3-D STRUCTURE OF THE PION AND KAON FROM QCD'S DYSON-SCHWINGER EQUATIONS.

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TMD PDFs

- The TMD PDFs are defined with correlation function with finite transverse separation

$$\Phi_{ij}(x,\boldsymbol{k}_\perp,S) = \int \frac{\mathrm{d}\xi^- \mathrm{d}^2\boldsymbol{\xi}_\perp}{(2\pi)^3} e^{i(k^+\xi_- - \boldsymbol{k}_\perp\cdot\boldsymbol{\xi}_\perp)} \langle P,S|\overline{\psi}_j(0)\mathcal{U}^{n_-}_{(0,+\infty)}\mathcal{U}^{n_-}_{(+\infty,\xi)}\psi_i(\xi)|P,S\rangle\Big|_{\xi^+=0},$$

The TMD PDFs enter the general decomposition of the correlation function.

$$\Phi(x,\boldsymbol{k}_\perp,S) = \frac{1}{2}\left\{ f_1 \not{n}_+ - f_{1T}^\perp \frac{\epsilon_T^{ij}\boldsymbol{k}_\perp^i S_\perp^j}{M}\not{n}_+ + \Lambda g_{1L}\gamma_5\not{n}_+ + \frac{(\boldsymbol{k}_\perp\cdot\boldsymbol{S}_\perp)}{M} g_{1T}\gamma_5\not{n}_+ + h_{1T}\frac{[\not{S}_\perp,\not{n}_+]}{2}\gamma_5 \right. \textbf{+...}$$

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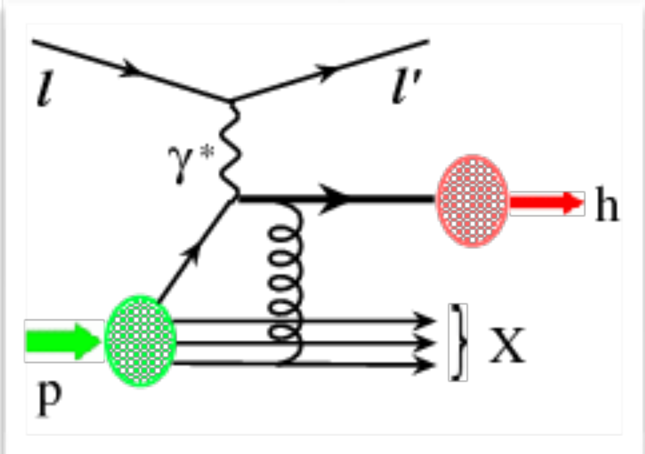

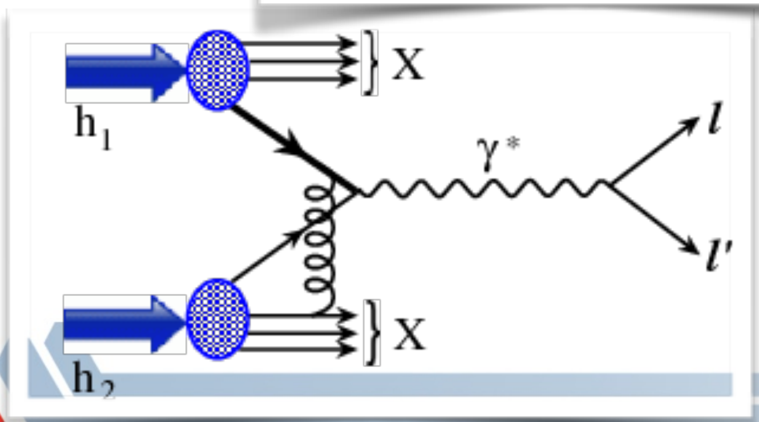

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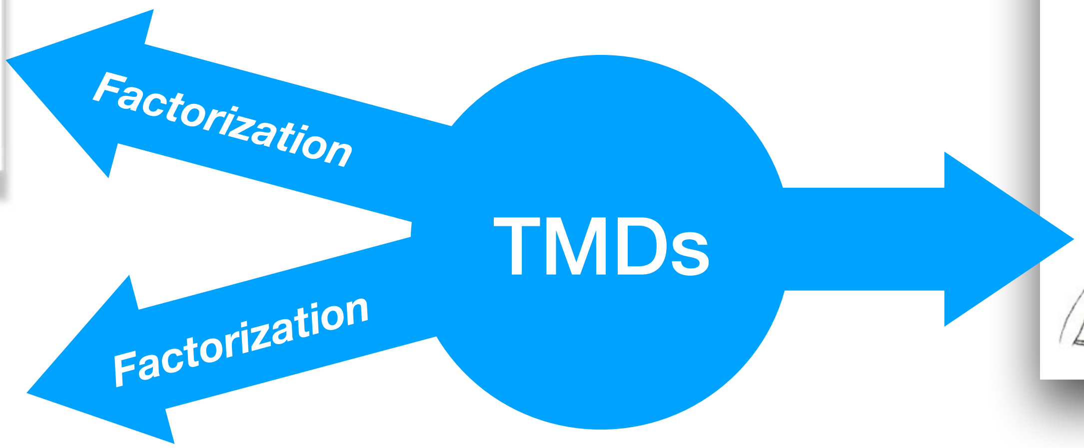
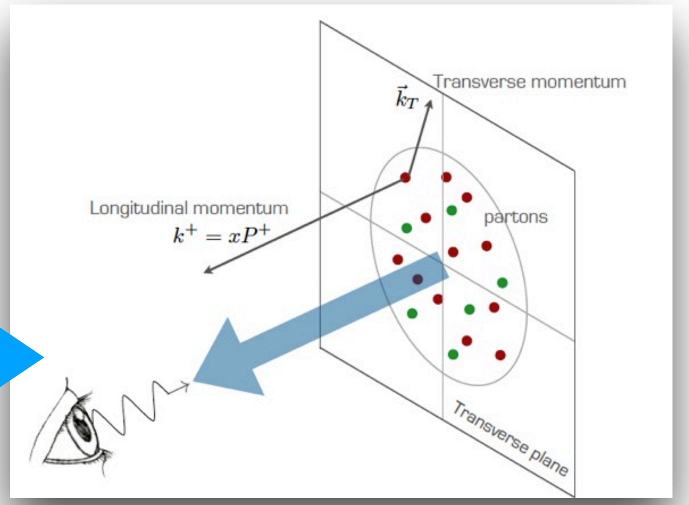




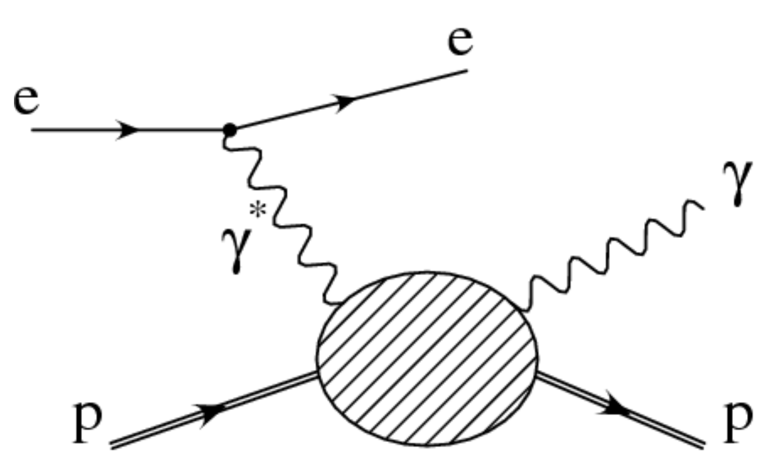
GPDs

- The generalized parton distribution introduces a finite momentum transfer Δ to the parent hadron, i.e, $\Delta \neq 0$.

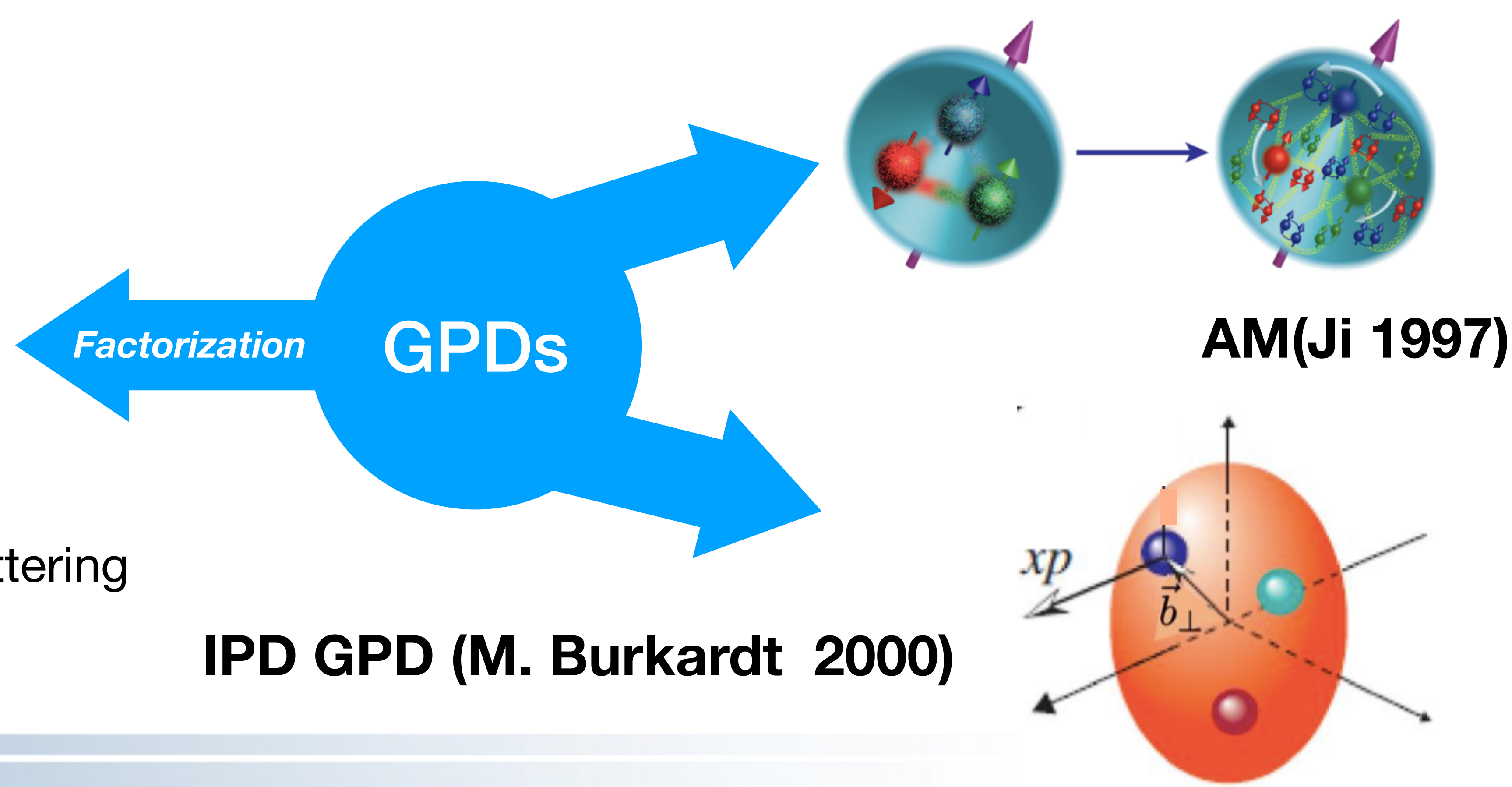
$$\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle P - \frac{\Delta}{2} | \bar{\psi}_i(0)\gamma^+\psi_j(z) | P + \frac{\Delta}{2} \rangle \Big|_{z^+=z_\perp=0}$$

$$= \frac{1}{P^+}\left[H^q(x,\xi,t)\bar{u}\left(P+\frac{\Delta}{2}\right)\gamma^+ u\left(P-\frac{\Delta}{2}\right) + E^q(x,\xi,t)\bar{u}(P+\frac{\Delta}{2})\frac{i\sigma^{+\mu}\Delta_\mu}{2M} u\left(P-\frac{\Delta}{2}\right)\right]$$

- GPDs show up in the factorization of DVCS et al. It encodes important information of hadrons, e.g., AM decomposition and spatial density distribution in the transverse plane.



Deeply virtual Compton scattering



Nonperturbative QCD (starting point of evolution)

QCD

$$\mathcal{L}_{QCD} = \bar{q}_i \gamma^\mu (i\partial_\mu - g_s t^a A^a_\mu - m_i) q_i - \frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu}$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_s f^{abc} A^b_\mu A^c_\nu \qquad \alpha_s = \frac{g_s^2}{4\pi}$$

Nonperturbative QCD methods

1. ADS/QCD
2. Dyson-Schwinger equations.
3. Effective theories and models, e.g., NJL model...
4. Light front QCD.
5. Lattice QCD.

etc...

Transverse momentum dependent distributions (TMD)
3-D tomography in the momentum space.

Generalized parton distributions (GPD)
3-D picture of hadrons in the mixed spatial-momentum space.

