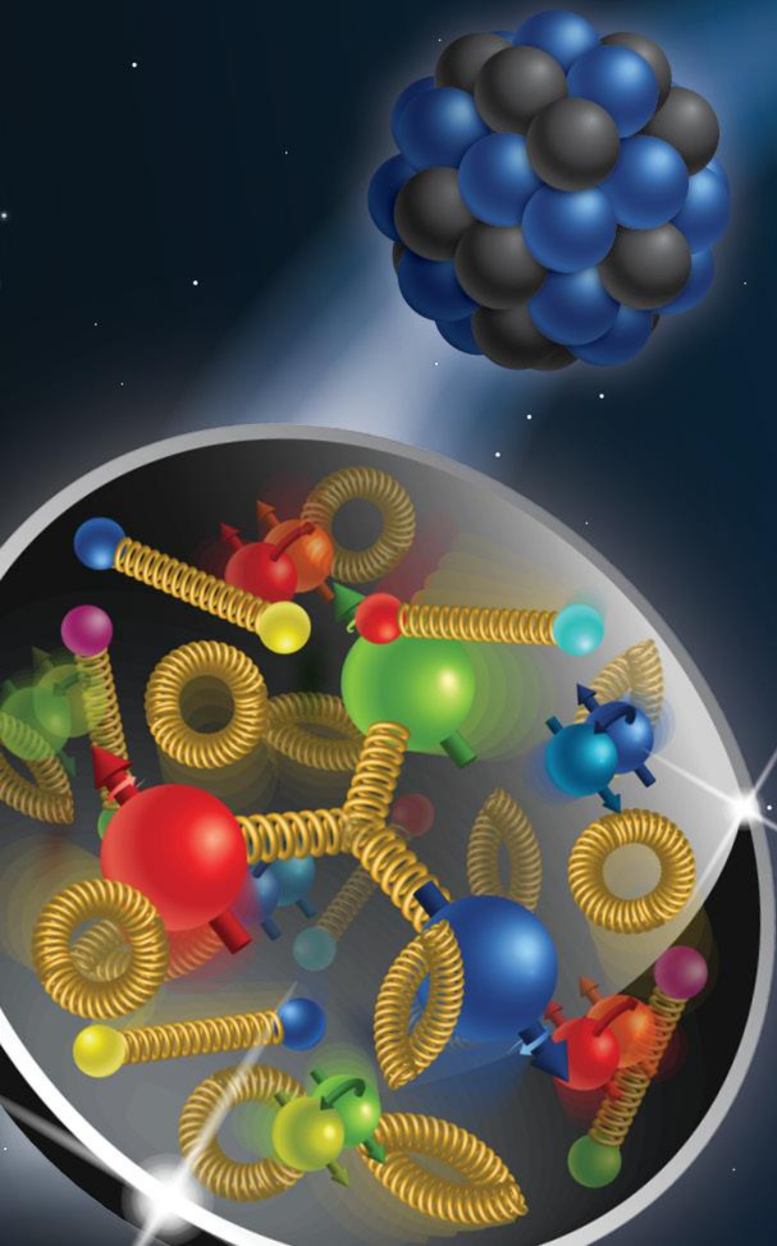

Electron Heating by Ions in Cooling Rings

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Electron Ion Collider – eRHIC

BROOKHAVEN NATIONAL LABORATORY
U.S. DEPARTMENT OF ENERGY | Office of Science

Outline

❑ Background

- Cooling for EIC
- Cooling & heating

❑ Electron Beam Heating by Ions

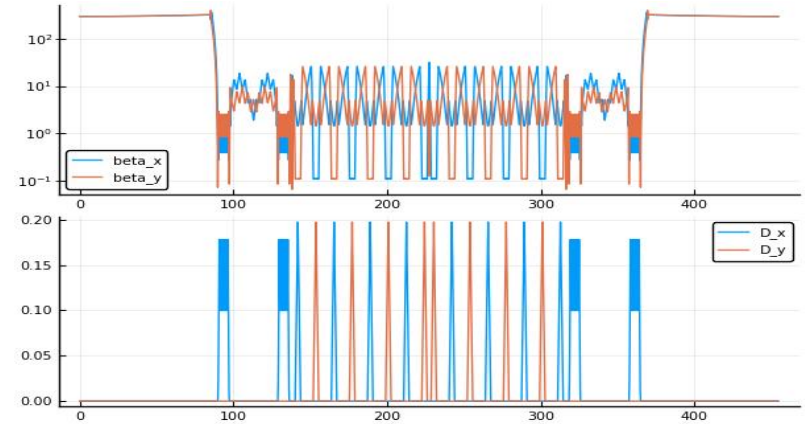
- Spitzer formula
- Kinetic theory (gas model)
- Calculations

