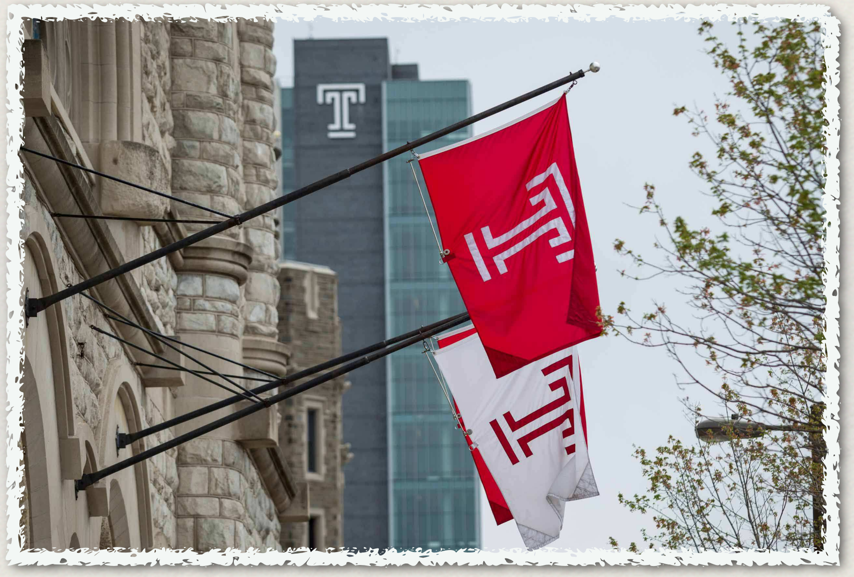
Successes and Challenges in quasi-PDFs

Martha Constantinou

Temple University



QCD Evolution
Argonne National Laboratory
May 13, 2019

Work within ETMC: Extended Twisted Mass Collaboration

C. Alexandrou	Univ. of Cyprus/Cyprus Institute
K. Cichy	Adam Mickiewicz University
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Relevant publications:

- M. Constantinou, H. Panagopoulos,
  Phys. Rev. D 96 (2017) 054506, [arXiv:1705.11193]
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  First complete work at physical point

- C. Alexandrou, K. Cichy, M. Constantinou, K. Jansen, A. Scapellato, F. Steffens,
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OUTLINE

A. Introduction

B. quasi-PDFs on the lattice

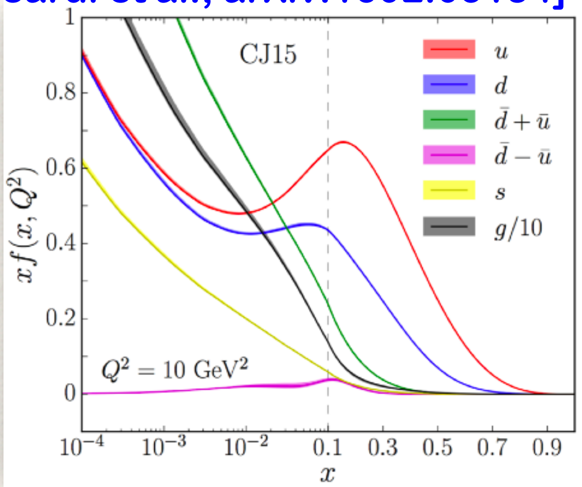
C. Success and Challenge of lattice quasi-PDFs

1. Lattice Matrix Elements
2. Systematic uncertainties
3. Renormalization
4. Fourier transform
5. Matching
6. Comparison with global fits

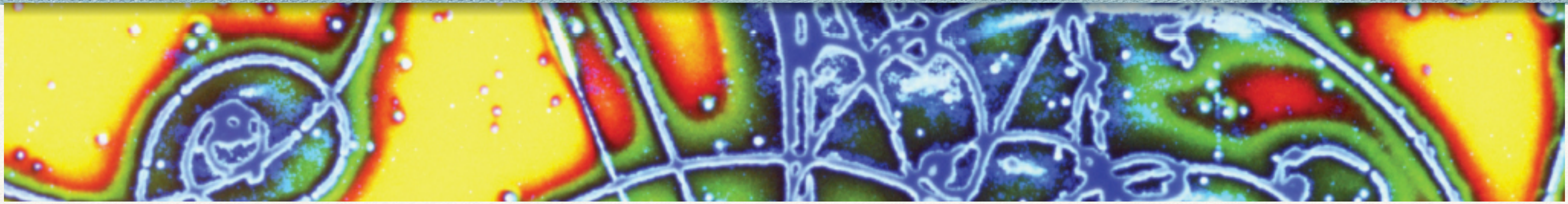
D. Summary

Parton Distribution Functions

A. Accardi et al., arXiv:1602.03154]



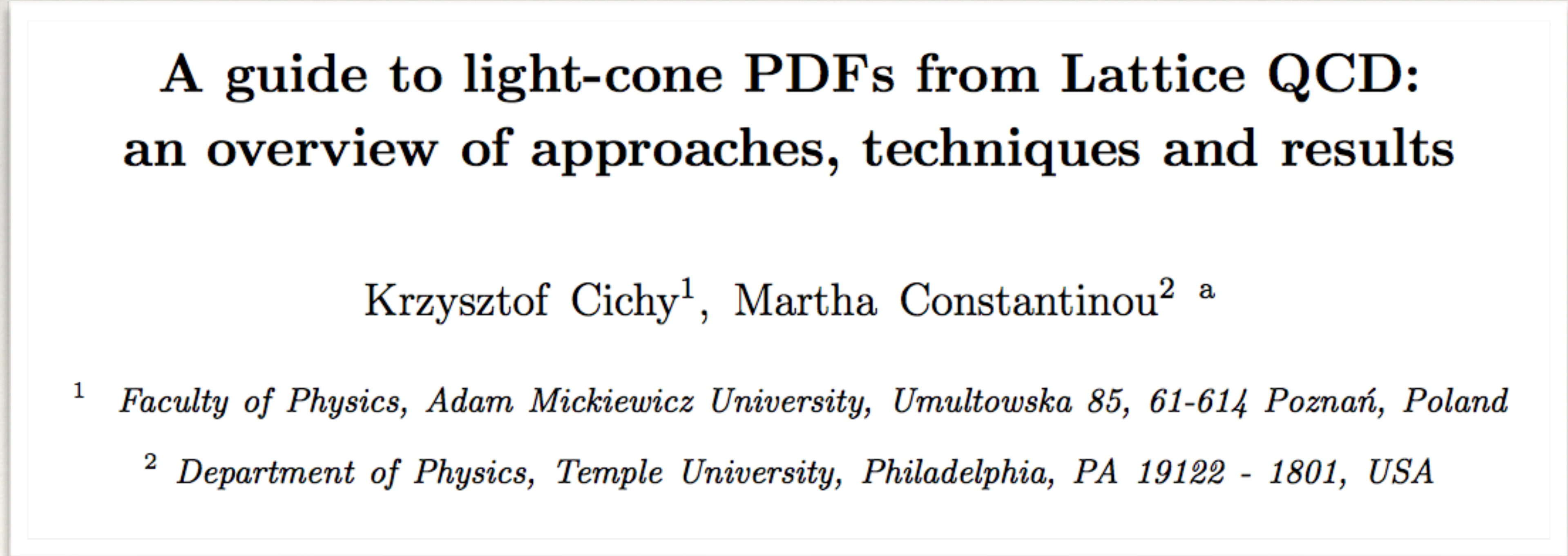
- **Universal tools to study hadron structure (1-D)**
- **Global fit analyses of DIS data: main source of information**
- **Global fits not without ambiguities** → **Calculation from first principle imperative**
- **PDFs parameterized in terms of off-forward matrix elements of non-local light-cone operators (Not accessible in Euclidean lattice)**
- **Lattice QCD: long-standing history of moments of PDFs (via OPE), but reconstruction of PDFs not feasible (gauge noise, mixing)**
- **Alternative approaches proposed, e.g.: quasi-PDFs, good lattice cross secti high moments (auxiliary heavy quark), hadronic tensor, OPE w/o OPE**
- **All methods are under investigation in lattice QCD**
  (See talks of this meeting)



Advances in High Energy Physics

Invited review in special issue:

"Transverse Momentum Dependent Observables from Low to High Energy: Factorization, Evolution, and Global Analyses"



A guide to light-cone PDFs from Lattice QCD:
an overview of approaches, techniques and results

Krzysztof Cichy[1], Martha Constantinou[2] [a]

[1] *Faculty of Physics, Adam Mickiewicz University, Umultowska 85, 61-614 Poznań, Poland*

[2] *Department of Physics, Temple University, Philadelphia, PA 19122 - 1801, USA*

Accepted in Advances in HEP, arXiv:1811.07248

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Access of PDFs on a Euclidean Lattice

Matrix elements of spatial operators with fast moving hadrons

$$\tilde{q}(x,\mu^2,P_3) = \int \frac{dz}{4\pi} e^{-i\,x\,P_3\,z} \langle N(P_3)\,|\,\bar{\Psi}(z)\,\Gamma\,\mathcal{A}(z,0)\Psi(0)\,|\,N(P_3)\rangle_{\mu^2}$$

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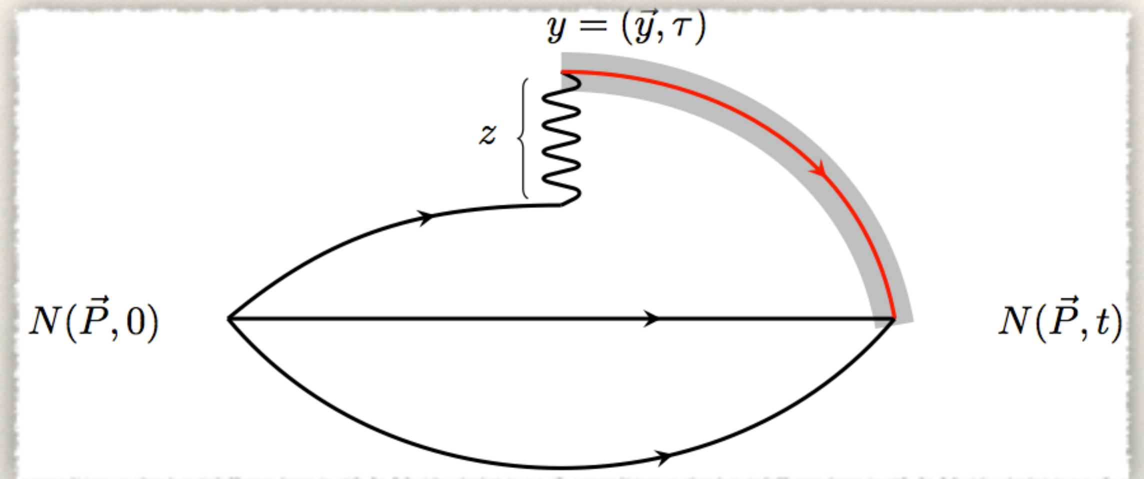

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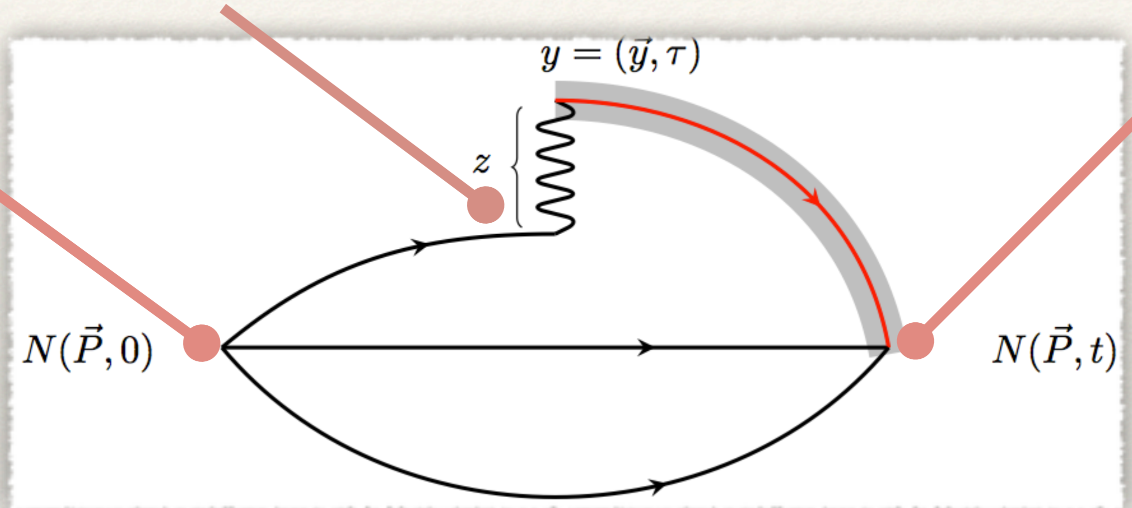

Current insertion

Sink

Source

- Separation between source and sink: excited states investigation
- Current insertion: unpolarized, helicity, transversity

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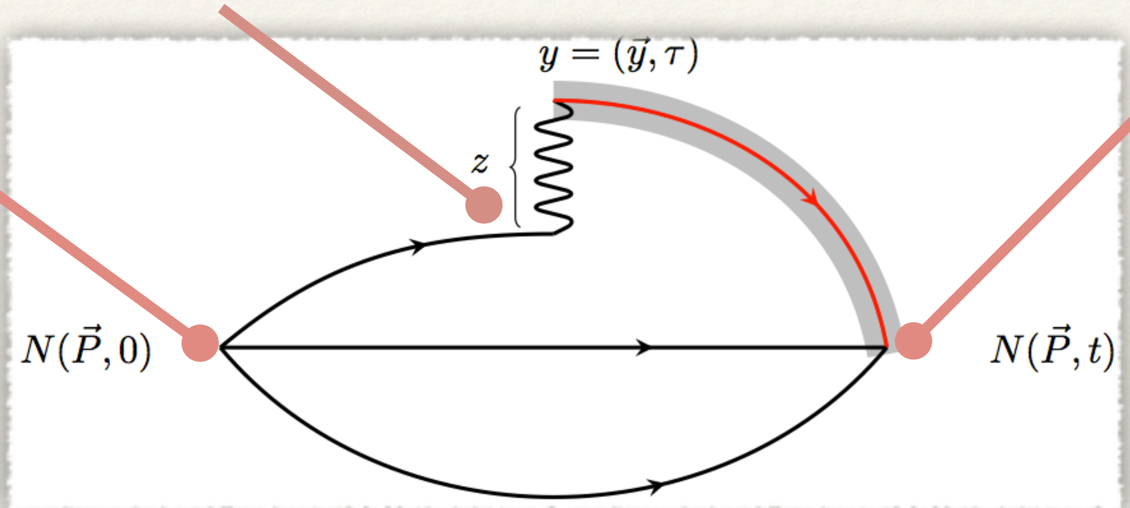

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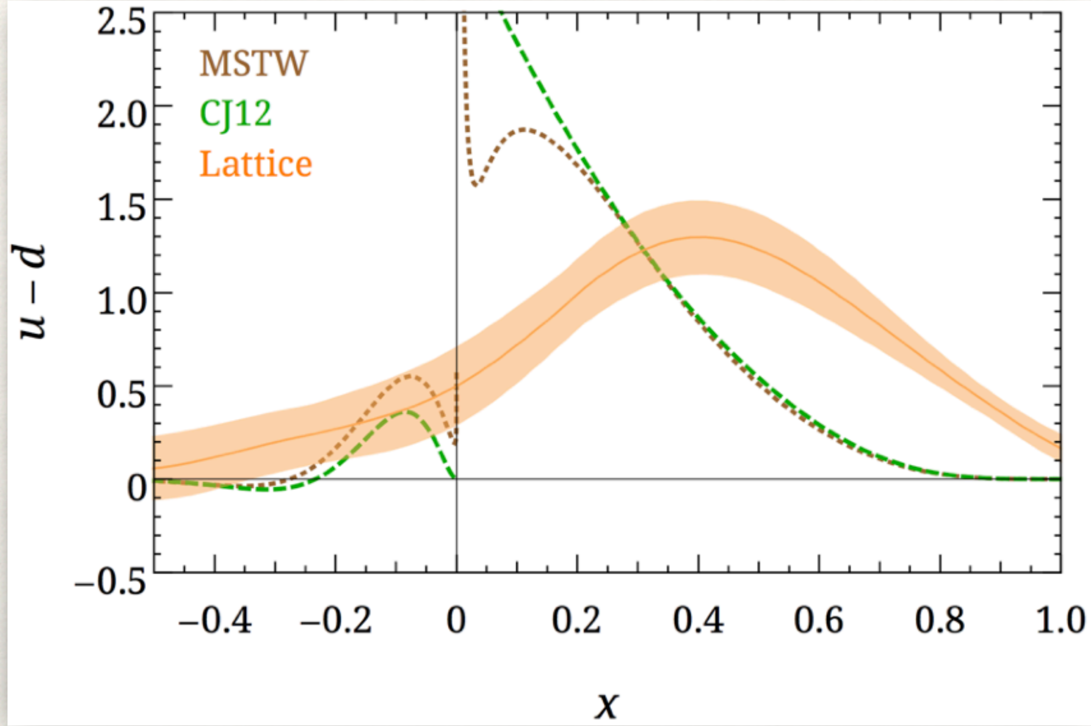
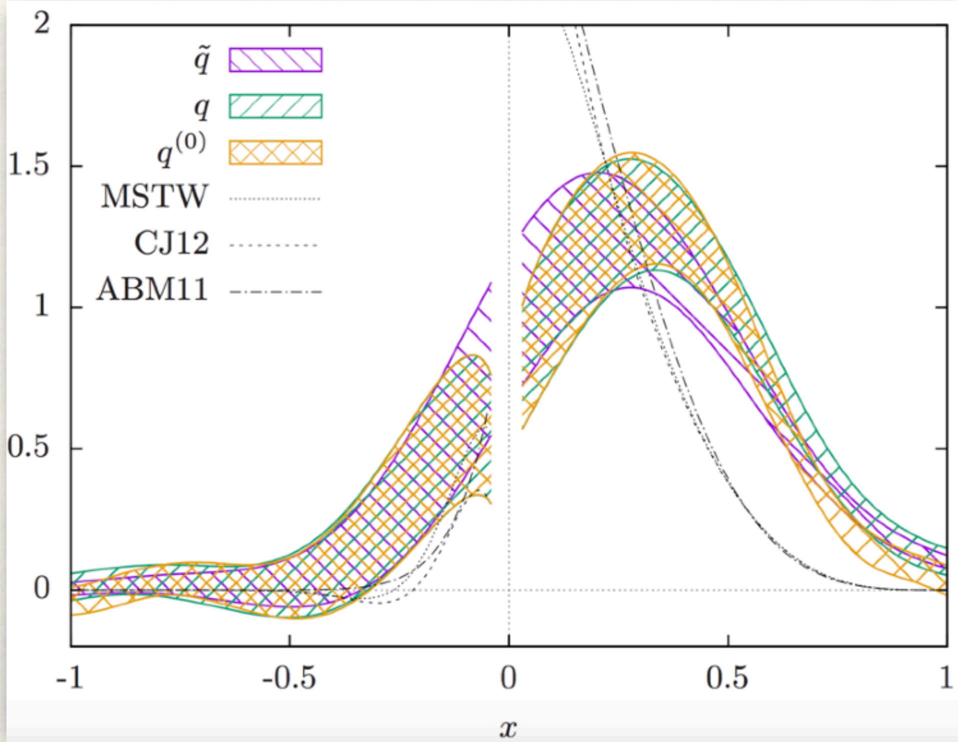
Contact with light-cone PDFs feasible:

