Higher-Twist Fragmentation Functions in SIDIS, e^+e^-, and pp

Daniel Pitonyak

Lebanon Valley College, Annville, PA

Workshop on Novel Probes of the Nucleon Structure (FF2019)

Duke University

Durham, NC

March 15, 2019

Multi-Parton Fragmentation Functions in SIDIS, e^+e^-, and pp

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Outline

- Definitions of *collinear* twist-3 FFs
- Important relations: EoMRs and LIRs
- SIDIS, e^+e^-, and *pp* observables that probe twist-3 FFs
- Connection of *TMD* FFs to *collinear* twist-3 FFs
- Open issues
- Summary

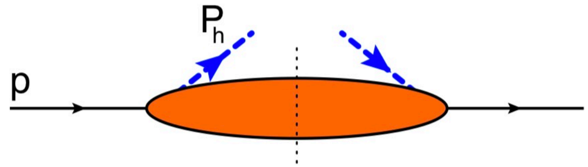
- Interesting topics not covered in this talk
  - Observables for twist-3 *TMD* FF (see, e.g., data from CLAS and HERMES; Yang, Lu, Schmidt (2016),…)
  - Observables for twist-3 *DiFF* (e.g., $\tilde{G}^{\sphericalangle}$ probed in single-longitudinal dihadron production in SIDIS (data from COMPASS; Yang, Wang, Yang, Lu (2019),…))
  - Connection of the chiral-odd twist-3 FF E and $\tilde{E}$ to dynamical chiral symmetry breaking (Accardi & Signori (2018, 2019)) and probing transversity in inclusive DIS (Accardi & Bacchetta (2017)))

Definitions of Collinear Twist-3 FFs

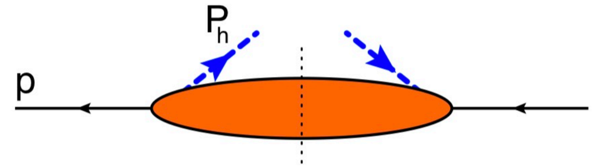
(Gamberg, Kang, DP, Schlegel, Yoshida JHEP **1901** (2019))

Intrinsic FF

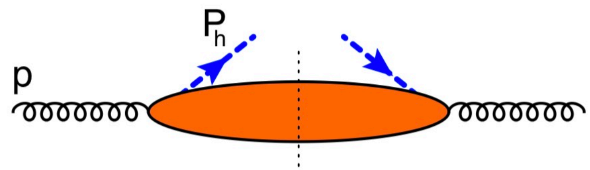
($p = P_h/z$)



(a) Quark fragmentation



(b) Antiquark fragmentation



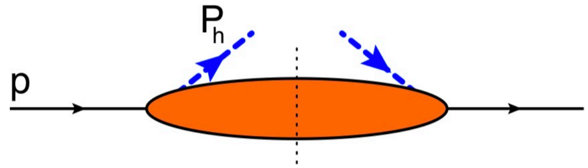
(c) Gluon fragmentation

$$\Delta^q_{ij}(z) = \frac{z^{2\varepsilon}}{z}\Big(\not{P}_h\, D_1^q(z) - S_{hL}\, \not{P}_h \gamma_5\, G_1^q(z) - \tfrac{1}{2}[\not{P}_h, \not{S}_h]\gamma_5\, H_1^q(z)$$
$$- M_h \epsilon^{P_h n \alpha S_h}\gamma_\alpha\, \boldsymbol{D_T^q(z)} - M_h \not{S}_{hT}\gamma_5\, \boldsymbol{G_T^q(z)} + M_h\, \boldsymbol{E^q(z)}$$
$$- M_h\, S_{hL}\, i\gamma_5\, \boldsymbol{E_L^q(z)} + M_h \tfrac{i}{2}[\not{P}_h, \not{n}]\, \boldsymbol{H^q(z)} + M_h\, S_{hL}\, \tfrac{1}{2}[\not{P}_h, \not{n}]\gamma_5\, \boldsymbol{H_L^q(z)} \Big)$$

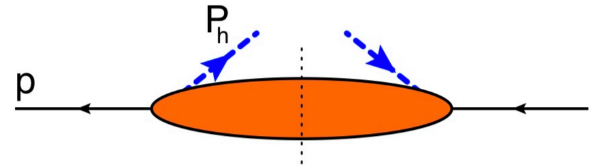
$$\Delta^{g;\mu\nu}(z) = \frac{z^{2\varepsilon}}{z^2}\Big(- g_T^{\mu\nu}\, D_1^g(z) - S_L\, i\epsilon^{P_h n \mu\nu}\, G_1^g(z)$$
$$- M_h\, n^{\{\mu}\epsilon^{\nu\} P_h n S_{hT}}\, \boldsymbol{D_T^g(z)} - i\, M_h\, n^{[\mu}\epsilon^{\nu] P_h n S_{hT}}\, \boldsymbol{G_T^g(z)} \Big)$$

Kinematical FF

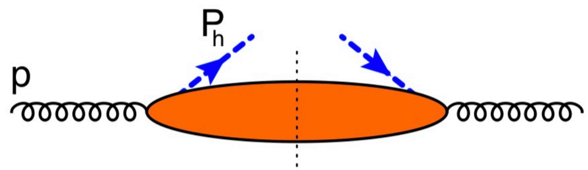
$(p = P_h/z + p_T)$



(a) Quark fragmentation



(b) Antiquark fragmentation



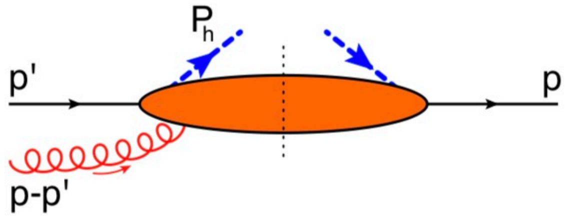
(c) Gluon fragmentation

$$\Delta^{q;\rho}_{\partial;ij}(z) = \frac{z^{2\varepsilon}}{z} M_h \Big(\epsilon^{P_h n \rho S_{hT}} \not{P}_h \, \boldsymbol{D_{1T}^{\perp(1),q}(z)} - S_{hT}^{\rho} \not{P}_h \gamma_5 \, \boldsymbol{G_{1T}^{\perp(1),q}(z)} + \frac{i}{2} [\not{P}_h, \gamma_T^{\rho}] \, \boldsymbol{H_1^{\perp(1),q}(z)} + \frac{1}{2} S_L \, [\not{P}_h, \gamma_T^{\rho}] \gamma_5 \, \boldsymbol{H_{1L}^{\perp(1),q}(z)} \Big)$$

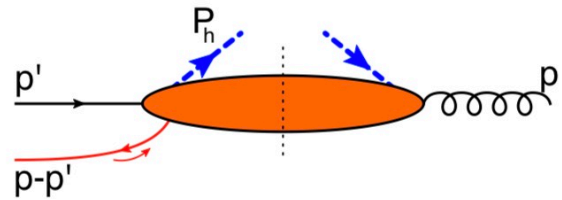
$$\Delta^{g;\mu\nu;\rho}_{\partial}(z) = \frac{z^{2\varepsilon}}{z^2} M_h \Big(g_T^{\mu\nu} \, \epsilon^{P_h n \rho S_h} \, \boldsymbol{D_{1T}^{\perp(1)g}(z)} + i\epsilon^{P_h n \mu\nu} \, S_{hT}^{\rho} \, \boldsymbol{G_{1T}^{\perp(1)g}(z)} - \frac{1}{2} \Big(g_T^{\rho\{\mu} \epsilon^{\nu\} P_h n S_h} + S_{hT}^{\{\mu} \epsilon^{\nu\} P_h n \rho} \Big) \boldsymbol{H_1^{(1)g}(z)} \Big)$$

Dynamical FF

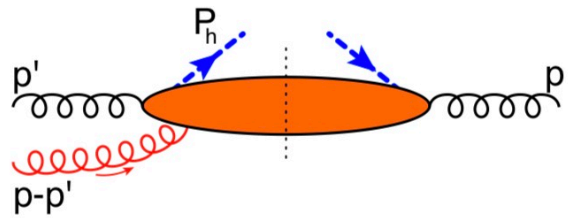
($p = P_h/z$; $p' = P_h/z'$)



