Physics insights from quasi-PDFs

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Work within ETMC: Extended Twisted Mass Collaboration

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Introduction

Light-cone and off light-cone distributions

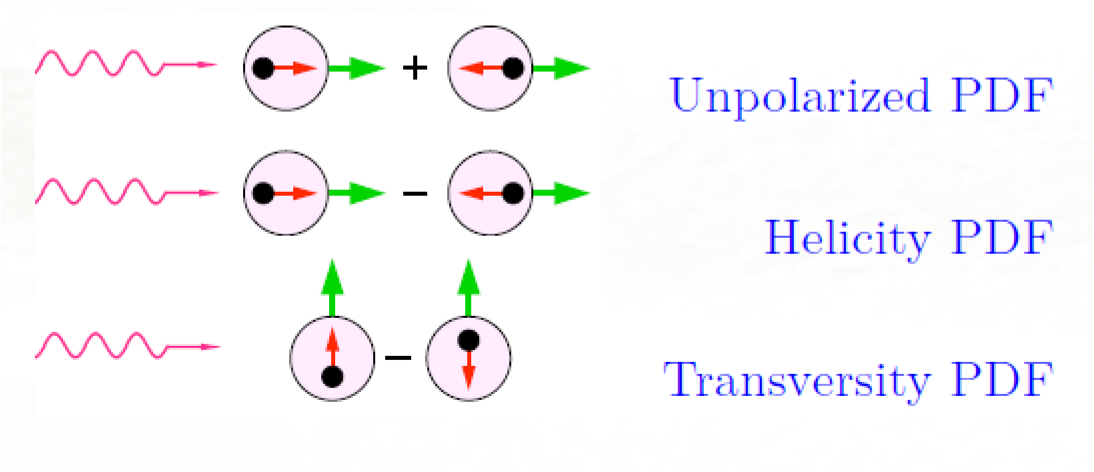
Sea asymmetries and chiral loops

Transversity

Summary

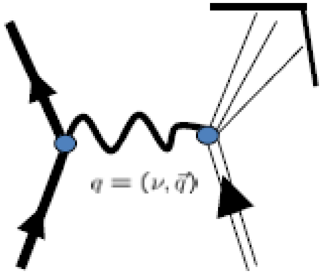
Introduction

Complete set of twist-2 parton distribution functions



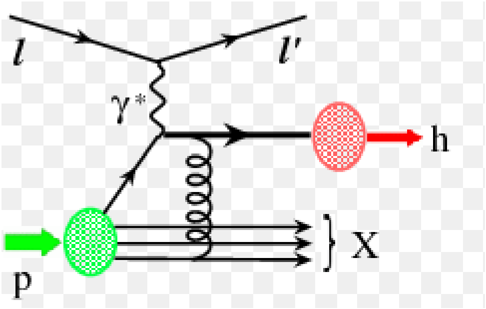
Cross sections are measured

Totally inclusive



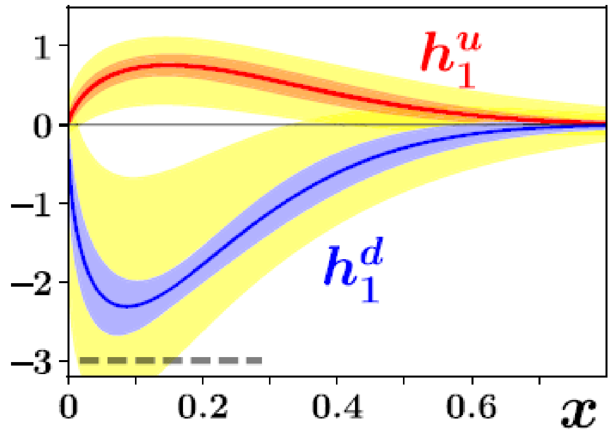
Have access to the chiral-even distributions $f_1(x)$ (unpolarized) and $g_1(x)$ (helicity)

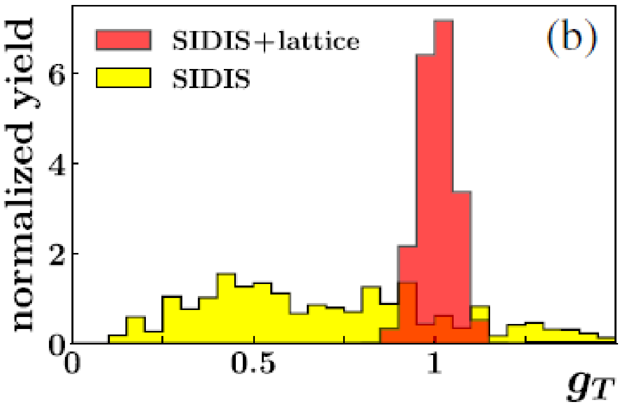
Semi-inclusive



Have access to the chiral-odd distribution $h_1(x)$ (transversity). Naturally more difficult to obtain data on transversity

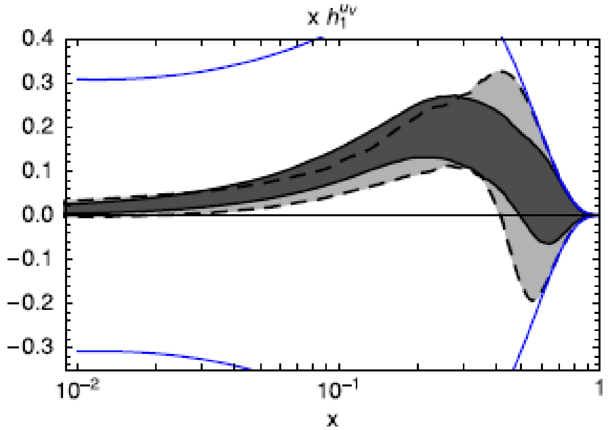
Transversity: two recent extractions

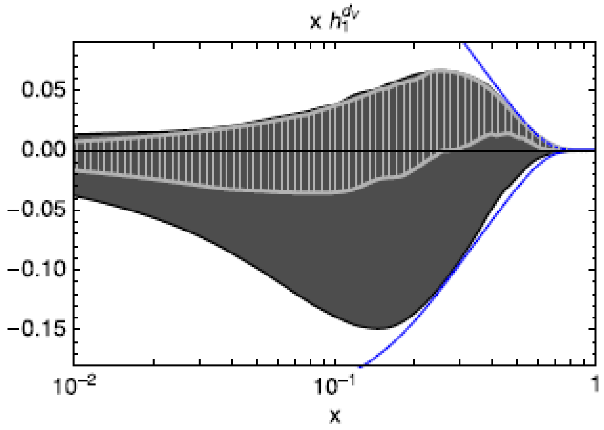




$$g_T = \int_0^1 dx \left(h_1^u(x) - h_1^d(x) \right) = 1.0(1)$$

H.-W. Lin PRL 120, 152502 (2018)





$$g_T = 0.53(25)$$

Radici and Bacchetta PRL 120, 192001 (2018)

Can we have an *ab initio* calculation of $h_1^q(x)$?

