Magnetized dynamic friction for times short compared to the plasma period

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Outline

- Parameter regime for relativistic electron coolers
- Details of the calculation
- Preliminary results
- Comparison with other models
- Future work

Relativistic cooling → short interaction time

- Prototyping is done in the parameter regime of Fedotov *et al*. (2006)

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Numerical study of the magnetized friction force

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Analysis of the magnetized friction force *

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- For testing, we considered the following beam frame parameters:
  - *e^- density, $n_e = 2x10^{15}\ m^{-3}$*
  - *ideal solenoid, $B = 5\ T$*
  - *interaction time, $\tau_{int} = 4x10^{-10}\ s \sim 56\ T_L \sim 0.16\ T_{pl}$*
    - **16% of a plasma period → no shielding of the interaction**
  - *distance to nearest e^-, $r_1 \sim 4.9x10^{-6}\ m \sim 10\ r_L$*
    - **small Larmor radius → strong B-field assumption is reasonable**

Our approach is motivated by the work of *Ya. Derbenev*

Ya. Derbenev, "Theory of Electron Cooling," arXiv (2017); https://arxiv.org/abs/1703.09735

THEORY OF ELECTRON COOLING

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- The E-fields associated with friction must be carefully identified
  - *these are the fields generated by the presence of the ion*

bulk fields | friction | statistical fluctuations

$$\vec{E}(\vec{r},\vec{v},t) = \langle \vec{E}^0 \rangle(\vec{r},t) + \langle \Delta\vec{E} \rangle(\vec{r},\vec{v},t) + \vec{E}^{fl}(\vec{r},\vec{v},t) \qquad (1.1)$$

- Friction force must be calculated along the ion trajectory:

$$\vec{F} = -ze\langle \Delta\vec{E} \rangle(\vec{r},\vec{v},t)\Big|_{\vec{r}=\vec{r}(t),\dot{\vec{r}}(t)=\vec{v}} \qquad (1.2)$$

  - *we do this numerically for each individual ion-electron interaction*
    - **total force obtained by summing over e^- distribution (i.e. no shielding)**
  - *bulk forces are removed by subtracting force from unperturbed e^-'s*

Gyrokinetic averaging yields 1D e^- oscillations

- Hamiltonian perturbation theory for single ion & e^-
  - *unperturbed motion: drifting ion and magnetized e^-*
  - *primary assumption: D (impact parameter) >> r_L (Larmor radius)*
  - *longitudinal dynamics: $V_{ion,\perp} = 0$ (to be relaxed in future work)*
- e^- gyrocenters stay on cylinder of constant radius D (different for different e-'s)
- choose ion to be stationary at the origin (convenient)
  - *gyrocenters move in a 1D potential:*
  - *weak nonlinearity (larger amplitudes => longer periods)*

$$\ddot{z}(t) = -\frac{Ze^2}{4\pi\epsilon_0 m_e}\frac{z}{(D^2+z^2)^{3/2}}$$

- shortest oscillation period:

$$T_{lin} = \frac{2\pi}{c}\sqrt{\frac{D^3}{Zr_e}}$$

  - *both trapped and passing orbits*
  - *1D numerical simulations are required to capture these effects*

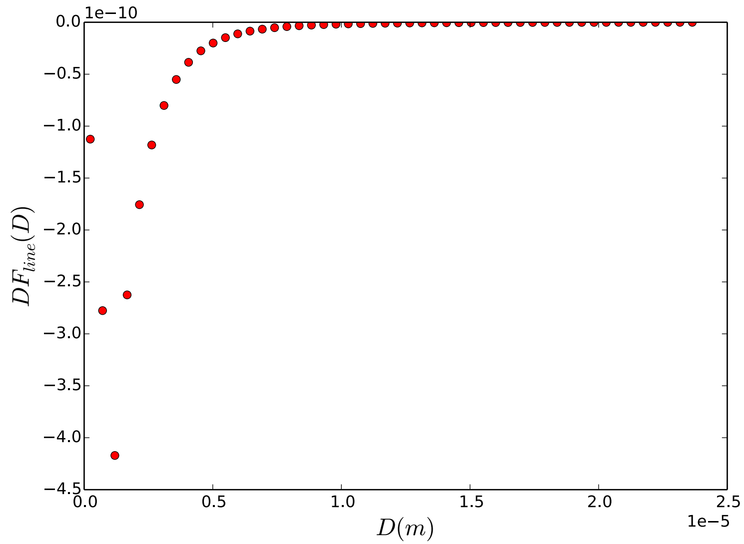
Key aspects of the numerical simulations

