Generalized Wandzura Wilczek Relations and Partonic Orbital Angular Momentum

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Thanks!!

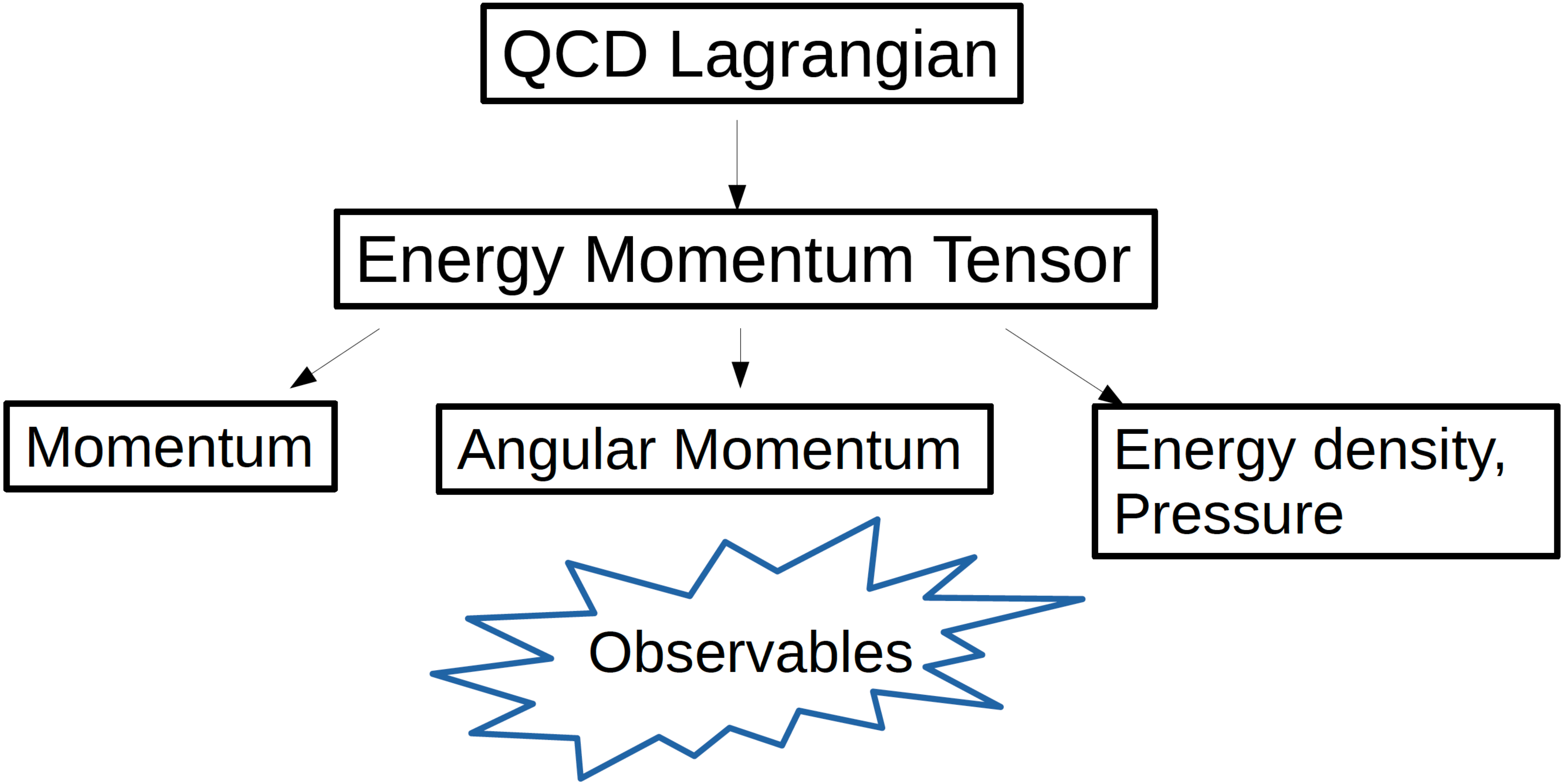
Outline

- Partonic Orbital Angular Momentum
- Wandzura Wilczek and genuine twist three contributions to twist 3 GPDs
- Role of the gauge link
- Extending to chiral odd sector and twist four

AR, Engelhardt and Liuti PRD 98 (2018)

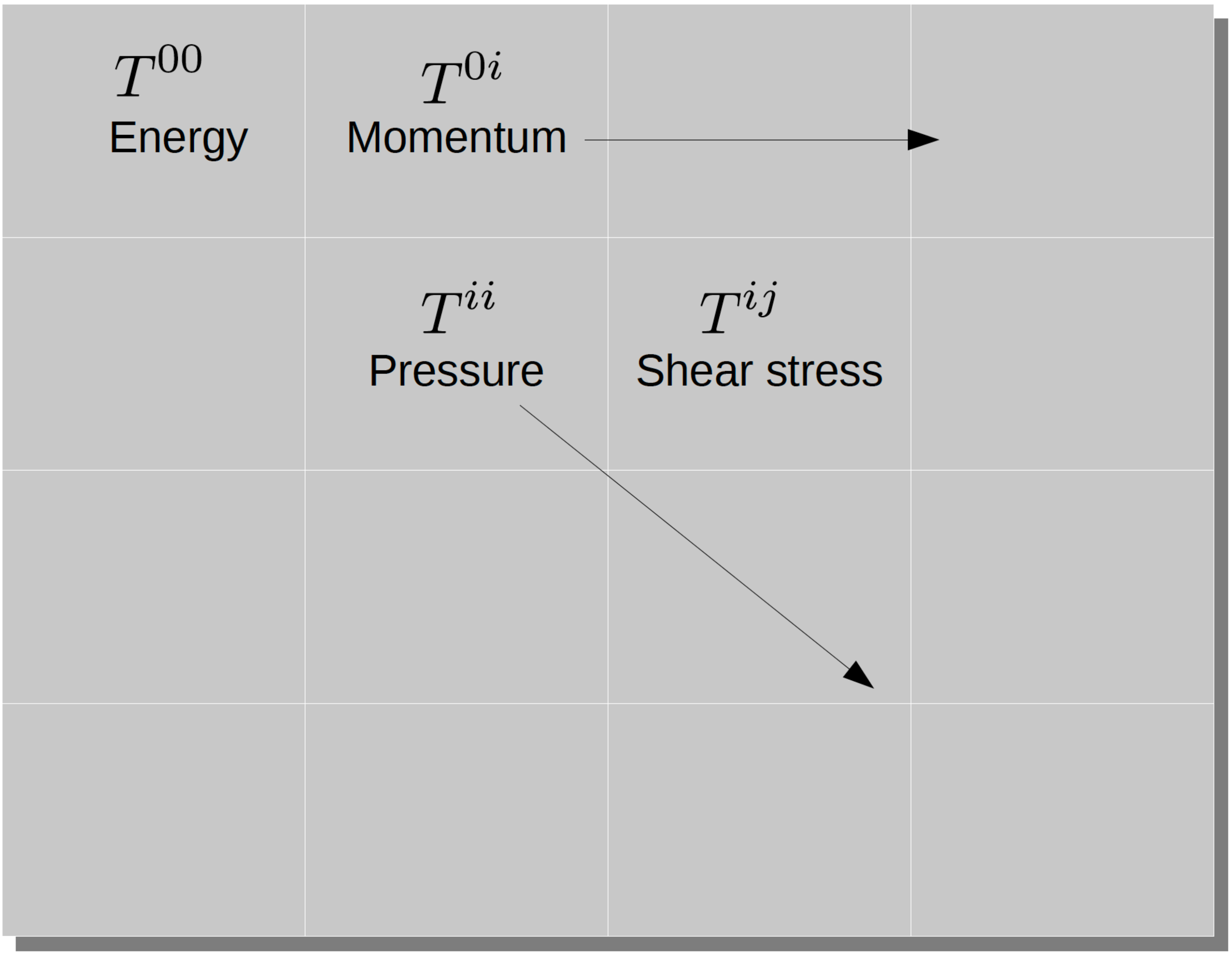
AR, Courtoy, Engelhardt and Liuti PRD 94 (2016)

QCD Energy Momentum Tensor



Deeply Virtual Compton Scattering, moments of GPDs etc.

QCD Energy Momentum Tensor



GPD based definition of Angular Momentum

$$J^i_{q,g} = \frac{1}{2}\epsilon^{ijk}\int d^3x\left(T^{0k}_{q,g}x^j - T^{0j}_{q,g}x^k\right)$$

$$\vec{J}_q = \int d^3x\psi^\dagger\left[\vec{\gamma}\gamma_5 + \vec{x}\times i\vec{D}\right]\psi \qquad \vec{J}_g = \int d^3x\left(\vec{x}\times\left(\vec{E}\times\vec{B}\right)\right)$$

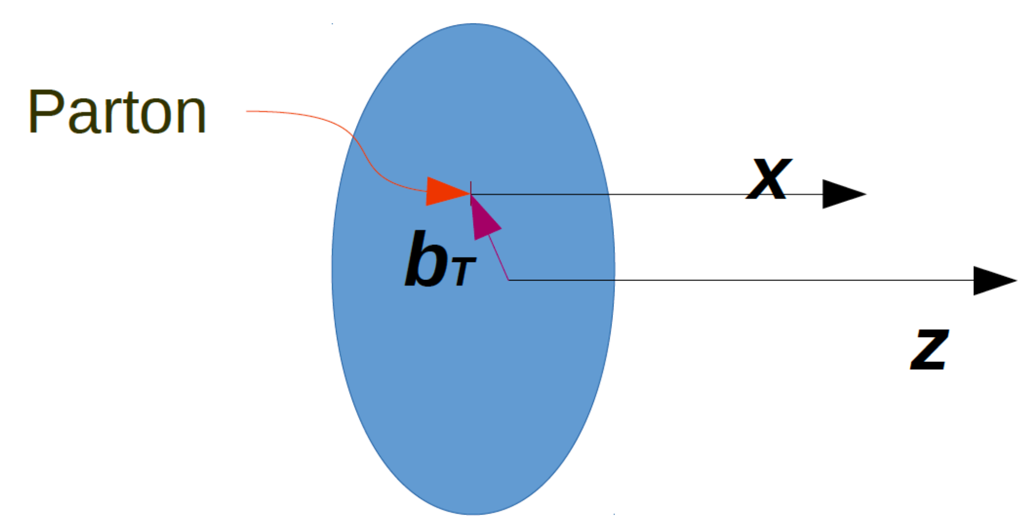
$$J_q = \frac{1}{2}\int dxx(H_q(x,0,0) + E_q(x,0,0))$$

