05219

ENTANGLEMENT OF PARTONS?

Fragmentation

- Fragmentation also leads to an ensemble that (within the constraints such as charge, flavor, spin) appears to maximize entropy.

  F Berges, S Floerchinger, R Venugopalan, NPA 982 (2019) 819

- Possibly explains success of statistical hadronization model
  - not due to thermalization by collisions
  - note presence of ${}^{3}_{\Lambda}\mathrm{H}$ (> 5 fm)

    A Andronic, P Braun-Munzinger, K Redlich, J Stachel, 1710.09425

- Conjecture: Entanglement of partons! Involving color as well as space-time freedom!

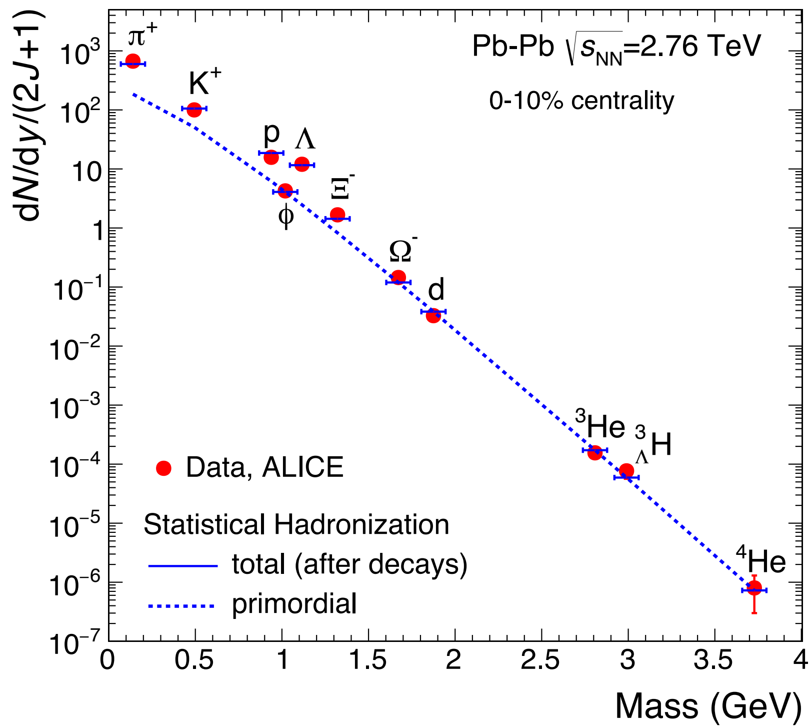


FIG. 2. Mass dependence of hadron yields compared with predictions of the statistical hadronization model. Only particles, no anti-particles, are included and the yields are divided by the spin degeneracy factor $(2J+1)$. Data are from the ALICE collaboration for central Pb–Pb collisions at the LHC. For the statistical hadronization approach, plotted are the "total" yields, including all contributions from high-mass resonances (for the Λ hyperon, the contribution from the electromagnetic decay $\Sigma^0 \to \Lambda\gamma$, which cannot be resolved experimentally, is also included), and the ("primordial") yields prior to strong and electromagnetic decays. For more details see text.

Statistical model for hadronization

proton-proton scattering

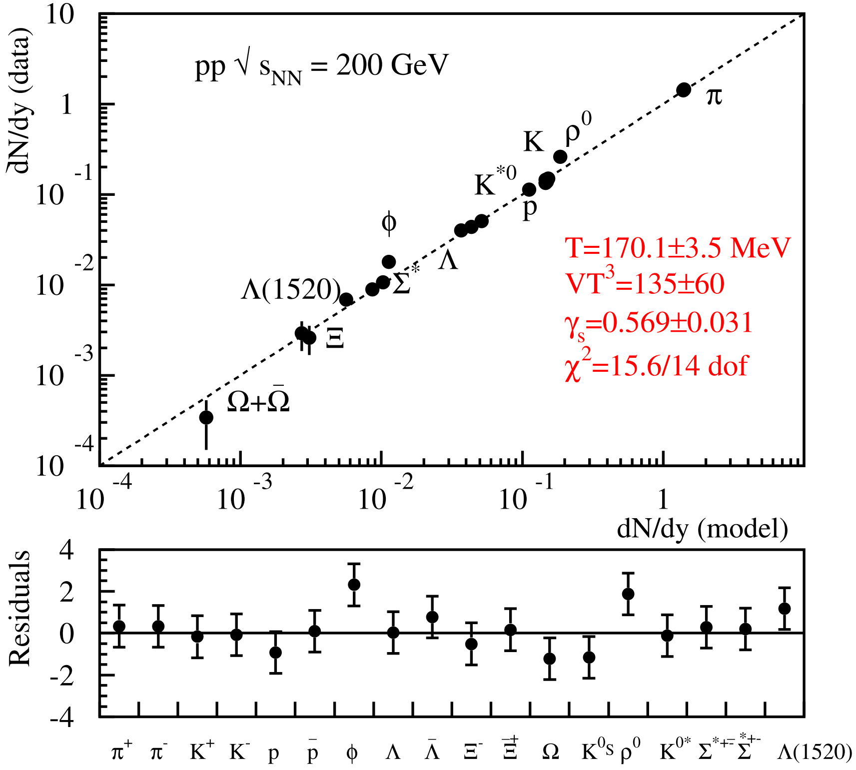
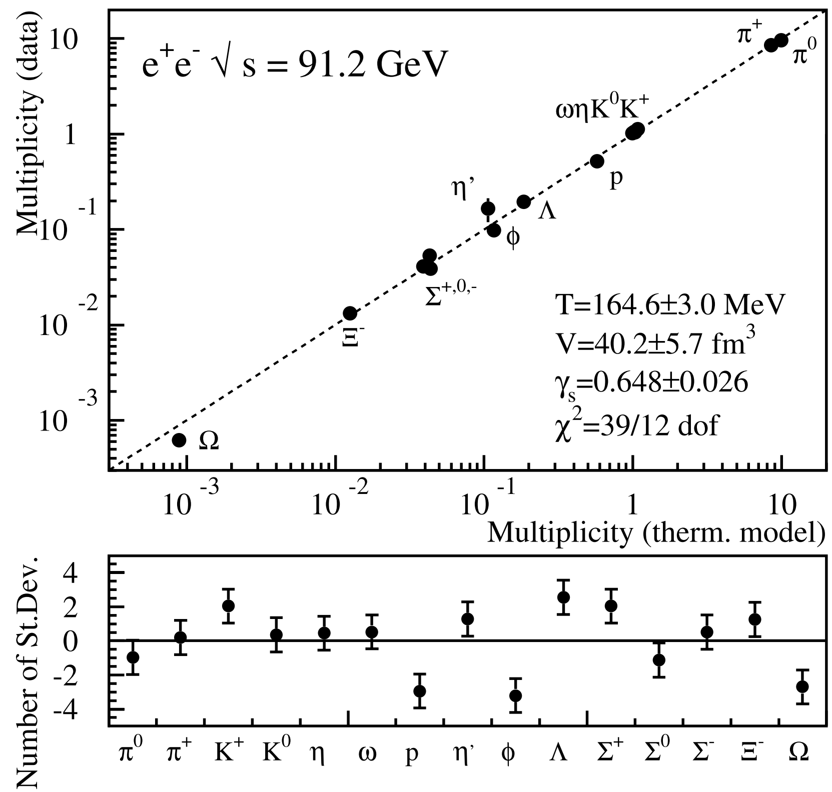


