Quark and gluon contributions to the QCD trace anomaly

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Outline

- Trace anomaly in QCD
- Decomposing the trace anomaly
- Applications

Refs.
YH, Abha Rajan, Kazuhiro Tanaka, JHEP 1812 (2018) 008
YH, Di-Lun Yang, PRD98 (2018) 074003
Tanaka, JHEP 1901 (2019) 120

Quantum Chromodynamics

$$L = -\frac{1}{4} F_a^{\mu\nu} F_{\mu\nu}^a + \overline{q}(i\gamma^\mu D_\mu - m)q$$

Theory of the strong interaction.
Non-abelian gauge theory based on color SU(3)
Approximate conformal symmetry at the classical level.

Energy momentum tensor (EMT)

$$T^{\mu\nu} = -F^{\mu\lambda} F^{\nu}{}_{\lambda} + \frac{\eta^{\mu\nu}}{4} F^2 + i\bar{q}\gamma^{(\mu} D^{\nu)} q$$

Vanishing trace in the chiral limit

$$T_\mu^\mu = m\bar{q}q$$

The trace anomaly

Conformal symmetry explicitly broken by the quantum effects.

$$T^{\mu}_{\mu} = \frac{\beta(g)}{2g} F^2 + m(1 + \gamma_m(g))\bar{q}q$$

Collins, Duncan, Joglekar (1977)
N.K. Nielsen (1977)

Fundamentally important in QCD. Trace anomaly is the origin of hadron masses

$$\langle P | T^{\mu\nu} | P \rangle = 2P^{\mu} P^{\nu}$$

$$\langle P | T^{\mu}_{\mu} | P \rangle = 2M^2$$

NAS report
07/24/2018

An Assessment of U.S.-Based Electron-Ion Collider Science

Committee on U.S.-Based Electron-Ion Collider Science Assessment

Board on Physics and Astronomy

Division on Engineering and Physical Sciences

A Consensus Study Report of

The National Academies of
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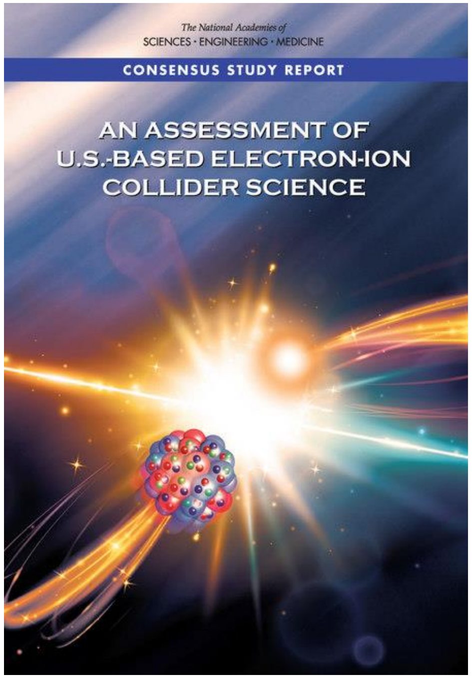


Finding 1: An EIC can uniquely address three profound questions about nucleons-protons—and how they are assembled to form the nuclei of atoms:

- How does the mass of the nucleon arise?
- How does the spin of the nucleon arise?
- What are the emergent properties of dense systems of gluons?

A naïve question

triggered by correspondence from Maxim Polyakov

The EMT consists of quark and gluon parts.

$$T^{\mu\nu} = \underbrace{-F^{\mu\lambda}F^{\nu}{}_{\lambda} + \frac{\eta^{\mu\nu}}{4}F^2}_{T_g^{\mu\nu}} + \underbrace{i\bar{q}\gamma^{(\mu}D^{\nu)}q}_{T_q^{\mu\nu}}$$

$$T^{\mu}_{\mu} = (T_q)^{\mu}_{\mu} + (T_g)^{\mu}_{\mu} = \frac{\beta}{2g}F^2 + m(1+\gamma_m)\bar{\psi}\psi$$

Can we compute $(T_q)^{\mu}_{\mu}$ and $(T_g)^{\mu}_{\mu}$ separately?

Is this a well-defined problem?
Maybe yes, but why do we care?

Nucleon gravitational form factors

The QCD energy momentum tensor contains rich information about the structure of the nucleon.