(a) qg fragmentation



(b) $q\bar{q}$ fragmentation



(c) gg fragmentation

$\beta = z/z'$

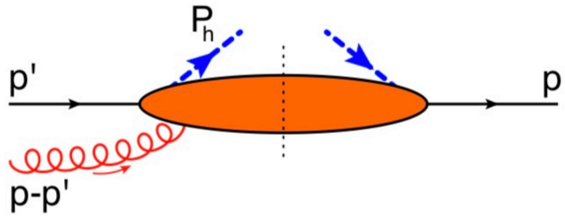
$$\Delta_{F;ij}^{qg;\rho}(z,\beta) = z^{2\varepsilon}\,\frac{M_h}{z}\Big(\epsilon^{P_h n \rho S_h}\,\not{P}_h\, i(\hat{D}_{FT}^{qg})^*(z,\beta) + S_{hT}^{\rho}\,\not{P}_h\gamma_5\,(\hat{G}_{FT}^{qg})^*(z,\beta) + \frac{i}{2}[\not{P}_h,\gamma_T^{\rho}]\, i(\hat{H}_{FU}^{qg})^*(z,\beta) - \frac{1}{2}S_{hL}[\not{P}_h,\gamma_T^{\rho}]\gamma_5\,(\hat{H}_{FL}^{qg})^*(z,\beta)\Big)$$

$$\Delta_{F;ij}^{q\bar{q};\rho}(z,\beta) = (\hat{D}_{FT}^{qg},\hat{G}_{FT}^{qg},\hat{H}_{FU}^{qg},\hat{H}_{FL}^{qg}) \rightarrow (\hat{D}_{FT}^{q\bar{q}},\hat{G}_{FT}^{q\bar{q}},\hat{H}_{FU}^{q\bar{q}},\hat{H}_{FL}^{q\bar{q}})$$

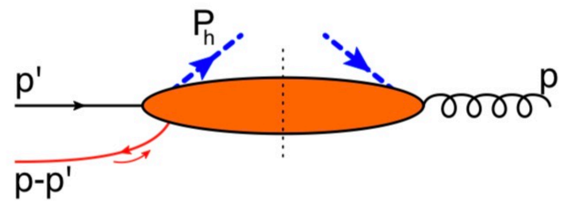
$$\Delta_{F;ij}^{\bar{q}q;\rho}(z,\beta) = (\hat{D}_{FT}^{qg},\hat{G}_{FT}^{qg},\hat{H}_{FU}^{qg},\hat{H}_{FL}^{qg}) \rightarrow (\hat{D}_{FT}^{\bar{q}q},-\hat{G}_{FT}^{\bar{q}q},\hat{H}_{FU}^{\bar{q}q},\hat{H}_{FL}^{\bar{q}q})$$

Dynamical FF

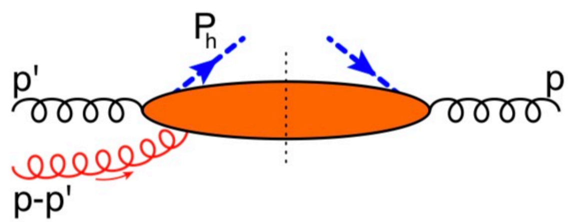
($p = P_h/z$; $p' = P_h/z'$)



(a) qg fragmentation



(b) $q\bar{q}$ fragmentation



(c) gg fragmentation

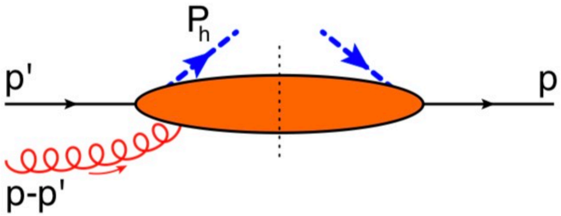
$\beta = z/z'$

$$\Delta_{F;ij}^{qg;\rho}(z,\beta) = z^{2\varepsilon}\frac{M_h}{z}\Big(\epsilon^{P_h n \rho S_h}\,\not{P}_h\, i(\hat{D}_{FT}^{qg})^*(z,\beta) + S_{hT}^{\rho}\,\not{P}_h\gamma_5\,(\hat{G}_{FT}^{qg})^*(z,\beta) + \frac{i}{2}[\not{P}_h,\gamma_T^{\rho}]\,i(\hat{H}_{FU}^{qg})^*(z,\beta) - \frac{1}{2}S_{hL}[\not{P}_h,\gamma_T^{\rho}]\gamma_5\,(\hat{H}_{FL}^{qg})^*(z,\beta)\Big)$$

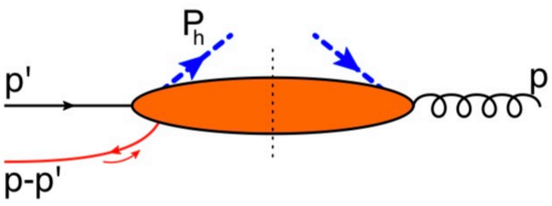
$$\Delta_F^{gg;\mu\nu\rho}(z,\beta) = -z^{2\varepsilon}\frac{M_h}{z^2}\Big[g_T^{\mu\nu} i\epsilon^{P_h n \rho S_h}\,\hat{N}_2^*(z,\beta) - g_T^{\mu\rho} i\epsilon^{P_h n \nu S_h}\,\hat{N}_2^*(z,1-\beta) - g_T^{\nu\rho} i\epsilon^{P_h n \mu S_h}\,\hat{N}_1^*(z,\beta)\Big]$$

Dynamical FF

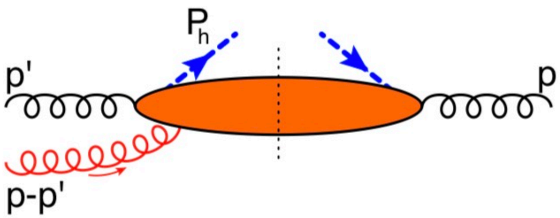
($p = P_h/z$; $p' = P_h/z'$)



(a) qg fragmentation



(b) $q\bar{q}$ fragmentation



(c) gg fragmentation

We note that the (gluonic and fermionic) *poles* of the dynamical FF *vanish* (Meissner & Metz (2009)). This makes the calculation of twist-3 fragmentation effects different from the calculation of soft-gluon and soft-fermion poles on the PDF side.

Important Relations: EoMRs and LIRs

QCD equation of motion relations (EoMRs) and Lorentz invariance relations (LIRs) are necessary to guarantee

- EM and color gauge invariance of the cross section
- Frame independence of the cross section

(Kanazawa, Metz, DP, Schlegel, PLB **742** (2015); Kanazawa, Metz, DP, Schlegel, PLB **744** (2015); Koike, DP, Takagi, Yoshida PLB **752** (2016); Koike, DP, Yoshida PLB **759** (2016); Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016); Gamberg, Kang, DP, Schlegel, Yoshida JHEP **1901** (2019))

They are known for both twist-3 PDFs and FFs in the quark sector

EoMRs are known in the gluon sector but LIRs have not been derived

EoMR

$$H^q(z) = -2z H_1^{\perp(1),q}(z) + 2z \int_z^\infty \frac{dz_1}{z_1^2} \frac{1}{\frac{1}{z} - \frac{1}{z_1}} \hat{H}_{FU}^{qg,\Im}(z, z_1)$$

LIR

$$\frac{H^q(z)}{z} = -\left(1 - z\frac{d}{dz}\right) H_1^{\perp(1),q}(z) - \frac{2}{z} \int_z^\infty \frac{dz_1}{z_1^2} \frac{\hat{H}_{FU}^{qg,\Im}(z, z_1)}{(1/z - 1/z_1)^2}$$

(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))

EoMR

$$H^q(z) = -2z H_1^{\perp(1),q}(z) + 2z\int_z^\infty \frac{dz_1}{z_1^2}\frac{1}{\frac{1}{z}-\frac{1}{z_1}}\hat{H}_{FU}^{qg,\mathfrak{S}}(z,z_1)$$

LIR

$$\frac{H^q(z)}{z} = -\left(1 - z\frac{d}{dz}\right)H_1^{\perp(1),q}(z) - \frac{2}{z}\int_z^\infty \frac{dz_1}{z_1^2}\frac{\hat{H}_{FU}^{qg,\mathfrak{S}}(z,z_1)}{(1/z-1/z_1)^2}$$