❑ IBS

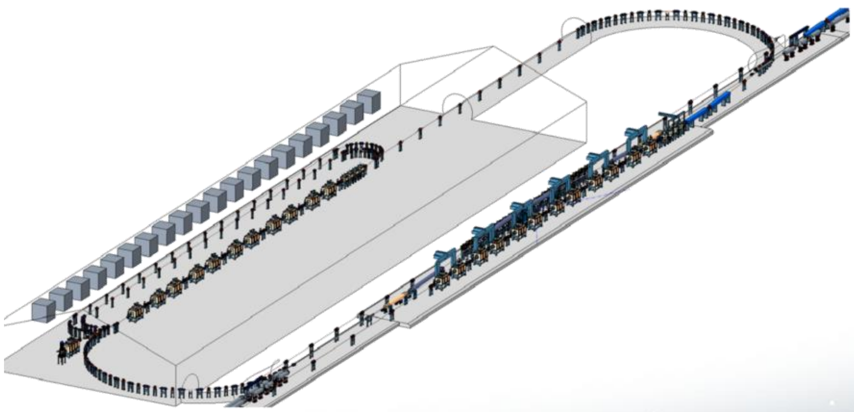
❑ Summary

Cooling Ring

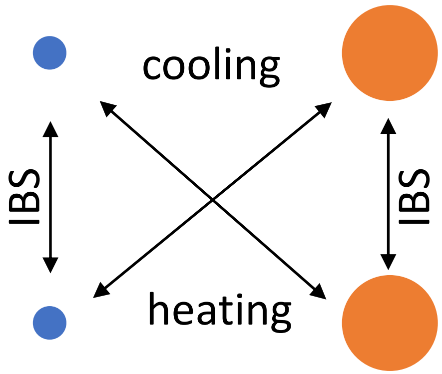
- **Beam cooling**
  - Storage ring with high energy, high power electron beam.
  - <2 hour cooling rate with bunched electron beam.
  - The IBS effect is the most important heating effect in the ion beam.



- **E-beam quality**
  - The single bunch e-beam will interact with ions many turns instead of once.
  - Heating: IBS, heating by ions, space charge
  - Cooling: synchrotron damping



Cooling & Heating effect



Cooling

Magnetic cooling force (Parkhomchuk's formula)

Non-magnetic cooling force

Intra-Beam Scattering

Piwinski, Martini, B-M and Jie-Wei model

Nagaitsev model (considered dispersions and simplified the integration)

Coupling effect in transverse

E-beam heating by ions

1. How to calculate the electron heating rate (model)?
2. How large of it comparing with IBS effect?

Spitzer Formula

- Based on energy exchange between electrons and ions
- Maxwellian velocity distribution with different kinetic temperature T_i , T_e

$$\frac{dT_e}{dt} = \frac{T_i - T_e}{\tau_{eq}} \qquad \tau_{eq} = \frac{3Mm(4\pi\varepsilon_0)^2}{8n_i z^2 e^2 L_{\mathrm{e}} (2\pi)^{1/2}} \left(\frac{kT_{\mathrm{e}}}{m} + \frac{kT_i}{M}\right)^{3/2}$$

E-beam heating rate by ion

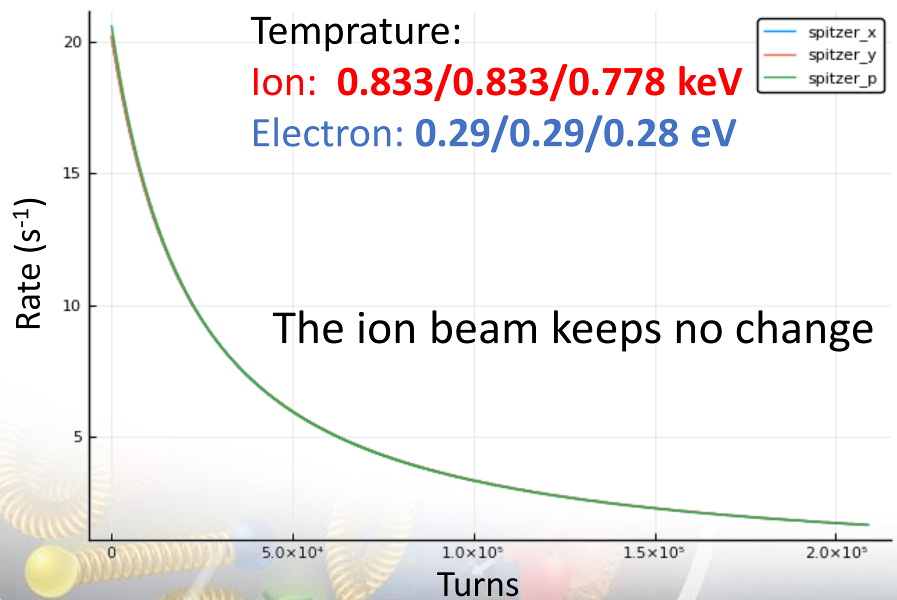
$$\begin{cases} \dfrac{d\epsilon_{x,y}}{\epsilon_{x,y} dt} = \dfrac{T_{ix,iy}/T_{ex,ey} - 1}{\gamma \tau_{eq}} \\ \dfrac{d(\frac{\Delta p}{p})^2}{(\frac{\Delta p}{p})^2 dt} = \dfrac{T_{i//}/T_{e//} - 1}{\gamma \tau_{eq}} \end{cases}$$