Nowadays, it is a standard practice to discuss the quark and gluon parts separately
→ partonic decomposition of the nucleon mass and spin.

most general parametrization

$$\langle P'|T^{\mu\nu}_{q,g}|P\rangle = \bar{u}(P')\Big[A_{q,g}\gamma^{(\mu}\bar{P}^{\nu)} + B_{q,g}\frac{\bar{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M} + D_{q,g}\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{4M} + \bar{C}_{q,g}M\eta^{\mu\nu}\Big]u(P)$$

$$\Delta^\mu = P'^\mu - P^\mu$$

All the form factors should be renormalized $A_{q,g}(\Delta^2) \to A^R_{q,g}(\Delta^2,\mu)$, etc.

All the form factors are interesting.

Physics of $\bar{C}_{q,g}$?

Non-conservation of the quark/gluon parts

$$\langle \partial_\mu T^{\mu\nu}_{q,g} \rangle \sim \Delta^\nu \bar{C}_{q,g}$$

$\bar{C}_q + \bar{C}_g = 0$ because the total EMT is conserved.

Related to the quark and gluon parts of the trace anomaly

$$\langle P | (T_{q,g})^\mu_\mu | P \rangle = 2M^2 (A_{q,g} + 4\bar{C}_{q,g})$$

Poorly understood, but potentially very interesting in connection with the nucleon mass problem!

Renormalization of $\bar{C}_{q,g}$: a first look

Polyakov, Son, arXiv.1808.00155

Because of the relation $\langle \partial_\mu T^{\mu\nu}_{q,g} \rangle \sim \Delta^\nu \bar{C}_{q,g}$, the scale dependence of $\bar{C}_{q,g}$ is governed by the second anomalous dimension of the energy momentum tensor.

$$\frac{\partial}{\partial \ln \mu} \bar{C}^R_q = -\frac{\alpha_s}{4\pi} \left(\frac{16}{3} C_F + \frac{4n_f}{3} \right) \bar{C}^R_q + \mathcal{O}(m) + \mathcal{O}(\alpha_s^2)$$

This implies (in the chiral limit) $\bar{C}^R_{q,g}(\mu \to \infty) \to 0$??

Derivation of trace anomaly: a reminder

First choose a regularization scheme.
e.g., dimensional regularization (DR), Pauli-Villars, etc.

Trace anomaly shows up by exploiting the pathologies of the chosen scheme.

→ The decomposition $T^\mu_\mu = (T_g)^\mu_\mu + (T_q)^\mu_\mu$ is scheme dependent.

In the following, I consider only DR.

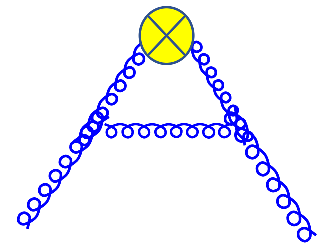
In $4-2\epsilon$ dimensions,

$$T^\mu_\mu = -2\epsilon \frac{F^2}{4} + m\bar{q}q$$

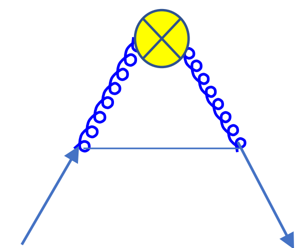
from $(T_g)^\mu_\mu$ from $(T_q)^\mu_\mu$

Operator renormalization and mixing

The bare operator F^2 is divergent   c.f. Tarrach (1982)



$$F^2 = \left(1 + \beta_0 \frac{\alpha_s}{4\pi\epsilon}\right) (F^2)_R - \frac{2\gamma_m^0}{\epsilon} (m\bar{q}q)_R + \cdots$$



$$T_\mu^\mu = -2\epsilon \frac{F^2}{4} + m\bar{q}q$$
$$= \frac{\beta(g)}{2g} (F^2)_R + \gamma_m (m\bar{q}q)_R + (m\bar{q}q)_R$$

from $(T_g)_\mu^\mu$ from $(T_q)_\mu^\mu$

For the bare EMT, in DR, the anomaly entirely comes from the gluon part $T_g^{\mu\nu}$

Renormalized trace $(T^R_{q,g})^\alpha_\alpha$: my three mistakes (1/3)

RG-invariant in DR

For the bare operators,

$$(T_q)^\mu_\mu = (m\bar{q}q)_R = m\bar{q}q$$

$$(T_g)^\mu_\mu = \frac{\beta}{2g}(F^2)_R + \gamma_m (m\bar{q}q)_m = \frac{\beta}{2g}F^2 + \gamma_m m\bar{q}q$$

Does this mean

$$(T^R_q(\mu))^\alpha_\alpha = (T_q)^\alpha_\alpha$$

$$(T^R_g(\mu))^\alpha_\alpha = (T_g)^\alpha_\alpha$$

??

