QCD Evolution 2019, 13-17 May 2019, Argonne National Laboratory, IL USA

Gravitational form factors and their applications in hadronic physics

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Outline

- **Introduction**
  importance of energy momentum tensor (EMT)
  D-term last unknown global property

- **What do we know from theory & experiment**
  Belle $\gamma\gamma^* \to \pi^0\pi^0$: π^0 D-term Kumano et al
  DVCS nucleon: Kumericki, Müller; Burkert et al, Nature (2018)

- **What do we learn about hadrons**
  3D densities: valid for nucleon & nuclei (Breit frame) $\to$ PS
  mechanical properties of light mesons $\to$ Adam Freese
  other frames, 2D, higher spin $\to$ Cedric Lorcé

- **Applications**
  insights in forces: stability
  large-N_c baryons, hadrocharmonia $\to$ PS
  largest hadron: neutron star $\to$ Simonetta Liuti

- **Outlook**

- based on: M.V.Polyakov, PS, IJMPA **33**, 1830025 (2018) `arXiv:1805.06596`
  (and many works before, and many after it!)

supported by NSF

Introduction Energy momentum tensor (EMT)

- **theory** $\mathcal{L} = \mathcal{L}(\partial_\kappa \phi_i, \phi_i)$

- **canonical EMT:** Noether current of translations $\rightarrow$ $\widehat{T}_{\mu\nu} = \partial_\mu \phi_i \dfrac{\partial \mathcal{L}}{\partial(\partial^\nu \phi_i)} - g_{\mu\nu}\,\mathcal{L}$

- **symmetric EMT:** couple to classical (symm.) gravitational background metric field

- **EMT conservation:** $\partial^\mu \hat{T}_{\mu\nu} = 0$

- **Poincaré group generators:** $\widehat{P}^\mu = \int \mathrm{d}^3x\; \widehat{T}^{0\mu}$, $\widehat{M}^{\kappa\nu} = \int \mathrm{d}^3x\; (x^\kappa \widehat{T}^{0\nu} - x^\nu \widehat{T}^{0\kappa})$

  Poincaré algebra: $[\widehat{P}^\mu, \widehat{P}^\nu] = 0$, $[\widehat{M}^{\mu\nu}, \widehat{P}^\kappa] = i(g^{\mu\kappa}\widehat{P}^\nu - g^{\nu\kappa}\widehat{P}^\mu)$, $[\widehat{M}^{\mu\nu}, \widehat{M}^{\kappa\sigma}] = i(g^{\mu\kappa}\widehat{M}^{\nu\sigma} - g^{\nu\kappa}\widehat{M}^{\mu\sigma} - g^{\mu\sigma}\widehat{M}^{\nu\kappa} + g^{\nu\sigma}\widehat{M}^{\mu\kappa})$

- **Casimir operators:** $\widehat{P}^\mu \widehat{P}_\mu \rightarrow m^2$, $\widehat{W}^\mu \widehat{W}_\mu \rightarrow m^2\, J(J+1)$ where $\widehat{W}^\kappa = -\frac{1}{2}\varepsilon^{\kappa\mu\nu\sigma}\widehat{M}_{\mu\nu}\widehat{P}_\sigma$ $\rightarrow$ **mass & spin**

- **in QCD** $\widehat{T}_{\mu\nu} = \sum_q T^q_{\mu\nu} + \widehat{T}^g_{\mu\nu}$ quark, gluon parts each gauge-invariant, but not conserved

- classically **scale transformations** $\rightarrow$ $j^\mu = x_\nu \widehat{T}^{\mu\nu}$ conserved $\partial_\mu j^\mu = \widehat{T}^\mu_\mu = \sum_q m_q \bar{\psi}_q \psi_q$ for $m_q \rightarrow 0$

- quantum corrections: **trace anomaly** $\widehat{T}^\mu_\mu \equiv \dfrac{\beta(g)}{2g}\, F^{a,\mu\nu} F^a{}_{\mu\nu} + (1+\gamma_m) \sum_q m_q \bar{\psi}_q \psi_q$ in QED, QCD

  S.L.Adler, J.C.Collins and A.Duncan, PRD **15**, 1712 (1977). N.K.Nielsen, NPB **120**, 212 (1977).
  J.C.Collins, A.Duncan and S.D.Joglekar, PRD **16**, 438 (1977).

EMT form factors of nucleon (Kobzarev & Okun 1962, Pagels 1966)

$$\langle p'|\hat{T}^{a}_{\mu\nu}|p\rangle = \bar{u}(p')\Bigg[A^a(t,\mu^2)\,\frac{P_\mu P_\nu}{M} + J^a(t,\mu^2)\,\frac{i(P_\mu\sigma_{\nu\rho}+P_\nu\sigma_{\mu\rho})\Delta^\rho}{2M} + D^a(t,\mu^2)\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4M} + \bar{c}^a(t,\mu^2)g_{\mu\nu}\Bigg]u(p)$$

- conserved external current $\partial_\mu \hat{T}^{\mu\nu} = 0$, $\hat{T}_{\mu\nu} = \sum_a \hat{T}^a_{\mu\nu}$ $(a = q, g)$
- $A(t) = \sum_a A^a(t,\mu^2)$, $J(t)$, $D(t)$ scale invariant, $\sum_a \bar{c}^a(t,\mu^2) = 0$
- constraints: **mass** $\Leftrightarrow$ $A(0) = 1$ $\Leftrightarrow$ quarks + gluons carry 100 % of nucleon momentum

  spin $\Leftrightarrow$ $J(0) = \frac{1}{2}$ $\Leftrightarrow$ quarks + gluons carry 100 % of nucleon spin *

  D-term $\Leftrightarrow$ $D(0) \equiv D$ $\to$ unconstrained! **Last global unknown!**

$$2P = (p'+p)\qquad \Delta = (p'-p)\qquad t = \Delta^2$$

notation: $A^q(t) + B^q(t) = 2\,J^q(t)$

$D^q(t) = \frac{4}{5}\,d_1^q(t) = \frac{1}{4}\,C^q(t)$ or $C^q(t)$

$A^q(t) = M_2^q(t)$

* or: total anomalous gravitomagnetic moment vanishes (Gordon identity)

Studying hadrons:

$|N\rangle$ = **strong**–interaction particle. Use other forces to probe it!

em:	$\partial_\mu J^\mu_{\rm em} = 0$	$\langle N'\vert J^\mu_{\rm em}\vert N\rangle$	$\longrightarrow$	$G_E(t),\ G_M(t)$	$\longrightarrow$	$Q,\ \mu,\ \ldots$
weak:	PCAC	$\langle N'\vert J^\mu_{\rm weak}\vert N\rangle$	$\longrightarrow$	$G_A(t),\ G_P(t)$	$\longrightarrow$	$g_A,\ g_p,\ \ldots$
gravity:	$\partial_\mu T^{\mu\nu}_{\rm grav} = 0$	$\langle N'\vert T^{\mu\nu}_{\rm grav}\vert N\rangle$	$\longrightarrow$	$A(t),\ J(t),\ D(t)$	$\longrightarrow$	$M,\ J,\ D,\ \ldots$

global properties:

$$
\begin{aligned}
Q_{\rm prot} &= 1.602176487(40) \times 10^{-19}\,{\rm C} \\
\mu_{\rm prot} &= 2.792847356(23)\,\mu_N \\
g_A &= 1.2694(28) \\
g_p &= 8.06(0.55) \\
M &= 938.272013(23)\ {\rm MeV} \\
J &= \tfrac{1}{2} \\
D &= \ ?
\end{aligned}
$$

and more:
t-dependence ... ...
parton structure, etc ...

$\hookrightarrow$ D = *"last" global unknown*

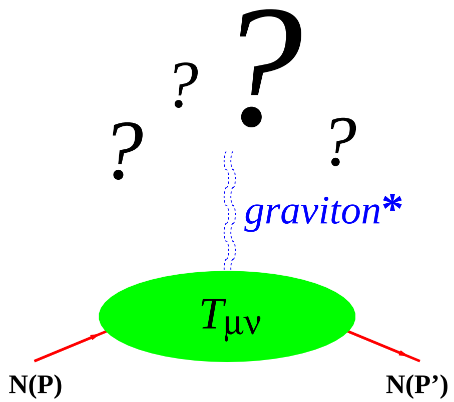
which value does it have?

how can it be measured?

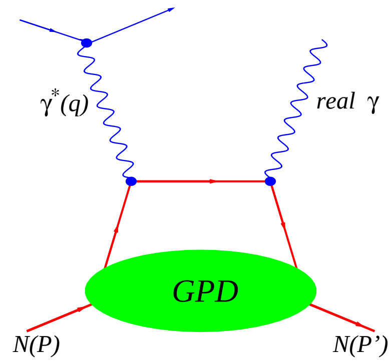
what does it mean?

How to measure?