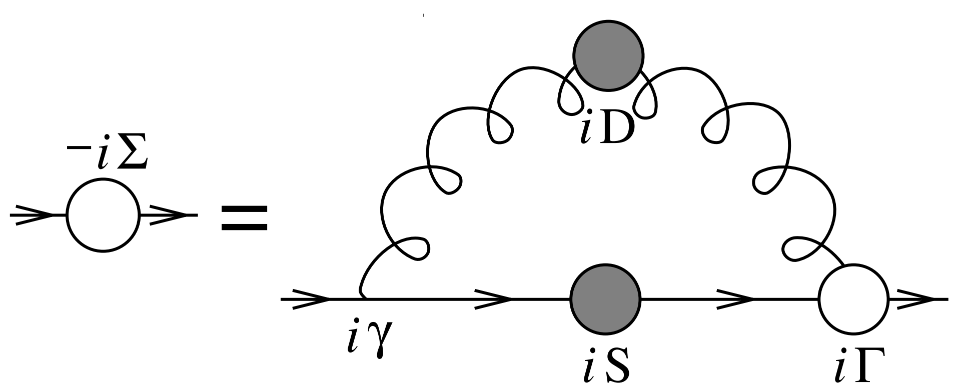
QCD's DSEs

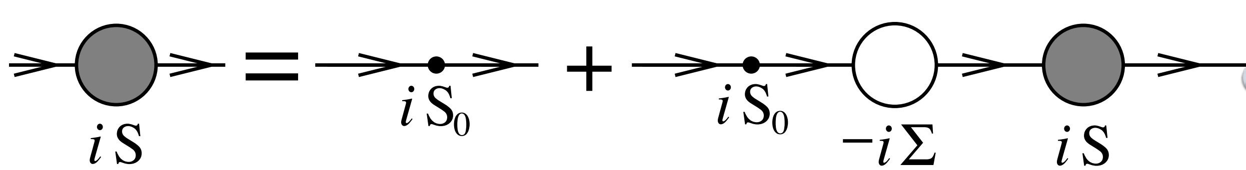
- Dyson-Schwinger equations: general relations between Green functions in quantum field theories.

✓Quantum Field Theory
✓Path Integral formulation

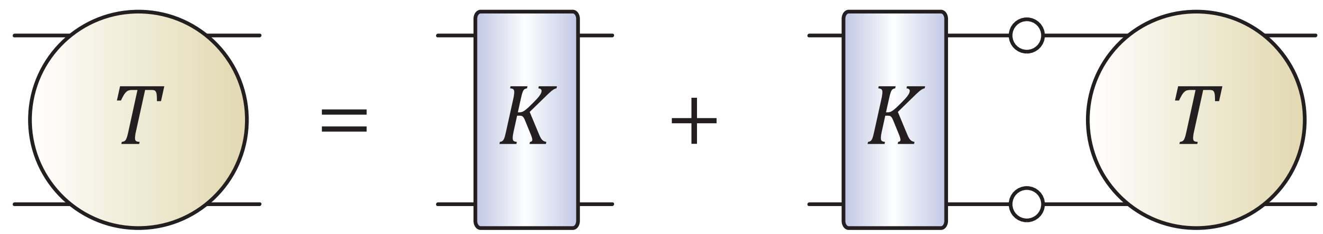
Non-perturbative

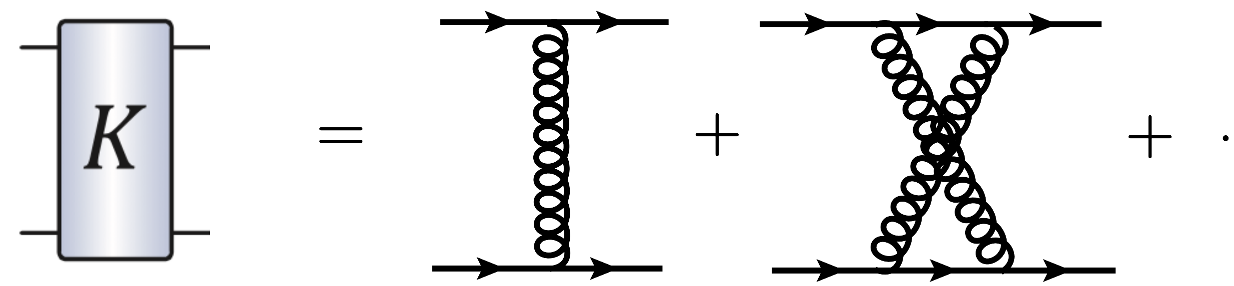
- Quark DSE:



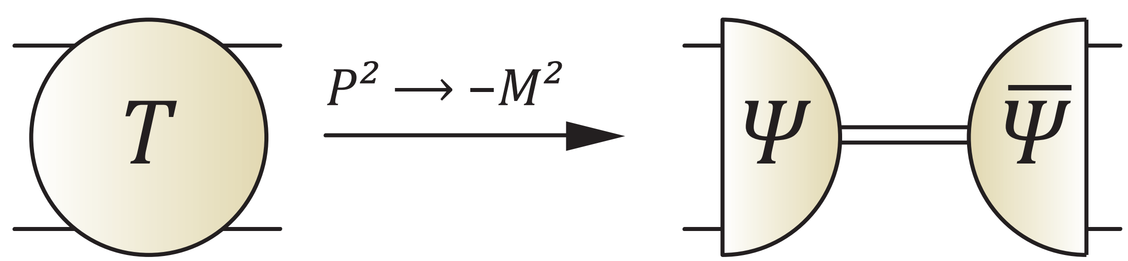


- The connected 4-quark scattering amplitude satisfies the Dyson equation

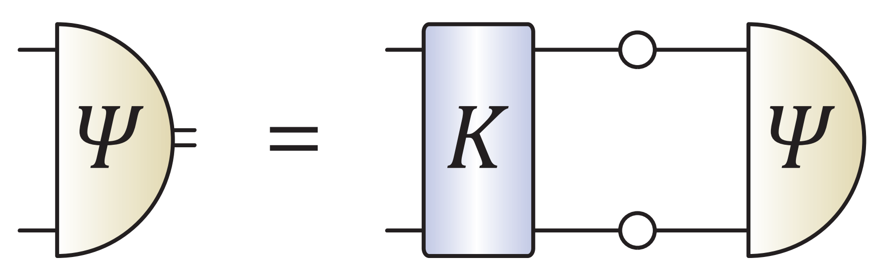




- Near mass pole, the quark scattering amplitude is dominated by hadron's Bethe-Salpeter WF.

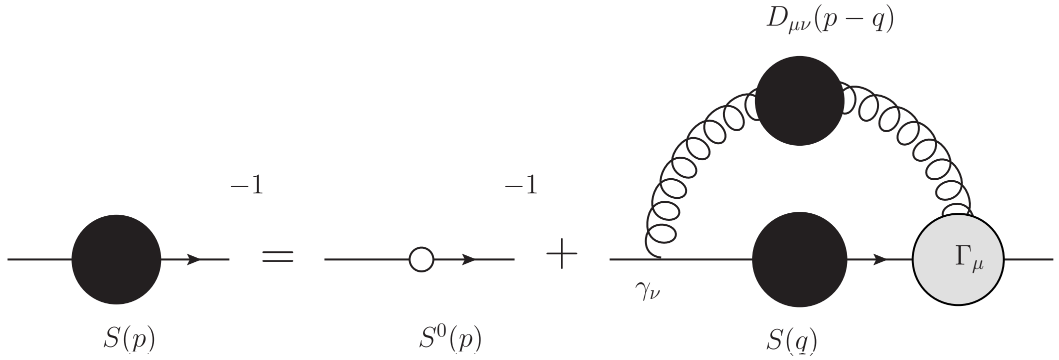


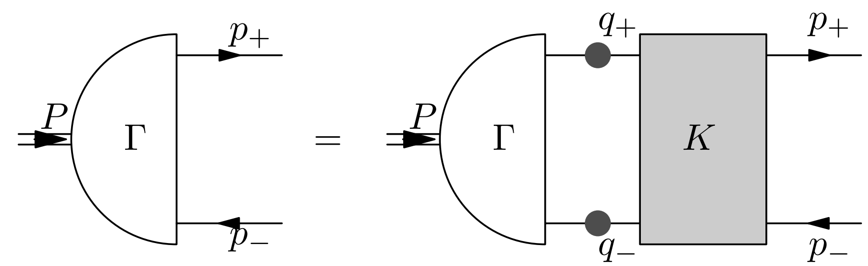
- Meson Bethe-Salpeter equation.



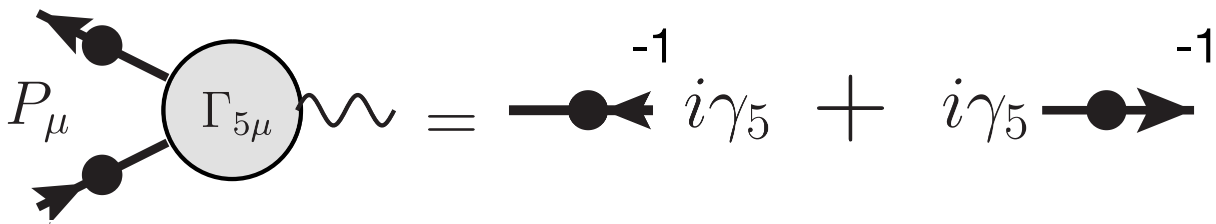
Chiral symmetry and AVWTI

- The hadron wave function can be solved by aligning the quark DSE and hadron BSE.

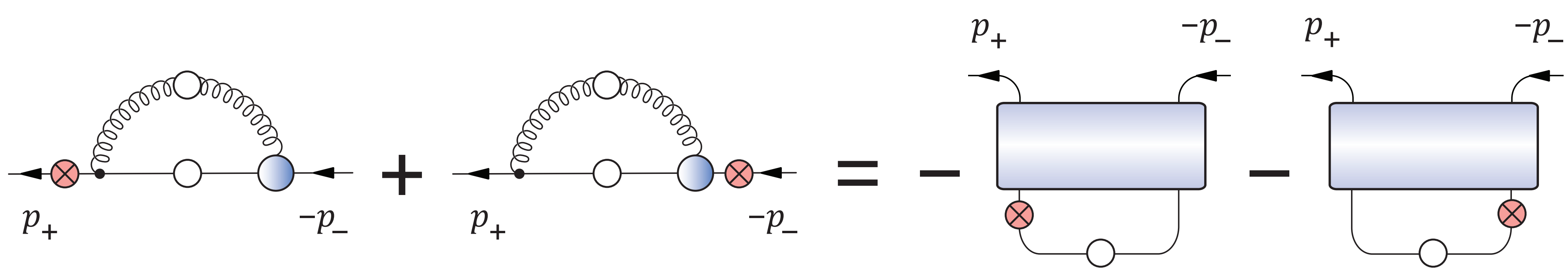




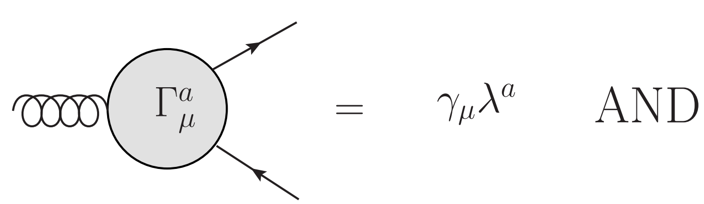
- To solve these equations, truncation is needed for the vertex and scattering kernel. A physically reasonable truncation scheme should preserve QCD's (nearly) chiral symmetry by respecting the Axial-Vector Ward-Takahashi Identity

