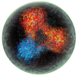
Quark fragmentation and dynamical mass generation

Andrea Signori

Argonne National Laboratory

QCD evolution 2019

May 13, 2019



Outline

Hadronization

Momentum sum rules

Phenomenology

Outline

Hadronization

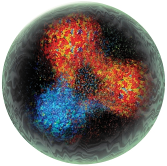
Momentum sum rules

Phenomenology

Hadronization and QCD

Hadronization is directly connected to the dynamical generation of some of the hadronic properties, e.g.:

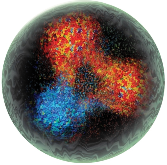
- the mass
- the spin
- the size of hadrons



Hadronization and QCD

Hadronization is directly connected to the dynamical generation of some of the hadronic properties, e.g.:

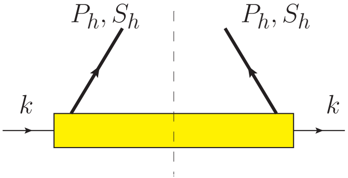
- the mass
- the spin
- the size of hadrons



Hadronization is also connected to the confinement of partons and to the chiral symmetry, and is thus one of the most interesting phenomena within QCD.

Quark fragmentation functions

$$\Delta_{ij}(k, P_h, S_h) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{ikx}\, \frac{\mathrm{Tr}_c}{N_c} \langle\Omega|\hat{T} W_1(\infty,\xi)\psi_i(\xi)\, a^\dagger a\, \overline{\psi}_j(0) W_2(0,\infty)|\Omega\rangle$$



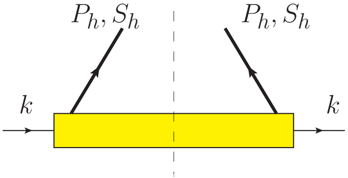
quark pol. / hadron pol.

hadron pol. \ quark pol.	U	L	T
U	D_1		$H_1^\perp$
L		G_{1L}	$H_{1L}^\perp$
T	$D_{1T}^\perp$	G_{1T}	$H_1,\ H_{1T}^\perp$

$$\Delta_{ij}(z, \mathbf{P}_{h\perp}) = \mathrm{Disc} \int \frac{dk^+}{2z} [\Delta_{ij}(k, P_h)]_{k^- = P_h^- / z} = \frac{\gamma^+}{2} D_1(z, P_{h\perp}^2) + \cdots$$

Quark fragmentation functions

$$\Delta_{ij}(k, P_h, S_h) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{ikx}\, \frac{\mathrm{Tr}_c}{N_c} \langle \Omega | \hat{T} W_1(\infty, \xi) \psi_i(\xi)\, a^\dagger a\, \overline{\psi}_j(0) W_2(0, \infty) | \Omega \rangle$$



quark pol.

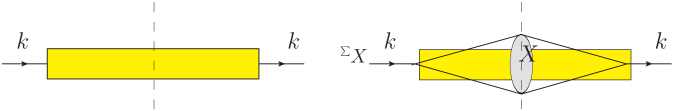
hadron pol.	U	L	T
U	D_1		$H_1^\perp$
L		G_{1L}	$H_{1L}^\perp$
T	$D_{1T}^\perp$	G_{1T}	H_1, $H_{1T}^\perp$

$$\Delta_{ij}(z, \mathbf{P}_{h\perp}) = \mathrm{Disc} \int \frac{dk^+}{2z} [\Delta_{ij}(k, P_h)]_{k^- = P_h^-/z} = \frac{\gamma^+}{2} D_1(z, P_{h\perp}^2) + \cdots$$

Here we consider quark to single-hadron fragmentation functions (FFs), but the argument can be extended, in principle, to di-hadron FFs and gluon FFs.