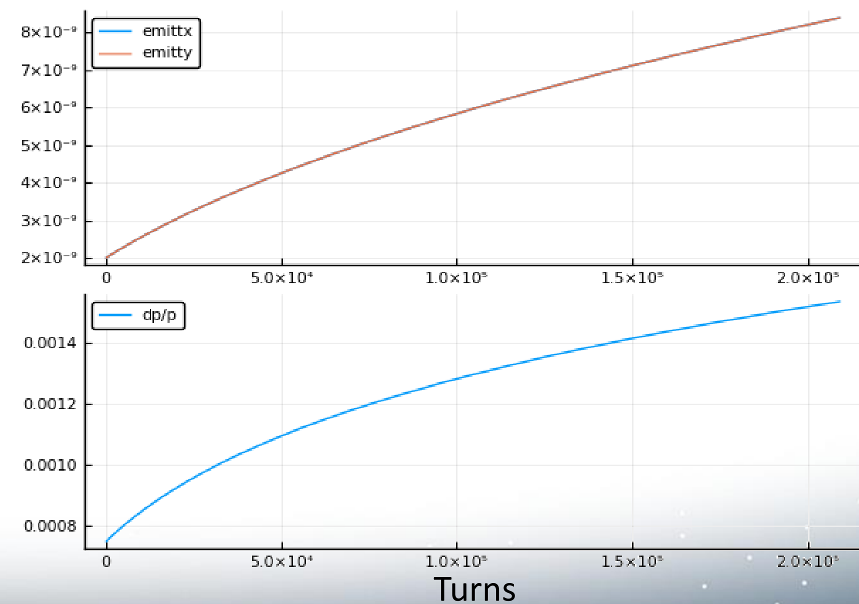
- The temperature of the beam in each dimension is independent.
- Only good for the beam (electron and ion) with same temperature in 3D.

Calculation with heating

- Only electron heating effect in the calculation.
- Both ion and electron beam have the same temperature in each dimension.
- The heating rate in each dimension keeps the same during the evolution.

	Ion (proton)	electron
Energy	$\gamma = 293.1$	$\gamma = 293.1$
ε_x (rad.m)	3.1e-9	2.0e-9
ε_y(rad.m)	3.1e-9	2.0e-9
dp/p	9e-4	7.5e-4
Bunch length (m)	0.263	0.07
N	5e10	1.2e11





Kinetic theory

$f(x, y, s, p_x, p_y, p_s, t)$: Distribution function:

Boltzmann transport equation:

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \vec{v} \cdot \nabla f + \vec{F} \cdot \frac{\partial f}{\partial \vec{p}} = C(f)$$

Diffusion (freely moving) — External force (electromagnetic field) — Collision (between particles)

Only consider the collision between a and b

$$\frac{\partial f_a}{\partial t} = \sum_b C_{ab}$$

Coulomb collision operator

$$U_{\alpha,\beta} = \frac{u^2 \delta_{\alpha,\beta} - u_\alpha u_\beta}{u^3}$$

$$C_{ab} = \frac{\gamma_{ab}}{2} \frac{\partial}{\partial v_\alpha} \int d^3 v' U_{\alpha,\beta} \left(f_b' \frac{\partial f_a}{\partial v_\beta} - \frac{m_a}{m_b} f_a \frac{\partial f_b'}{\partial v_\beta'}\right)$$

$$\gamma_{ab} = \frac{e_a^2 e_b^2 \ln\Lambda}{4\pi \varepsilon_0^2 m_a^2}$$

The heating rate $\left\langle \dot{v_x^2} \right\rangle$

$$\int d^3 v \frac{\partial f_a}{\partial t} v_x^2 = \left\langle \dot{v_x^2} \right\rangle n_a$$

Gas heating

Gas Heating Rate

$$\left\langle \dot{v}_x^2 \right\rangle n_a = \int d^3v \frac{\partial f}{\partial t} v_x^2$$

$$= \gamma_{ab} \int d^3v \int d^3v' \{ \frac{u^2 - u_x^2}{u^3} \frac{v_x^2}{\sigma_{v_x}^2} - \frac{u_x u_y}{u^3} \frac{v_x v_y}{\sigma_{v_y}^2} - \frac{u_x u_s}{u^3} \frac{v_x v_s}{\sigma_{v_s}^2} \} f_a(v) f_b(v')$$

$$= \gamma_{ab} \int d^3u \int d^3v \{ \frac{u^2 - u_x^2}{u^3} \frac{v_x^2}{\sigma_{v_x}^2} - \frac{u_x u_y}{u^3} \frac{v_x v_y}{\sigma_{v_y}^2} - \frac{u_x u_s}{u^3} \frac{v_x v_s}{\sigma_{v_s}^2} \} f_a(v) f_b(u+v)$$

- Here we consider the beam in 6-D phase space is independent and Gaussian distribution.
- No coupling in the cooling section and no beam rotates in phase space.