- Matching procedure in large momentum EFT (LaMET) to relate quasi-PDFs to light-cone PDF
- Difference reduced as P_3 increases $\mathcal{O}\left(\Lambda^2_{\rm QCD}/P_3^2, m_N^2/P_3^2\right)$

First Success: exploratory studies feasible



[H.W. Lin et al., Phys. Rev. D 91, 054510 (2015), arXiv:1402.1462]



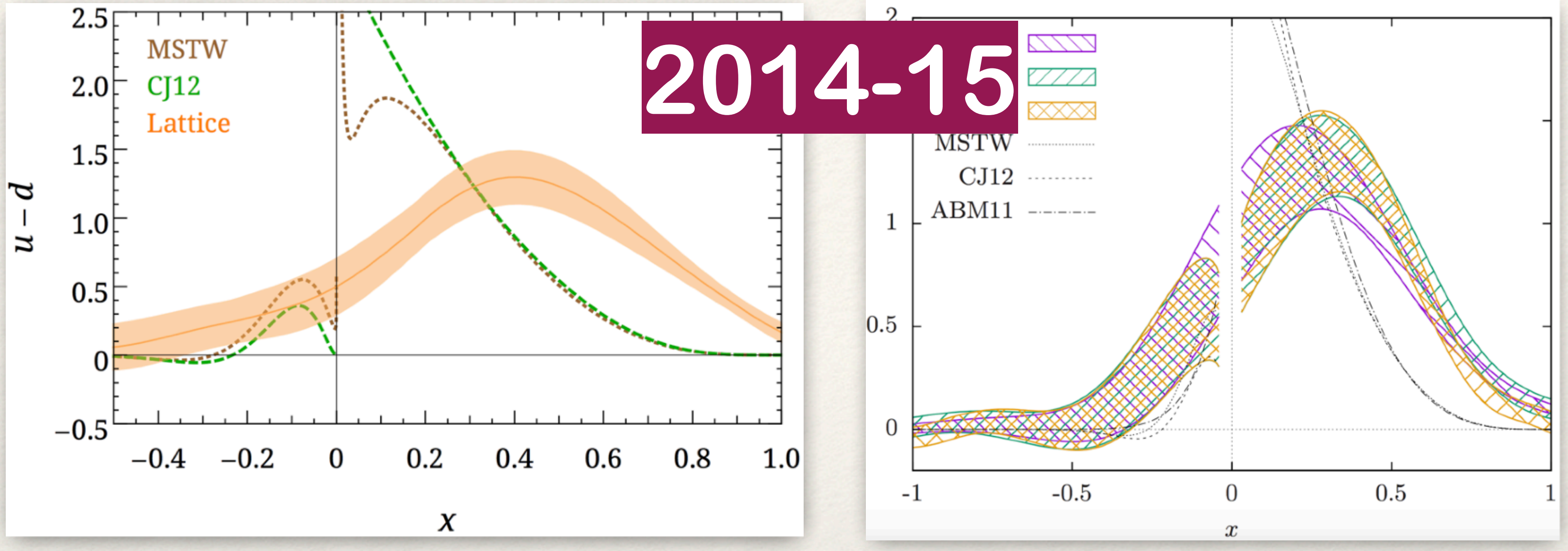
[C. Alexandrou, Phys. Rev. D 92, 014502 (2015), arXiv:1504.07455]

- Prior 2017 lattice calculations missing two main ingredients, preventing comparison with phenomenological data on PDFs
  - Renormalizability / renormalization
  - Appropriate matching expressions for lattice data

- Calculations significantly improved and extended to other hadrons

Recent review: K. Cichy, M. Constantinou, AHEP, [arXiv:1811.07248]

Lattice studies of quasi-PDFs



2014-15

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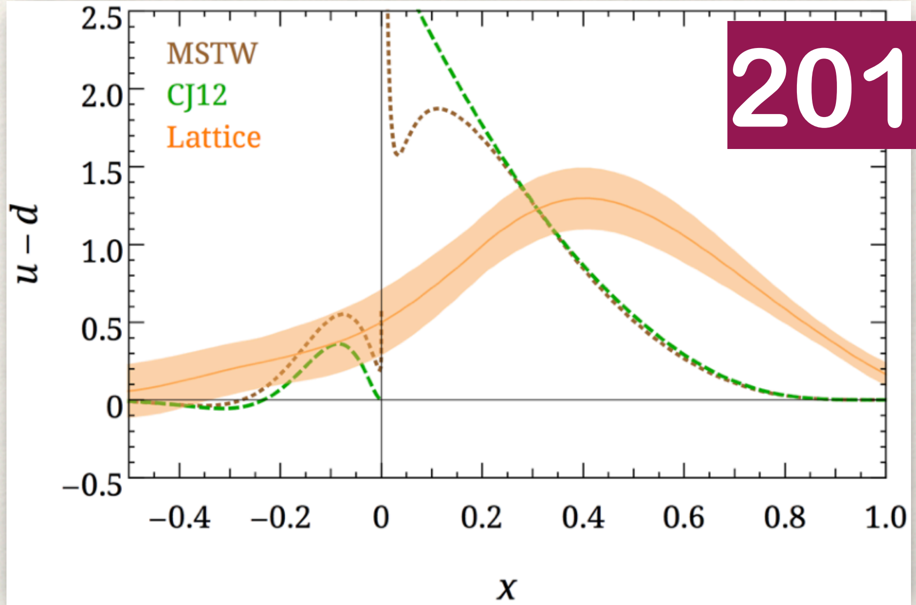

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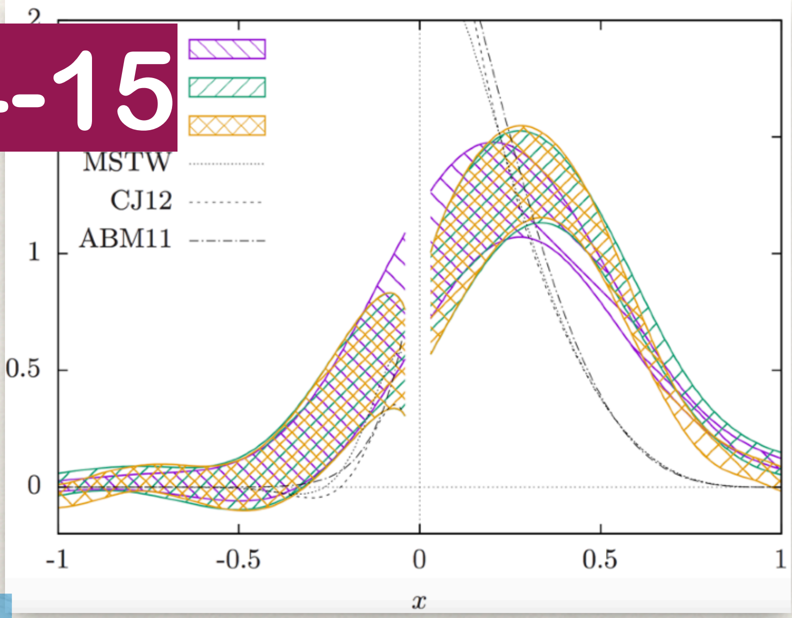
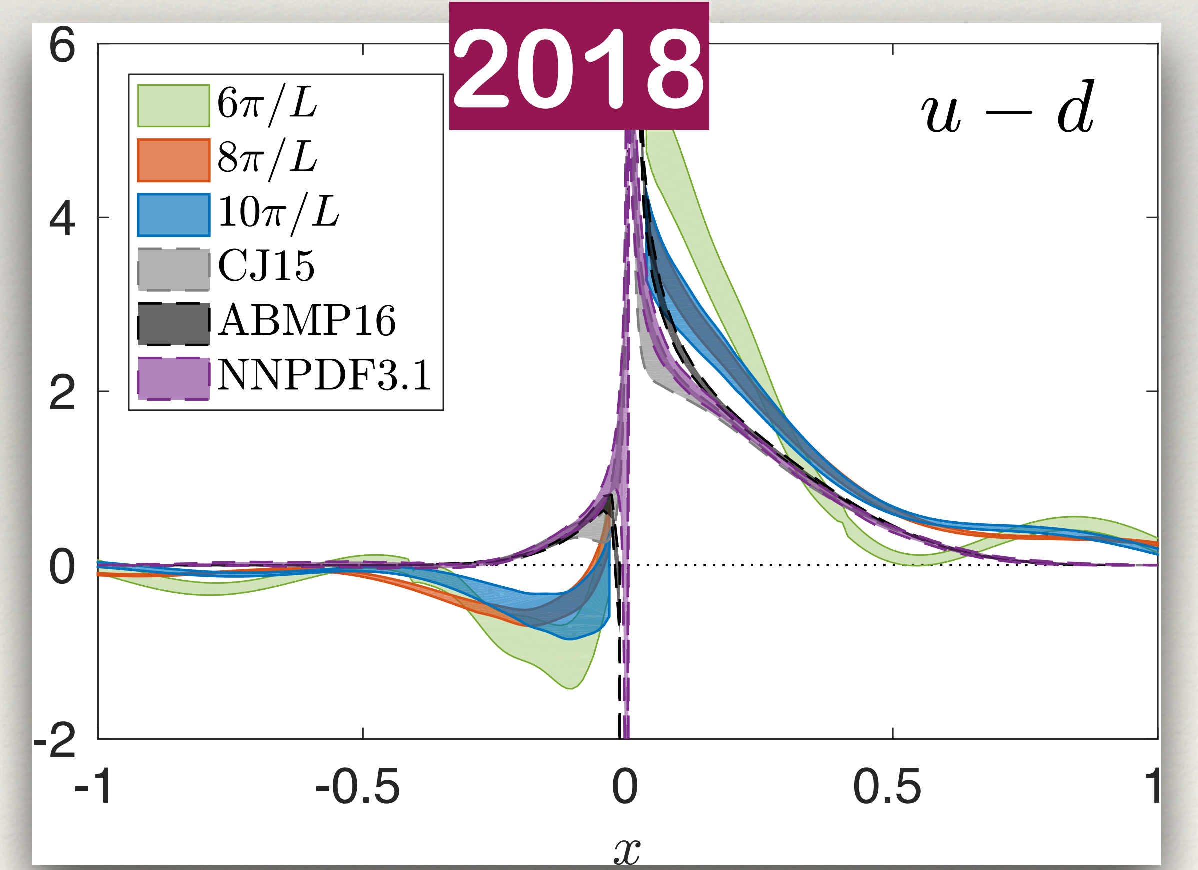


2018



Progress due to:

- ★ Simulations at physical point
- ★ Renormalization
- ★ Matching

C. Alexandrou, K. Cichy, M. Constantinou, K. Jansen, A. Scapellato, F. Steffens, PRL 121 (2018) 112001, [arXiv:1803.02685]

Multi-component calculation of quasi-PDFs

A. Calculation of matrix elements with fast moving hadrons

$$C^{2pt} = \langle N | N \rangle \qquad C^{3pt} = \langle N | \overline{\psi}(z)\Gamma\mathscr{A}(z,0)\psi(0) | N \rangle$$

B. Construction of ratios in forward limit

$$\frac{C^{3pt}(t,\tau,0,\overrightarrow{P})}{C^{2pt}(t,0,\overrightarrow{P})} \overset{0<<\tau<<t}{=} h_0(P_3, z)$$

C. Renormalization (complex functions, presence of mixing)

D. Fourier transform to momentum space (*x*)

$$\tilde{q}(x,\mu^2,P_3) = \int \frac{dz}{4\pi}\, e^{ixP_3z} \langle N | \overline{\psi}(z)\Gamma\mathscr{A}(z,0)\psi(0) | N \rangle$$

E. Matching to light-cone PDFs (LaMET)

$$q(x,\mu) = \int_{-\infty}^{\infty} \frac{d\xi}{|\xi|}\, C\left(\xi, \frac{\mu}{xP_3}\right) \tilde{q}\left(\frac{x}{\xi}, \mu, P_3\right)$$

F. Target mass corrections (elimination of residual m_N/P_3)

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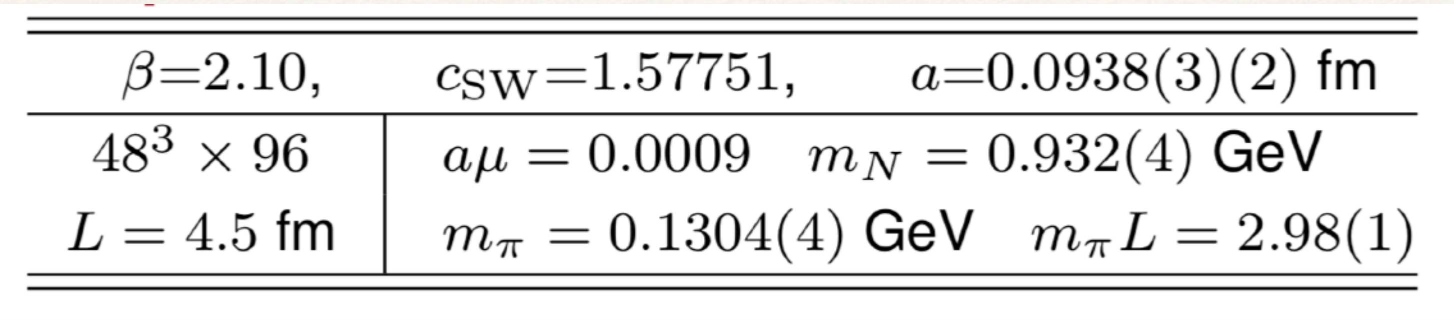
Each step has systematic uncertainties and challenges !

Parameters of ETMC calculation

[C. Alexandrou et al., (PRL), arXiv:1803.02685], [C. Alexandrou et al., arXiv:1807.00232]

- Nf=2 twisted mass fermions & clover term
- Ensemble parameters:



β=2.10,	$c_{\rm SW}$=1.57751,	a=0.0938(3)(2) fm
$48^3 \times 96$	$a\mu = 0.0009$	$m_N = 0.932(4)$ GeV
$L = 4.5$ fm	$m_\pi = 0.1304(4)$ GeV	$m_\pi L = 2.98(1)$

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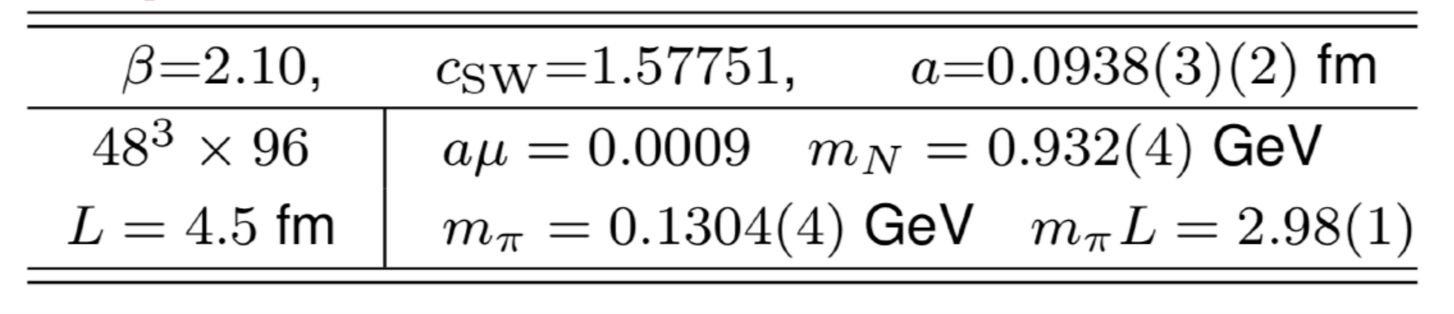

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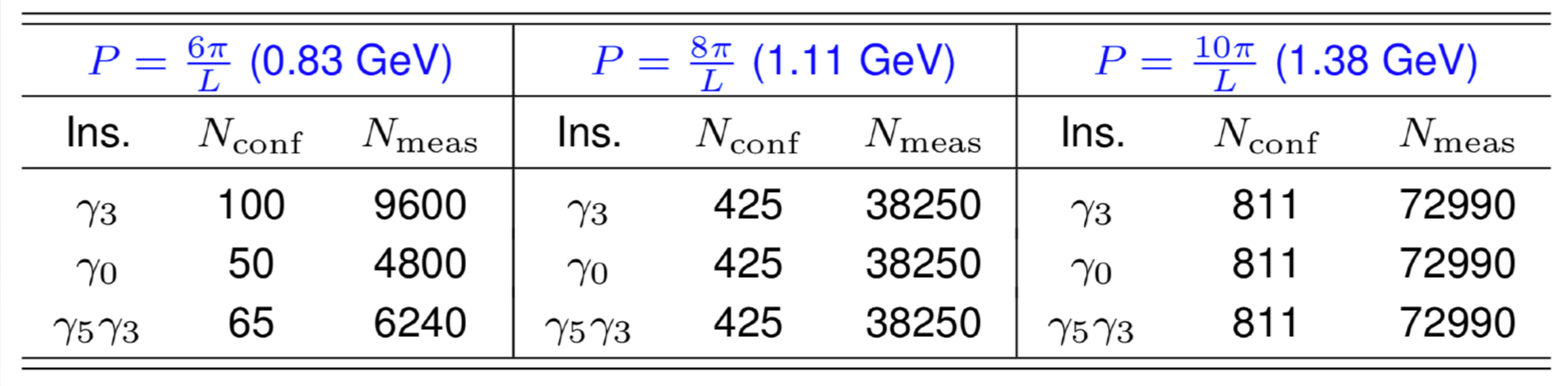
- Nucleon momentum & statistics:



$P = \frac{6\pi}{L}$ (0.83 GeV)			$P = \frac{8\pi}{L}$ (1.11 GeV)			$P = \frac{10\pi}{L}$ (1.38 GeV)		
Ins.	$N_{\rm conf}$	$N_{\rm meas}$	Ins.	$N_{\rm conf}$	$N_{\rm meas}$	Ins.	$N_{\rm conf}$	$N_{\rm meas}$
γ_3	100	9600	γ_3	425	38250	γ_3	811	72990
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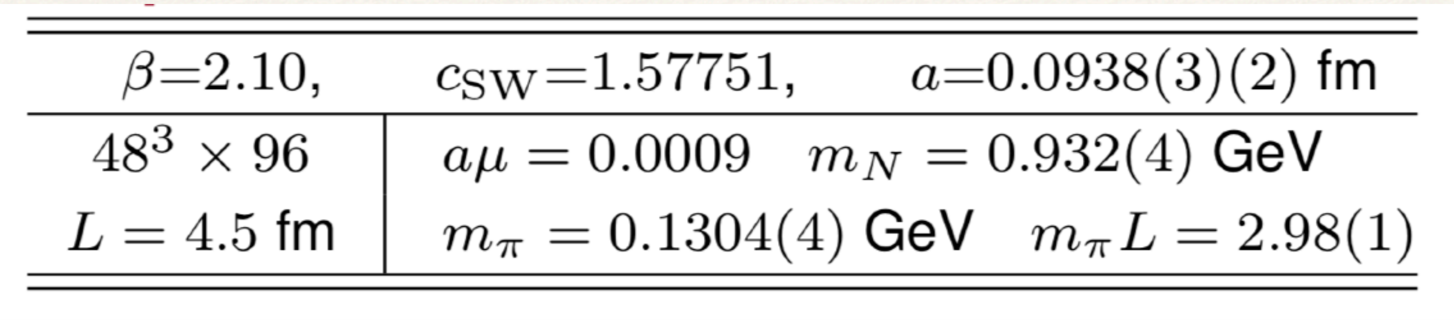