⬇

$$H^q(z) = \int_z^1 dz_1 \int_{z_1}^\infty \frac{dz_2}{z_2^2} 2\left[\frac{\left(2(\frac{2}{z_1}-\frac{1}{z_2}) + \frac{1}{z_1}\left(\frac{1}{z_1}-\frac{1}{z_2}\right)\delta\left(\frac{1}{z_1}-\frac{1}{z}\right)\right)\hat{H}_{FU}^{qg,\mathfrak{S}}(z_1,z_2)}{\left(\frac{1}{z_1}-\frac{1}{z_2}\right)^2}\right]$$

$$H_1^{\perp(1),q}(z) = -\frac{2}{z}\int_z^1 dz_1 \int_{z_1}^\infty \frac{dz_2}{z_2^2}\frac{\left(\frac{2}{z_1}-\frac{1}{z_2}\right)}{\left(\frac{1}{z_1}-\frac{1}{z_2}\right)^2}\hat{H}_{FU}^{qg,\mathfrak{S}}(z_1,z_2)$$

(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))

EoMR

$$D_T^q(z) = -z D_{1T}^{\perp(1),q}(z) + z \int_z^\infty \frac{dz_1}{z_1^2} \frac{\left(\hat{D}_{FT}^{qg,\mathfrak{S}}(z,z_1) - \hat{G}_{FT}^{qg,\mathfrak{S}}(z,z_1)\right)}{\frac{1}{z} - \frac{1}{z_1}}$$

LIR

$$\frac{D_T^q(z)}{z} = -\left(1 - z\frac{d}{dz}\right) D_{1T}^{\perp(1),q}(z) - \frac{2}{z}\int_z^\infty \frac{dz_1}{z_1^2} \frac{\hat{D}_{FT}^{qg,\mathfrak{S}}(z,z_1)}{(1/z - 1/z_1)^2}$$

$$D_T^q(z) = -z \int_z^1 \frac{dz_1}{z_1} \int_{z_1}^\infty \frac{dz_2}{z_2^2} \left[\frac{\left(1 + \frac{1}{z_1}\delta\left(\frac{1}{z_1} - \frac{1}{z}\right)\right) \hat{G}_{FT}^{qg,\mathfrak{S}}(z_1,z_2)}{\frac{1}{z_1} - \frac{1}{z_2}} - \frac{\left(\frac{3}{z_1} - \frac{1}{z_2} + \frac{1}{z_1}\left(\frac{1}{z_1} - \frac{1}{z_2}\right)\delta\left(\frac{1}{z_1} - \frac{1}{z}\right)\right) \hat{D}_{FT}^{qg,\mathfrak{S}}(z_1,z_2)}{\left(\frac{1}{z_1} - \frac{1}{z_2}\right)^2} \right]$$

$$D_{1T}^{\perp(1),q}(z) = \int_z^1 \frac{dz_1}{z_1} \int_{z_1}^\infty \frac{dz_2}{z_2^2} \left[\frac{\hat{G}_{FT}^{qg,\mathfrak{S}}(z_1,z_2)}{\frac{1}{z_1} - \frac{1}{z_2}} - \frac{\left(\frac{3}{z_1} - \frac{1}{z_2}\right) \hat{D}_{FT}^{qg,\mathfrak{S}}(z_1,z_2)}{\left(\frac{1}{z_1} - \frac{1}{z_2}\right)^2} \right]$$

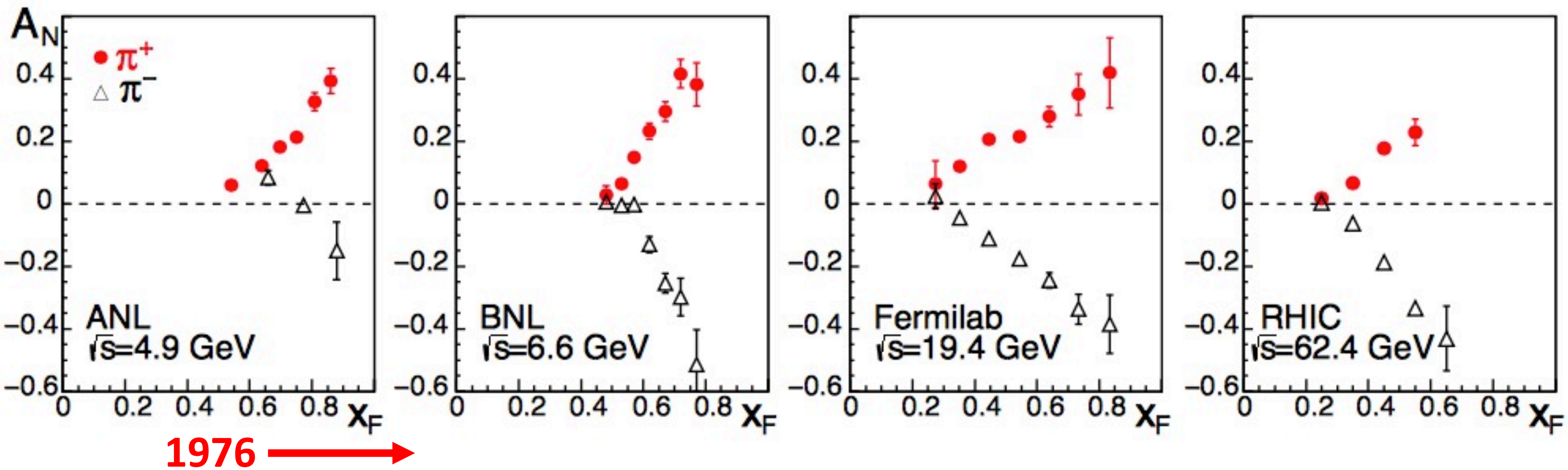
(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))

All kinematical and intrinsic functions can be written in terms of dynamical functions (multi-parton correlators)!

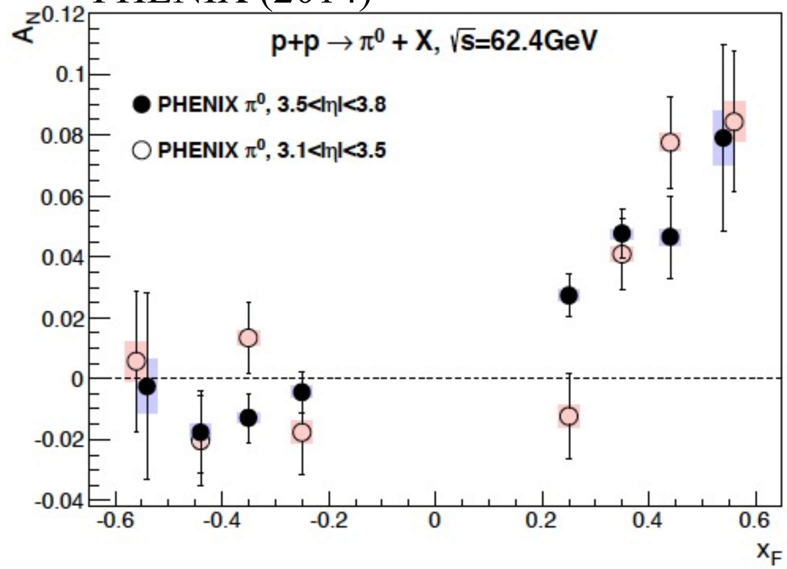
(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))

Observables that Probe Twist-3 FFs

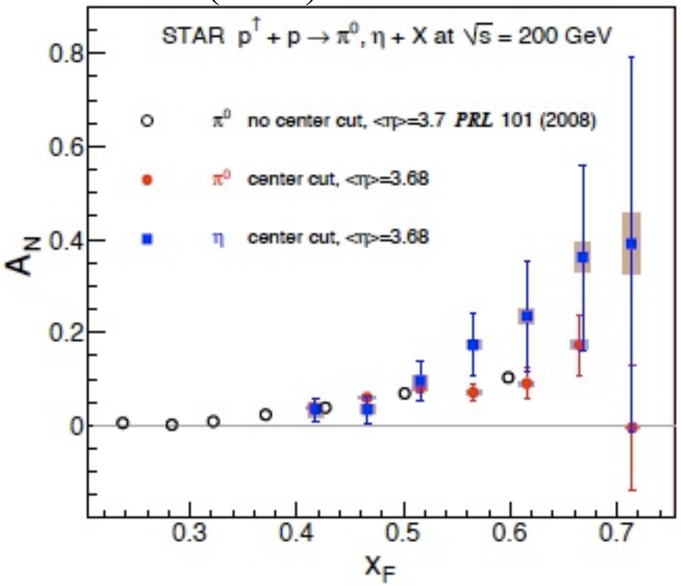
A_N in $pp \to \pi X$ – PUZZLE FOR 40+ YEARS!