The cut quark propagator

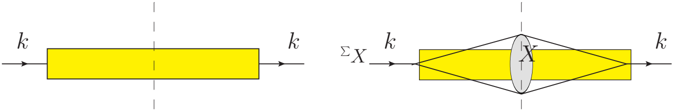
$$\Xi_{ij}(k;v) = \mathrm{Disc} \int \frac{d^4\xi}{(2\pi)^4}\, e^{ikx}\, \frac{\mathrm{Tr}_c}{N_c} \langle\Omega|\hat{T} W_1(\infty,\xi;v)\psi_i(\xi)\overline{\psi}_j(0) W_2(0,\infty;v)|\Omega\rangle$$



- Partonic picture: gauge invariant dressed quark correlator
  - only the discontinuity is considered → on-shellness
  - the color is neutralized by the average

The cut quark propagator

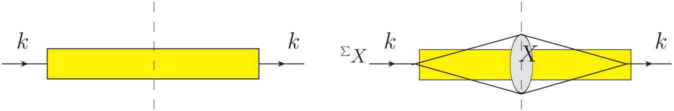
$$\Xi_{ij}(k;v) = \text{Disc} \int \frac{d^4\xi}{(2\pi)^4}\, e^{ikx}\, \frac{\text{Tr}_c}{N_c} \langle\Omega|\hat{T} W_1(\infty,\xi;v)\psi_i(\xi)\overline{\psi}_j(0)W_2(0,\infty;v)|\Omega\rangle$$



- Partonic picture: gauge invariant dressed quark correlator
  - only the discontinuity is considered $\rightarrow$ on-shellness
  - the color is neutralized by the average
- Hadronic picture: "fully inclusive jet" correlator
  - X: the complete set of hadronization products crossing the cut
  - no hadrons are measured $\rightarrow$ no need for algorithms to define a jet
  - the scale is defined by the end-point kinematics

The cut quark propagator

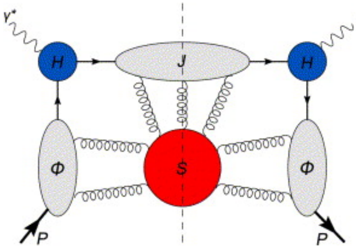
$$\Xi_{ij}(k;v) = \mathsf{Disc} \int \frac{d^4\xi}{(2\pi)^4}\, e^{ikx}\, \frac{\mathsf{Tr}_c}{N_c} \langle\Omega|\hat{T} W_1(\infty,\xi;v)\psi_i(\xi)\overline{\psi}_j(0) W_2(0,\infty;v)|\Omega\rangle$$



- Partonic picture: gauge invariant dressed quark correlator
  - only the discontinuity is considered → on-shellness
  - the color is neutralized by the average
- Hadronic picture: “fully inclusive jet” correlator
  - X: the complete set of hadronization products crossing the cut
  - no hadrons are measured → no need for algorithms to define a jet
  - the scale is defined by the end-point kinematics
- insights into dynamical generation of mass and momentum and chiral symmetry breaking

The cut quark propagator

$$\Xi_{ij}(k;v) = \text{Disc} \int \frac{d^4\xi}{(2\pi)^4}\, e^{ikx}\, \frac{\text{Tr}_c}{N_c} \langle \Omega | \hat{T} W_1(\infty,\xi;v) \psi_i(\xi) \overline{\psi}_j(0) W_2(0,\infty;v) | \Omega \rangle$$

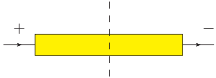


See Sterman NPB 281 ('87) 310, Chen et al. NPB 763 ('07) 183, Accardi et al. - 0805.1496, Collins et al. - 0708.2833 (and refs. therein)
(figure from Chen et al.)

- Ξ emerges in the factorization theorem for DIS at *large x*, where a new semi-hard scale appears
- Ξ captures the physics at $Q^2(1-x) \sim Q\Lambda_{QCD}$, which becomes increasingly non-perturbative at low energy and large x
- the end-point factorization should be extend to different processes (e.g. e^+e^-)
- here we study the properties of Ξ and Δ regardless of processes

The quark-jet mass

$$M_j(k^-) \sim \int dk^+ \mathrm{Tr}_D \left[\Xi \, \mathbb{I} \right] \quad \sim$$



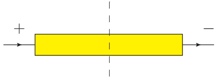
Mass associated with the scalar term (chiral-odd) of the cut quark propagator:

- inclusive "jet mass" *or* color-screened dressed quark mass

The quark-jet mass

$$M_j(k^-) \sim \int dk^+ \mathrm{Tr}_D \left[\Xi \, \mathbb{I} \right]$$

$\sim$



Mass associated with the scalar term (chiral-odd) of the cut quark propagator:

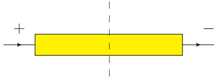
- inclusive "jet mass" *or* color-screened dressed quark mass

In the light-cone gauge we can relate it to the chiral-odd spectral function for the quark propagator:

$$M_j = \int_0^{+\infty} d\mu^2 \, \sqrt{\mu^2} \, \rho_1^{lcg}(\mu^2)$$

The quark-jet mass

$$M_j(k^-) \sim \int dk^+ \mathrm{Tr}_D \left[\Xi\, \mathbb{I}\right] \qquad \sim$$



Mass associated with the scalar term (chiral-odd) of the cut quark propagator:

- inclusive "jet mass" *or* color-screened dressed quark mass

In the light-cone gauge we can relate it to the chiral-odd spectral function for the quark propagator:

$$M_j = \int_0^{+\infty} d\mu^2 \sqrt{\mu^2}\, \rho_1^{lcg}(\mu^2)$$

This mass term:

- gauge-invariant
- renormalization scale dependent
- calculable via the spectral functions of the cut quark propagator
- measurable via momentum sum rules for twist-3 FFs

Outline

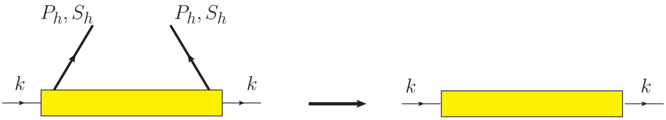
Hadronization

Momentum sum rules

Phenomenology

Master sum rule - operator level

$$\sum_{h\,S_h} \int \frac{d^4 P_h}{(2\pi)^3} \delta(P_h^2 - M_h^2)\, P_h^\mu\, \Delta^h(k, P_h, S_h) = k^\mu\, \Xi^{\text{uncut}}(k)$$

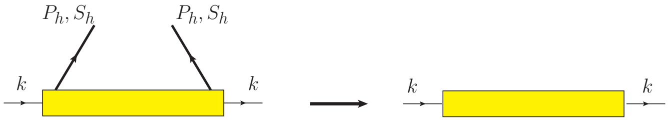


Dressed quark propagator
as the "average" on-shell four-momentum produced by hadronization

The discontinuity and the Dirac projections of both sides give the momentum sum rules for the FFs

Master sum rule - operator level

$$\sum_{h\,S_h} \int \frac{d^4 P_h}{(2\pi)^3} \delta(P_h^2 - M_h^2)\, P_h^\mu\, \Delta^h(k, P_h, S_h) = k^\mu\, \Xi^{\text{uncut}}(k)$$

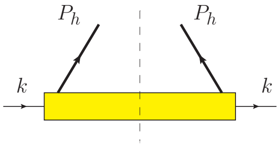


Dressed quark propagator
as the "average" on-shell four-momentum produced by hadronization

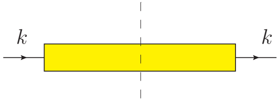
The discontinuity and the Dirac projections of both sides give the momentum sum rules for the FFs

Accardi, AS - 1903.04458 + work in progress
extend work by Meissner et al. - 1002.4393

Unpolarized sector



$$\Delta^h(z,\boldsymbol{P}_\perp) = \frac{1}{2}\gamma^+ D_1^h(z,P_{h\perp}^2) + \frac{M_h}{2P_h^-}E^h(z,P_{h\perp}^2)\,\mathbb{I} + \frac{\not{\boldsymbol{P}}_{h\perp}}{2zP_h^-}D^{\perp h} + \text{quark-pol.}$$



$$4(2\pi)^3\,J(k^-,\boldsymbol{k}_T) = \{\gamma^+ + \frac{M_j}{k^-}\,\mathbb{I} + \frac{\not{\boldsymbol{k}}_T}{k^-}\}\theta(k^-) + \text{higher twist}$$

Mass sum rule

The $\mathbb{I}$ projection of the operatorial sum rule yields (Accardi, AS - 1903.04458):

$$\boxed{\sum_{h\, S_h} \int dz M_h E^h(z) = M_j}$$

average of produced hadron masses
weighted by chiral-odd E FF

Mass sum rule

The $\mathbb{I}$ projection of the operatorial sum rule yields (Accardi, AS - 1903.04458):

$$\sum_{h\,S_h} \int dz M_h E^h(z) = M_j$$

average of produced hadron masses
weighted by chiral-odd E FF

Separation of the jet/quark mass into current and dynamical components:

Mass sum rule

The $\mathbb{I}$ projection of the operatorial sum rule yields (Accardi, AS - 1903.04458):

$$\sum_{h\, S_h} \int dz M_h E^h(z) = M_j$$

average of produced hadron masses
weighted by chiral-odd E FF

Separation of the jet/quark mass into current and dynamical components:

EOM relations: $E^h = \tilde{E}^h + \frac{m_q}{M_h} z D_1^h$

Mass sum rule

The $\mathbb{I}$ projection of the operatorial sum rule yields (Accardi, AS - 1903.04458):

$$\sum_{h\, S_h} \int dz M_h E^h(z) = M_j$$

average of produced hadron masses
weighted by chiral-odd E FF

Separation of the jet/quark mass into current and dynamical components:

EOM relations: $E^h = \tilde{E}^h + \frac{m_q}{M_h} z D_1^h$

WW approximation: $\tilde{E}^h = 0 \implies M_j = m_q$

Mass sum rule

The $\mathbb{I}$ projection of the operatorial sum rule yields (Accardi, AS - 1903.04458):

$$\sum_{h\,S_h} \int dz M_h E^h(z) = M_j$$

average of produced hadron masses
weighted by chiral-odd E FF

Separation of the jet/quark mass into current and dynamical components:

EOM relations: $E^h = \tilde{E}^h + \frac{m_q}{M_h} z D_1^h$

WW approximation: $\tilde{E}^h = 0 \implies M_j = m_q$

In the full QCD, instead, we decompose $M_j = m_q + m_q^{corr}$, where

$$\sum_{h\,S_h} \int dz M_h \tilde{E}^h(z) = M_j - m_q = m_q^{corr}$$

Mass sum rule

The $\mathbb{I}$ projection of the operatorial sum rule yields (Accardi, AS - 1903.04458):

$$\sum_{h\,S_h} \int dz M_h E^h(z) = M_j$$

average of produced hadron masses
weighted by chiral-odd E FF

Separation of the jet/quark mass into current and dynamical components:

EOM relations: $E^h = \tilde{E}^h + \frac{m_q}{M_h} z D_1^h$

WW approximation: $\tilde{E}^h = 0 \implies M_j = m_q$

In the full QCD, instead, we decompose $M_j = m_q + m_q^{corr}$, where

$$\sum_{h\,S_h} \int dz M_h \tilde{E}^h(z) = M_j - m_q = m_q^{corr}$$

We expect m_q^{corr} not to vanish in the chiral limit

Full set of momentum sum rules

$$\sum_{h}\sum_{S_h}\int dz\, z\, D_1^h(z) = 1$$

$$\sum_{h}\sum_{S_h}\int dz\, M_h E^h(z) = M_j \qquad \sum_{h}\sum_{S_h}\int dz\, M_h \tilde{E}^h(z) = m_q^{corr}$$

$$\sum_{h}\sum_{S_h}\int dz\, M_h H^h(z) = 0 \qquad \sum_{h}\sum_{S_h}\int dz\, M_h \tilde{H}^h(z) = 0$$

$$\sum_{h}\sum_{S_h}\int dz\, z\, M_h H_1^{\perp\,(1)\,h}(z) = 0$$

$$\sum_{h}\sum_{S_h}\int dz\, M_h^2 D^{\perp\,(1)\,h}(z) = 0 \qquad \sum_{h}\sum_{S_h}\int dz\, M_h^2 \tilde{D}^{\perp\,(1)\,h}(z) = -\frac{1}{2}\langle \boldsymbol{P}_\perp^2/z\rangle$$

$$\sum_{h}\sum_{S_h}\int dz\, M_h^2 G^{\perp\,(1)\,h}(z) = 0 \qquad \sum_{h}\sum_{S_h}\int dz\, M_h^2 \tilde{G}^{\perp\,(1)\,h}(z) = 0$$

In red the ones connected to dynamical quantities

The sum rules for D_1, $H_1^\perp$, $\tilde{H}$ are already known in literature

Caveats and remarks

"*Excusatio non petita accusatio manifesta*", but still ... :