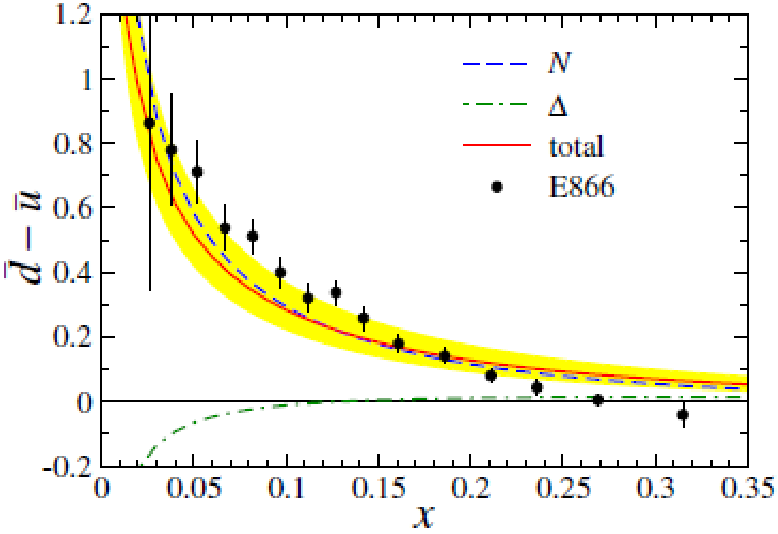
Nucleon sea

Gluons are flavour blind

But, from NMC data

$$\int_0^1 \frac{dx}{x}\left(F_2^p(x) - F_2^n(x)\right) = \frac{1}{3} + \frac{2}{3}\int_0^1 dx(\bar{u}(x) - \bar{d}(x)) = 0.235 \pm 0.026$$

And from E886 data



$\bar{d}(x) \neq \bar{u}(x)$ has a nonperturbative origin

What is the physics behind it? Chiral Loops?

P. C. Barry, N. Sato, W. Melnitchouk and C. R. Ji, PRL **121** (2018) no.15, 152001

Light-cone and off light-cone PDFs

Quark distribution is given by a light-cone correlation

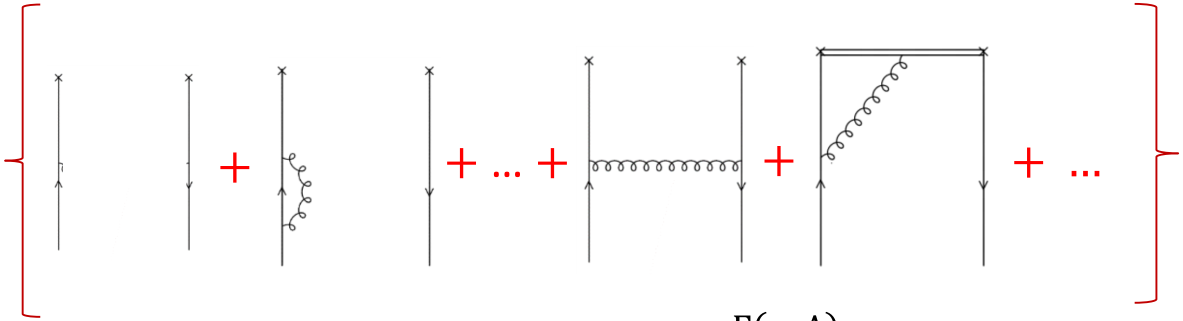
$$q(x) = \frac{1}{4\pi} \int e^{-ixP\cdot\zeta} \langle P | \bar{\psi}(n\cdot\zeta) \gamma^{+} W(n\cdot\zeta, 0) \psi(0) | P \rangle \,, \qquad n\cdot\zeta = z^{-}$$

Dirac Structure

Wilson line

Perturbative correction to isovector quark distributions:

$q(x,\Lambda)=$



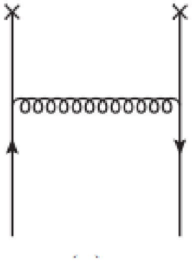
$\delta(1-y)$ $\Pi(\Lambda)\delta(1-y)$ $\Gamma(y,\Lambda)$

$\otimes q_{bare}(y,\Lambda) + \mathcal{O}(\alpha_s^2)$

Regulator of IR and UV divergences

$$q(x,\Lambda) = q_{bare}(x)\left\{1 + \frac{\alpha_s}{2\pi}\Pi(\Lambda)\right\} + \frac{\alpha_s}{2\pi}\int_x^1 \Gamma\left(\frac{x}{y},\Lambda\right) q_{bare}(y)\frac{dy}{y} + \mathcal{O}(\alpha_s^2)$$

On the light-cone



$$= -ig^2 C_F \int \frac{dk^+ dk^- d^2k_\perp}{(2\pi)^4} \frac{\bar{u}(p)\gamma^\mu k\cdot\gamma\gamma^+ k\cdot\gamma\gamma_\mu u(p)}{(k^2)^2(p-k)^2}\,\delta\left(y-\frac{k^+}{p^+}\right)$$

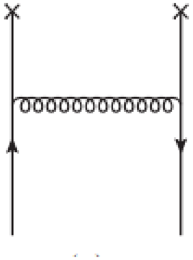
$$p = (\xi P^+, 0,0,0);\;\; \xi = \frac{p^+}{P^+}$$

$$k^2 + i\epsilon = 2yp^+\left(k^- - \frac{k_\perp^2}{2yp^+} + i\epsilon\right)$$

$$(p-k)^2 + i\epsilon = -2p^+(1-y)\left(k^- + \frac{k_\perp^2}{2p^+(1-y)} - i\epsilon\right)$$

For $0 < y < 1$, one pole in the upper half and other in the lower half of the complex plane

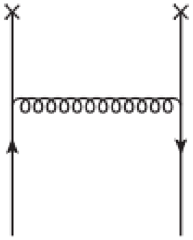
For y> 1 or y< 0 , the poles are either on the lower half or on the upper half of the complex plane



$$= 2\alpha_s C_F(1-y)\int \frac{d^2k_\perp}{(2\pi)^2}\frac{\bar{u}(p)\gamma^+u(p)}{k_\perp^2} = \frac{\alpha_s}{2\pi}(1-y)\left(\frac{1}{\epsilon_{UV}} - \frac{1}{\epsilon_{IR}} + \ln\left(\frac{\mu^2}{\mu_F^2}\right)\right)$$

DR used for IR and UV divergences

Infinite momentum frame (IMF)

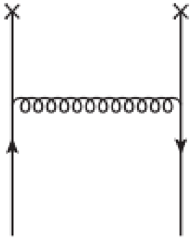


$$= -ig^2 C_F \int \frac{dk^0 dk^3 d^2k_\perp}{(2\pi)^4} \frac{\bar{u}(p)\gamma^\mu k\cdot\gamma\gamma^3 k\cdot\gamma\gamma_\mu u(p)}{(k^2)^2(p-k)^2}\,\delta\left(y-\frac{k^3}{p^3}\right)$$

$$k^2 + i\epsilon = \left(k^0 - \sqrt{k_\perp^2 + y^2(p^3)^2} + i\epsilon\right)\left(k^0 + \sqrt{k_\perp^2 + y^2(p^3)^2} - i\epsilon\right)$$

$$(\hat{p}-k)^2 + i\epsilon = \left(k^0 - p^3 - \sqrt{k_\perp^2 + (1-y)^2(p^3)^2} + i\epsilon\right)\left(k^0 - p^3 + \sqrt{k_\perp^2 + (1-y)^2(p^3)^2} - i\epsilon\right)$$