Fig. 1 *Above*: fitted vs. measured midrapidity densities in *pp* collisions at $\sqrt{s} = 200$ GeV. *Below*: residual distributions

electron-positron annihilation



F Becattini, P Castorina, J Manninen and H Satz, EPJ C 56 (2008) 493

F Becattini, P Castorina, A Milov and H Satz, EPJ C 66 (2010) 377

Statistical model for hadronization

- Precise details of model not so relevant

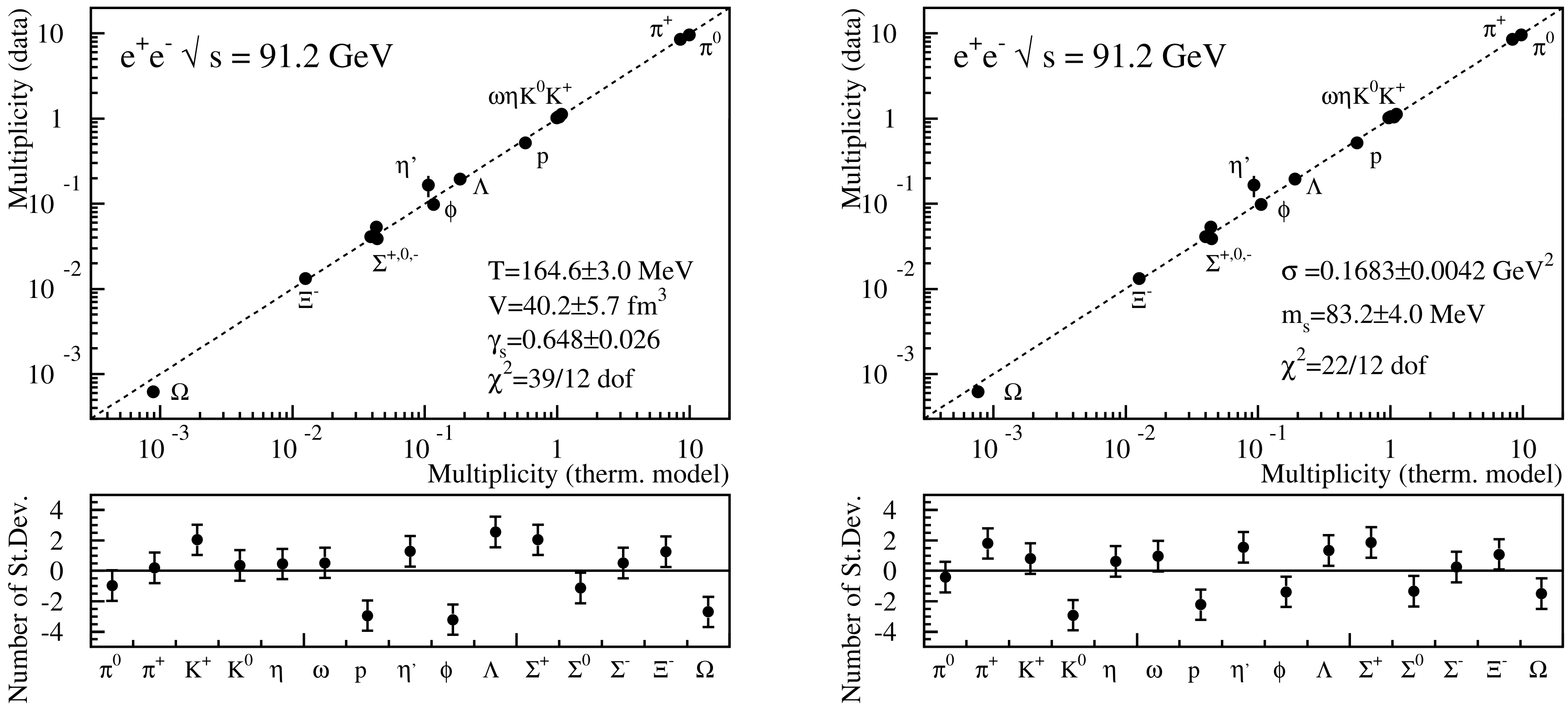


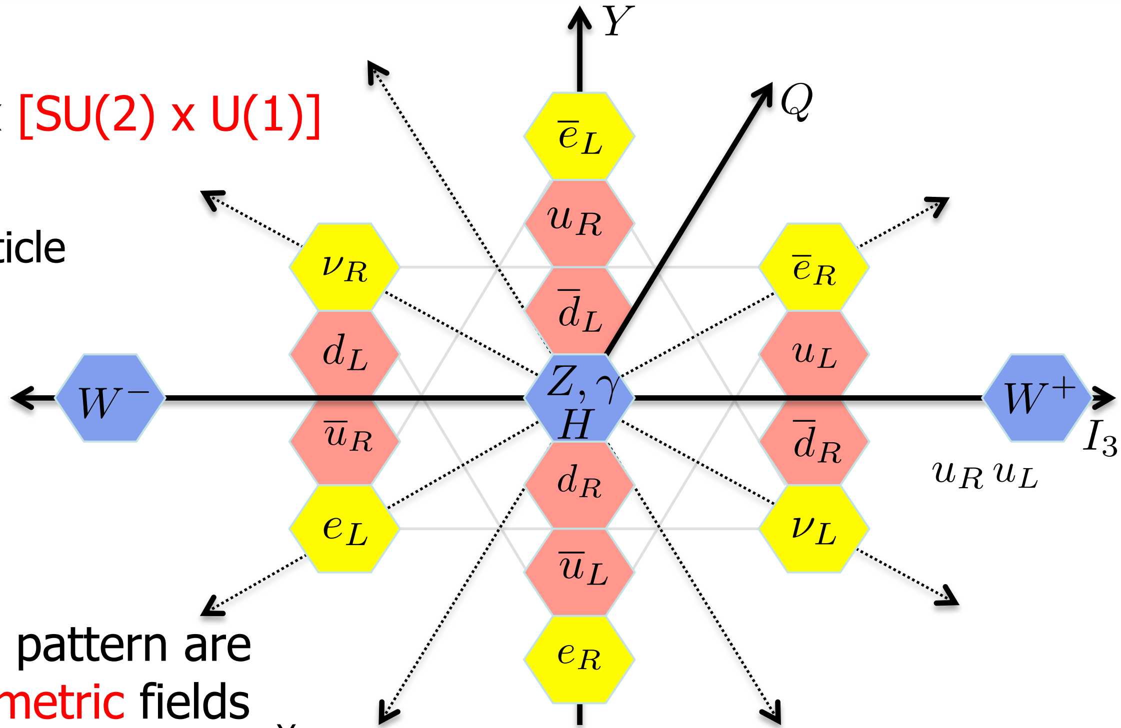
Fig. 4 Comparison between measured and fit multiplicities of long-lived hadronic species in e^+e^- collisions at $\sqrt{s} = 91.25$ GeV. *Left*: statistical hadronization model with one temperature. *Right*: Hawking–Unruh radiation model

