

*QCD Evolution 2019, Argonne National Lab, May 14, 2019*

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# The transverse nucleon spin asymmetry in photon SIDIS in the collinear twist-3 formalism

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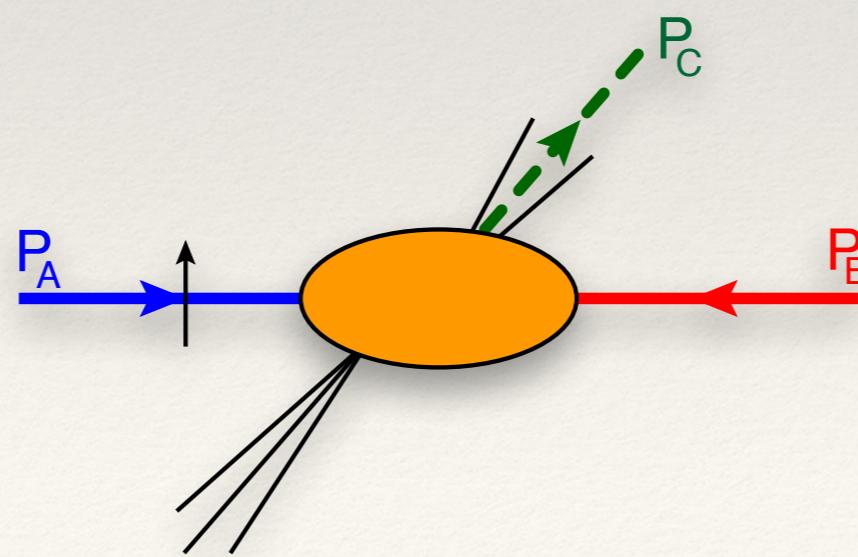
Marc Schlegel  
Department of Physics  
New Mexico State University

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with W. Albaltan, A. Prokudin

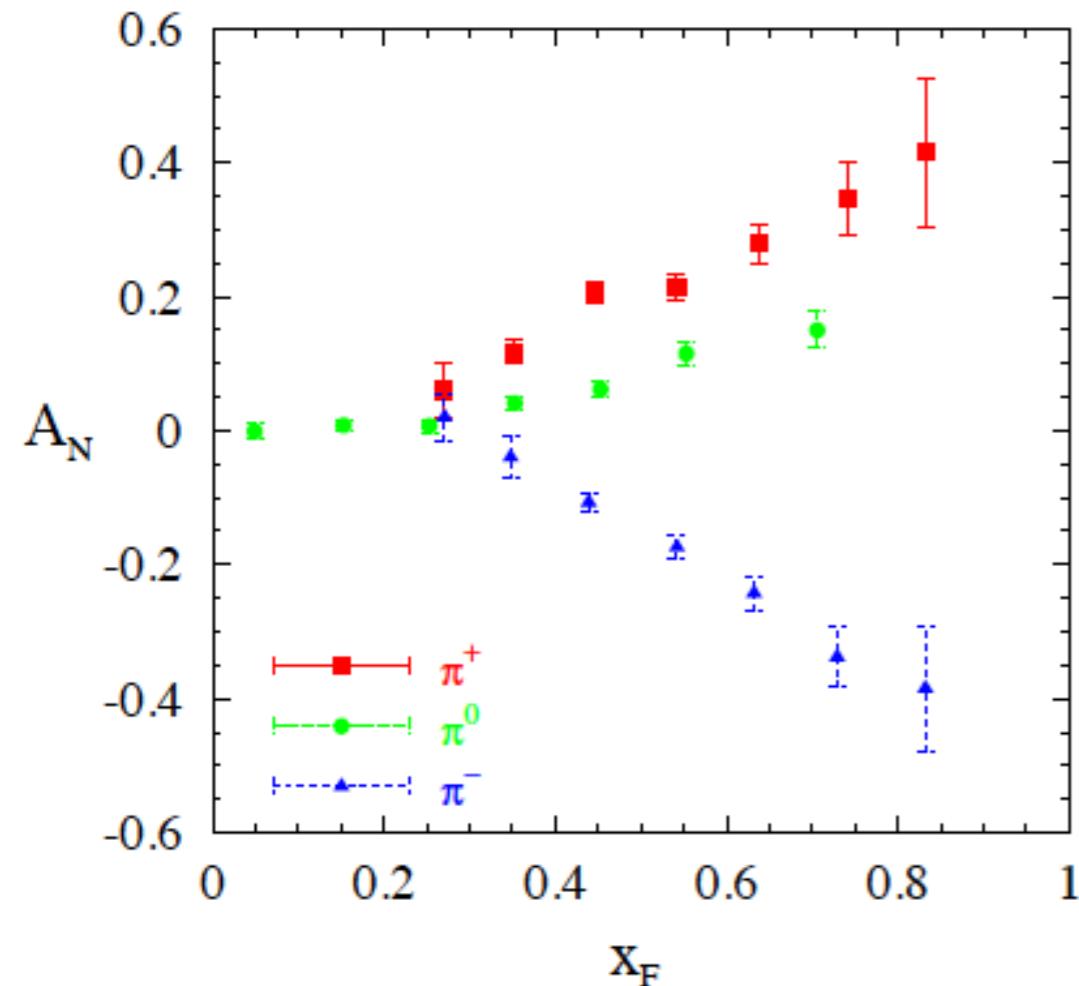
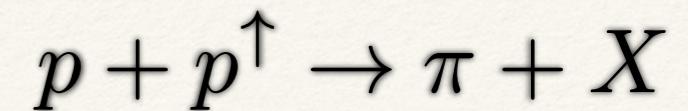
# Transverse Spin Effects in Single-Inclusive Hard Processes