Integrating in k^0 and taking the $p^3 \to \infty$ limit:



$$= 2\alpha_s C_F (1-y)\int \frac{d^2k_\perp}{(2\pi)^2}\frac{\bar{u}(p)\gamma^3 u(p)}{k_\perp^2}$$

LC and IMF have the same IR and UV behaviour and are equivalente

Unfortunately, they can not be computed within LQCD

What if p_3 is kept finite?

$$\tilde{q}(x,\Lambda) = q_{bare}(x)\left\{1 + \frac{\alpha_s}{2\pi}\tilde{\Pi}(\Lambda)\right\} + \frac{\alpha_s}{2\pi}\int_x^1 \tilde{\Gamma}\left(\frac{x}{y},\Lambda\right) q_{bare}(y)\frac{dy}{y} + \mathcal{O}(\alpha_s^2)$$

Poles as last slide, however finite p^3 finite makes the tranverse momentum integrals finite for $y < 0$ or for $y > 1$;

$$\tilde{q}(x) = \frac{1}{4\pi}\int e^{-ixP^3 z}\left\langle P\middle|\bar{\psi}(z)\gamma^0 W\,(z,0)\psi(0)\middle|P\right\rangle$$

Pure spatial correlation
Can be computed within LQCD

X. Ji, PRL 110 (2013) 262002.

Keeping p_3 finite, makes the resulting expressions more complex

Feynman gauge used

$$\tilde{\Gamma}_{\gamma^0}\left(\xi,\frac{p_3}{\mu_F}\right) = \begin{cases} \dfrac{1+\xi^2}{1-\xi}\ln\left(\dfrac{\xi}{\xi-1}\right) + 1\,, & \xi > 1\,, \\ \dfrac{1+\xi^2}{1-\xi}\left(-\dfrac{1}{\epsilon_{IR}} + \ln\left(\dfrac{4\xi(1-\xi)(p_3)^2}{\mu_F^2}\right)\right) - \dfrac{2\xi}{1-\xi} + \xi, & 0 < \xi < 1\,, \\ \dfrac{1+\xi^2}{1-\xi}\ln\left(\dfrac{\xi-1}{\xi}\right) - 1\,, & \xi < 0\,. \end{cases}$$

IR divergence in the $0 < \xi < 1$ only

Exactly the same IR of the light cone case

IMF and finite momentum frames differ in the UV only

$$\begin{aligned} \tilde{\Pi}_{\gamma^0}^{\overline{MS}}(\xi, p_3/\mu_F, \mu/\mu_F) &= \frac{5}{2} + \frac{3}{2}\ln\left(\frac{\mu_F^2}{4\mu^2}\right) \\ &- \int d\xi \begin{cases} \dfrac{1+\xi^2}{1-\xi}\ln\dfrac{\xi}{\xi-1} + 1 - \dfrac{3}{2\xi}, & \xi > 1 \\ \dfrac{1+\xi^2}{1-\xi}\left(-\dfrac{1}{\epsilon_{IR}} + \ln\dfrac{4\xi(1-\xi)(p_3)^2}{\mu_F^2}\right) - \dfrac{2\xi}{1-\xi} + \xi, & 0 < \xi < 1 \\ \dfrac{1+\xi^2}{1-\xi}\ln\dfrac{\xi-1}{\xi} - 1 - \dfrac{3}{2(1-\xi)}, & \xi < 0 \end{cases} \end{aligned}$$

Matching

$$q_{\gamma^0}(x,\mu\ ) = \int_{-\infty}^{+\infty} \frac{dy}{|y|}\, C_{\gamma^0}^{\overline{MS}}\left(\frac{x}{y},\frac{\mu}{p_3},\frac{\mu}{\mu_F}\right) \tilde{q}_{\gamma^0}(y,P_3,\mu)$$

$$C_{\gamma^0}^{\overline{MS}}(\xi) = \delta(1-\xi) - \frac{\alpha_s}{2\pi} C_F \left(\left(\tilde{\Pi}_{\gamma^0}^{\overline{MS}} - \Pi_{\gamma^0}^{\overline{MS}}\right)\delta(1-\xi) + \tilde{\Gamma}_{\gamma^0}(\xi) - \Gamma_{\gamma^0}^{\overline{MS}}(\xi)\right)$$

$$\begin{aligned} C_{\gamma^0}^{\overline{\rm MS}}\left(\xi,\frac{\bar\mu}{p_3},\frac{\bar\mu}{\mu_F}\right) &= \delta\left(1-\xi\right) \\ &+ \frac{\alpha_s C_F}{2\pi} \begin{cases} \left(\frac{1+\xi^2}{1-\xi}\ln\left(\frac{\xi}{\xi-1}\right) + 1 + \frac{3}{2\xi}\right)_{+(1)} - \frac{3}{2\xi}, & \xi > 1 \\ \left(\frac{1+\xi^2}{1-\xi}\left[\ln\left(\frac{p_3^2}{\bar\mu^2}\right) + \ln\left(4\xi(1-\xi)\right)\right] - \frac{\xi(1+\xi)}{1-\xi}\right)_{+(1)}, & 0 < \xi < 1 \\ \left(-\frac{1+\xi^2}{1-\xi}\ln\left(\frac{-\xi}{1-\xi}\right) - 1 + \frac{3}{2(1-\xi)}\right)_{+(1)} - \frac{3}{2(1-\xi)}, & \xi < 0 \end{cases} \\ &+ \frac{\alpha_s C_F}{2\pi}\delta(1-\xi)\left(\frac{3}{2}\ln\left(\frac{\mu_F^2}{4\bar\mu^2}\right) + \frac{5}{2}\right). \end{aligned}$$

$\ln(\xi)$ divergent as $\xi \to \pm\infty$ when convoluted

$\tilde{\Gamma}_{\gamma^0}(\xi)$ UV finite, diverges when integrated over ξ

C. Alexandrou, K. Cichy, M. Constantinou, K. Hadjiyiannakou, K. Jansen, A. Scapellato and F. Steffens, arXiv:1902.00587 [hep-lat]

T.Izubuchi, X.Ji, L.Jin, I.W.Stewart and Y.Zhao, PRD 98 056004 (2018)

X. Xiong, X. Ji, J. H. Zhang and Y. Zhao, PRD 90 014051 (2014)
C.Alexandrou, K.Cichy, V.Drach, E.Garcia-Ramos, K.Hadjiyiannakou, K.Jansen, F.Steffens and C.Wiese, PRD 92 014502 (2015)
W. Wang, S. Zhao and R. Zhu, Eur. Phys. J. C78 (2018) 147;
W. Stewart, Y. Zhao, PRD 97 054512 (2018
C.Alexandrou, K.Cichy, M.Constantinou, K.Jansen, A.Scapellato and F.Steffens, PRL 121, (2018), 112001.