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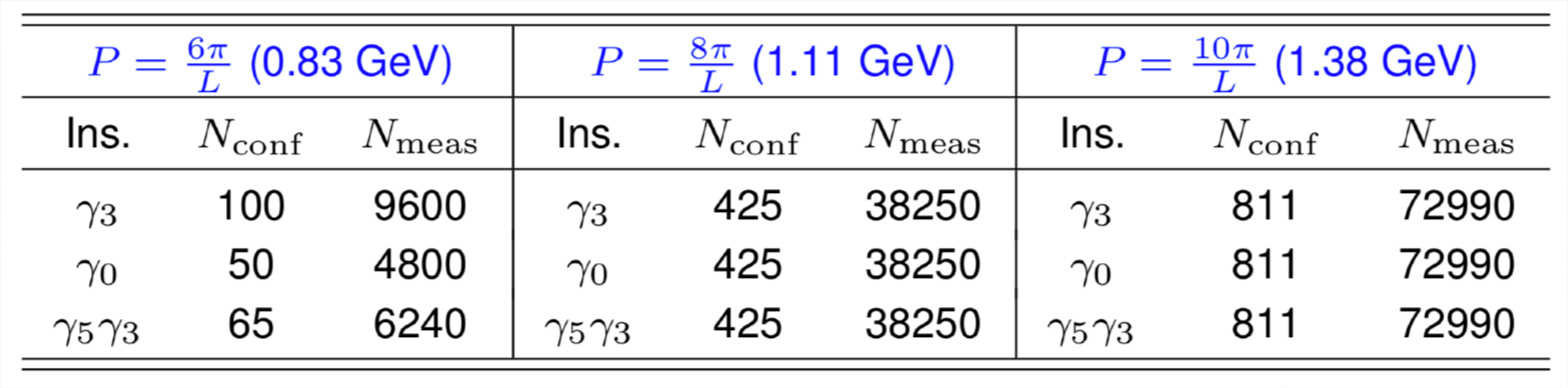

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- Excited states investigation:

$T_{\rm sink} = 8a, 9a, 10a, 12a,\qquad (T_{\rm sink} = 0.75, 0.84, 0.94, 1.13,\ {\rm fm})$

Investigation of systematic uncertainties

On a single ensemble:

- Excited states contamination
- Pion mass (with simulations at physical point)
- Renormalization and mixing
- Reconstruction of PDFs

Using multiple ensembles:

- Cut-off effects due to finite lattice spacing
- Finite volume effects
- Pion mass dependence

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Effects reduced in single ensemble with appropriate parameters

Challenges of calculation

Noise-to-signal ratio increases with:

- ★ Hadron momentum boost
- ★ Simulations at the physical point
- ★ Source-sink separation

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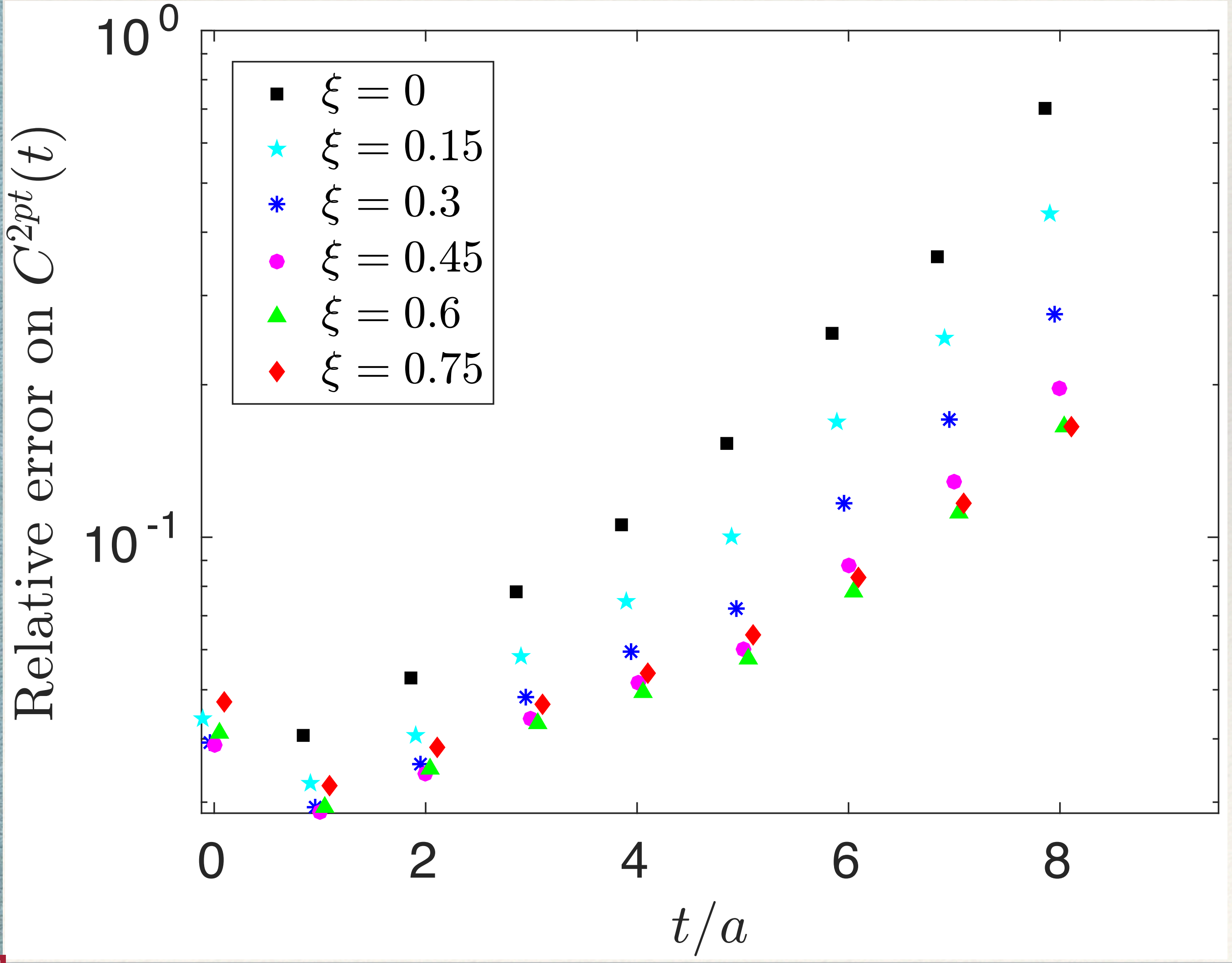

Momentum smearing

[G. Bali et al., PRD93, 094515 (2016)]

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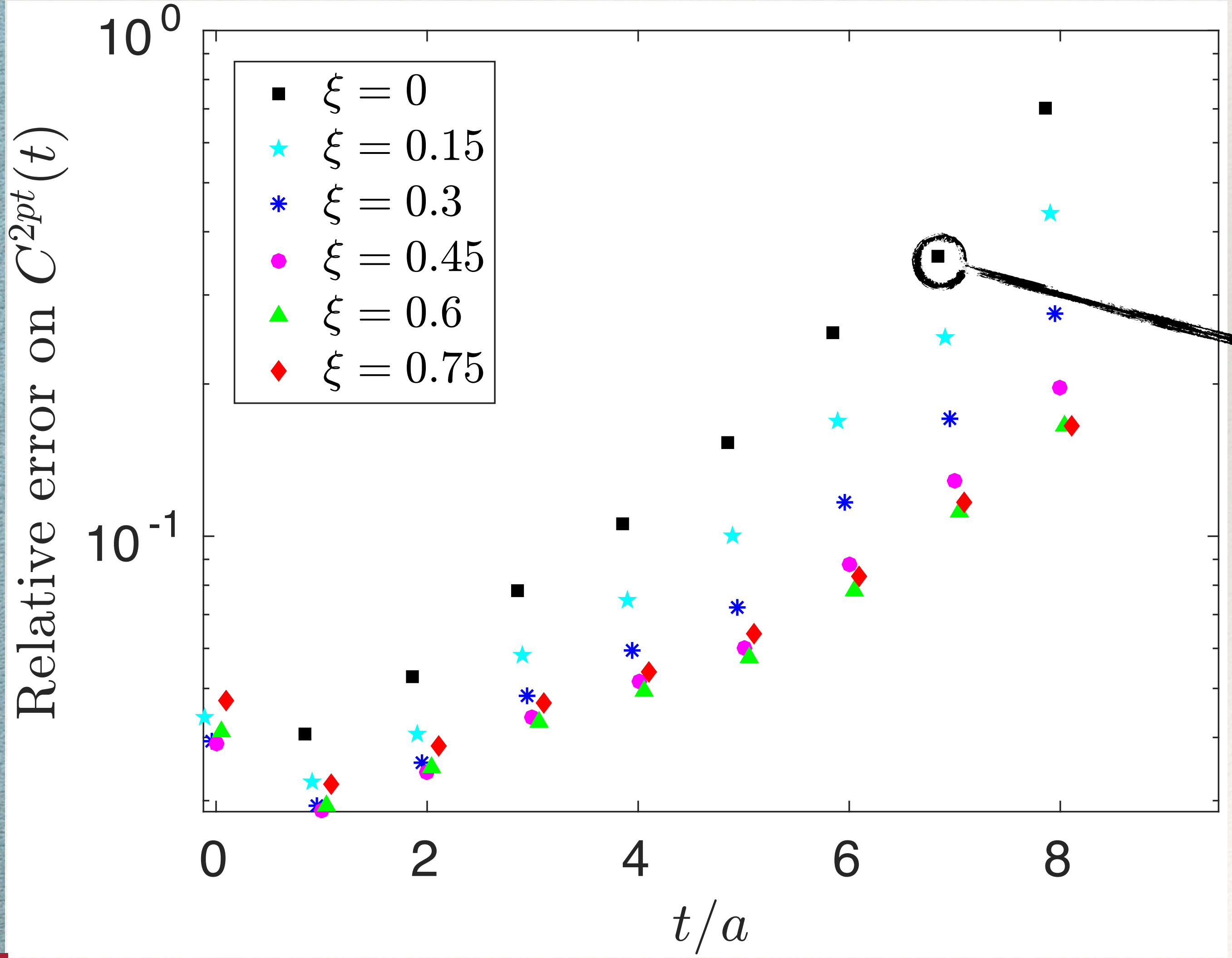

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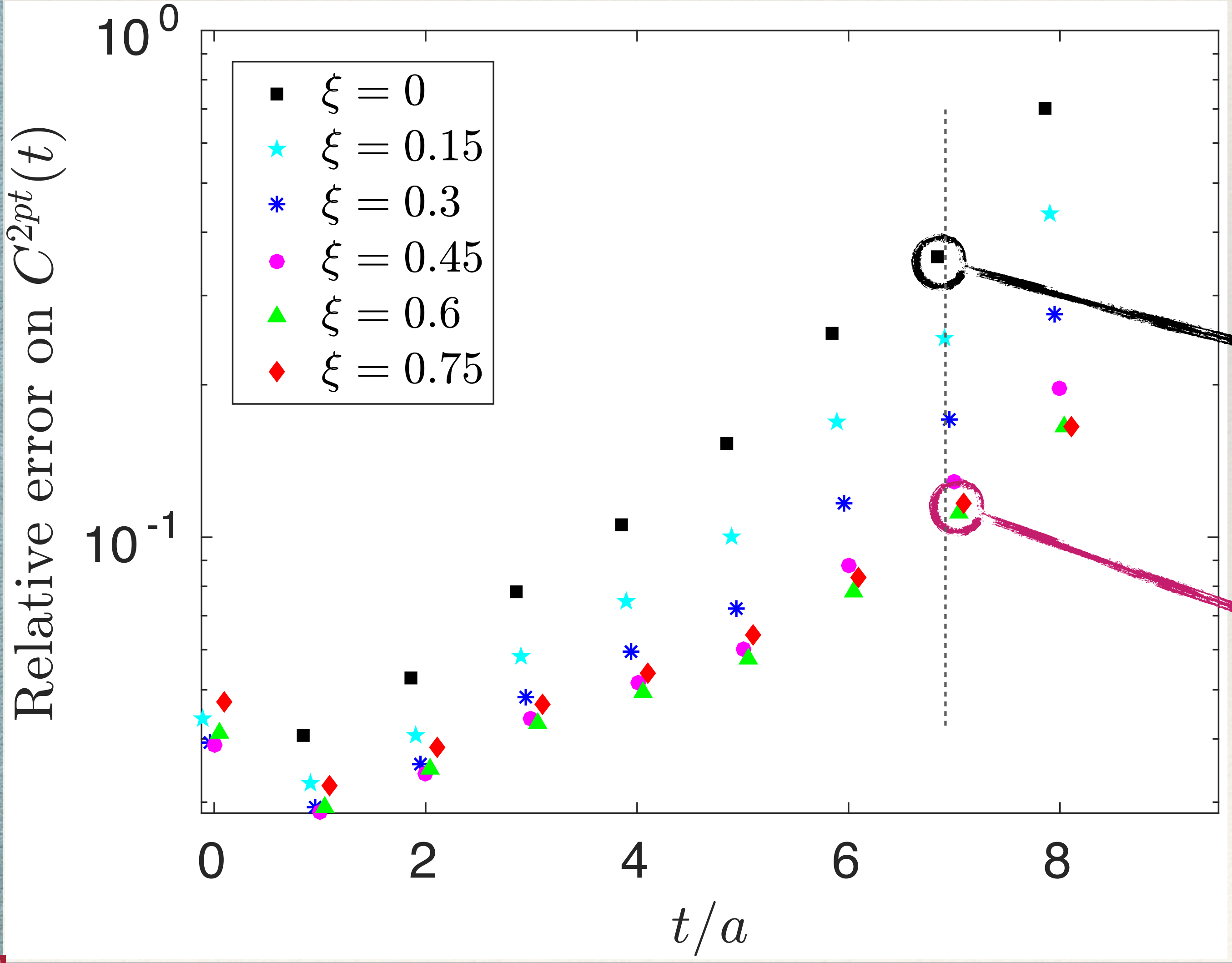

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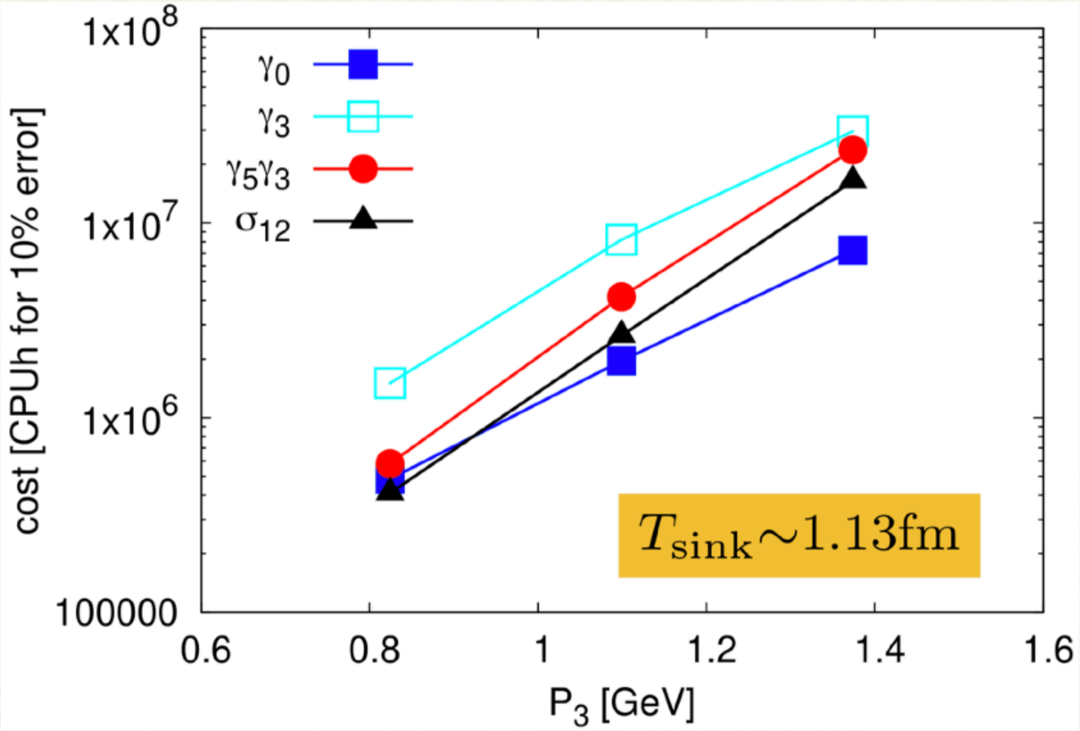
No momentum smearing

Momentum smearing recedes noise, helps reach higher momenta

Challenges of calculation



Despite the improvement in the signal, there are limitations in maximum momentum due to computational cost

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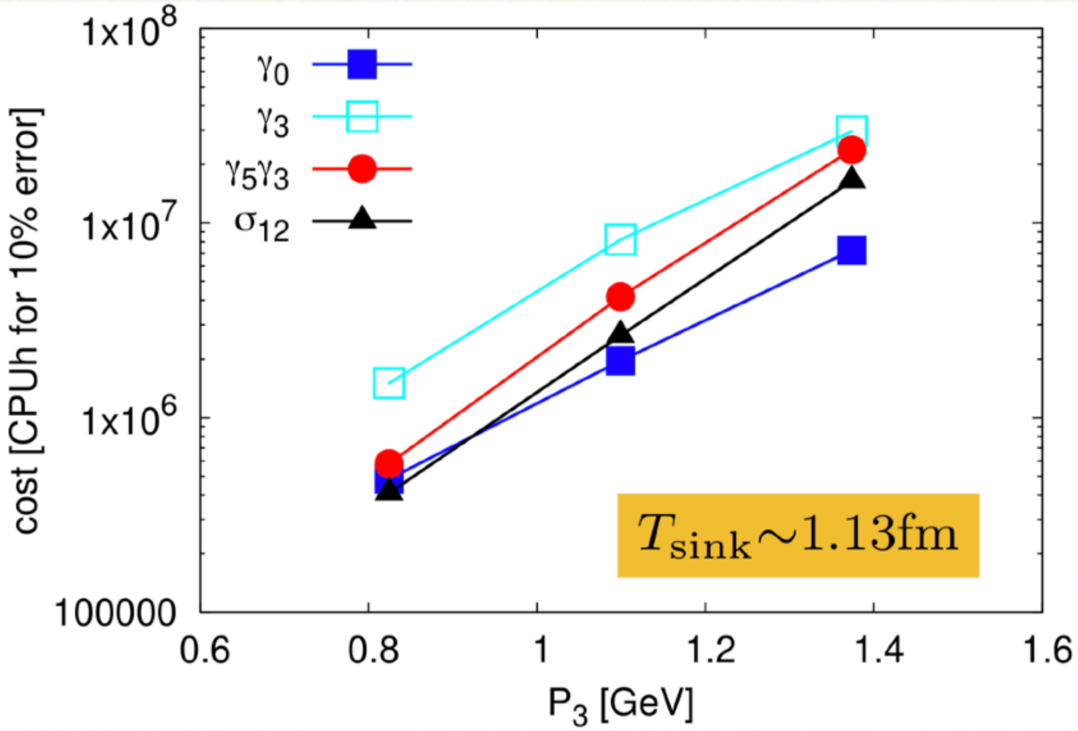