$$P_A^\uparrow + P_B \rightarrow P_C + X$$

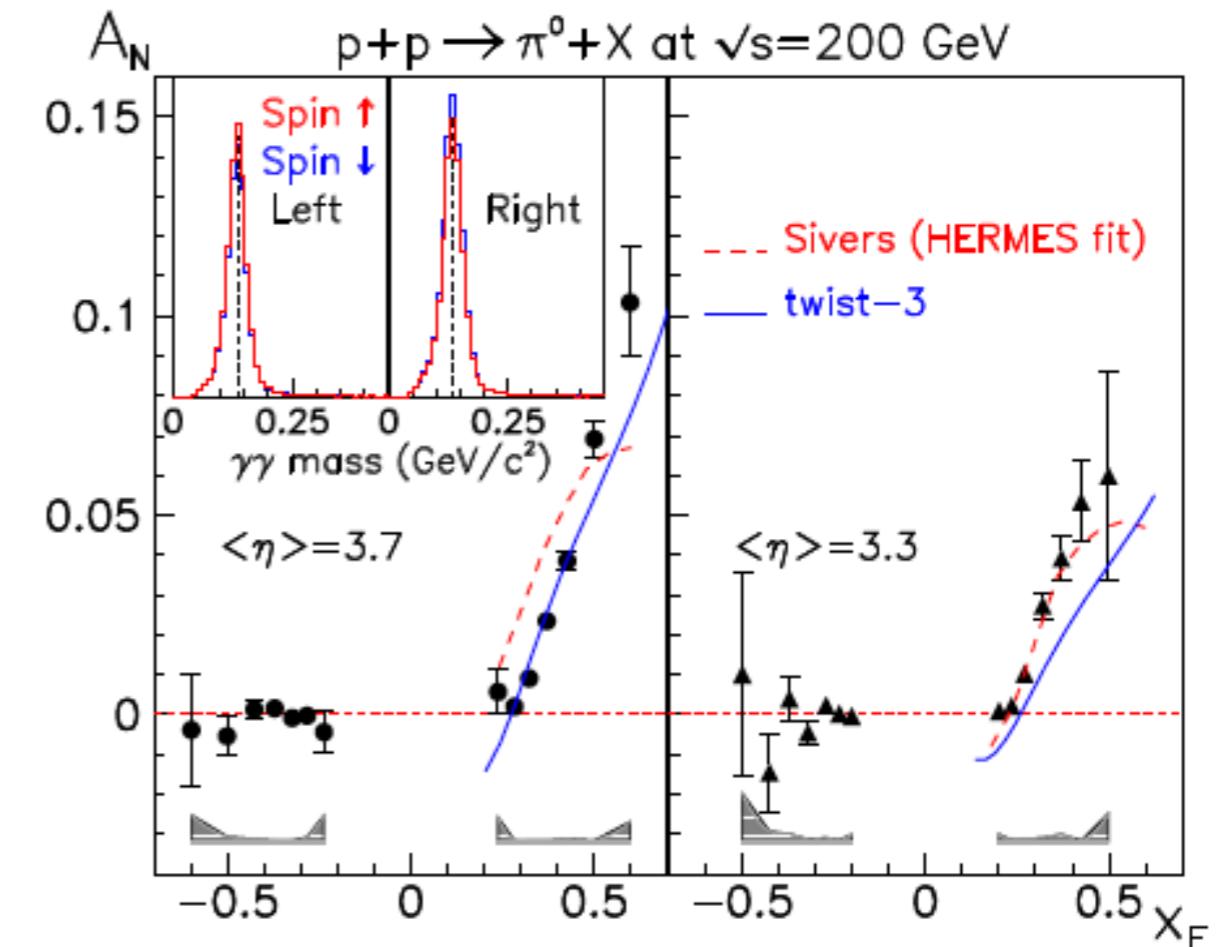


# “Show-off” Transverse SSA

$$A_N = \frac{\sigma^{\uparrow} - \sigma^{\downarrow}}{\sigma^{\uparrow} + \sigma^{\downarrow}}$$



$\sqrt{s} = 20 \text{ GeV}$  [E704 coll. (1991)]



$\sqrt{s} = 200 \text{ GeV}$  [STAR coll. (2008)]

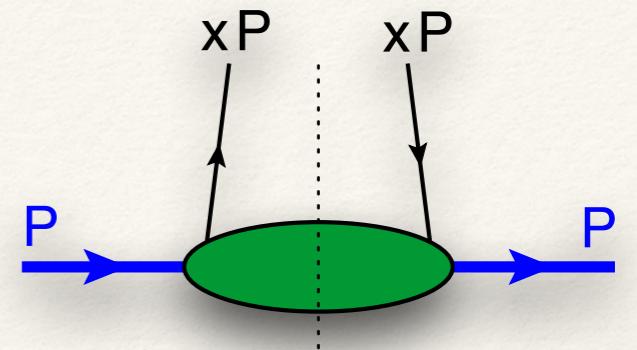
large effects  
cannot be explained in the standard parton model  
(using transversity)  
→ collinear Twist-3 Formalism  
(Efremov, Teryaev, Qiu, Sterman)

Collinear twist-3 formalism: several types of matrix elements compete

## Collinear twist-3 formalism: several types of matrix elements compete *intrinsic* twist-3 PDF

$$g_T^q(x) = -\frac{1}{M} \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P, S_T | \bar{q}(0) \not{S}_T \gamma_5 q(\lambda n) | P, S_T \rangle$$