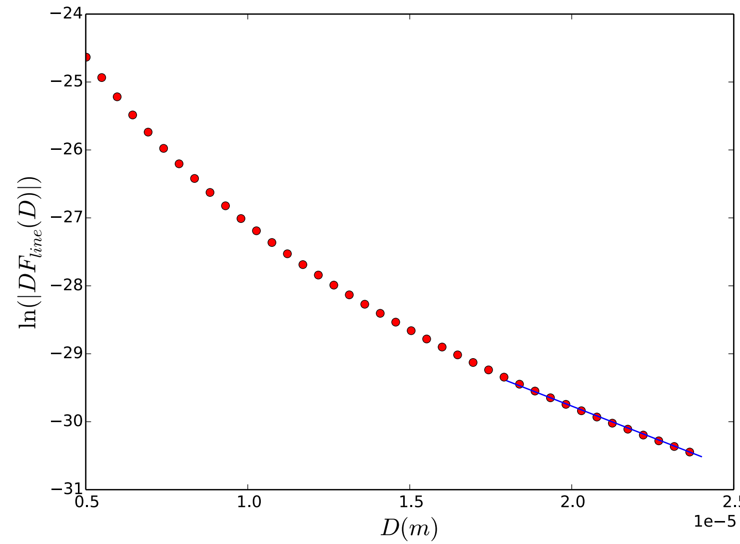
- Work in the system of reference where the ion is at rest
  - *assume ion velocity along the field lines of B (→ axial symmetry)*
  - *cold electrons → all have the same initial velocity w.r.t. ion*
  - *momentum kicks add up*
- Dynamical friction comes from ion-induced *density perturbation*
  - *force is the difference between perturbed & unperturbed e^- effects*
    - **hence, we track pairs of electrons with identical initial conditions**
  - *this approach eliminates all bulk forces, both physical and numerical*
- Compute ensemble-average expectation value of friction
  - *we assume a locally-uniform electron density n_e*
  - *transversely, e^--s are uniformly distributed on lines of constant D*
    - **there is no logarithmic singularity for D→0, nor for D→∞**
  - *longitudinal distribution is uniform in initial z position, z_{ini}*
    - **finite range of z_{ini} values contribute non-negligibly to the friction force**
    - **range depends on: D (impact parameter), V_{ion}, Z (ion charge state)**
- Thermal e- effects are obtained via convolution

Finite friction for all ρ *(no logarithmic singularities)*

- First add up contributions to the friction force from initial conditions on lines of constant D, then integrate over the impact parameter:

$$F_{\|}(V_{\perp}=0) = 2\pi n_e \int_0^{\infty} dD\, D F_{line}(D) \equiv 2\pi n_e \int_0^{\infty} dD\, D \int_{-\infty}^{\infty} dz_{ini} F_{i-e}(z_{ini}, D)$$

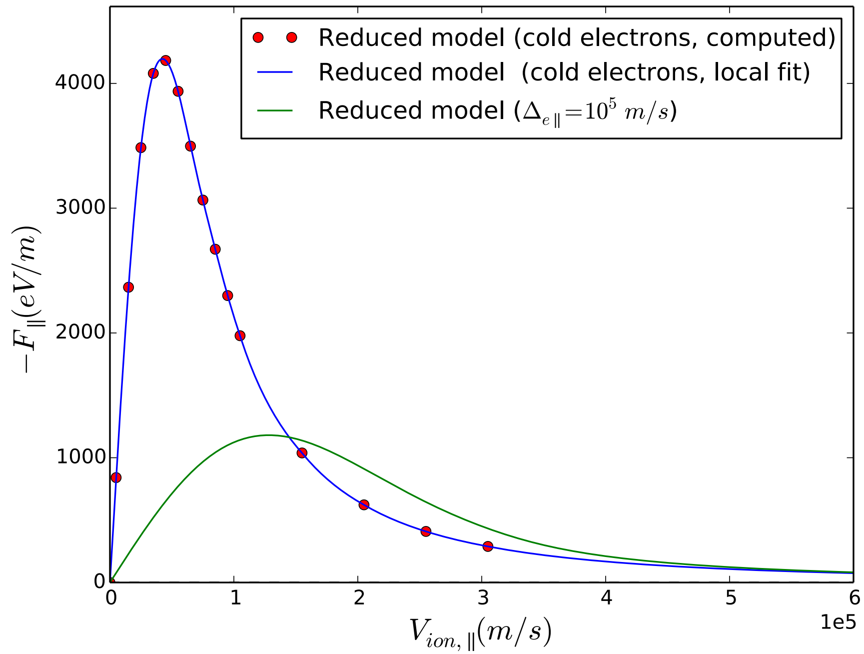




- Integrand is finite for small D & tails off exponentially => finite $F_{\|}$
- Exponential fall-off for large D makes it possible to correct (analytically) for a finite values of D_{max} in simulations
- Repeat for different values of $V_{ion,\|}$ to compute $F_{\|}$ ($V_{ion,\|}$)

$F_{\parallel}(V_{ion,\parallel})$ for warm electrons constructed via convolution with electron distribution density

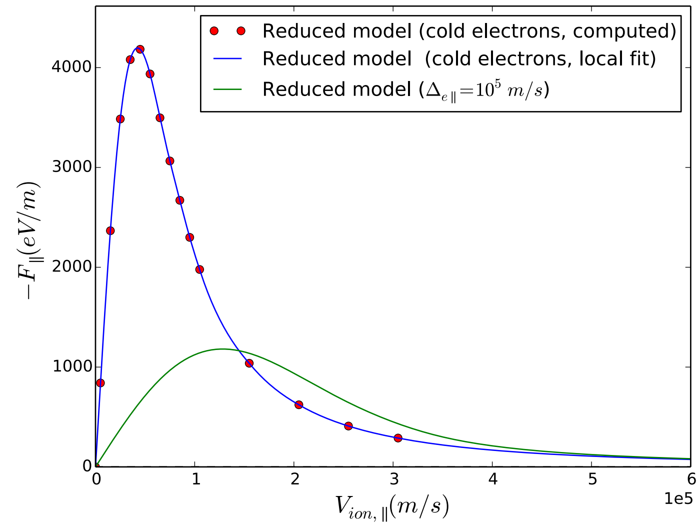
- For cold electrons and Au^{+79} ion, correct/expected qualitative behavior of $F_{\parallel}(V_{ion,\parallel})$ seen for both small and large $V_{ion,\parallel}$:
  - *linear in V for small V*
  - *$1/V^2$ for large V*
- For an arbitrary distribution $f(v_{e,\parallel})$ of *warm* electrons, $F_{\parallel}(V_{ion,\parallel})$ is computed by convolution of $f(v_{e,\parallel})$ with $F_{\parallel}(V_{ion,\parallel})$ for cold electrons