Xiangdong Ji, PRL 78.610,1997

To access OAM, we take the difference between total angular momentum and spin

$$\mathcal{L}_q = J_q - \frac{1}{2}\Delta\Sigma$$

OAM Total Spin



Direct description of OAM

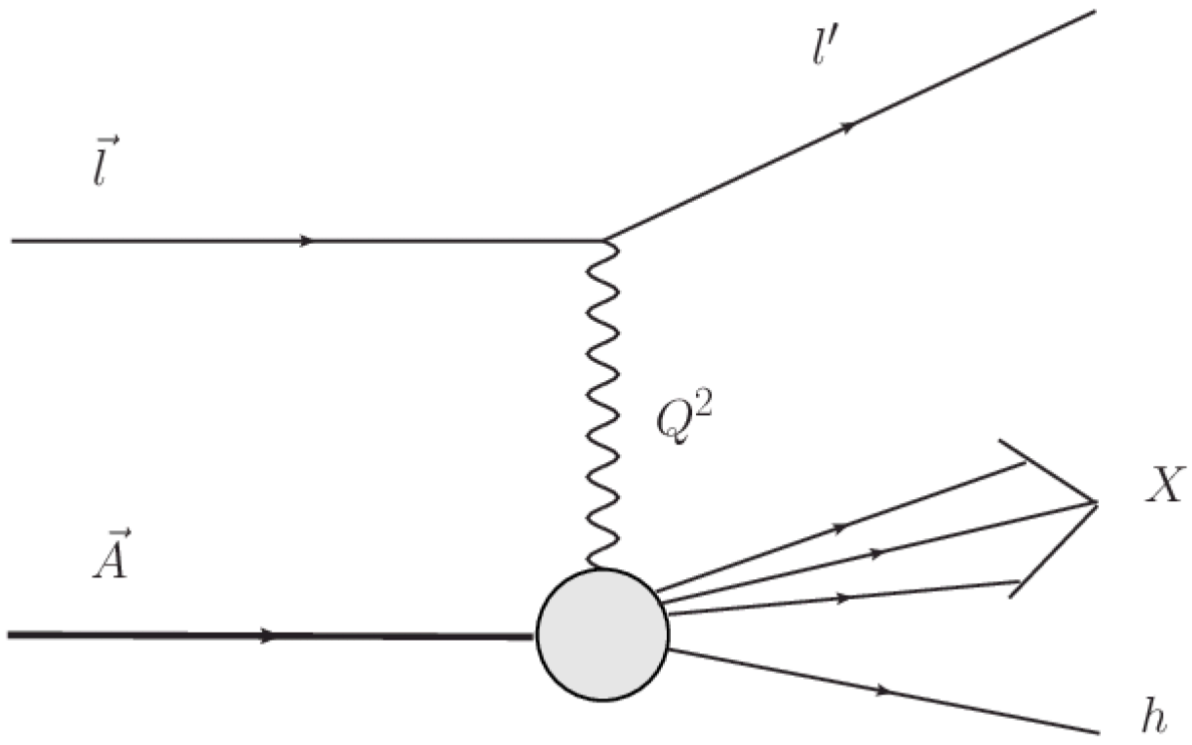
$$\int dx\, x G_2 = \int dx\, x(H+E) - \int dx\, \tilde{H}$$

$$G_2 \equiv \tilde{E}_{2T} + H + E$$

Kiptily and Polyakov, Eur Phys J C 37 (2004)
Hatta and Yoshida, JHEP (1210), 2012

- The moment in x of the GPD G_2 shown to be OAM

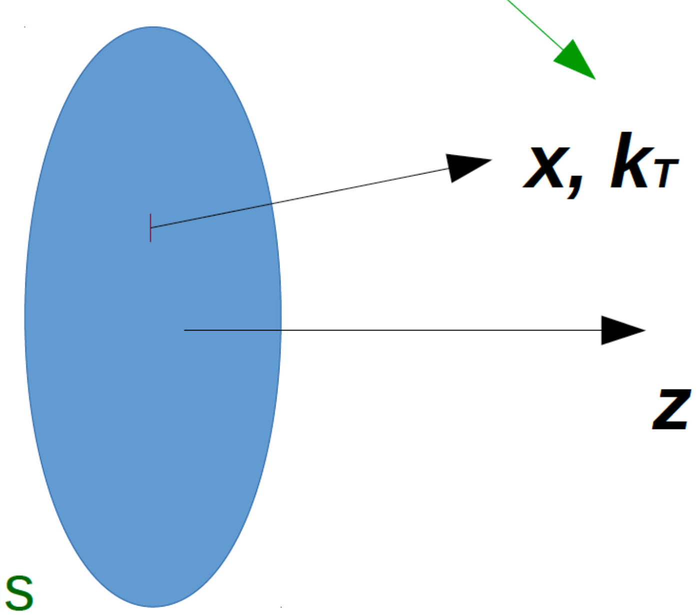
Intrinsic Transverse Momentum



The partonic transverse momentum is correlated with the momentum of the observed hadron

Semi inclusive Deep Inelastic Scattering

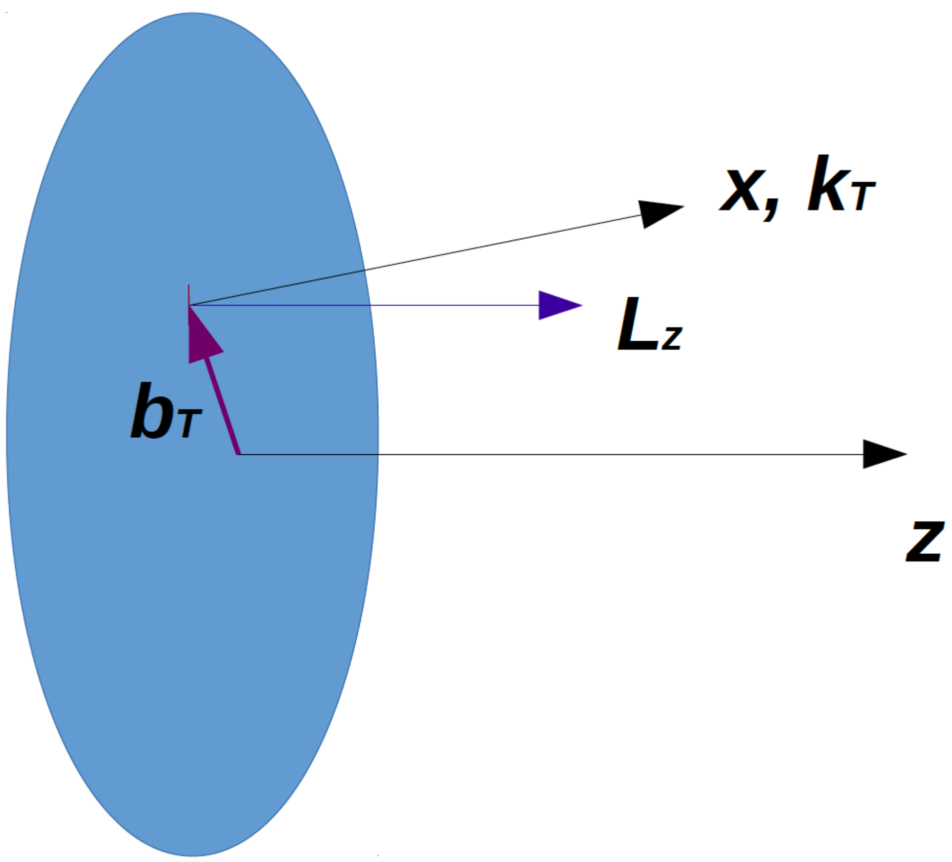
However, the target does not remain intact, no access to the spatial distribution of partons



Transverse Momentum Distributions

Partonic Orbital Angular Momentum II

- Consider measuring both the intrinsic transverse momentum and the spatial distribution of partons
- $L_{q,z}$ = $\mathbf{b}_T \times \mathbf{k}_T$



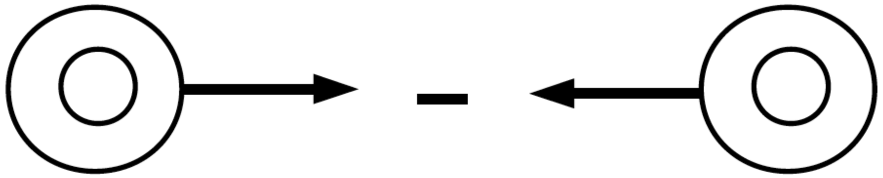
$$W_{\Lambda,\Lambda'}^{[\gamma^+]} = \frac{1}{2M}\bar{U}(p',\Lambda')[F_{11} + \frac{i\sigma^{i+}k_T^i}{\bar{p}_+}F_{12} + \frac{i\sigma^{i+}\Delta_T^i}{\bar{p}_+}F_{13} + \frac{i\sigma^{ij}k_T^i\Delta_T^j}{M^2}F_{14}]U(p,\Lambda)$$

Generalized Transverse Momentum Distributions (related by Fourier transform to Wigner Distributions)

Meissner Metz and Schlegel, JHEP 0908 (2009)

GTMDs that describe OAM

- How does F_{14} connect to OAM ?



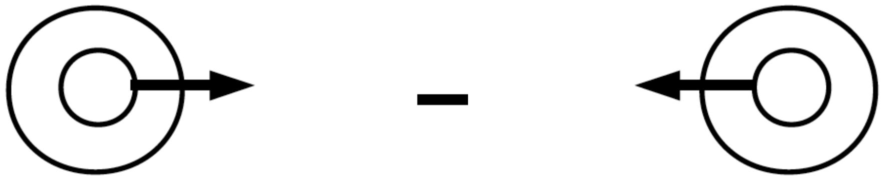
Unpolarized quark in a longitudinally polarized proton

$$\mathcal{W}(x, \mathbf{k}_T, \mathbf{b}) = \int \frac{d^2\Delta_T}{(2\pi)^2} e^{ib.\Delta_T} \left[W^{\gamma^+}_{++} - W^{\gamma^+}_{--} \right]$$

$$L = \int dx \int d^2k_T \int d^2\mathbf{b} (\mathbf{b} \times \mathbf{k}_T) \mathcal{W}(x, \mathbf{k}_T, \mathbf{b}) = -\int dx \int d^2k_T \frac{k_T^2}{M^2} F_{14}$$

Lorce et al PRD84, (2011)

- Another GTMD relevant to OAM



G_{11} describes a longitudinally polarized quark in an unpolarized proton. Measures spin orbit correlation.