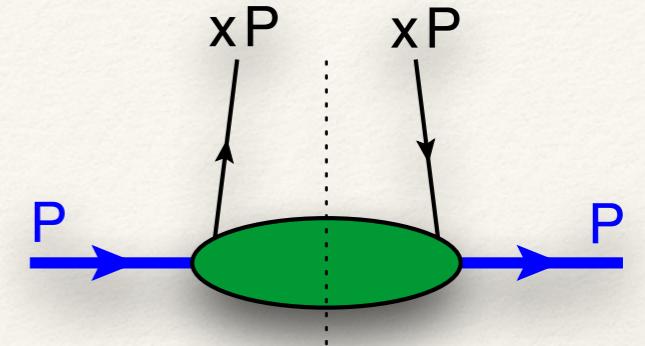
- sensitive to ‘bad quark field components’,
- twist-3 characteristics hidden in Dirac structure
- generates the  $g_2$  structure function in DIS
- No probabilistic interpretation



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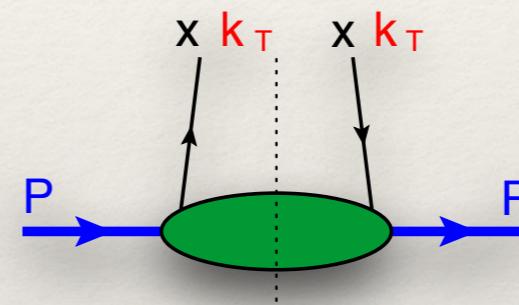
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*kinematical* twist-3 PDFs:

Small transverse quark/gluon momenta  $k_T$ :



$$(\not{k}_T \times S_T) f_{1T}^{\perp, q}(x, \not{k}_T^2) \propto \int \frac{d\lambda d^2 z_T}{(2\pi)^3} e^{i\lambda x + i\not{k}_T \cdot z_T} \langle P, S_T | \bar{q}(0) \not{\gamma} \mathcal{W} q(\lambda n + z_T) | P, S_T \rangle$$

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Sivers function

‘transhelicity’

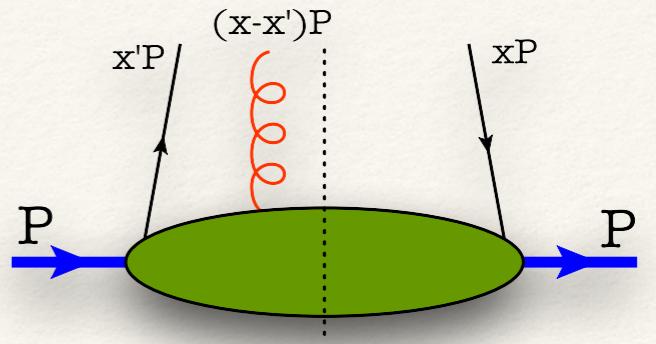
*Collinear* twist-3 formalism: TMD moments are needed

$$f_{1T}^{\perp, (1)}(x) = \int d^2 k_T \frac{\not{k}_T^2}{2M^2} f_{1T}^{\perp}(x, \not{k}_T^2)$$

$$g_{1T}^{(1)}(x) = \int d^2 k_T \frac{\not{k}_T^2}{2M^2} g_{1T}(x, \not{k}_T^2)$$

→ twist-3 characteristics through small transverse parton momentum  $k_T$

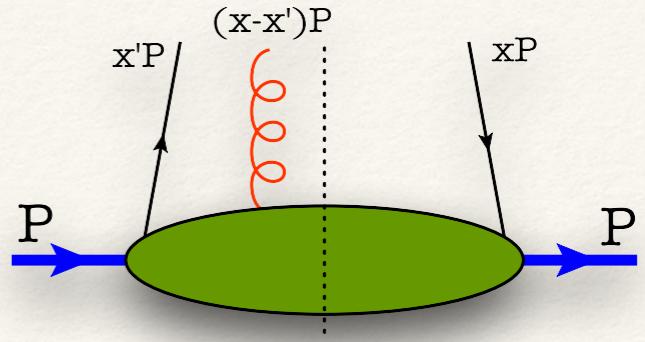
## Dynamical twist-3: Quark - Gluon - Quark Correlations (ETQS-matrix elements)



$$2M i\epsilon^{Pn\rho S} F_{FT}^q(x, x') = \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x'} e^{i\mu(x-x')} \langle P, S_T | \bar{q}(0) \not{\epsilon} ig F^{n\rho}(\mu n) q(\lambda n) | P, S_T \rangle$$

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'dynamical twist - 3'

→ 3 - parton correlator: suppression by additional propagator

→ Quark-Gluon-Quark correlation functions  
drive x-dependence of TMDs like Sivers function, transhelicity, etc.

→ so far: only “diagonal support”  $\pi F_{FT}(x, x) = f_{1T}^{\perp(1)}(x)$  constraint by SIDIS data

→ ‘integrated’  $F_{FT}(x, x')$ : average transverse color Lorentz force on struck quark  
[Burkardt, PRD88, 114502], see talk by M. Burkardt

$$F^{n\rho} = [\vec{E} + \vec{n} \times \vec{B}]^\rho \propto \int dx \int dx' F_{FT}(x, x') \propto \int dx x^2 g_T(x)$$

# QCD EoM relation & Lorentz-Invariance Relations

[Kanazawa, Koike, Metz, Pitonyak, MS, PRD 2016]

## QCD EoM for Twist-3 PDFs

$$g_{1T}^{(1)}(x) = x g_T(x) - \frac{m_q}{M} h_1(x)$$
$$+ \int_{-1}^1 dx' \frac{F_{FT}(x, x') - G_{FT}(x, x')}{x - x'}$$

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LIR for Twist-3 PDFs  
based on translation invariance

$$g_T(x) = g_1(x) + \frac{d}{dx} g_{1T}^{(1)}(x) - 2 \int_{-1}^1 dx' \frac{G_{FT}(x, x')}{(x - x')^2}$$

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Two equations, three functions → eliminate ‘intrinsic & kinematical twist-3’