Renormalized trace $(T^R_{q,g})^\alpha_\alpha$: my three mistakes (2/3)

By definition, $\langle P|(T^R_{q,g})^\mu_\mu|P\rangle = 2M^2(A^R_{q,g} + 4\bar{C}^R_{q,g})$

Consider the $\mu \longrightarrow \infty$ limit of this.

Well-known asymptotic formulas: $A^R_q(\infty) = \frac{3n_f}{16+3n_f}$ $\quad A^R_g(\infty) = \frac{16}{16+3n_f}$

According to Polyakov, Son (2018), $\bar{C}_{q,g}(\infty) = 0$ in the chiral limit

Therefore, in the chiral limit,

$$(T_q)^\alpha_\alpha = (T^R_q(\infty))^\alpha_\alpha \sim \frac{3n_f}{16+3n_f}$$

$$(T_g)^\alpha_\alpha = (T^R_g(\infty))^\alpha_\alpha \sim \frac{16}{16+3n_f}$$

??

Renormalized trace $(T^R_{q,g})^\alpha_\alpha$: my three mistakes (3/3)

Repeat the derivation of trace anomaly, but now with the renormalized operators

$$T^{\mu\nu}_{gR} = -(F^{\mu\lambda}F^{\nu}{}_{\lambda})_R + \frac{\eta^{\mu\nu}}{4}(F^2)_R$$

$$T^{\mu\nu}_{qR} = i(\bar{\psi}\gamma^\mu D^\nu \psi)_R$$

$(F^2)_R$ is now a finite operator. $d \to 4$ limit can be safely taken

$$(T^R_g)^\mu_\mu = 0$$

$$(T^R_q)^\mu_\mu = (m\bar{\psi}\psi)_R$$

??

Renormalized trace $(T^R_{q,g})^\alpha_\alpha$: the correct result

Choose the basis of operators

$$
\begin{aligned}
O_1 &= -F^{\mu\lambda}F^{\nu}{}_{\lambda}, \\
O_2 &= \eta^{\mu\nu}F^2, \\
O_3 &= i\bar{\psi}\gamma^{(\mu}\overleftrightarrow{D}^{\nu)}\psi, \\
O_4 &= \eta^{\mu\nu}m\bar{\psi}\psi.
\end{aligned}
$$

$$T^{\mu\nu} = O_1 + \frac{O_2}{4} + O_3.$$

Mixing under renormalization

$$
\begin{aligned}
O_1^R &= Z_T O_1 + Z_M O_2 + Z_L O_3 + Z_S O_4, \\
O_2^R &= Z_F O_2 + Z_C O_4, \\
O_3^R &= Z_\psi O_3 + Z_K O_4 + Z_Q O_1 + Z_B O_2, \\
O_4^R &= O_4.
\end{aligned}
$$

Impose two conditions. First condition is simply $T^{\mu\nu} = T^{\mu\nu}_R$

Second condition:

$O_1^R - (\text{trace})$ and $O_3^R - (\text{trace})$ satisfy the usual twist-2, spin-2, RG equation

$$\frac{\partial}{\partial \ln \mu}\begin{pmatrix} A_q^R \\ A_g^R \end{pmatrix} = \frac{\alpha_s}{4\pi}\begin{pmatrix} -\frac{16}{3}C_F & \frac{4n_f}{3} \\ \frac{16}{3}C_F & -\frac{4n_f}{3} \end{pmatrix}\begin{pmatrix} A_q^R \\ A_g^R \end{pmatrix},$$

cf. Peskin's Eq.(18.186)

BEWARE, in DR, trace operation and renormalization do not commute

$$\eta_{\mu\nu}(F^{\mu\lambda}F^{\nu}{}_{\lambda})_R \neq (F^{\mu\lambda}F_{\mu\lambda})_R \qquad \eta_{\mu\nu}(\bar{\psi}\gamma^{\mu}D^{\nu}\psi)_R \neq (\bar{\psi}\not{D}\psi)_R$$

Introduce more unknown constants

$$\eta_{\mu\nu}(F^{\mu\lambda}F^{\nu}{}_{\lambda})_R = x(F^2)_R + y(m\bar{\psi}\psi)_R$$