Define a new scheme, where the remaining divergences are subtracted outside the physical region only, resulting in a minimal modification of $\overline{MS}$: Modified $\overline{MS}$ ($M\overline{MS}$):

$$\tilde{Z}^{\mathrm{M\overline{MS}}}_{\Gamma_{\gamma^0}}(\xi) = 1 - \frac{\alpha_s}{2\pi}C_F\frac{3}{2}\left(-\frac{1}{\xi}\theta(\xi-1) - \frac{1}{1-\xi}\theta(-\xi)\right) - \frac{\alpha_s C_F}{2\pi}\delta(1-\xi)\left(\frac{3}{2}\ln\left(\frac{1}{4}\right)+\frac{5}{2}\right)$$

Momentum space

$$\begin{aligned} Z^{\mathrm{M\overline{MS}}}_{\Gamma_{\gamma^0}}(z\bar{\mu}) \; &= \; 1 - \frac{\alpha_s}{2\pi}C_F\left(\frac{3}{2}\ln\left(\frac{1}{4}\right)+\frac{5}{2}\right) \\ &+ \; \frac{3}{2}\frac{\alpha_s}{2\pi}C_F\left(i\pi\frac{|z\bar{\mu}|}{2z\bar{\mu}} - \mathrm{Ci}(z\bar{\mu}) + \ln(z\bar{\mu}) - \ln(|z\bar{\mu}|) - i\mathrm{Si}(z\bar{\mu})\right) \\ &- \; \frac{3}{2}\frac{\alpha_s}{2\pi}C_F e^{iz\bar{\mu}}\left(\frac{2\mathrm{Ei}(-iz\bar{\mu}) - \ln(-iz\bar{\mu}) + \ln(iz\bar{\mu}) + i\pi\mathrm{sgn}(z\bar{\mu})}{2}\right) \end{aligned}$$

Position space

In the limit of $z \to 0$

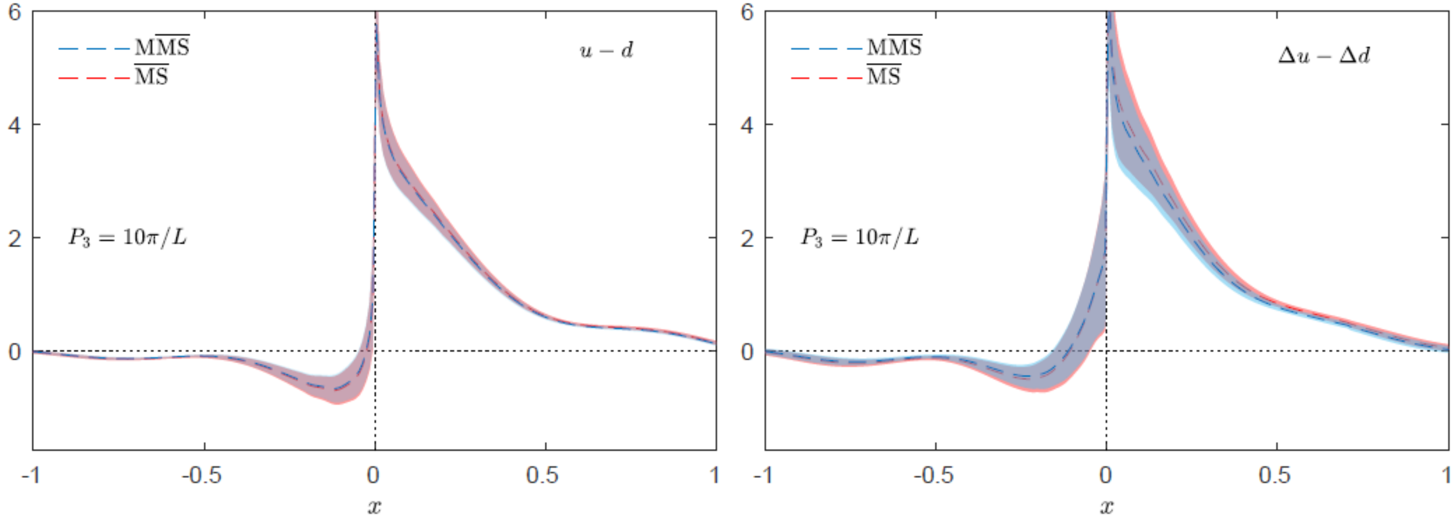
$$Z^{\mathrm{M\overline{MS}}}_{\Gamma_{\gamma^0}}(z\to 0) = 1 - \frac{\alpha_s C_F}{2\pi}\left(\frac{3}{2}\ln\left(\frac{\bar{\mu}^2 z^2 e^{2\gamma_E}}{4}\right)+\frac{5}{2}\right) = Z^{\mathrm{ratio}}_{\Gamma_{\gamma^0}}(z\bar{\mu})$$

Ratio scheme, introduced
by Izubuchi et al. arXiv:1801.03917

It subtracts the $\ln(z^2 \to 0)$ divergence present in the $\overline{MS}$ scheme

$$q^{\overline{\mathrm{MS}}}(x,\bar{\mu}) = \int_{-\infty}^{+\infty} \frac{dy}{|y|} C^{\mathrm{M}\overline{\mathrm{MS}}}\left(\frac{x}{y}, \frac{\bar{\mu}}{p_3}\right) \tilde{q}^{\mathrm{M}\overline{\mathrm{MS}}}(y, P_3, \bar{\mu})$$

$$C^{\mathrm{M}\overline{\mathrm{MS}}}_{\gamma^0,\gamma^3,\gamma^3\gamma^5}\left(\xi, \frac{\bar{\mu}}{p_3}\right) = \delta\left(1-\xi\right) + \frac{\alpha_s C_F}{2\pi} \begin{cases} \left(\frac{1+\xi^2}{1-\xi}\ln\left(\frac{\xi}{\xi-1}\right) + 1 + \frac{3}{2\xi}\right)_{+(1)}, & \xi > 1, \\ \left(\frac{1+\xi^2}{1-\xi}\left[\ln\left(\frac{p_3^2}{\bar{\mu}^2}\right) + \ln\left(4\xi(1-\xi)\right)\right] - \frac{\xi(1+\xi)}{1-\xi} + 2\iota(1-\xi)\right)_{+(1)}, & 0 < \xi < 1 \\ \left(-\frac{1+\xi^2}{1-\xi}\ln\left(\frac{-\xi}{1-\xi}\right) - 1 + \frac{3}{2(1-\xi)}\right)_{+(1)}, & \xi < 0, \end{cases}$$



Antiquarks Quarks Antiquarks Quarks

$\overline{MS}$: Previous calculation (C.Alexandrou et al., PRL 121, (2018), 112001), where $Z^{M\overline{MS}}$ had not been applied to lattice data

