Transverse Force Tomography

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disentangling $x-\xi$ dependence of GPDs from QCD evolution

- $\Re\mathcal{A}_{DVCS} \longrightarrow \int dx \frac{GPDs(x,\xi,t)}{x\pm\xi}$
- $\Im\mathcal{A}_{DVCS} \longrightarrow GPDs(\xi,\xi,t)$
- polynomiality insufficient to uniquely extract $GPDs(x,\xi,t)$
- additional input needed to disentangle $x-\xi$ dependence

$\hookrightarrow$ Q^2 dependence described by known evolution kernels!

$\hookrightarrow$ use Q^2 dependence of $\mathcal{A}_{DVCS}$ to further constrain $GPDs(x,\xi,t)$

- twist-3 effects may need to be included in such a program!

nucleon structure

- twist-2 observables tell us what nucleon structure is
- twist-3 helps us understand what makes nucleon structure

transverse force tomography (this talk)

- twist-3 PDFs: $d_2 \longrightarrow \perp$ force
- twist-3 GPDs: $\perp$ position space resolved force

weird stuff going on at twist 3 $\longrightarrow$ Fatma Aslan

- discontinuities in GPDs
- $\delta(x)$ in PDFs

yes, measuring twist-3 GPDs will be hard, but...

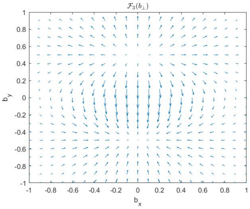
- lattice QCD can provide (genuine) twist 3 info much sooner

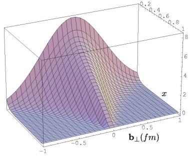
Outline

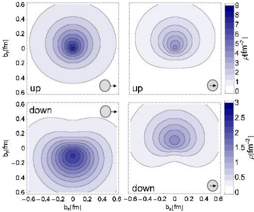
- motivation
- twist-3 PDFs $\longrightarrow$ 'the force'
- GPDs $\longrightarrow$ 3D imaging of the nucleon

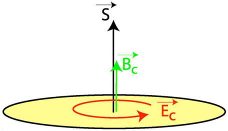
$\hookrightarrow$ twist-3 GPDs $\longrightarrow$ distribution of 'the force' in $\perp$ plane

- summary









$d_2 \leftrightarrow$ average $\perp$ force on quark in DIS from $\perp$ pol target

polarized DIS:

- $\sigma_{LL} \propto g_1 - \frac{2Mx}{\nu} g_2$
- $\sigma_{LT} \propto g_T \equiv g_1 + g_2$

$\hookrightarrow$ 'clean' separation between g_2 and $\frac{1}{Q^2}$ corrections to g_1

- $g_2 = g_2^{WW} + \bar{g}_2$ with $g_2^{WW}(x) \equiv -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y)$

$$d_2 \equiv 3 \int dx\, x^2 \bar{g}_2(x) = \frac{1}{2MP^{+2}S^x} \left\langle P,S \left| \bar{q}(0)\gamma^+ g F^{+y}(0) q(0) \right| P,S \right\rangle$$

color Lorentz Force on ejected quark (MB, PRD 88 (2013) 114502)

$$\sqrt{2}F^{+y} = F^{0y} + F^{zy} = -E^y + B^x = -\left(\vec{E} + \vec{v} \times \vec{B}\right)^y \quad \text{for } \vec{v} = (0,0,-1)$$

matrix element defining $d_2 \leftrightarrow 1^{st}$ integration point in QS-integral

$d_2 \Rightarrow \perp$ force $\leftrightarrow$ QS-integral $\Rightarrow \perp$ impulse

sign of d_2

- $\perp$ deformation of $q(x, \mathbf{b}_\perp)$

$\hookrightarrow$ sign of d_2^q: opposite Sivers

magnitude of d_2

- $\langle F^y \rangle = -2M^2 d_2 = -10 \frac{GeV}{fm} d_2$
- $|\langle F^y \rangle| \ll \sigma \approx 1 \frac{GeV}{fm} \Rightarrow d_2 = \mathcal{O}(0.01)$

