Micro-bunched Electron Cooling for JLEIC parameters

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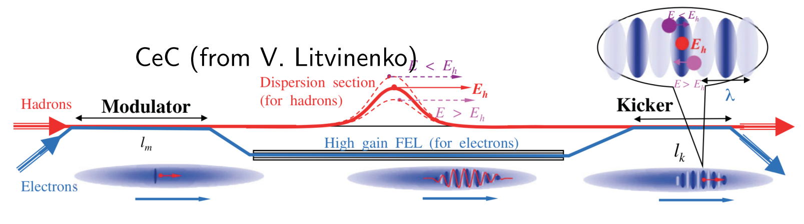
Cooling options for EIC

Traditional stochastic cooling is two slow for EIC.

- Conventional electron cooling (adopted for JLEIC)
- Coherent electron cooling (CeC) with an FEL amplifier [Proposed in Refs.[1].]

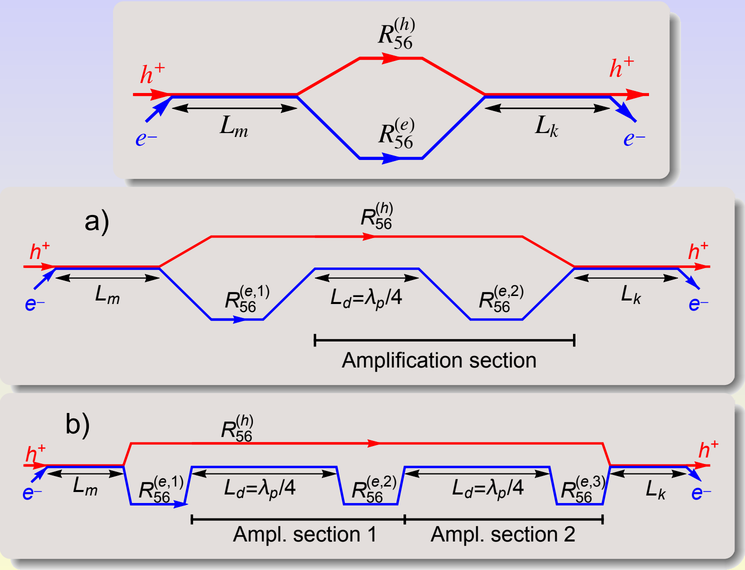


An FEL is a narrow-band amplifier and the gain is limited by the saturation effects.

- Microbunched coherent electron cooling (MBEC). Proposed in Ref.[2].
- Optical stochastic cooling.

[1] Derbenev, AIP Conf. Proc. **253**, 103 (1992); Litvinenko, Derbenev. PRL, **102**, 114801 (2009).
[2] D. Ratner, PRL, **111**, 084802 (2013).

Microbunched electron cooling (MBEC)



Theoretical analysis

In this analysis we used the Vlasov equation to track the dynamics of microscopic 1D fluctuations in the electron and hadron beams during their interaction and propagation through the system.

Assumptions:

- 1D model: hadrons and electrons are treated as infinitely thin slices of charge Ze ($-e$ for electrons) with a Gaussian transverse charge distribution (round beams).
- Perfect overlap of the electron and hadron beams in the modulator and the kicker.
- Particles (slices) do not shift relative to each longitudinally during the interaction in the modulator and the kicker.
- Chicanes shift particles in the longitudinal direction by $R_{56}\eta$.
- There is a perfect mixing in the hadron beam on the scale Δz_{int} during one revolution in the ring.