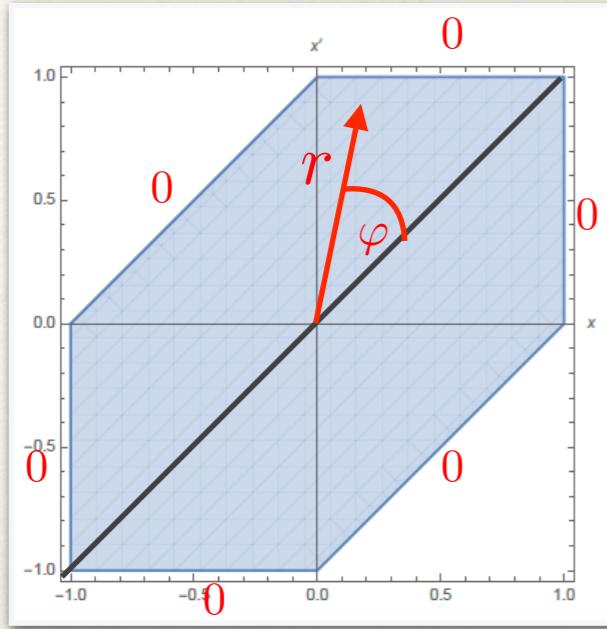
$$g_T(x) = \int_x^1 \frac{dy}{y} g_1(y) + \frac{m_q}{M} \left( \frac{1}{x} h_1(x) - \int_x^1 \frac{dy}{y^2} h_1(y) \right)$$

$$+ \int_x^1 \frac{dy}{y^2} \int_{-1}^1 dz \left[ \frac{(1-y\delta(y-x)) F_{FT}(y, z)}{y-z} - \frac{(3y-z-y(y-z)\delta(y-x)) G_{FT}(y, z)}{(y-z)^2} \right]$$

EoM & LIR relation crucial for gauge invariance, invariance of LC vector n

## Support properties

$$-1 \leq x, x' \leq 1 \quad |x - x'| \leq 1 \quad \text{and continuous}$$

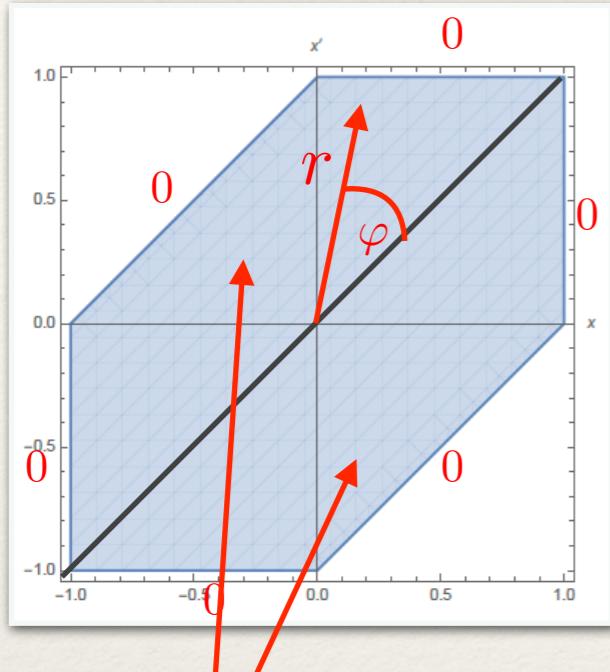


$$F_{FT}(x, x') = +F_{FT}(x', x) \implies \sum_n a_n(r) \cos(n\varphi)$$

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$$\pi F_{FT}^q(x, x) = f_{1T}^{\perp(1), q}(x)$$

$$\pi F_{FT}^q(-x, -x) = f_{1T}^{\perp(1), \bar{q}}(x)$$

Fixes  $a_0, a_1$

Gluons poles 'known' from SIDIS experiments

$$F_{FT}(x, x') = \left( \frac{(1-x^2)(1-x'^2)(1-(x-x'))}{(1-xx')^2} \right)^\delta (a_0(r) + a_1(r) \cos(\varphi))$$