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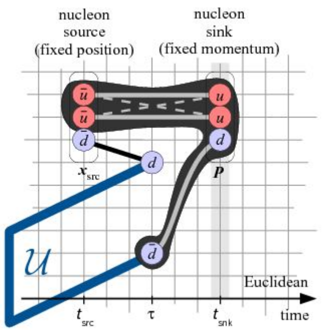
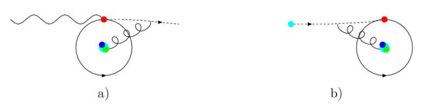
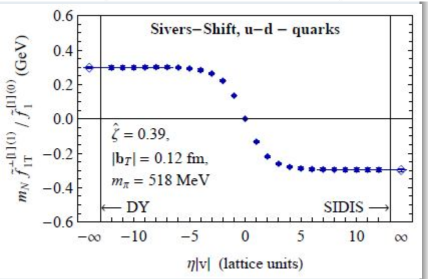
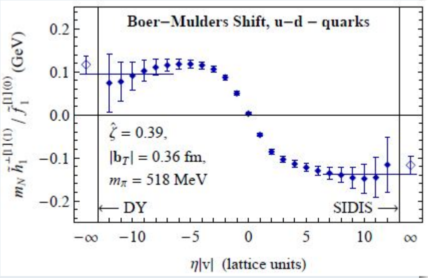
consitent with experiment (JLab,SLAC), model calculations (Weiss), and lattice QCD calculations (Göckeler et al., 2005)







$f_{1T}^{\perp}(x, \mathbf{k}_\perp)$ is $\mathbf{k}_\perp$-odd term in quark-spin averaged momentum distribution in $\perp$ polarized target



Force Operator W.Armstrong, F.Aslan, MB, S.Liuti, M.Engelhardt

slope at length =0

chirally even spin-dependent twist-3 PDF $g_2(x)$

MB, PRD 88 (2013) 114502

- $\int dx\, x^2 g_2(x) \Rightarrow \perp$ force on unpolarized quark in $\perp$ polarized target

$\hookrightarrow$ 'Sivers force'

scalar twist-3 PDF $e(x)$

MB, PRD 88 (2013) 114502

- $\int dx\, x^2 e(x) \Rightarrow \perp$ force on $\perp$ polarized quark in unpolarized target

$\hookrightarrow$ 'Boer-Mulders force'

chirally odd spin-dependent twist-3 PDF $h_2(x)$

M.Abdallah & MB, PRD94 (2016) 094040

- $\int dx\, x^2 h_2(x) = 0$

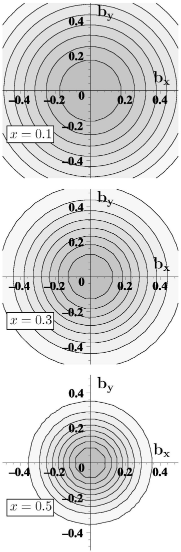
$\hookrightarrow$ $\perp$ force on $\perp$ pol. quark in long. pol. target vanishes due to parity

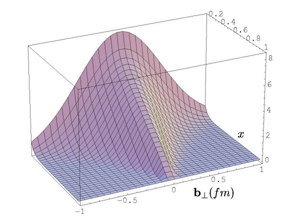
- $\int dx\, x^3 h_2(x) \Rightarrow$ long. gradient of $\perp$ force on $\perp$ polarized quark in long. polarized target

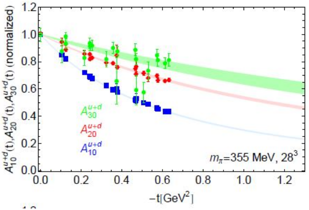
$\hookrightarrow$ chirally odd 'wormgear force'

Physics of GPDs: 3D Imaging MB,PRD 62, 071503 (2000)

$q(x, \mathbf{b}_\perp)$ for unpol. p







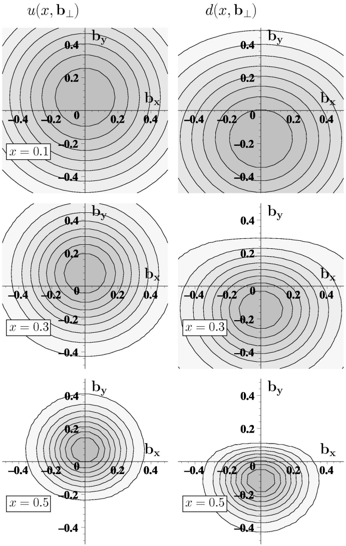
unpolarized proton

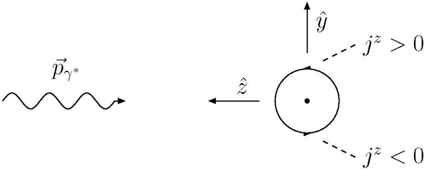
- $q(x, \mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H(x, 0, -\mathbf{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp \cdot \mathbf{\Delta}_\perp}$

$\hookrightarrow$ probabilistic interpretation

- $F_1(-\mathbf{\Delta}_\perp^2) = \int dx H(x, 0, -\mathbf{\Delta}_\perp^2)$
- x = momentum fraction of the quark
- $\mathbf{b}_\perp$ relative to $\perp$ center of momentum
- small x: large 'meson cloud'
- larger x: compact 'valence core'
- $x \to 1$: active quark = center of momentum