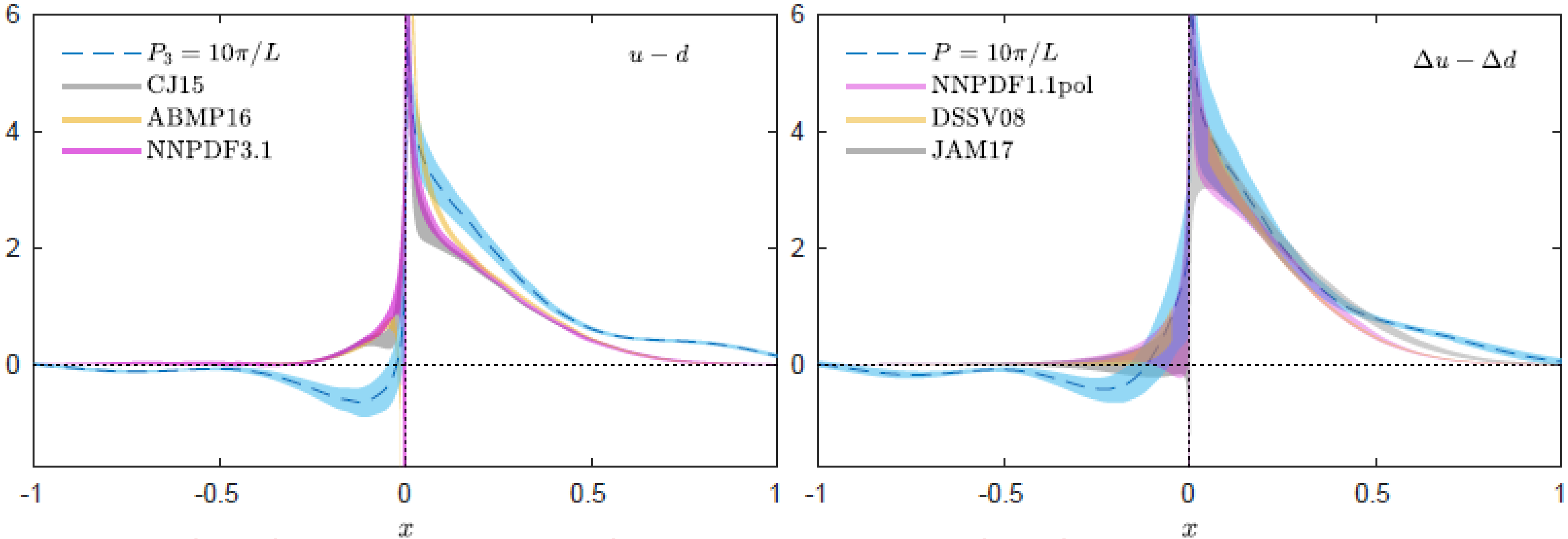
$M\overline{MS}$: Extra subtraction consistently applied

Isovector unpolarized and helicity distributions

Unpolarized

$P_3 = 1.38\ GeV$

Helicity



Antiquarks

Quarks

Antiquarks

Quarks

C. Alexandrou, K. Cichy, M. Constantinou, K. Hadjiyiannakou, K. Jansen, A. Scapellato and F. Steffens, arXiv:1902.00587 [hep-lat].

$\bar{d}(x) - \bar{u}(x) < 0$ induced by the finite number of points in the Fourier transform? Can we still say something about $\bar{d}(x) - \bar{u}(x)$?

$m_\pi \cong 130$ MeV

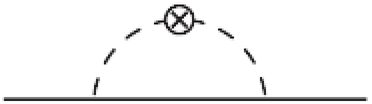
$48^3 \times 96$ lattice

$a \cong 0.093$ fm

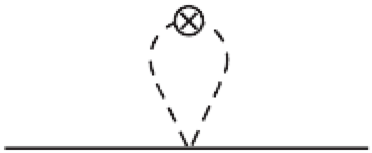
See M. Constantinou talk

Sea asymmetries and chiral loops

$$\mathcal{L}_{\pi N} = \frac{g_A}{2f_\pi}\bar{\psi}_N\gamma^\mu\gamma_5\boldsymbol{\tau}\cdot\partial_\mu\boldsymbol{\pi}\psi_N - \frac{1}{(2f_\pi)^2}\bar{\psi}_N\gamma^\mu\boldsymbol{\tau}\cdot(\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi})\psi_N,$$



Usual Rainbow



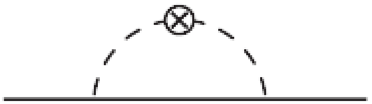
Weiberg-Tomozawa

Contributions from nucleon and delta intermediate states

$$(\bar{d} - \bar{u})^p(x) = 2[(f_{N\to N\pi} - f_{N\to\Delta\pi})\otimes\overline{q_v}^{\pi}](x)$$

Antiquark distributions in the pion

Loop correction very much as before, with nucleons and pions replacing quarks and gluons:



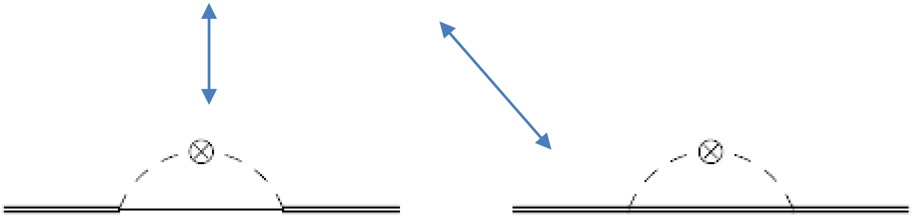
$$= 4M\left(\frac{g_A}{2f_\pi}\right)^2\int\frac{d^4k}{(2\pi)^4}\bar{u}(p)(\gamma_5 k\cdot\gamma)\frac{i(p\cdot\gamma - k\cdot\gamma - M)}{D_N}(\gamma_5 k\cdot\gamma)u(p)\frac{i}{D_\pi}\frac{i}{D_\pi}\frac{2k^+}{p^+}\delta\left(y - \frac{k^+}{p^+}\right)$$

$$D_\pi = k^2 - m_\pi^2 + i\epsilon$$

$$D_N = (p-k)^2 - M^2 + i\epsilon$$

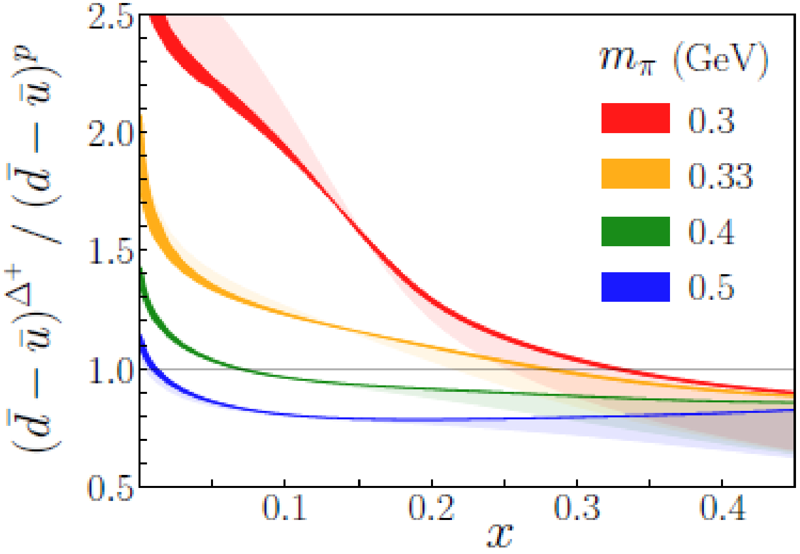
$\bar{d}(x) - \bar{u}(x)$ asymmetry in the Delta

$$(\bar{d} - \bar{u})^{\Delta^+}(x) = [(f_{\Delta\to N\pi} + 2 f_{\Delta\to\Delta\pi}) \otimes \overline{q_v}^{\pi}](x)$$



$$f_{\Delta\to\Delta\pi}(y) = \frac{g_A^2}{50 M_\Delta^2 (4\pi f_\pi)^2} \int dk_\perp^2 \frac{y}{1-y} \times \left[\frac{m_\pi^2 [m_\pi^2 (2M_\Delta^2 - m_\pi^2) - 10 M_\Delta^4]}{D_{\Delta\Delta}^2} + \frac{m_\pi^2 (4M_\Delta^2 - 3m_\pi^2) - 10 M_\Delta^4}{D_{\Delta\Delta}} \right];$$

$$f_{\Delta\to N\pi}(y) = \frac{g_A^2}{50 M_\Delta^2 (4\pi f_\pi)^2} \int dk_\perp^2 \frac{y(\overline{M}^2 - m_\pi^2)}{(1-y)} \times \left[\frac{(\overline{M}^2 - m_\pi^2)(\Delta^2 - m_\pi^2)}{D_{\Delta N}^2} - \frac{\overline{M}^2 - 3m_\pi^2 + 2\Delta^2}{D_{\Delta N}} \right]$$