$$f(v) = \frac{n0}{\sqrt{(2\pi)^3} \sigma_{v_x} \sigma_{v_y} \sigma_{v_s}} \mathrm{Exp}(-\frac{v_x^2}{2\sigma_{v_x}^2} - \frac{v_y^2}{2\sigma_{v_y}^2} - \frac{v_s^2}{2\sigma_{v_s}^2})$$

$$n(x,y,s) = \frac{N}{\sqrt{(2\pi)^3} \sigma_x \sigma_y \sigma_s} \mathrm{Exp}\left(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} - \frac{s^2}{2\sigma_s^2}\right) = \mathrm{n0}\ \mathrm{Exp}\left(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} - \frac{s^2}{2\sigma_s^2}\right)$$

Gas Heating Rate

$$\left\langle \dot{v_x^2} \right\rangle n_a = \frac{\gamma_{ab} n0_b}{(2\pi)^3 \sigma_{v_{ax}} \sigma_{v_{ay}} \sigma_{v_{as}} \sigma_{v_{bx}} \sigma_{v_{by}} \sigma_{v_{bs}}} \int d^3u \{$$

$$\frac{u^2 - u_x^2}{u^3} \frac{1}{\sigma_{v_{ax}}^2} I(\frac{1}{2\sigma_{v_{ax}}^2}, \frac{1}{2\sigma_{v_{bx}}^2}, u_x, 2) \cdot I(\frac{1}{2\sigma_{v_{ay}}^2}, \frac{1}{2\sigma_{v_{by}}^2}, u_y, 0) \cdot I(\frac{1}{2\sigma_{v_{as}}^2}, \frac{1}{2\sigma_{v_{bs}}^2}, u_s, 0)$$

$$-\frac{u_x u_y}{u^3} \frac{1}{\sigma_{v_{ay}}^2} I(\frac{1}{2\sigma_{v_{ax}}^2}, \frac{1}{2\sigma_{v_{bx}}^2}, u_x, 1) \cdot I(\frac{1}{2\sigma_{v_{ay}}^2}, \frac{1}{2\sigma_{v_{by}}^2}, u_y, 1) \cdot I(\frac{1}{2\sigma_{v_{as}}^2}, \frac{1}{2\sigma_{v_{bs}}^2}, u_s, 0)$$

$$-\frac{u_x u_s}{u^3} \frac{1}{\sigma_{v_{as}}^2} I(\frac{1}{2\sigma_{v_{ax}}^2}, \frac{1}{2\sigma_{v_{bx}}^2}, u_x, 1) \cdot I(\frac{1}{2\sigma_{v_{ay}}^2}, \frac{1}{2\sigma_{v_{by}}^2}, u_y, 0) \cdot I(\frac{1}{2\sigma_{v_{as}}^2}, \frac{1}{2\sigma_{v_{bs}}^2}, u_s, 1)\}$$

$$I(\alpha, \beta, u, r) = \int_{-\infty}^{\infty} v^r \exp\left\{-\beta(u+v)^2 - \alpha v^2\right\} dv$$

$$I(\alpha, \beta, u, 1) = -\sqrt{\frac{\pi}{\alpha+\beta}} \frac{\beta u}{\alpha+\beta} \exp(-\frac{\alpha\beta}{\alpha+\beta} u^2)$$

$$I(\alpha, \beta, u, 0) = \sqrt{\frac{\pi}{\alpha+\beta}} \exp(-\frac{\alpha\beta}{\alpha+\beta} u^2)$$

$$I(\alpha, \beta, u, 2) = [\frac{1}{2(\alpha+\beta)} + \frac{\beta^2 u^2}{(\alpha+\beta)^2}] \sqrt{\frac{\pi}{\alpha+\beta}} \exp(-\frac{\alpha\beta}{\alpha+\beta} u^2)$$