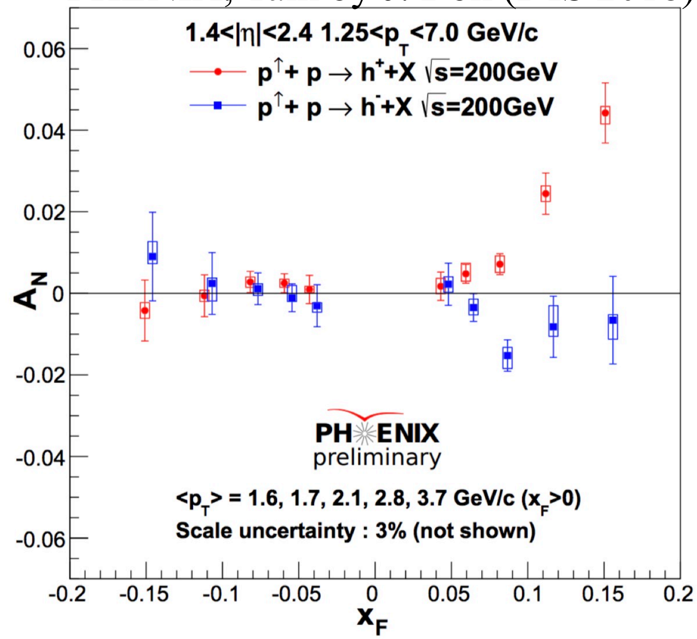
PHENIX (2014)



STAR (2012)



PHENIX, Talk by J. Bok (DIS 2018)



$$d\Delta\sigma^{\pi} \sim H \otimes f_1 \otimes \boldsymbol{F_{FT}(x,x)}$$

$$E_\ell \frac{d^3\Delta\sigma(\vec{s}_T)}{d^3\ell} = \frac{\alpha_s^2}{S} \sum_{a,b,c} \int_{z_{\rm min}}^1 \frac{dz}{z^2} D_{c\to h}(z) \int_{x'_{\rm min}}^1 \frac{dx'}{x'} \frac{1}{x'S + T/z} \phi_{b/B}(x') \times \sqrt{4\pi\alpha_s} \left(\frac{\epsilon^{\ell s_T n \bar{n}}}{z\hat{u}}\right) \frac{1}{x} \left[T_{a,F}(x,x) - x\left(\frac{d}{dx}T_{a,F}(x,x)\right)\right] H_{ab\to c}(\hat{s},\hat{t},\hat{u})$$

$$F_{FT} \sim T_F$$

(Qiu & Sterman (1999), Kouvaris, et al. (2006))

For many years the Qiu-Sterman/Sivers-type contribution was thought to be the dominant source of TSSAs in $\boldsymbol{p^{\uparrow}p \longrightarrow \pi\, X}$

$$d\Delta\sigma^{\pi} \sim H \otimes f_1 \otimes \boldsymbol{F_{FT}(x,x)}$$

(Kang, Qiu, Vogelsang, Yuan (2011); Kang and Prokudin (2012); Metz, DP, Schäfer, Schlegel, Vogelsang, Zhou (2012))

$$\sout{d\Delta\sigma^{\pi} \sim H \otimes f_1 \otimes \boldsymbol{F_{FT}(x,x)}}$$

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes S \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{H}, \int \frac{dz_1}{z_1^2} \frac{\boldsymbol{\hat{H}_{FU}^{\Im}}}{(1/z - 1/z_1)^2} \right)$$

$$E_h \frac{d\Delta\sigma^{Frag}(S_T)}{d^3\vec{P}_h} = -\frac{4\alpha_s^2 M_h}{S}\, \epsilon^{P'PP_hS_T} \sum_i \sum_{a,b,c} \int_0^1 \frac{dz}{z^3} \int_0^1 dx' \int_0^1 dx\; \delta(\hat{s}+\hat{t}+\hat{u}) \frac{1}{\hat{s}\,(-x'\hat{t} - x\hat{u})}$$
$$\times\; h_1^a(x)\, f_1^b(x') \left\{ \left[H_1^{\perp(1),c}(z) - z\frac{dH_1^{\perp(1),c}(z)}{dz} \right] S_{H_1^{\perp}}^i + \frac{1}{z} H^c(z)\, S_H^i \right.$$
$$\left. + \frac{2}{z} \int_z^{\infty} \frac{dz_1}{z_1^2} \frac{1}{\left(\frac{1}{z} - \frac{1}{z_1}\right)^2} \hat{H}_{FU}^{c,\Im}(z,z_1)\, S_{\hat{H}_{FU}}^i \right\}$$

Non-pole matrix element!

(Metz & DP - PLB **723** (2013))

$$\sout{d\Delta\sigma^\pi \sim H \otimes f_1 \otimes \boldsymbol{F_{FT}(x,x)}}$$

$$d\Delta\sigma^\pi \sim \boldsymbol{h_1} \otimes S \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{H}, \int \frac{dz_1}{z_1^2} \frac{\boldsymbol{\hat{H}_{FU}^{\Im}}}{(1/z - 1/z_1)^2} \right)$$

$$E_h \frac{d\Delta\sigma^{Frag}(S_T)}{d^3\vec{P}_h} = -\frac{4\alpha_s^2 M_h}{S}\,\epsilon^{P'PP_hS_T} \sum_i \sum_{a,b,c} \int_0^1 \frac{dz}{z^3} \int_0^1 dx' \int_0^1 dx\; \delta(\hat{s}+\hat{t}+\hat{u}) \frac{1}{\hat{s}\,(-x'\hat{t} - x\hat{u})}$$
$$\times\; h_1^a(x)\, f_1^b(x') \left\{ \left[H_1^{\perp(1),c}(z) - z\frac{dH_1^{\perp(1),c}(z)}{dz} \right] S^i_{H_1^\perp} + \frac{1}{z} H^c(z)\, S^i_H \right.$$
$$\left. + \frac{2}{z} \int_z^\infty \frac{dz_1}{z_1^2} \frac{1}{\left(\frac{1}{z} - \frac{1}{z_1}\right)^2} \hat{H}_{FU}^{c,\Im}(z,z_1)\, S^i_{\hat{H}_{FU}} \right\}$$

(Metz & DP - PLB **723** (2013))

We now believe the TSSAs in $p^\uparrow p \to \pi\, X$ are due to fragmentation effects as the partons form pions in the final state

(Kanazawa, Koike, Metz, DP, PRD **89**(RC) (2014);
Gamberg, Kang, DP, Prokudin, PLB **770** (2017))

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes S \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{H}, \int \frac{dz_1}{z_1^2} \frac{\boldsymbol{\hat{H}_{FU}^{\Im}}}{(1/z - 1/z_1)^2} \right)$$

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes S \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{H}, \int \frac{dz_1}{z_1^2} \frac{\boldsymbol{\hat{H}_{FU}^{\Im}}}{(1/z - 1/z_1)^2} \right)$$

EoMR & LIR

$$2z \int_z^{\infty} \frac{dz_1}{z_1^2} \frac{1}{\frac{1}{z} - \frac{1}{z_1}} \hat{H}_{FU}^{qg,\Im}(z, z_1)$$

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes \tilde{S} \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{\tilde{H}} \right)$$

$$E_h \frac{d\Delta\sigma^{Frag}(S_T)}{d^3\vec{P}_h} = -\frac{4\alpha_s^2 M_h}{S} \epsilon^{P'PP_hS_T} \sum_i \sum_{a,b,c} \int_0^1 \frac{dz}{z^3} \int_0^1 dx' \int_0^1 dx \; \delta(\hat{s}+\hat{t}+\hat{u}) \frac{1}{\hat{s}}$$
$$\times \; h_1^a(x)\, f_1^b(x') \left\{ \left[H_1^{\perp(1),c}(z) - z\frac{dH_1^{\perp(1),c}(z)}{dz} \right] \tilde{S}^i_{H_1^\perp} + \left[-2H_1^{\perp(1),c}(z) + \frac{1}{z}\tilde{H}^c(z) \right] \tilde{S}^i_H \right\}$$