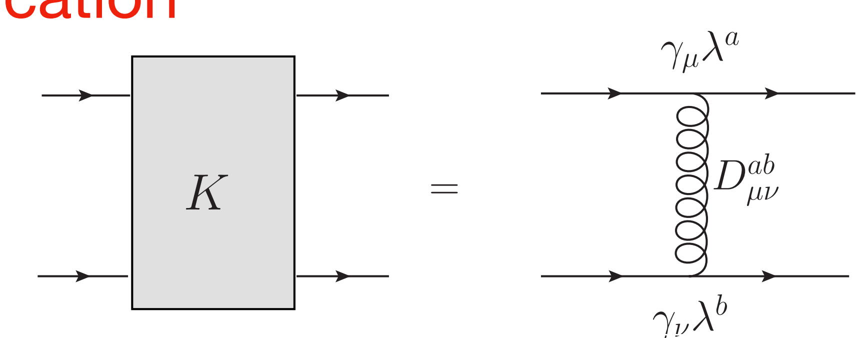


- The AVWTI relates the vertex Γ and kernel K



- The simplest is the Rainbow-Ladder truncation





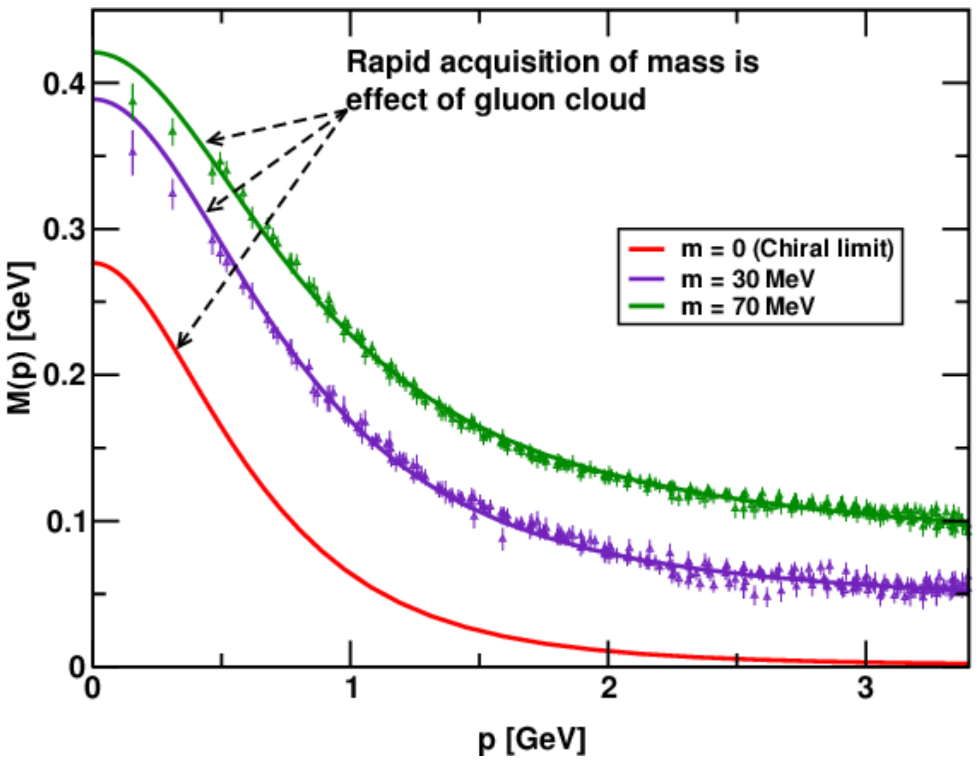
DSEs highlights and status

- Dynamical chiral symmetry breaking

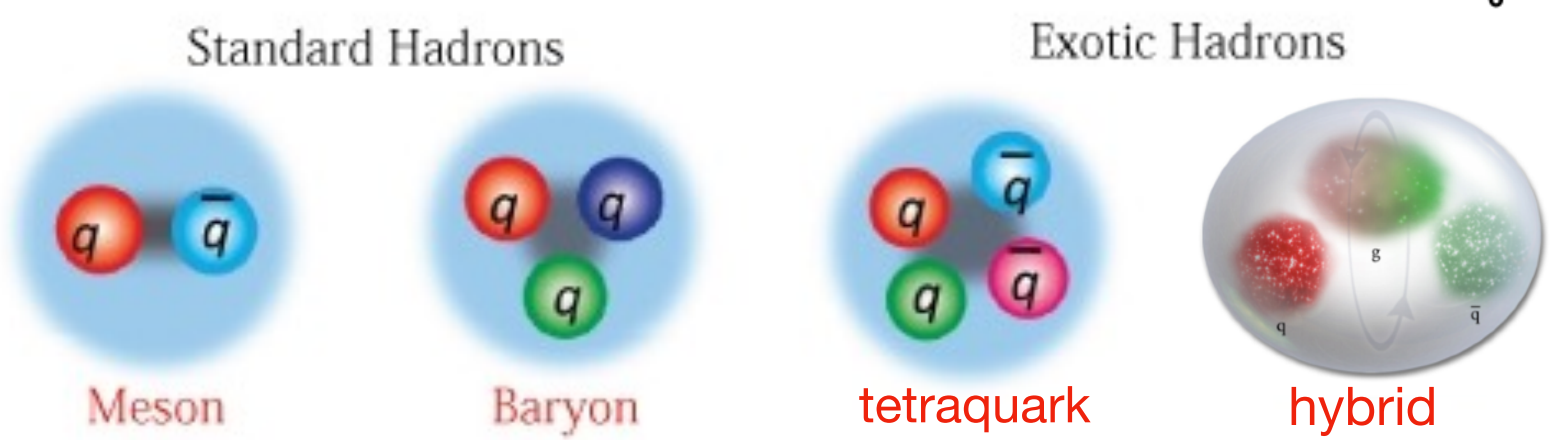
$$S(p) = \frac{Z(p^2)}{i\not{p} + M(p^2)}$$

  - m=0
  - M>>0

(M.S. Bhagwat et al, PRC2003)



- Hadron spectrum



Pieter Maris and Craig D. Roberts, PRC 1997

Gernot Eichmann, PRL 2010

Jorge Segovia, et al, PRL 2015

G Eichmann, C S. Fischer, W Heupel , PLB 2016

Shu-Sheng Xu, eta al 2018

- Form factors and parton distribution
  - Elastic and transition form factor
  - Parton distribution amplitude
  - Parton distribution function
  - GPD & TMD

Pieter Maris and Peter Tandy, PRC 2000, 2002,

Lei Chang, et al PRL 2013, G Eichmann PRD2011

Lei Chang, et al PRL 2013, Ian Cloet, et al, PRL 2013,

Chao Shi et al PLB 2014, Cedric Mezrag, et al PLB 2019

Trang Nguyen, et al PRD 2011, Kyle Bednar, et al PRL (in review) 2019

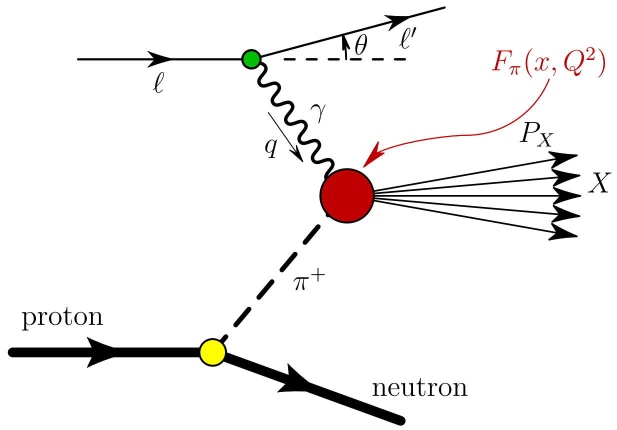
(Cedric Mezrag et al PLB 2015, Chao Shi, et al PRL 2019)

Why pion and kaon?

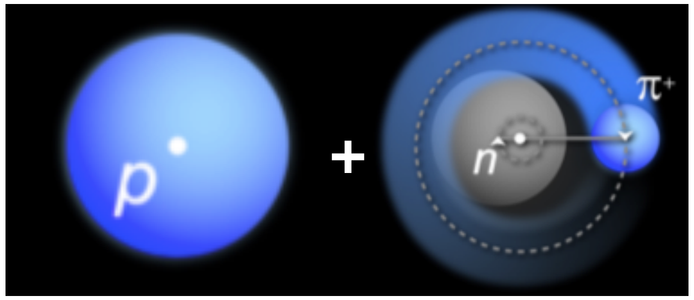
- Pion (and kaon) has the dual roles of being both a QCD bound state and also the Goldstone boson of DCSB. DCSB contributes 99% mass in visible universe. The massness of proton and masslessness of pion are closely related and both deserve studying.

- Pion (and kaon) is among the few hadrons whose parton structure can be experimentally measured, through, e.g., Drell-Yan and Sullivan process (off-shell pion).

- Pion also enters the description of nucleon by meson cloud. For a quark-core nucleon, pion cloud reduces its mass by ~20%, modifies nucleon's EM radius, and provides the sea quark content hence its asymmetry. $\bar{u}_p(x) \neq \bar{d}_p(x)$



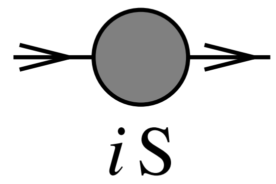
Sullivan process



- Theoretically, pion and kaon have been well studied in DSEs, there is no free parameter, the TMDs and GPDs pose new challenge.

TMDs & GPDs: Covariant approach

DSEs:



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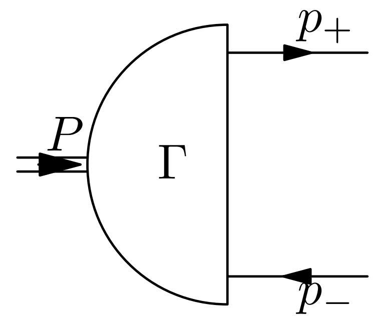
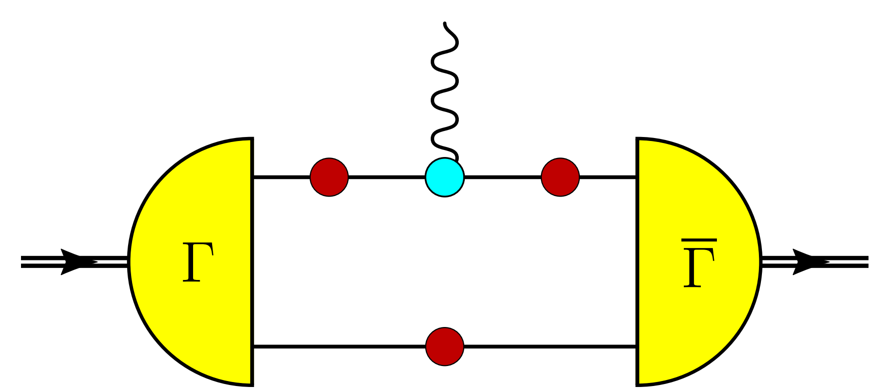


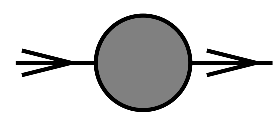
- *Covariant approach: Compute the triangle diagrams in terms of fully covariant propagators/vertices with appropriate truncations.*
- *Light-front approach: Extract from pion's Bethe-Salpeter wave functions the LFWFs and calculate TMDs and GPDs using overlap representation.*

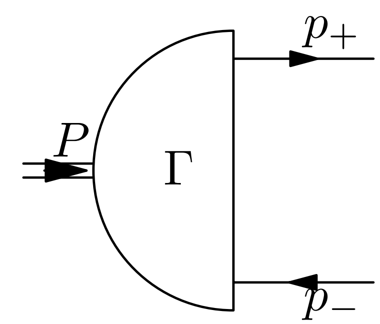
TMD & GPD

Impulse Approximation:



TMDs & GPDs: Light-front approach

DSEs: 

& 

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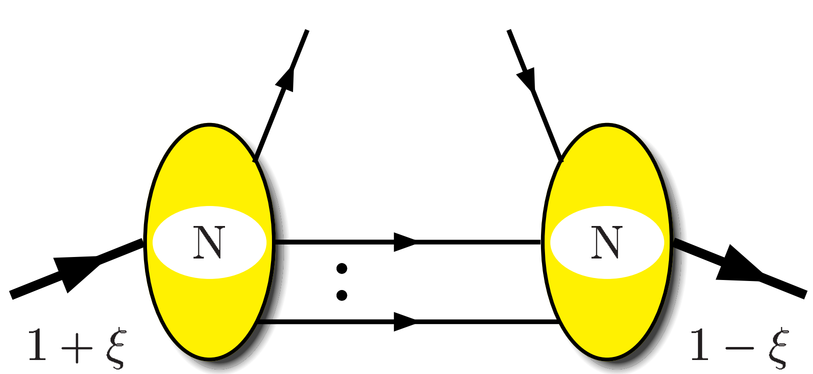
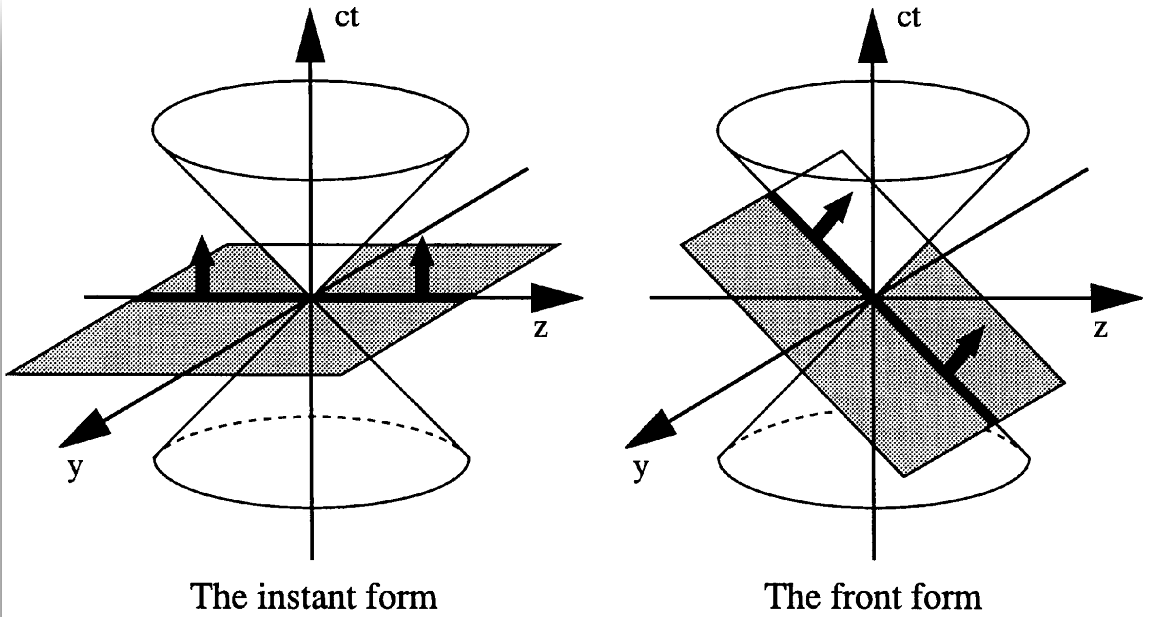
TMD & GPD



Light-front QCD

- QCD quantized in light front coordinate. A natural formalism in describing hard hadron scattering. The PDF, GPDs and TMDs are defined on (near) the null plane of light front. $\xi^+ = 0$
- In the light-front formalism, the hadronic state take a Fock-state expansion, characterized by light front wave functions (LFWFs).

$$|\pi^+\rangle = |u\bar{d}\rangle + |u\bar{d}\,g\rangle + |u\bar{d}\,gg\rangle + \ldots + |u\bar{d}\,q\bar{q}\rangle + |u\bar{d}\,q\bar{q}\,g\rangle + \ldots$$



$\tilde{x}^0 = ct$, $\tilde{x}^1 = x$, $\tilde{x}^2 = y$, $\tilde{x}^3 = z$

$\tilde{x}^0 = ct+z$, $\tilde{x}^1 = x$, $\tilde{x}^2 = y$, $\tilde{x}^3 = ct-z$

- The LFWFs encode all the non-perturbative information of the hadron's internal structure. $\psi(x, k_\perp)$ with $x = k_i^+/P^+$ They are boost invariant (since e.g., $k^+ \to e^\omega k^+$) and therefore provide relativistic description of bound systems in terms of quantum-mechanical-like wave functions.

- To calculate the LFWFs, the standard way is to diagonalize the light-cone Hamiltonian. However, this is very difficult.

Schwinger-Dyson approach

- There is an alternative way to calculate the LFWFs, using DSEs!