The Two Definitions

- Weighted average of b_T X k_T

GTMD

$$L_z = -\int dx \int d^2k_T \frac{k_T^2}{M^2} F_{14}$$

Lorce, Pasquini (2011)

- Difference of total angular momentum and spin

$$\mathcal{L}_q = J_q - \frac{1}{2}\Delta\Sigma$$

GPD

$$\frac{1}{2}\int_{-1}^{1} dx x(H_q + E_q) \qquad \frac{1}{2}\int_{-1}^{1} dx \tilde{H}_q$$

The Two Definitions

$F_{14}^{(1)}$

- Weighted average of b_T X k_T

GTMD

$$L_z = -\int dx \boxed{\int d^2 k_T \frac{k_T^2}{M^2} F_{14}}$$

Lorce, Pasquini (2011)

- Difference of total angular momentum and spin

GPD

$$\mathcal{L}_q = J_q - \frac{1}{2}\Delta\Sigma$$

$$\frac{1}{2}\int_{-1}^{1} dx x (H_q + E_q) \qquad \frac{1}{2}\int_{-1}^{1} dx \tilde{H}_q$$

Is there a connection ?

- We find that

$$F_{14}^{(1)}(x) = \int_x^1 dy \left(\tilde{E}_{2T}(y) + H(y) + E(y) \right)$$

AR, Engelhardt and Liuti PRD 98 (2018)

AR, Courtoy, Engelhardt and Liuti PRD 94 (2016)

- This is a form of Lorentz Invariant Relation (LIR)
- This is a distribution of OAM in x
- Derived for a straight gauge link

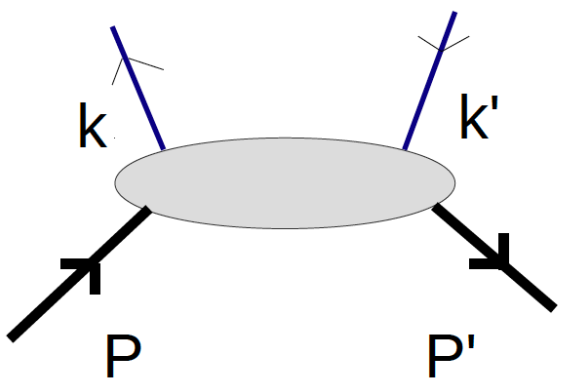
Higher Twist

$$\int \frac{dz_-}{2\pi} e^{ixP^+z^-} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\Gamma\psi(z/2) \mid p, \Lambda \rangle_{z^+=z_T=0}$$

$$\gamma^+, \gamma^+\gamma^5, \sigma^{i+}\gamma^5$$

Leading twist – twist 2

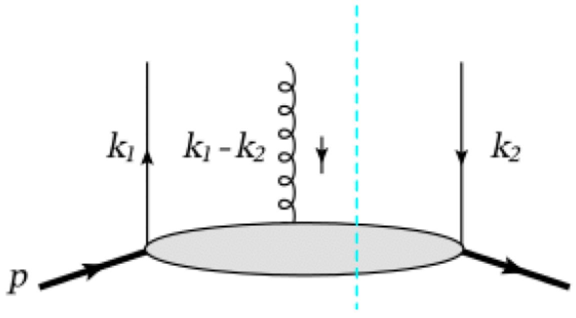
- Involve only good components
- Simple interpretation in terms of parton densities



$$\gamma^i, \gamma^i\gamma^5, \sigma^{ij}\gamma^5, 1, \gamma^5, \sigma^{+-}\gamma^5$$

Higher twist – twist 3

- Involve one good and one bad component
- The bad component represents a quark gluon composite



Derivation of Generalized LIRs

To derive these we look at the parameterization of the quark quark correlator function at different levels

$$\int \frac{d^4z}{2\pi} e^{ik.z} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\Gamma\psi(z/2) \mid p, \Lambda\rangle$$

Generalized Parton Correlation Functions (GPCFS)

Integrate over k^-

Meissner Metz and Schlegel, JHEP 0908 (2009)

$$\int \frac{dz_- d^2z_T}{2\pi} e^{ixP^+z^- - k_T.z_T} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\Gamma\psi(z/2) \mid p, \Lambda\rangle_{z^+=0}$$

GTMDs

Integrate over k_T

$$\int \frac{dz_-}{2\pi} e^{ixP^+z^-} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\Gamma\psi(z/2) \mid p, \Lambda\rangle_{z^+=z_T=0}$$

GPDs

Generalized Lorentz Invariance Relations

- Parametrization of the quark quark correlator at different levels
- LIRs occur because the number of GPCFs is less than the number of GTMDs.

$$\mathcal{W}^{[\gamma^\mu]}_{\Lambda\Lambda'} = \frac{\bar{U}U}{M}(P^\mu A_1^F + k^\mu A_2^F + \Delta^\mu A_3^F) + i\frac{\bar{U}\sigma^{\mu k}U}{M}A_5^F + i\frac{\bar{U}\sigma^{\mu\Delta}U}{M}A_6^F + i\frac{\bar{U}\sigma^{k\Delta}U}{M^3}(P^\mu A_8^F + k^\mu A_9^F + \Delta^\mu A_{17}^F)$$

Integrate over k^-

Explicit k_T coefficient

$$W^{[\gamma^+]}_{\Lambda,\Lambda'} = \frac{1}{2M}\bar{U}(p',\Lambda')[F_{11} + \frac{i\sigma^{i+}k_T^i}{\bar{p}_+}F_{12} + \frac{i\sigma^{i+}\Delta_T^i}{\bar{p}_+}F_{13} + \frac{i\sigma^{ij}k_T^i\Delta_T^j}{M^2}F_{14}]U(p,\Lambda)$$

Integrate over k_T

$$F^{[\gamma^i]}_{\Lambda,\Lambda'} = \frac{1}{2(P^+)^2}\bar{U}\left[i\sigma^{+i}H_{2T} + \frac{\gamma^+\Delta_T^i}{2M}E_{2T} + \frac{P^+\Delta_T^i}{M^2}\tilde{H}_{2T} - \frac{P^+\gamma^i}{M}\tilde{E}_{2T}\right]U$$

$$\int \frac{d^4z}{2\pi}e^{ik.z}\langle p',\Lambda' \mid \bar{\psi}(-z/2)\Gamma\psi(z/2) \mid p,\Lambda\rangle$$

Generalized Lorentz Invariance Relations

- As the quark quark correlator is non-local, the parametrization depends on choice of gauge link
- At the completely unintegrated level, we have no knowledge of the light-cone direction for a straight gauge link, hence fewer functions occur at this level for this case as compared to staple gauge link case

Gauge link

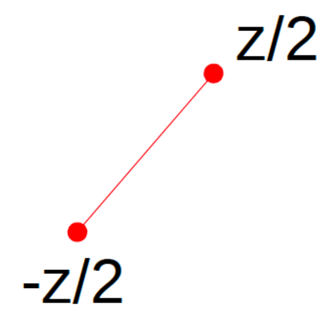
$$\int \frac{d^4 z}{2\pi} e^{ik.z} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\mathcal{U}\Gamma\psi(z/2) \mid p, \Lambda\rangle$$

Non local operator

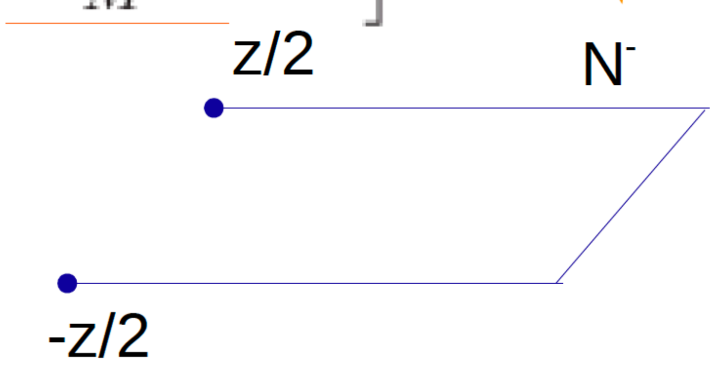
Generalized Lorentz Invariance Relations

$$\int \frac{d^4 z}{2\pi} e^{ik.z} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\mathcal{U}\Gamma\psi(z/2) \mid p, \Lambda\rangle$$

$$\begin{aligned}\mathcal{W}^{[\gamma^\mu]}_{\Lambda\Lambda'} \; &= \; \frac{\bar{U}U}{M}(P^\mu A_1^F + k^\mu A_2^F + \Delta^\mu A_3^F) + i\frac{\bar{U}\sigma^{\mu k}U}{M}A_5^F + i\frac{\bar{U}\sigma^{\mu\Delta}U}{M}A_6^F \\ &+ \; i\frac{\bar{U}\sigma^{k\Delta}U}{M^3}(P^\mu A_8^F + k^\mu A_9^F + \Delta^\mu A_{17}^F)\end{aligned}$$



$$\begin{aligned}&W^{[\gamma^\mu]}_{\lambda\lambda'}(P,k,\Delta,N;\eta) \\ &= \bar{u}(p',\lambda')\Big[\frac{P^\mu}{M}A_1^F + \frac{k^\mu}{M}A_2^F + \frac{\Delta^\mu}{M}A_3^F + \frac{N^\mu}{M}A_4^F + \frac{i\sigma^{\mu k}}{M}A_5^F + \frac{i\sigma^{\mu\Delta}}{M}A_6^F + \frac{i\sigma^{\mu N}}{M}A_7^F \\ &\quad + \frac{P^\mu\, i\sigma^{k\Delta}}{M^3}A_8^F + \frac{k^\mu\, i\sigma^{k\Delta}}{M^3}A_9^F + \frac{N^\mu\, i\sigma^{k\Delta}}{M^3}A_{10}^F + \frac{P^\mu\, i\sigma^{kN}}{M^3}A_{11}^F + \frac{k^\mu\, i\sigma^{kN}}{M^3}A_{12}^F \\ &\quad + \frac{N^\mu\, i\sigma^{kN}}{M^3}A_{13}^F + \frac{P^\mu\, i\sigma^{\Delta N}}{M^3}A_{14}^F + \frac{\Delta^\mu\, i\sigma^{\Delta N}}{M^3}A_{15}^F + \frac{N^\mu\, i\sigma^{\Delta N}}{M^3}A_{16}^F\Big]u(p,\lambda)\,, \qquad (2.19)\end{aligned}$$