F Becattini, P Castorina, J Manninen and H Satz, EPJ C 56 (2008) 493

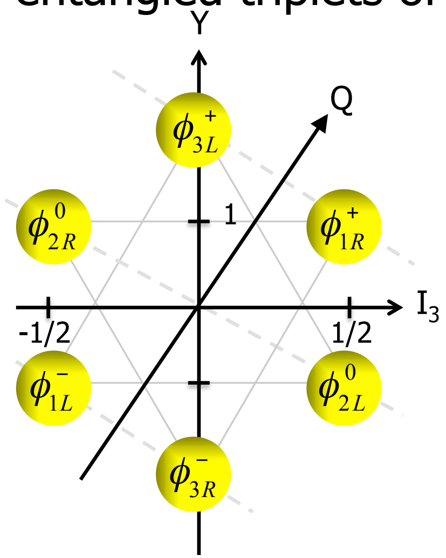
EMERGENT GAUGE SYMMETRIES

entanglement of quarks and leptons (another perspective)

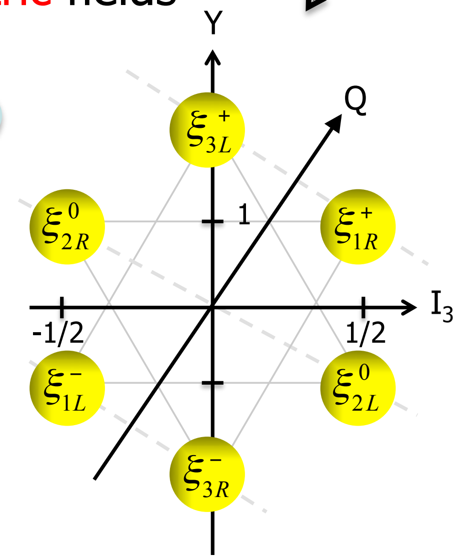
- Standard model:
  - symmetry pattern: SU(3) x [SU(2) x U(1)]
  - but … complex
    - right-left, particle-antiparticle
    - color
    - families
- Emergence?



- Conjecture: at the basis of this pattern are entangled triplets of supersymmetric fields



A x B x C



Emergent symmetries in a SUSY setting

- Hilbert space (0D field theory)
  $\{(a^\dagger)^n|0\rangle, b^\dagger|0\rangle\}$ $\qquad$ $[a, a^\dagger] = 1, \; \{b, b^\dagger\} = 1$
- Supercharges
  $Q^\dagger_{ik} = b_i\, a^\dagger_k$ and $Q_{ik} = b^\dagger_i a_k$ $\qquad$ $\{Q^\dagger_{ik}, Q_{jl}\} = \frac{1}{2}\,\delta_{ij}\{a^\dagger_l, a_k\} + \frac{1}{2}\,\delta_{kl}[b^\dagger_i, b_j]$

  $a^\dagger_k \xrightarrow{Q_{ik}} b^\dagger_i \qquad a^\dagger_k \xleftarrow{Q^\dagger_{ik}} b^\dagger_i$

  hamiltonian/number operators (i=j, k=l)
  & unitary rotations
- For boson and fermion fields
  $$\varphi = \frac{1}{\sqrt{2\omega}}(a + a^\dagger) \quad \text{and} \quad \xi = \frac{1}{\sqrt{2}}(b + b^\dagger)$$
  $$Q = \sqrt{\omega}(a^\dagger b - b^\dagger a)$$

$$[Q, \varphi] = \xi \qquad \{Q, \xi\} = \{Q, [Q, \varphi]\} = F = iD\varphi$$
$$[Q, F] = [Q, \{Q, \xi\}] = iD\xi$$

Single (free) field
$$F = [\varphi, H]$$
$$= iD\varphi = i\dot{\varphi}$$

$$iD = i\partial + gA$$
↑ unitary rotations

- Implement symmetries via constraints F
  ... and a nontrivial vacuum (not everything is for free!)

$$\phi(t) = \mathcal{T} \exp\left(-i\int_0^t ds\cdot D\right)\phi$$

Emerging symmetries and space-time

Fields	Generators Space-time	&	Internal multipartite setting
■ Real/Majorana: $\phi \quad \xi \quad \text{and } \langle\phi\rangle = 1$	■ H		
■ $\phi_{R/L} \quad \xi_{R/L} \quad \text{and } \langle\phi_R\rangle = \langle\phi_L\rangle = 1/\sqrt{2}$ (Wess-Zumino)	■ P^+, P^-		K, <u>SU(3)</u>

$U(1)_R$ x $U(1)_L$ x SU(3)

Emerging symmetries and space-time

Fields	Generators Space-time & Internal	
■ Real/Majorana: $\phi \quad \xi \quad \text{and } \langle\phi\rangle = 1$	■ H	
■ $\phi_{R/L} \quad \xi_{R/L} \quad \text{and } \langle\phi_R\rangle = \langle\phi_L\rangle = 1/\sqrt{2}$ (Wess-Zumino → gauge theory)	■ P+, P–	K, SU(3)
		Z(2)
■ 1D: $\phi_S \quad \phi_P \longrightarrow A_3^a \quad \psi$	■ H, P, K	SU(3)

$$iD_\sigma\psi^i = i\partial_\sigma\psi^i + g_0 \sum_{a=1,\ldots 8} A_\sigma^a (T_a)^i_j \psi^j$$

P(1,1) x SU(3)

Emerging symmetries and space-time

Fields

- Real/Majorana: $\phi \quad \xi \quad \text{and } \langle\phi\rangle = 1$
- $\phi_{R/L} \quad \xi_{R/L} \quad \text{and } \langle\phi_R\rangle = \langle\phi_L\rangle = 1/\sqrt{2}$
  (Wess-Zumino → gauge theory)
- 1D: $\phi_S \quad \phi_P \longrightarrow A_3^a \quad \psi$

$$iD_\sigma\psi^i = i\partial_\sigma\psi^i + g_0 \sum_{a=1,\ldots 8} A_\sigma^a (T_a)^i_j \psi^j$$

- 3D: $\phi_S \quad A_k^a \quad \psi$

$$iD_\mu\psi^i = i\partial_\mu\psi^i + g \sum_{a=1,2,3,8} A_\mu^a (T_a)^i_j \psi^j$$

Generators

Space-time & Internal

- H
- P⁺, P⁻ — K, <u>SU(3)</u>

Z(2)

- H, P, K — <u>SU(3)</u> = [<u>SO(3)</u>,<u>SU(2)xU(1)</u>]

Z(3)

- H, **P**, **K**, **J** — <u>SU(2)xU(1)</u>

P(1,3) x SU(2) x U(1)