- direct probe:
  graviton scattering (in principle)



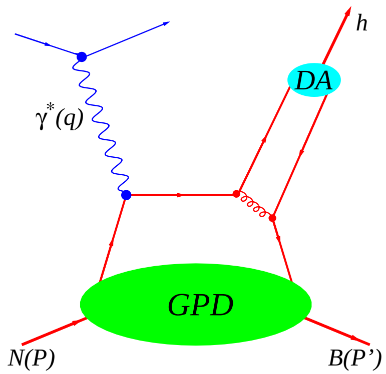
- indirect access through: **GPDs**
  D.Müller, D.Robaschik, B.Geyer, F.-M.Dittes, J.Hořejši, Fortsch. Phys. **42**, 101 (1994)



$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle N'(p')|\overline{\psi}_q(-\tfrac{\lambda n}{2})\, n_\mu \gamma^\mu\, \mathcal{W}(-\tfrac{\lambda n}{2}, \tfrac{\lambda n}{2})\, \psi_q(\tfrac{\lambda n}{2})\,|N(p)\rangle$$

$$= \overline{u}(p')\left[n_\mu \gamma^\mu H^q(x,\xi,t) + \overline{u}(p') \frac{i\sigma^{\mu\nu} n_\mu \Delta_\nu}{2M} E^q(x,\xi,t)\right] u(p)$$

- deeply virtual Compton Scattering
  X.D.Ji, PRL **78**, 610 (1997); PRD **55**, 7114 (1997).
  A.V.Radyushkin, PLB **380**, 417 & **385**, 333 (1996).
  J.C.Collins and A.Freund, PRD **59**, 074009 (1999).



- hard exclusive meson production
  J.C.Collins, L.Frankfurt, M.Strikman, PRD **56**, 2982 (1997).

- bonus: gravity couples to total EMT
  hard exclusive reactions distinguish q, g
  (indebted to M. Diehl for this wisecrack)

polynomiality

$$\int dx\; x\, H^q(x,\xi,t) = \boldsymbol{A^q(t)} + \xi^2 \boldsymbol{D^q(t)}$$

$$\int dx\; x\, E^q(x,\xi,t) = \boldsymbol{B^q(t)} - \xi^2 \boldsymbol{D^q(t)}$$

- **extraction** of GPDs & EMT form factors: non-trivial task
  GPDs are convoluted. In DVCS "Compton form factors:"

$$\mathcal{H}(\xi,t,\mu^2)=\sum_q e_q^2\int\mathrm{d}x\left[\frac{1}{x-\xi-i\varepsilon}-\frac{1}{x+\xi-i\varepsilon}\right]H^q(x,\xi,t,\mu^2)$$

in LO (analog $\mathcal{E}$)

- **dispersion relations** $\to$ for $D(t)$ situation better:

$$\mathfrak{Re}\,\mathcal{H}(\xi,t,\mu^2)\;=\frac{1}{\pi}\,\mathrm{PV}\int\mathrm{d}x\left[\frac{1}{\xi-x}-\frac{1}{\xi+x}\right]\,\mathcal{Im}\,\mathcal{H}(x,t,\mu^2)-\Delta(t,\mu^2)$$

$$\Delta(t,\mu^2)=4\sum_q e_q^2\left[d_1^q(t,\mu^2)+d_3^q(t,\mu^2)+d_5^q(t,\mu^2)+\dots\right]$$

Teryaev hep-ph/0510031
Anikin, Teryaev, PRD76 (2007)
Diehl and Ivanov, EPJC52 (2007)
Radyushkin, PRD83, 076006 (2011)
M.V.Polyakov, PLB 555 (2003) small x

$$\lim_{\mu\to\infty}d_1^Q(t,\mu^2)=d_1(t)\,\frac{N_f}{N_f+4C_F}\qquad\qquad \frac{4}{5}\,d_1(t)=D(t)$$

$$\lim_{\mu\to\infty}d_1^g(t,\mu^2)=d_1(t)\,\frac{4C_F}{N_f+4C_F}\qquad\qquad C_F=\frac{N_c^2-1}{2N_c}$$

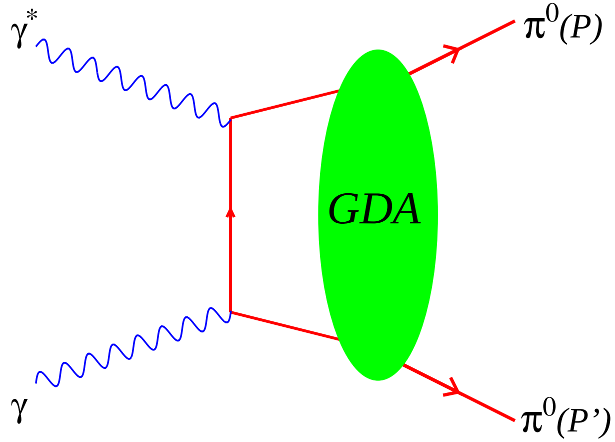
$$\lim_{\mu\to\infty}d_i^a(t,\mu^2)\to 0\quad\text{for } i=3,5,\dots$$

- beam-spin asymmetry in DVCS $\rightsquigarrow$ $\mathcal{Im}\,\mathcal{H}$
  unpolarized DVCS cross section $\rightsquigarrow$ $\mathcal{Re}\,\mathcal{H}$

- **generalized distribution amplitudes**

analytic continuation of GPDs
to timelike region
$t > 2m^2$

opportunity to learn about EMT
of unstable particles e.g. π^0



What do we know about the D-term from theory?

Spin-0 particles

- free Klein-Gordon field $\boldsymbol{D = -1}$
  (Pagels 1966; Hudson, PS 2017)

- Goldstone bosons of chiral symmetry breaking $\boldsymbol{D = -1}$
  Novikov, Shifman; Voloshin, Zakharov (1980); Polyakov, Weiss (1999)
  soft pion theorems (needed for decays of $\psi' \to J/\psi\,\pi\pi$ in 1980s)
  light Higgs $\to \pi\pi$ (Donoghue, Gasser, Leutwyler 1990)

- chiral perturbation theory for π, K, η
  Donoghue, Leutwyler (1991)

$$
\begin{aligned}
D_\pi &= -1 + 16\,a\,\frac{m_\pi^2}{F^2} + \frac{m_\pi^2}{F^2}\,I_\pi - \frac{m_\pi^2}{3F^2}\,I_\eta + \mathcal{O}(E^4) \\
D_K &= -1 + 16\,a\,\frac{m_K^2}{F^2} + \frac{2m_K^2}{3F^2}\,I_\eta + \mathcal{O}(E^4) \\
D_\eta &= -1 + 16\,a\,\frac{m_\eta^2}{F^2} - \frac{m_\pi^2}{F^2}\,I_\pi + \frac{8m_K^2}{3F^2}\,I_K + \frac{4m_\eta^2 - m_\pi^2}{3F^2}\,I_\eta + \mathcal{O}(E^4)
\end{aligned}
$$

where
$a = L_{11}(\mu) - L_{13}(\mu)$
$I_i = \frac{1}{48\pi^2}\,(\log\frac{\mu^2}{m_i^2} - 1)$
$i = \pi,\ K,\ \eta.$

$D_\pi = -0.97 \pm 0.01$
$D_K = -0.77 \pm 0.15$
$D_\eta = -0.69 \pm 0.19$
estimates Hudson, PS
PRD **96** (2017) 114013

$D(t)$ of spin $\frac{1}{2}$ particle: generated by dynamics!

- **free spin $\frac{1}{2}$ fermion:** $D = 0$

  Dirac equation predicts $g = 2$ anomalous magnetic moment
  analoguously it predicts $D = 0$ for non-interacting fermion
  implicit: e.g. Donoghue et al, PLB**529**, 132 (2002)
  explicit: Hudson, PS PRD97 (2018) 056003

- **interacting fermion I:** $D = -\frac{1}{3} F_\pi^2 M_N \int d^3r\; r^2\, P_2(\cos\theta)\, \mathrm{tr}_F[\nabla^k U][\nabla^k U^\dagger] + \mathcal{O}(\nabla U)^3$

  nucleon in chiral quark-soliton model (Δ resonance)
  Diakonov, Petrov, Pobylitsa, NPB 306, 809 (1988)
  $\mathcal{L}_{\text{eff}} = \bar\Psi\,(i\,\partial\!\!\!/ - M\,U^{\gamma_5})\Psi,\;\; U^{\gamma_5} = \exp(i\gamma_5\tau^a\pi^a/f_\pi)$

  "switch off chiral interaction"
  $\Leftrightarrow$ pion fields $U = \exp(i\tau^a\pi^a/F_\pi) \to 1$
  $\Rightarrow \mathcal{L}_{\text{eff}} \to \bar\Psi\,(i\,\partial\!\!\!/ - M)\Psi$
  $\Rightarrow D \to 0$

  expansion to leading order in ∇U
  PS, Radici, Boffi, PRD66 (2002)
  all-order resummed (in large-N_c)
  Goeke et al PRD75 (2007)
  (see next page)

- **interacting fermion II:** $D = -N_c^2 \underbrace{\left(\frac{4\pi^2 - 15}{45}\right)}_{\approx 0.54}$ for $R \to \infty$ or $m_q \to \infty$

  free fermion $\to$
  introduce boundary condition
  $\to$ interaction (even confining)
  $\to$ generates a non-zero D-term!