Recent papers

- 1D theory of longitudinal cooling without amplification stages (G. Stupakov, PRAB, **21**, 114402 (2018))
- 1D theory of longitudinal cooling with one and two amplification cascades (G. Stupakov, P. Baxevanis, PRAB, **22**, 034401, 2019)
- Theory of transverse cooling (P. Baxevanis, G. Stupakov, in preparation)
- 3D theory, work in progress (G. Stupakov, P. Baxevanis, IPAC 2019)

Representative set of parameters for JLEIC MBEC

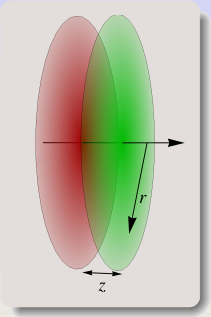
In numerical estimates we assume the following set of parameters for the hadron and electron cooler beams:

Collider circumference [m]	2154
Proton energy [GeV]	100
Proton relative energy spread, $\sigma_{\eta h}$	6×10^{-4}
Electron energy [MeV]	54
Electron relative energy spread, $\sigma_{\eta e}$	1×10^{-4}
Electron beam charge [pC]	250
Electron beam peak current [A]	7.5
Repetition rate [MHz]	476.3
Horizontal/vertical proton emittance [nm]	9.4/1.9
RMS beam size in mod. and kicker, Σ_x/Σ_y, [mm]	0.68/0.31
L_m, L_k [m]	40

The electron bunch length, $\sigma_{ze} \approx 4$ mm, is shorter than the proton bunch length, $\sigma_{ze} \lesssim \sigma_{zh} = 2$ cm.

The cooler-beam current is ~ 120 mA.

The model



Each hadron and electron are treated as infinitely thin slices of charge Ze or $-e$ respectively with a Gaussian transverse charge distribution over the surface of the slice,

$$\frac{Ze}{2\pi\Sigma_x\Sigma_y}e^{-x^2/2\Sigma_x^2-y^2/2\Sigma_y^2}$$

(this is justifiable if hadrons and electron execute several betatron oscillations during interaction). Here Σ_x and Σ_y are the rms transverse sizes of the beam.

We assume the beta functions in the modulator and the kicker, $\beta_x = \beta_y = 50$ m. For the nominal emittance of the proton beam (9.4 nm/1.9 mn), $\Sigma_x = 0.68$ mm, $\Sigma_y = 0.31$ mm, with the ratio $\Sigma_y/\Sigma_x = 0.45$.

Longitudinal cooling time, no amplification

The rate of energy spread change is

$$\frac{d\sigma_{\eta h}^2}{dt} = -\frac{\sigma_{\eta h}^2}{t_\mathrm{c}} + 2D$$

The cooling time depends on $R_{56}^{(e)}$ and $R_{56}^{(h)}$. The optimal values are: $R_{56}^{(e)} = 0.46\Sigma_x/\sigma_{\eta e}\gamma = 3$ cm, $R_{56}^{(h)} = 0.46\Sigma_x/\sigma_{\eta h}\gamma = 0.5$ cm, with

$$N_\mathrm{c}^{-1} \equiv \frac{T}{t_\mathrm{c}} = \frac{0.31}{\sigma_{\eta h}\sigma_{\eta e}} \frac{1}{\gamma^3} \frac{I_e}{I_A} \frac{r_h L_m L_k}{\Sigma_x^3}$$

Here, $I_A \approx 17$ kA is the Alfven current and $r_h = (Ze)^2/m_h c^2$ is the classical radius for hadrons ($\approx 1.5 \times 10^{-18}$ m for protons) and T is the revolution period.
The factor 0.31 is due to the ellipticity of the beam (for a round beam we had $0.1/\Sigma^3$ instead of $0.31/\Sigma_x^3$).