$\hookrightarrow$ $\vec{b}_\perp \to 0$ (narrow distribution) for $x \to 1$





proton polarized in $+\hat{x}$ direction

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H_q(x,0,-\Delta_\perp^2) e^{-i\mathbf{b}_\perp\cdot\Delta_\perp}$$

$$-\frac{1}{2M}\frac{\partial}{\partial b_y} \int \frac{d^2\Delta_\perp}{(2\pi)^2} E_q(x,0,-\Delta_\perp^2) e^{-i\mathbf{b}_\perp\cdot\Delta_\perp}$$

- relevant density in DIS is $j^+ \equiv j^0 + j^z$ and left-right asymmetry from j^z
- av. shift model-independently related to anomalous magnetic moments:

$$\langle b_y^q \rangle \equiv \int dx \int d^2b_\perp q(x, \mathbf{b}_\perp) b_y = \frac{1}{2M} \int dx E_q(x,0,0) = \frac{\kappa_q}{2M}$$

twist-2 GPDs → Impact Parameter Dependent PDFs

⊥ localized state

$$|\mathbf{R}_\perp = 0, p^+, \Lambda\rangle \equiv \mathcal{N} \int d^2\mathbf{p}_\perp |\mathbf{p}_\perp, p^+, \Lambda\rangle$$

⊥ charge distribution (unpolarized quarks)

$$\begin{aligned}
\rho_{\Lambda'\Lambda}(\mathbf{b}_\perp) &\equiv \langle \mathbf{R}_\perp = 0, p^+, \Lambda' | \bar{q}(\mathbf{b}_\perp)\gamma^+ q(\mathbf{b}_\perp) | \mathbf{R}_\perp = 0, p^+, \Lambda\rangle \\
&= |\mathcal{N}|^2 \int d^2\mathbf{p}_\perp \int d^2\mathbf{p}'_\perp \langle \mathbf{p}'_\perp, p^+, \Lambda' | \bar{q}(0)\gamma^+ q(0) | \mathbf{p}_\perp, p^+, \Lambda\rangle e^{i\mathbf{b}_\perp\cdot(\mathbf{p}_\perp - \mathbf{p}'_\perp)} \\
&= |\mathcal{N}|^2\, 2P^+ \int d^2\mathbf{P}_\perp \int d^2\mathbf{\Delta}_\perp F_{\Lambda'\Lambda}(-\mathbf{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp\cdot\mathbf{\Delta}_\perp} \\
&= \int d^2\mathbf{\Delta}_\perp F_{\Lambda'\Lambda}(-\mathbf{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp\cdot\mathbf{\Delta}_\perp}
\end{aligned}$$

- crucial: $\langle \mathbf{p}'_\perp, p^+, \Lambda' | \bar{q}(0)\gamma^+ q(0) | \mathbf{p}_\perp, p^+, \Lambda\rangle$ depends only on $\mathbf{\Delta}_\perp$
- $F_{\Lambda'\Lambda}(-\mathbf{\Delta}_\perp^2)$ some linear combination of F_1 & F_2 - depending on Λ, Λ'
- similar for various polarized quark densities
- similar for x-dependent densities $\longrightarrow$ GPDs

localized state - an attempt (for simplicity no spin)

- $|\vec{R}=0\rangle \equiv \mathcal{N}\int \frac{d^3p}{\sqrt{2\omega(\vec{p})}}|\vec{p}\rangle$

charge distribution in that state

$$\langle \vec{R}=0|\bar{q}(\vec{r})\gamma^0 q(\vec{r})|\vec{R}=0\rangle$$
$$\sim \int \frac{d^3p'}{\sqrt{2\omega(\vec{p}')}}\frac{d^3p}{\sqrt{2\omega(\vec{p})}}\left(\omega(\vec{p})+\omega(\vec{p}')\right)F(t)$$

- additional ωs in $t=\left(\omega(\vec{p})-\omega(\vec{p}')\right)^2-\vec{\Delta}^2$

$\hookrightarrow$ not possible to factorize into $\int d^3\Delta$ and $\int d^3P$

- except if you simply assume $\omega(\vec{p})=\omega(\vec{p}')$ and call it Breit 'frame'

not possible to construct state in which the charge distribution equals the 3D Fourier transform of the form factor $\int d^3\Delta F(-\vec{\Delta}^2)e^{-i\vec{\Delta}\cdot\vec{r}}$

$\perp$ force distribution (unpolarized quarks)

$$F^i_{\Lambda'\Lambda}(\mathbf{b}_\perp) \equiv \langle \mathbf{R}_\perp = 0, p^+, \Lambda' | \bar{q}(\mathbf{b}_\perp)\gamma^+ g F^{+i}(\mathbf{b}_\perp) q(\mathbf{b}_\perp) | \mathbf{R}_\perp = 0, p^+, \Lambda\rangle$$
$$= |\mathcal{N}|^2 \int d^2\mathbf{p}_\perp \int d^2\mathbf{p}'_\perp \langle \mathbf{p}'_\perp, p^+, \Lambda | \bar{q}(0)\gamma^+ g F^{+i}(0) q(0) | \mathbf{p}_\perp, p^+, \Lambda\rangle e^{i\mathbf{b}_\perp\cdot(\mathbf{p}_\perp - \mathbf{p}'_\perp)}$$