Calculation with heating

- Same temperature (3D) of ion and electron

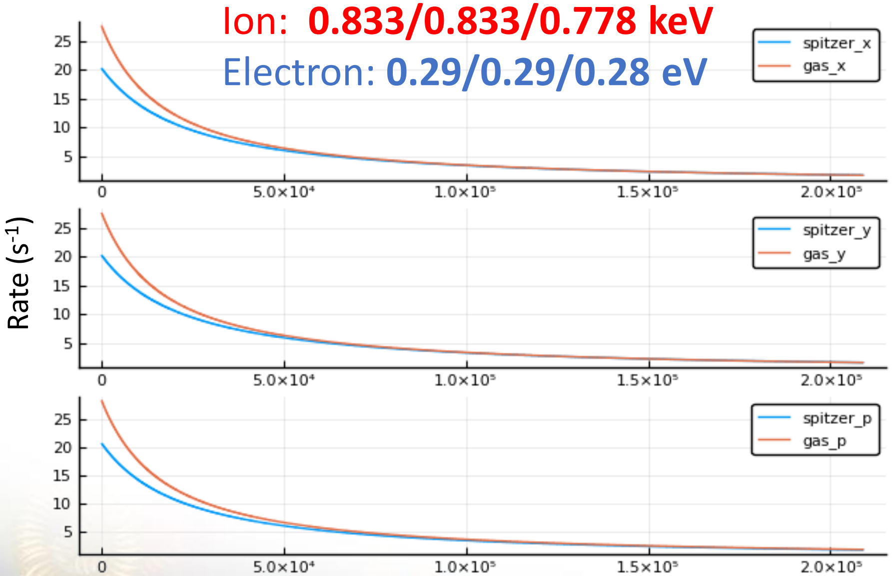
- Spitzer model vs. gas model

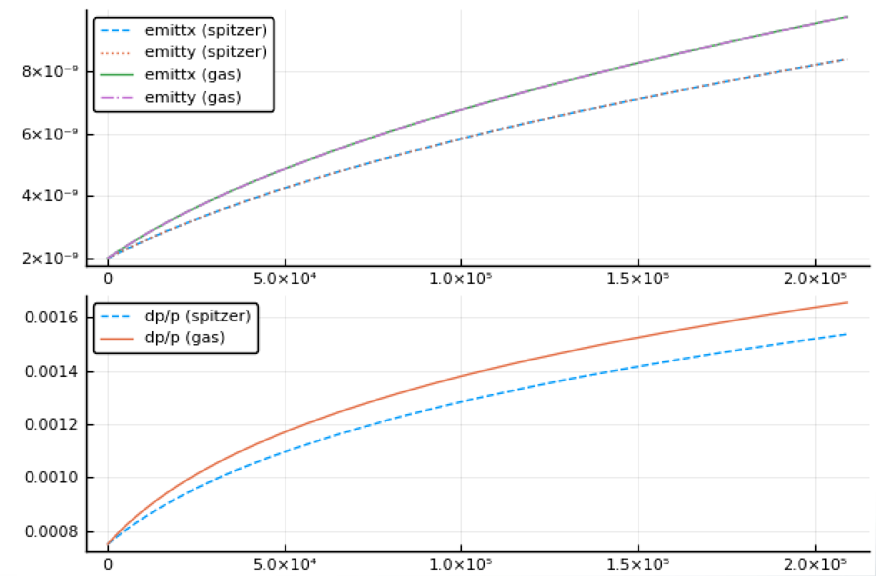
A little difference between the two models

	Ion (proton)	electron
Energy	$\gamma = 293.1$	$\gamma = 293.1$
ε_x (rad.m)	3.1e-9	2.0e-9
ε_y(rad.m)	3.1e-9	2.0e-9
dp/p	9e-4	7.5e-4
Bunch length (m)	0.263	0.07
N	5e10	1.2e11

Ion: **0.833/0.833/0.778 keV**

Electron: **0.29/0.29/0.28 eV**





Calculation with heating

- Different temperature (3D) of electron
  - Spitzer model vs. gas model

These two models are consistent with each other

	Ion (proton)	electron
Energy	$\gamma = 293.1$	$\gamma = 293.1$
ε_x (rad.m)	3.1e-9	2.0e-9
ε_y(rad.m)	3.1e-9	8.0e-9
dp/p	9e-4	5.0e-4
Bunch length (m)	0.263	0.07
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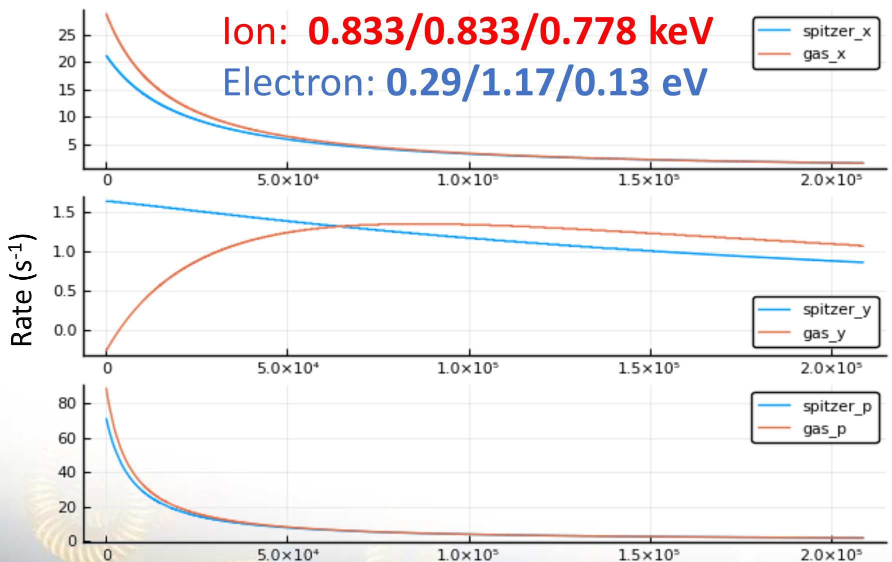
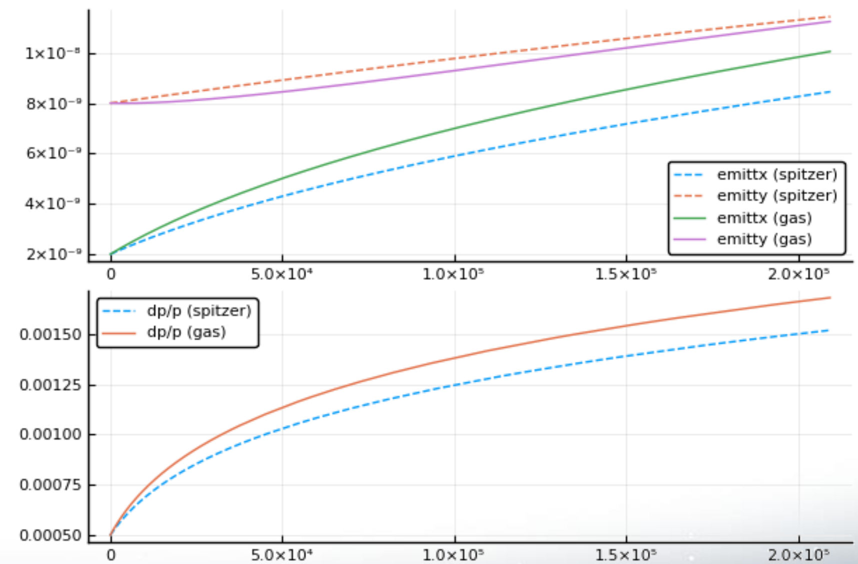