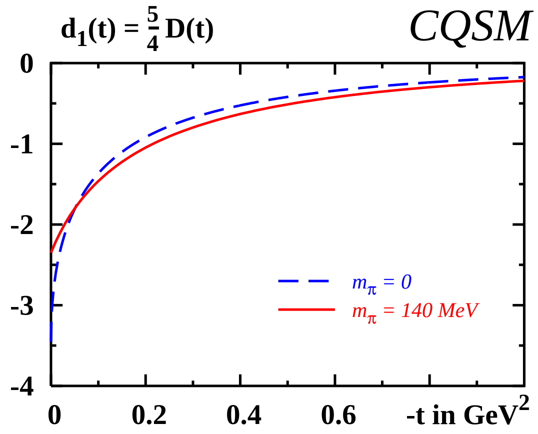
  respects large N_c
  also non-relativistic limit
  Hudson, PS PRD97 (2018)

- **distinction bosons and fermions:** $\begin{cases} \text{non-interacting boson } D = -1 \\ \text{non-interacting fermion } D = 0 \end{cases}$

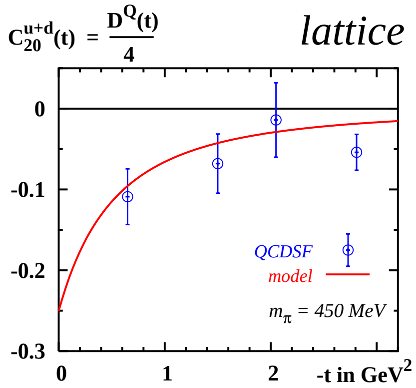
$D(t) = \frac{4}{5}\,d_1(t) = 4C(t)$ from models lattice, dispersion relations

- **bag model** $D = -1.145 < 0$ (Ji, Melnitchouk, Song (1997), Neubelt, Sampino et al (2019))

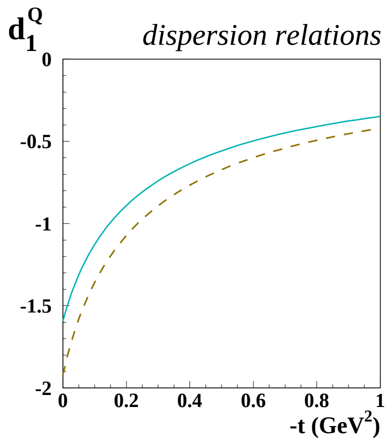
- **chiral quark soliton**
  Goeke et al, PRD75 (2007)
  $d_1(m_\pi) = \overset{\circ}{d}_1 + \frac{5k\,g_A^2 M}{64\,\pi\, f_\pi^2}\, m_\pi + \ldots$
  $\overset{\circ}{d}{}_1'(0) = -\frac{k\, g_A^2\, M}{32\,\pi f_\pi^2\, m_\pi} + \ldots \quad k = \begin{cases} 1, & N_c < \infty \\ 3, & N_c \to \infty \end{cases}$
  χPT cannot predict nucleon D-term
  Belitsky, Ji (2002), Diehl et al (2006)



- **lattice:** QCDSF
  Göckeler et al, PRL92 (2004)
  $\mu = 2\,\mathrm{GeV}$, $m_\pi = 450\,\mathrm{MeV}$
  disconnected diagrams neglected
  recently:
  $D^g(t) < 0$ with $|D^g(t)| > |D^Q(t)|$
  Shanahan, Detmold, PRD99 (2019)



- **dispersion relations** $d_1^Q(t)$
  Pasquini, Polyakov, Vanderhaeghen (2014)
  pion PDFs are input, scale $\mu^2 = 4\,\mathrm{GeV}^2$



What do we know about D-term from experiment?

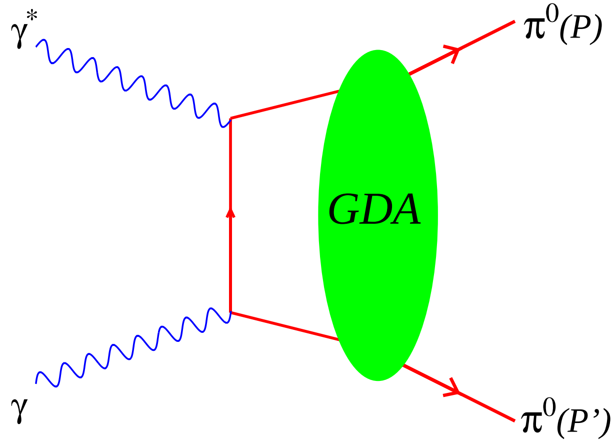
first insights from experiment

- **D-term of π^0**

  access EMT form factors of unstable particles through generalized distribution amplitudes (analytic continuation of GPDs) via $\gamma\gamma^* \to \pi^0\pi^0$ in e^+e^-
  Masuda et al (Belle), PRD 93, 032003 (2016)

  best fit to Belle data $\rightarrow$ $D^Q_{\pi^0} \approx -0.7$ at $\langle Q^2 \rangle = 16.6\ \mathrm{GeV}^2$
  Kumano, Song, Teryaev, PRD97, 014020 (2018)

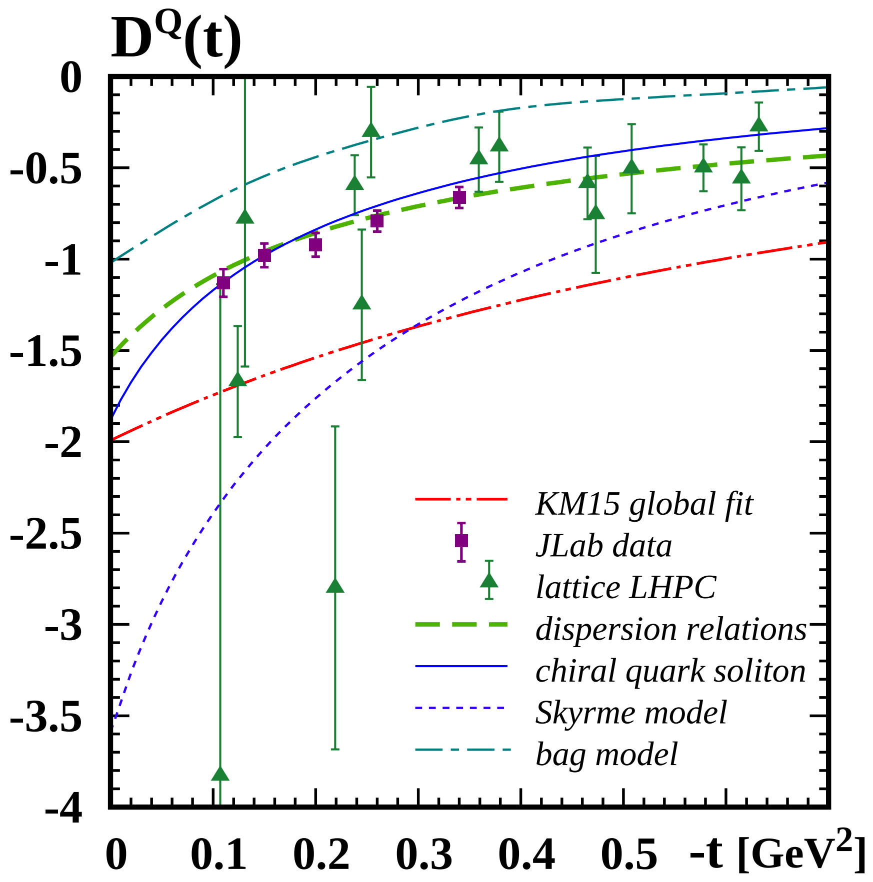
  compatible with soft pion theorem: $D_{\pi^0} \approx -1$
  (assuming gluons contribute the rest)



- D-term of nucleon

Burkert, Elouadrhiri, Girod, **Nature 557, 396 (2018)** based on:
Girod et al PRL 100 (2008) 162002 and Jo et al PRL 115 (2015) 212003

beam–spin asymmetry→$\mathcal{I}m\,\mathcal{H}$ unpol. cross section→$\mathcal{R}e\,\mathcal{H}$



$$\Delta(t,\mu^2) = 4\sum_q e_q^2 \left[d_1^q(t,\mu^2) + \ldots\right]$$

assumptions:

- neglect power corrections, NLO corrections at $E_{\text{beam}} = 6$ GeV and $\langle Q^2\rangle = 1.5\,\text{GeV}^2$
- only $\mathcal{H}$, neglect $\mathcal{E}$, etc
- $\Delta(t,\mu^2) = 4\sum_q e_q^2 d_1^q(t,\mu^2)$
  with $d_i^q(t,\mu^2)$ for $i = 3, 5, \ldots$ neglected
  (in CQSM $d_3^Q/d_1^Q \sim 0.3$, $d_5^Q/d_1^Q \sim 0.1$
  (Kivel, Polyakov, Vanderhaeghen (2001))
- assume $d_1^u \approx d_1^d$
  (okay in CQSM, to be tested in experiment)

  $\rightsquigarrow\ D^Q(t,\mu^2) \approx \frac{18}{25}\Delta(t,\mu^2)$
- how good are these approximations?
  will see: JLab12, COMPASS, EIC, future experiments

⇒ CLAS, KM-fits, dispersion relations, models, lattice: ***D*-term negative**

interpretation what do we learn from EMT form factors?