Same calculation that reproduces $(\bar{d} - \bar{u})^p$

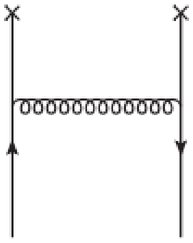
Enhancement from the opening of the decay channel when $m_\pi \sim M_\Delta - M$

Can be tested in a Lattice computation!

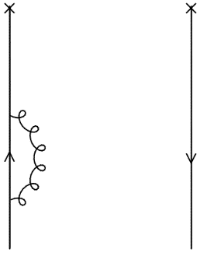
J. J. Ethier, W. Melnitchouk, F. Steffens and A. W. Thomas, arXiv:1809.06885 [hep-ph].

Transversity case

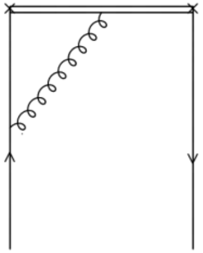
$$\tilde{h}_1(x) = \frac{1}{4\pi}\int e^{-ixP^3z}\,\langle P|\bar{\psi}(z)\gamma^3\gamma^j W\,(z,0)\psi(0)|P\rangle$$



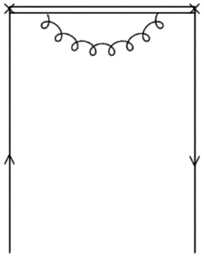
$= 0$



$\neq 0$



$\neq 0$



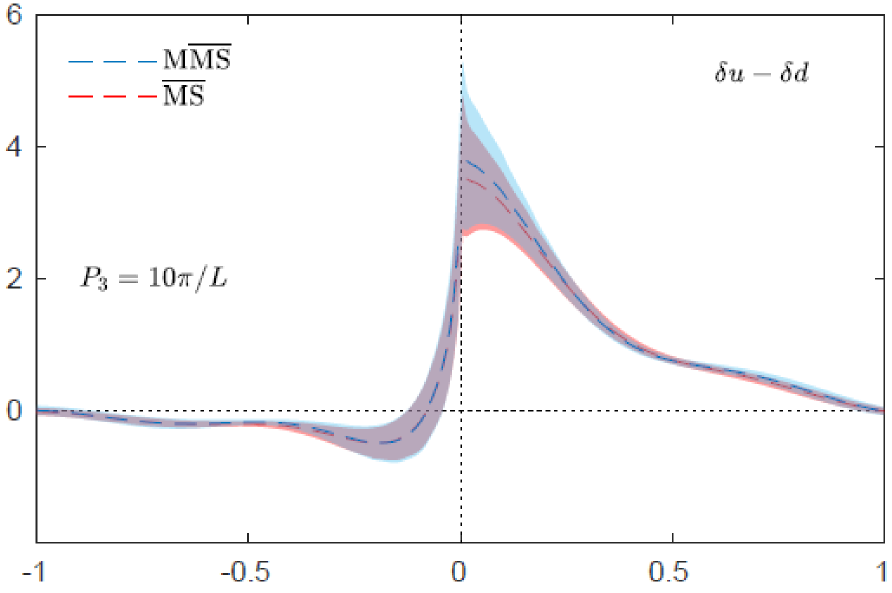
$\neq 0$

Easier than previous case

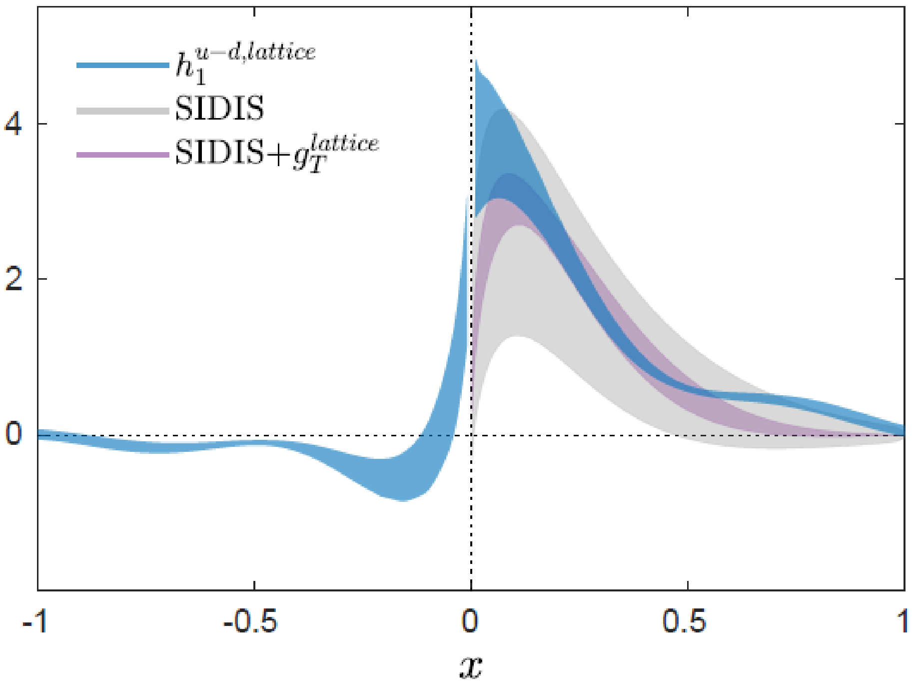
$$\Gamma^{\overline{MS}}_{\gamma^3\gamma^j}\left(\xi,\frac{\mu}{\mu_F}\right)=\frac{2\xi}{1-\xi}\left(-\frac{1}{\epsilon_{IR}}+\ln\frac{\mu^2}{\mu_F^2}\right)$$

$$\tilde{\Gamma}_{\gamma^3\gamma^j}(\xi,p_3/\mu_F)=\begin{cases}\dfrac{2\xi}{1-\xi}\ln\dfrac{\xi}{\xi-1}, & \xi>1\\[2ex] \dfrac{2\xi}{1-\xi}\left(-\dfrac{1}{\epsilon_{IR}}+\ln\dfrac{4\xi(1-\xi)(p_3)^2}{\mu_F^2}\right)-\dfrac{2\xi}{(1-\xi)}, & 0<\xi<1\\[2ex] \dfrac{2\xi}{1-\xi}\ln\dfrac{\xi-1}{\xi}, & \xi<0\end{cases}$$

$$C^{\mathrm{M}\overline{\mathrm{MS}}}_{\gamma^3\gamma^j}\left(\xi,\frac{\bar{\mu}}{p_3}\right)=\delta(1-\xi)+\frac{\alpha_s}{2\pi}C_F\begin{cases}\left[\frac{2\xi}{1-\xi}\ln\left(\frac{\xi}{\xi-1}\right)+\frac{2}{\xi}\right]_{+(1)}, & \xi>1,\\ \left[\frac{2\xi}{1-\xi}\left(\ln\left(\frac{p_3^2}{\bar{\mu}^2}\right)+\ln(4\xi(1-\xi))\right)-\frac{2\xi}{1-\xi}\right]_{+(1)}, & 0<\xi<1\\ \left[-\frac{2\xi}{1-\xi}\ln\left(\frac{\xi}{\xi-1}\right)+\frac{2}{1-\xi}\right]_{+(1)}, & \xi<0\,.\end{cases}$$