Caveats and remarks

"*Excusatio non petita accusatio manifesta*", but still ... :

- we are working with bare quantities, but since we operate on the basis of some of the symmetries of QCD (Lorentz invariance, P, T invariance) and the argument is related to the conservation of the partonic four-momentum, we expect the equations to be valid *in form* at the renormalized level

Caveats and remarks

"Excusatio non petita accusatio manifesta", but still ... :

- we are working with bare quantities, but since we operate on the basis of some of the symmetries of QCD (Lorentz invariance, P, T invariance) and the argument is related to the conservation of the partonic four-momentum, we expect the equations to be valid *in form* at the renormalized level
- the proof should anyway be generalized to the renormalized case
- quantitative predictions in QCD should consider the renormalization of these operators ($\Longrightarrow$ running of the jet/quark mass, evolution of the (TMD) FFs)

Caveats and remarks

"Excusatio non petita accusatio manifesta", but still ... :

- we are working with bare quantities, but since we operate on the basis of some of the symmetries of QCD (Lorentz invariance, P, T invariance) and the argument is related to the conservation of the partonic four-momentum, we expect the equations to be valid *in form* at the renormalized level
- the proof should anyway be generalized to the renormalized case
- quantitative predictions in QCD should consider the renormalization of these operators ($\Longrightarrow$ running of the jet/quark mass, evolution of the (TMD) FFs)
- keeping the Wilson lines on the light-cone has the advantage that the structures associated to $\not{w} = \not{n}_+$ in the quark propagator emerge only at twist 4
- if $\not{w} = \not{n}_+$ one has to renormalize the TMD FFs on the light-cone (SCET literature)

Outline

Hadronization

Momentum sum rules

Phenomenology

The NJL model of QCD

The Nambu–Jona-Lasinio (NJL) model of QCD is a chiral effective theory which is useful to help understand non-perturbative phenomena in low energy QCD. In particular:

- it encapsulates dynamical chiral symmetry breaking (gap equation)
- it mimics confinement

The NJL model of QCD

The Nambu–Jona-Lasinio (NJL) model of QCD is a chiral effective theory which is useful to help understand non-perturbative phenomena in low energy QCD. In particular:

- it encapsulates dynamical chiral symmetry breaking (gap equation)
- it mimics confinement

contact four-fermion interaction $\Longrightarrow$ *non-renormalizable*
Proper-time regularization scheme: it can incorporate aspects of confinement

The NJL model of QCD

The Nambu–Jona-Lasinio (NJL) model of QCD is a chiral effective theory which is useful to help understand non-perturbative phenomena in low energy QCD. In particular:

- it encapsulates dynamical chiral symmetry breaking (gap equation)
- it mimics confinement

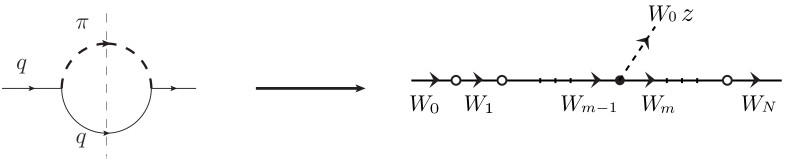
contact four-fermion interaction $\Longrightarrow$ *non-renormalizable*
Proper-time regularization scheme: it can incorporate aspects of confinement

The NJL model has been used to describe:

- hadrons as bound states of quarks
- nuclear matter and nuclei in terms of quarks (medium modifications)
- phases of strongly interacting matter at high densities (e.g. neutron stars, etc.)