- Breit frame $\Delta^\mu = (0, \vec{\Delta})$ and $t = -\vec{\Delta}^2$

- analog to electric form factor, for proton:

$$G_E(\vec{\Delta}^2) = \int d^3\vec{r}\, \rho_E(\vec{r})\, e^{i\vec{\Delta}\vec{r}} = 1 - \frac{1}{6}\vec{\Delta}^2 \underbrace{\langle r_{ch}^2 \rangle}_{\approx (0.8\ldots\,\text{fm})^2} + \ldots$$

$\rightarrow$ charge distribution $Q = \int d^3\vec{r}\, \rho_E(\vec{r})$ Sachs, PR126 (1962) 2256

- **3D density**

  popular concept

- limitations:
  3D densities not exact, "relativistic corrections" for $r \lesssim \lambda_{\text{Compt}} = \frac{\hbar}{mc}$
  2D densities exact partonic probability densities (better concept)

limitations known since Sachs (1962). Discussed e.g. in:

- Belitsky & Radyushkin, Phys. Rept. 418, 1 (2005), Sec. 2.2.2
- X.-D. Ji, PLB254 (1991) 456 (Skyrme model, not a big effect)
- G. Miller, PRC80 (2009) 045210 (toy model, dramatic effect)
- Hudson, PS PRD **96** (2017) 114013 (not a big effect)

- important distinction:

  2D densities = **partonic probability densities** (unitarity)
  must (and better be) exact! $\rightarrow$ M. Burkardt (2000)
  apply to any particle (including the light pion)

  vs

  3D densities = **mechanical response functions**
  correlation functions ($\neq$ probabilities!)
  if reasonably small corrections have to be tolarated: here "ok"

- **attention:**
  heavy constituents $\rightarrow$ sufficient condition (not necessary!)
  despite light quarks: in QCD nucleon heavy (necessary condition)

- mathematically correct,
  physically justified for heavy particles:

  relative correction for $\langle r_E^2\rangle = \int d^3r\, r^2 T_{00}(r)/m$ is $\delta_{\rm rel} = 1/(2m^2R^2)$ Hudson, PS PRD (2007)
  numerically $\underbrace{\text{pion}}_{220\,\%}$, $\underbrace{\text{kaon}}_{25\,\%}$, $\underbrace{\text{nucleon}}_{3\,\%}$, $\underbrace{\text{deuterium}}_{1\times10^{-3}}$, $\underbrace{^{4}\text{He}}_{5\times10^{-4}}$, $\underbrace{^{12}\text{C}}_{3\times10^{-5}}$, $\underbrace{^{20}\text{Ne}}_{6\times10^{-6}}$, $\underbrace{^{56}\text{Fe}}_{5\times10^{-7}}$, $\underbrace{^{132}\text{Xe}}_{6\times10^{-8}}$, $\underbrace{^{208}\text{Pb}}_{2\times10^{-8}}$

- for nucleon strictly correct in large-N_c limit
  "$1/N_c$ only small parameter in QCD at all energies" (S. Coleman, Aspects of Symmetry)

- **interpretation as 3D-densities**
  M.V.Polyakov, PLB 555 (2003) 57

  Breit frame with $\Delta^\mu = (0, \vec{\Delta})$: static EMT $\boldsymbol{T_{\mu\nu}(\vec{r})} = \int \frac{d^3\vec{\Delta}}{2E(2\pi)^3}\, e^{i\vec{\Delta}\,\vec{r}}\, \langle P'|\hat{T}_{\mu\nu}|P\rangle$
  formulae correct, interpretation has grain of salt

$$\int d^3r\, T_{00}(\vec{r}) = M \qquad \text{known}$$

$$\int d^3r\, \varepsilon^{ijk}\, s_i\, r_j\, T_{0k}(\vec{r},\, \vec{s}) = \frac{1}{2} \qquad \text{known}$$

$$-\frac{2}{5} M \int d^3r \left(r^i r^j - \frac{r^2}{3}\delta^{ij}\right) T_{ij}(\vec{r}) \equiv \boldsymbol{D} \qquad \textbf{new!}$$

with: $$\boldsymbol{T_{ij}(\vec{r}) = s(r)\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\delta_{ij}\right) + p(r)\,\delta_{ij}} \qquad \textbf{stress tensor}$$

$\boldsymbol{s(r)}$ related to distribution of *shear forces*
$\boldsymbol{p(r)}$ distribution of *pressure* inside hadron
$\Big\} \longrightarrow$ **"mechanical properties"**

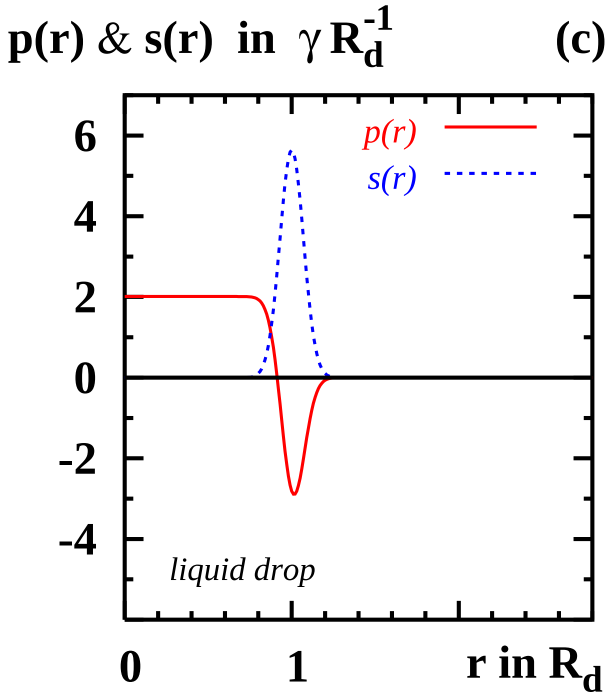
- **relation to stability**: EMT conservation $\Leftrightarrow$ $\partial^\mu \hat{T}_{\mu\nu} = 0$ $\Leftrightarrow$ $\nabla^i T_{ij}(\vec{r}) = 0$

$\hookrightarrow$ necessary condition for stability $\int_0^\infty \mathrm{d}r\, \boldsymbol{r^2\, p(r)} = 0$ (von Laue, 1911)

$$\boldsymbol{D} = -\frac{16\pi}{15}\, m \int_0^\infty \mathrm{d}r\, r^4 s(r) = 4\pi m \int_0^\infty \mathrm{d}r\, \boldsymbol{r^4\, p(r)}$$ $\rightarrow$ shows how internal forces balance
(already the sign insightful! So far always negative)

intuition from models:

- **liquid drop model of nucleus**



radius $R_A = R_0\, A^{1/3}$, $m_A = m_0\, A$

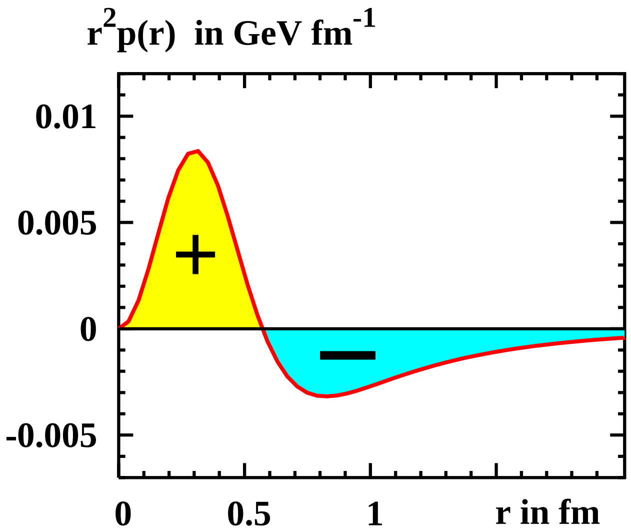
surface tension $\gamma = \frac{1}{2} p_0 R_A$, $s(r) = \gamma\, \delta(r - R_A)$

pressure $p(r) = p_0\, \Theta(R_A - r) - \frac{1}{3} p_0 R_A\, \delta(r - R_A)$

D-term $\boldsymbol{D = -\frac{4\pi}{3}\, m_A\, \gamma\, R_A^4 \approx -0.2\, A^{7/3}}$

M.V.Polyakov PLB555 (2003);
tested in Walecka model Guzey, Siddikov (2006)
alternative result in Liuti, Taneja, PRC 72 (2005)

- **chiral quark soliton model of nucleon**



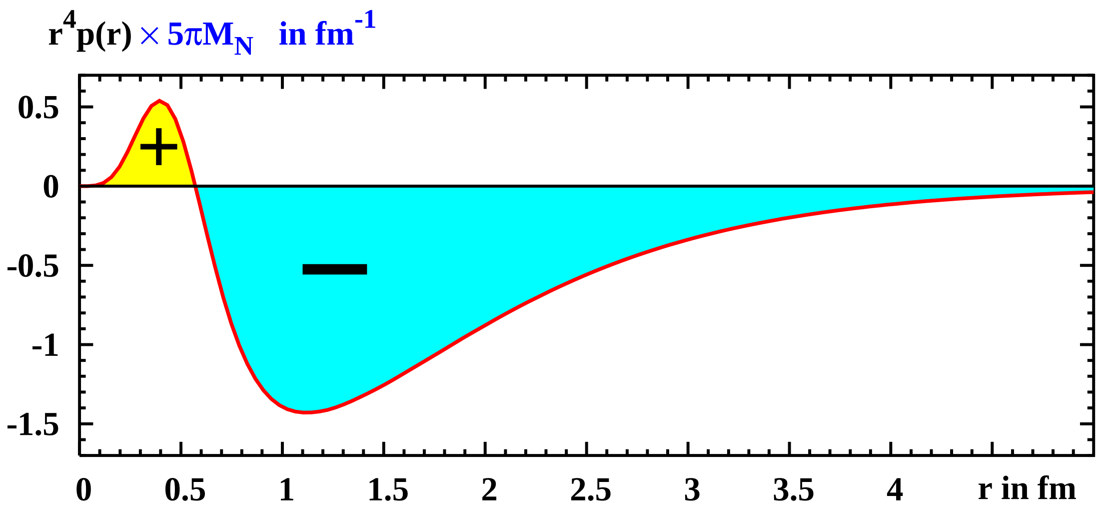
- $p(0) = 0.23\,\text{GeV/fm}^3 \approx 4 \times 10^{34}\,\text{N/m}^2$

  $\gtrsim$ 10-100×(pressure in center of neutron star)

- $p(r) = 0$ at $r = 0.57\,\text{fm}$ change of sign in pressure

- $p(r) = \left(\frac{3g_A^2}{8\pi f_\pi}\right)^2 \frac{1}{r^6}$ at large r in chiral limit $m_\pi \to 0$