Comparing with phenomenology:



C. Alexandrou et al., PRD98, 091503 (2018)

$$g_T = \int_{-1}^{+1} dx h_1^{u-d} = 1.09(11)$$

This should be compared to: $g_T = 1.06(1)$ from dedicated lattice QCD calculation

C. Alexandrou et al., PRD95, 114514 (2017)

$g_T = 1.0(1)$ from Monte Carlo global analysis

H.-W. Lin PRL 120, 152502 (2018)

$g_T = 0.53(25)$ from global analysis of ep and pp data

Radici and Bacchetta PRL 120, 192001 (2018)

A new direction?

Lattice QCD results to be treated with same status of experimental results

Use lattice data as experimental points in a global fitting analysis. How compatible is the lattice data with, for example, the E866 data?

Fourier transform using finite number of points is avoided

Work in progress in collaboration with JAM

Summary

Matching between quasi-PDFs and light-cone PDFs does not preserve the norm of the distribution In the $\overline{MS}$ scheme;

Extra subtraction in the unphysical region restores the norm preservation;

$\bar{d}(x) - \bar{u}(x) < 0$ probably induced by finite number of points used in the Fourier transform;

We have strong hints that the non-trivial structure of the nucleon sea is deeply connected to the breaking of chiral symmetry;

Same physics that describes $\bar{d}(x) - \bar{u}(x)$ can can be applied to the Δ^+;

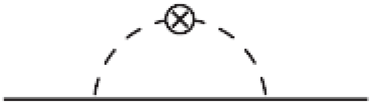
Strong enhancement of the asymmetry found close to the Δ^+ decay channel;

Quasi-PDFs can be used to test the role of chiral symmetry and the pion cloud in the generation of a nonperturbative sea in baryons;

Transversity poorly constrained by current experimental data;

g_T has been shown to greatly improve the fitting. Would the LQCD z dependent data points improve the fitting further?

Hoe compatible is lattice data with other sets of data (E866 data, for example) in a global fitting analysis?



$$= \frac{g_A^2 M^2}{(4\pi f_\pi)^2} \int dk_\perp^2 \frac{y(k_\perp^2 + y^2 M^2)}{(1-y)^2 D_{NN}^2}$$

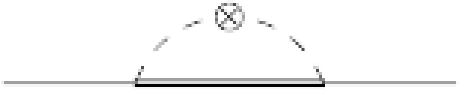
Same UV and IR structure seen before

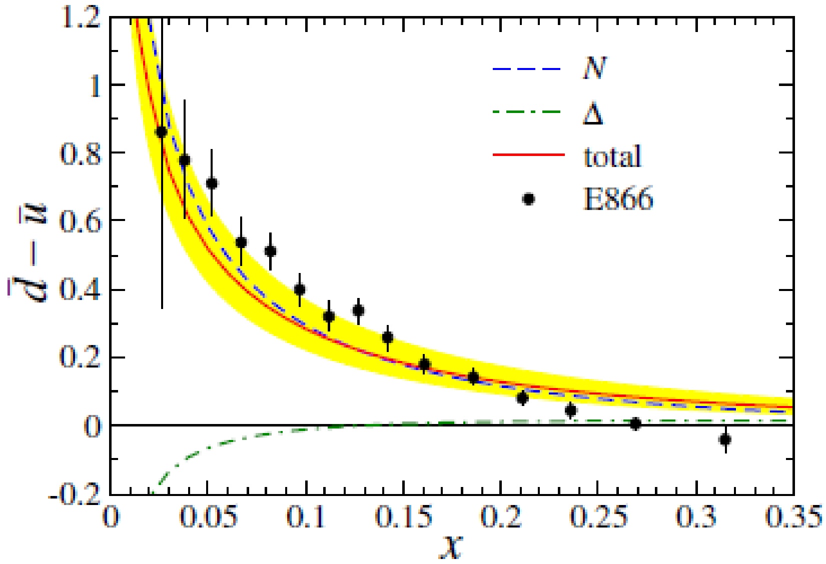
$$D_{BB'} = -\frac{k_\perp^2 - y(1-y)M_B^2 + yM_{B'}^2 + (1-y)m_\pi^2}{(1-y)}; \quad D_{NN} \to -\frac{k_\perp^2 + y^2 M_N^2}{(1-y)}$$

For $m_\pi^2 \to 0$, LNA behaviour $\sim m_\pi^2 \ln(m_\pi^2$

Thomas, Melnitchouk, Steffens PRL 85 (2000)

Similar computation for $N \to \Delta\pi$



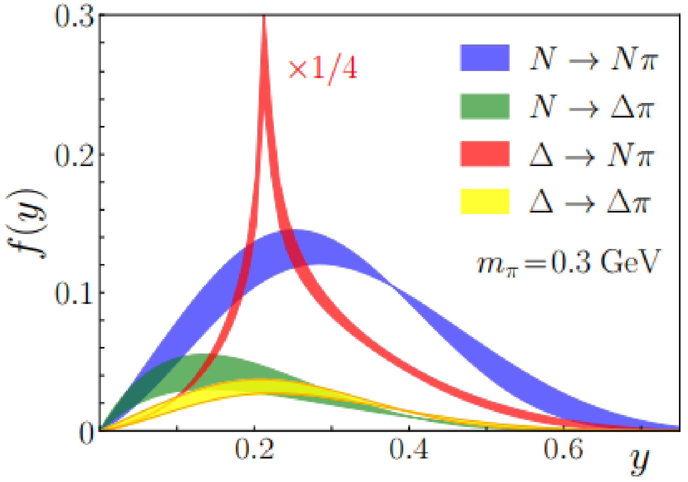


Yellow band express uncertainties from the UV regulator used (different phenomenological πN form factors, for example)