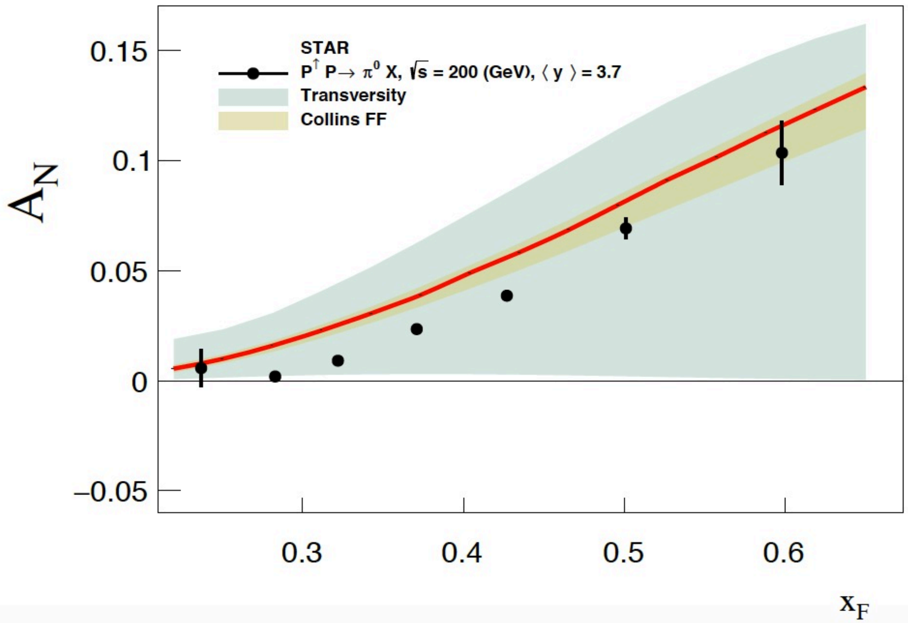
where $\tilde{S}^i_{H_1^\perp} \equiv \dfrac{S^i_{H_1^\perp} - S^i_{H_{FU}}}{-x'\hat{t} - x\hat{u}}$ and $\tilde{S}^i_H \equiv \dfrac{S^i_H - S^i_{H_{FU}}}{-x'\hat{t} - x\hat{u}}$

(Gamberg, Kang, DP, Prokudin, PLB **770** (2017))

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes S \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{H}, \int \frac{dz_1}{z_1^2} \frac{\boldsymbol{\hat{H}_{FU}^{\Im}}}{(1/z - 1/z_1)^2} \right)$$

EoMR & LIR

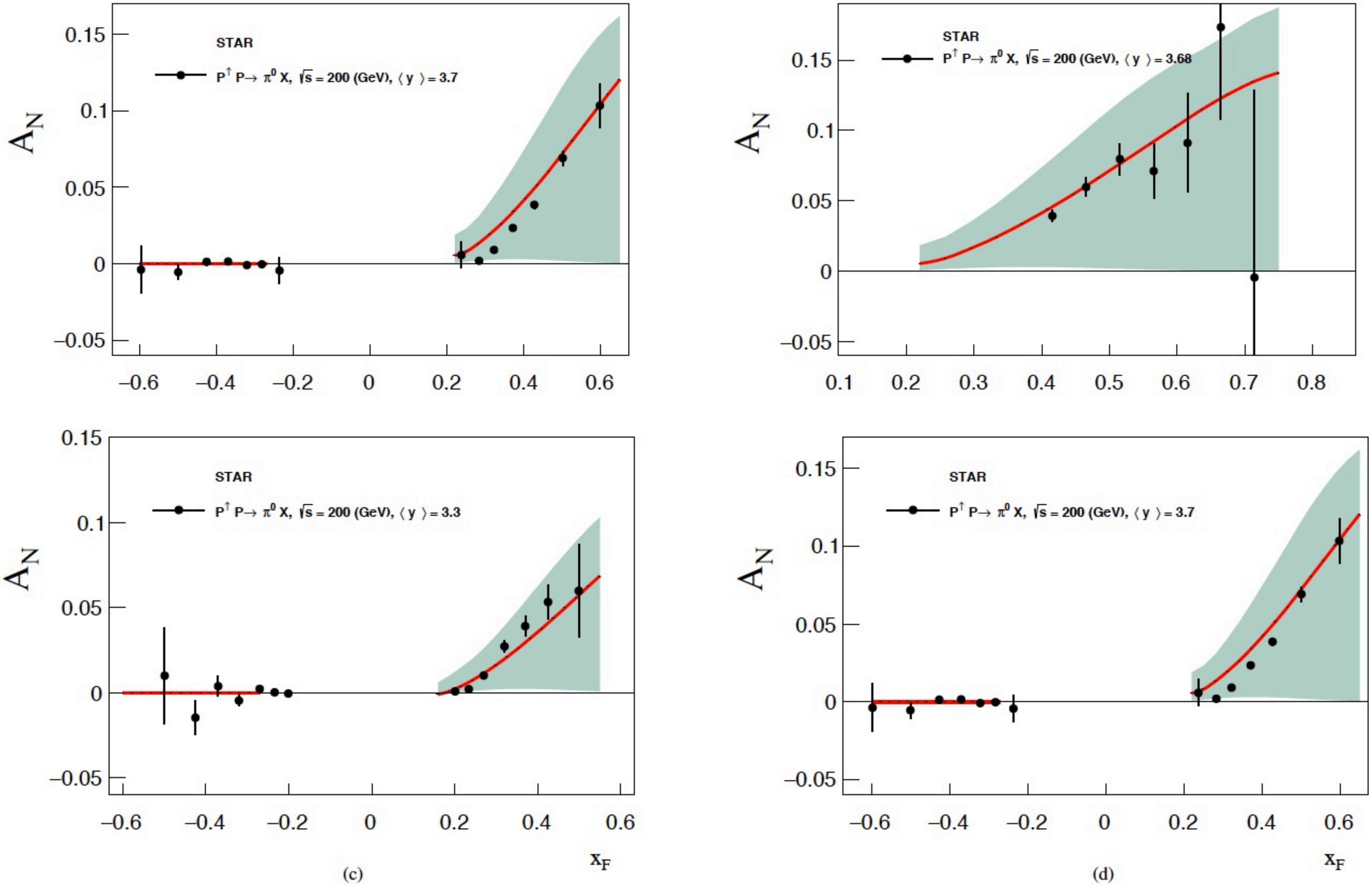
$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes \tilde{S} \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{\tilde{H}} \right)$$



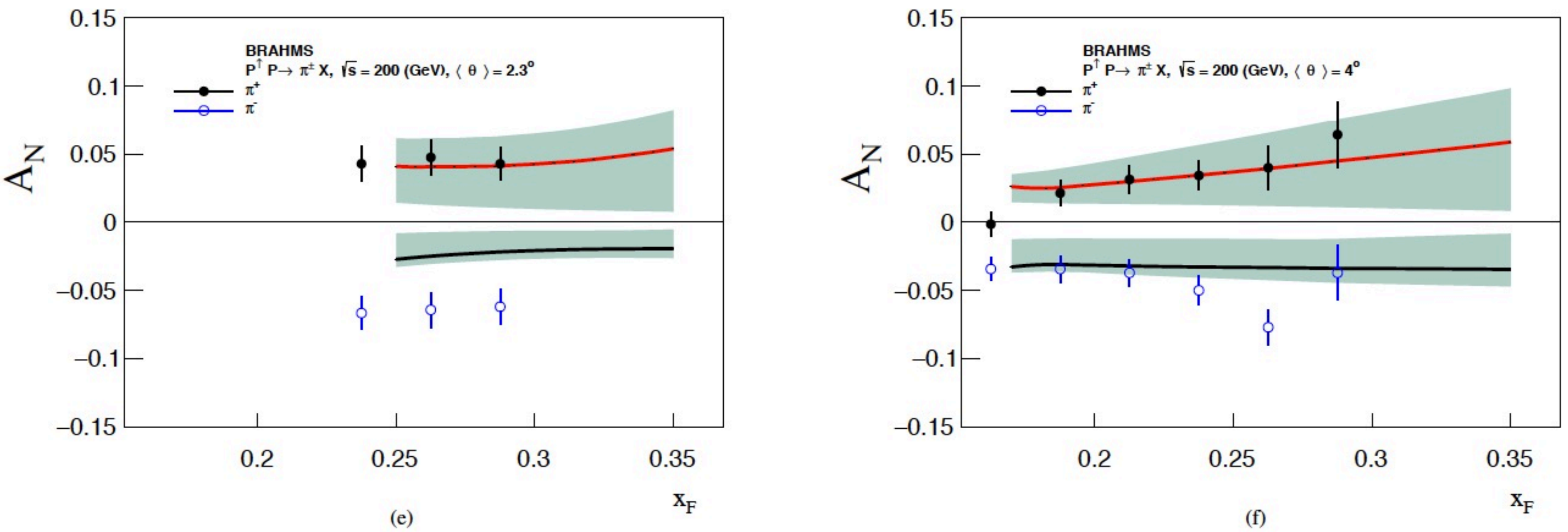
Fragmentation term is the main cause of A_N in *pp -> π X*

The ***A_N data from RHIC*** can be used to constrain ***transversity at large x!***

(Gamberg, Kang, DP, Prokudin, PLB **770** (2017))



(Gamberg, Kang, DP, Prokudin, PLB **770** (2017))



(Gamberg, Kang, DP, Prokudin, PLB **770** (2017))

A_N in $e\,p^{\uparrow} \to \pi\,X$

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes S \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{\tilde{H}}\right)$$

(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))

*Note: Only the fragmentation terms are shown.