Result in $\overline{\mathrm{MS}}$ at one-loop

YH, Rajan, Tanaka, JHEP 1812 (2018) 008

$$\eta_{\mu\nu} T^{\mu\nu}_{gR} = \frac{\alpha_s}{4\pi}\left(-\frac{11C_A}{6}(F^2)_R + \frac{14C_F}{3}(m\bar{\psi}\psi)_R\right),$$

$$\eta_{\mu\nu} T^{\mu\nu}_{qR} = (m\bar{\psi}\psi)_R + \frac{\alpha_s}{4\pi}\left(\frac{n_f}{3}(F^2)_R + \frac{4C_F}{3}(m\bar{\psi}\psi)_R\right)$$

n_f term in the 1-loop beta function

$$\lim_{\mu\to\infty}(T_q^R(\mu))^\alpha_\alpha \neq (T_q)^\alpha_\alpha$$

$$\lim_{\mu\to\infty}(T_g^R(\mu))^\alpha_\alpha \neq (T_g)^\alpha_\alpha$$

Finite renormalization

Result in $\overline{\mathrm{MS}}$ at two-loops

YH, Rajan, Tanaka, JHEP 1812 (2018) 008

$$\eta_{\mu\nu}\left(T_g^{\mu\nu}\right)_R = \frac{\alpha_s}{4\pi}\left(\frac{14}{3}C_F\left(m\bar{\psi}\psi\right)_R - \frac{11}{6}C_A\left(F^2\right)_R\right) + \left(\frac{\alpha_s}{4\pi}\right)^2 \times\left[\left(C_F\left(\frac{812C_A}{27} - \frac{22n_f}{27}\right) + \frac{85C_F^2}{27}\right)\left(m\bar{\psi}\psi\right)_R + \left(\frac{28C_An_f}{27} - \frac{17C_A^2}{3} + \frac{5C_Fn_f}{54}\right)\left(F^2\right)_R\right]$$

n_f terms contributes to the gluon part

$$\eta_{\mu\nu}\left(T_q^{\mu\nu}\right)_R = \left(m\bar{\psi}\psi\right)_R + \frac{\alpha_s}{4\pi}\left(\frac{4}{3}C_F\left(m\bar{\psi}\psi\right)_R + \frac{1}{3}n_f\left(F^2\right)_R\right) + \left(\frac{\alpha_s}{4\pi}\right)^2 \times\left[\left(C_F\left(\frac{61C_A}{27} - \frac{68n_f}{27}\right) - \frac{4C_F^2}{27}\right)\left(m\bar{\psi}\psi\right)_R + \left(\frac{17C_An_f}{27} + \frac{49C_Fn_f}{54}\right)\left(F^2\right)_R\right]$$

Result in $\overline{\mathrm{MS}}$ at three-loops

Tanaka, JHEP 1901 (2019) 120

$$\eta_{\mu\nu}\left(T_g^{\mu\nu}\right)_R = \frac{\alpha_s}{4\pi}\left(\frac{14}{3}C_F\left(m\bar{\psi}\psi\right)_R - \frac{11}{6}C_A\left(F^2\right)_R\right) + \left(\frac{\alpha_s}{4\pi}\right)^2$$
$$\times\left[\left(C_F\left(\frac{812C_A}{27} - \frac{22n_f}{27}\right) + \frac{85C_F^2}{27}\right)\left(m\bar{\psi}\psi\right)_R + \left(\frac{28C_An_f}{27} - \frac{17C_A^2}{3} + \frac{5C_Fn_f}{54}\right)\left(F^2\right)_R\right]$$
$$+\left(\frac{\alpha_s}{4\pi}\right)^3\left[\left\{n_f\left(\left(\frac{368\zeta(3)}{9} - \frac{25229}{729}\right)C_F^2 - \frac{2}{243}(4968\zeta(3)+1423)C_AC_F\right)\right.\right.$$
$$+\left(\frac{32\zeta(3)}{3} - \frac{91753}{1458}\right)C_AC_F^2 + \left(\frac{294929}{1458} - \frac{32\zeta(3)}{9}\right)C_A^2C_F - \frac{554}{243}C_Fn_f^2$$
$$\left.+\left(\frac{95041}{729} - \frac{64\zeta(3)}{9}\right)C_F^3\right\}\left(m\bar{\psi}\psi\right)_R$$
$$\left.+\left\{n_f\left(\left(\frac{1123}{162} - \frac{52\zeta(3)}{9}\right)C_AC_F + \left(4\zeta(3) + \frac{293}{36}\right)C_A^2 + \frac{16}{729}(81\zeta(3)-98)C_F^2\right) + n_f^2\left(\frac{655C_A}{2916} - \frac{361C_F}{729}\right) - \frac{2857C_A^3}{108}\right\}\left(F^2\right)_R\right]$$