  Goeke et al, PRD75 (2007) 094021



recall: $\int_0^\infty dr\, \boldsymbol{r^2\, p(r)} = 0$

$\boldsymbol{D} = 4\pi m \int_0^\infty dr\, \boldsymbol{r^4\, p(r)} < 0$

negative sign of D $\Leftrightarrow$ **stability** (necessary condition)

(see also Nature article Burkert et al, also lattice Shanahan, Detmold)

mechanical radius

- $T_{ij}(\vec{r}) = s(r)\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\delta_{ij}\right) + p(r)\,\delta_{ij} =$ symmetric 3×3 matrix $\rightarrow$ diagonalize:

  $\frac{2}{3}\,s(r) + p(r) =$ normal force (eigenvector $\vec{e}_r$)

  $-\frac{1}{3}\,s(r) + p(r) =$ tangential force ($\vec{e}_\theta$, $\vec{e}_\phi$, degenerate for spin 0 and $\frac{1}{2}$)

- mechanical stability $\Leftrightarrow$ normal force directed towards outside

  $\Leftrightarrow T^{ij} e_r^j\, dA = \underbrace{[\frac{2}{3}\,s(r) + p(r)]}_{>0} e_r^i\, dA \quad \Rightarrow \quad \boldsymbol{D < 0}$ **(proof!)** Perevalova et al (2016)

- define: $\langle \boldsymbol{r^2} \rangle_{\text{mech}} = \dfrac{\int d^3r\; r^2[\frac{2}{3}\,s(r) + p(r)]}{\int d^3r\; [\frac{2}{3}\,s(r) + p(r)]} = \dfrac{6D(0)}{\int_{-\infty}^{0} dt\, D(t)}$ vs $\langle r^2_{\text{ch}} \rangle = \frac{6\,G_E'(0)}{G_E(0)}$ "anti-derivative"

  intuitive result for large nucleus $\frac{2}{3}\,s(r) + p(r) = p_0\,\Theta(R_A - r) \rightarrow \langle r^2 \rangle_{\text{mech}} = \frac{3}{5}\,R_A^2$

  M.Polyakov, PS arXiv:1801.05858 (Kumano, Song, Teryaev PRD (2018) used $D'(0)$ but inadequate)

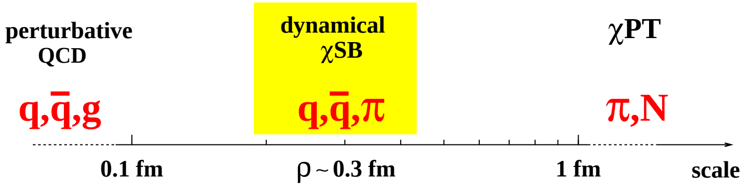
- in chiral limit $\langle r^2 \rangle_{\text{mech}}$ finite vs $\langle r^2_{\text{ch}} \rangle$ divergent (better concept)

- neutron $\langle r^2 \rangle_{\text{mech}}$ same as proton(!) $\langle r^2_{\text{ch}} \rangle = -0.11\,\text{fm}^2$ inappropriate concept for neutron size (see also recent work Lorcé, Moutarde, Trawiński)

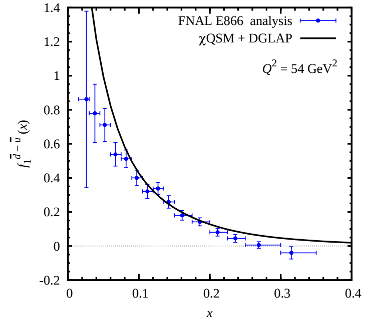
- proton: $\langle \boldsymbol{r^2} \rangle_{\text{mech}} \approx \boldsymbol{0.75}\,\langle \boldsymbol{r^2_{\text{ch}}} \rangle$ in chiral quark soliton model (why always this model?)

brief interlude: chiral quark soliton model

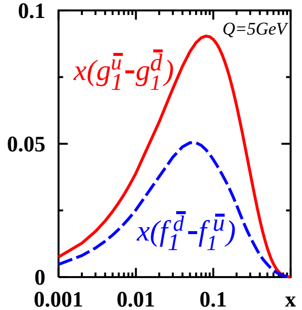
- spontanous breaking of chiral symmetry
  effective degrees of freedom
  q, $\bar{q}$, Goldstone bosons



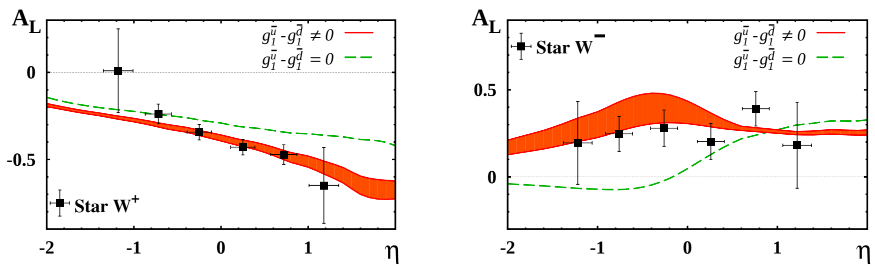
- **postdiction:** flavor asymmetry in unpolarized sea
  Pobylitsa et al, Phys.Rev.D59 (1999) 034024



- **prediction:** even larger asymmetry in helicity sea
  Diakonov et al, Nucl.Phys.B480 (1996) 341



- **prediction:** A_L in $W^{\pm}$ at RHIC:
  Dressler et al, EPJC 18 (2001) 719
  confirmed: STAR, PRL 113, 072301 (2014)



Applications of EMT

- hadronic decays of charmonia $\Psi(2S) \to J/\Psi\,\pi\,\pi$: $\langle \pi\,\pi\,|T^{\mu\nu}|0\rangle \;\leftrightarrow\; \langle \pi\,|T^{\mu\nu}|\,\pi\rangle$
  Voloshin, Zakharov (1980), Novikov, Shifman (1981), Voloshin, Dolgov (1982)

- hadrocharmonia
  Voloshin (1982), Eides, Petrov, Polyakov (2017) $\leftarrow$ here

- mass decomposition
  X. D. Ji, PRL 74, 1071 (1995), PRD 52, 271 (1995)

- spin decomposition
  X. D. Ji, Phys. Rev. Lett. 78, 610 (1997), Leader, Lorcé, Phys. Rept. 541, 163 (2014)

- forces inside hadrons
  M. V. Polyakov, Phys. Lett. B 555, 57 (2003)

- q- and g-contributions to trace anomaly
  Tuesday talk by Yoshitaka Hatta

- Thursday morning: talks by Simonetta Liuti, Adam Freese, Cedric Lorcé
  also: Constantia Alexandrou, Matthias Burkardt, Sylvester Joosten, Andreas Metz, Chao Shi
  in principle applications in any other talk too! Pay attention to all talks!! Stay tuned!!!

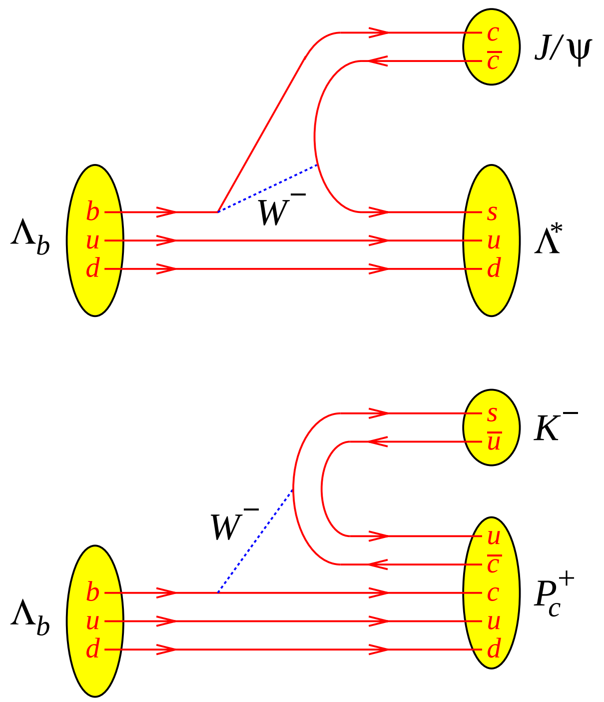
Application: hidden-charm pentaquarks as hadrocharmonia

$\Lambda_b^0 \longrightarrow J/\Psi\, p\, K^-$ seen
Aaij *et al.* PRL 115 (2015)

Λ_b^0 $m = 5.6$ GeV, $\tau = 1.5$ ps
J/Ψ $m = 3.1$ GeV, $\Gamma = 93$ keV, $\Gamma_{\mu^+\mu^-} = 6\,\%$
Λ^* $m = 1.4$ GeV or more, $\Lambda^* \to K^- p$ in 10^{-23}s

$\longrightarrow J/\Psi\, \Lambda^*$

$\longrightarrow P_c^+\, K^-$



state	m [MeV]	Γ [MeV]	$\Gamma_{\rm rel}$	mode	J^P
$P_c^+(4380)$	$4380 \pm 8 \pm 29$	$205 \pm 18 \pm 86$	$(4.1 \pm 0.5 \pm 1.1)\,\%$	$J/\psi\, p$	$\frac{3}{2}^{\mp}$ or $\frac{5}{2}^{+}$
$P_c^+(4450)$	$4450 \pm 2 \pm 3$	$39 \pm 5 \pm 19$	$(8.4 \pm 0.7 \pm 4.2)\,\%$	$J/\psi\, p$	$\frac{5}{2}^{\pm}$ or $\frac{3}{2}^{-}$