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(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))

A_{LT} in $\vec{e}\,p^\uparrow \to \pi\,X$ $\quad d\Delta\sigma^\pi \sim \boldsymbol{h_1} \otimes S \otimes \boldsymbol{E}$

(Kanazawa, Metz, DP, Schlegel PLB **742** (2015))

A_{LT} in $\vec{p}\,p^\uparrow \to \pi\,X$ $\quad d\Delta\sigma^\pi \sim g_1 \otimes \boldsymbol{h_1} \otimes S \otimes \boldsymbol{E}$

(Koike, DP, Takagi, Yoshida PLB **752** (2016))

***Note**: Only the fragmentation terms are shown.

A_N **in** $e\, p^\uparrow \to \pi\, X$ $\quad d\Delta\sigma^\pi \sim h_1 \otimes S \otimes \left(H_1^{\perp(1)}, \tilde{H}\right)$

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(Koike, DP, Takagi, Yoshida PLB **752** (2016))

A_N **in** $e\, p \to \Lambda^\uparrow X$ $\quad d\Delta\sigma^{\Lambda^\uparrow} \sim f_1 \otimes S \otimes \left(D_{1T}^{\perp(1)}, D_T\right)$

(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))

A_N **in** $pp \to \Lambda^\uparrow X$ $\quad d\Delta\sigma^{\Lambda^\uparrow} \sim f_1 \otimes S \otimes \left(D_{1T}^{\perp(1)}, D_T, \int \frac{dz_1}{z_1} \frac{\hat{D}_{FT}^{\Im} + \hat{G}_{FT}^{\Im}}{1/z - 1/z_1}\right)$

(only qq and qgq FF)

(Koike, Metz, DP, Yabe, Yoshida PRD **1901** (2017))

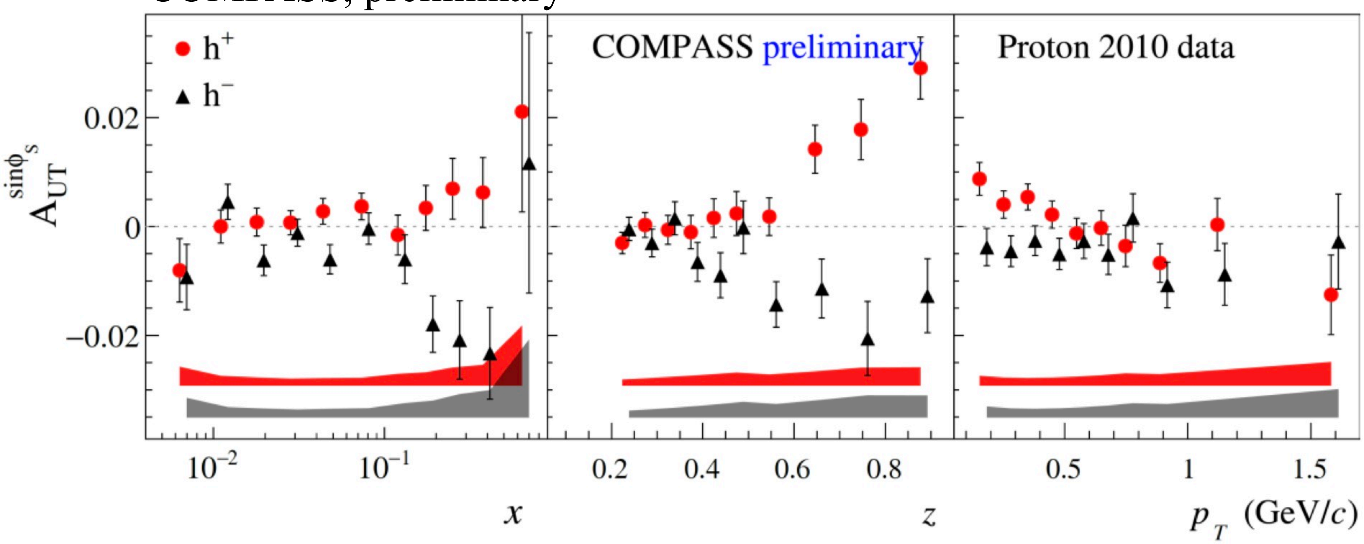
*Note: Only the fragmentation terms are shown.

$A_{UT}^{\sin\phi_S}$ in SIDIS integrated over P_T

$$F_{UT}^{\sin\phi_S} \propto \sum_a e_a^2 \frac{2M_h}{Q} h_1^a(x) \frac{\tilde{H}^a(z)}{z}$$

(Mulders, Tangerman (1996); Bacchetta, et al. (2007); Wang & Lu (2016))

COMPASS, preliminary



A_N in $e^+e^- \to h\,X$

$$\frac{E_h\, d\sigma(S_h)}{d^3\vec{P}_h} = \sigma_0\,(1-2v)\,\frac{8M_h}{z_h\,s^2}\,\epsilon^{ll'P_h S_h}\sum_{f=q,\bar{q}} e_f^2\,\frac{D_T^f(z_h)}{z_h}$$

(Boer, Jakob, Mulders (1997); Gamberg, Kang, DP, Schlegel, Yoshida JHEP **1901** (2019))

NLO calculation is available => evolution of D_T
(Gamberg, Kang, DP, Schlegel, Yoshida JHEP **1901** (2019))

Note that this observable probes the *intrinsic FF* D_T and NOT the polarizing FF $D_{1T}^{\perp}$

A_N in $e^+e^- \to h\,X$

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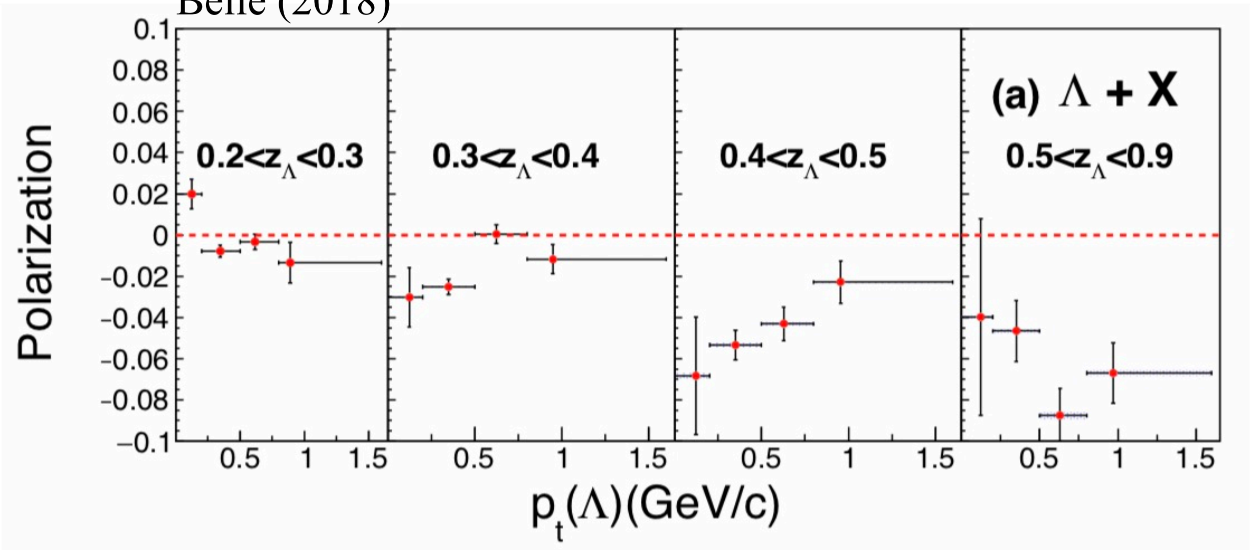
Belle (2018)