Cooling rate

The electron beam overlaps only with a small fraction of the hadron beam. Over many revolutions, hadrons move longitudinally due to the synchrotron oscillations. One needs to average the cooling rate over the length of the electron bunch,

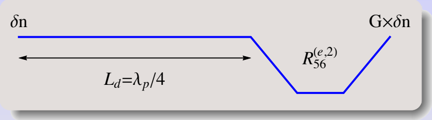
$$N_{\mathrm{c}}^{-1} = \frac{0.31}{\sigma_{\eta h}\sigma_{\eta e}} \frac{1}{\gamma^3} \frac{cQ_e}{\sqrt{2\pi}\sigma_{zh} I_A} \frac{r_h L_m L_k}{\Sigma_x^3}$$

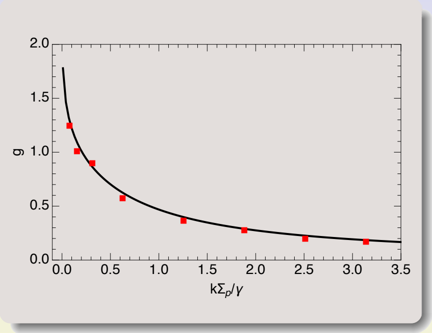
the cooling time is

$$T_{\mathrm{c}} \approx 0.7 \text{ h}$$

(for eRHIC this number was 41 h).

Amplification of microbunching in the electron beam[3]





In 1D model, the amplification factor $G(k)$ is derived theoretically. For the optimized chicane strength (note the minus sign in G—this is for $R_{56}^{(e,2)} > 0$),

$$G(k) = -\frac{1}{\sigma_{\eta e}}\sqrt{\frac{I_e}{I_A\gamma}}g\left(\frac{k\Sigma_p}{\gamma}\right)$$

where Σ_p is the beam radius.

We also simulated g solving equations of motion for electrons in the drift with account of the Coulomb interactions. Red dots—the result of simulations.

This is a broadband amplifier. Unfortunately, small k (long period) plasma oscillations have small plasma frequency.

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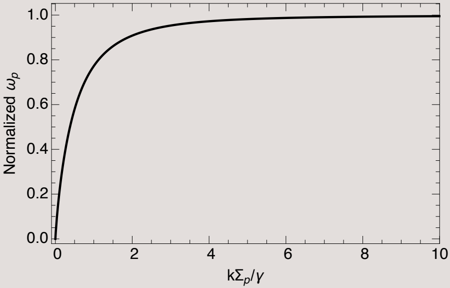
[3] Schneidmiller and Yurkov, PRSTAB **13**, 110701 (2010); Dohlus, Schneidmiller and Yurkov, PRSTAB **14** 090702 (2011); Marinelli et al., PRL **110**, 264802 (2013).

Dispersion of plasma oscillations

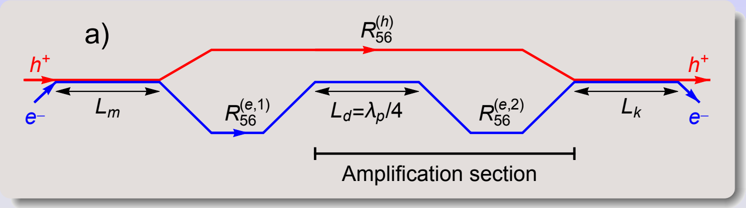


Plasma frequency is normalized by

$$\omega_{p0} = \frac{c}{\Sigma_p}\left(\frac{I_e}{I_A\gamma^3}\right)^{1/2}$$

This makes the optimal length of the amplification section longer than follows from simple estimates.

MBEC amplification using plasma oscillations[4]



In this analysis we assumed a round electron beam in the amplification section, $\Sigma_p = 0.14$ mm, $\Sigma_p/\Sigma_x = 0.2$. Analytic theory predicts for the amplification factor for the beam current I_e

$$G \approx 0.64 \frac{1}{\sigma_{\eta e}} \sqrt{\frac{I_e}{\gamma I_A}} = 13$$

To estimate the cooling time we need to average over the length of the electron bunch

$$N_c^{-1} = 0.041 \frac{(Q_e c)^{3/2} r_h L_m L_k}{(\sigma_z^{(e)})^{1/2} \sigma_z^{(h)} \Sigma_x^3 \gamma^{7/2} I_A^{3/2} \sigma_{\eta e}^2 \sigma_{\eta h}}, \qquad T_c \approx 4 \text{ min}$$

[4] G. Stupakov, P. Baxevanis, PRAB, **22**, 034401, 2019.