  "...he ('t Hooft) did not use the light–cone formalism and which nowadays might be called standard. Instead, he started from covariant equations... The light–cone Schrodinger equation was then obtained by projecting the Bethe–Salpeter equation onto hyper-surfaces of equal light–cone time. In this way, one avoids to explicitly derive the light–cone Hamiltonian, which, as explained above, can be a tedious enterprise in view of complicated constraints one has to solve..." (Thomas Heinzl)

  What we do: solve the BS equation first and then project the BS wave functions on to the light front!

- Connect modern DSEs study with the light-front QCD.

  Intrinsic Transverse Motion of the Pion's Valence Quarks
  Chao Shi and Ian C. Cloët,
  Phys.Rev.Lett. 122 (2019) no.8, 082301

- Lagrangian formalism (DSEs) + Hamiltonian formalism (LF QCD).

LFWFs & Bethe-Salpeter wave function

- Leading light front Fock components of pion (and kaon)

LFWFs, spin anti-parallel/parallel

$$|\pi^+(P)\rangle = |\pi^+(P)\rangle_{l_z=0} + |\pi^+(P)\rangle_{|l_z|=1}$$

$$|\pi^+(P)\rangle_{l_z=0} = i\int \frac{d^2k_\perp}{2(2\pi)^3}\frac{dx}{\sqrt{x\bar{x}}}\psi_0(x,k_\perp^2)\frac{\delta_{ij}}{\sqrt{3}}\frac{1}{\sqrt{2}}[b^\dagger_{u\uparrow i}(x,k_\perp)d^\dagger_{d\downarrow j}(\bar{x},\bar{k}_\perp) - b^\dagger_{u\downarrow i}(x,k_\perp)d^\dagger_{d\uparrow j}(\bar{x},\bar{k}_\perp)]|0\rangle,$$

$$|\pi^+(P)\rangle_{|l_z|=1} = i\int \frac{d^2k_\perp}{2(2\pi)^3}\frac{dx}{\sqrt{x\bar{x}}}\psi_1(x,k_\perp^2)\frac{\delta_{ij}}{\sqrt{3}}\frac{1}{\sqrt{2}}[k_\perp^- b^\dagger_{u\uparrow i}(x,k_\perp)d^\dagger_{d\uparrow j}(\bar{x},\bar{k}_\perp) + k_\perp^+ b^\dagger_{u\downarrow i}(x,k_\perp)d^\dagger_{d\downarrow j}(\bar{x},\bar{k}_\perp)]|0\rangle,$$

- Correlation function & LFWFs:

$$\langle 0|\bar{d}_+(0)\gamma^+\gamma_5 u_+(\xi^-,\xi_\perp)|\pi^+(P)\rangle = i\sqrt{6}P^+\psi_0(\xi^-,\xi_\perp),$$

$$\langle 0|\bar{d}_+(0)\sigma^{+i}\gamma_5 u_+(\xi^-,\xi_\perp)|\pi^+(P)\rangle = -i\sqrt{6}P^+\partial^i\psi_1(\xi^-,\xi_\perp).$$

(M. Burkardt et al, PLB 2002)

- LFWFs & BS wave function:

BS wave function

Project on to the light front (light front time $\xi^+=0$)

$$\psi_0(x,\boldsymbol{k}_T^2) = \sqrt{3}\,i\int \frac{dk^+dk^-}{2\pi} \times \mathrm{Tr}_D\left[\gamma^+\gamma_5\chi(k,p)\right]\delta\left(x\,p^+ - k^+\right),$$

$$\psi_1(x,\boldsymbol{k}_T^2) = -\sqrt{3}\,i\int \frac{dk^+dk^-}{2\pi}\frac{1}{\boldsymbol{k}_T^2} \times \mathrm{Tr}_D\left[i\sigma_{+i}\boldsymbol{k}_T^i\,\gamma_5\chi(k,p)\right]\delta\left(x\,p^+ - k^+\right),$$

Spin configuration

LFWFs: $\psi_0(x, k_\perp^2)$ & $\psi_1(x, k_\perp^2)$

- ψ_0 and ψ_1 are comparable in strength, suggesting the spin parallel $q\bar{q}$ has considerable contribution (relativistic system).

- Strong support at infrared k_T, a consequence of the DCSB which generates significant strength in the infrared region of BS wave function.

- At ultraviolet of k_T, ψ_0 scale as $1/k_T^2$ and ψ_1 scale as $1/k_T^4$, as has been predicted by pQCD.

- The x and k_T dependence in the LFWFs are un-factorizable, namely, the shape of LFWFs in x changes as k_T varies.

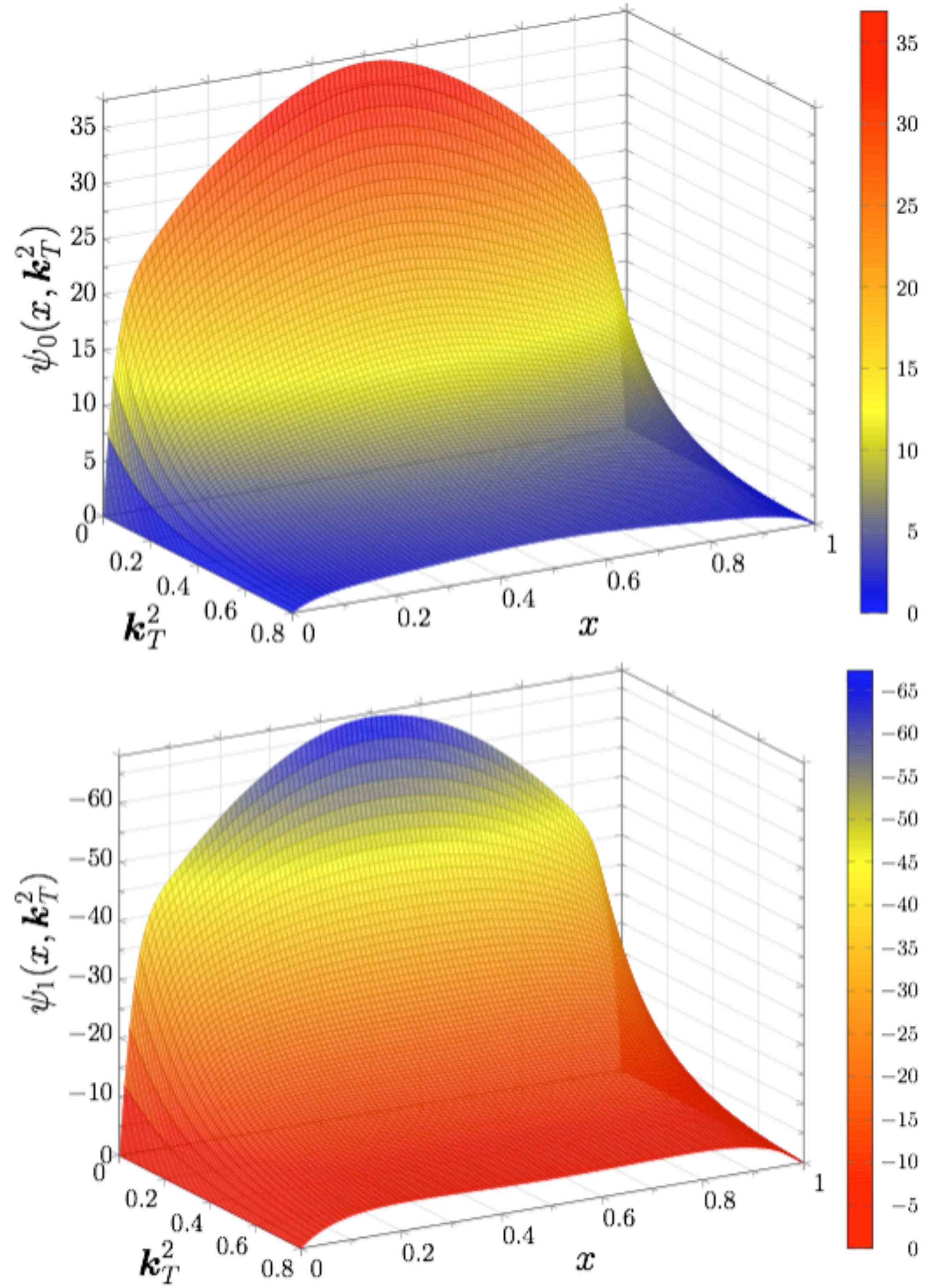


Figure 1. *Upper panel:* DSE result using the DCSB-improved kernel for the pion's $l_z = 0$ minimal ($\bar{q}q$) Fock-state LFWF. *Lower panel:* Analogous result for the pion's $|l_z| = 1$ minimal Fock-state LFWF. The LFWFs are given in units of GeV^{-2} and $\boldsymbol{k}_T^2$ in GeV^2.

TMD light-front overlap representation

(M. Burkardt et al, PLB 2002)

Fock Expansion

$$f_{1,\pi}(x,k_\perp^2) = \int \frac{d\xi^- d^2\boldsymbol{\xi}_\perp}{(2\pi)^3} e^{i(\xi^- k^+ - \boldsymbol{\xi}_\perp\cdot\mathbf{k}_\perp)} \langle\pi(P)|\bar{q}(0)\gamma^+ q(\xi^-,\boldsymbol{\xi}_\perp)|\pi(P)\rangle.$$

Decomposition

$$q_{(+)}(\xi^-,\xi_\perp) + q_{(-)}(\xi^-,\xi_\perp)$$

canonical expansion

$$q_{(+)}(\xi^+=0,\xi^-,\xi_\perp) = \int \frac{d^2k_\perp}{(2\pi)^3}\frac{dk^+}{2k^+}\sum_\lambda [b_\lambda(k)u(k\lambda)e^{-i(k^+\xi^- - \vec{k}_\perp\vec{\xi}_\perp)} + d_\lambda^+(k)\nu(k\lambda)e^{i(k^+\xi^- - \vec{k}_\perp\vec{\xi}_\perp)}]$$

$$|\pi^+(P)\rangle_{L_z=0} = \int \frac{d^2k_\perp}{2(2\pi)^3}\frac{dx}{\sqrt{x(1-x)}}\psi_{\uparrow\downarrow}(x,k_\perp)[b^\dagger_{u\uparrow i}(x,k_\perp)d^\dagger_{d\downarrow i}(1-x,-k_\perp) - b^\dagger_{u\downarrow i}(x,k_\perp)d^\dagger_{d\uparrow i}(1-x,-k_\perp)]|0\rangle$$

$$|\pi^+(P)\rangle_{|L_z|=1} = \int \frac{d^2k_\perp}{2(2\pi)^3}\frac{dx}{\sqrt{x(1-x)}}\psi_{\uparrow\uparrow}(x,k_\perp)[(k_1-ik_2)b^\dagger_{u\uparrow i}(x,k_\perp)d^\dagger_{d\uparrow i}(1-x,-k_\perp) + + (k_1+ik_2)b^\dagger_{u\downarrow i}(x,k_\perp)d^\dagger_{d\downarrow i}(1-x,-k_\perp)]|0\rangle$$

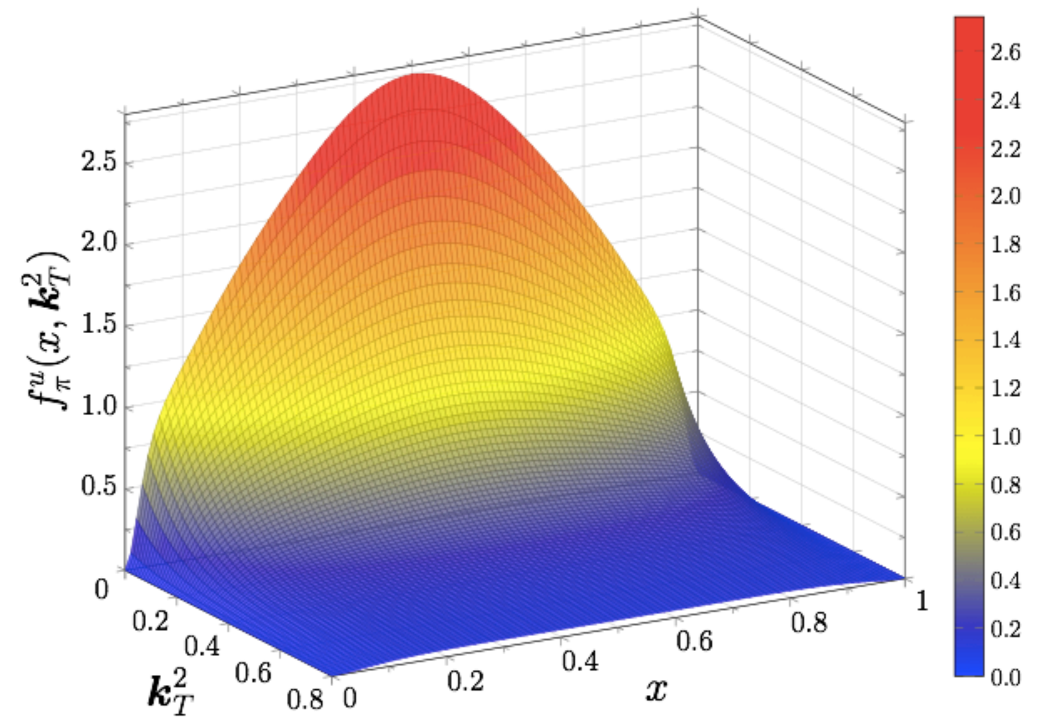
★ $$f_{1,\pi}(x,\mathbf{k}_\perp^2) = |\psi_{\uparrow\downarrow}(x,k_\perp^2)|^2 + k_\perp^2|\psi_{\uparrow\uparrow}(x,k_\perp^2)|^2$$