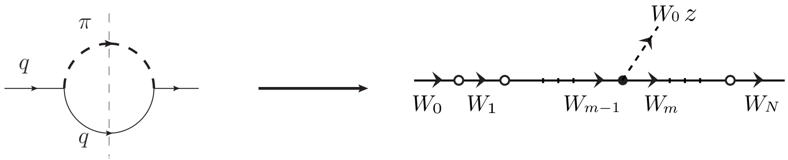
(Klevansky - Rev.Mod.Phys. 64 (1992) 649-708)

The NJL-jet model for FFs



- Within the NJL it is possible to calculate PDFs and FFs by calculating and regularizing the associated Feynman diagrams
- A more realistic model of FFs: take into account that the fragmentation process occurs as a *cascade*: the NJL-jet (Ito et al. - 0906.5362)

The NJL-jet model for FFs



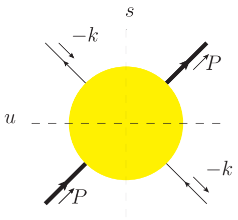
- Within the NJL it is possible to calculate PDFs and FFs by calculating and regularizing the associated Feynman diagrams
- A more realistic model of FFs: take into account that the fragmentation process occurs as a *cascade*: the NJL-jet (Ito et al. - 0906.5362)

$$D_q^\pi(z) = \sum_{m=1}^{N} \int_0^1 d\eta_1 \cdots \int_0^1 d\eta_N \, 6^N \sum_{Q_N} d_q^{Q_1}(\eta_1) \cdots d_{Q_{m-1}}^{\pi}(z) \cdots d_{Q_{N-1}}^{Q_N}(\eta_N)$$

The physical FF D_q^π can be calculated from the *elementary* d_q^π solving two integral Volterra equations

A single QCD scattering amplitude

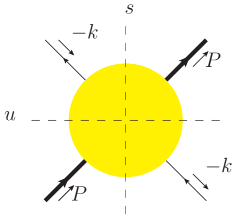
Parton distribution functions (PDFs) and FFs:
discontinuity of the same QCD scattering amplitude $\mathcal{A}(k^2, s, u)$ evaluated in different kinematic regions ($|x| < 1$ for PDFs and $|x = 1/z| > 1$ for FFs)



$$\Phi(x) = \theta(x)\theta(1-x)\mathsf{D}_{[s]}\mathcal{A} + \theta(-x)\theta(1+x)\mathsf{D}_{[u]}\mathcal{A}$$
$$\Delta(x) = \theta(x-1)\mathsf{D}_{[s]}\mathcal{A} + \theta(-1-x)\mathsf{D}_{[u]}\mathcal{A}$$

A single QCD scattering amplitude

Parton distribution functions (PDFs) and FFs:
discontinuity of the same QCD scattering amplitude $\mathcal{A}(k^2, s, u)$ evaluated in different kinematic regions ($|x| < 1$ for PDFs and $|x = 1/z| > 1$ for FFs)



$$\Phi(x) = \theta(x)\theta(1-x)\mathsf{D}_{[s]}\mathcal{A} + \theta(-x)\theta(1+x)\mathsf{D}_{[u]}\mathcal{A}$$
$$\Delta(x) = \theta(x-1)\mathsf{D}_{[s]}\mathcal{A} + \theta(-1-x)\mathsf{D}_{[u]}\mathcal{A}$$

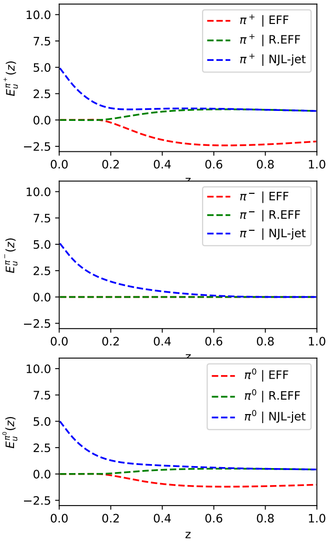
Moreover, the Drell-Levy-Yan (DLY) correspondence (which pre-dates QCD) allows one to connect (unpolarized) PDFs and FFs

(in pQCD discussed at the collinear level and up to twist-2)

$$\Delta^{[\Gamma]}(z) = \frac{z}{2N_c}\Phi^{[\Gamma]}(x = 1/z) \quad \text{with} \quad \Gamma = \{\gamma^+, \mathbb{I}\} \to \{D_1(z), E(z)\}$$

- Gamberg, Mukherjee, Mulders - 1010.4556
- Ito et al. - 0906.5362, Blüemlein et al. - Nucl.Phys. B586 (2000) 349-381