An analogy

- The proton electromagnetic current is parameterized by the Dirac and Pauli form factors

$$J^\mu = e\overline{U}(P', S')\left[\gamma^\mu F_1 + \frac{i\sigma^{\mu\Delta}}{2M}F_2\right]U(P, S)$$

- We know that the vector GPDs should integrate to some combination of the same form factors irrespective of twist

$$\int dx H(x, 0, t) = F_1(t)$$

Generalized Lorentz Invariance Relations

- The same set of As describe the whole vector sector.

$$\boxed{\gamma^+}$$

$$F_{14}^{(1)} = \int d\sigma d\sigma' d\tau \frac{M^3}{2} J \left[A_8^F + x A_9^F\right]$$

$$J = \sqrt{x\sigma - \tau - \frac{x^2 P^2}{M^2} - \frac{\Delta_T^2 \sigma'^2}{M^2}}$$

$$H + E = \int d\sigma d\sigma' d\tau \frac{M^3}{J} \sigma' A_5^F + A_6^F + \left(\frac{\sigma}{2} - \frac{x P^2}{M^2}\right)\left(A_8^F + x A_9^F\right)$$

$$\boxed{\gamma_T^i}$$

$$\tilde{E}_{2T} = \int d\sigma d\sigma' d\tau \frac{M^3}{J}\left[\left(x\sigma - \tau - \frac{x^2 P^2}{M^2} - \frac{\Delta_T^2 \sigma'^2}{M^2}\right) A_9^F - \sigma' A_5^F - A_6^F\right]$$

$$\sigma \equiv \frac{2k.P}{M^2}, \qquad \tau \equiv \frac{k^2}{M^2}, \qquad \sigma' \equiv \frac{k.\Delta}{\Delta^2} = \frac{k_T.\Delta_T}{\Delta_T^2}$$

$$-\frac{dF_{14}^{(1)}}{dx} = \tilde{E}_{2T} + H + E$$

$$F_{14}^{(1)}(x) = \int_x^1 dy \left(\tilde{E}_{2T}(y) + H(y) + E(y)\right)$$

Distribution of OAM in x !

k_T^2 moment of a twist two function

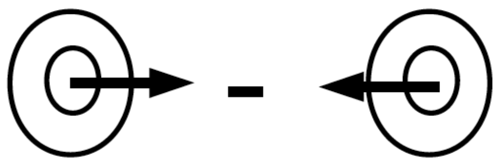
Twist three function

Generalized Lorentz Invariance Relations

Axial Vector

$$\frac{dG_{11}^{e(1)}}{dx} = -\left(2\tilde{H}_{2T}' + E_{2T}'\right) - \tilde{H}$$

$$\frac{dG_{12}^{e(1)}}{dx} = H_{2T}' - \frac{\Delta_T^2}{4M^2}E_{2T}' - \left(1 + \frac{\Delta_T^2}{2M^2}\right)\tilde{H}$$

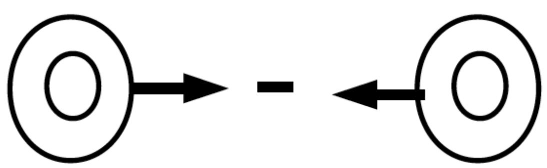


Twist two

Twist three

Vector

$$\frac{dF_{14}^{e(1)}}{dx} = \tilde{E}_{2T} + H + E$$



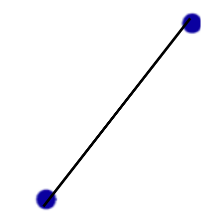
The GTMDs are complex in general.

$$X = X^e + iX^o$$

The imaginary part integrates to zero, on integration over k_T.

LIR violating term

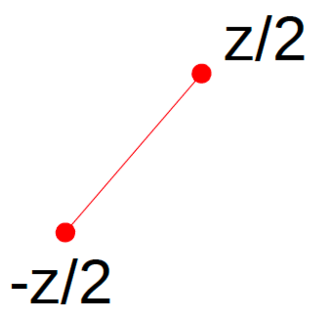
$$\frac{dF_{14}^{(1)}}{dx} = \widetilde{E}_{2T} + H + E$$



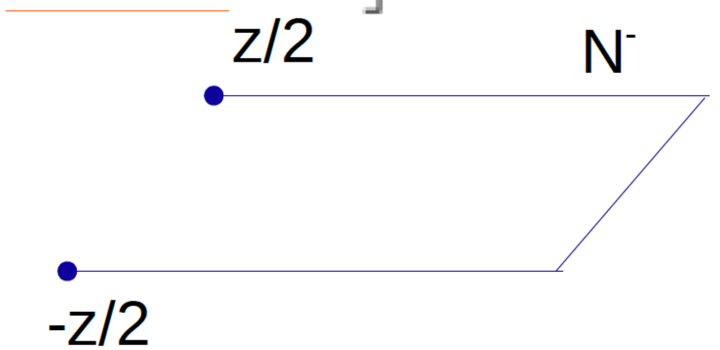
Generalized Lorentz Invariance Relations

$$\int \frac{d^4 z}{2\pi} e^{ik.z} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\mathcal{U}\Gamma\psi(z/2) \mid p, \Lambda\rangle$$

$$\begin{aligned} \mathcal{W}^{[\gamma^\mu]}_{\Lambda\Lambda'} &= \frac{\bar{U}U}{M}(P^\mu A_1^F + k^\mu A_2^F + \Delta^\mu A_3^F) + i\frac{\bar{U}\sigma^{\mu k}U}{M} A_5^F + i\frac{\bar{U}\sigma^{\mu\Delta}U}{M} A_6^F \\ &+ i\frac{\bar{U}\sigma^{k\Delta}U}{M^3}(P^\mu A_8^F + k^\mu A_9^F + \Delta^\mu A_{17}^F) \end{aligned}$$

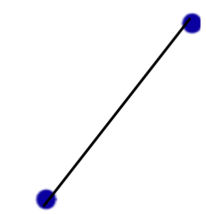


$$\begin{aligned} & W^{[\gamma^\mu]}_{\lambda\lambda'}(P, k, \Delta, N; \eta) \\ & = \bar{u}(p', \lambda') \left[\frac{P^\mu}{M} A_1^F + \frac{k^\mu}{M} A_2^F + \frac{\Delta^\mu}{M} A_3^F + \frac{N^\mu}{M} A_4^F + \frac{i\sigma^{\mu k}}{M} A_5^F + \frac{i\sigma^{\mu\Delta}}{M} A_6^F + \frac{i\sigma^{\mu N}}{M} A_7^F \right. \\ & \quad + \frac{P^\mu\, i\sigma^{k\Delta}}{M^3} A_8^F + \frac{k^\mu\, i\sigma^{k\Delta}}{M^3} A_9^F + \frac{N^\mu\, i\sigma^{k\Delta}}{M^3} A_{10}^F + \frac{P^\mu\, i\sigma^{kN}}{M^3} A_{11}^F + \frac{k^\mu\, i\sigma^{kN}}{M^3} A_{12}^F \\ & \quad \left. + \frac{N^\mu\, i\sigma^{kN}}{M^3} A_{13}^F + \frac{P^\mu\, i\sigma^{\Delta N}}{M^3} A_{14}^F + \frac{\Delta^\mu\, i\sigma^{\Delta N}}{M^3} A_{15}^F + \frac{N^\mu\, i\sigma^{\Delta N}}{M^3} A_{16}^F \right] u(p, \lambda) , \end{aligned} \quad (2.19)$$