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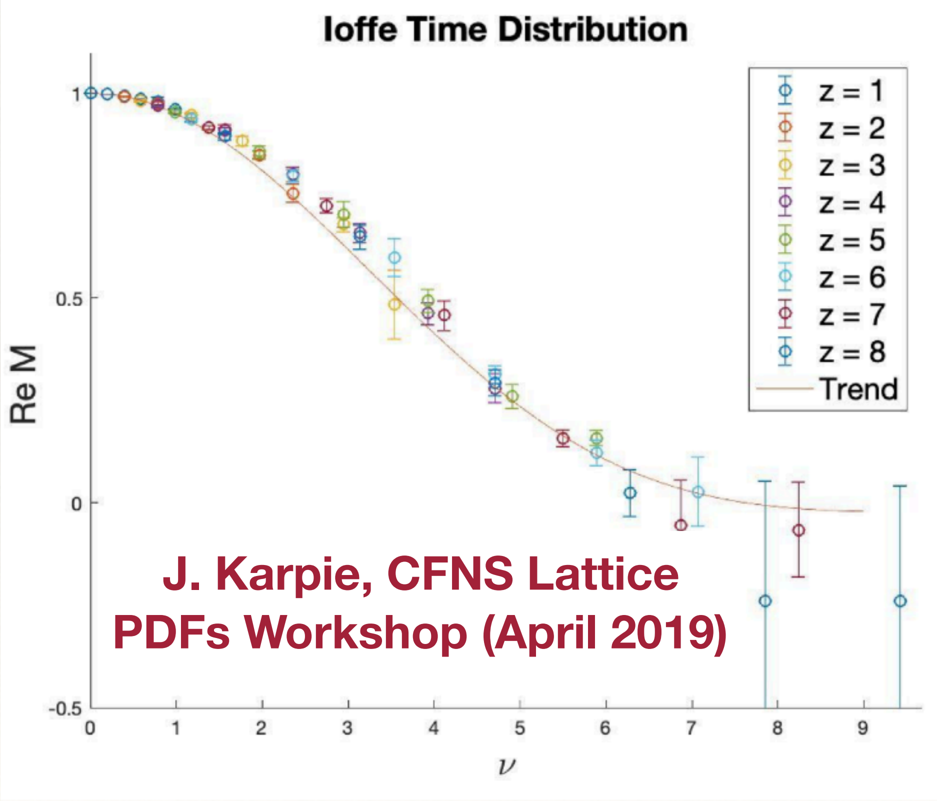

J. Karpie, CFNS Lattice PDFs Workshop (April 2019)

★ **pseudo-PDFs:**

m_π=440MeV, P3 < 2GeV, Tsink~1.3fm

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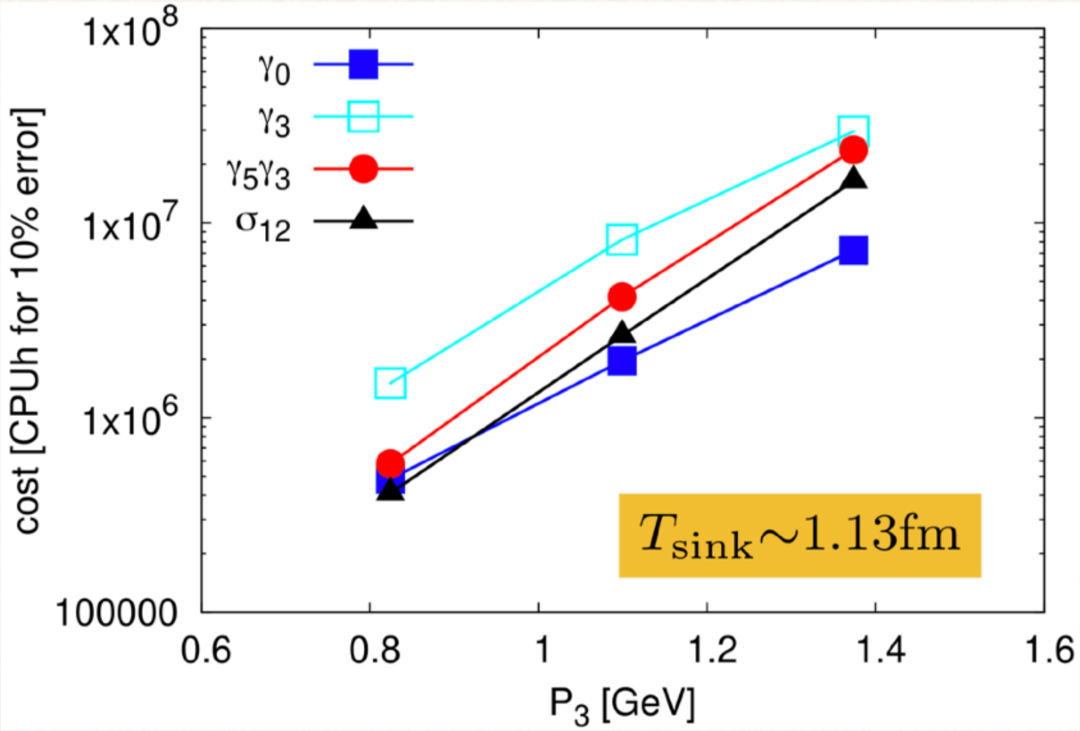

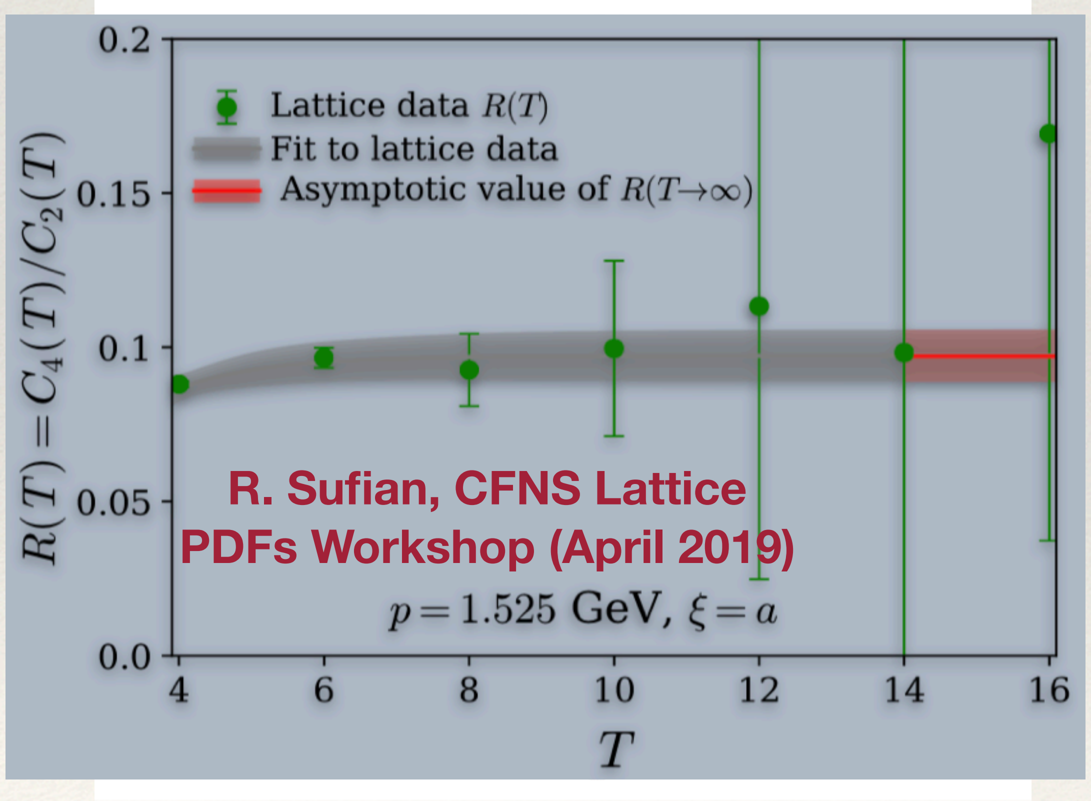

R. Sufian, CFNS Lattice PDFs Workshop (April 2019)

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- ★ **pseudo-PDFs:**
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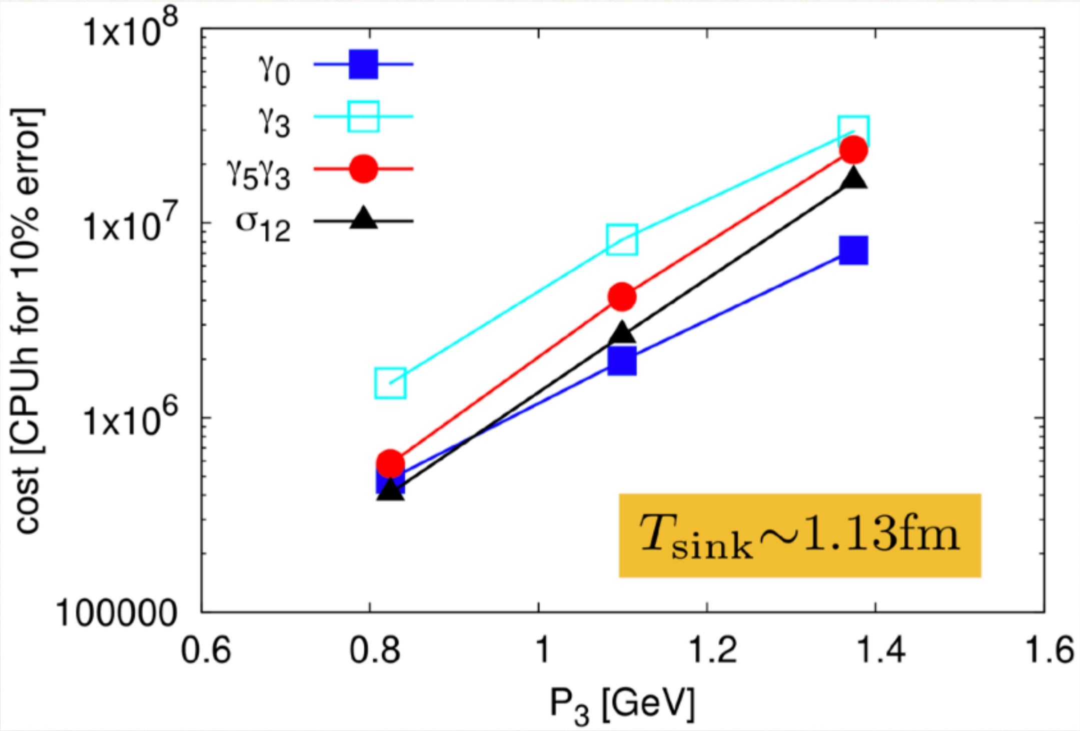
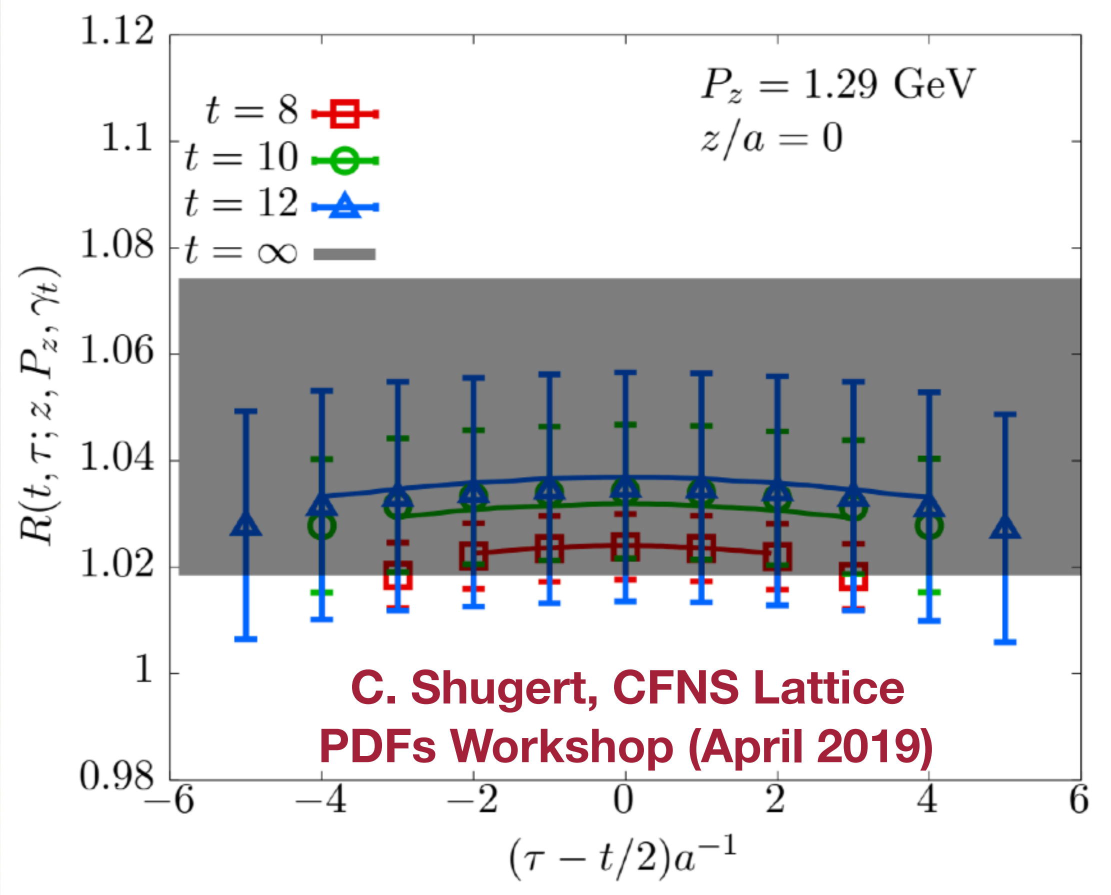
- ★ **Good lattice cross-sections:**
  m_π=413MeV, T_{sink}<2fm, P_3=1.53 GeV

Challenges of calculation





C. Shugert, CFNS Lattice PDFs Workshop (April 2019)

Despite the improvement in the signal, there are limitations in maximum momentum due to computational cost

- **pseudo-PDFs:**
  m_π=440MeV, P3 < 2GeV, Tsink~1.3fm
- **Good lattice cross-sections:**
  m_π=413MeV, $T_{\rm sink}$<2fm, P_3=1.53 GeV
- **Valence Pion PDF**
  m_π=300MeV, $T_{\rm sink}$=0.72fm, P_3=1.7MeV

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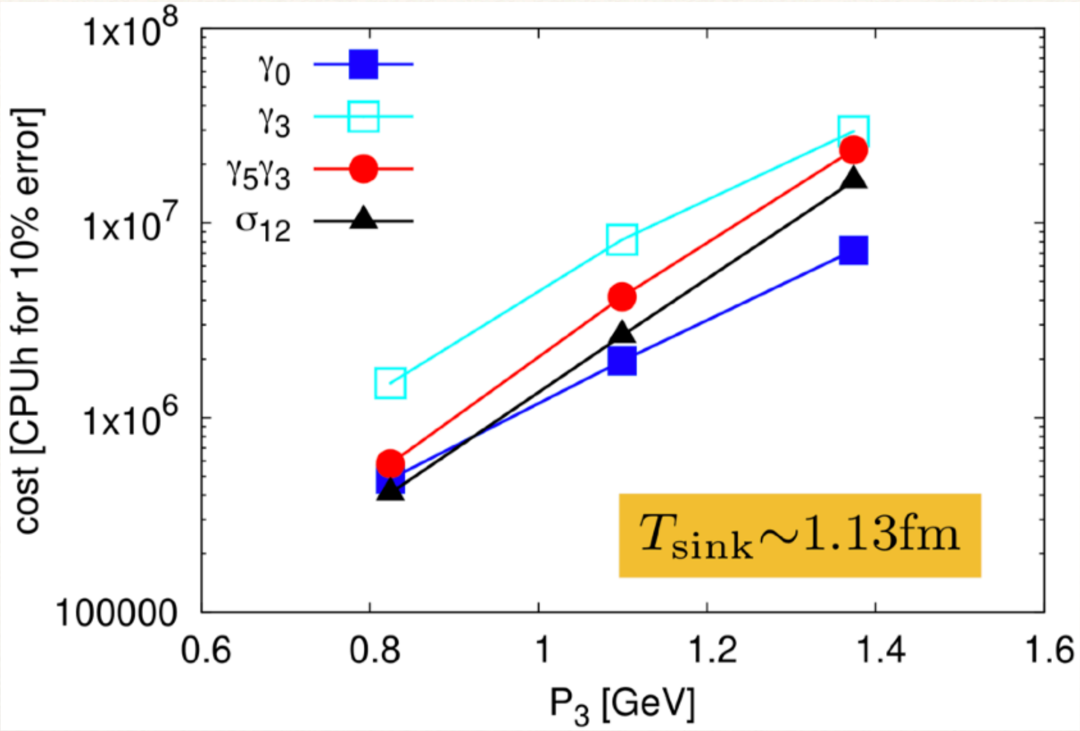

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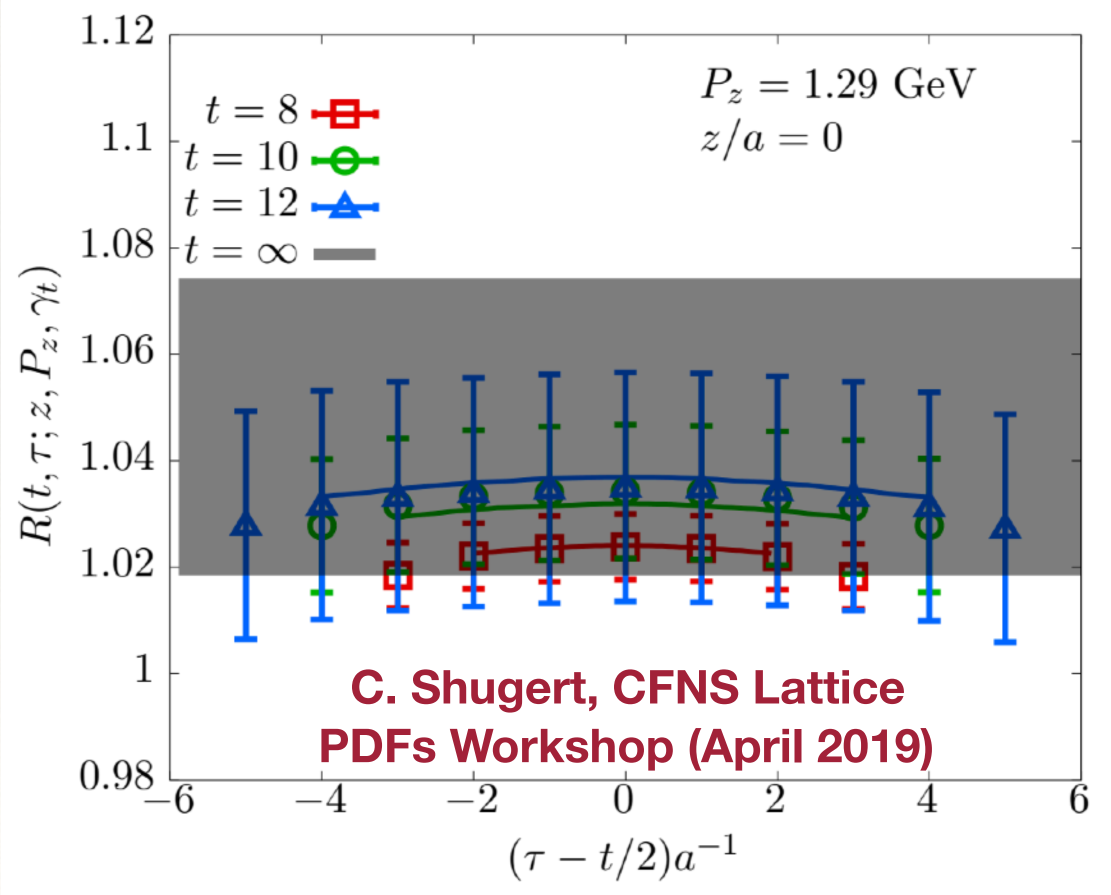