TMD PDFs

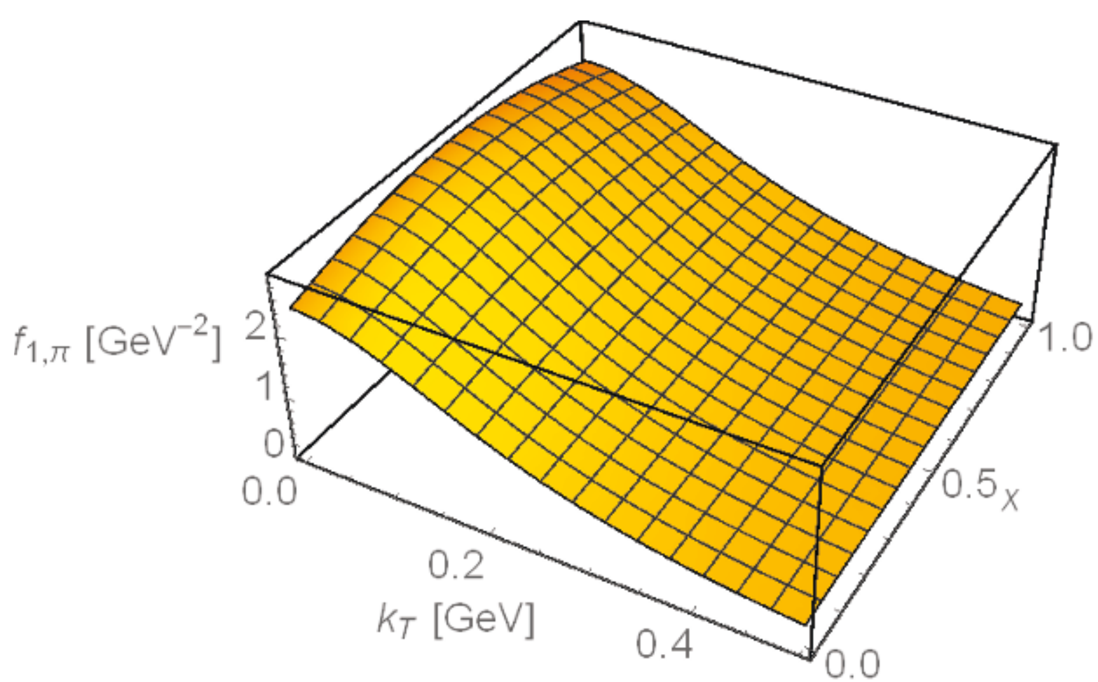
$$f_{1,\pi}(x, \mathbf{k}_\perp^2) = |\psi_{\uparrow\downarrow}(x, k_\perp^2)|^2 + k_\perp^2 |\psi_{\uparrow\uparrow}(x, k_\perp^2)|^2$$

- Significant support at low k_T. Unfactorizable x and k_T dependence.
- End point behavior ~$(1-x)^2$, following counting rule.. Stanley J. Brodsky and Feng Yuan, PRD **74,** 094018 (2006)
- Qualitatively, low k_T behavior resembles Gaussian form.
- Quantitatively, in the b-space, the exponential behavior exp(-λb) is favored as compared to Gaussian form exp(-λ^2 b) at large b.

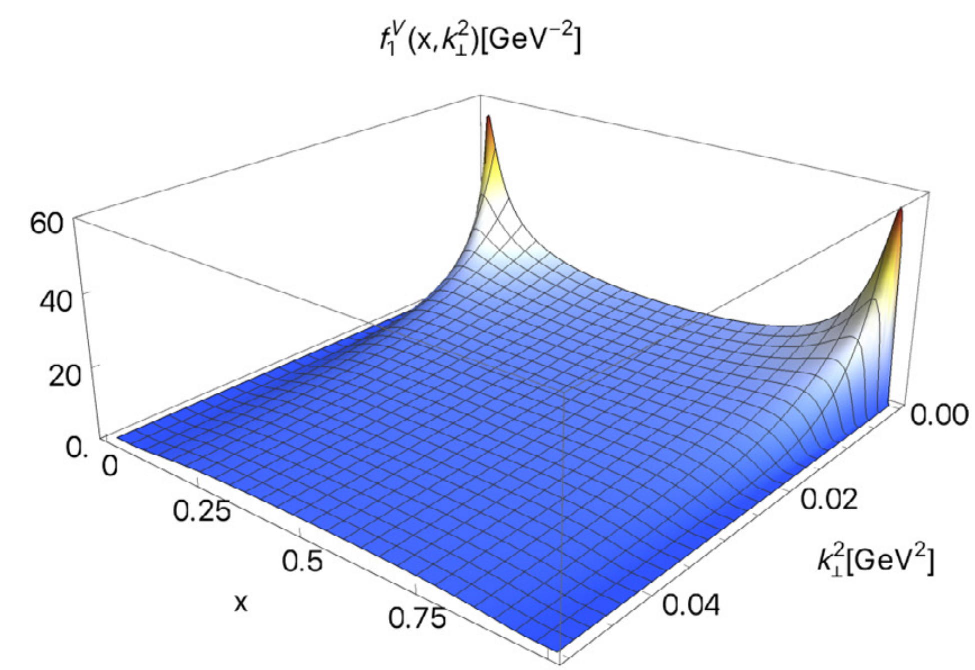
Ignazio Scimemi and Alexey Vladimirov Eur. Phys. J. C (2018) 78:89



DSEs+LF QCD

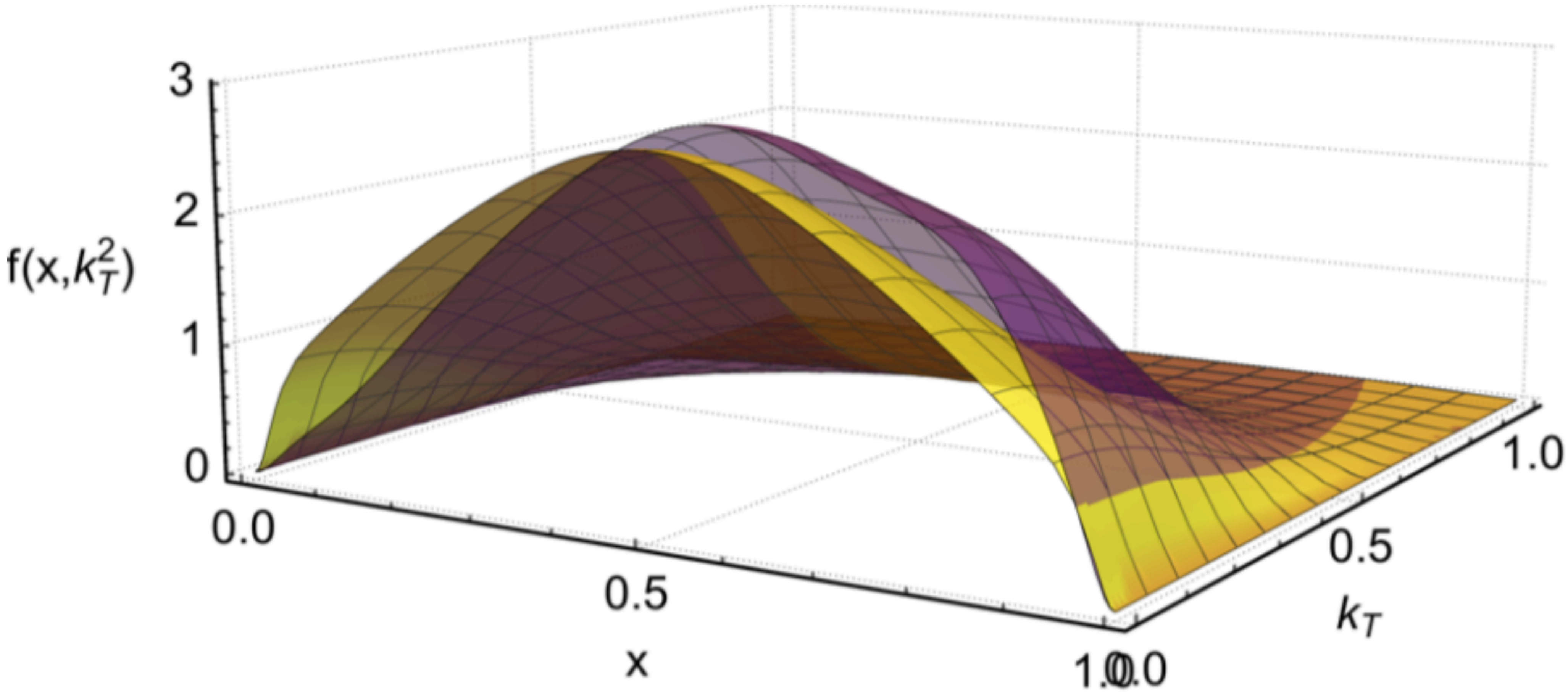


NJL model (Santiago Noguera and Sergio Scopetta, PLB2017)



Holographic QCD(Alessandro Bacchetta, Sabrina Cotogno, Barbara Pasquini, PLB2017)

Kaon TMD PDF



- Non-symmetric in x=0.5, skewed with s quark carrying more longitudinal momentum fraction.
- The width of transverse momentum increases by about 10%, m_u/m_s gets masked by DCSB effect.
- $f^u(x, k_\perp^2) = f^s(1-x, k_\perp^2)$, flavor dependence in k_T.

TMD evolution

Renormalization group (RG) equation:

$$\mu^2 \frac{d}{d\mu^2} F_{f\leftarrow h}(x, \vec{b}; \mu, \zeta) = \frac{1}{2} \gamma_F^f(\mu, \zeta) F_{f\leftarrow h}(x, \vec{b}; \mu, \zeta),$$

$$\zeta \frac{d}{d\zeta} F_{f\leftarrow h}(x, \vec{b}; \mu, \zeta) = -\mathcal{D}^f(\mu, \vec{b}) F_{f\leftarrow h}(x, \vec{b}; \mu, \zeta).$$

Anomalous Dimension

TMD PDF in the coordinate space

The scale μ is the standard RG scale, with the additional rapidity factorization scale ζ to regularize the light-cone divergence arising from Wilson lines. They were usually chosen to be the same order of scattering scale.

Solution:

$$F_{f\leftarrow h}(x, \vec{b}; \mu_f, \zeta_f) = \exp\left[\int_P \left(\gamma_F^f(\mu, \zeta) \frac{d\mu}{\mu} - \mathcal{D}^f(\mu, \vec{b}) \frac{d\zeta}{\zeta}\right)\right] F_{f\leftarrow h}(x, \vec{b}; \mu_i, \zeta_i)$$

TMD evolution:

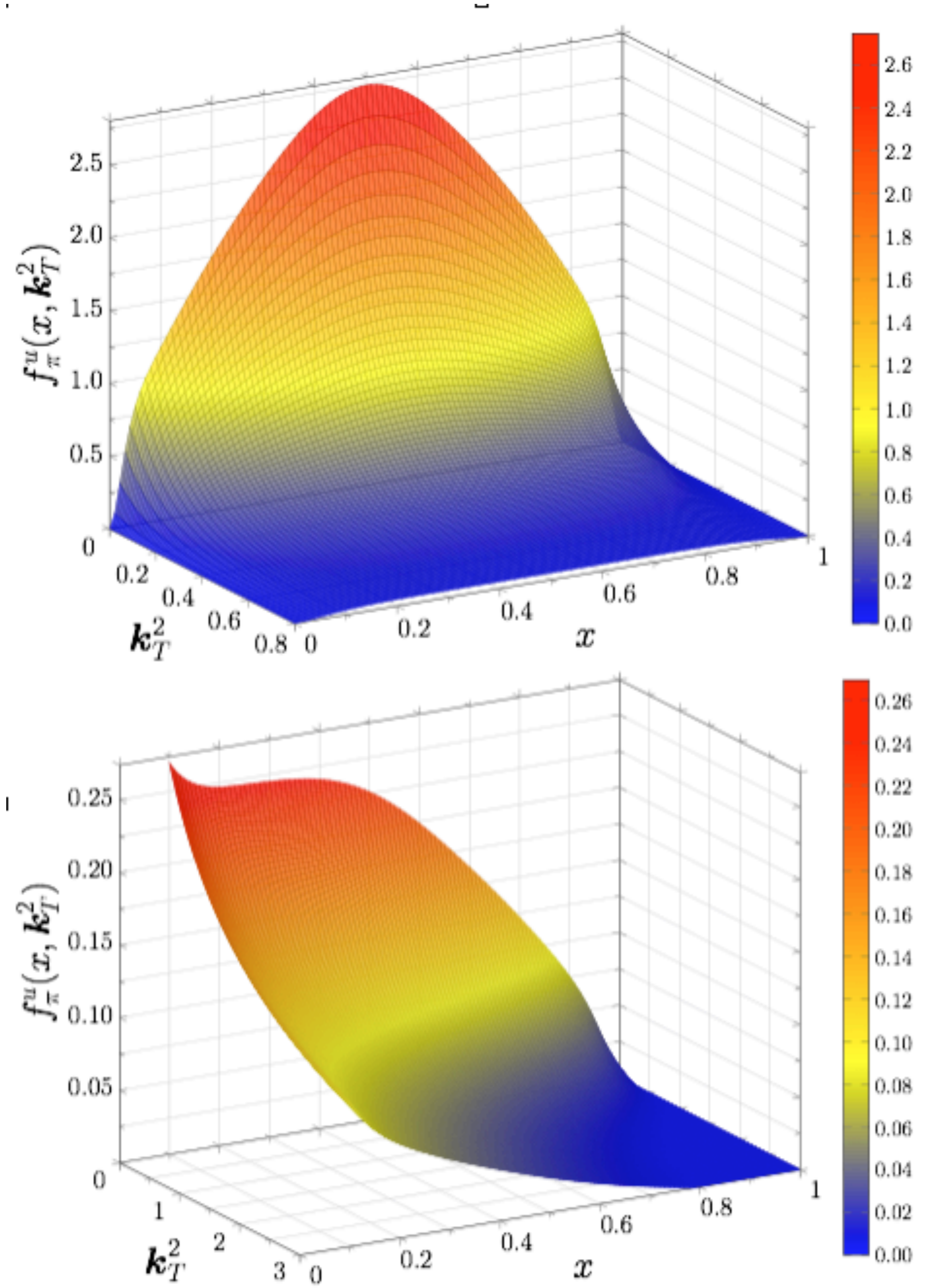
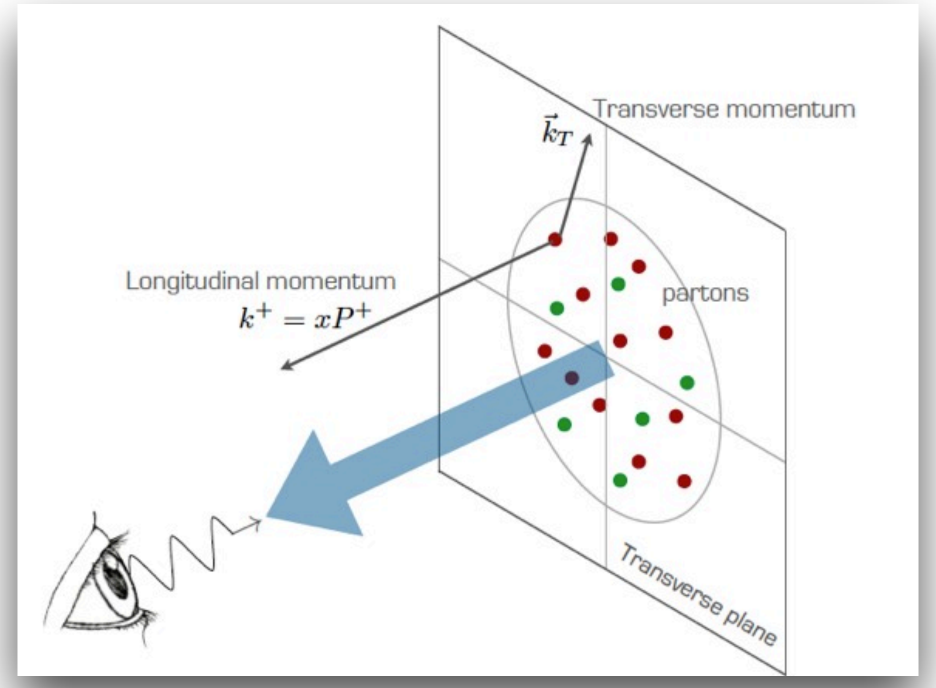