Emerging symmetries and space-time

Fields

- Real/Majorana: $\phi \quad \xi$ and $\langle\phi\rangle = 1$
- $\phi_{R/L} \quad \xi_{R/L}$ and $\langle\phi_R\rangle = \langle\phi_L\rangle = 1/\sqrt{2}$
  (Wess-Zumino → gauge theory)
- 1D: $\phi_S \quad \phi_P \longrightarrow A_3^a \quad \psi$

$$iD_\sigma\psi^i = i\partial_\sigma\psi^i + g_0 \sum_{a=1,\ldots 8} A_\sigma^a (T_a)^i_j \psi^j$$

- 3D: $\phi_S \quad A_k^a \quad \psi$

$$iD_\mu\psi^i = i\partial_\mu\psi^i + g \sum_{a=1,2,3,8} A_\mu^a (T_a)^i_j \psi^j$$

- and ....

$$n_\pm^\sigma \longrightarrow n_\alpha^\mu \qquad \gamma^\sigma = \begin{bmatrix} 0 & n_-^\sigma \\ n_+^\sigma & 0 \end{bmatrix} \longrightarrow \gamma^\mu = \begin{bmatrix} 0 & \bar{\sigma}^\mu \\ \sigma^\mu & 0 \end{bmatrix}$$

in order to match space-time and field symmetries and respect Coleman-Mandula

Generators

Space-time & Internal

- H
- P^+, P^- — K, SU(3)
- Z(2)
- H, P, K — SU(3) = [SO(3),SU(2)xU(1)]
- Z(3)
- H, **P**, **K**, **J** — SU(2)xU(1)

P(1,3) x SU(2) x U(1)

A(4)

DYNAMICS

Dynamics

- Right-Left symmetry
- Supersymmetry (Wess-Zumino structure)
  - Bosons: $\phi\sqrt{2} = e^{i\pi/4}\phi_R + e^{-i\pi/4}\phi_L$ $= \phi_S + i\phi_P = \chi\, e^{i\theta}$
  - Fermions: $\xi\sqrt{2} = \begin{bmatrix} \xi_R \\ -i\,\xi_L \end{bmatrix}$
- Wess-Zumino in 1+1 dim including potential and vev determining symmetry

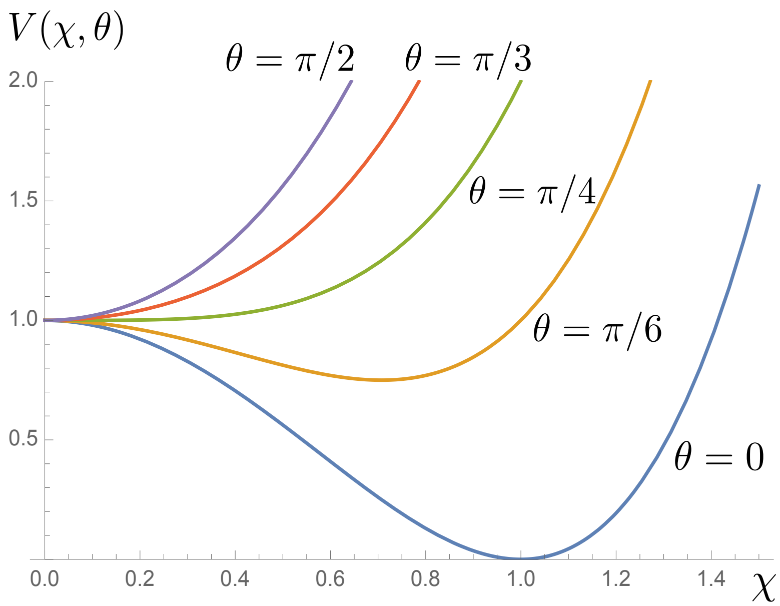
$$V(\phi) = \tfrac{1}{8}M^2\left(4\phi_S^2\phi_P^2 + (1-\phi_S^2+\phi_P^2)^2\right)$$
$$= \tfrac{1}{8}M^2\left(\chi^4\sin^2(2\theta) + (\chi^2\cos(2\theta)-1)^2\right)$$

- Pseudoscalar fields (θ) → gauge fields

$$\phi^\dagger\partial_\sigma\phi = \tfrac{1}{2}\,\chi^T D_\sigma\chi$$

- ... + masses through symmetry breaking
- Link to gravity (?) via constraint

$$\lambda\left(\chi^2\cos^2(2\theta)-1\right)$$



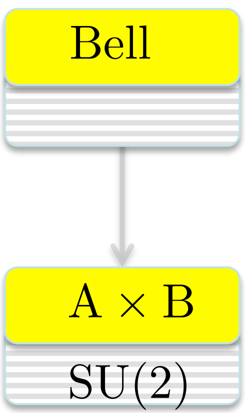
SO(3) symmetry via vev

FERMIONS AND BOSONS AS TRIPARTITE STATES

Bipartite entangled states

- Bell states are maximally entangled (MaxEnt) states in product space $\mathcal{H}^A \otimes \mathcal{H}^B$:
  $|RR\rangle + e^{i\varphi}|LL\rangle$ or $|RL\rangle + e^{i\varphi}|LR\rangle$

- They belong to the same class (SLOCC, for us local unitary, local = subspace)



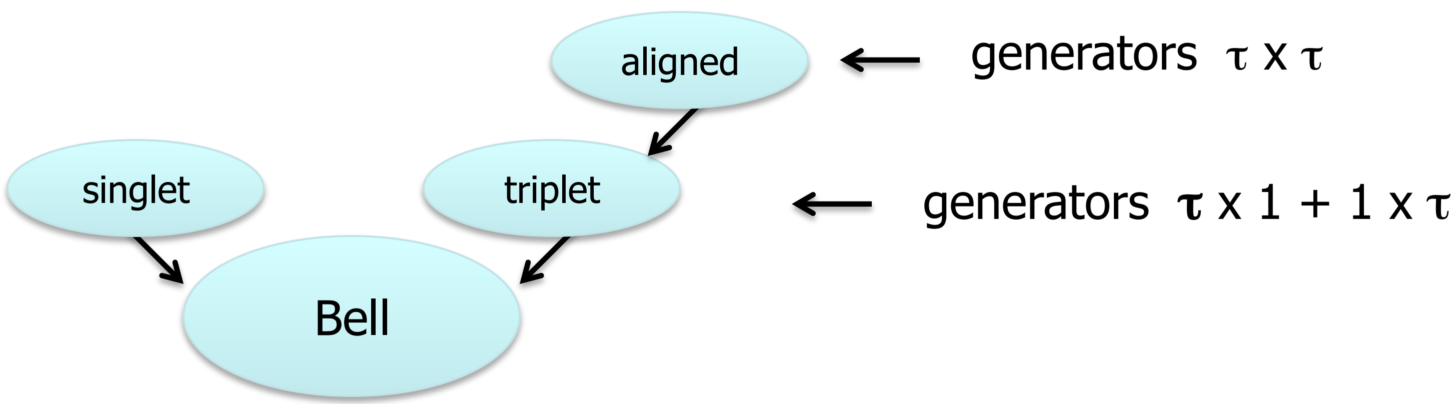
$$\rho = |\text{Bell}\rangle\langle\text{Bell}| \implies \rho_A = \tfrac{1}{2}\left(|R\rangle\langle R| + |L\rangle\langle L|\right)$$

$$|\Phi\rangle = a|RR\rangle + b|RL\rangle + c|LR\rangle + d|LL\rangle$$
$$= \sqrt{p_1}|a_1 b_1\rangle + \sqrt{p_2}|a_2 b_2\rangle \quad \text{(Schmidt decomp.)}$$

entanglement measure:

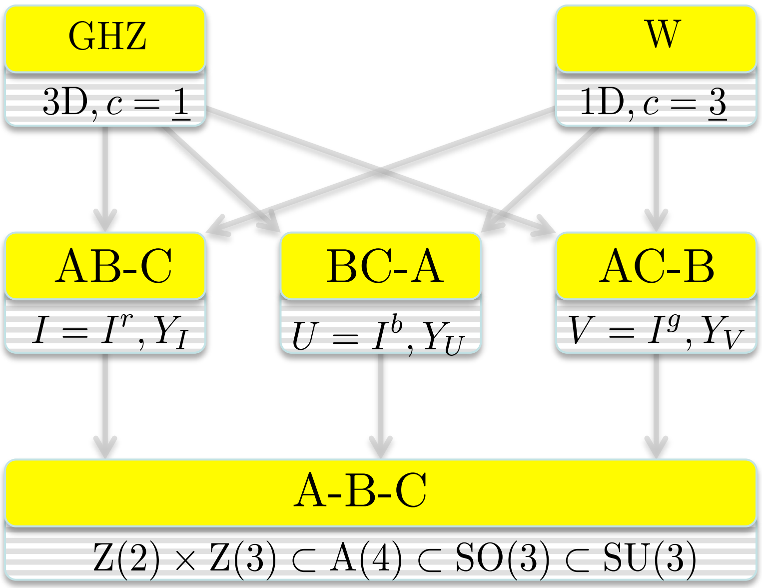
$$0 \le \Delta = \sqrt{2(1 - \mathrm{Tr}(\rho^2))} = 2|ad - bc| \to 2\sqrt{p_1 p_2} \le 1$$

- Symmetry eigenstates can be aligned or entangled



Tripartite entangled chiral states

- Two classes of maximally entangled tripartite ABC states: (Dur, Vidal, Cirac 2000)



$$|\mathrm{GHZ}\rangle = \frac{1}{\sqrt{2}}(|RRR\rangle + |LLL\rangle)$$

$$\rho_{AB} = \frac{1}{2}\left(|RR\rangle\langle RR| + |LL\rangle\langle LL|\right)$$

GHZ: fragile

$$|\mathrm{W}\rangle = \frac{1}{\sqrt{3}}(|LRR\rangle + |RLR\rangle + |RRL\rangle)$$

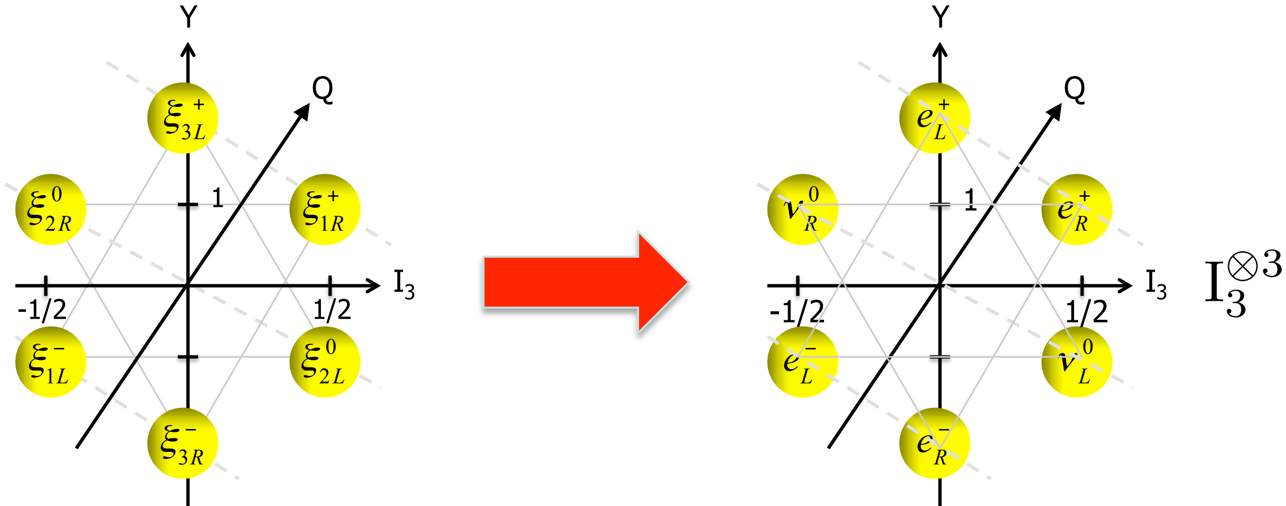
$$\rho_{AB} = \frac{2}{3}|\mathrm{Bell}\rangle\langle\mathrm{Bell}| + \frac{1}{3}|RR\rangle\langle RR|$$

W: robust

- Beyond tripartites there is an infinite number of classes!

Leptons

- GHZ class, $|\mathrm{GHZ}\rangle = \frac{1}{\sqrt{2}}(|RRR\rangle + |LLL\rangle)$ has same symmetry as basis (in particular chirality)



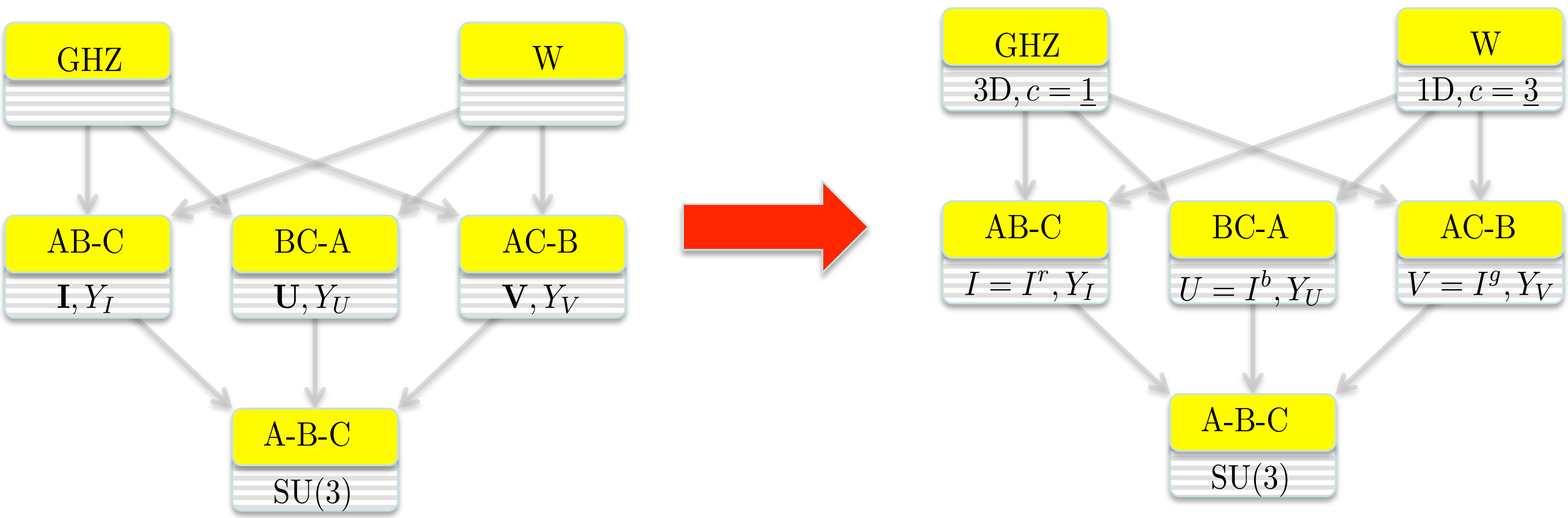
- Using $t_I \equiv (\mathbf{I}, Y_I)$ note that $t_I \otimes t_U \otimes t_V$ is LU equivalent to $t_I \otimes t_I \otimes t_I$ and the aligned GHZ states can be SO(3) multiplets (living in 3D) identified with leptons (cf HO)
- Embedding symmetry A(4) has three singlet representations: families
- This gives tri-bimaximal family – electroweak mixing [slightly different from the way obtained by Fritsch & Xing, or Harrison, Perkins & Scott]