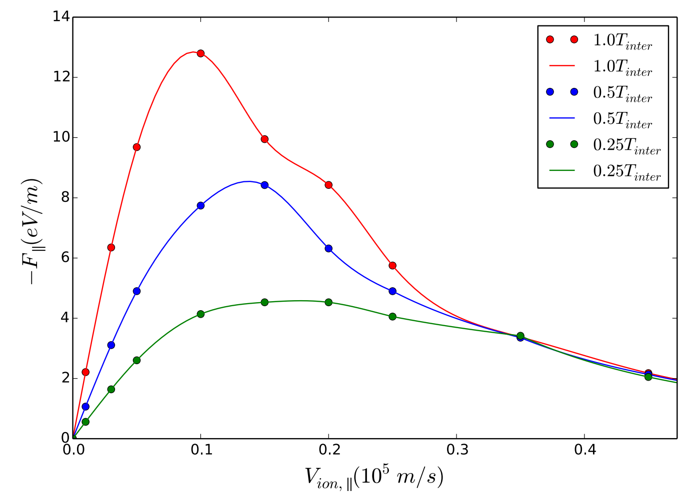


- Convolution with $f(v_{e,\parallel})$ acts as a smoothing filter => peak of $F_{\parallel}(V_{ion,\parallel})$ for warm electrons is lower and shifted towards larger $V_{ion,\parallel}$
- Just as for cold e^- gas, for warm electrons $F_{\parallel}(V_{ion,\parallel})$ is linear in $V_{ion,\parallel}$ for small $V_{ion,\parallel}$ and scales as $1/V^2$ in the large $V_{ion,\parallel}$ region
- As expected, $F_{\parallel}(V_{ion,\parallel})$ for different electron temperatures converge as $V_{ion,\parallel}$ gets larger

$F_{||}(V_{ion,||})$ for cold electrons: scaling in Z and T_{int}

- For cold electrons, looked at protons and Au^{+79} ion and different interaction times in the cooler (still interaction-time-averaged force):
  - *for small V,* $dF_{||}(V)/dV \approx 2Z\, n_e\, m_e\, r_e\, c^2\, T_{int}$
  - *large-V tail is well approximated by* $F_{||} \approx 2\pi Z^2 n_e m_e (r_e c^2)^2 / V^2$, with no dependence on T_{int}
  - *for a given* T_{int}*, peak friction force scales as* $Z^{4/3}$
- For $T_{int} < T_{pl}$ and small-to-moderate V_{ion}, $F_{||}(V_{ion,||})$ goes up with interaction time; large-V tail is T_{int} –independent
- $F_{||}(V_{ion,||})$ is linear in n_e by construction





Asymptotic model for cold, strongly magnetized electrons

$$F_{\parallel} = -\frac{3}{2}\omega_{pe}^2 \frac{(Ze)^2}{4\pi\varepsilon_0}\left[\ln\left(\frac{\rho_{\max}^A}{\rho_{\min}^A}\right)\left(\frac{V_\perp}{V_{ion}}\right)^2 + \frac{2}{3}\right]\frac{V_\parallel}{V_{ion}^3}$$

$$r_L = V_{rms,e,\perp} / \Omega_L(B_\parallel)$$

$$\rho_{\min}^A = \max(r_L, \rho_{\min})$$

$$\rho_{\max}^A = \min(r_{beam}, \rho_{\max})$$

$$\rho_{\max} = V_{rel} / \max(\omega_{pe}, 1/\tau)$$

$$V_{rel} = \max(V_{ion}, V_{e,rms,\parallel})$$

$$V_{ion}^2 = V_\parallel^2 + V_\perp^2$$

or, for large V_{ion} parallel to B

$$F_\parallel(V_\perp = 0) = -4\pi Z^2 n_e m_e (r_e c^2)^2 \frac{1}{V_\parallel^2}$$

Ya. S. Derbenev and A.N. Skrinsky, "The Effect of an Accompanying Magnetic Field on Electron Cooling," Part. Accel. **8** (1978), 235.

Ya. S. Derbenev and A.N. Skrinskii, "Magnetization effects in electron cooling," Fiz. Plazmy **4** (1978), p. 492;   Sov. J. Plasma Phys. **4** (1978), 273.