MBEC transverse cooling

- For the hadron beam, we only consider betatron motion in the vertical (y) direction.
- For the hadron transport line between the modulator and the kicker, including the hadron chicane, the four-dimensional transfer matrix is given by

$$R = \begin{pmatrix} R_{33} & R_{34} & 0 & R_{36} \\ R_{43} & R_{44} & 0 & R_{46} \\ R_{53} & R_{54} & 1 & R_{56} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

  which is supposed to act on the combined vector $\mathbf{y} = (y, \theta_y, z, \eta)$.
- We adopt a simplified model in which $\beta_1 = \beta_2 = \beta$, $\alpha_1 = \alpha_2 = 0$, $D_1 = D_2 = D$ and $D_1' = D_2' = 0$ (1 and 2 refer to the modulator and the kicker).

Cooling analysis, one amplification section

- Neglecting diffusion effects, the cooling equations for energy spread and emittance are
$$\frac{d(\sigma_{\eta h})^2}{dt} = -\frac{(\sigma_{\eta h})^2}{N_c^{\eta} T}$$
and
$$\frac{d\epsilon}{dt} = -\frac{\epsilon}{N_c^{\epsilon} T},$$
where we recall that T is the revolution period.
- The scaled cooling times N_c^{η} and N_c^{ϵ} are given by $1/N_c^{\eta} = A_0 I_{\eta}$ and $1/N_c^{\epsilon} = A_0 I_{\epsilon}$, where
$$A_0 = \frac{4 I_e^{eff} L_m L_k r_h}{\pi \Sigma_x^3 \gamma^3 I_A \sigma_{\eta e} \sigma_{\eta h}} \left(\frac{1}{\sigma_{\eta e}} \sqrt{\frac{2 I_e^{eff}}{\gamma I_A}} \right)$$
is a dimensionless pre-factor. We derived analytical expressions for the cooling integrals I_{η} and I_{ϵ}.

Optimization of the cooling rate

How do we optimize the emittance cooling rate? Qualitatively, the cooling rate is redistributed between the longitudinal and transverse degrees of freedom.

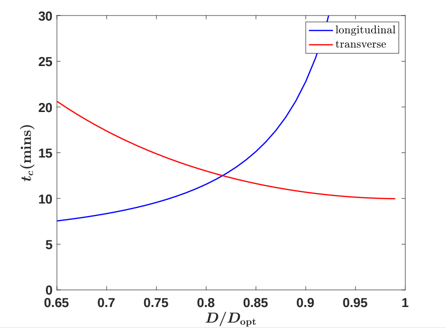
- $\varepsilon = \sqrt{\Sigma_y^2 + \sigma_\eta^2 D^2}/\Sigma_x$ is the ellipticity parameter (or aspect ratio) including the effect of dispersion.
- Zero cooling for the energy spread!
- Then, we need to maximize the modified cooling integral I_ϵ.

In terms of real parameters, this translates to:

- Optimum dispersion $D \approx 1.08$ m.
- Phase advance $\mu \approx 0.75$.
- Chicane strengths: $R_{56}^h = 1.6$ cm and $R_{56}^e = 5.3$ cm.
- Length of amplification drift $L_p = 26.3$ m.

Cooling rates as a function of the dispersion

We average the cooling rate over the hadron and electron bunches.



- We scan the dispersion around the optimum value while keeping everything else constant.
- A minimum is observed for the emittance cooling time (at $D = D_{\rm opt} = 1.08$ m).
- Minimum cooling time in y direction ~ 10 min.

Adding one more amplification stage would make the cooling time even smaller (roughly by order of magnitude), but amplification of the noise and saturation of the amplified signal may play a role.