$$U_{\mathrm{TB}} = WU_Q = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ \omega^2 & 1 & \omega \\ \omega & 1 & \omega^2 \end{bmatrix} \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 & -i \\ 0 & 1 & 0 \\ 1 & 0 & i \end{bmatrix} = \begin{bmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{bmatrix}$$

Quarks

- For W-class chirality is more complex
- again employ SU(3) and SU(2) x U(1) subgroups (I, U, V) in bipartite classes
- $t_I \otimes t_U \otimes t_V \rightarrow t_I^r \otimes t_I^g \otimes t_I^b$

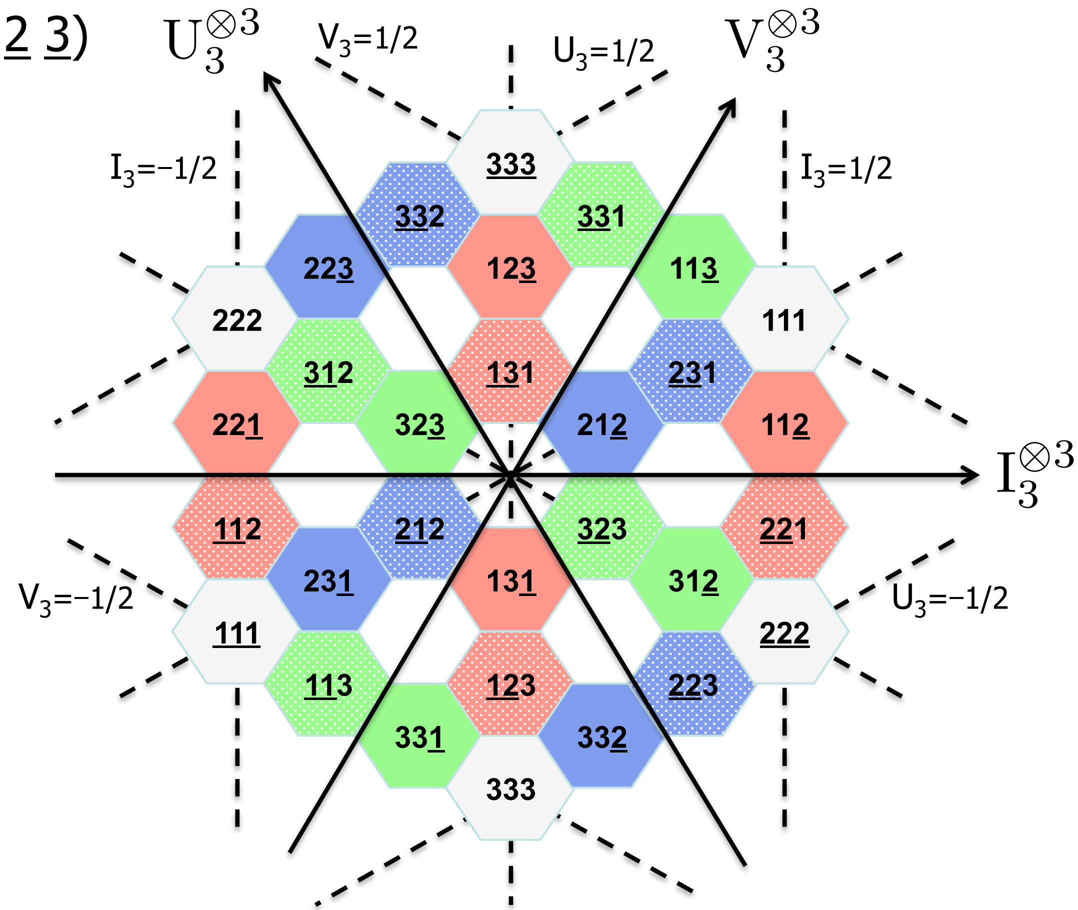
$$|\mathrm{GHZ}\rangle = \frac{1}{\sqrt{2}}(|RRR\rangle + |LLL\rangle)$$
$$|\mathrm{W}\rangle = \frac{1}{\sqrt{3}}(|LRR\rangle + |RLR\rangle + |RRL\rangle)$$



- A(4) symmetry: three singlets and three triplets
- Construct SU(3) root diagram to see all GHZ- and W-states