Figure 2. *Upper panel:* DSE result using the DCSB-improved kernel for the time-reversal even u-quark TMD of the pion, $f_\pi^u(x, \boldsymbol{k}_T^2)$, at the model scale of $\mu_0^2 = 0.52\,\text{GeV}^2$. *Lower panel:* Analogous result evolved to a scale of $\mu = 6\,\text{GeV}$ using TMD evolution with the b^* prescription and $g_2 = 0.09\,\text{GeV}$ [43]. The TMDs are given in units of GeV^{-2} and $\boldsymbol{k}_T^2$ in GeV^2.



- Evolution has a significant effect, leading to approximately an order of magnitude of suppression at small k_T , and broader in k_T.
- Experiment?

π-N Drell-Yan

Experiment (E615)

Transverse momentum dependence parameterized by function *P(qT;xF ,mμμ)*

$$\frac{d^3\sigma}{dx_\pi dx_N dq_T} = \frac{d^2\sigma}{dx_\pi dx_N} P(q_T; x_F, m_{\mu\mu}).$$

$$q^0 = \frac{\sqrt{s}}{2}(x_\pi + x_N)$$

$$q^3 = \frac{\sqrt{3}}{2}(x_\pi - x_N)$$

"Experimental study of muon pairs produced by 252-GeV pions on tungsten", Conway, J.S. et al. Phys.Rev. D39 (1989) 92-122.

Theory

$$\frac{d^3\sigma}{dx_\pi dx_N dq_T} \propto |q_T| F^1_{uu}(x_\pi, x_N, q_T)$$

(leading twist)

TMD formalism: $F^1_{UU}(x_1, x_2, q_T) = \frac{1}{N_c}\sum_a e_a^2 \int d^2\boldsymbol{k}_{1\perp} d^2\boldsymbol{k}_{2\perp} \delta^{(2)}(\boldsymbol{q}_T - \boldsymbol{k}_{1\perp} - \boldsymbol{k}_{2\perp}) f^{\bar{a}}_{1,\pi}(x_1, \boldsymbol{k}^2_{1\perp}) f^a_{1,N}(x_2, \boldsymbol{k}^2_{2\perp})$

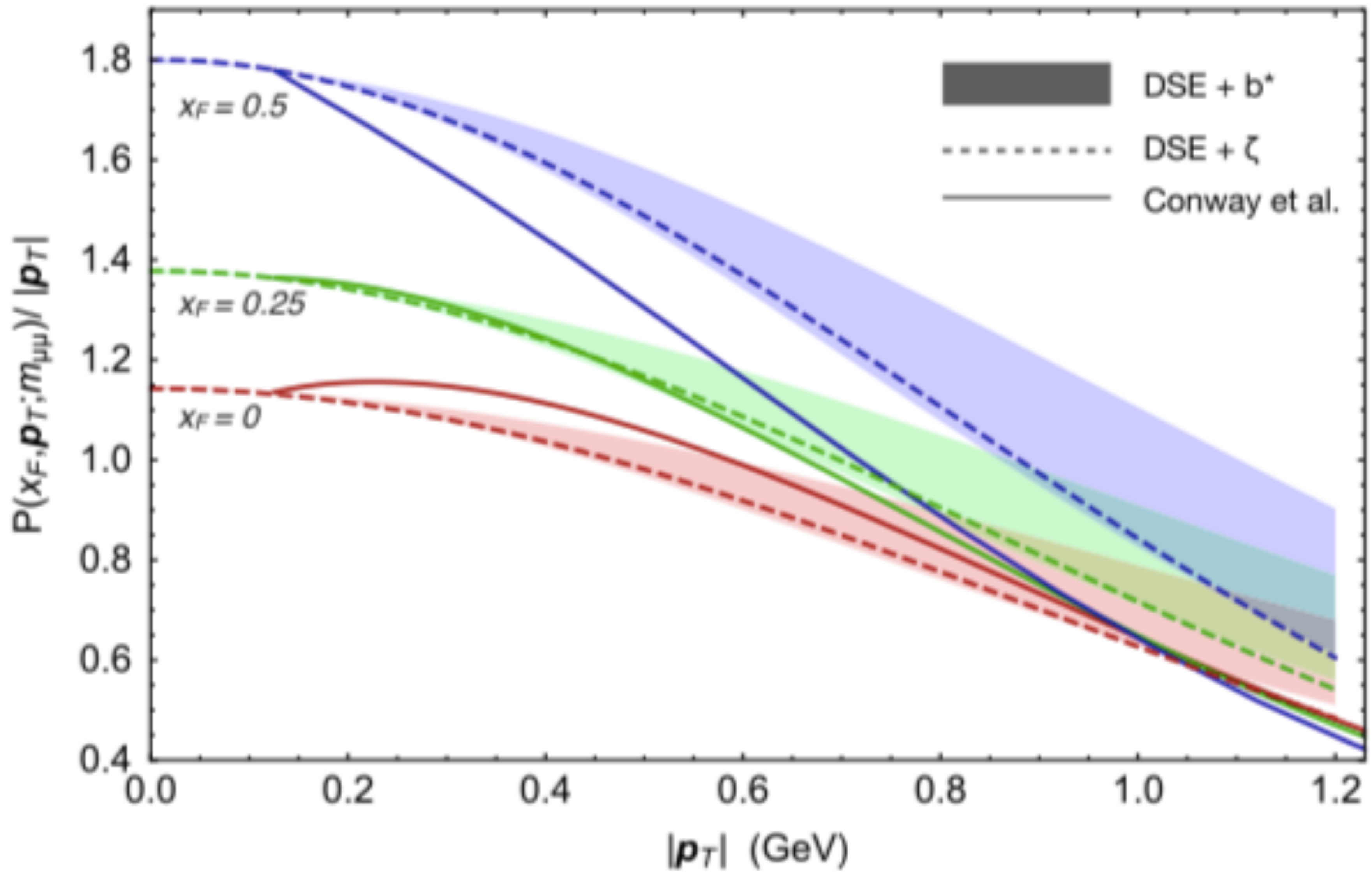
offered by DSEs&evolution

borrow from global fits

Alessandro Bacchetta, Filippo Delcarro, et al, JHEP06(2017)081
Ignazio Scimemi and Alexey Vladimirov Eur. Phys. J. C (2018) 78:89

Examine: $P(q_T; x_F, m_{\mu\mu}) \propto |q_T| F^1_{UU}(q_T; x_F, \tau)$

E615:



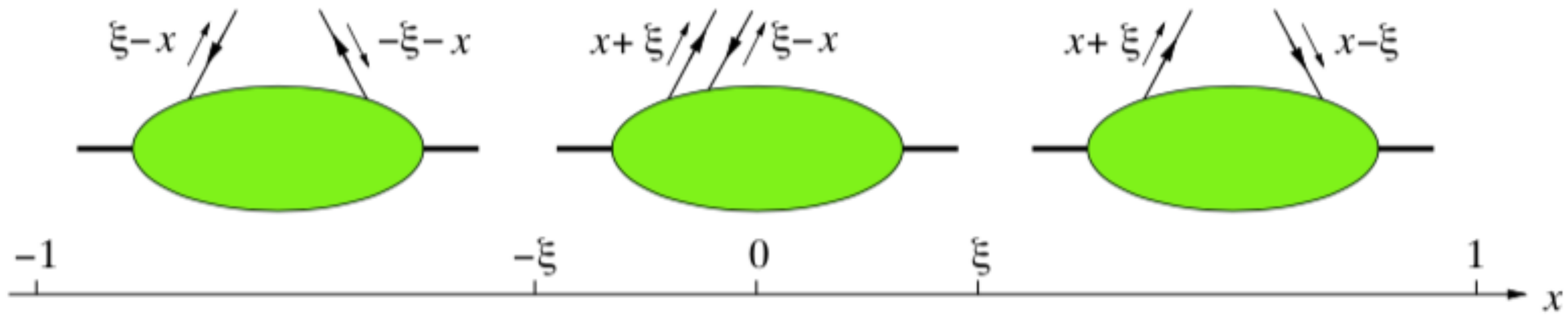
- Our results using two evolution schemes generally agree with E615 measurement. In particular, when g2 goes to zero as suggested by ζ-prescription at higher order.
- The deviation is less than 10%for x_F =0 and 0.25, and increases to 30% for $x_F = 0.5$. Higher Fock state effects, higher twist effect, or both?

Pion GPD

- Quark GPD of pion at leading twist

$$H_\pi^q(x,\xi,t) = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p_2|\bar{\psi}^q(-\frac{z}{2})\gamma^+\psi^q(\frac{z}{2})|p_1\rangle|_{z^+=z_\perp=0}$$

- There are two regions, ERBL and DGLAP region, named after their evolution in limiting cases



- In the DGLAP region (1>=|x|>=|ξ|):

$$H^u_{\pi^+}(x,\xi,t)\Big|_{\xi\le x} = \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}\left[\Psi^*_{l=0}\left(\frac{x-\xi}{1-\xi},\hat{\mathbf{k}}_\perp\right)\Psi_{l=0}\left(\frac{x+\xi}{1+\xi},\tilde{\mathbf{k}}_\perp\right) + \hat{\mathbf{k}}_\perp\cdot\tilde{\mathbf{k}}_\perp\,\Psi^*_{l=1}\left(\frac{x-\xi}{1-\xi},\hat{\mathbf{k}}_\perp\right)\Psi_{l=1}\left(\frac{x+\xi}{1+\xi},\tilde{\mathbf{k}}_\perp\right)\right]$$

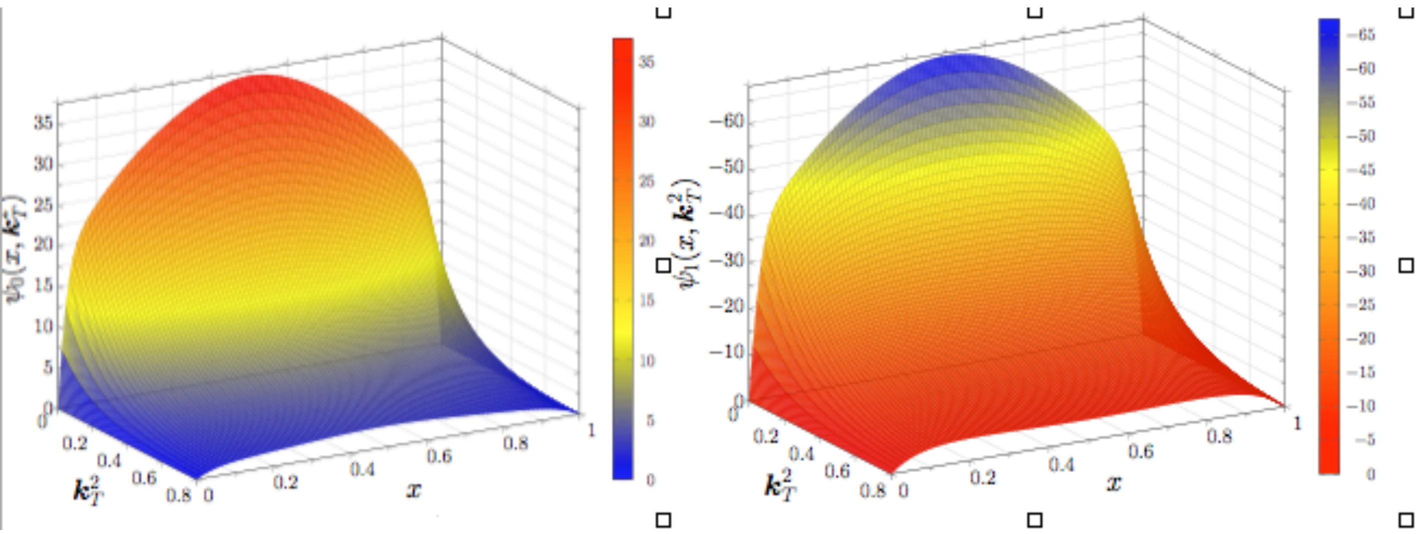
- Impact Parameter dependent GPD:

$$q(x,b_T) = \int \frac{d^2\Delta_T}{(2\pi)^2} e^{-i\Delta_T\cdot b_T} H(x,0,-\Delta_T^2)$$