$$\eta_{\mu\nu}\left(T_q^{\mu\nu}\right)_R = \left(m\bar{\psi}\psi\right)_R + \frac{\alpha_s}{4\pi}\left(\frac{4}{3}C_F\left(m\bar{\psi}\psi\right)_R + \frac{1}{3}n_f\left(F^2\right)_R\right) + \left(\frac{\alpha_s}{4\pi}\right)^2$$
$$\times\left[\left(C_F\left(\frac{61C_A}{27} - \frac{68n_f}{27}\right) - \frac{4C_F^2}{27}\right)\left(m\bar{\psi}\psi\right)_R + \left(\frac{17C_An_f}{27} + \frac{49C_Fn_f}{54}\right)\left(F^2\right)_R\right]$$
$$+\left(\frac{\alpha_s}{4\pi}\right)^3\left[\left\{n_f\left(\left(\frac{64\zeta(3)}{9} - \frac{8305}{729}\right)C_F^2 - \frac{2}{243}(864\zeta(3)+1079)C_AC_F\right)\right.\right.$$
$$-\frac{8}{729}(972\zeta(3)+143)C_AC_F^2 + \left(\frac{32\zeta(3)}{9} + \frac{6611}{729}\right)C_A^2C_F - \frac{76}{243}C_Fn_f^2$$
$$\left.+\frac{8}{729}(648\zeta(3)-125)C_F^3\right\}\left(m\bar{\psi}\psi\right)_R$$
$$+\left\{n_f\left(\left(\frac{52\zeta(3)}{9} - \frac{401}{324}\right)C_AC_F + \left(\frac{134}{27} - 4\zeta(3)\right)C_A^2 + \left(\frac{2407}{1458} - \frac{16\zeta(3)}{9}\right)C_F^2\right)\right.$$
$$\left.\left.+n_f^2\left(-\frac{697C_A}{729} - \frac{169C_F}{1458}\right)\right\}\left(F^2\right)_R\right],$$

Renormalization of $\bar{C}_{q,g}$: the real thing

Return to

$$\frac{\partial}{\partial \ln \mu}\bar{C}_q^R = -\frac{\alpha_s}{4\pi}\left(\frac{16}{3}C_F + \frac{4n_f}{3}\right)\bar{C}_q^R + \mathcal{O}(m) + \mathcal{O}(\alpha_s^2)$$

Correct result at $\mathcal{O}(\alpha_s)$

$$\frac{\partial \bar{C}_q^R}{\partial \ln \mu} = -\frac{\alpha_s}{4\pi}\left(\frac{16C_F}{3} + \frac{4n_f}{3}\right)\bar{C}_q^R + \frac{\alpha_s}{4\pi}\left[\frac{4C_F}{3}\frac{\langle P|(m\bar{\psi}\psi)_R|P\rangle}{2M^2} + \frac{n_f}{3}\left(\frac{\langle P|(m\bar{\psi}\psi)_R|P\rangle}{2M^2} - 1\right)\right]$$

$\mathcal{O}(m)$

Naively $\mathcal{O}(\alpha_s^2)$, but promoted to $\mathcal{O}(\alpha_s)$ due to trace anomaly!

$$\alpha_s F^2 \sim \mathcal{O}(1)$$

$$\bar{C}_q^R(\mu) = -\frac{1}{4}\left(\frac{n_f}{4C_F+n_f}+\frac{2n_f}{3\beta_0}\right)+\frac{1}{4}\left(\frac{2n_f}{3\beta_0}+1\right)\frac{\langle P|\left(m\bar{\psi}\psi\right)_R|P\rangle}{2M^2}$$
$$-\frac{4C_F A_q^R(\mu_0)+n_f\left(A_q^R(\mu_0)-1\right)}{4(4C_F+n_f)}\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\frac{8C_F+2n_f}{3\beta_0}}$$
$$+\frac{\alpha_s(\mu)}{4\pi}\left[\frac{n_f\left(-\frac{34C_A}{27}-\frac{49C_F}{27}\right)}{4\beta_0}+\frac{\beta_1 n_f}{6\beta_0^2}\right.$$
$$\left.+\frac{1}{4}\left(\frac{n_f\left(\frac{34C_A}{27}+\frac{157C_F}{27}\right)}{\beta_0}+\frac{4C_F}{3}-\frac{2\beta_1 n_f}{3\beta_0^2}\right)\frac{\langle P|\left(m\bar{\psi}\psi\right)_R|P\rangle}{2M^2}\right]+\cdots,$$

$$\simeq -0.146 - 0.25\left(A_q^R(\mu_0)-0.36\right)\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\frac{50}{81}} - 0.01\alpha_s(\mu)$$
$$+\left(0.306+0.08\alpha_s(\mu)\right)\frac{\langle P|\left(m\bar{\psi}\psi\right)_R|P\rangle}{2M^2},$$