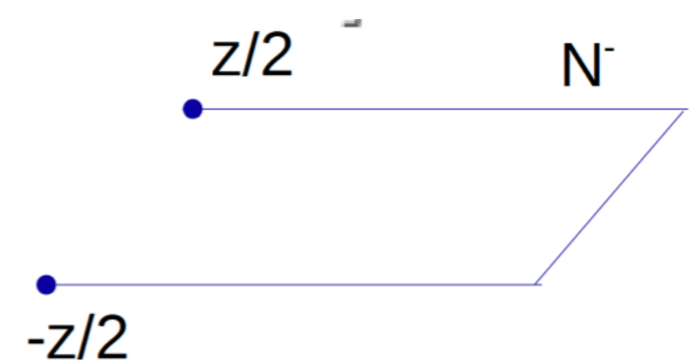
LIR violating term

$$\frac{dF_{14}^{(1)}}{dx} = \widetilde{E}_{2T} + H + E$$



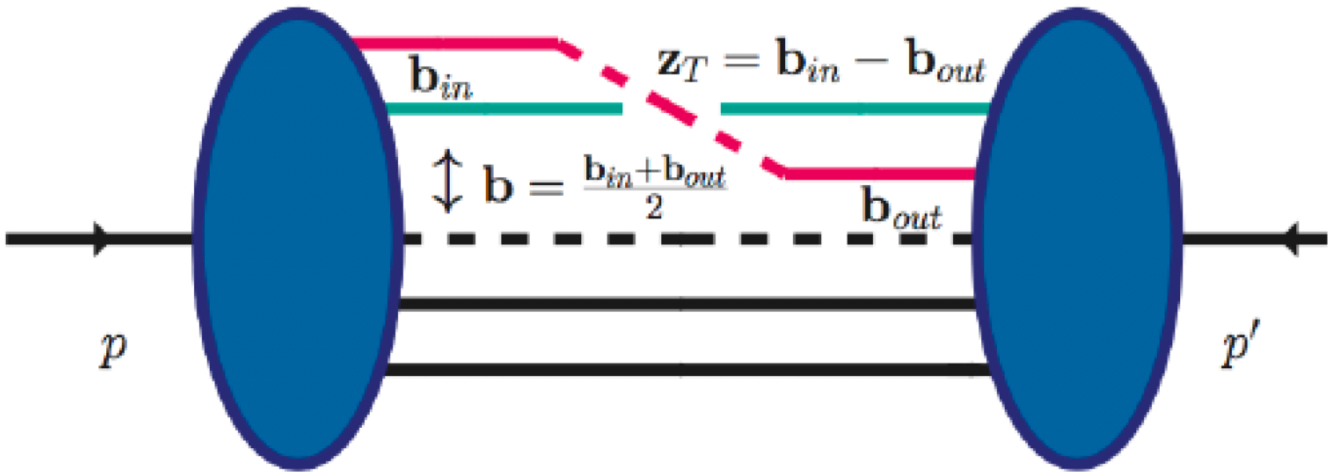
LIR violating term

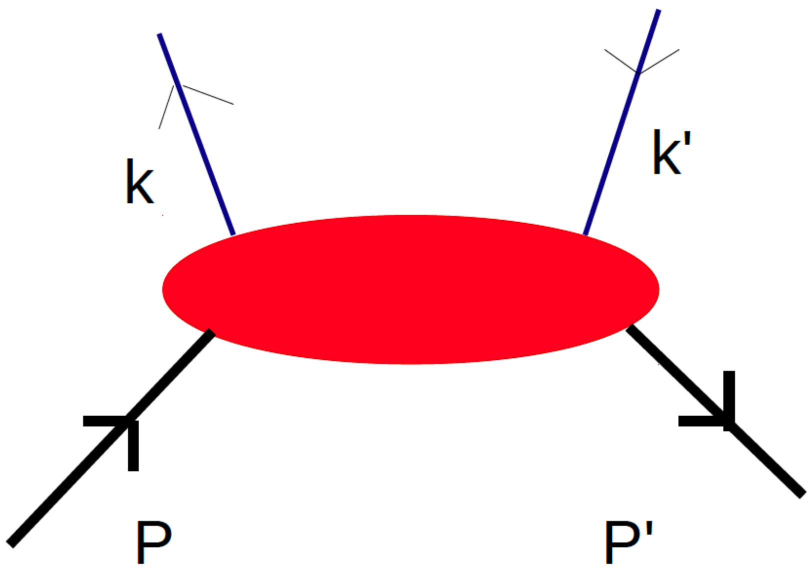
$$\frac{dF_{14}^{(1)}}{dx} = \widetilde{E}_{2T} + H + E + \mathcal{A}_{F_{14}}$$



$$\begin{aligned}
\mathcal{A}_{F_{14}}(x) &\equiv v^- \frac{(2P^+)^2}{M^2}\int d^2k_T \int dk^- \Bigg[\frac{k_T\cdot\Delta_T}{\Delta_T^2}(A_{11}+xA_{12})+A_{14} \\
&+ \frac{k_T^2\Delta_T^2-(k_T\cdot\Delta_T)^2}{\Delta_T^2}\left(\frac{\partial A_8}{\partial(k\cdot v)}+x\frac{\partial A_9}{\partial(k\cdot v)}\right)\Bigg] \\
&= \frac{dF_{14}^{(1)}}{dx} - \left.\frac{dF_{14}^{(1)}}{dx}\right|_{v=0}
\end{aligned}$$

Intrinsic Momentum vs Momentum Transfer Δ





$$k \longleftrightarrow z$$

$$\Delta \longleftrightarrow b$$

Courtoy et al PhysLett B731, 2013

Burkardt,Phys Rev D62, 2000

$$\int \frac{dz_-}{2\pi} e^{ixP^+z^-} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\Gamma\psi(z/2) \mid p, \Lambda\rangle_{z^+=z_T=0}$$

Equations of Motion Relations

How do we obtain these ?

$$
\begin{aligned}
(i\not{D} - m)\psi(z_{out}) &= (i\not{\partial} + g\not{A} - m)\psi(z_{out}) = 0, \\
\bar{\psi}(z_{in})(i\overleftarrow{\not{D}} + m) &= \bar{\psi}(z_{in})(i\overleftarrow{\not{\partial}} - g\not{A} + m) = 0
\end{aligned}
$$

Equations of Motion Relations

How do we obtain these ?

$$\begin{aligned}
\mathcal{U} i\sigma^{i+}\gamma_5 (i\not{D} - m)\psi(z_{out}) &= \mathcal{U} i\sigma^{i+}\gamma_5 (i\not{\partial} + g\not{A} - m)\psi(z_{out}) = 0, \\
\bar{\psi}(z_{in})(i\overleftarrow{\not{D}} + m) i\sigma^{i+}\gamma_5 \mathcal{U} &= \bar{\psi}(z_{in})(i\overleftarrow{\not{\partial}} - g\not{A} + m) i\sigma^{i+}\gamma_5 \mathcal{U} = 0
\end{aligned}$$

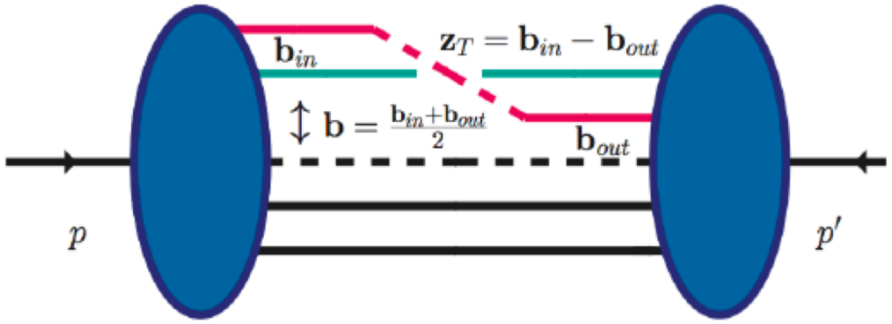
Equations of Motion Relations

How do we obtain these ?

$$\mathcal{U} i\sigma^{i+}\gamma_5 (i\not{D} - m)\psi(z_{out}) = \mathcal{U} i\sigma^{i+}\gamma_5 (i\not{\partial} + g\not{A} - m)\psi(z_{out}) = 0,$$

$$\bar{\psi}(z_{in})(i\overleftarrow{\not{D}} + m) i\sigma^{i+}\gamma_5 \mathcal{U} = \bar{\psi}(z_{in})(i\overleftarrow{\not{\partial}} - g\not{A} + m) i\sigma^{i+}\gamma_5 \mathcal{U} = 0$$

$$b = \frac{z_{in} + z_{out}}{2}, \qquad z = z_{in} - z_{out}$$



$$\int db^- d^2 b_T e^{-ib\cdot\Delta} \int dz^- d^2 z_T e^{-ik\cdot z} \langle p', \Lambda' | \bar{\psi} \left[(i\overleftarrow{D} + m) i\sigma^{i+}\gamma^5 \pm i\sigma^{i+}\gamma^5 (i\overrightarrow{D} - m) \right] \psi | p, \Lambda \rangle = 0$$

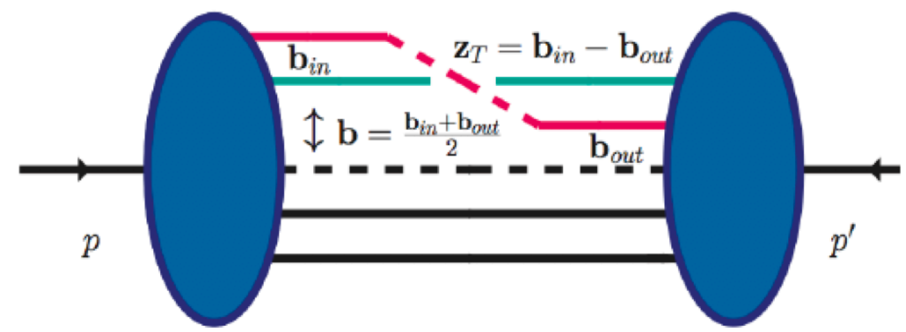
Equations of Motion R

Crucial for understanding qgq contribution to GPDs!!

How do we obtain these ?

$$\begin{aligned}
\mathcal{U} i\sigma^{i+}\gamma_5 (i\not{D} - m)\psi(z_{out}) &= \mathcal{U} i\sigma^{i+}\gamma_5 (i\not{\partial} + g\not{A} - m)\psi(z_{out}) = 0, \\
\bar{\psi}(z_{in})(i\overleftarrow{\not{D}} + m) i\sigma^{i+}\gamma_5 \mathcal{U} &= \bar{\psi}(z_{in})(i\overleftarrow{\not{\partial}} - g\not{A} + m) i\sigma^{i+}\gamma_5 \mathcal{U} = 0
\end{aligned}$$

$$b = \frac{z_{in} + z_{out}}{2}, \qquad z = z_{in} - z_{out}$$



$$\int db^- d^2 b_T e^{-ib\cdot\Delta} \int dz^- d^2 z_T e^{-ik\cdot z} \langle p', \Lambda' | \bar{\psi} \left[(i\overleftarrow{D} + m) i\sigma^{i+}\gamma^5 \pm i\sigma^{i+}\gamma^5 (i\overrightarrow{D} - m) \right] \psi | p, \Lambda \rangle = 0$$

EoM relations for Orbital Angular Momentum

no mass term!

$$x\tilde{E}_{2T} = -\tilde{H} + \boxed{2\int d^2k_T \frac{k_T^2 sin^2\phi}{M^2} F_{14}} + \frac{\Delta^i}{\Delta_T^2}\int d^2k_T(\mathcal{M}^{i,S}_{++} - \mathcal{M}^{i,S}_{--})$$

Twist 3

Twist 2

Genuine Twist 3 (explicit gluon)

$$\boxed{\frac{dF_{14}^{(1)}}{dx} = \tilde{E}_{2T} + H + E}$$

$$\mathcal{M}^{i,S}_{\Lambda'\Lambda} = \frac{i}{4}\int \frac{dz^- d^2z_T}{(2\pi)^3} e^{ixP^+z^- - ik_T\cdot z_T} \langle p',\Lambda' \mid \overline{\psi}\left(-\frac{z}{2}\right)\left[(\overrightarrow{\partial} - ig\not{A})\mathcal{U}\Gamma\Big|_{-z/2} + \Gamma\mathcal{U}(\overleftarrow{\partial} + ig\not{A})\Big|_{z/2}\right]\psi\left(\frac{z}{2}\right) \mid p,\Lambda\rangle_{z^+=0}$$

$$\int dx \int d^2k_T\, \mathcal{M}^{i,S}_{\Lambda'\Lambda} = i\epsilon^{ij} g v^- \frac{1}{2P^+}\int_0^1 ds\, \langle p',\Lambda'|\bar{\psi}(0)\gamma^+ U(0,sv)F^{+j}(sv)U(sv,0)\psi(0)|p,\Lambda\rangle$$