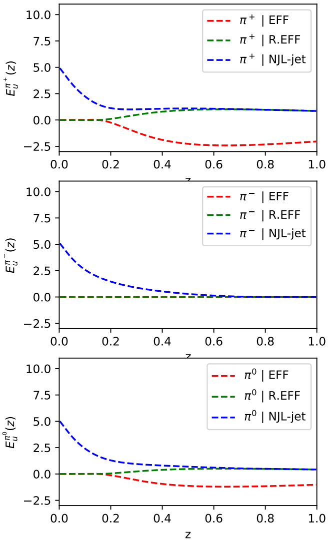
An estimate of M_j



PRELIMINARY

We estimate M_j
for an up quark in the pion sector
at the low-energy model scale (< 0.6 GeV)

An estimate of M_j

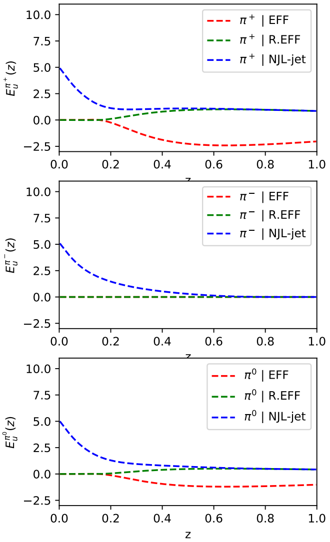


PRELIMINARY

We estimate M_j
for an up quark in the pion sector
at the low-energy model scale (< 0.6 GeV)

$$M^u_{j(\pi)} = \sum_{h=\pi^{+,0,-}} m_\pi \int dz E^h_u(z) \sim 0.44 \text{ GeV}$$

An estimate of M_j



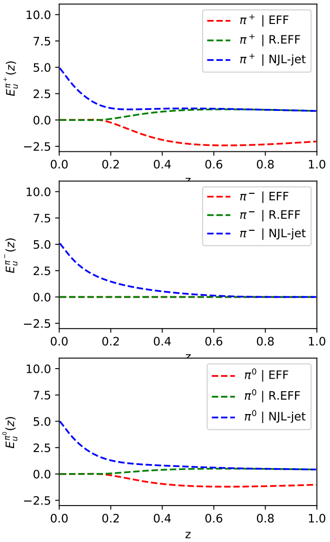
PRELIMINARY

We estimate M_j
for an up quark in the pion sector
at the low-energy model scale (< 0.6 GeV)

$$M^u_{j(\pi)} = \sum_{h=\pi^{+,0,-}} m_\pi \int dz E^h_u(z) \sim 0.44 \text{ GeV}$$

- estimate the current quark mass from the gap equation and calculate m_u^{corr}

An estimate of M_j



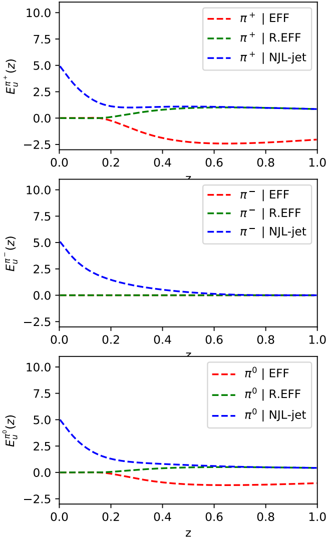
PRELIMINARY

We estimate M_j
for an up quark in the pion sector
at the low-energy model scale (< 0.6 GeV)

$$M^u_{j(\pi)} = \sum_{h=\pi^{+,0,-}} m_\pi \int dz E^h_u(z) \sim 0.44 \text{ GeV}$$

- estimate the current quark mass from the gap equation and calculate m_u^{corr}
- study the chiral limit

An estimate of M_j



PRELIMINARY

We estimate M_j
for an up quark in the pion sector
at the low-energy model scale (< 0.6 GeV)

$$M^u_{j(\pi)} = \sum_{h=\pi^{+,0,-}} m_\pi \int dz E^h_u(z) \sim 0.44 \text{ GeV}$$

- estimate the current quark mass from the gap equation and calculate m_u^{corr}
- study the chiral limit
- compare to (NJL) gap equation: $M_q = m_q - 4G_\pi \langle q\bar{q} \rangle$