(fermionic) root diagram: tripartite entanglement

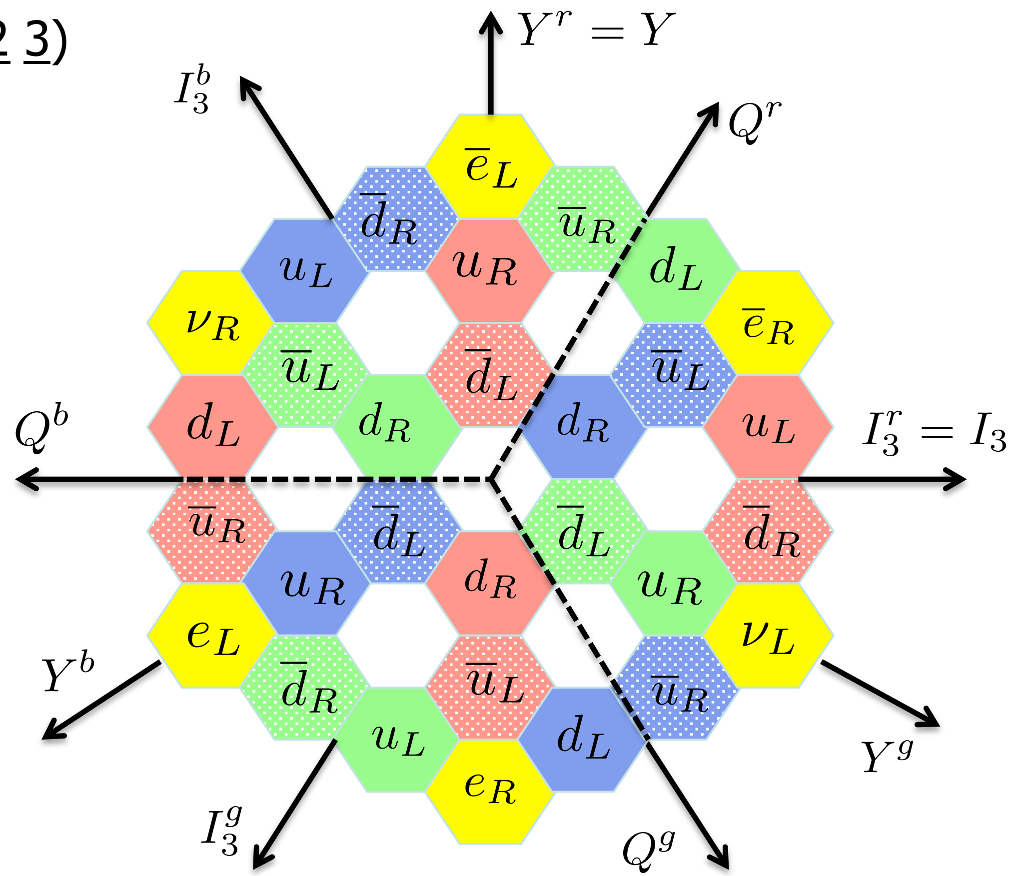
- Tripartite states (R: 1 2 3 & L: 1 2 3)
- Aligned (RRR, LLL): GHZ states
  - I, U, *and* V allowed
  - SO(3) → asymptotic/space
  - Three A(4) singlets
- Mingled (RRL, RLL): W-states
  - I, U, *or* V allowed
  - non-asymptotic
  - Three A(4) triplets (color)



(fermionic) root diagram: electroweak identification

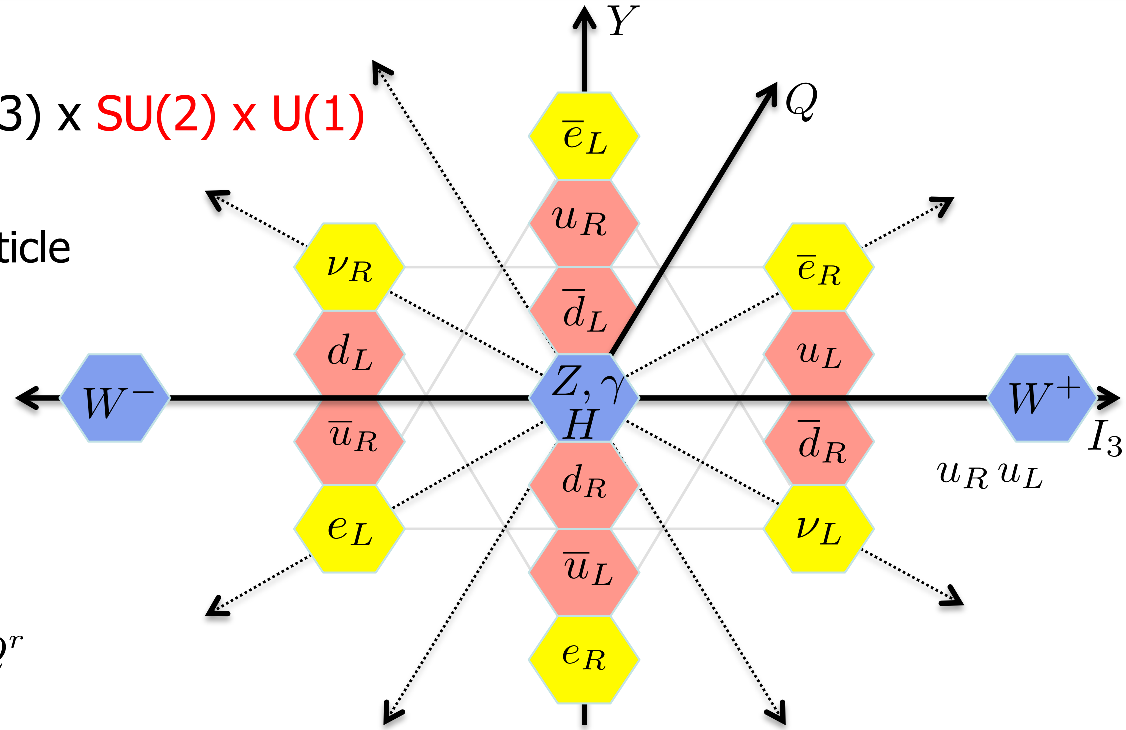
- Tripartite states (R: 1 2 3 & L: $\underline{1}$ $\underline{2}$ $\underline{3}$)
- Aligned (RRR, LLL): LEPTONS
  - I, U, *and* V allowed
  - SO(3) → asymptotic/space
  - Three A(4) singlets
- Mingled (RRL, RLL): QUARKS
  - I, U, *or* V allowed
  - non-asymptotic
  - Three A(4) triplets (color)
- Resembles the rishon model

Harari & Seiberg 1982, Shupe 1979



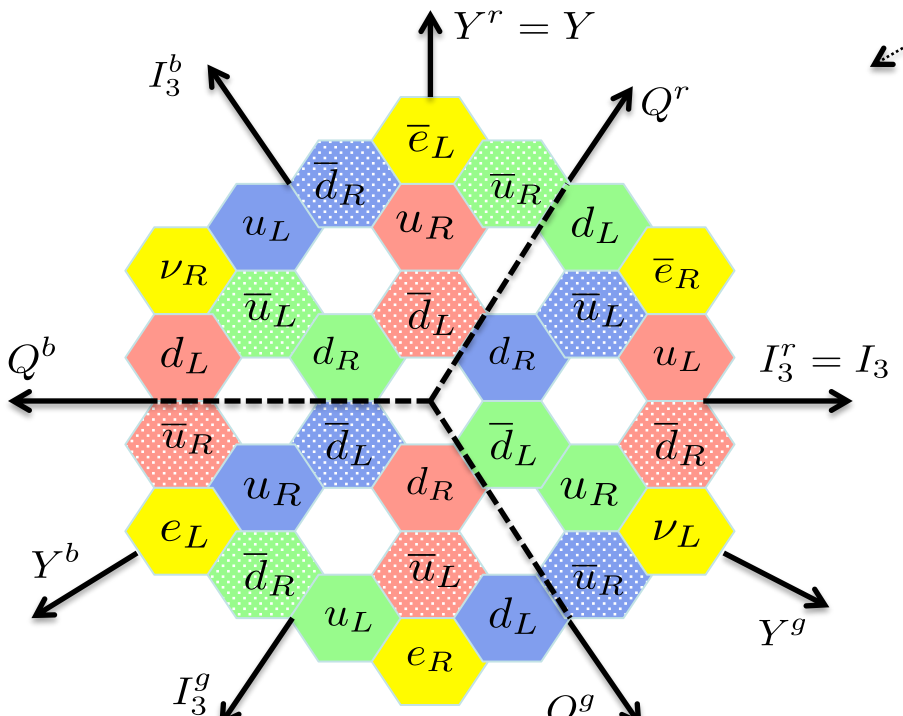
Entanglement offers new perspective quarks and leptons