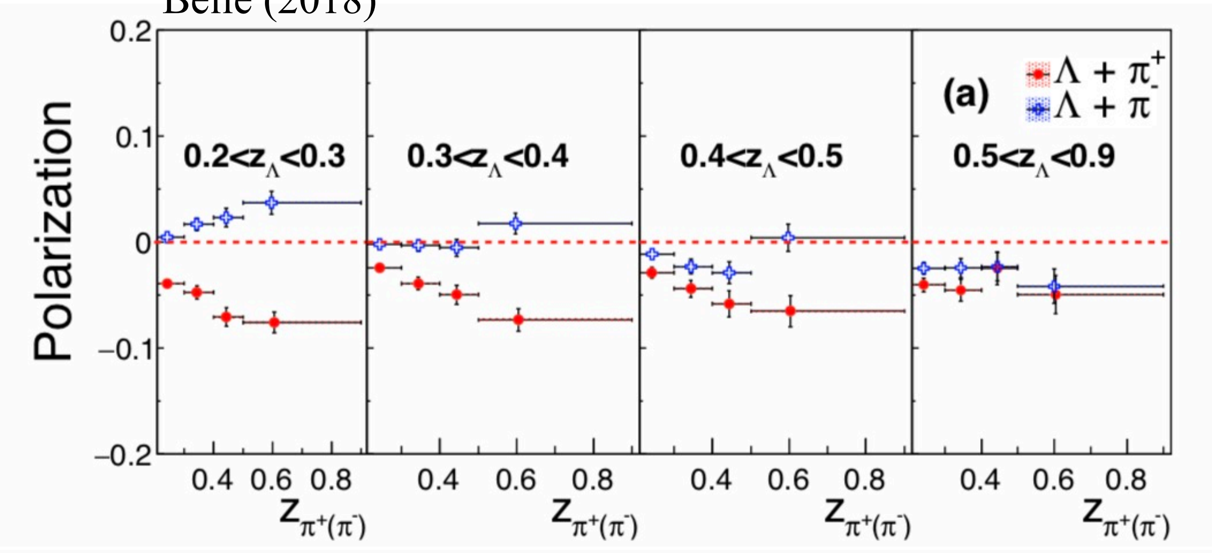


$A_{UT}^{\sin\phi_S}$ in $e^+e^- \to h_1 h_2 X$ integrated over q_T

$$F_{UT}^{\sin\phi_S} \propto \sum_{a,\bar{a}} e_a^2 \left(\frac{2M_2}{Q} D_1^a(z_1) \frac{D_T^{\bar{a}}(z_2)}{z_2} + \frac{2M_1}{Q} \frac{\tilde{H}^a(z_1)}{z_1} H_1^{\bar{a}}(z_2) \right)$$

(Boer, Jakob, Mulders (1997))

Belle (2018)



Connecting TMD FFs to Collinear (Twist-3) FF

TMD (b-space)

Kinematical FF

$$\tilde{H}_1^{\perp(1)}(z, b_T; Q^2, \mu_Q) \sim H_1^{\perp(1)}(z; \mu_{b_*}) \exp\left[-S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q) - S_{NP}^{H_1^\perp}(b_T, Q)\right]$$

$$S_{NP}^{H_1^\perp} \to g_{H_1^\perp}(z, b_T) + g_K(b_T)\ln(Q/Q_0)$$

(Echevarria, Idilbi, Scimemi (2014); Kang, Prokudin, Sun, Yuan (2016))

The ***collinear* twist-3 functions** (along with the NP g-function) are what get extracted in analyses of transverse-spin ***TMD processes***!

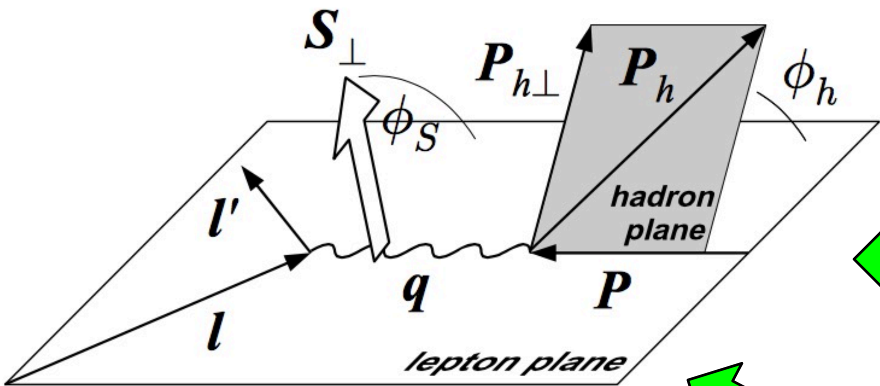
$$\tilde{H}_1^{\perp(1)}(z, b_T; Q^2, \mu_Q) \sim \boxed{H_1^{\perp(1)}(z;\mu_{b_*})}\exp\left[-S_{pert}(b_*(b_T);\mu_{b_*},Q,\mu_Q) - S_{NP}^{H_1^\perp}(b_T,Q)\right]$$

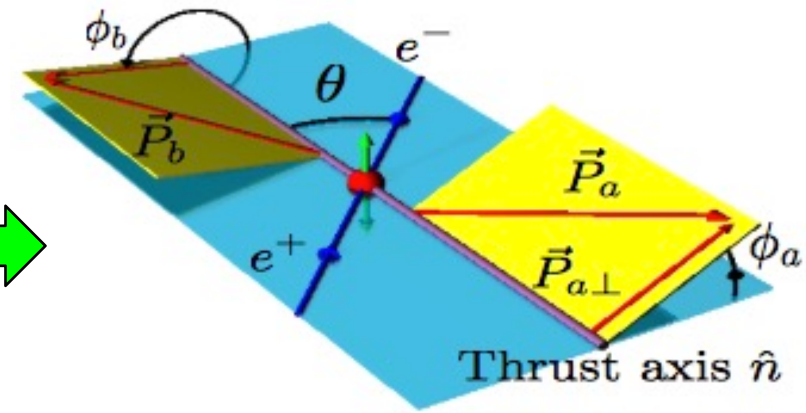
$$H_1^{\perp(1),q}(z) = -\frac{2}{z}\int_z^1 dz_1 \int_{z_1}^\infty \frac{dz_2}{z_2^2}\,\frac{\left(\frac{2}{z_1}-\frac{1}{z_2}\right)}{\left(\frac{1}{z_1}-\frac{1}{z_2}\right)^2}\,\hat{H}_{FU}^{qg,\Im}(z_1,z_2)$$

$$\tilde{D}_{1T}^{\perp(1)}(z, b_T; Q^2, \mu_Q) \sim \boxed{D_{1T}^{\perp(1)}(z;\mu_{b_*})}\exp\left[-S_{pert}(b_*(b_T);\mu_{b_*},Q,\mu_Q) - S_{NP}^{D_{1T}^\perp}(b_T,Q)\right]$$

$$D_{1T}^{\perp(1),q}(z) = \int_z^1 \frac{dz_1}{z_1}\int_{z_1}^\infty \frac{dz_2}{z_2^2}\left[\frac{\hat{G}_{FT}^{qg,\Im}(z_1,z_2)}{\frac{1}{z_1}-\frac{1}{z_2}} - \frac{\left(\frac{3}{z_1}-\frac{1}{z_2}\right)\hat{D}_{FT}^{qg,\Im}(z_1,z_2)}{\left(\frac{1}{z_1}-\frac{1}{z_2}\right)^2}\right]$$