Model ansätze

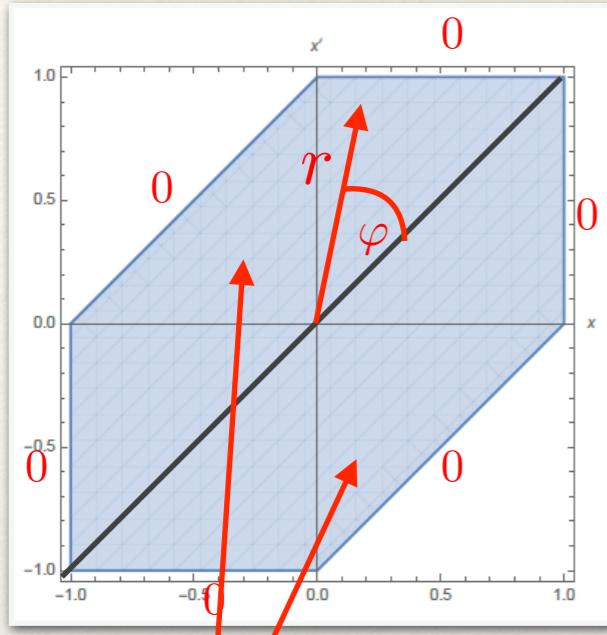
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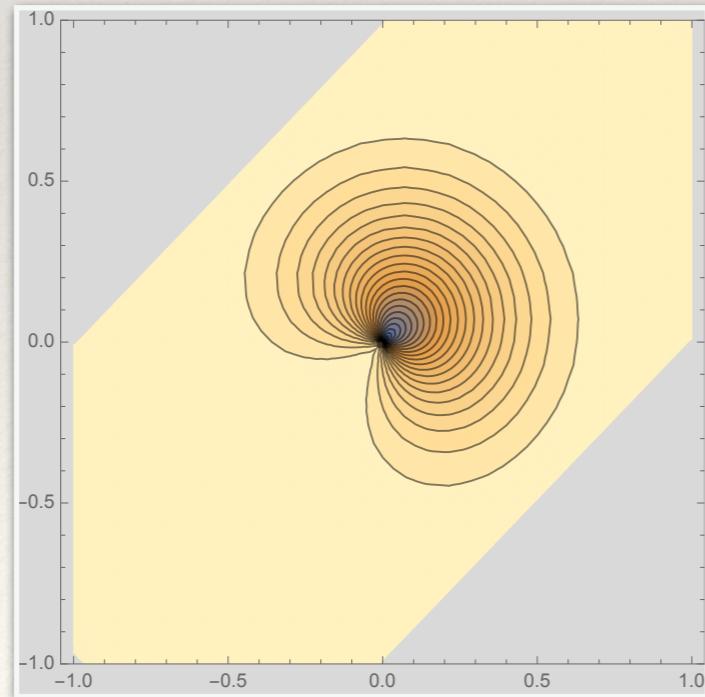
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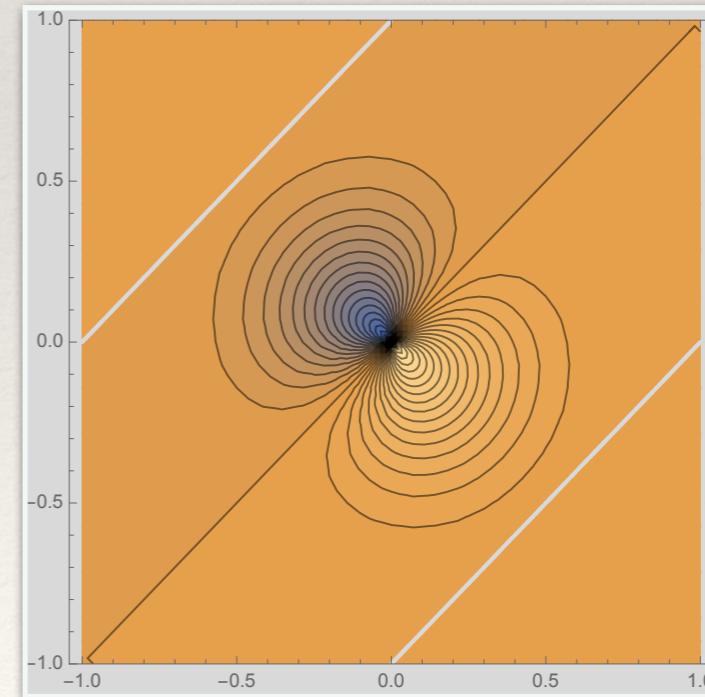
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$$G_{FT}(x, x')$$



# **Transverse Spin Asymmetries in Photon SIDIS**

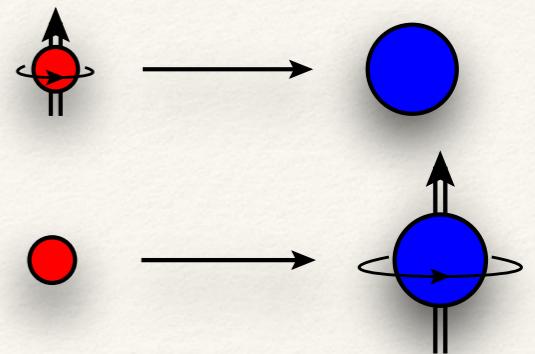
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⇒ New effects for a Two-Photon Exchange!

$$A_N = \frac{\sigma^{\uparrow} - \sigma^{\downarrow}}{\sigma^{\uparrow} + \sigma^{\downarrow}}$$

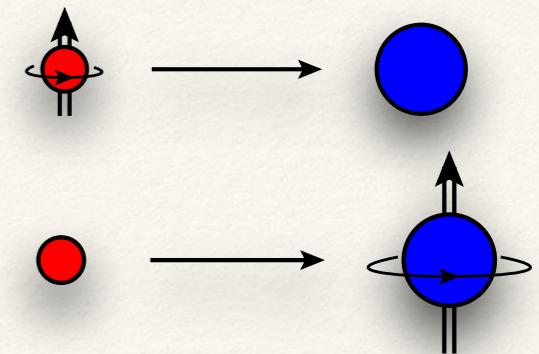


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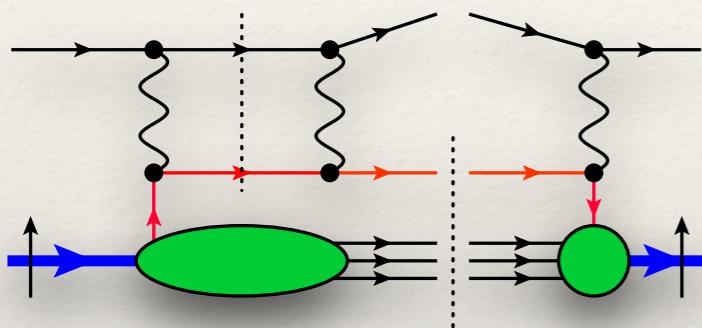
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## Transverse Target Single Spin Asymmetry in DIS

[Metz, M.S., Goeke, PLB 2006]



$$A_{UT} \propto \alpha_{\text{em}} \frac{M}{Q} \left( \frac{a}{\varepsilon} + b \right) \sum_q e_q^3 x_B g_T^q(x_B)$$