I. Meshkov, "Electron Cooling; Status and Perspectives," Phys. Part. Nucl. **25** (1994), 631.

Including thermal effects

$$\mathbf{F} = -\frac{1}{\pi}\omega_{pe}^2 \frac{(Ze)^2}{4\pi\varepsilon_0} \ln\left(\frac{\rho_{\max} + \rho_{\min} + r_L}{\rho_{\min} + r_L}\right) \frac{\mathbf{V}_{ion}}{\left(V_{ion}^2 + V_{eff}^2\right)^{3/2}}$$

$$\rho_{\min} = \left(Ze^2/4\pi\varepsilon_0\right)/m_e V_{ion}^2$$

$$\rho_{\max} = V_{ion}/\max\left(\omega_{pe}, 1/\tau\right)$$

$$r_L = V_{rms,e,\perp}/\Omega_L\left(B_{\parallel}\right)$$

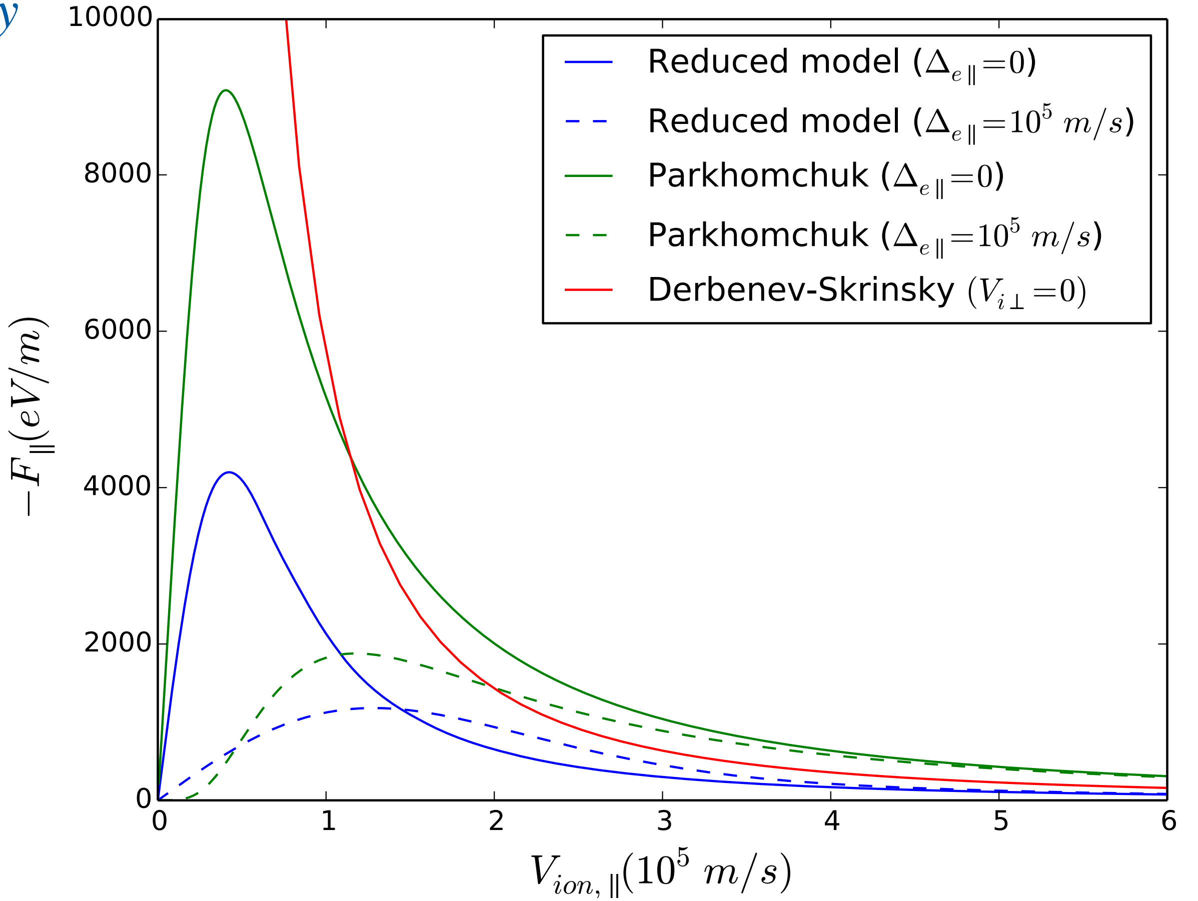
$$V_{eff}^2 = V_{e,rms,\parallel}^2 + \Delta V_{\perp e}^2$$

V.V. Parkhomchuk, “New insights in the theory of electron cooling,” Nucl. Instr. Meth. in Phys. Res. **A 441** (2000).

I. Meshkov, “Electron Cooling; Status and Perspectives,” Phys. Part. Nucl. **25** (1994), 631.

Compare with Derbenev-Skrinsky and Parkhomchuk (1)

- Comparison of new model for an Au^{+79} ion, with:
  - *Derbenev and Skrinsky (D-S) for $V_{ion,\perp} = 0$ and large $V_{ion,\parallel}$*
  - *Parkhomchuk (P) with 0 and finite effective longitudinal e- temperature*
- Consistently lower force than D-S and P for larger $V_{ion,\parallel}$
  - *for lower ion velocity and warm electrons, details depend on **Z***