Wandzura Wilczek Relations

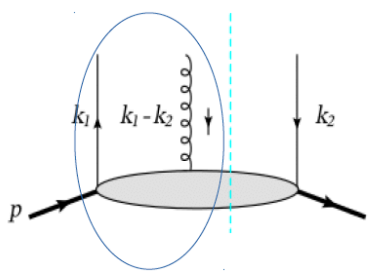
$$\tilde{E}_{2T} = -\int_x^1 \frac{dy}{y}(H+E) + \left[\frac{\tilde{H}}{x} - \int_x^1 \frac{dy}{y^2}\tilde{H}\right] + \left[\frac{1}{x}\mathcal{M}_{F_{14}} - \int_x^1 \frac{dy}{y^2}\mathcal{M}_{F_{14}}\right]$$

Twist three vector GPD

Twist two

Axial vector GPD contributes to a vector GPD

Genuine Tw 3



AR, Engelhardt and Liuti PRD 98 (2018)

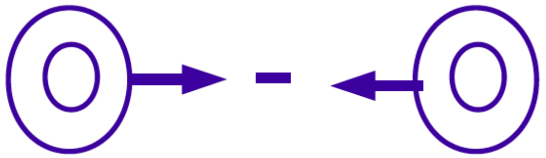
$$g_2(x) = -g_1(x) + \int_x^1 \frac{dy}{y} g_1(x) + \bar{g}_2(x)$$

Twist three PDF

Twist two

Genuine Tw 3

Moments of twist three GPDs -Quark gluon structure



$$\int dx \widetilde{E}_{2T} = -\int dx (H+E) \quad \Rightarrow \int dx \left(\widetilde{E}_{2T} + H + E \right) = 0$$

$$\int dx\, \underline{x}\, \widetilde{E}_{2T} = -\frac{1}{2}\int dx\, x (H+E) - \frac{1}{2}\int dx \widetilde{H}$$

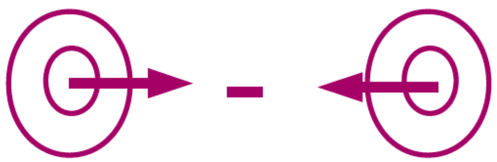
OAM

$$\int dx\, \underline{x}^2\, \widetilde{E}_{2T} = -\frac{1}{3}\int dx\, x^2 (H+E) - \frac{2}{3}\int dx\, x \widetilde{H} - \boxed{\frac{2}{3}\int dx\, x\, \mathcal{M}_{F_{14}}\Big|_{v=0}}$$

Genuine Twist Three

$$\int dx\, x \int d^2 k_T\, \mathcal{M}^{i,S}_{\Lambda'\Lambda} = \frac{ig}{4(P^+)^2} \langle p', \Lambda' | \bar{\psi}(0) \gamma^+ \gamma^5 F^{+i}(0) \psi(0) | p, \Lambda \rangle$$

Moments of twist three GPDs -Quark gluon structure



$$\int dx \left(E'_{2T} + 2\widetilde{H}'_{2T}\right) = -\int dx \widetilde{H} \qquad \Rightarrow \int dx \left(E'_{2T} + 2\widetilde{H}'_{2T} + \widetilde{H}\right) = 0$$

$$\int dx\underline{x} \left(E'_{2T} + 2\widetilde{H}'_{2T}\right) = -\frac{1}{2}\int dx x \widetilde{H} - \frac{1}{2}\int dx H + \boxed{\frac{m}{2M}\int dx (E_T + 2\widetilde{H}_T)}$$

mass term

$$\int dx\, \underline{x^2} \left(E'_{2T} + 2\widetilde{H}'_{2T}\right) = -\frac{1}{3}\int dx x^2 \widetilde{H} - \frac{2}{3}\int dx x H + \boxed{\frac{2m}{3M}\int dx x (E_T + 2\widetilde{H}_T)}$$

$$- \boxed{\frac{2}{3}\int dx x \mathcal{M}_{G_{11}}\Big|_{v=0}}$$

Genuine Twist Three d_2

$$\int dx\, x \int d^2 k_T\, \mathcal{M}^{i,A}_{\Lambda'\Lambda} = \frac{g}{4(P^+)^2}\epsilon^{ij}\langle p', \Lambda' | \bar{\psi}(0)\gamma^+ F^{+j}(0)\psi(0) | p, \Lambda\rangle$$

Back to staple gauge link and LIR violating term

Equation of Motion

$$0 = \boxed{x\widetilde{E}_{2T} + \widetilde{H}} - F_{14}^{(1)} + \int d^2k_T \frac{\Delta^i}{\Delta_T^2} \left(\mathcal{M}_{++}^{i,S} - \mathcal{M}_{--}^{i,S}\right)$$

GPDs are not affected by staple

Back to staple gauge link and LIR violating term

Equation of Motion

$$0 = \boxed{x\widetilde{E}_{2T} + \widetilde{H}} - F_{14}^{(1)} + \int d^2k_T \frac{\Delta^i}{\Delta_T^2} \left(\mathcal{M}_{++}^{i,S} - \mathcal{M}_{--}^{i,S}\right)$$

GPDs are not affected by staple

Eliminated using LIR

$$\frac{dF_{14}^{(1)}}{dx} = \widetilde{E}_{2T} + H + E + \mathcal{A}_{F_{14}}$$

Back to staple gauge link and LIR violating term

Equation of Motion

$$0 = \boxed{x\widetilde{E}_{2T} + \widetilde{H}} - F_{14}^{(1)} + \int d^2k_T \frac{\Delta^i}{\Delta_T^2}\left(\mathcal{M}_{++}^{i,S} - \mathcal{M}_{--}^{i,S}\right)$$

GPDs are not affected by staple

Eliminated using LIR

$$\frac{dF_{14}^{(1)}}{dx} = \widetilde{E}_{2T} + H + E + \mathcal{A}_{F_{14}}$$

$$\mathcal{A}_{F_{14}}(x) = \frac{d}{dx}\left(\mathcal{M}_{F_{14}} - \mathcal{M}_{F_{14}}|_{v=0}\right)$$

Connects completely unintegrated quark quark correlator to qgq correlator!

$$\begin{aligned}\mathcal{A}_{F_{14}}(x) \equiv\ & v^- \frac{(2P^+)^2}{M^2}\int d^2k_T \int dk^- \left[\frac{k_T\cdot\Delta_T}{\Delta_T^2}(A_{11} + xA_{12}) + A_{14}\right. \\ + & \left.\frac{k_T^2\Delta_T^2 - (k_T\cdot\Delta_T)^2}{\Delta_T^2}\left(\frac{\partial A_8}{\partial(k\cdot v)} + x\frac{\partial A_9}{\partial(k\cdot v)}\right)\right]\end{aligned}$$

B. Kriesten, S. Liuti (in progress)

What violates the WW relations?

$$\widetilde{E}_{2T} = -\int_x^1 \frac{dy}{y}(H+E) - \left[\frac{\widetilde{H}}{x} - \int_x^1 \frac{dy}{y^2}\widetilde{H}\right] - \left[\frac{1}{x}\mathcal{M}_{F_{14}} - \int_x^1 \frac{dy}{y^2}\mathcal{M}_{F_{14}}\right] - \int_x^1 \frac{dy}{y}\mathcal{A}_{F_{14}}$$

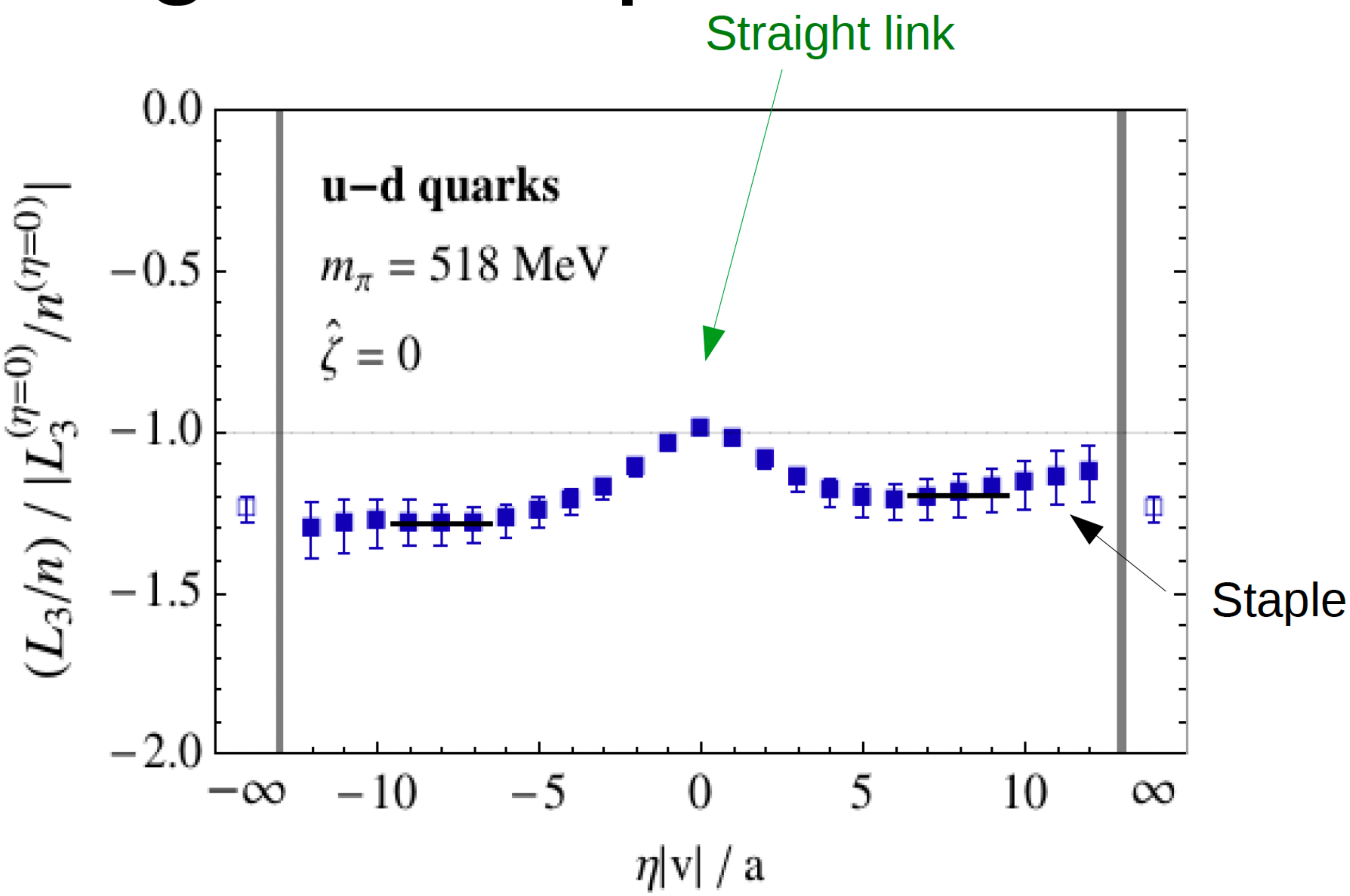
$$\begin{aligned} 2\widetilde{H}'_{2T} + E'_{2T} = & -\int_x^1 \frac{dy}{y}\widetilde{H} - \left[\frac{H}{x} - \int_x^1 \frac{dy}{y^2}H\right] + \frac{m}{M}\left[\frac{1}{x}(2\widetilde{H}_T + E_T) - \int_x^1 \frac{dy}{y^2}(2\widetilde{H}_T + E_T)\right] \\ & - \left[\frac{1}{x}\mathcal{M}_{G_{11}} - \int_x^1 \frac{dy}{y^2}\mathcal{M}_{G_{11}}\right] + \int_x^1 \frac{dy}{y}\mathcal{A}_{G_{11}} \end{aligned}$$

$$\widetilde{E}_{2T} = \widetilde{E}_{2T}^{WW} + \widetilde{E}_{2T}^{(3)} + \widetilde{E}_{2T}^{LIR}$$

$$\overline{E}'_{2T} = \overline{E}'^{WW}_{2T} + \overline{E}'^{(3)}_{2T} + \overline{E}'^{LIR}_{2T} + \overline{E}'^{m}_{2T}$$