Hadrocharmonium approach Eides, Petrov, Polyakov, PRD93, 054039 (2016)

- **theoretical approach:** in heavy quark limit $\Rightarrow$ $V_{\rm eff} = -\frac{1}{2}\,\alpha\,\vec{E}^2$ Voloshin, Yad. Fiz. **36**, 247 (1982)

- **chromoelectric polarizability** property of charmonium

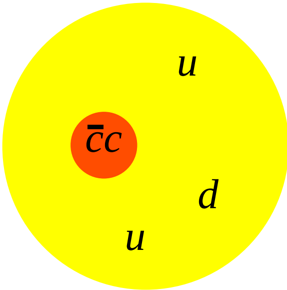
  $\alpha(1S) \approx (1.6 \pm 0.8)\,\text{GeV}^{-3}$ Polyakov, PS PRD98 (2018); Sugiura et al, arXiv:1711.11219
  $\alpha(2S) \approx 17\,\text{GeV}^{-3}$ Eides et al; Perevalova, Polyakov, PS, PRD 94, 054024 (2016)
  $|\alpha(2S \to 1S)| \approx 2\,\text{GeV}^{-3}$ Voloshin, Prog. Part. Nucl. Phys. 61 (2008) 455
  cf. quarkonia = Coulomb systems Peskin NPB 156 (1979) 365

- **chromoelectric field strength:** $\vec{E}^2 \to T_{00}(r),\ p(r)$ from CQSM, Skyrme

- **compute quarkonium-nucleon bound state:** $\left(-\frac{\vec{\nabla}^2}{2\mu} + V_{\rm eff}(r)\right)\psi = E_{\rm bind}\,\psi$

- **results:** N and J/ψ form no bound state

  N and $\psi(2S)$ form **two** bound states
  with nearly degenerate masses $\sim 4450\,\text{MeV}$
  mass-splitting $\mathcal{O}(10\text{–}20)\,\text{MeV}$, $J^P = \frac{1}{2}^-$ and $\frac{3}{2}^-$,
  important: partial width $\Gamma = |\alpha(2S \to 1S)|^2 \times \cdots =$ few tens of MeV



- **update:** `26 March 2019, Observation of new pentaquarks, Moriond conference`:

  previous $P_c^+(4450)$ resolved in:
  $P_c^+(4440)$ $m = (4440.3 \pm 1.3^{+4.1}_{-4.7})$MeV, $\Gamma = (20.6 \pm 4.9^{+8.7}_{-10.1})$MeV
  $P_c^+(4457)$ $m = (4457.3 \pm 0.6^{+4.1}_{-1.7})$MeV, $\Gamma = (6.4 \pm 2.0^{+5.7}_{-1.9})$MeV **exciting!**

- **predictions:** bound states of $\psi(2S)$ with Δ and hyperons $\leftarrow$ test approch

Summary & Outlook

- **GPDs, GDAs** $\rightarrow$
  form factors of **energy momentum tensor**
- **D-term**: last unknown global property,
  related to forces, attractive and physically appealling $\rightarrow$ "motivation"
- **first results**(!) from experiment/phenomenology for proton, π^0
  compatible with results from theory and models
- define **pressure, forces & mechanical radius**
  $\rightarrow$ unique, appealing, complementary information!
- **applications:**
  imaging of nucleon structure
  hadrocharmonia pentaquarks & tetraquarks
  more applications (see other talks at workshop)
- rich **potential**, new **predictions**, some work is done
  lots of work still ahead of us
- I hope this talk showed:
  appealing, interesting topics, to be continued!

Summary & Outlook

- **GPDs, GDAs** $\rightarrow$
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- I hope this talk showed:
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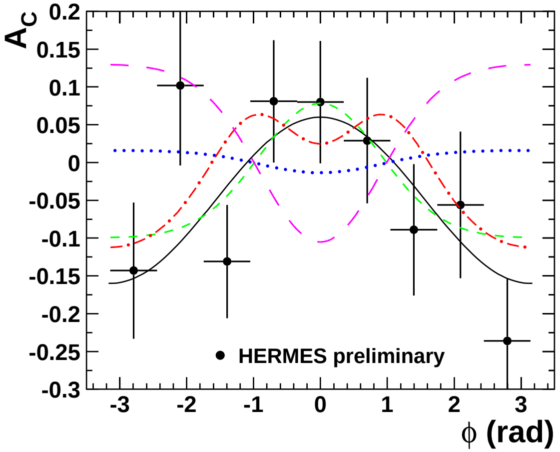
- **looking forward to other talks! Stay tuned!!**

Thank you!

Support slides

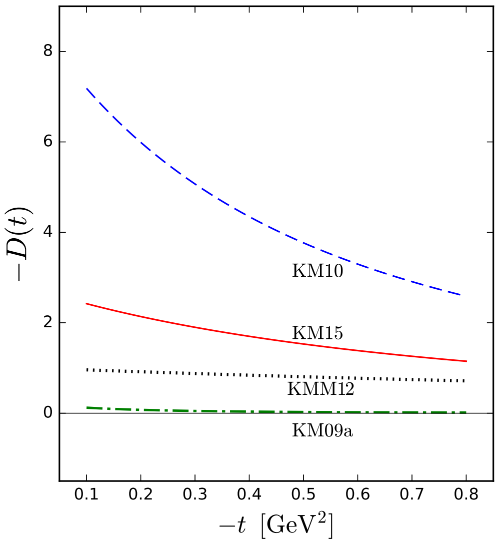
Results from experiment & phenomenology

- HERMES proceeding NPA711, 171 (2002) (model-dependent)



beam charge asymmetry (DVCS e^+ vs e^-)
dotted line: VGG model without D-term (ruled out)
dashed line: VGG model + positive D-term (ruled out)
dashed-dotted: VGG model + **negative** D-term (yeah!)
(cf. Belitsky, Müller, Kirchner, NPB 629 (2002) 323)

- fits by Kresimir Kumerički, Dieter Müller et al: $D < 0$ needed! (model-independent)



DVCS parametrizations from:
Kumerički, Müller, NPB 841 (2010) 1
Kumerički, Müller, Murray, Phys. Part. Nucl. 45 (2012) 723
Kumerički, Müller, EPJ Web Conf. 112 (2016) 01012
Fig. 9 in ECT* workshop proceeding 1712.04198
statistical uncertainty of D in KMM12: $\sim 50\,\%$,
statistical uncertainty of D in KM15: $\sim 20\,\%$.
unestimated systematic uncertainty
K.Kumerički private communication

EMT form factors of nucleon I (Kobzarev & Okun 1962, Pagels 1966)

$$\langle p'|\hat{T}^a_{\mu\nu}|p\rangle = \bar{u}(p')\Bigg[A^a(t,\mu^2)\,\frac{\gamma_\mu P_\nu + \gamma_\nu P_\mu}{2} + B^a(t,\mu^2)\,\frac{i(P_\mu\sigma_{\nu\rho} + P_\nu\sigma_{\mu\rho})\Delta^\rho}{4M} + D^a(t,\mu^2)\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4M} + \bar{c}^a(t,\mu^2)g_{\mu\nu}\Bigg]u(p)$$

- conserved external current $\partial_\mu \hat{T}^{\mu\nu} = 0$, $\hat{T}_{\mu\nu} = \sum_a \hat{T}^a_{\mu\nu}$ $(a = q, g)$
- $A(t) = \sum_a A^a(t,\mu^2)$, $B(t)$, $D(t)$ scale invariant, $\sum_a \bar{c}^a(t,\mu^2) = 0$
- constraints: **mass** $\Leftrightarrow$ $A(0) = 1$ $\Leftrightarrow$ quarks + gluons carry 100 % of nucleon momentum

  spin $\Leftrightarrow$ $B(0) = 0$ $\Leftrightarrow$ total anomalous gravitomagnetic moment vanishes *

  D-term $\Leftrightarrow$ $D(0) \equiv \boldsymbol{D}$ $\rightarrow$ unconstrained! **Last global unknown!**

$$2P = (p' + p)$$
$$\Delta = (p' - p)$$
$$t = \Delta^2$$

notation: $A^q(t) + B^q(t) = 2\,J^q(t)$

$D^q(t) = \frac{4}{5}\,d_1^q(t) = \frac{1}{4}\,C^q(t)$ or $C^q(t)$

$A^q(t) = M_2^q(t)$

* equivalent to: total nucleon spin $J^q + J^g = \frac{1}{2}$ is due to quarks + gluons (via Gordon identity)

EMT form factors of nucleon II (Kobzarev & Okun 1962, Pagels 1966)

$$\langle p'|\hat{T}^{a}_{\mu\nu}|p\rangle = \bar{u}(p')\Bigg[A^a(t,\mu^2)\,\frac{P_\mu P_\nu}{M} + J^a(t,\mu^2)\,\frac{i(P_\mu\sigma_{\nu\rho}+P_\nu\sigma_{\mu\rho})\Delta^\rho}{2M} + D^a(t,\mu^2)\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4M} + \bar{c}^a(t,\mu^2)g_{\mu\nu}\Bigg]u(p)$$

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$$2P = (p' + p) \qquad \Delta = (p' - p) \qquad t = \Delta^2$$

notation: $A^q(t) + B^q(t) = 2\,J^q(t)$

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$A^q(t) = M_2^q(t)$

* equivalent to: nucleons total anomalous gravitomagnetic moment vanishes (Gordon identity)

Nuclei

- nuclei in liquid drop model $\boldsymbol{D \approx -0.2 \times A^{7/3}}$ $\rightarrow$ potential for DVCS with nuclei!
  Maxim Polyakov (2002) (see below)