Cloët, AS - work in progress

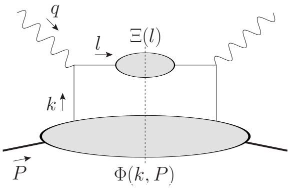
Semi-inclusive processes

- We can study the phenomenology of the jet/quark dressed mass in (semi-) inclusive hard processes applying the mass sum rule
- interesting but challenging: work in the chiral-odd sector at least at twist-3
- working in collinear factorization :

Semi-inclusive processes

- We can study the phenomenology of the jet/quark dressed mass in (semi-) inclusive hard processes applying the mass sum rule
- interesting but challenging: work in the chiral-odd sector at least at twist-3
- working in collinear factorization :

$$\ell N^{\uparrow} \to \ell h X\colon\; h_1(x) \otimes \tilde{E}(z) \quad \xrightarrow[\text{sum rule}]{\text{mass}} \quad \ell N^{\uparrow} \to \ell X\colon\; h_1(x) \otimes m_q^{corr}$$

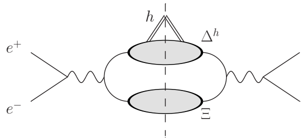


Contribution to the g_2 structure function in inclusive DIS
Accardi, Bacchetta - 1706.02000

Semi-inclusive processes

- We can study the phenomenology of the jet/quark dressed mass in (semi-) inclusive hard processes applying the mass sum rule
- interesting but challenging: work in the chiral-odd sector at least at twist-3
- working in collinear factorization :

$$e^+e^- \to h_1^\uparrow h_2 X:\ H_1(z_1) \otimes \tilde{E}(z_2) \xrightarrow[\text{sum rule}]{\text{mass}} e^+e^- \to h^\uparrow X:\ H_1(z) \otimes m_q^{corr}$$



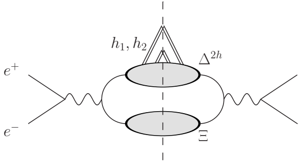
Requires both lepton and hadron polarization

Accardi, Bacchetta, Radici, AS - work in progress

Semi-inclusive processes

- We can study the phenomenology of the jet/quark dressed mass in (semi-) inclusive hard processes applying the mass sum rule
- interesting but challenging: work in the chiral-odd sector at least at twist-3
- working in collinear factorization :

$$e^+e^- \to \{h_1 h_2\} h_3 X:\ H_1^{\sphericalangle} \otimes \tilde{E}(z_3) \quad \xrightarrow[\text{sum rule}]{\text{mass}} \quad e^+e^- \to \{h_1 h_2\} X:\ H_1^{\sphericalangle} \otimes m_q^{corr}$$



Requires lepton polarization (?)
Accardi, Bacchetta, Radici, AS - work in progress

Semi-inclusive processes

- We can study the phenomenology of the jet/quark dressed mass in (semi-) inclusive hard processes applying the mass sum rule
- interesting but challenging: work in the chiral-odd sector at least at twist-3
- working in collinear factorization :
  - (?) $pp^{\uparrow} \to h_1 h_2 X \quad \xrightarrow[\text{sum rule}]{\text{mass}} \quad f_1(x_1) \otimes h_1(x_2) \otimes D_1(z) \otimes m_q^{corr}$ (fixed-target configuration at LHC)
- (?) potentially also TMD factorization
- in order to make quantitative predictions and extractions the factorization of these processes has to be addressed

Conclusions

- we can quantitatively connect quark fragmentation to the dynamical generation of mass and transverse momentum
  - gauge-invariant definition for jet/color-screend quark mass, M_j
  - its dynamical component m_q^{corr} is recognized as an observable order parameter for chiral-simmetry breaking
  - calculate/measure $\tilde{E}$: obtain dynamical mass m_q^{corr}
- momentum sum rules: powerful tool, investigate also renormalization properties
- within the WW approximation one fails to account -in principle- for the dynamical components of the mass and the transverse momentum
- phenomenology of the dressed quark mass in semi-inclusive processes: M_j can serve as a handle to access the chiral-odd sector of hadron structure and hadronization: work in progress
- possibility to measure M_j, m_q^{corr} in these processes

Hadronization and fellowships

From Sept. 2019 I will start a research program in partnership between the University of Pavia and JSA/JLab centered around hadronization

Inputs/ideas/discussions are welcome!