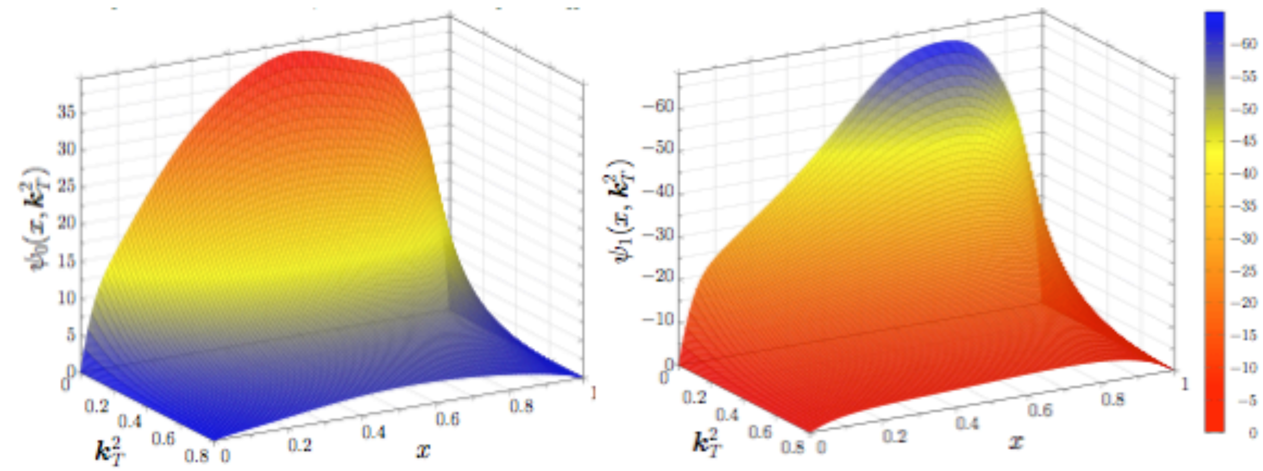
x: longitudinal momentum fraction carried by quark
b_T: transverse separation between the parton and hadron's center of transverse momentum.

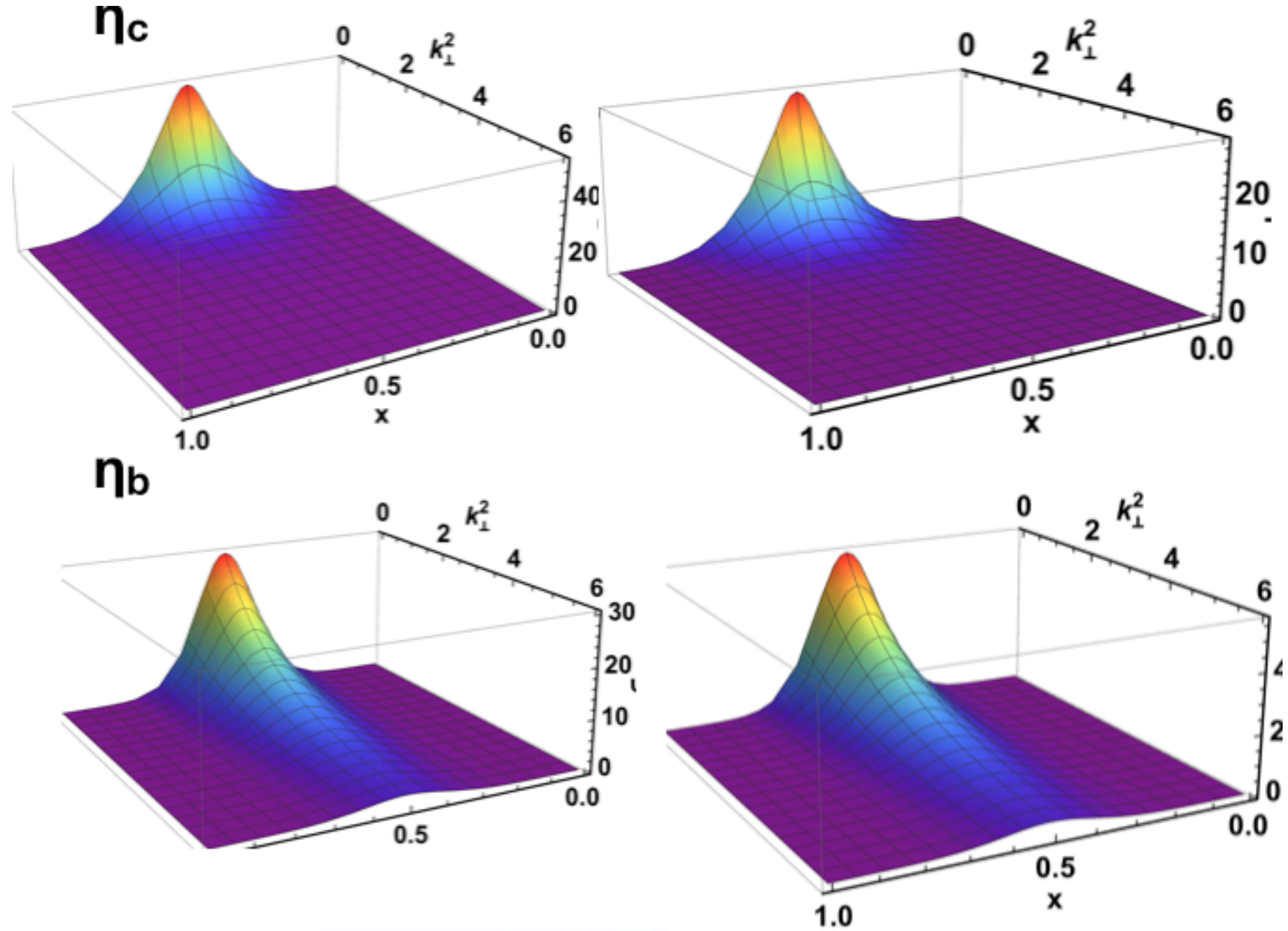
LFWFs : $\psi_0(x, k_\perp^2)$ & $\psi_1(x, k_\perp^2)$ (pseudo-scalar)

Pion

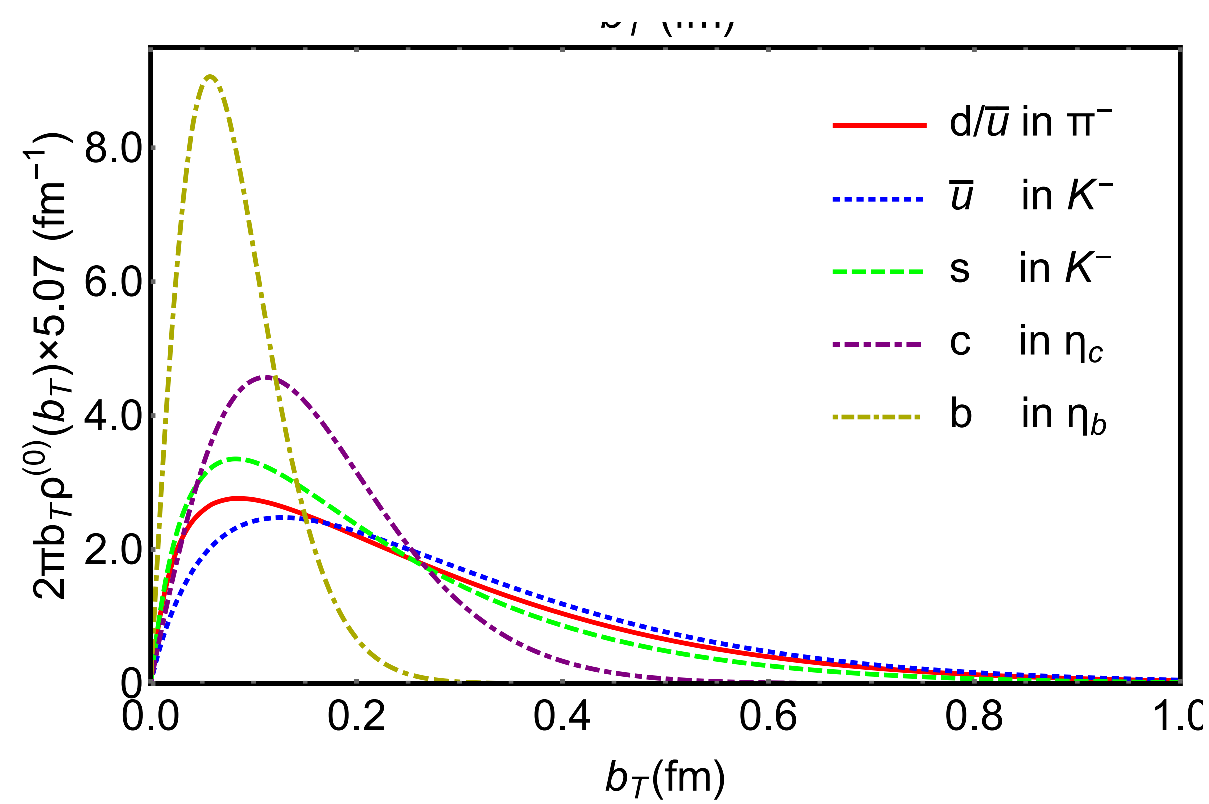


Kaon





Density distribution $\int_0^1 dx b_T \rho(x, b_T)$

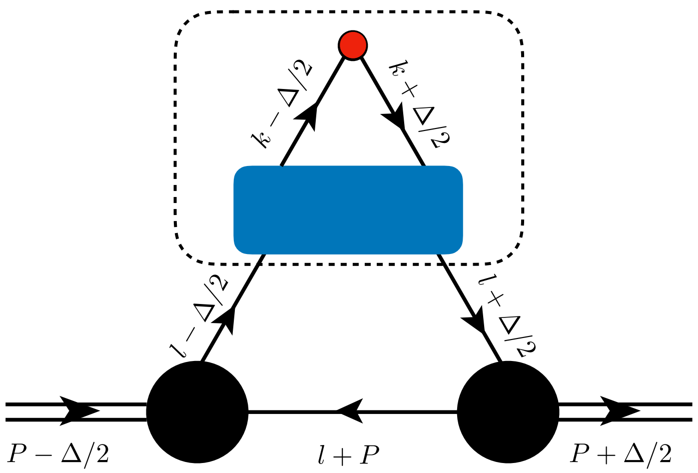


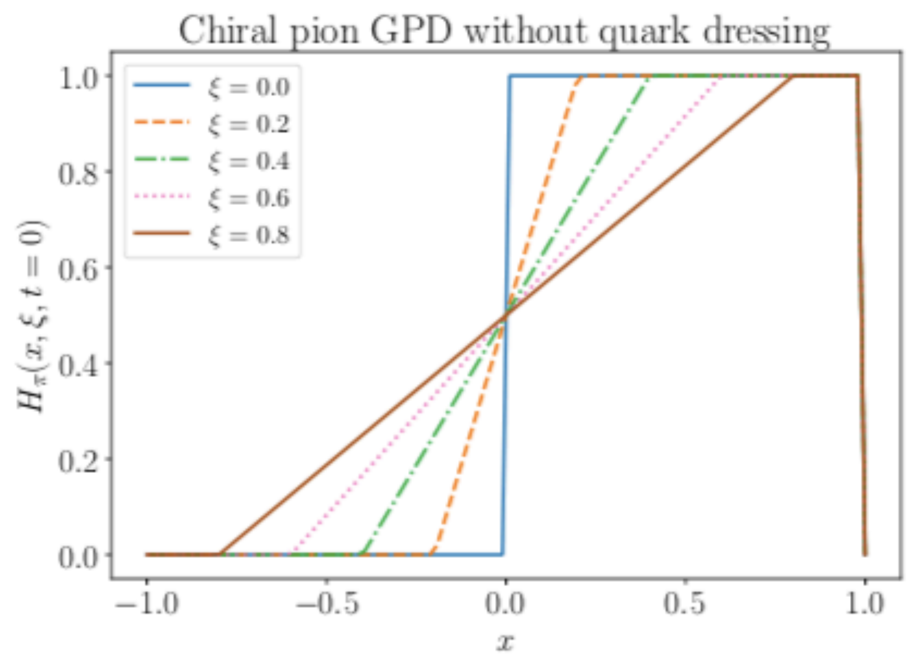
GPD in ERBL region (-ξ<=x<=ξ)

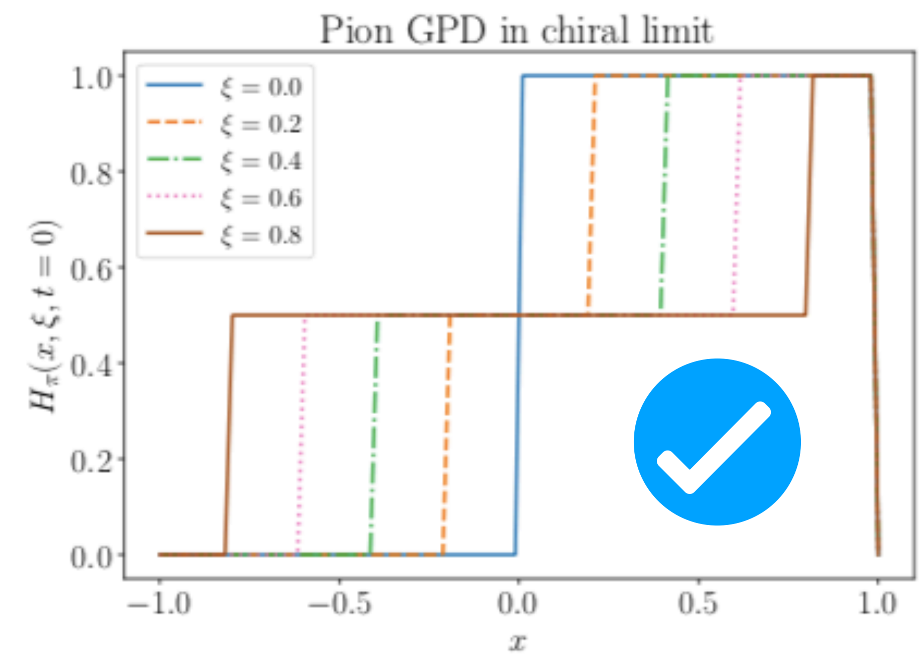
- Electro-Magnetic form factor

$$F(t) = \int_{-1}^{1} dx H(x, \xi, t)$$

- NJL calculation (Adam Freese et al) shows the dressing of the vertex insertion is crucial for ERBL region.