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No shortcuts to reliable estimates

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Success: Simulations at physical point

Unpolarized:

- Initial studies used γ^μ in same direction with Wilson line
- Mixing with higher twist revealed perturbatively

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[M. Constantinou, H. Panagopoulos, Phys. Rev. D 96 (2017) 054506, [arXiv:1705.11193]

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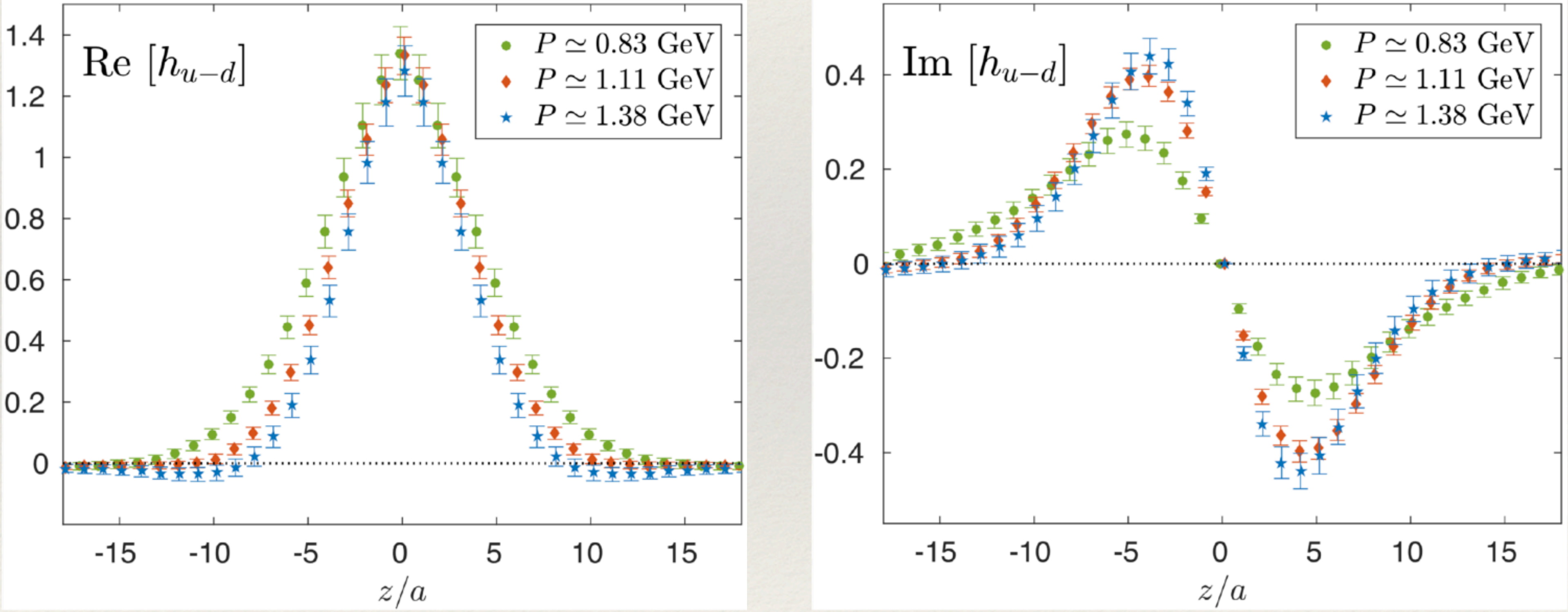


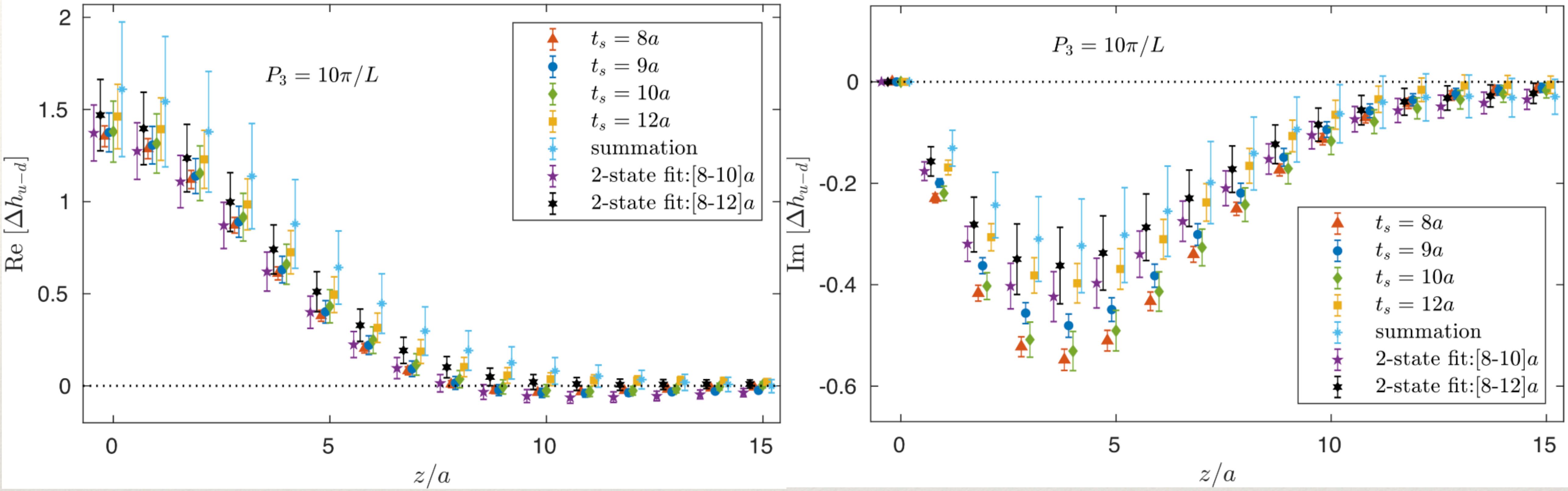
- Similar general features for polarized and transversity
- Highest priority: deliver reliable results

Challenge #1

How do we control contamination from excited states effects for fast moving nucleons?

Excited states contamination

Analyses techniques: Single-state fit, Two-state fit, Summation method



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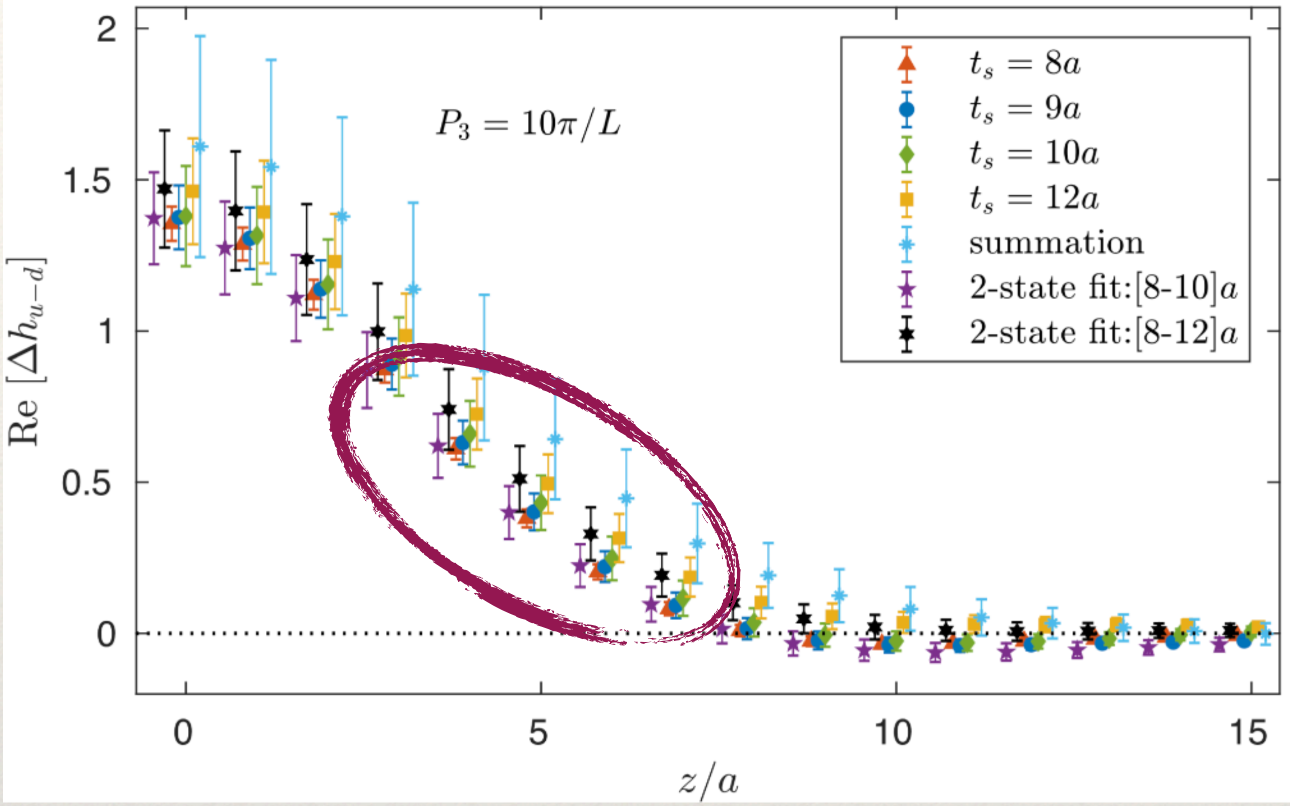

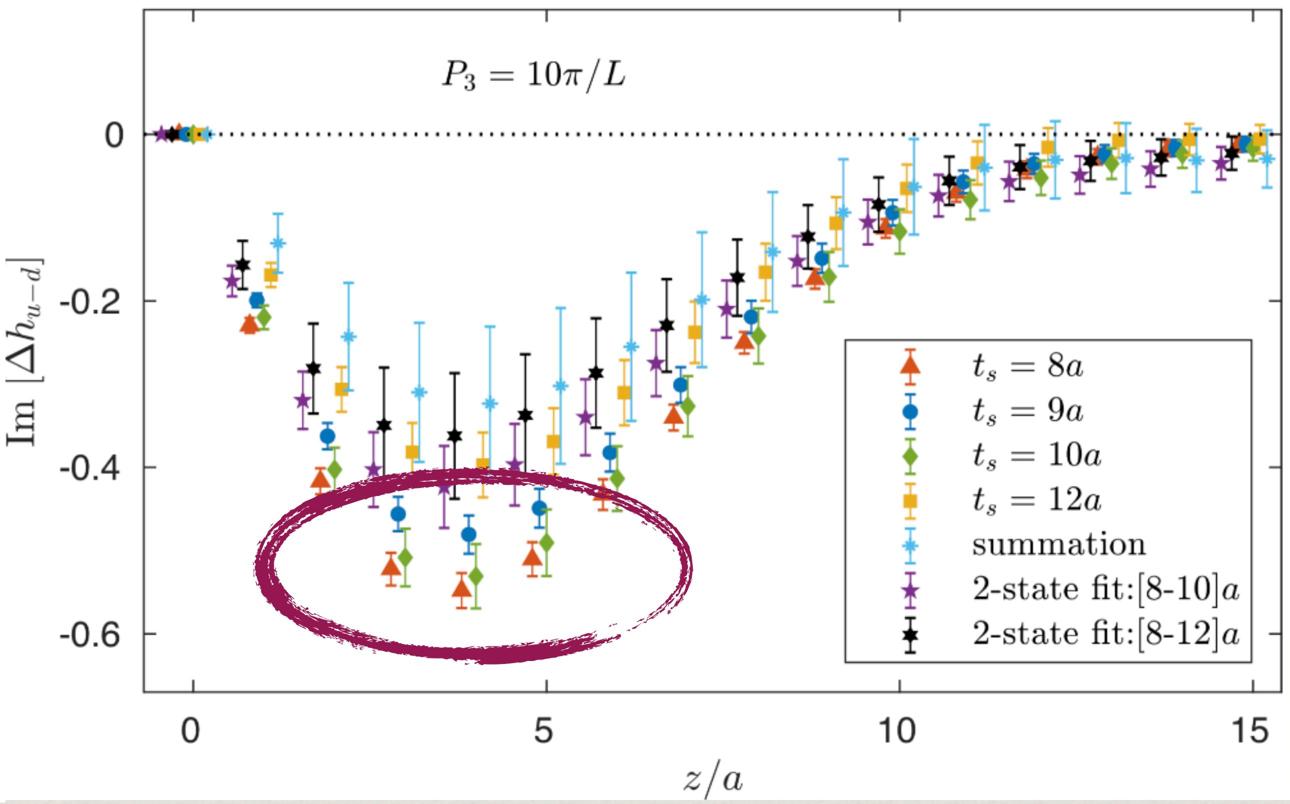

Conclusions:

- T_{sink}=8a heavily contaminated by excited states
- T_{sink}=9a-10a not consistent with T_{sink}=12 within uncertainties

! Crucial to have same error for reliable 2-state fit

! Excited states worsen as momentum *P* increases

! For momenta in this work, T_{sink}=1fm is safe

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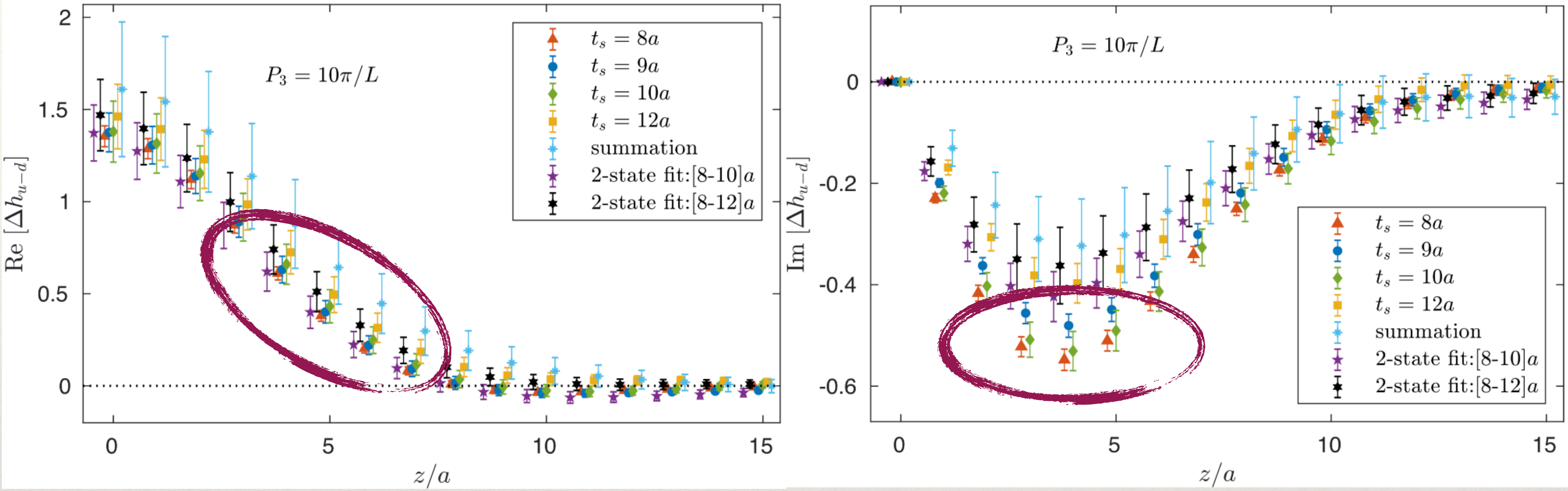

Such level of information is necessary to study excited states

Conclusions:

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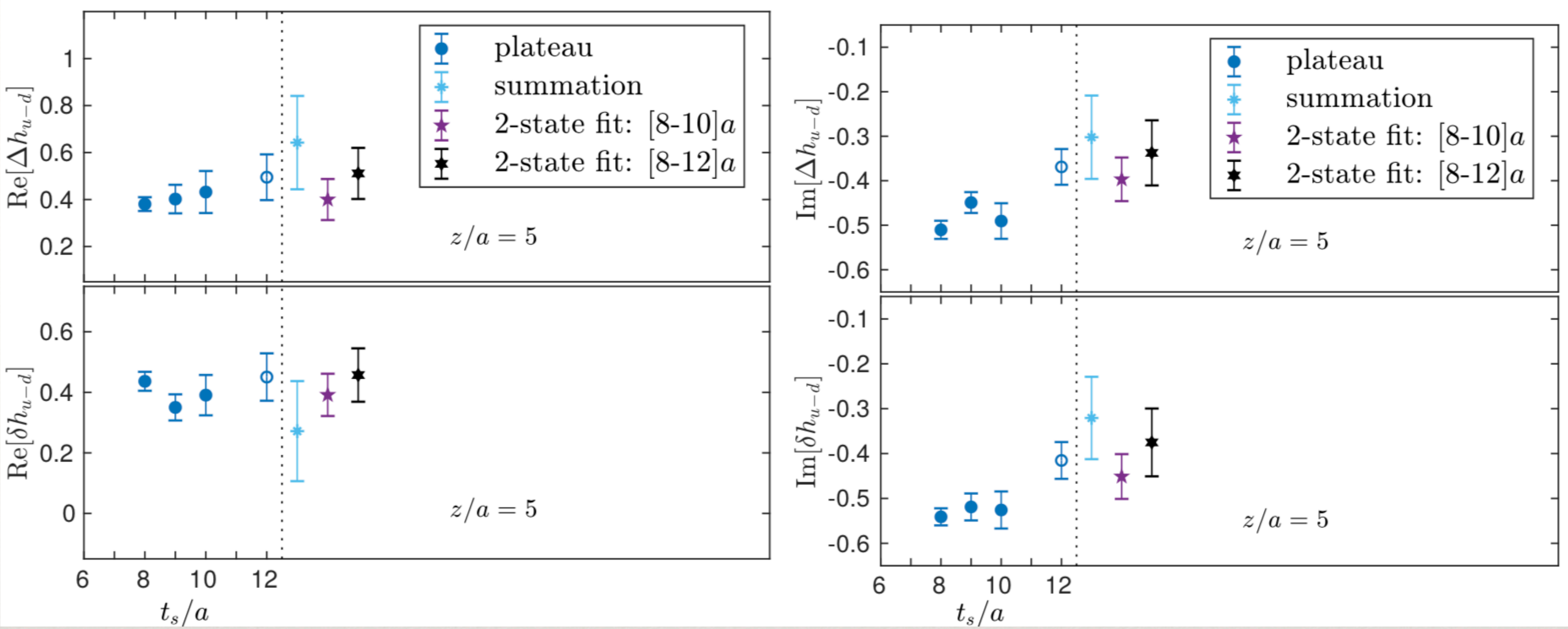