- Standard model:
  - beautifully symmetric: SU(3) x SU(2) x U(1)
  - but … complex:
    - right-left, particle-antiparticle
    - color
    - families
- Emergence ...
- ... in a tripartite Hilbert space



SU(2) x U(1) in SU(3) → sin Θ_W = ½

Weinberg 1972



triality Z(3) ...
reflected in family structure & color

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Bosons

- Boson fields appear as Higgs field and in covariant derivatives:

$$\phi\sqrt{2} = \chi\, e^{i\theta} \qquad \phi^\dagger \partial_\sigma \phi = \frac{1}{2}\chi^T D_\sigma \chi$$

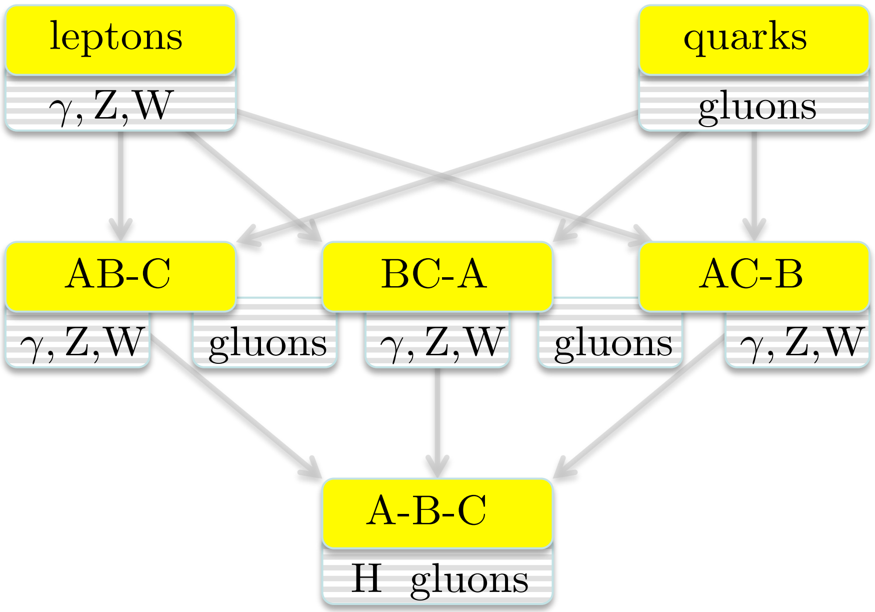
- Depending on implementation:

1D $$iD_\sigma \psi^i = i\partial_\sigma \psi^i + g_0 \sum_{a=1,\ldots 8} A^a_\sigma (T_a)^i_j \psi^j$$

3D $$iD_\mu \psi^i = i\partial_\mu \psi^i + g \sum_{a=1,2,3,8} A^a_\mu (T_a)^i_j \psi^j$$

- Gauge fields linked to symmetry generators

- More or less like SM starting with
  - $\sin\theta_W = 1/2$
  - $M_Z\sqrt{2} = M_H = M_{\rm top}/\sqrt{2}$
  - Need for radiative corrections

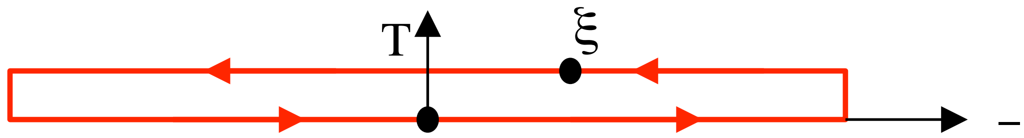


Entanglement and 3D composites (preliminary)

- Strong Interactions: resembles $XQCD_{1+1}$ (analogous to Kaplan 1306.5818), while dynamics governed via Wilson loop (including freezing of color at small x/high energies)

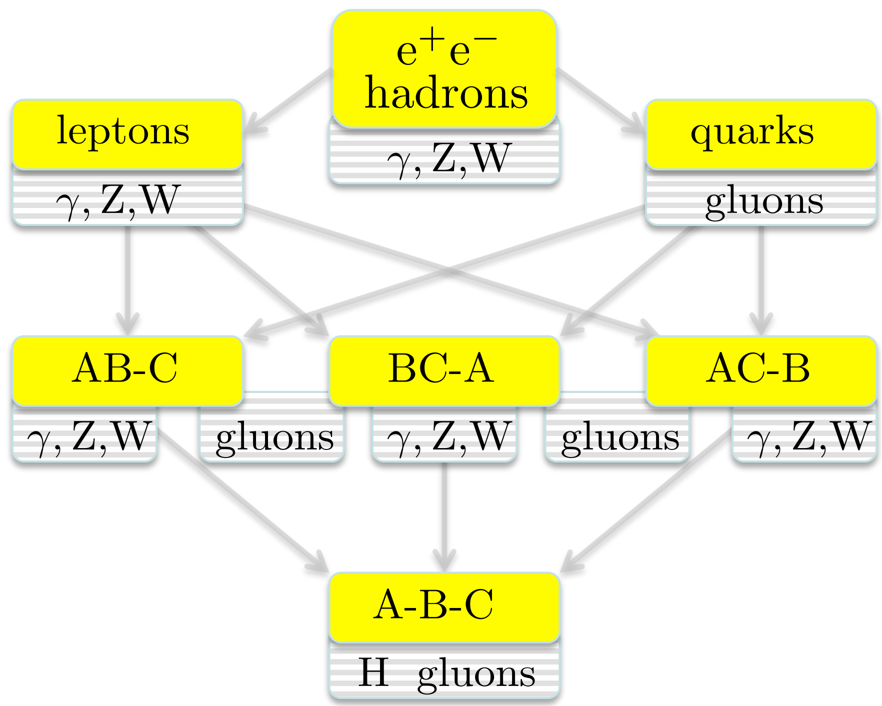
$$W[C] = \exp\left(-ig \oint_C ds^\mu A_\mu(s)\right)$$

$$gF_{\tau\sigma} = \delta W[C] / \delta\sigma^{\tau\sigma}$$



- Elementary constituents only involve entanglement in discrete R/L and 1,2,3 degrees of freedom; composites involve entanglement in the continuous space-time freedom

- Quark-entangled states form hadrons that are global SU(3) color singlets appearing in 3D (more or less following the rules of the NRQM)
  - valence – current quarks (ontological basis choice)
- More on QCD:
  - collinearity and TMD
  - light-front dominance, OPE
  - jets, SCET
  - AdS/CFT
  - color-kinematic dualities



Concluding remarks

- Entanglement of hadrons: PDFs
  - Simplifications of gluon TMDs at small x: linearly polarized gluon distributions in unpolarized and transversely polarized nucleons dominate over circularly polarized distributions.
  - Wilson loop matrix elements crucial in low x domain.
  - Access to C-odd matrix elements via nuclear density profiles

- Fragmentation linked to entanglement of partons

- Emergent gauge symmetries
  - Use of supercharges and emergence of space-time symmetries
  - Classes of tripartite entanglement: GHZ and W
  - Leptons and quarks in different classes with different implementations of basic symmetries