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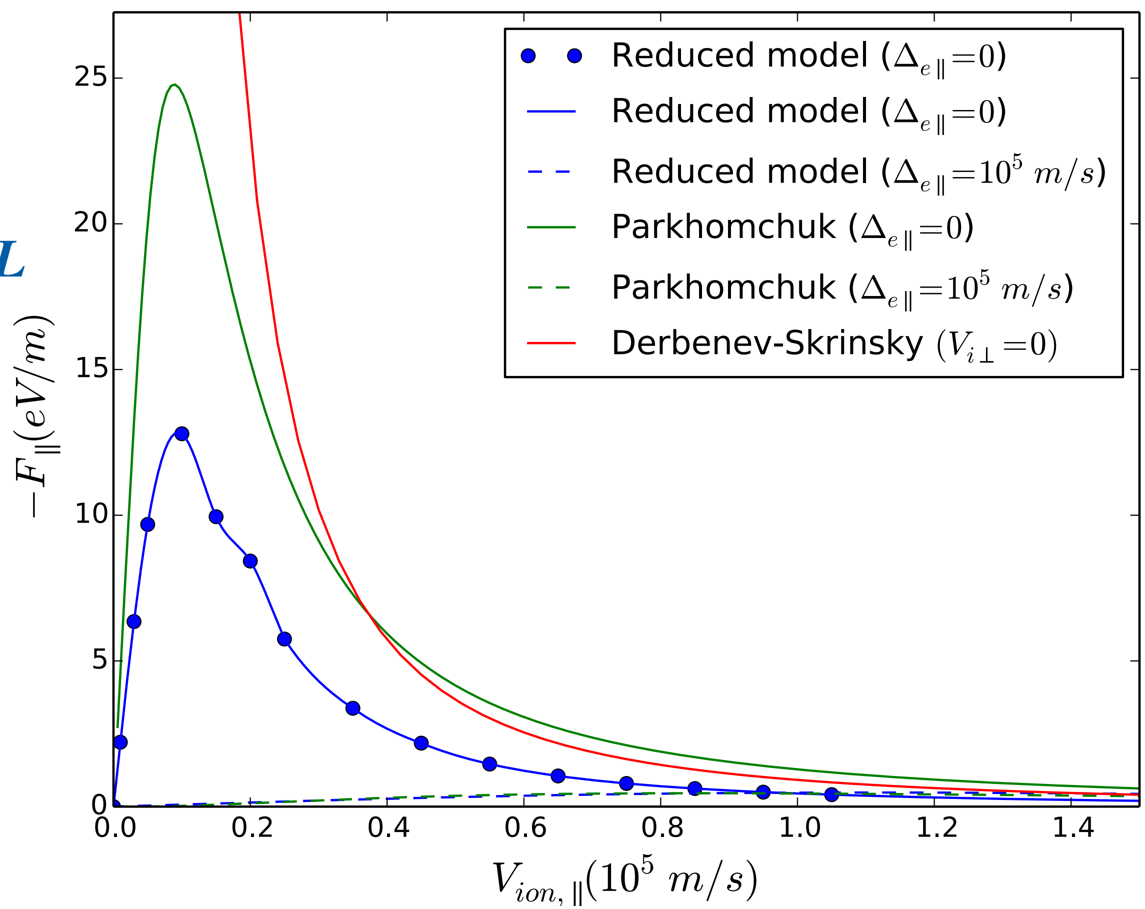
Compare with Derbenev-Skrinsky and Parkhomchuk (2)

- Comparison of new model for protons, with:
  - *Derbenev and Skrinsky (D-S) for $V_{ion,\perp} = 0$ and large $V_{ion,\parallel}$*
  - *Parkhomchuk (P) with 0 and finite effective longitudinal e- temperature*
- Consistently lower force than D-S and P for larger $V_{ion,\parallel}$
- For cold electrons:
  - *new model shows consistently lower force values than Parkhomchuk at **ALL** velocities*



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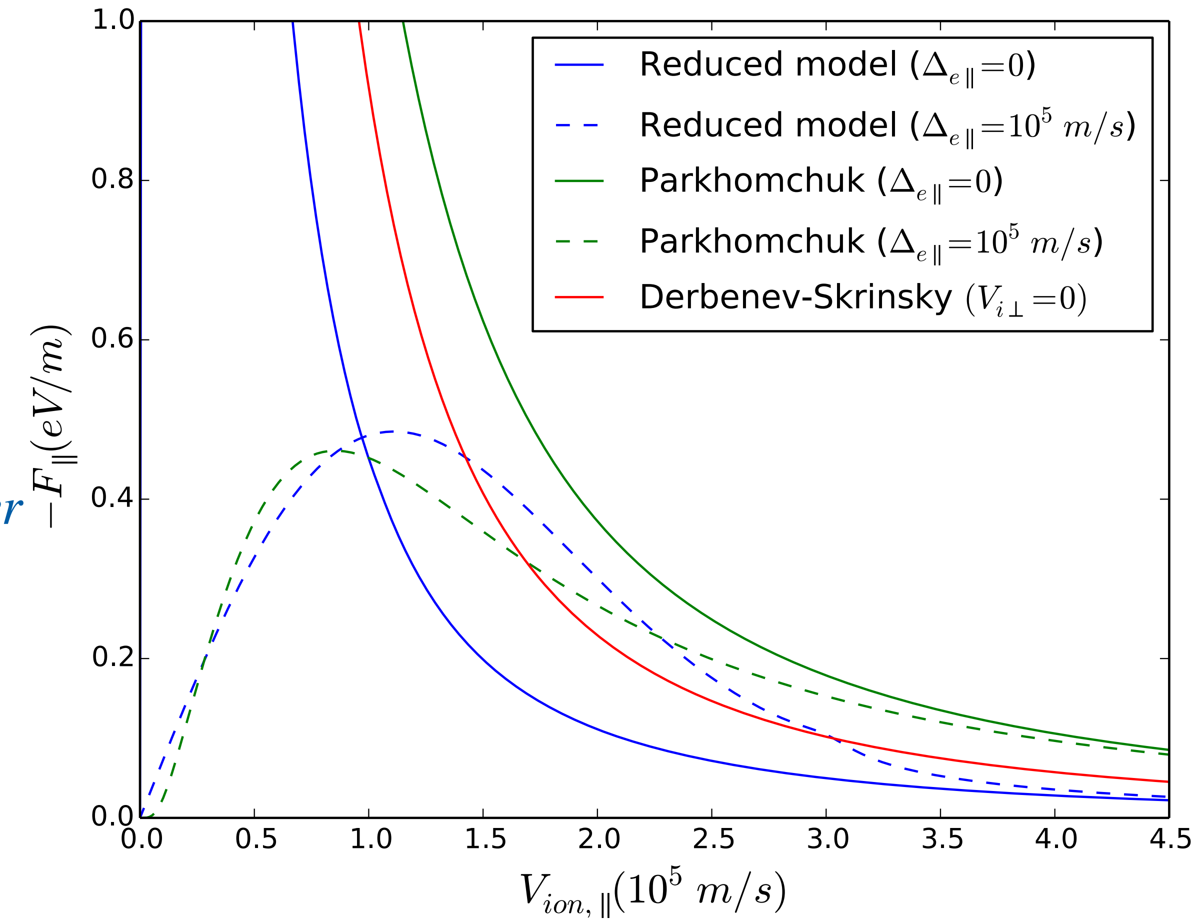
Compare with Derbenev-Skrinsky and Parkhomchuk (3)