Can be used within global analises to determine pion PDFs

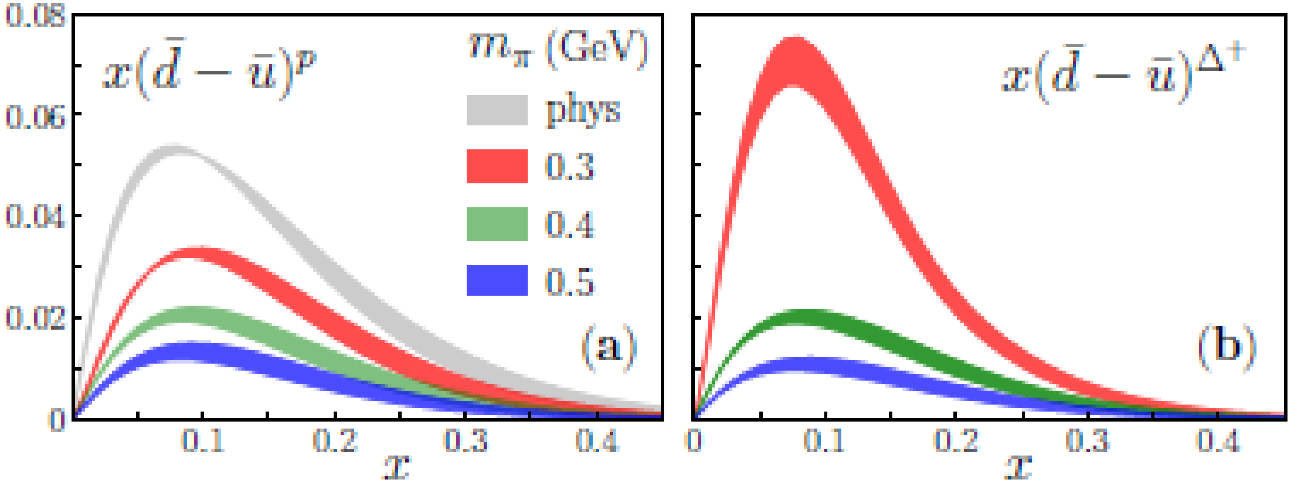
P. C. Barry, N. Sato, W. Melnitchouk and C. R. Ji, PRL **121** (2018) no.15, 152001

Spliting functions



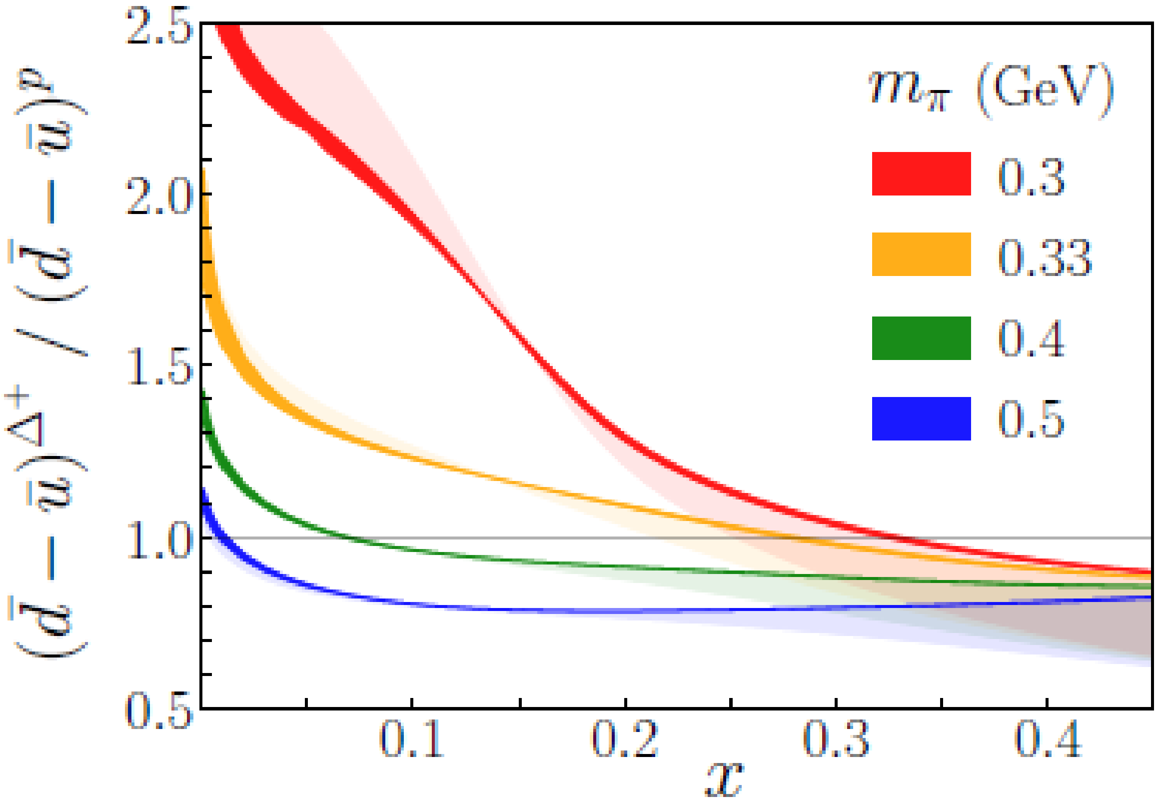
Uncertainty band from the use of different form factors regulators

Asymmetries in the Proton and in the Delta



Enhancement from the opening of the decay channel when $m_\pi \sim M_\Delta - M$

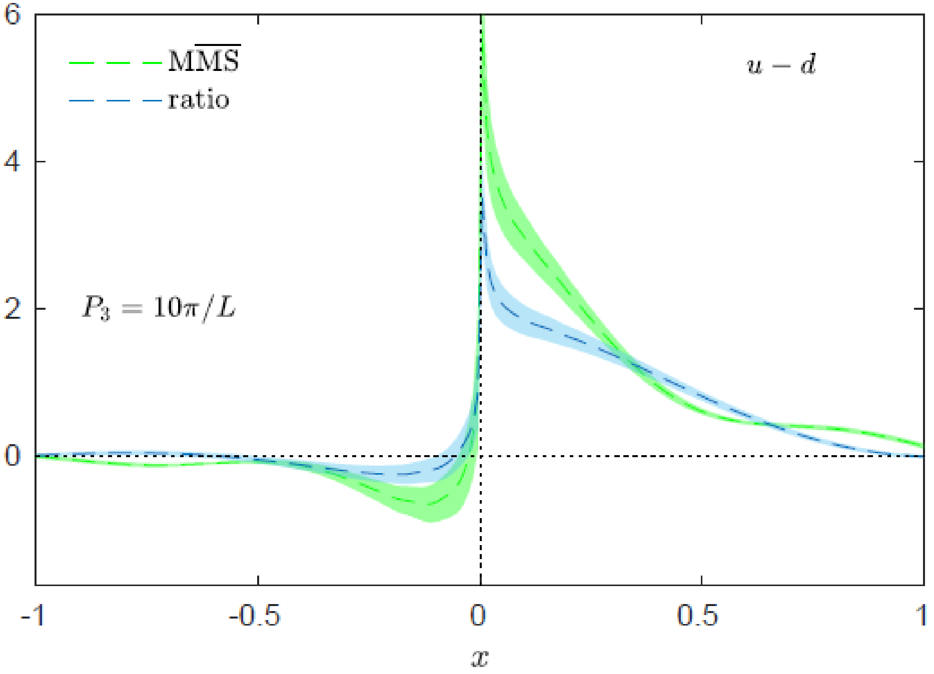
Ratio of the asymmetries



Darker bands: uncertainties on the pion PDFs

Ligher bands: dependence on the choice of regulator

J. J. Ethier, W. Melnitchouk, F. Steffens and A. W. Thomas, arXiv:1809.06885 [hep-ph].



$\mathrm{M}\overline{\mathrm{MS}}$
ratio
$u-d$
$P_3 = 10\pi/L$
6
4
2
0
-1
-0.5
0
0.5
1
x