Asymptotic value in the chiral limit $(n_f = 3)$

cf. $\bar{C}_q^R(\mu\sim 0)\approx 0.014$ from instantons Polyakov, Son (2018)

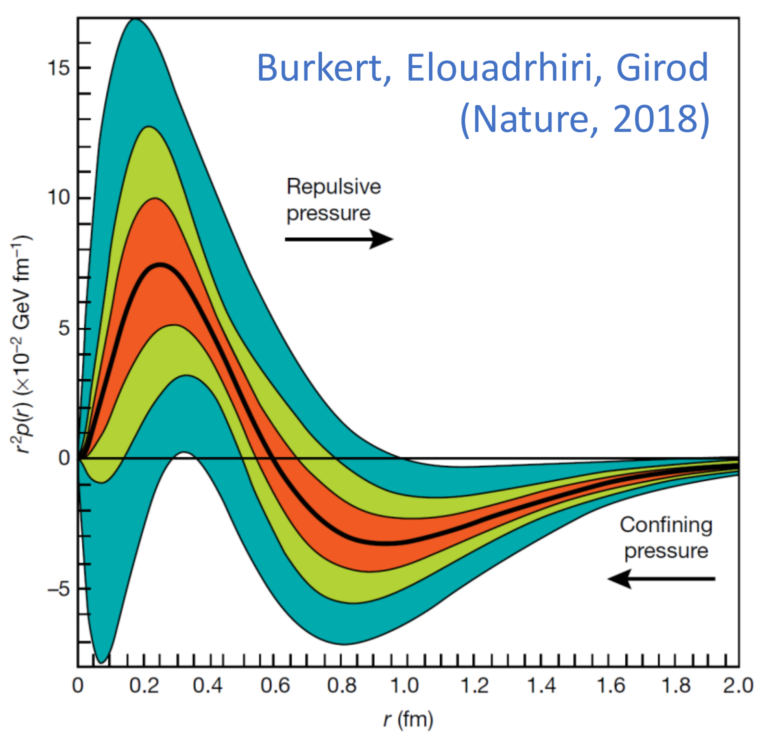
Applications

`Pressure' inside the proton from quark and gluon subsystems

Polyakov, Schweitzer (2018)

$$p_{q,g}(r) = \frac{1}{6Mr^2}\frac{d}{dr}r^2\frac{d}{dr}D_{q,g}(r) - M\bar{C}_{q,g}(r)$$

$$\bar{C}_q(\Delta = 0) = -\bar{C}_g(\Delta = 0) < 0$$



Discussed in Polyakov, Son (2018),
Our result 10 times larger and has an opposite sign.

A correction to the Ji sum rule

Decomposition of the transversely polarized proton

$$J^{q,g} = \frac{1}{2}(A_{q,g} + B_{q,g}) \quad + \frac{P^3}{2(P^0 + M)}\bar{C}_{q,g}$$

Ji (1996)
Ji, Xiong, Yuan (2012)

YH, Tanaka, Yoshida (2012)
Leader (2012)
Chakrabarti, Mondal, Mukherjee (2015)

Asymptotically,

$$\frac{1}{2}(A_q + B_q) \approx 0.18 \qquad \bar{C}_q \approx -0.146$$

The correction is sizable for a relativistic proton.

Measuring the trace anomaly $\langle P|F^{\mu\nu}F_{\mu\nu}|P\rangle$ in experiments

The operator $F^{\mu\nu}F_{\mu\nu}$ is twist-four,
highly suppressed in high energy scattering.

Purely gluonic operator, very difficult to compute in lattice QCD

Instead, we should look at low-energy scattering.

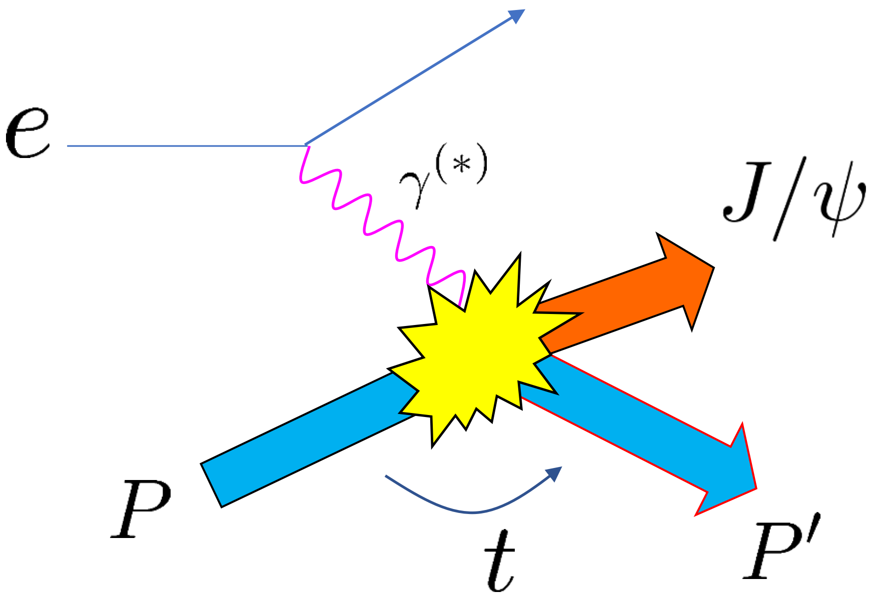
Purely gluonic operator. Use quarkonium as a probe.