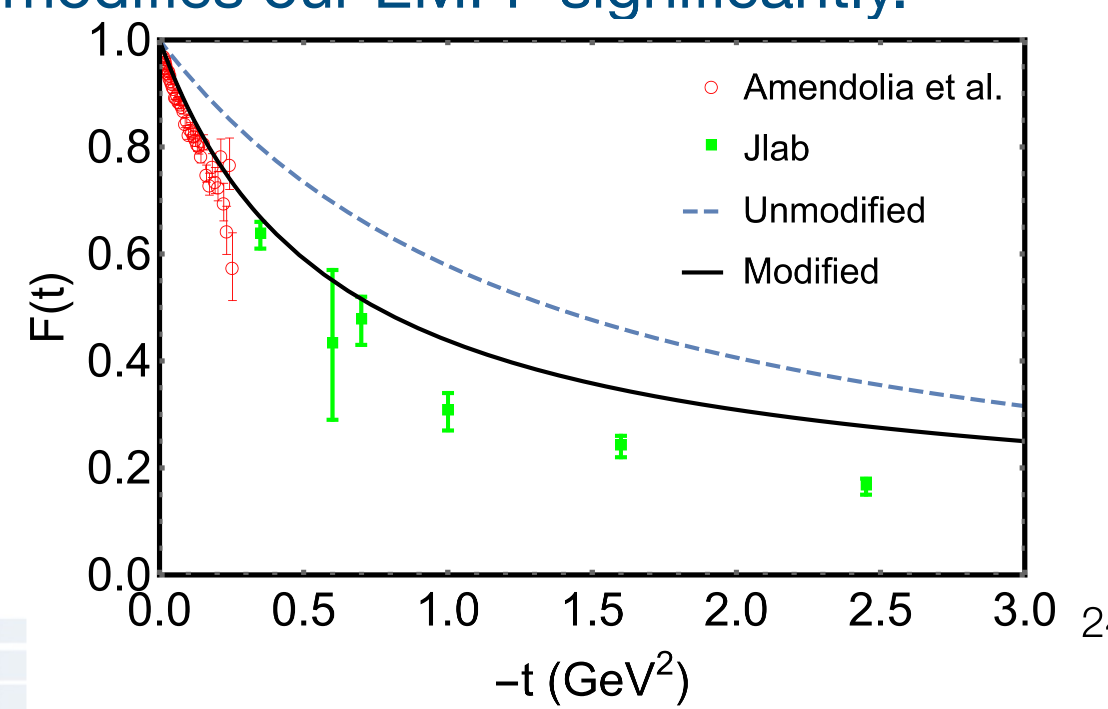






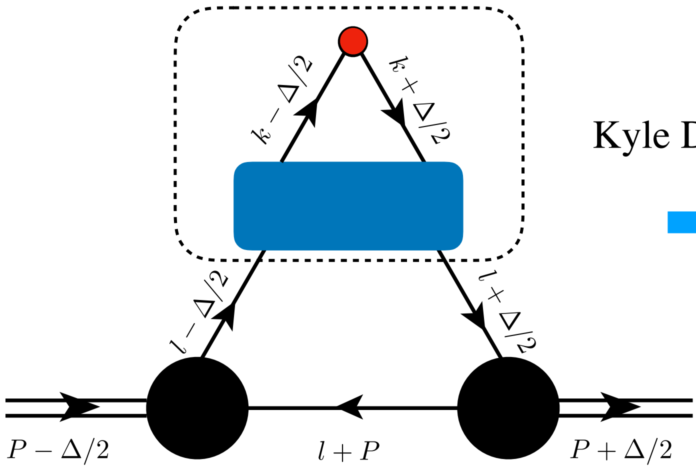
- In the absence of gluons, the NJL model finds a "hidden" ERBL region contribution at even zero skewness, proportional to $\delta(x)$, which modifies our EMFF significantly.

- Conclusion
  - *The LF approach is convenient in sketching DGLAP region GPD (including the TMD).*
  - *To reveal GPD in the ERBL region, the covariant approach is needed. (Higher Fock states implicitly involved)*

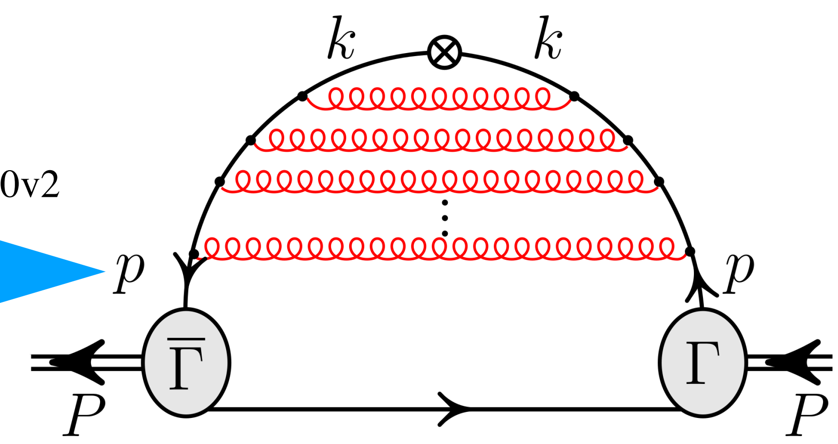


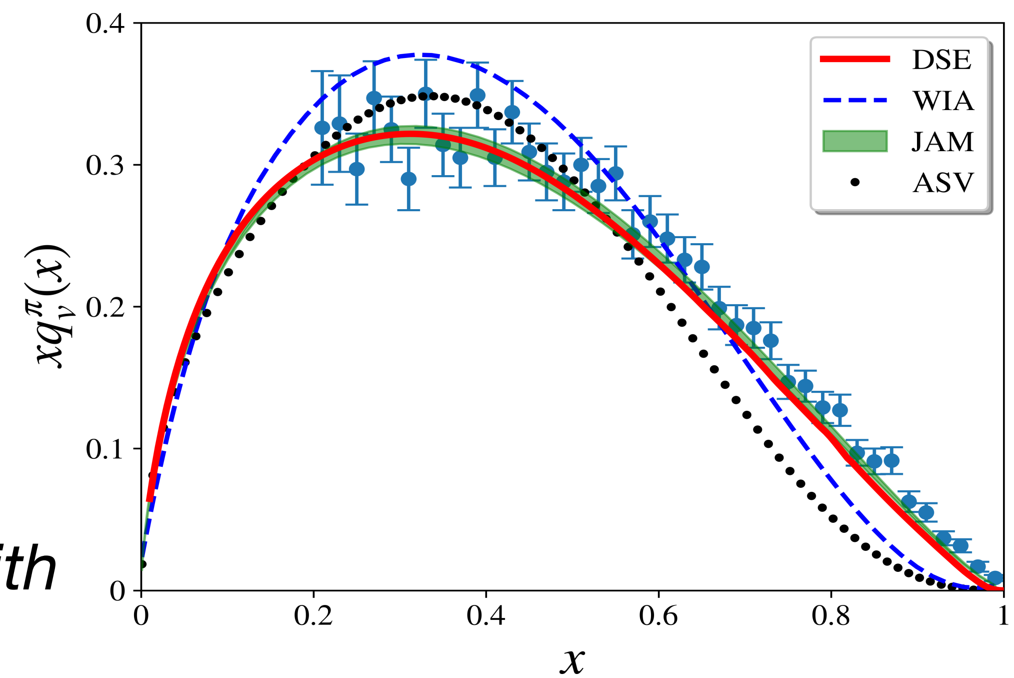
Covariant approach and higher Fock states

- Covariant approach: Rainbow-Ladder DSE calculation of pion PDF



Kyle D. Bednar, Ian C. Cloët, and Peter C. Tandy, arXiv:1811.12310v2

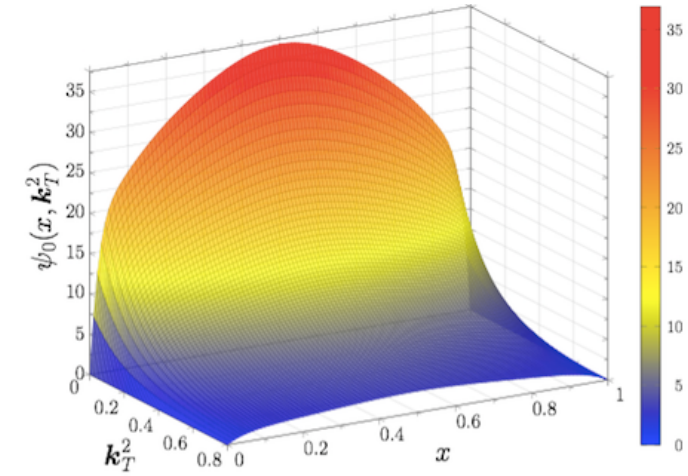


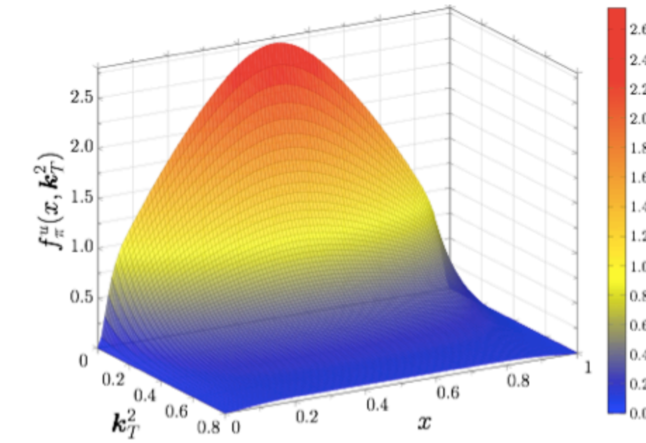


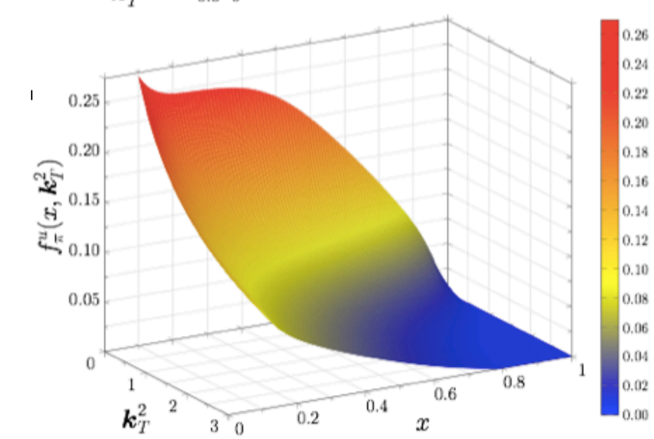
- *Within the DSEs, For pion, gluon carries around 30% momentum at hadron scale.*
- *For DSEs with gluon d.o.f, higher Fock components with gluons are involved in the covariant approach.*
- *This allows better precision by going to a larger (safer) initial evolution scale, e.g., 500 MeV ---> 800 MeV.*

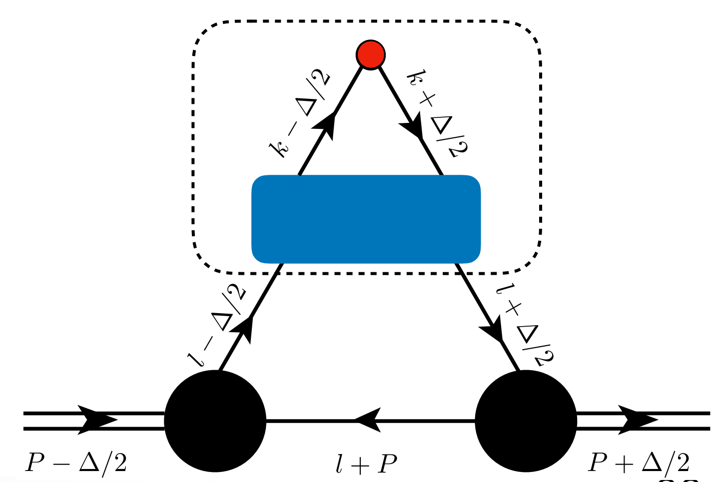
Conclusions and Outlook

- DSEs, starting with the quark and gluon degrees of freedom, provide a fully covariant solution to a variety of hadron problems with very few parameters.
- The Bethe-Salpeter wave functions can be projected on to the light front, which provides a unique chance to calculate the light front wave functions.
- The pion unpolarized TMD PDF calculated from DSE+LF is in good agreement with experiment data within the TMD formalism.
- The GPD in the DGLAP region can be studied using LF approach, but ERBL region is lacking. Covariant approach is necessary.
- TMD and GPD calculation could be refined with the covariant approach within DSEs, by incorporating many higher Fock states and pushing to a larger and safer initial evolution scale.
- Nucleon is readily to be studied.










Thank
You