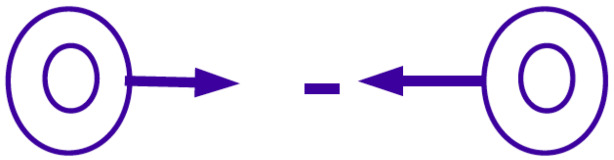
Calculating the torque from Lattice

F_{14}



Michael Engelhardt

Phys. Rev. D95 (2017)



Longitudinally polarized proton

$$\mathcal{L}_{JM} - \mathcal{L}_{Ji} = \mathcal{T}$$

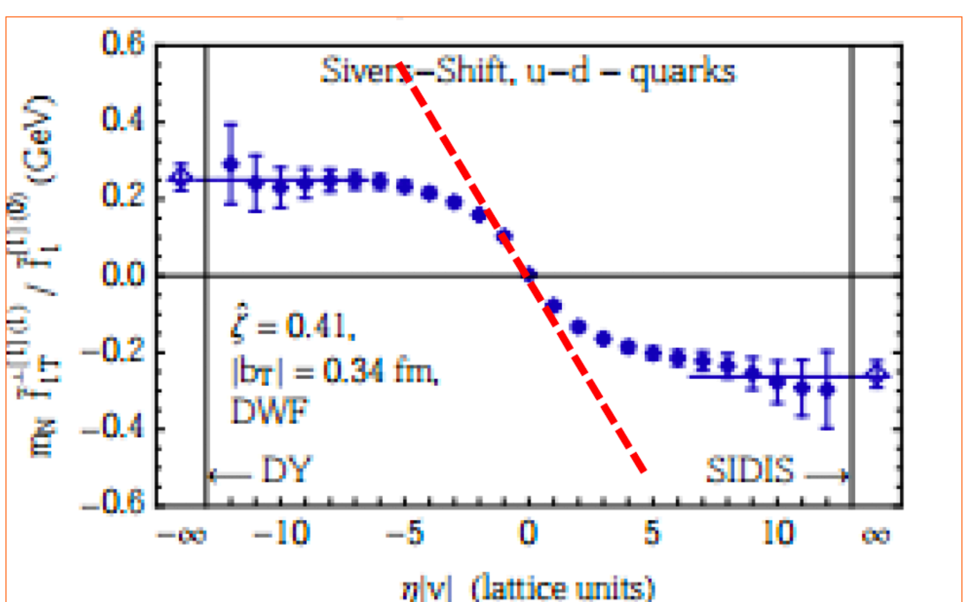
Torque!

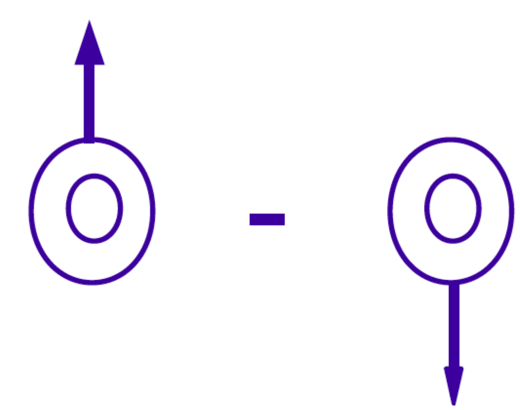
Calculating the force from Lattice data – Sivers function

$$\frac{d}{dv^-}\int dx F_{12}^{(1)}\bigg|_{v^-=0} = \frac{d}{dv^-}\int dx \mathcal{M}_{F_{12}}\bigg|_{v^-=0} = i(2P^+)\int dx\, x\frac{\Delta_i}{\Delta_T^2}\left(\mathcal{M}_{++}^{i,A} - \mathcal{M}_{--}^{i,A}\right) = \mathcal{M}_{G_{12}}^{n=3}$$

$$\frac{d}{dv^-}\int dx G_{12}^{(1)}\bigg|_{v^-=0} = \frac{d}{dv^-}\int dx \mathcal{M}_{G_{12}}\bigg|_{v^-=0} = i(2P^+)\int dx\, x\frac{\Delta_i}{\Delta_T^2}\left(\mathcal{M}_{++}^{i,S} + \mathcal{M}_{--}^{i,S}\right) = \mathcal{M}_{F_{12}}^{n=3}$$

The derivative with respect to the gauge link direction gives the force!





Transversely polarized proton

W. Armstrong, F. Aslan, M. Burkardt, M. Engelhardt, B. Kriesten, S. Liuti and A. Rajan (in progress)

Extending to the Chiral Odd Sector

$$\boxed{\frac{dh_{1L}^{\perp(1)}}{dx} = h_1 - h_L}$$

$$\Big\downarrow \text{Off forward}$$

LIR

$$\boxed{\frac{dH_{17}^{(1)}}{dx} = H_T - \frac{\Delta_T^2}{4M^2}E_T - \tilde{H}_2'}$$

$$\boxed{-x\tilde{H}_2' - \int d^2k_T \frac{(k_T \times \Delta_T)^2}{M^2\Delta_T^2} H_{17} + \frac{m}{M}\tilde{H} - \frac{1}{2M}\int d^2k_T \left(\mathcal{M}_{++}^{\gamma^+\gamma^5 A} - \mathcal{M}_{--}^{\gamma^+\gamma^5 A}\right) = 0}$$

EoM

$$\Gamma = \gamma^+, \gamma^+\gamma^5$$

for connecting chiral odd GPDs and GTMDs

$$0 = \int \frac{dz_{in}^- \, d^2 z_{in,T}}{(2\pi)^3} \int \frac{dz_{out}^- \, d^2 z_{out,T}}{(2\pi)^3} e^{ik(z_{out}-z_{in})+i\Delta(z_{out}+z_{in})/2}$$

$$\cdot \langle p', \Lambda' \mid \bar{\psi}(z_{in}) \left[(i\overleftarrow{\not{D}} + m)\Gamma\mathcal{U} \pm \Gamma\mathcal{U}(i\not{D} - m) \right] \psi(z_{out}) \mid p, \Lambda \rangle \Big|_{z_{in}^+ = z_{out}^+ = 0}$$

LIRs between Twist 3 and Twist 4 distributions

$$W^{[\gamma_T^j]} = \frac{M}{P^+}\left[\left(\frac{k_T^j}{M}f^\perp + \frac{i\Lambda\epsilon^{ij}k_T^i}{M}f_L^\perp\right)\delta_{\Lambda\Lambda'} + \left(\frac{k^j(\Lambda k_1 + ik_2)}{M^2}f_T^\perp + (\Lambda\delta_{j1} + i\delta_{j2})f_T'\right)\delta_{-\Lambda\Lambda'}\right]$$

$$\xrightarrow{\int d^2k_T} -\frac{M}{P^+}(\Lambda\delta_{j1} + i\delta_{j2})H_{2T}\delta_{-\Lambda\Lambda'}$$

$$W^{[\gamma_T^j\gamma^5]} = \frac{M}{P^+}\left[\left(\frac{i\epsilon^{ij}k_T^i}{M}g^\perp + \Lambda\frac{k_T^j}{M}g_L^\perp\right)\delta_{\Lambda\Lambda'} + \left((\Lambda\delta_{j1} + i\delta_{j2})g_T' + \frac{k_T^j(\Lambda k_1 + ik_2)}{M^2}g_T^\perp\right)\delta_{-\Lambda\Lambda'}\right]$$

$$\xrightarrow{\int d^2k_T} \frac{M(\delta_{j1} + i\Lambda\delta_{j2})}{P^+}g_T\delta_{-\Lambda\Lambda'}$$

LIRs between Twist 3 and Twist 4 distributions

$$W^{[\gamma_T^j]} = \frac{M}{P^+}\left[\left(\frac{k_T^j}{M}f^\perp + \frac{i\Lambda\epsilon^{ij}k_T^i}{M}f_L^\perp\right)\delta_{\Lambda\Lambda'} + \left(\frac{k^j(\Lambda k_1 + ik_2)}{M^2}f_T^\perp + (\Lambda\delta_{j1} + i\delta_{j2})f_T'\right)\delta_{-\Lambda\Lambda'}\right]$$

$$\xrightarrow{\int d^2k_T} -\frac{M}{P^+}(\Lambda\delta_{j1} + i\delta_{j2})H_{2T}\delta_{-\Lambda\Lambda'}$$

$$W^{[\gamma_T^j\gamma^5]} = \frac{M}{P^+}\left[\left(\frac{i\epsilon^{ij}k_T^i}{M}g^\perp + \Lambda\frac{k_T^j}{M}g_L^\perp\right)\delta_{\Lambda\Lambda'} + \left((\Lambda\delta_{j1} + i\delta_{j2})g_T' + \frac{k_T^j(\Lambda k_1 + ik_2)}{M^2}g_T^\perp\right)\delta_{-\Lambda\Lambda'}\right]$$

$$\xrightarrow{\int d^2k_T} \frac{M(\delta_{j1} + i\Lambda\delta_{j2})}{P^+}g_T\delta_{-\Lambda\Lambda'}$$

$$W^{[\gamma^-]} = \frac{M^2}{(P^+)^2}\left[f_3\delta_{\Lambda\Lambda'} + \frac{\Lambda k_1 + ik_2}{M}f_{3T}^\perp\delta_{-\Lambda\Lambda'}\right]$$

$$\xrightarrow{\int d^2k_T} \frac{M^2}{(P^+)^2}f_3\delta_{\Lambda\Lambda'}$$

$$W^{[\gamma^-\gamma^5]} = \frac{M^2}{(P^+)^2}\left[\Lambda g_{3L}\delta_{\Lambda\Lambda'} + \frac{k_1 + i\Lambda k_2}{M}g_{3T}\delta_{-\Lambda\Lambda'}\right]$$

$$\xrightarrow{\int d^2k_T} \frac{M^2}{(P^+)^2}\Lambda g_{3L}\delta_{\Lambda\Lambda'}$$