- Comparison of new model for protons *(zoomed in)*, with:
  - *Derbenev and Skrinsky (D-S) for $V_{ion,\perp}$ = 0 and large $V_{ion,\parallel}$*
  - *Parkhomchuk (P) with 0 and finite effective* long. *e^- temp*
- Consistently lower force than D-S and P for larger $V_{ion,\parallel}$
- For warm electrons, new model agrees approximately with Parkhomchuk (but details depend on Z)
  - *not yet known how often (in what sub-domain of parameter space) this occurs*



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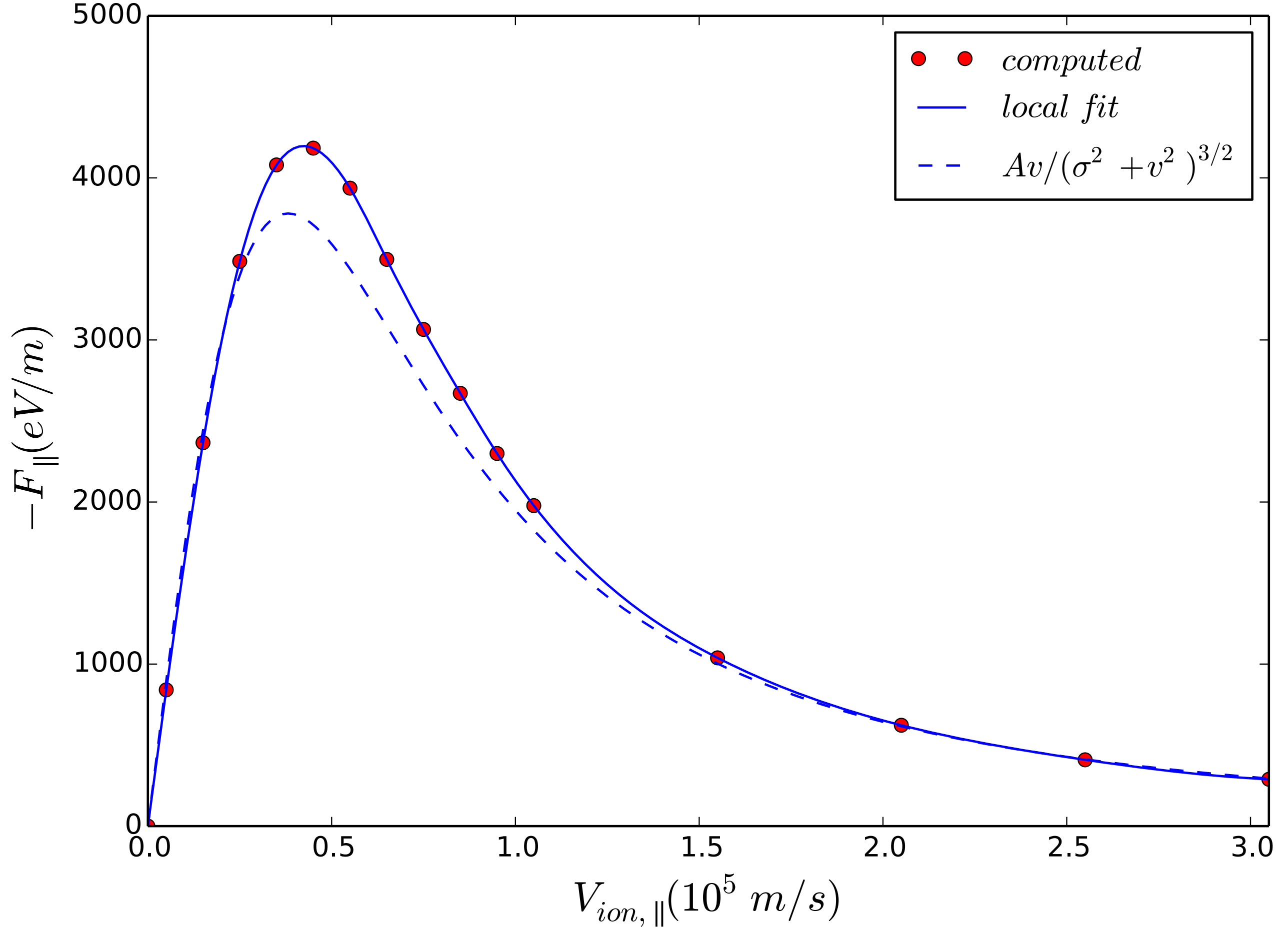
Simple, approximate 2-parameter model