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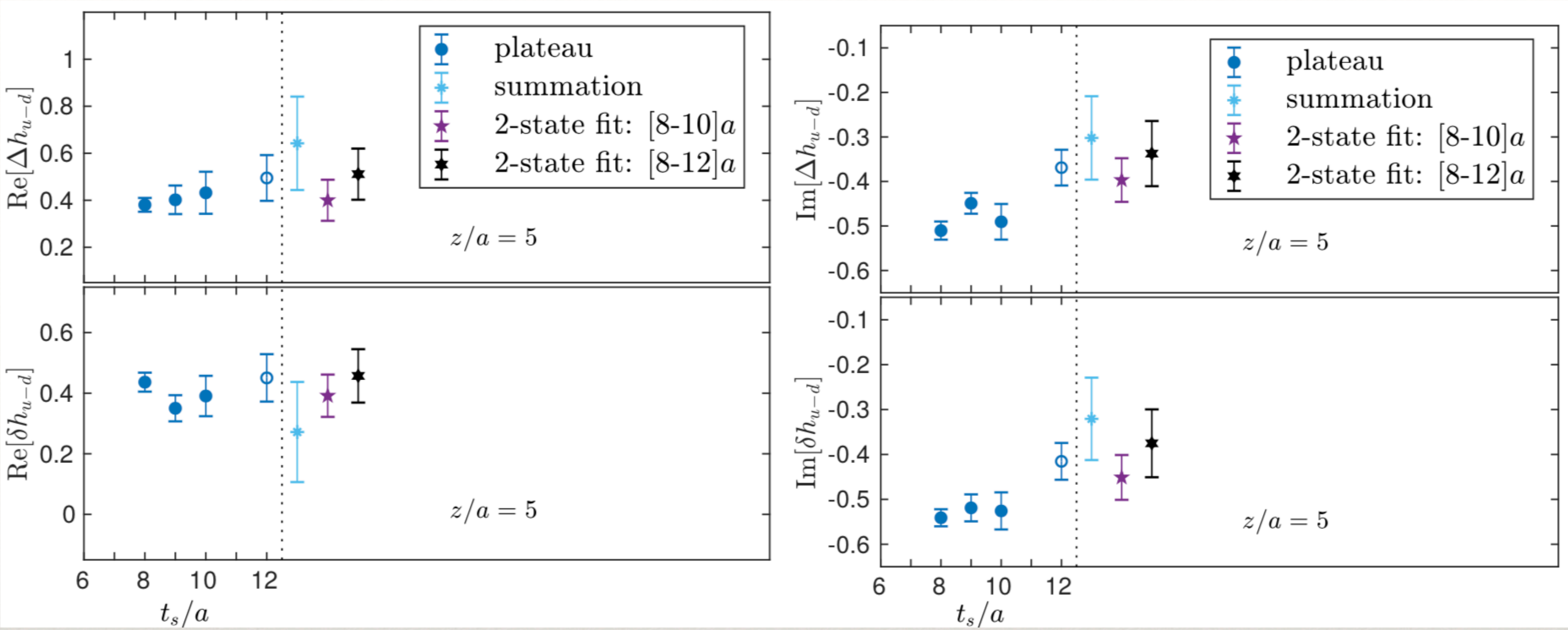
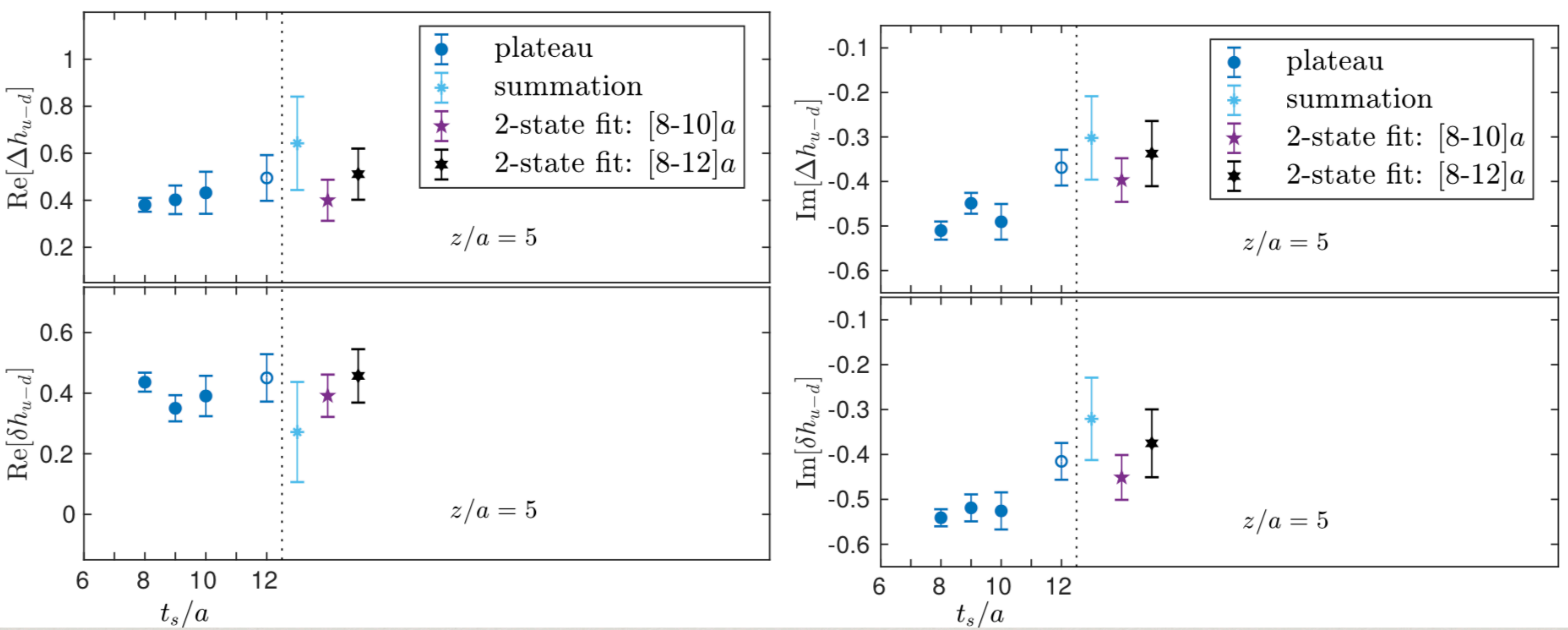


- **Non-predictable behavior (depends in *z* value)**
- **Real and imaginary part affected differently**

Excited states contamination



- **Non-predictable behavior (depends in *z* value)**
- **Real and imaginary part affected differently**

Conclusions:

- **Excited states uncontrolled for T_{sink} <1fm**
- **Multi-sink analysis <u>demands</u> same accuracy for all data**

Major Success: Renormalization

Critical part of calculation:

- Elimination of power and logarithmic divergences and dependence on regulator
- Renormalizability proven to all orders in perturbation theory

X. Ji, J. H. Zhang and Y. Zhao, Phys. Rev. Lett. 120, no. 11 (2018) 112001 [arXiv:1706.08962]

T. Ishikawa, Y. Q. Ma, J. W. Qiu, S. Yoshida, Phys. Rev. D 96, no. 9 (2017) 094019 [arXiv:1707.03107]

J. Green, K. Jansen, F. Steffens, Phys. Rev. Lett. 121 022004 (2018), [arXiv:1707.07152]

- Identification and elimination of mixing (lattice pert. theory)

M. Constantinou, “Renormalization Issues on Long-Link Operators”, GHP meeting, Feb. 2, 2017

[M. Constantinou, H. Panagopoulos, Phys. Rev. D 96 (2017) 054506, arXiv:1705.11193]

[C.Alexandrou et al., Nucl. Phys. B 923 (2017) 394 (Frontiers Article), arXiv:1706.00265]

- Proposed scheme (and variations) now used in most studies

Renormalization scheme:

$$\frac{Z_{\mathcal{O}}^{\mathrm{RI}'}(z,\mu_0,m_\pi)}{Z_q^{\mathrm{RI}'}(\mu_0,m_\pi)}\frac{1}{12}\mathrm{Tr}\left[\mathcal{V}(z,p,m_\pi)\left(\mathcal{V}^{\mathrm{Born}}(z,p)\right)^{-1}\right]\Big|_{p^2=\mu_0^2}=1$$

$$Z_q^{\mathrm{RI}'}(\mu_0,m_\pi)\frac{1}{12}\mathrm{Tr}\left[(S(p,m_\pi))^{-1}\,S^{\mathrm{Born}}(p)\right]\Big|_{p^2=\mu_0^2}=1$$

- RI’-type, employed non-perturbatively
- Applicable for cases of mixing
- Pert. theory used for conversion to MSbar scheme

Systematics in Renormalization functions

- Several ensembles for chiral extrapolation
- Several RI renormalization scales to convert to MSbar and remove residual dependence on initial scale
- Volume effects
- Conversion to Modified MS (MMS) scheme
- Subtraction of discretization artifacts in Z_q
  Subtraction to $\mathcal{O}(g^2\, a^\infty)$ completed

[Alexandrou et al (ETMC), arXiv:1902.00587]

Challenge #2

How to reconstruct the PDF (Fourier transform) from a discrete small number of data?

Alternative Fourier

Standard Fourier (SF):

$$\tilde{q}(x) = 2P_3 \int_{-z_{\max}}^{z_{\max}} \frac{dz}{4\pi} e^{ixzP_3} h(z)$$

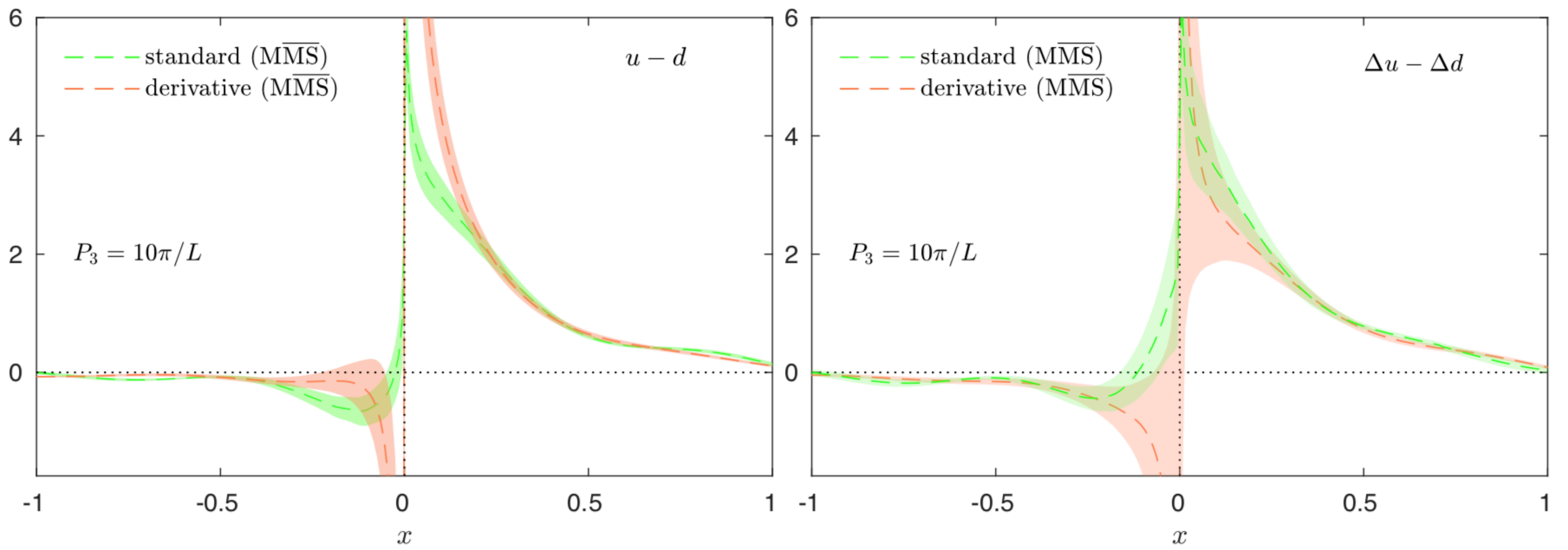
can be written using integration by parts (DF):

$$\tilde{q}(x) = h(z)\frac{e^{ixzP_3}}{2\pi i x}\Big|_{-z_{\max}}^{z_{\max}} - \int_{-z_{\max}}^{z_{\max}} \frac{dz}{2\pi} \frac{e^{ixzP_3}}{ix} h'(z)$$

[H.W. Lin et al., arXiv:1708.05301]

- Surface term ignored, but contribution non-negligible if matrix elements have not decayed to zero at some $z_{\max}$
- The 1/x in the surface term may lead to uncontrolled effect for small values of x

Alternative Fourier



Both SF and DF use the same lattice data

- **Truncation at z_{max} (SF) vs neglecting surface term (DF)** (latter non-negligible numerically)
- **Oscillations slightly reduced for DF, but small-x not well-behaved**
- **SF, DF different systematics, DF may have enhanced cut-off effects**

Advanced reconstruction promising approach:

J. Karpie et al., JHEP (in press), arXiv:1901.05408

Challenge #3

Is matching to light-cone PDFs unique?

Matching

- Variety of prescriptions for quasi-PDFs in:
  - MSbar scheme
  - RI-type scheme
  - MMS scheme
  - Ratio scheme

- Modified prescription (MMS) in
C. Alexandrou et al. (ETMC), Phys. Rev. Lett. 121, 112001 (2018), arXiv:1803.02685
compared to: T. Izubuchi et al., arXiv:1801.03917

- Matching MMS: normalization of the distributions preserved

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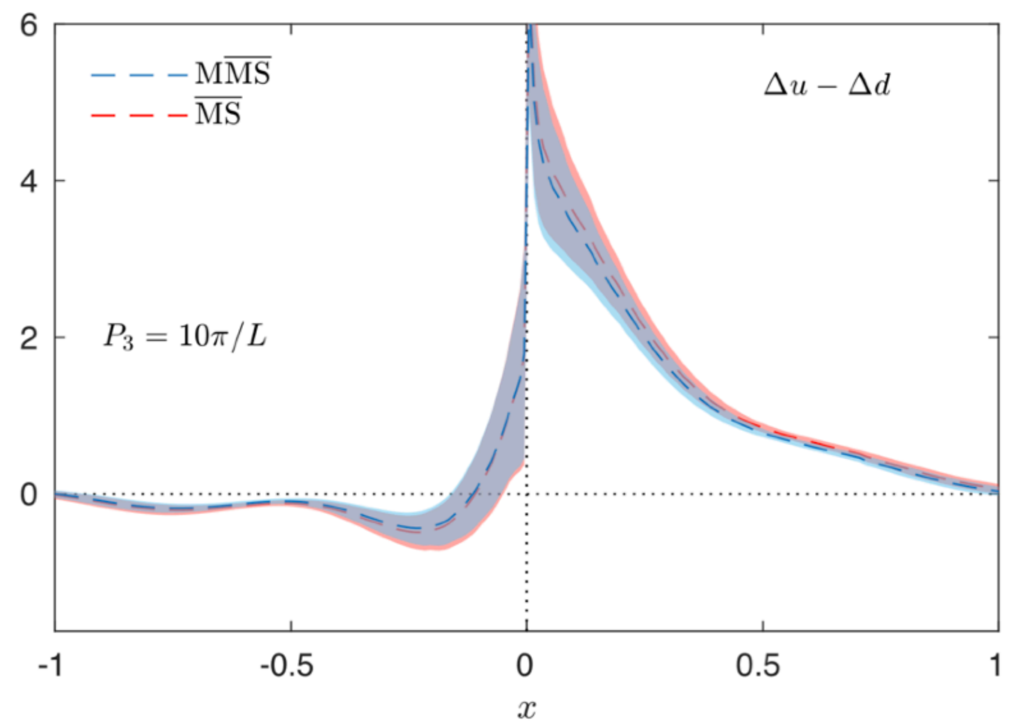
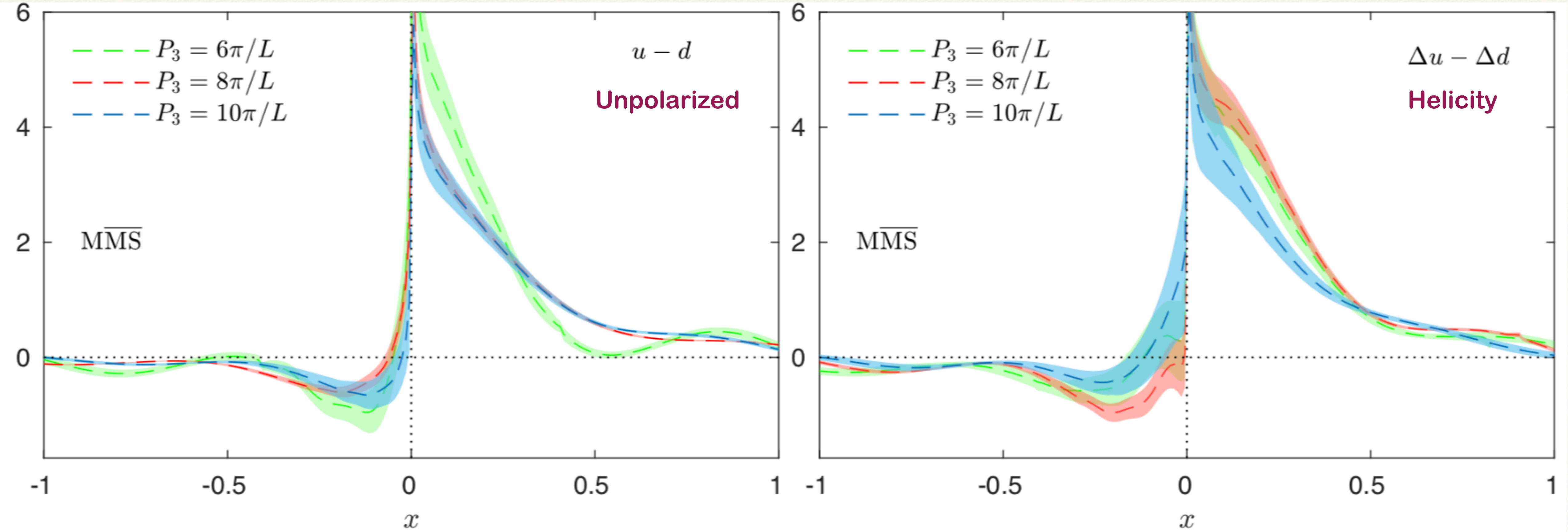