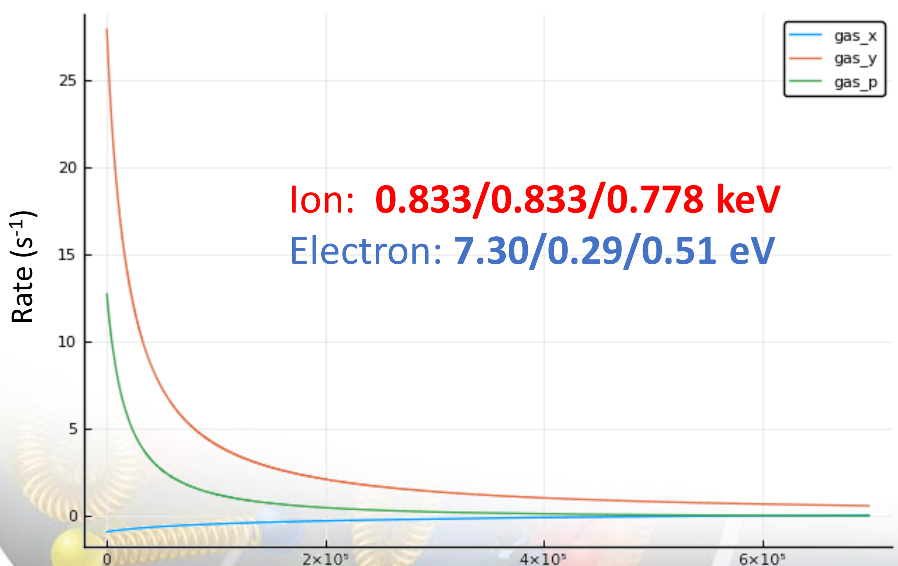


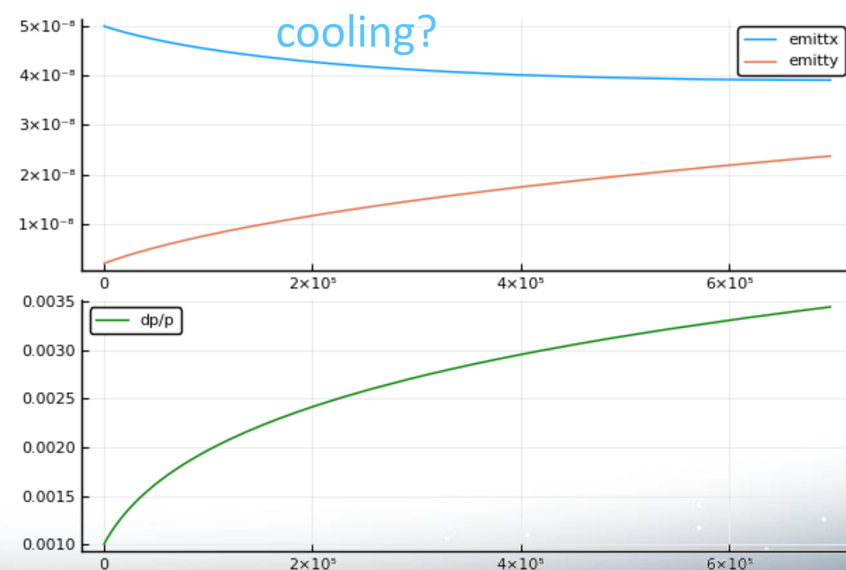
Calculation with heating

- Large temperature difference of electron

- The heating rate in horizontal is negative, which means cooling effect on the electron beam.