$$F_{\parallel}(v) = \frac{Av}{(\sigma^2 + v^2)^{3/2}}$$

$$A \approx 2\pi Z^2 n_e m_e (r_e c^2)^2$$

$$\sigma \approx (\pi Z r_e c^2 / T_{int})^{1/3}$$

- Large v:
  - $F_{\parallel} \sim A/v^2$
  - *A found via fit*
- Small v:
  - $dF/dv \sim A/\sigma^3$
  - σ *found via fit*
- Peak force is underestimated by ~10%



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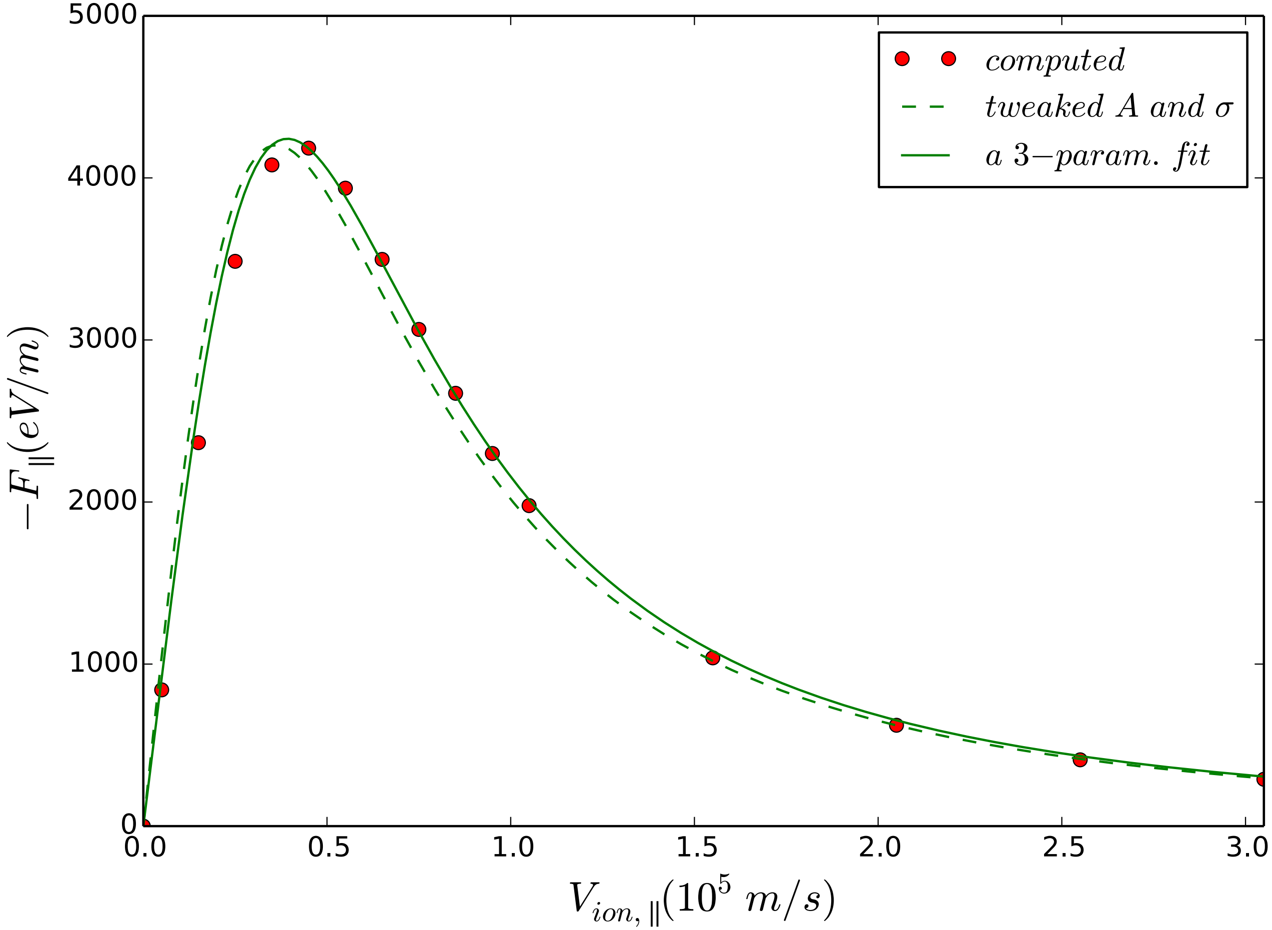
3-parameter model fits the calculations closely