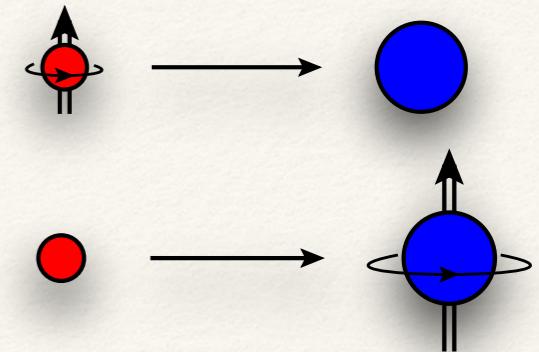
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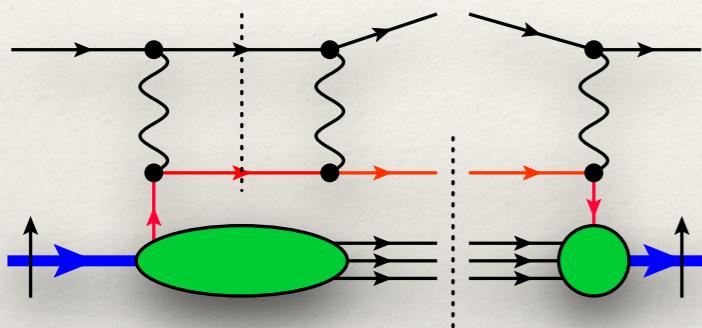
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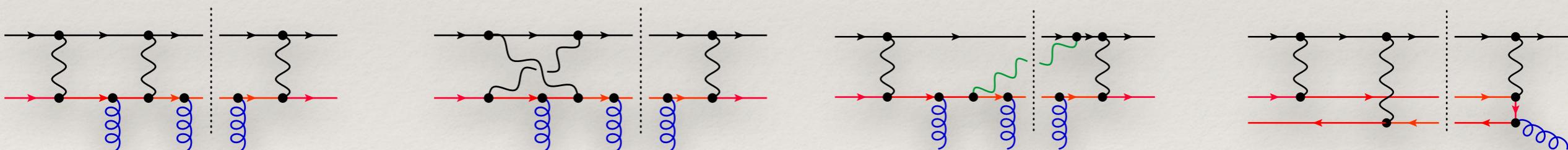
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## Inclusion of 'Quark - Gluon Correlations'



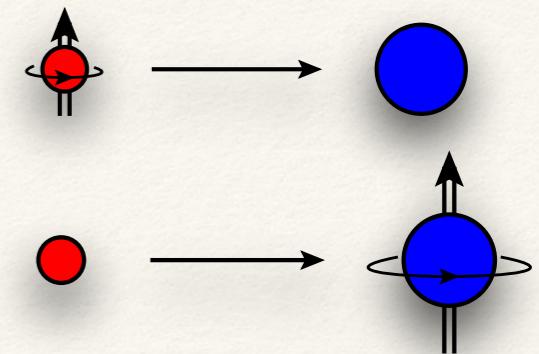
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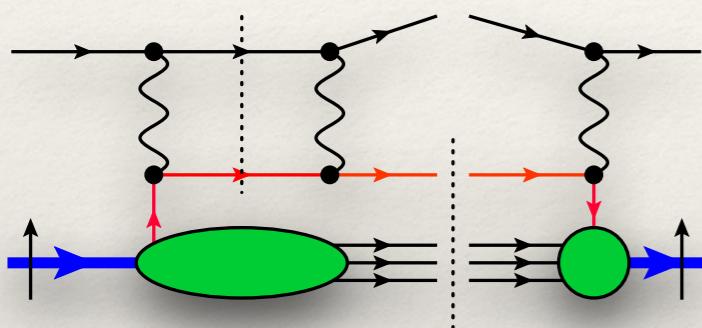
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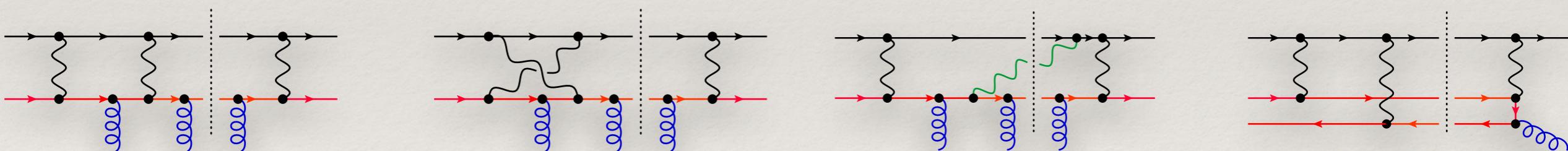
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**Result:** [MS; Metz, Pitonyak, Schäfer, Schlegel, Vogelsang, PRD 2012, 2013]

$$A_{UT} \propto \alpha_{\text{em}} \frac{M}{Q} \left[ \int dx' (\hat{\sigma}_+ F_{FT}(x_B, x') + \hat{\sigma}_- G_{FT}(x_B, x')) + \hat{\sigma}^\gamma F_{FT}^\gamma(x_B, x_B) \right]$$

# Photon SIDIS: $e + p \longrightarrow e + \gamma + X$

[Albaltan, Prokudin, M.S., in preparation]