Methodology extensively discussed by F. Steffens Tue @ 3pm

Momentum dependence



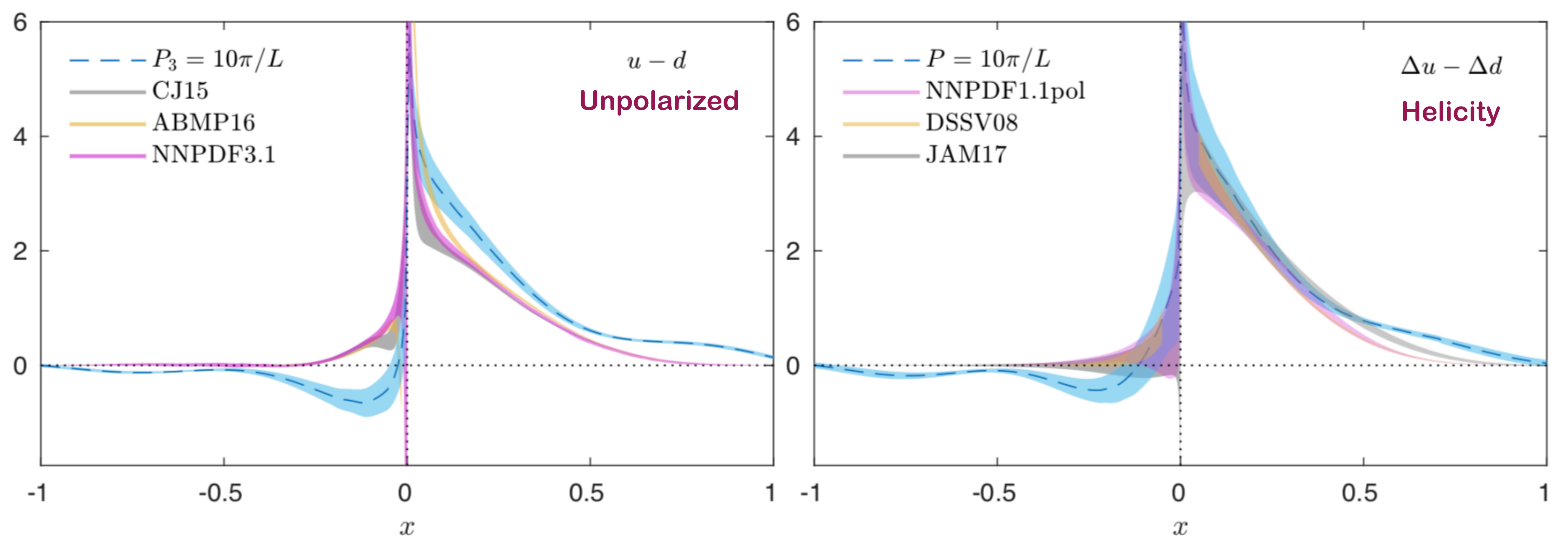
- **Increasing momentum leads to better agreement with the global fits**
- **Momentum dependence different for each type of PDFs**
  higher-twist effects within statistical uncertainties (~5%)
- **x~1: affected by finite nucleon momentum (milder for p=1.4 GeV)**

Comparison with global fits

Upon:

- Fourier transform of renormalized matrix elements
- Matching of quasi-PDFs (LaMET)
- Target Mass Corrections (m_N / P: finite)

[J.W. Chen et al., NPB 911 (2016) 246, arXiv:1603.06664]



- Lattice PDF approach phenomenological fits
- Negative x region: anti-quark contribution
  currently suffers from enhanced uncertainties

Transversity:
See talk by F. Steffens
Tue @ 3pm

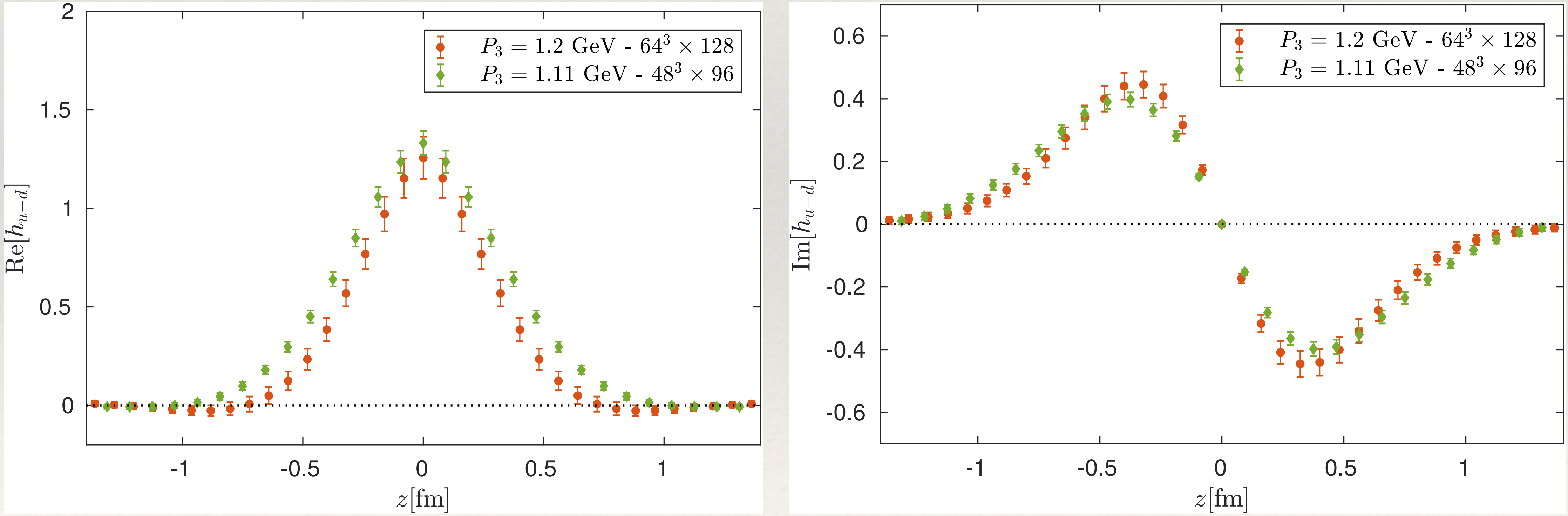
What is next?

Preliminary results for larger-volume ensemble

- Twisted Mass fermions with clover term

Ensemble	a [fm]	volume $L^3 \times T$	N_f	m_π [MeV]	Lm_π	L [fm]
cB211.64	0.081	$64^3 \times 128$	u, d, s, c	135	3.55	5.2

- “Similar” momenta and T_{sink} as our previous work (study systematics)
- Currently data production (statistics up to ~3,300)



Preliminary results for quasi-GPDs

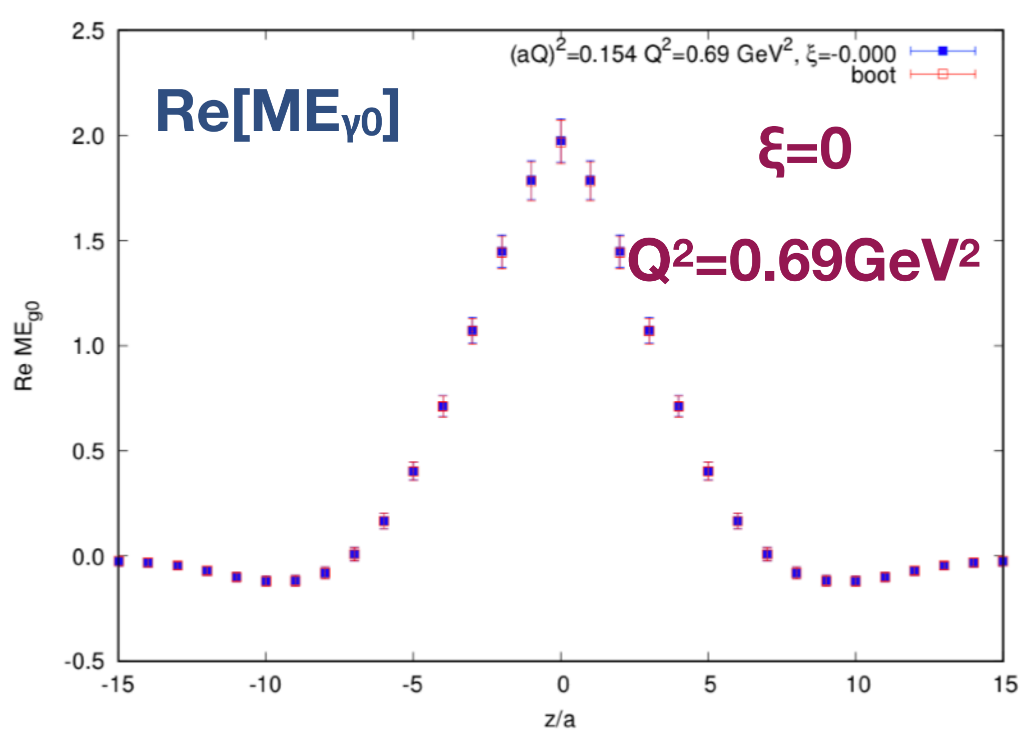
- Twisted Mass fermions with clover term

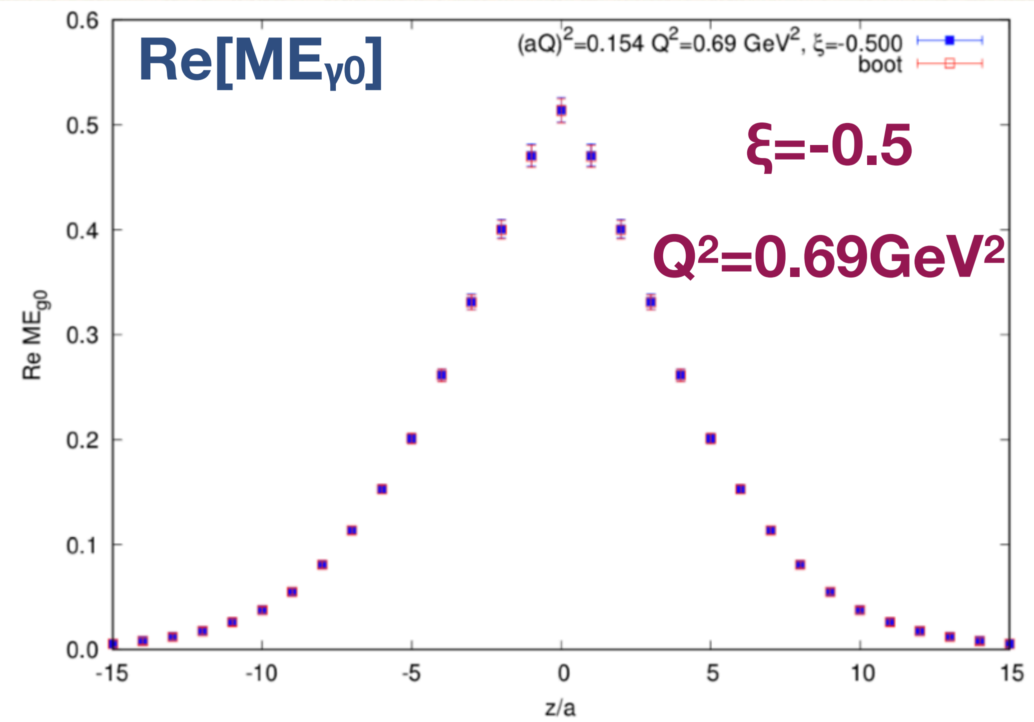
$$\xi \equiv -\frac{Q_3}{2P_3}$$

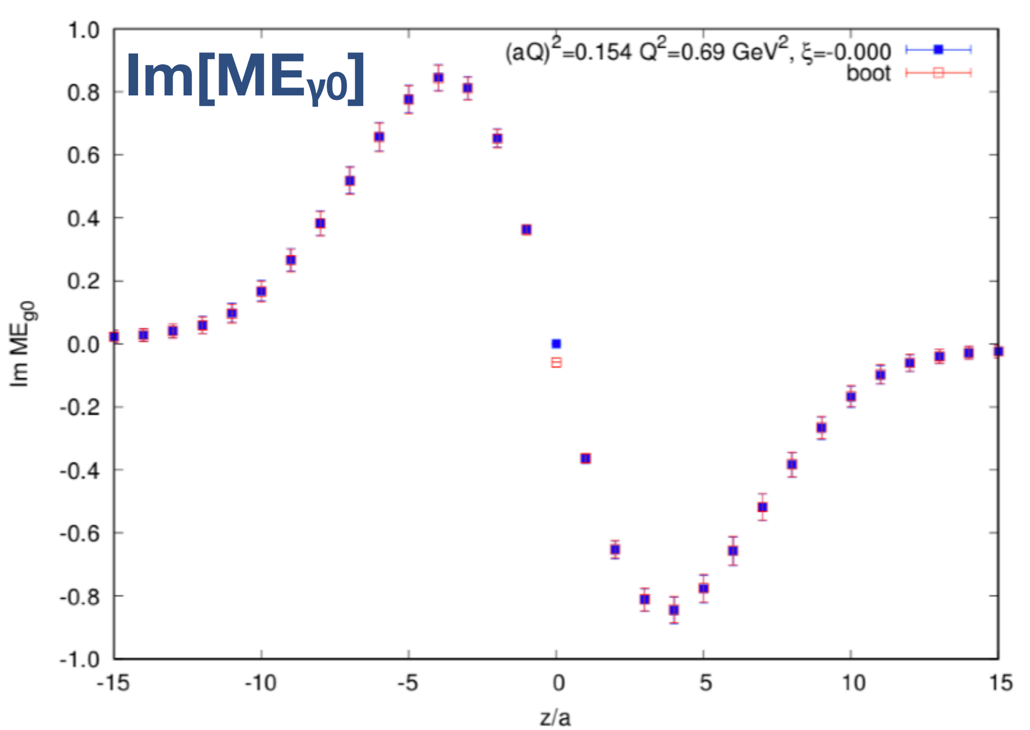
Quasi-skewness

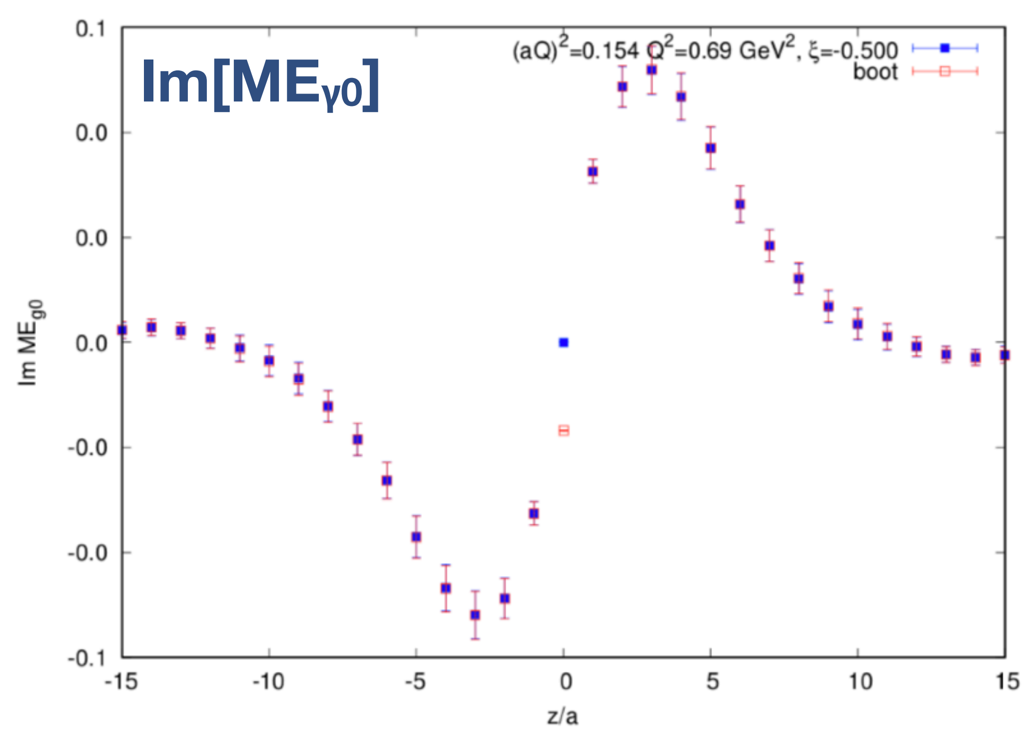
Ensemble	lattice spacing a	Volume $(L^3 \times T)$	m_π (MeV)	$m_\pi L$
cA211.32	0.093 fm	$32^3 \times 64$	270	4

P_3=1.25GeV









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- Identification of appropriate operators (no mixing)
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- Careful assessment of systematic uncertainties
  Fourier transform, volume & quenching effects, continuum limit, …
- Increase of momentum seems a natural next step
  BUT is a major challenge if reliable results are desired
- Other directions should be pursued, e.g. 2-loop matching

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Future of quasi-PDFs defined by reliable control of uncertainties

> ***“Aim for the sky, but move slowly, enjoying every step along the way. It is all those steps that make the journey complete”***
>
> *Chanda Kochhar*