- The physical system depends on 3 parameters:
  - n_e, Z, T_{int}
- Captured via perturbation of 2-parameter model:
  - *Tweak values of A and σ* or *add a small 3rd parameter*
- Improved parametric models are under development



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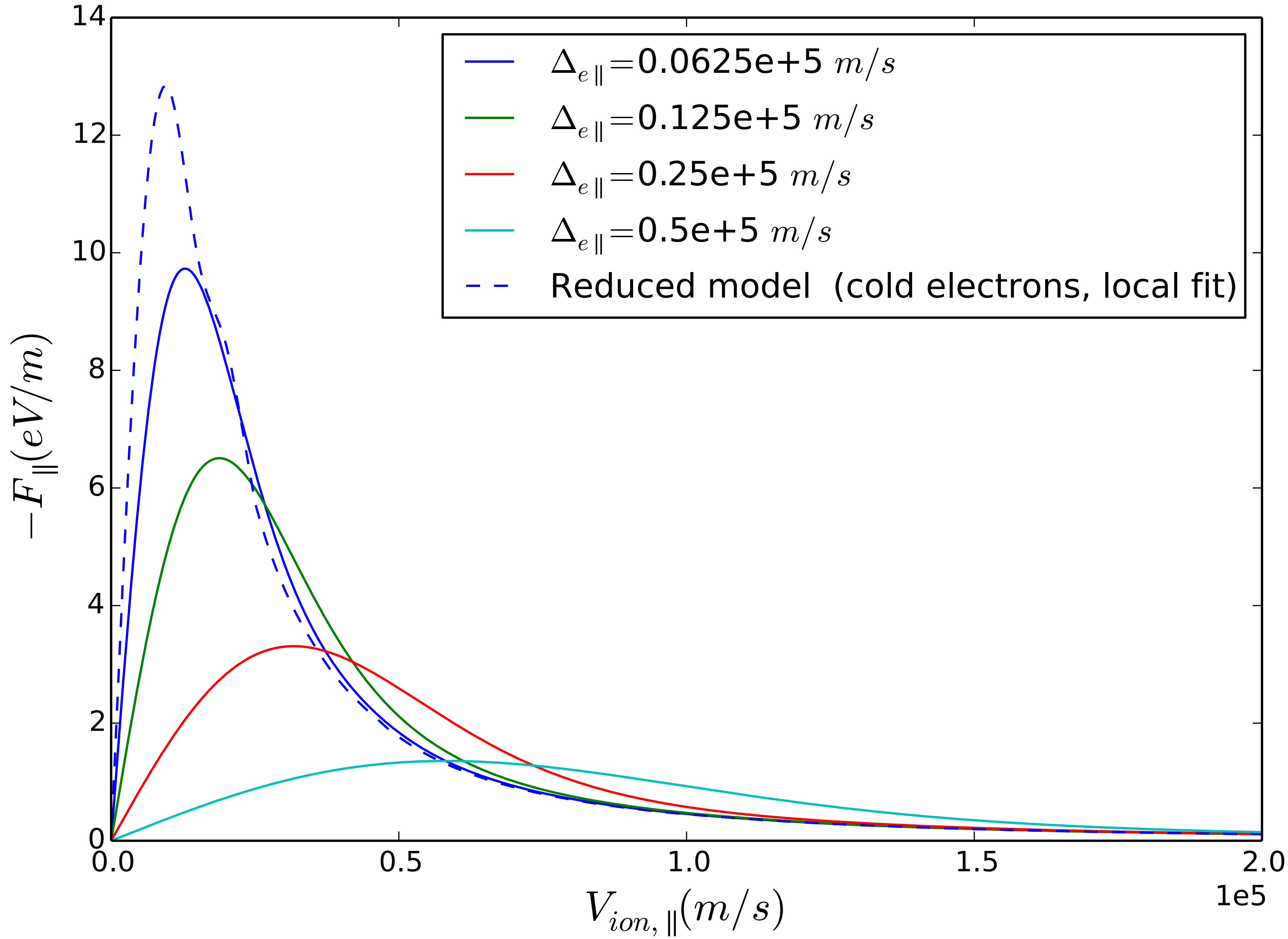
Work in progress and future plans

- Improved parametrized models for cold electrons, and parametrized models for non-zero electron temperature
- Better understanding of the role of trapped vs unbound electron orbits
- The case of finite B
- What happens to the magnitude of dynamic friction force as the interaction time approaches/exceeds T_{pl} ?
- Modeling transverse dynamic friction (have to work with non-zero electron temperature from the start)
- Statistical properties of $F(V)$: so far, only the expectation value was considered (in essence, the continuum limit)
- Adding new models to JSPEC as they become available, simulations in the JLEIC parameter regime
  - https://sirepo.com

Thank You!

Questions?



14
12
10
8
6
4
2
0
0.0
0.5
1.0
1.5
2.0
1e5
$-F_{\|}(eV/m)$
$V_{ion,\|}(m/s)$
$\Delta_{e\,\|}$=0.0625e+5 m/s
$\Delta_{e\,\|}$=0.125e+5 m/s
$\Delta_{e\,\|}$=0.25e+5 m/s
$\Delta_{e\,\|}$=0.5e+5 m/s
Reduced model (cold electrons, local fit)