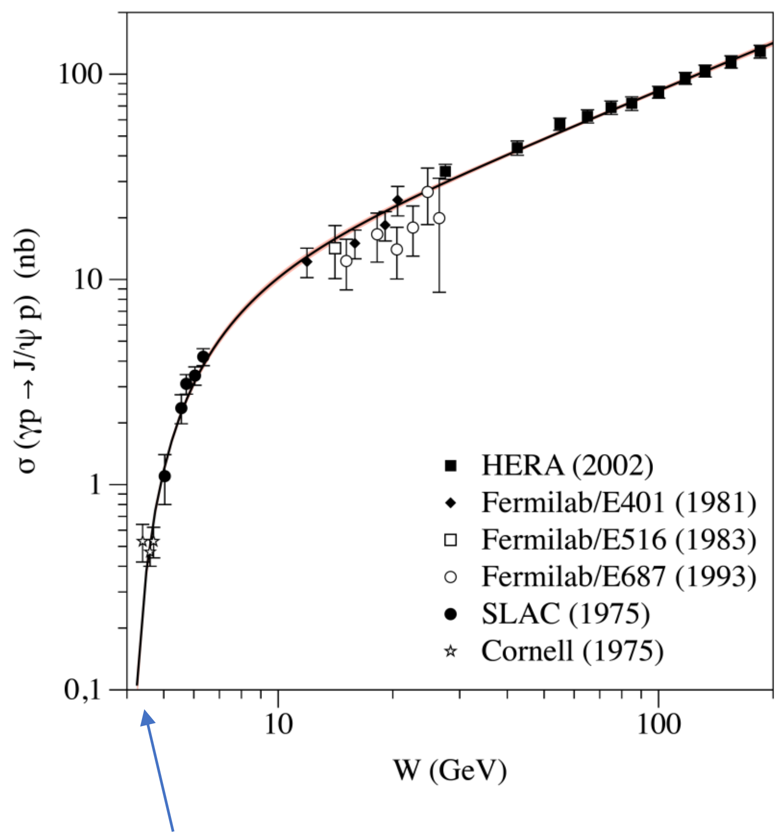
→ J/ψ photo-production near threshold.

Photo-production of J/ψ near threshold

Kharzeev, Satz, Syamtomov, Zinovjev (1998)
Brodsky, Chudakov, Hoyer, Laget (2000)

Sensitive to the non-forward matrix element $\langle P'|F^{\mu\nu}F_{\mu\nu}|P\rangle$





$W_{th} \approx 4.04\text{GeV}$

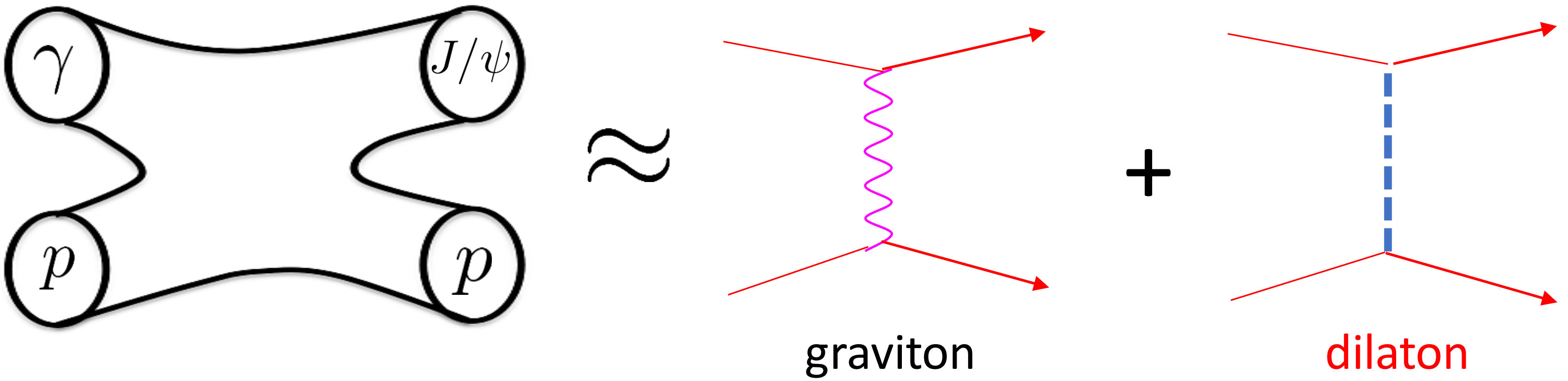
Straightforward to measure.
Ongoing experiments at Jlab.