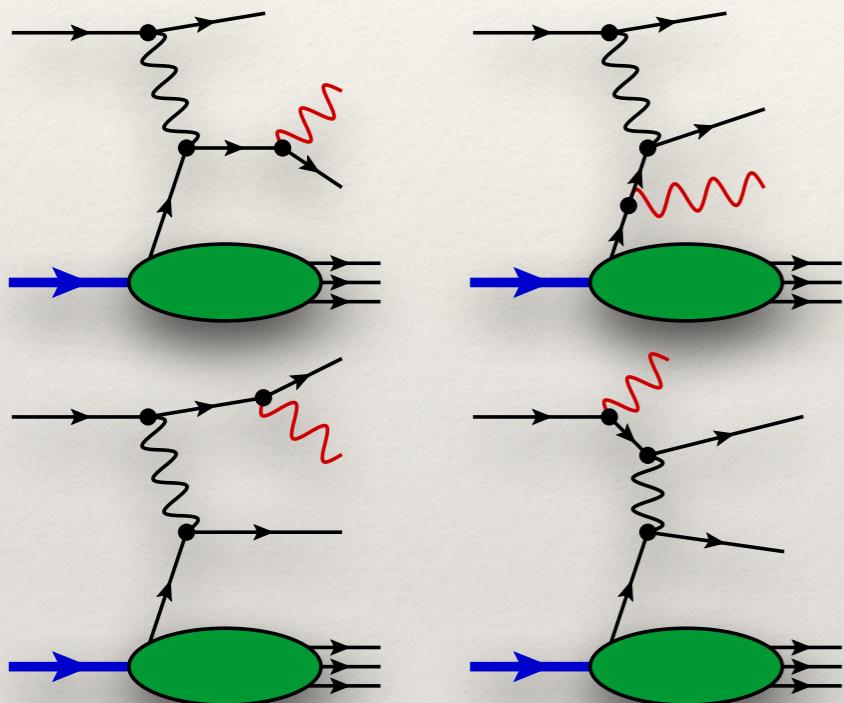
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Idea: Circumvent Christ - Lee theorem: hard, isolated real photon emission unpolarized cross section in the parton model

[Brodsky, Gunion, Jaffe, PRD 1972]



- avoid photon fragmentation: isolated photons
- collinear factorization:  
information on final quark is integrated out
- LO result:

$$E_\gamma E_e \frac{d\sigma_{UU}}{d^3\vec{P}_\gamma d^3\vec{P}_e} = \sum_q \left[ e_q^2 \hat{\sigma}_2 + e_q^3 \hat{\sigma}_3 + e_q^4 \hat{\sigma}_4 \right] f_1^q(\bar{x})$$

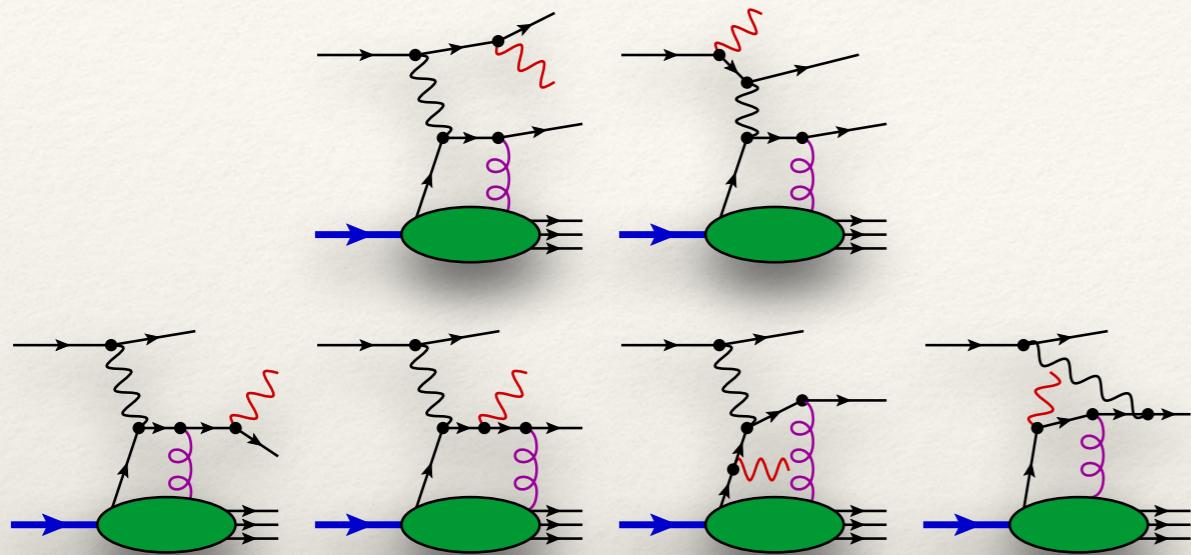
- two scales:  $Q^2 = -(l - l' - P_\gamma)^2$   $\tilde{Q}^2 = -(l - l')^2$

- two ‘Bjorken-x’:  
 $x_B = \frac{Q^2}{2P \cdot (l - l' - P_\gamma)}$   $\tilde{x}_B = \frac{\tilde{Q}^2}{2P \cdot (l - l')}$

BGJ - criterion for parton model dominance:  $Q^2, \tilde{Q}^2, |Q^2 - \tilde{Q}^2| \gg M^2$

# Transverse SSA in photon SIDIS

Include *intrinsic, kinematical & dynamical* twist - 3 contributions



At tree-level (LO):