Form factors of qgq correlator (F.Aslan, M.B., M.Schlegel arXiv:1904.03494)

$$\langle p', \lambda' | \bar{q}(0)\gamma^+ i g F^{+i}(0) q(0) | p, \lambda\rangle = \bar{u}(p', \lambda') \Big[\frac{P^+}{M}\gamma^+ \frac{\Delta^i}{M}\Phi_1(t) + \frac{P^+}{M} i\sigma^{+i}\Phi_2(t)$$
$$+ \frac{P^+}{M}\frac{\Delta^i}{M}\frac{i\sigma^{+\Delta}}{M}\Phi_3(t) + \frac{P^+}{M}\frac{\Delta^+}{M}\frac{i\sigma^{i\Delta}}{M}\Phi_4(t) + \frac{P_\perp \Delta^+ i\sigma^{+\Delta}}{M^3}\Phi_5(t)\Big] u(p, \lambda).$$

crucial:

- for $p^{+\prime} = p^+$, $\langle p', \lambda' | \bar{q}(0)\gamma^+ i g F^{+i}(0) q(0) | p, \lambda\rangle$ only depends on $\mathbf{\Delta}_\perp$

$\hookrightarrow$ similar to $\perp$ charge density ...

⊥ force distribution (unpolarized quarks)

$$F^i_{\Lambda'\Lambda}(\mathbf{b}_\perp) \equiv \langle \mathbf{R}_\perp = 0, p^+, \Lambda' | \bar{q}(\mathbf{b}_\perp)\gamma^+ g F^{+i}(\mathbf{b}_\perp) q(\mathbf{b}_\perp) | \mathbf{R}_\perp = 0, p^+, \Lambda\rangle$$
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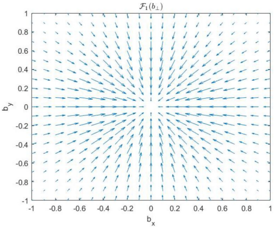
Φ_1

- unpolarized target
- axially symmetric 'radial' force



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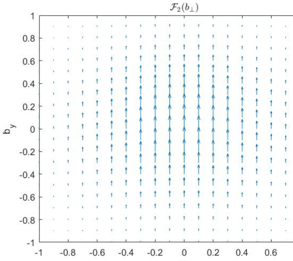
Φ_2

- ⊥ polarized target; force ⊥ to target spin

↪ spatially resolved Sivers force



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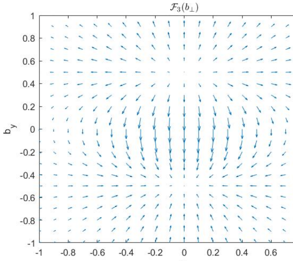
Φ_3

- tensor type force
- similar to charged particle flying through magnetic dipole field



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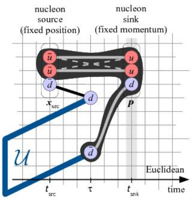
Φ_4 & Φ_5

- no contribution for $\Delta^+ = 0$

determining Φ_i

- match with x^2 moments of twist-3 GPDs (minus WW parts)
  F.Aslan, M.B. in progress
- experiments may take a few years, or immediately
- lattice QCD: fit to nonforward matrix elements of the 'force operator'
  in progress (J.Bickerton, R.Young, J.Zanotti)

the force operator



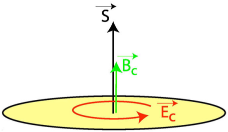
- form factor with quark denstity involving Wilson line staple
- take derivative w.r.t. staple length at length =0

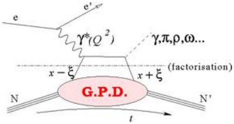
Summary

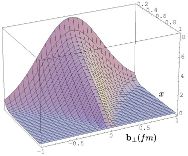
- GPDs $\xrightarrow{FT} q(x, \mathbf{b}_\perp)$ '3d imaging'
- x^2 moment of twist-3 PDFs $\rightarrow$ force
- x^2 moment of twist-3 GPDs:

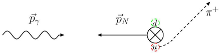
$\hookrightarrow$ $\bar{q}\gamma^+ F^{+\perp}\Gamma q$ distribution

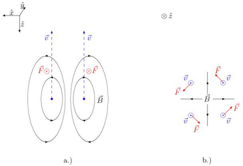
$\hookrightarrow$ $\perp$ force tomography











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