Difficult to compute from first principles
(need nonperturbative approaches)

Holographic approach

YH, Yang (2018)

The operator $F^{\mu\nu}F_{\mu\nu}$ is dual to a massless string called dilaton in AdS



Suppressed compared to graviton exchange at high energy, but not at very low energy!

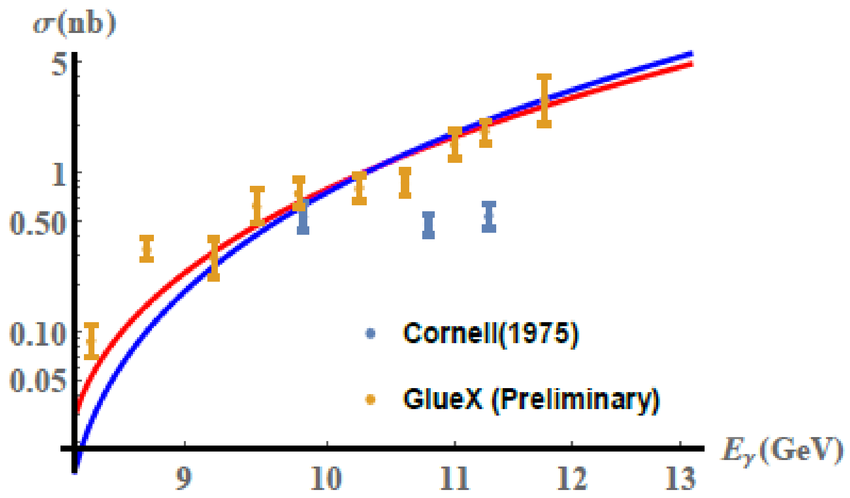
Non-forward matrix element difficult to deal with.
Relate to the gravitational form factor

$$\langle P'|F^{\mu\nu}F_{\mu\nu}|P\rangle \to A_{q,g}, B_{q,g}, D_{q,g}, \bar{C}_{q,g}$$

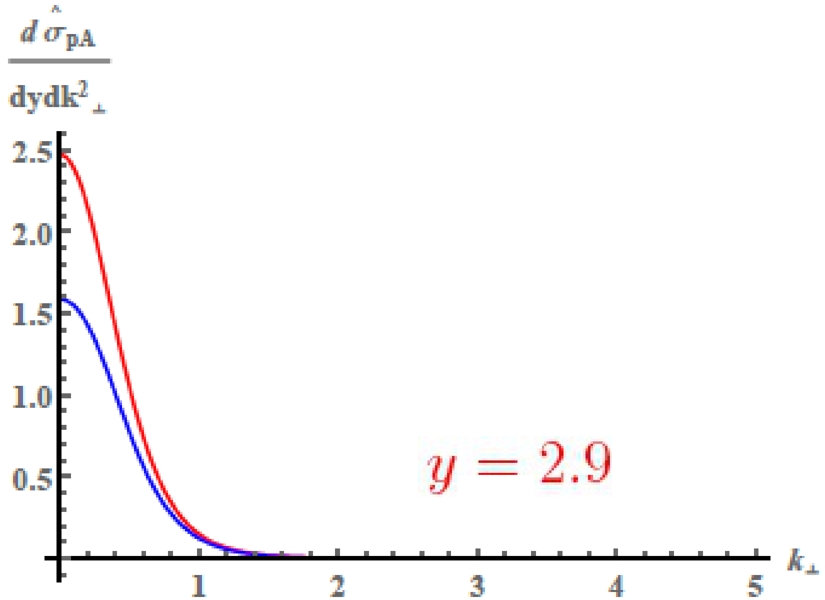
Fits and predictions

YH, Rajan, Yang, in preparation

J/ψ, JLab 12GeV, fixed target



Υ, RHIC 200GeV pA, ultraperipheral collisions



Red: with trace anomaly

Blue: without trace anomaly