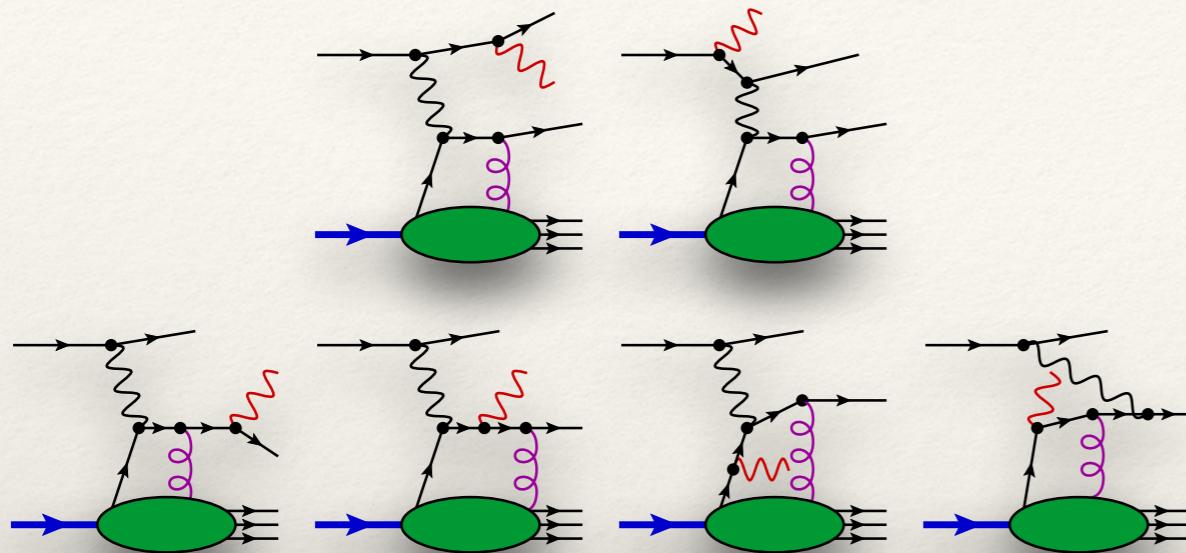
No contribution from  $g_T$  and  $g_{1T}^{(1)}$   
(no imaginary part)

Quark - Gluon correlations:

- 1) Soft Gluon Poles:  $F_{FT}(x_B, x_B)$
- 2) Soft Fermion Poles:  $F_{FT}(x_B, 0)$
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## LO Result:

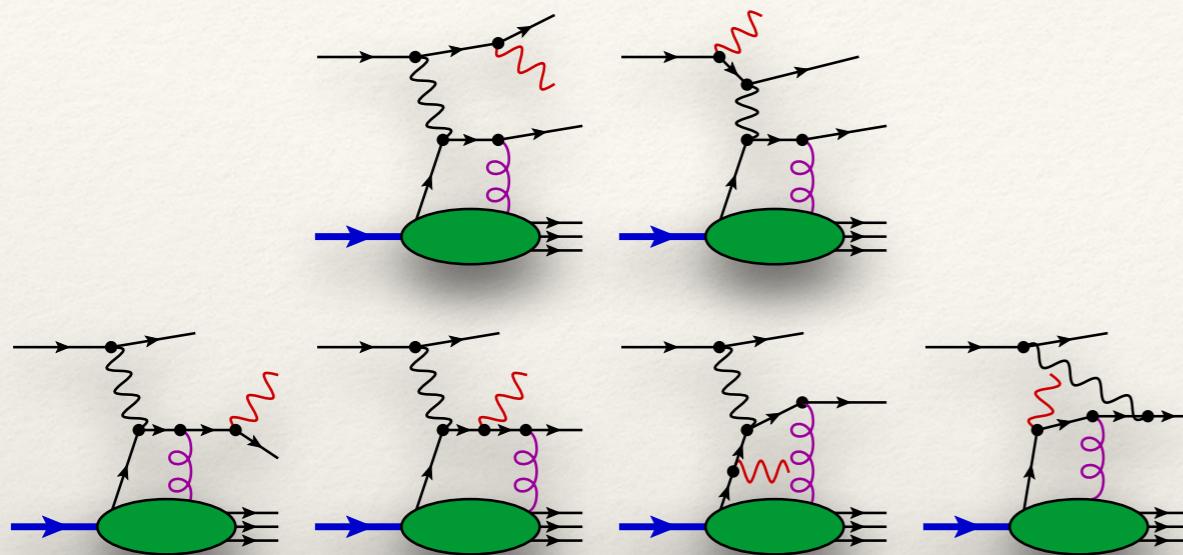
$$E_\gamma E' \frac{d\sigma_{UT}}{d^3\vec{P}_\gamma d^3\vec{P}_e} = \sum_q \left[ \hat{\sigma}_{+,HP} F_{FT}^q(x_B, \tilde{x}_B) + \hat{\sigma}_{+,SFP} F_{FT}^q(x_B, 0) + \hat{\sigma}_{-,HP} G_{FT}^q(x_B, \tilde{x}_B) + \hat{\sigma}_{-,SFP} G_{FT}^q(x_B, 0) \right]$$

Soft Gluon Poles vanish !

Bethe-Heitler contribution vanishes  
(Christ - Lee theorem)!

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⇒ unique process to directly study “off-diagonal” support  
of twist - 3 Quark - Gluon Correlation functions!

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Comparison to DIS cross section at EIC: suppression by  $\alpha = 1/137$

Event rate smaller

→  $(1/200 - 1/1000) \times$  DIS ‘total’ (binned) cross section at EIC

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Comparison to DIS cross section at EIC: suppression by  $\alpha = 1/137$

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→  $(1/200 - 1/1000) \times$  DIS ‘total’ (binned) cross section at EIC

Size of SSA

→ unknown, any estimate would be pure speculation  
→ Probably small, no BH contribution, (partial) cancellation of  
Compton & interference contribution (charge signs)

→ but too small?

Any experimental information would help...

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# Summary

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- ❖ Transverse Spin Polarization: Long history, measured in ep/ pp-collisions, theoretical treatment more complicated
- ❖ We can learn about the parton dynamics in the nucleon, e.g., transverse forces, non-perturbative QCD EoM and LIR are crucial
- ❖ Photon SIDIS: May be able to scan the support of dynamical twist-3 functions point-by-point at LO.
- ❖ Experimental opportunity at EIC, COMPASS, JLab
  - input would help our understanding of quark-gluon correlation
  - valuable for evolution of qqq functions and TMDs.