LIRs between Twist 3 and Twist 4 distributions

$$W^{[\gamma_T^j]} = \frac{M}{P^+}\left[\left(\frac{k_T^j}{M} f^\perp + \frac{i\Lambda\epsilon^{ij}k_T^i}{M} f_L^\perp\right)\delta_{\Lambda\Lambda'} + \left(\frac{k^j(\Lambda k_1 + ik_2)}{M^2} f_T^\perp + (\Lambda\delta_{j1} + i\delta_{j2}) f_T'\right)\delta_{-\Lambda\Lambda'}\right]$$

$$\xrightarrow{\int d^2k_T} -\frac{M}{P^+}(\Lambda\delta_{j1} + i\delta_{j2}) H_{2T}\,\delta_{-\Lambda\Lambda'}$$

$$W^{[\gamma_T^j\gamma^5]} = \frac{M}{P^+}\left[\left(\frac{i\epsilon^{ij}k_T^i}{M} g^\perp + \Lambda\frac{k_T^j}{M} g_L^\perp\right)\delta_{\Lambda\Lambda'} + \left((\Lambda\delta_{j1} + i\delta_{j2}) g_T' + \frac{k_T^j(\Lambda k_1 + ik_2)}{M^2} g_T^\perp\right)\delta_{-\Lambda\Lambda'}\right]$$

$$\xrightarrow{\int d^2k_T} \frac{M(\delta_{j1} + i\Lambda\delta_{j2})}{P^+} g_T\,\delta_{-\Lambda\Lambda'}$$

$$W^{[\gamma^-]} = \frac{M^2}{(P^+)^2}\left[\quad\right]$$

Measured in experiment F_L !

$$\xrightarrow{\int d^2k_T} \frac{M^2}{(P^+)^2} f_3\,\delta_{\Lambda\Lambda'}$$

$$W^{[\gamma^-\gamma^5]} = \frac{M^2}{(P^+)^2}\left[\Lambda g_{3L}\,\delta_{\Lambda\Lambda'} + \frac{k_1 + i\Lambda k_2}{M} g_{3T}\,\delta_{-\Lambda\Lambda'}\right]$$

$$\xrightarrow{\int d^2k_T} \frac{M^2}{(P^+)^2}\Lambda g_{3L}\,\delta_{\Lambda\Lambda'}$$

LIRs between Twist 3 and Twist 4 distributions

$$W^{[\gamma_T^j]} = \frac{M}{P^+}\left[\left(\frac{k_T^j}{M} f^\perp + \frac{i\Lambda\epsilon^{ij}k_T^i}{M} f_L^\perp\right)\delta_{\Lambda\Lambda'} + \left(\frac{k^j(\Lambda k_1 + ik_2)}{M^2} f_T^\perp + (\Lambda\delta_{j1} + i\delta_{j2}) f_T'\right)\delta_{-\Lambda\Lambda'}\right]$$

$$\xrightarrow{\int d^2k_T} -\frac{M}{P^+}(\Lambda\delta_{j1} + i\delta_{j2}) H_{2T}\delta_{-\Lambda\Lambda'}$$

$$W^{[\gamma_T^j\gamma^5]} = \frac{M}{P^+}\left[\left(\frac{i\epsilon^{ij}k_T^i}{M} g^\perp + \Lambda\frac{k_T^j}{M} g_L^\perp\right)\delta_{\Lambda\Lambda'} + \left((\Lambda\delta_{j1} + i\delta_{j2}) g_T' + \frac{k_T^j(\Lambda k_1 + ik_2)}{M^2} g_T^\perp\right)\delta_{-\Lambda\Lambda'}\right]$$

$$\xrightarrow{\int d^2k_T} \frac{M(\delta_{j1} + i\Lambda\delta_{j2})}{P^+} g_T\delta_{-\Lambda\Lambda'}$$

$$\boxed{2f_3 - f_1 = \frac{df^{\perp(1)}}{dx}}$$

$$\boxed{g_T + g_{3L} - \frac{1}{2}g_{1L} = \frac{1}{2}\frac{dg_L^{\perp(1)}}{dx}}$$

$$W^{[\gamma^-]} = \frac{M^2}{(P^+)^2}\left[\quad\right]$$

Measured in experiment F_L !

$$\xrightarrow{\int d^2k_T} \frac{M^2}{(P^+)^2} f_3\delta_{\Lambda\Lambda'}$$

$$W^{[\gamma^-\gamma^5]} = \frac{M^2}{(P^+)^2}\left[\Lambda g_{3L}\delta_{\Lambda\Lambda'} + \frac{k_1 + i\Lambda k_2}{M} g_{3T}\delta_{-\Lambda\Lambda'}\right]$$

$$\xrightarrow{\int d^2k_T} \frac{M^2}{(P^+)^2}\Lambda g_{3L}\delta_{\Lambda\Lambda'}$$

Summary

- Showed a way of deriving Wandzura Wilczek relations. Allows us to write out the quark gluon quark contribution to twist three GPDs precisely. Study the x dependence.

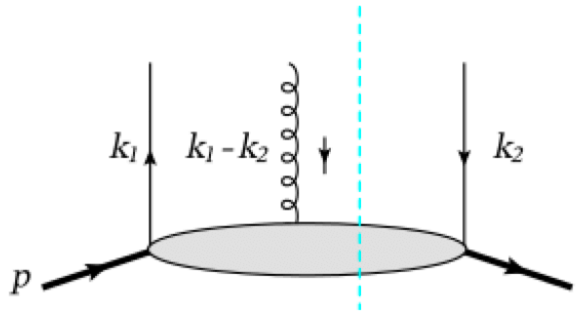
- Gluons play a key role in describing the properties of the nucleon.

Thanks!

Collinear Picture : Transverse Quark Current, Higher Twist

$$\bar{\psi}(-z/2)\gamma^{+}\psi(z/2) \longrightarrow$$ Leading order quark current

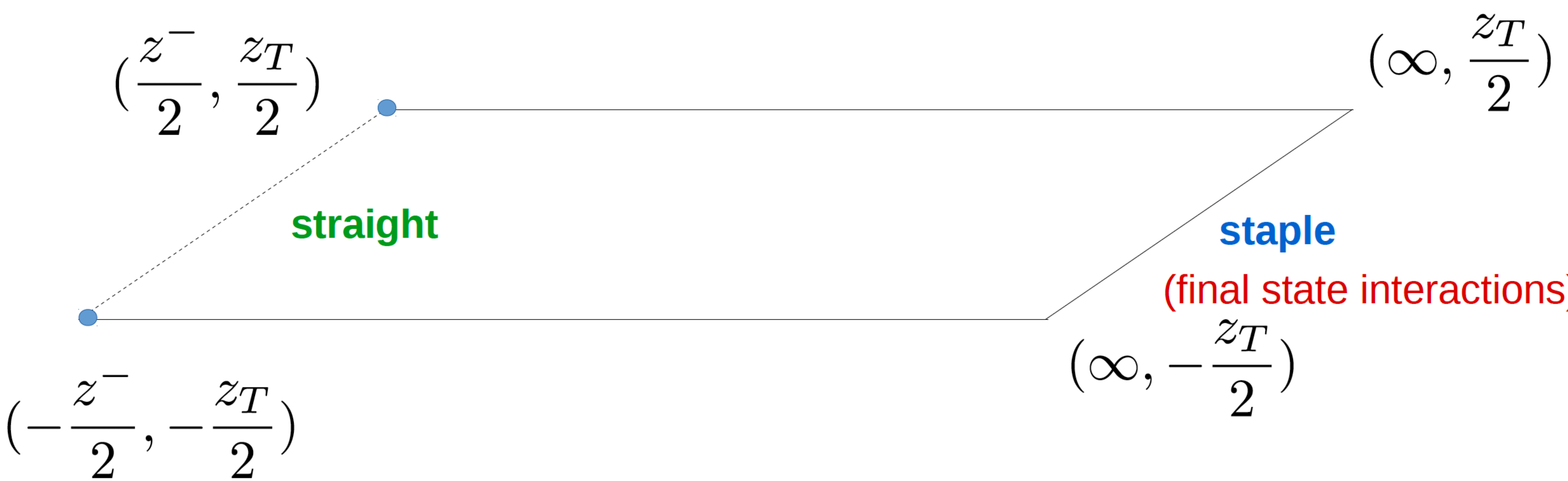
$$\bar{\psi}(-z/2)\gamma_T^{i}\psi(z/2) \longrightarrow$$ Transverse quark current, implicitly involves quark gluon interactions



Probabilistic parton model interpretation works well at leading order, with transverse quark projection operator need to include quark gluon interactions

Both in Collinear Picture

Quark gluon quark contributions



$$\int dx \int d^2k_T\, \mathcal{M}^{i,S}_{\Lambda'\Lambda} \quad =$$

$$i\epsilon^{ij} g v^- \frac{1}{2P^+} \int_0^1 ds\, \langle p', \Lambda' | \bar{\psi}(0)\gamma^+ U(0, sv) F^{+j}(sv) U(sv, 0)\psi(0) | p, \Lambda\rangle$$

$$\int dx \int d^2k_T\, \mathcal{M}^{i,A}_{\Lambda'\Lambda} \quad =$$

$$-g v^- \frac{1}{2P^+} \int_0^1 ds\, \langle p', \Lambda' | \bar{\psi}(0)\gamma^+\gamma^5 U(0, sv) F^{+i}(sv) U(sv, 0)\psi(0) | p, \Lambda\rangle$$