THANK YOU

U.S. DEPARTMENT OF ENERGY | Office of Science

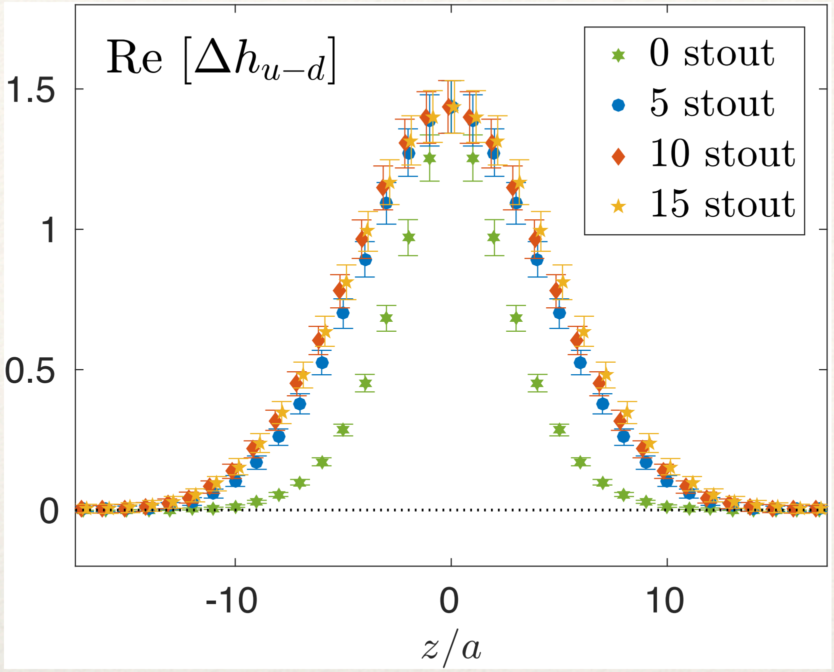
NSF

TMD Topical Collaboration

Grant No. PHY-1714407

BACKUP SLIDES

Renormalized Matrix Elements



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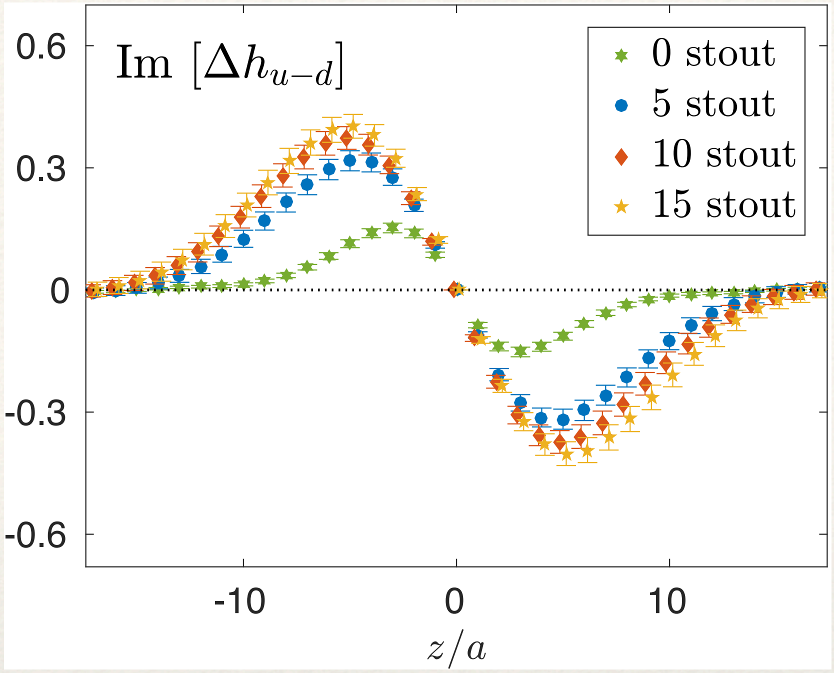
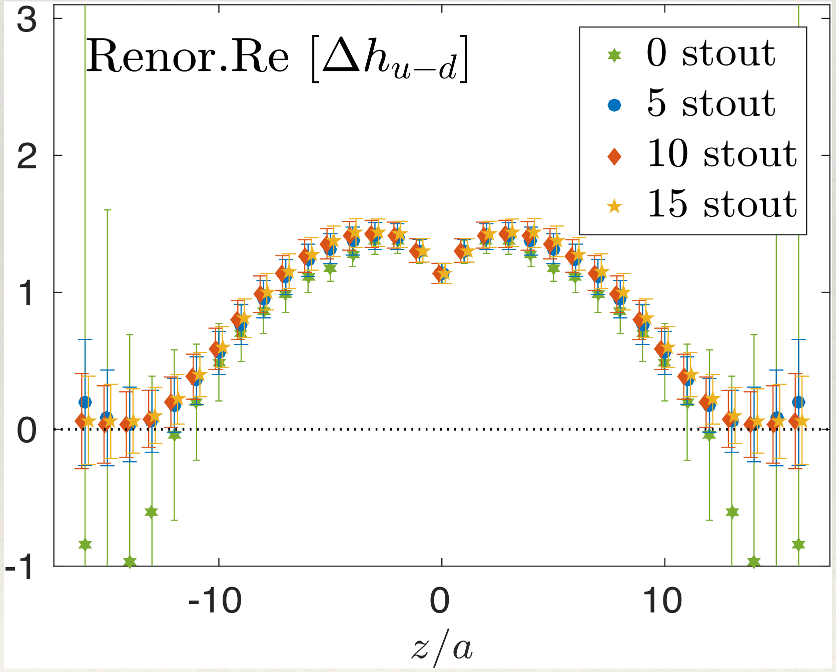
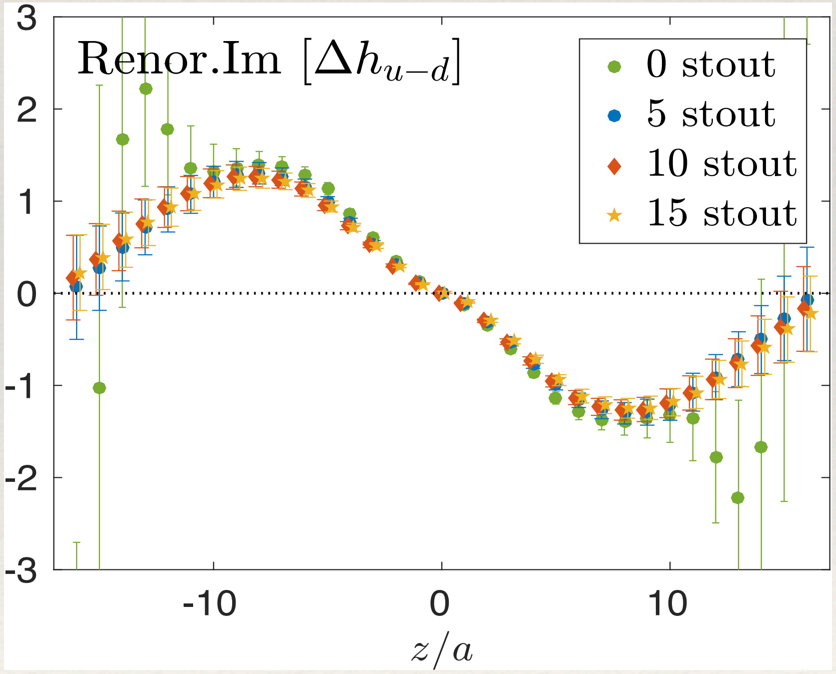
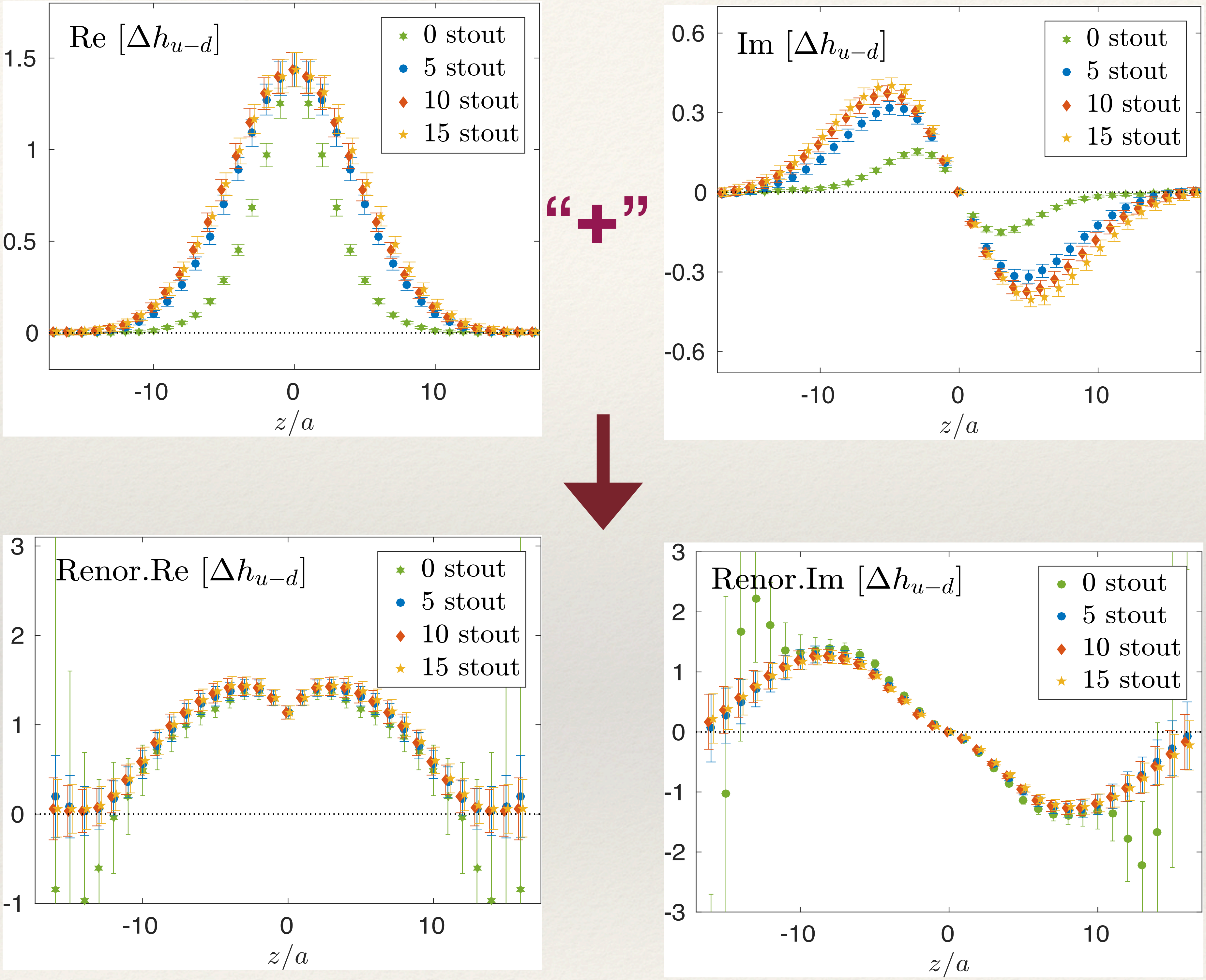






Renormalized ME do not dependent on stout smearing

Renormalized Matrix Elements

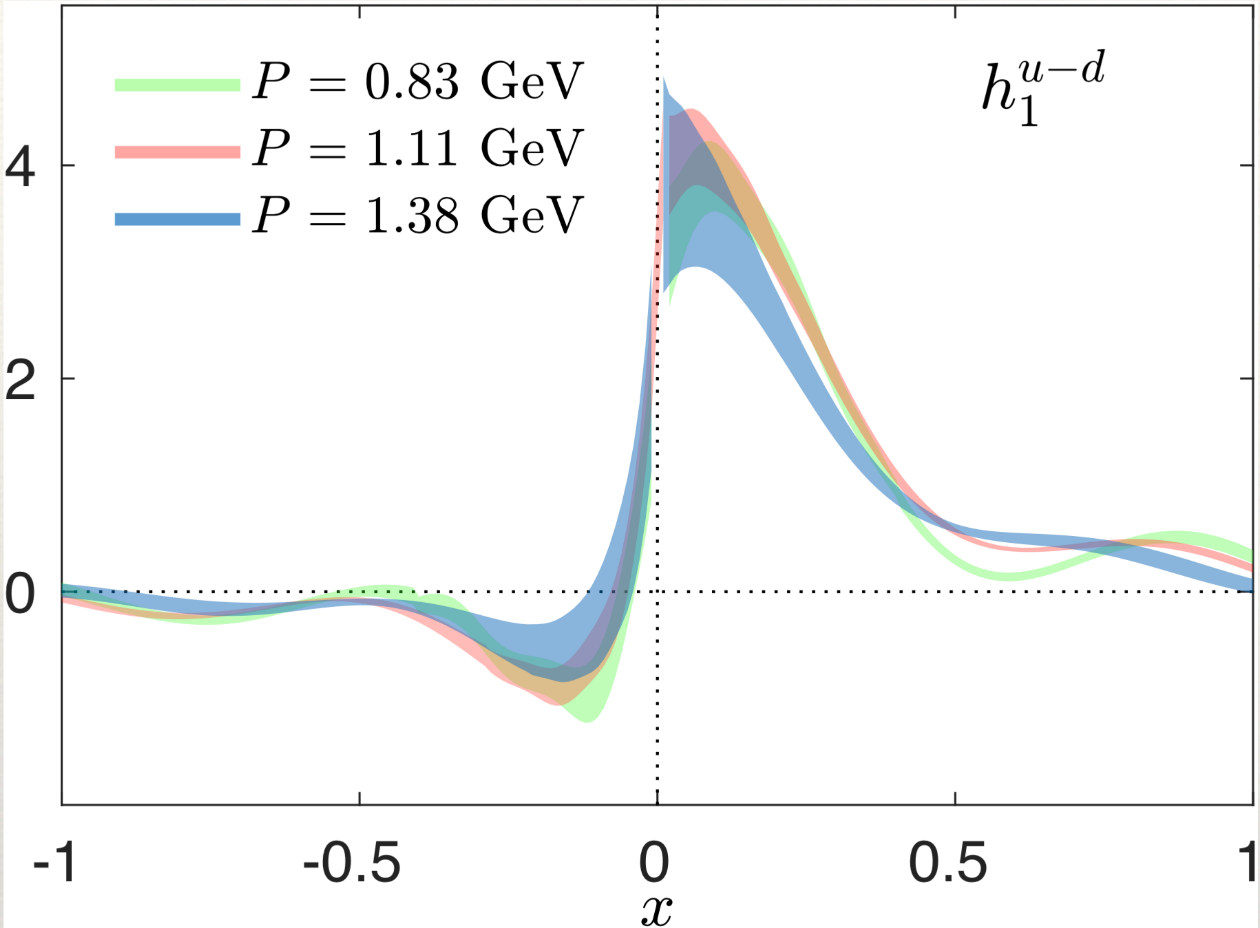


Renormalized ME do not dependent on stout smearing

[Alexandrou et al (ETMC), arXiv:1902.00587]

Transversity

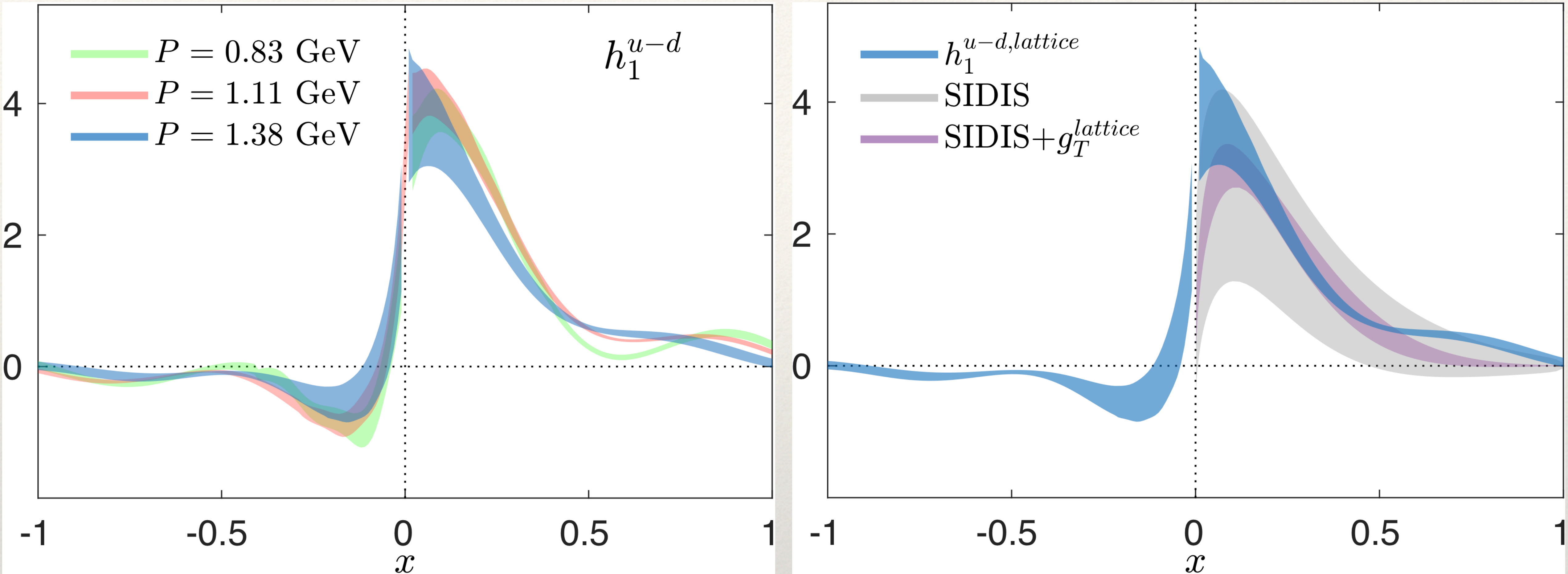
[C. Alexandrou et al., arXiv:1807.00232]



- Mild dependence on nucleon momentum
- Integral of PDF (g_T=1.09(11)) compatible with results from moments [C. Alexandrou et al., Phys. Rev. D95, 114514 (2017)]

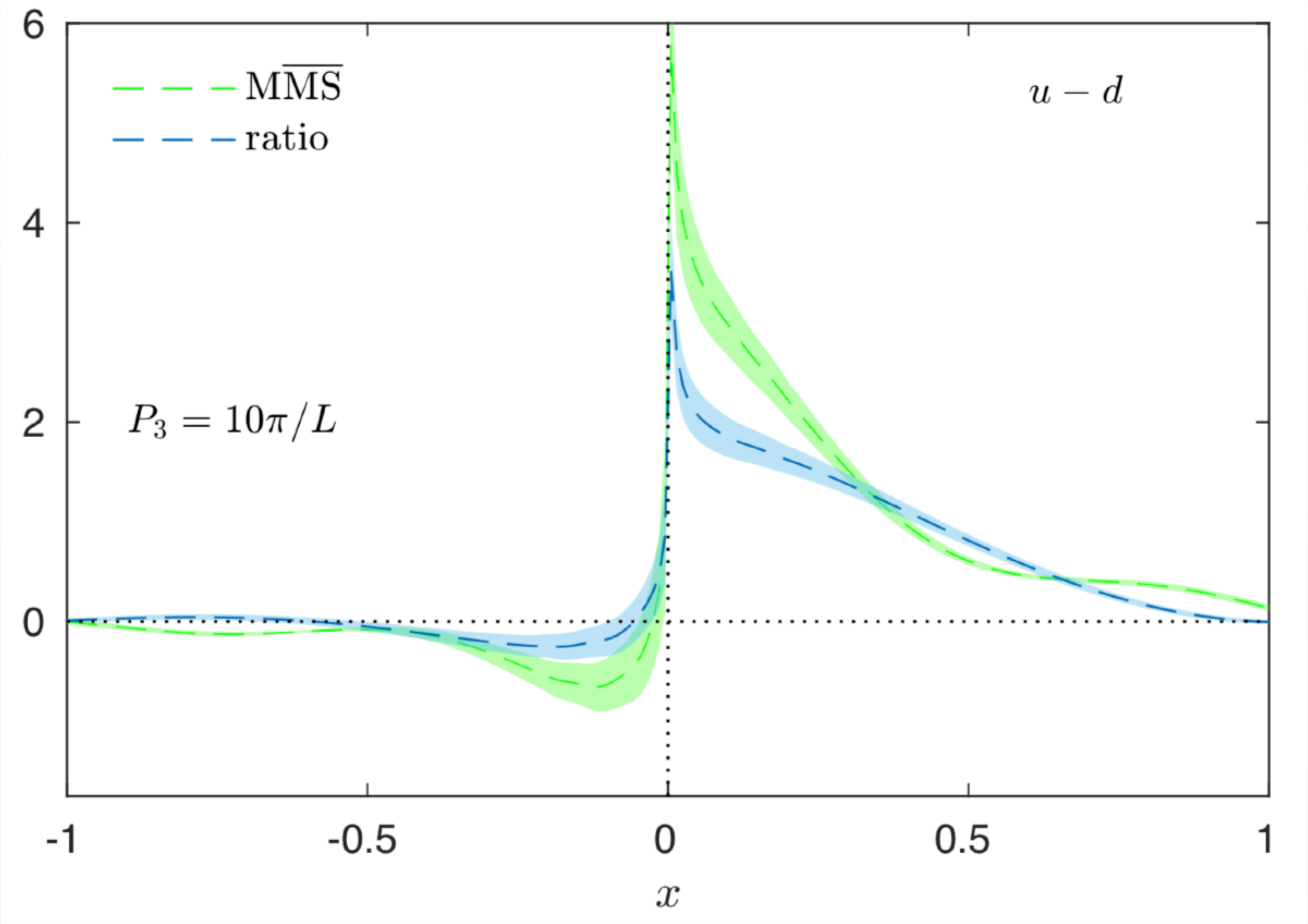
Transversity

[C. Alexandrou et al., arXiv:1807.00232]



- Mild dependence on nucleon momentum
- Integral of PDF (g_T=1.09(11)) compatible with results from moments [C. Alexandrou et al., Phys. Rev. D95, 114514 (2017)]
- Lattice data from quasi-PDFs more accurate that SIDIS
- SIDIS improved with g_T^{Lat} constraints, but *ab initio* quasi-PDFs statistically more accurate

“Ratio” scheme



Alternative way to achieve current conservation, which includes a modification of the physical region. Thus, the effect on the matched PDFs is expected to be larger numerically compared to MMS scheme