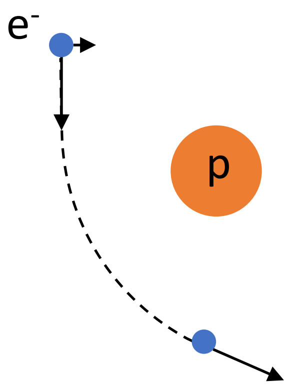
	Ion (proton)	electron
Energy	$\gamma = 293.1$	$\gamma = 293.1$
ε_x (rad.m)	3.1e-9	5.0e-8
ε_y(rad.m)	3.1e-9	2.0e-9
dp/p	9e-4	1.0e-3
Bunch length (m)	0.263	0.07
N	5e10	1.2e11





Calculation with heating

- Large temperature difference of electron



- The heating rate in horizontal is negative, which means cooling effect on the electron beam.
- The heating rates in spitzer model should be always positive because the high tem. of ion beam.

$$\frac{dT_e}{dt} = \frac{T_i - T_e}{\tau_{eq}} \qquad \tau_{eq} = \frac{3Mm(4\pi\varepsilon_0)^2}{8n_i z^2 e^2 L_e (2\pi)^{1/2}} \left(\frac{kT_e}{m} + \frac{kT_i}{M}\right)^{3/2}$$

- The negative heating rate is reasonable.

The gas model is good for the heating calculation

Are these calculation believable?

IBS calculation (gas model)

Boltzmann transport equation: $\frac{\partial f_a}{\partial t} = \sum_a C_{aa}$ (Collisions between electrons)

$$C_{aa} = \frac{\gamma_{aa}}{2} \frac{\partial}{\partial v_\alpha} \int d^3 v' U_{\alpha,\beta} (f_a' \frac{\partial f_a}{\partial v_\beta} - f_a \frac{\partial f_a'}{\partial v_\beta'})$$

Very simple model, no beam rotate in phase space, no dispersion and coupling etc.

Heating rate caused by IBS:

$$\left\langle \dot{v}_x^2 \right\rangle n_a = -\frac{\gamma_{aa} n_a(x,y,s)}{(2\pi)^3 \sigma_{v_{ax}}^2 \sigma_{v_{ay}}^2 \sigma_{v_{as}}^2} \int d^3 u \, \{$$

$$\frac{u^2 - u_x^2}{u^3} \frac{1}{\sigma_{v_{ax}}^2} I(\frac{1}{2\sigma_{v_{ax}}^2}, \frac{1}{2\sigma_{v_{ax}}^2}, u_x, 1) \cdot I(\frac{1}{2\sigma_{v_{ay}}^2}, \frac{1}{2\sigma_{v_{ay}}^2}, u_y, 0) \cdot I(\frac{1}{2\sigma_{v_{as}}^2}, \frac{1}{2\sigma_{v_{as}}^2}, u_s, 0)$$

$$-\frac{u_x u_y}{u^3} \frac{1}{\sigma_{v_{ay}}^2} I(\frac{1}{2\sigma_{v_{ax}}^2}, \frac{1}{2\sigma_{v_{ax}}^2}, u_x, 1) \cdot I(\frac{1}{2\sigma_{v_{ay}}^2}, \frac{1}{2\sigma_{v_{ay}}^2}, u_y, 0) \cdot I(\frac{1}{2\sigma_{v_{as}}^2}, \frac{1}{2\sigma_{v_{as}}^2}, u_s, 0)$$

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IBS calculation (gas model)

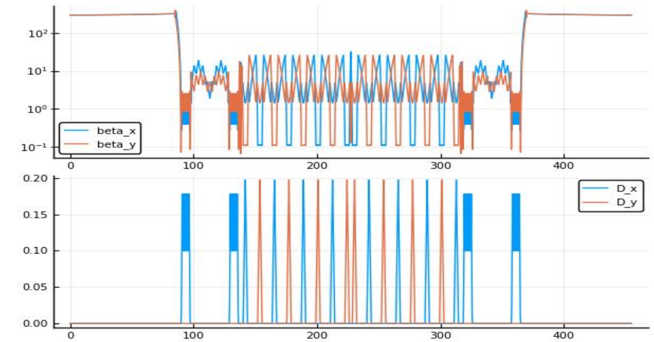
- Compared with the Nagaitsev's IBS model.
- Only one point of a ring ($\beta = 295m, \alpha = 0, D = 0$)
- The result is acceptable even though a little difference with Nagaitsev's model.