- nuclei in Walecka model
  Guzey, Siddikov (2006)

$$\begin{aligned} {}^{12}\mathrm{C}: \quad D &= -6.2 \\ {}^{16}\mathrm{O}: \quad D &= -115 \\ {}^{40}\mathrm{Ca}: \quad D &= -1220 \\ {}^{90}\mathrm{Zr}: \quad D &= -6600 \\ {}^{208}\mathrm{Pb}: \quad D &= -39000 \end{aligned}$$

Q-balls

- Q-balls, non-topological solitons, strongly interacting, $\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - V(|\phi|)$ S.Coleman (1985)

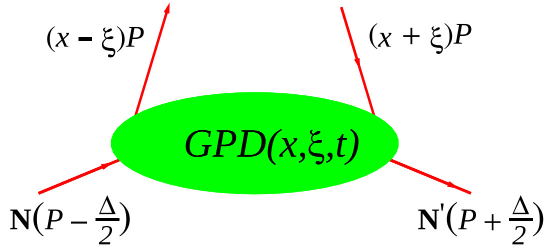
  ground states $90 \leq -D \leq \infty$
  Mai, PS PRD**86**, 076001 (2012)

  N^{th} excited Q-ball state (decay $\rightarrow$ ground states): $D = -\,\text{const}\, N^8$
  Mai, PS PRD**86**, 096002 (2012)

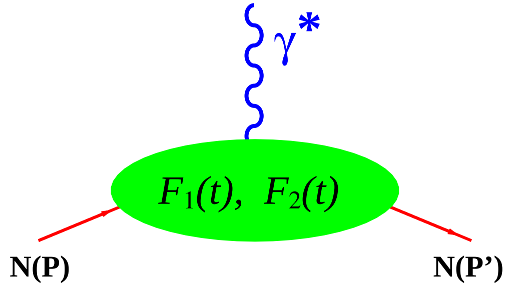
  Q-cloud limit, most extreme instability we could find: $D = -\,\text{const}/\varepsilon^2$ in the limit $\varepsilon \rightarrow 0$
  Cantara, Mai, PS NPA**953**, 1 (2016)

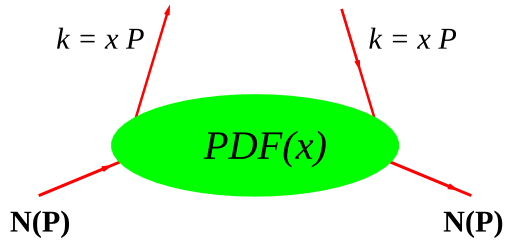
  Q-cloud excitations, even more extreme instability: $D < 0$ divergent and even more negative
  Bergabo, Cantara, PS, in preparation (2019)

GPDs



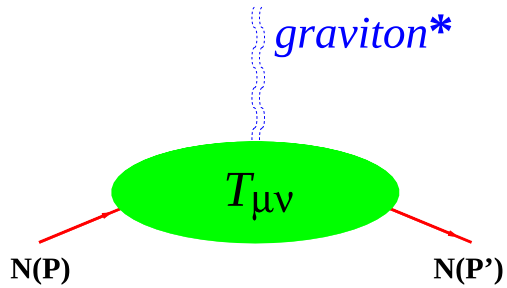
- **microsurgery**
  $\xi = (n \cdot \Delta)/(n \cdot P)$, $t = \Delta^2$
  $P = \frac{1}{2}(p' + p)$, $\Delta = p' - p$
  $n^2 = 0$, $n \cdot P = 2$, $k = xP$
  renormalization scale μ





- **generalize** PDFs, form factors
  $\int \mathrm{d}x\ H^q(x,\xi,t) = F_1^q(t)$
  $\int \mathrm{d}x\ E^q(x,\xi,t) = F_2^q(t)$
  $\lim_{\Delta \to 0} H^q(x,\xi,t) = f_1^q(x)$

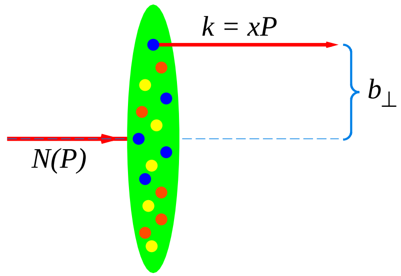
- **gravitational form factors (polynomiality)**
  $\int \mathrm{d}x\ x\, H^q(x,\xi,t) = \boldsymbol{A^q(t)} + \xi^2 \boldsymbol{D^q(t)}$
  $\int \mathrm{d}x\ x\, E^q(x,\xi,t) = \boldsymbol{B^q(t)} - \xi^2 \boldsymbol{D^q(t)}$



- **Ji sum rule**
  $\int \mathrm{d}x\ x\,(H^a + E^a)(x,\xi,t) = 2J^q(t) \xrightarrow{t\to 0} 2J^q(0)$

- **tomography** (M. Burkardt)

$$H^q(x,b_\perp) = \int \frac{\mathrm{d}^2\boldsymbol{\Delta}_\perp}{(2\pi)^2}\left[\lim_{\xi\to 0} H^q(x,\xi,t)\right] e^{i\boldsymbol{\Delta}_\perp \boldsymbol{b}_\perp}$$

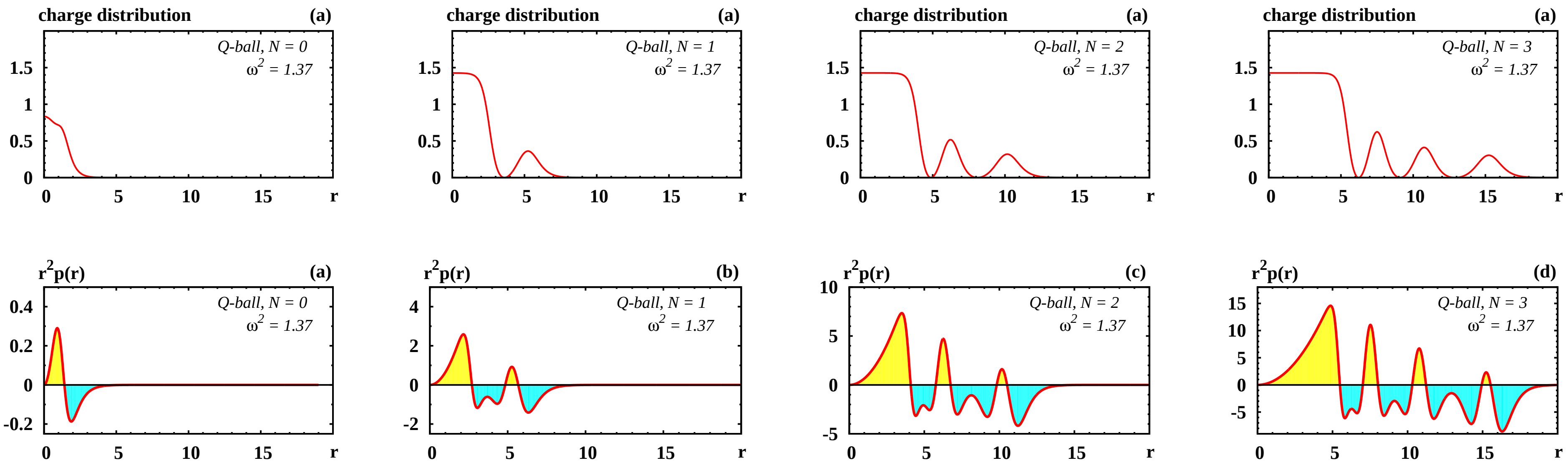


- **more intuition from toy system: Q-ball**

  $\mathcal{L} = \frac{1}{2}(\partial_\mu \Phi^*)(\partial^\mu \Phi) - V$ with U(1) global symm., $V = A\,(\Phi^*\Phi) - B\,(\Phi^*\Phi)^2 + C\,(\Phi^*\Phi)^3$, $\Phi(t,\vec{r}) = e^{i\omega t}\,\phi(r)$

  $N = 0$ ground state, $N = 1$ first excited state, etc Volkov & Wohnert (2002), Mai, PS PRD86 (2012)

  charge density exhibits N shells, $p(r)$ exhibits $(2N+1)$ zeros



  excited states unstable, but $\int_0^\infty \mathrm{d}r\; r^2 p(r) = 0$ always valid, and D-term always negative!

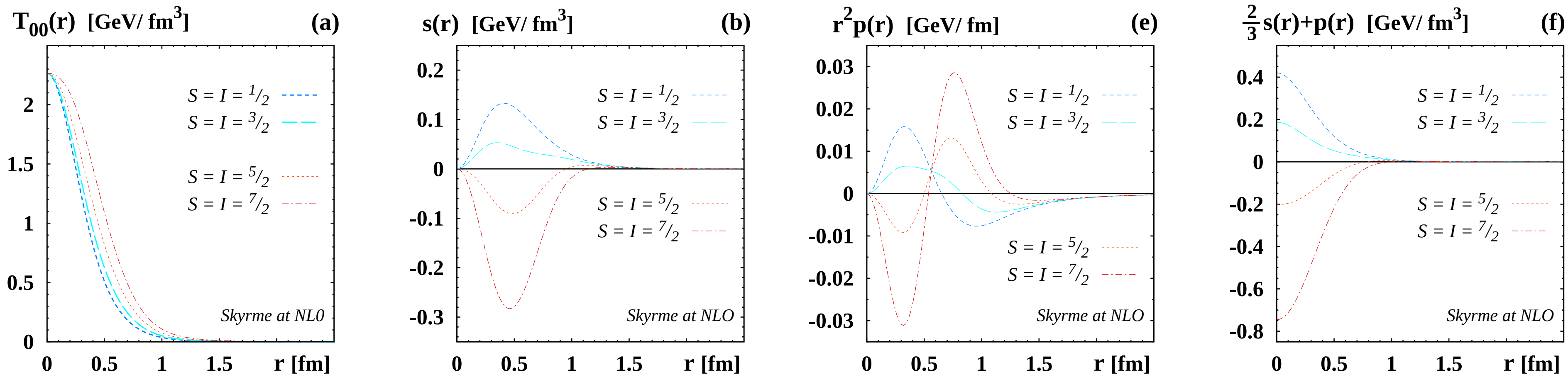
  so far **all D-terms** negative: pions, nucleons, nuclei, nucleons in nuclear matter, photons, Q-balls, Q-clouds

  could Roper resonance look like this? (possible to measure??) (transition GPDs???)
  However e.g. Δ-resonance, similar to nucleon! (lowest state for $J = T = \frac{3}{2}$, see below)

- side remark: you won't believe in how many models of the nucleon "the nucleon **does not** exist!!"
  (explodes or implodes within $t < 10^{-23}$s vs $t_{\text{proton}} > 10^{32}$ years ...)

Application I: nucleon, Δ, large-N_c artifacts   Witten 1979

in large N_c baryons $=$ rotational excitations of soliton with $S = I = \underbrace{\frac{1}{2}, \frac{3}{2}}_{\text{observed}}, \underbrace{\frac{5}{2}, \ldots}_{\text{artifacts}}$



$$M_B = M_{\text{sol}} + \frac{S(S+1)}{2\Theta}$$

nucleon $s(r) \neq \gamma\delta(r-R)$
Δ much more diffuse

$$\int_0^\infty \mathrm{d}r\, r^2 p(r) = 0$$

stability needs more:
$p(r) > 0$ in center,
negative outside
okay for nucleon, Δ
$\Longrightarrow$ implies $D < 0$

mechanical stability
$T^{ij}\, da^j \geq 0$
$\Leftrightarrow \ \frac{2}{3}\, s(r) + p(r) \geq 0$
artifacts do not satisfy!
$\Rightarrow$ **have positive D-term!!**
That's why they do not exist!
EMT: dynamical understanding
Perevalova et al (2016)

$\Rightarrow$ particles with positive D unphysical!!!

Application II: hidden-charm pentaquarks as hadrocharmonia

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Aaij *et al.* PRL 115 (2015)

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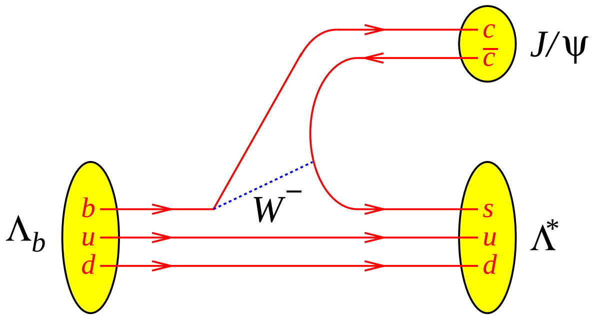
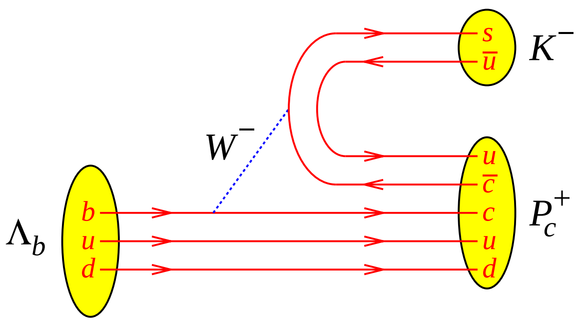
$\longrightarrow J/\Psi\, \Lambda^*$



$\longrightarrow K^-\, P_c^+$



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appealing approach to new pentaquarks

M. I. Eides, V. Y. Petrov and M. V. Polyakov, PRD93, 054039 (2016)

- **theoretical approach**

  $R_{c\bar{c}} \ll R_N \;\Rightarrow$ non-relativistic multipole expansion Gottfried, PRL 40 (1978) 598
  baryon-quarkonium interaction dominated by 2 virtual chromoelectric dipole gluons

  $V_{\rm eff} = -\frac{1}{2}\,\alpha\,\vec{\boldsymbol{E}}^2$ Voloshin, Yad. Fiz. **36**, 247 (1982)

- **chromoelectric polarizability**

$$\alpha(1S) \approx 0.2\,\text{GeV}^{-3}\ \text{(pert)},$$
$$\alpha(2S) \approx 12\,\text{GeV}^{-3}\ \text{(pert)},$$
$$\alpha(2S \to 1S) \approx \begin{cases} -0.6\,\text{GeV}^{-3}\ \text{(pert)}, \\ \pm 2\,\text{GeV}^{-3}\ \text{(pheno)}, \end{cases}$$

in heavy quark mass limit & large-N_c limit
$\rightsquigarrow$ "perturbative result" Peskin, NPB 156 (1979) 365

value for $2S \to 1S$ transition from
phenomenological analysis of $\psi' \to J/\psi\,\pi\,\pi$ data
Voloshin, Prog. Part. Nucl. Phys. 61 (2008) 455

- **chromoelectric field strength:**

$$\vec{\boldsymbol{E}}^2 = g^2\left(\frac{8\pi^2}{bg^2}\,T^{\mu}{}_{\mu} + T^G_{00}\right)$$

$b = \frac{11}{3}N_c - \frac{2}{3}N_F$ leading coeff. of β-function
$g =$ strong coupling at low (nucleon) scale $\lesssim 1\,\text{GeV}$
$g_s =$ strong coupling at scale of heavy quark ($g_s \neq g$)
$T^G_{00} = \xi T_{00}$ with $\xi =$ fractional contributions of gluon to M
$T^{\mu}{}_{\mu} = T^{00} - T^{ii}$, stress tensor $T^{ij} = \left(\frac{r^i}{r}\frac{r^j}{r} - \frac{1}{3}\delta^{ij}\right)s(r) + \delta^{ij}\,p(r)$

- **universal effective potential**

$$\boldsymbol{V}_{\rm eff} = -\frac{1}{2}\,\boldsymbol{\alpha}\,\frac{8\boldsymbol{\pi}^2}{\boldsymbol{b}}\,\frac{\boldsymbol{g}^2}{\boldsymbol{g}_s^2}\Big[\boldsymbol{\nu}\,\boldsymbol{T}_{00}(\boldsymbol{r}) + 3\boldsymbol{p}(\boldsymbol{r})\Big]\,, \quad \boldsymbol{\nu} = 1 + \boldsymbol{\xi}_s\,\frac{\boldsymbol{b}\,\boldsymbol{g}_s^2}{8\boldsymbol{\pi}^2}$$

$\boldsymbol{\nu \approx 1.5}$ estimate by Eides et al, op. cit.
Novikov & Shifman, Z.Phys.C8, 43 (1981);
X. D. Ji, Phys. Rev. Lett. **74**, 1071 (1995)

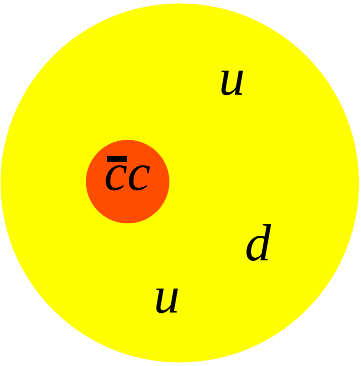
- **future application**: GPDs
  $\Rightarrow$ EMT form factors $\Rightarrow$ EMT densities
  $\Rightarrow$ universal potential V_{eff} for quarkonium-baryon interaction!

- **currently:** chiral quark soliton model (Eides et al, 2015); Skyrme (Perevalova et al 2016)

- **compute quarkonium-nucleon bound state**

  solve $\left(-\frac{\vec{\nabla}^2}{2\mu}+V_{\text{eff}}(r)\right)\psi = E_{\text{bind}}\,\psi$

  $\mu =$ reduced quarkonium-baryon mass



- **results:** N and J/ψ form no bound state
  N and $\psi(2S)$ form **two** bound states
  with nearly degenerate masses $\sim$4450 MeV
  mass-splitting $\mathcal{O}(10\text{–}20)$ MeV, $\alpha(2S) \approx 17\ \text{GeV}^{-3}$
  quantum numbers $J^P = \frac{1}{2}^-$ and $\frac{3}{2}^-$,
  important: partial width $\Gamma = |\alpha(2S \to 1S)|^2 \times \cdots =$ few tens of MeV

- **predictions:** for bound states of $\psi(2S)$ with Δ and hyperons $\leftarrow$ test approch

- **update** `26 March 2019, Observation of new pentaquarks, Moriond conference`:
  $P_c^+(4312)$ $m = (4311.9 \pm 0.7^{+6.8}_{-0.6})$MeV, $\Gamma = (9.8 \pm 2.7^{+3.7}_{-4.5})$MeV
  $P_c^+(4440)$ $m = (4440.3 \pm 1.3^{+4.1}_{-4.7})$MeV, $\Gamma = (20.6 \pm 4.9^{+8.7}_{-10.1})$MeV
  $P_c^+(4457)$ $m = (4457.3 \pm 0.6^{+4.1}_{-1.7})$MeV, $\Gamma = (6.4 \pm 2.0^{+5.7}_{-1.9})$MeV **exciting!**