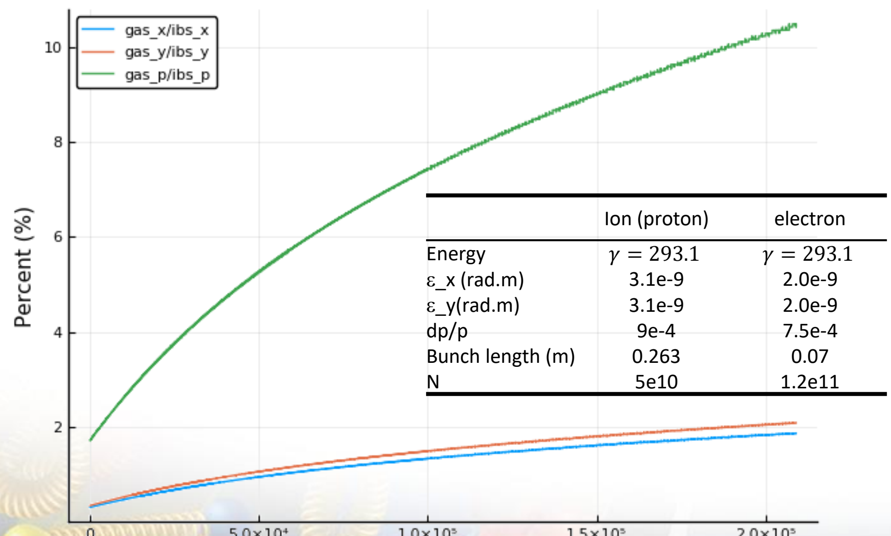
ε_x	ε_y	dp/p	Nagaitsev_x	Gas_IBS_x	Nagaitsev_y	Gas_IBS_y	Nagaitsev_s	Gas_IBS_s
2.00E-09	2.00E-09	7.50E-04	-0.8	-1.6	-0.8	-1.6	1.7	3.367
5.00E-09	2.00E-09	7.50E-04	-11.3	-18.9	12.3	23.03	14	26.43
1.00E-09	2.00E-09	7.50E-04	100	187.7	-26.8	-49	-23.8	-43.07
2.00E-09	2.00E-09	1.00E-04	-102	-183	-102	-183	11894	21548
2.00E-09	2.00E-09	1.00E-03	12.6	23.33	12.6	23.3	-14.6	-26.3

The gas heating model is good and reliable.

How large of electron heating rate by ions?

- The IBS heating rate is the average value of the ring lattice (Nagaitsev's model).
- Only gas heating rate is applied on beam in the simulation.
- The heating rate by ions can reach to 10% of the IBS heating rate in longitudinal, and it determined by the beam status.





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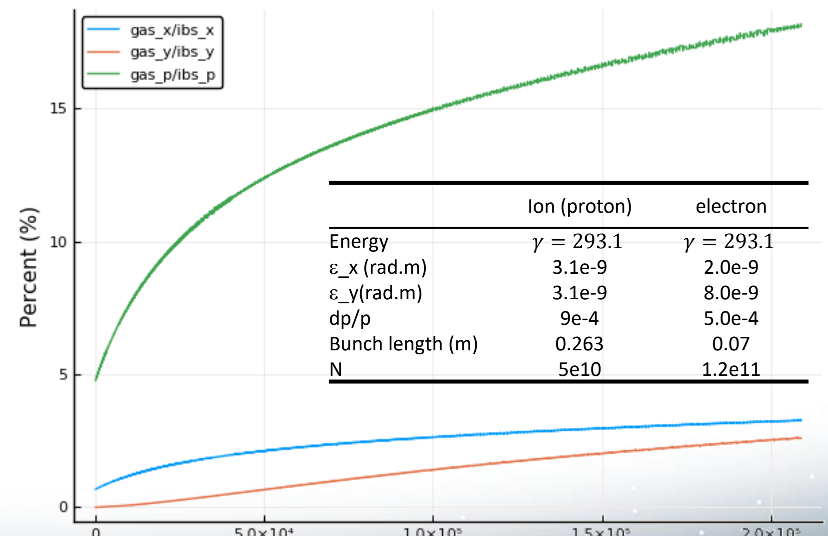

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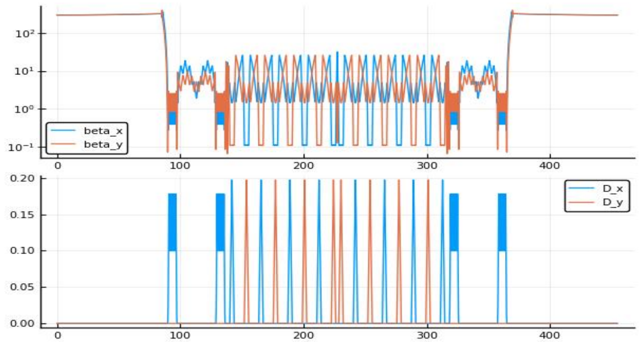
How large of electron heating rate by ions?



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- Only gas heating rate is applied on beam in the simulation.
- The heating rate by ions can reach to 10% of the IBS heating rate in longitudinal, and it determined by the beam status.

The electron beam status in cooling rings is mainly depended on the synchrotron damping, IBS and heating by ions. So, these effects should be considered seriously to study the electron beam quality, as well as cooling rate.

More works should be done in the future for the determination of electron beam parameters and cooling simulation.

Summary

- The electron beam heating by the ions was studied, and a good model is established for the calculation of the heating rate.
- Comparing with IBS, we think this heating effect could be an important factor to affect the electron beam quality.
- In the future, the heating by ions, synchrotron damping and IBS should be carefully considered for the estimation of electron beam parameters and cooling rate calculations.

Acknowledgement

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References

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