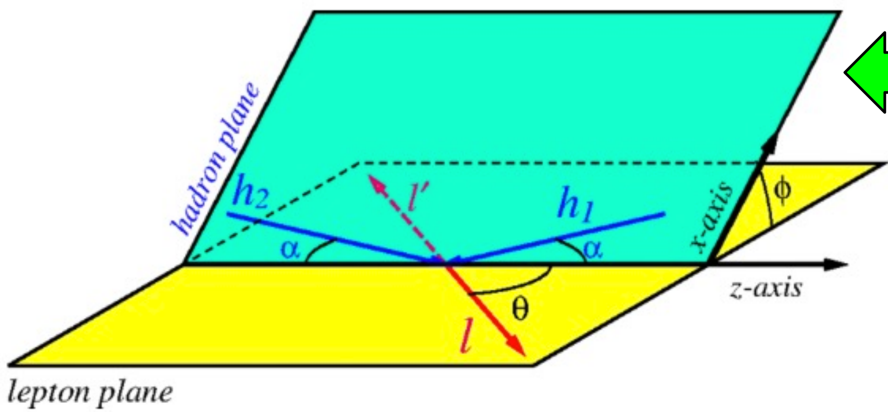
ALL ***transverse-spin observables are driven by multi-parton correlations!***

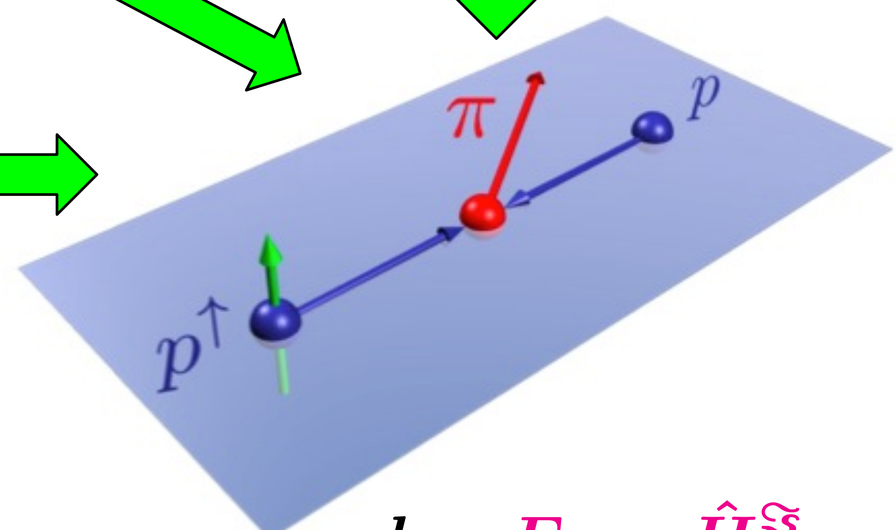




h_1, F_{FT}, $\hat{H}_{FU}^{\Im}$

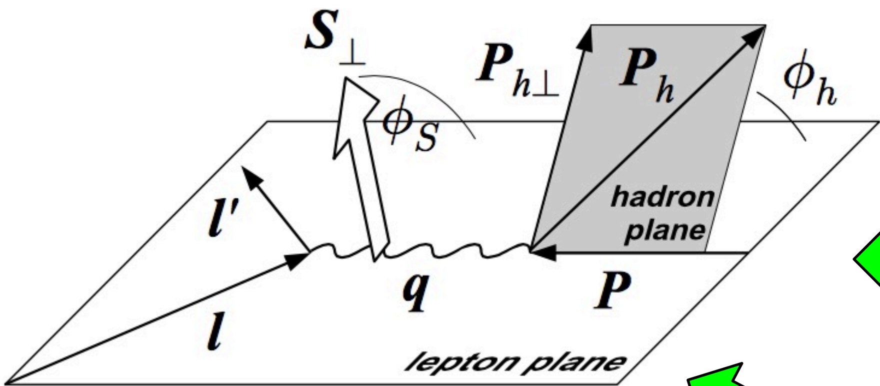
$\hat{H}_{FU}^{\Im}$



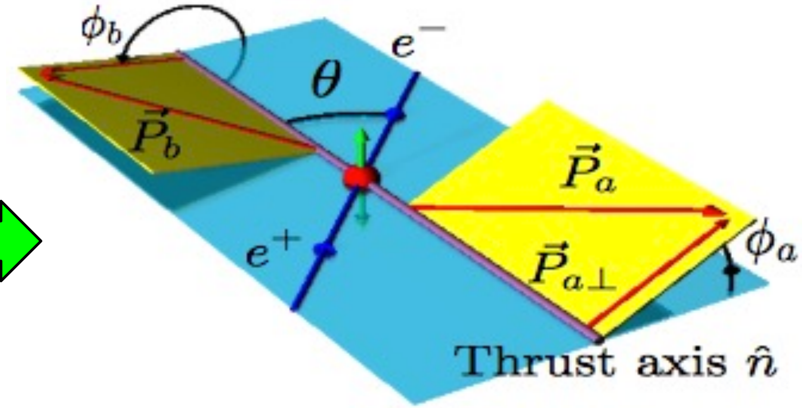


F_{FT}

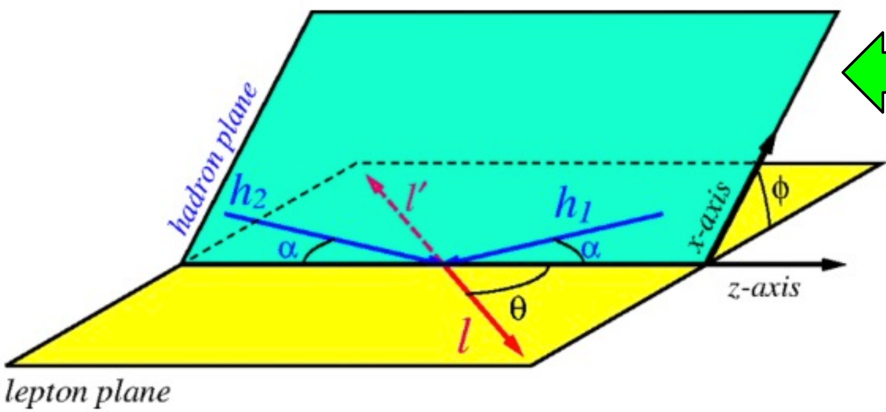
h_1, F_{FT}, $\hat{H}_{FU}^{\Im}$



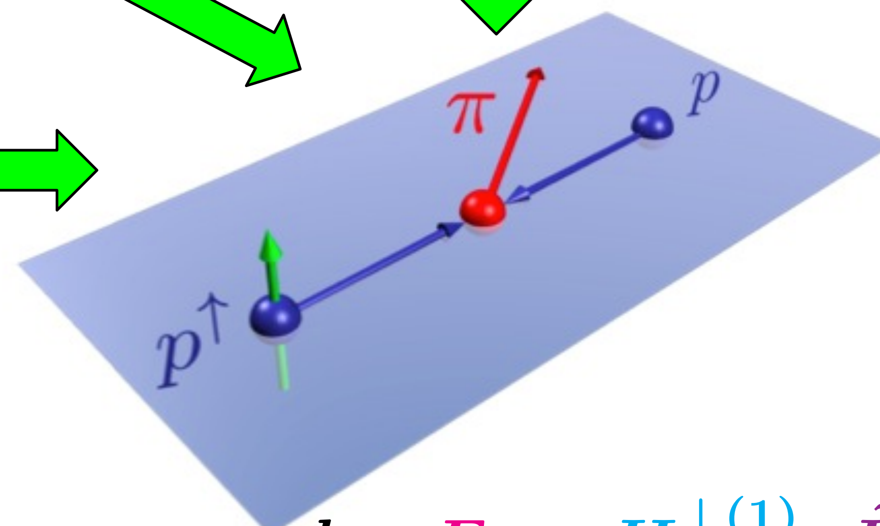
h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$



$H_1^{\perp(1)}$, $\tilde{H}$



F_{FT}



h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$

Open Issues

- NLO calculations of twist-3 observables (in particular those involving fragmentation) and fragmentation correlators

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- Derivation of LIRs for gluon FFs

- Can twist-3 fragmentation explain A_N results in pA from RHIC?
  (Hatta, Xiao, Yoshida, Yuan (2017); Bennic & Hatta (2019))

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- Global fit of *TMD and collinear* transverse spin observables (Gamberg, Kang, DP, Prokudin, Sato, …, on-going work)

Summary

- Collinear twist-3 FFs can be probed in a variety of different processes, and they are fundamentally different than their PDF counterparts (complex-valued, non-pole matrix elements,…).
- Both *TMD and collinear* functions that are relevant for transverse-spin observables are driven by multi-parton correlations – global analysis is possible!
- Several open issues remain for theory and phenomenology to validate the twist-3 framework and connect analytical calculations to experimental data.

Back-up Slides

- What follows are *very preliminary* results of a global fit of
  1) Collins effect in e^+e^-
  2) Collins effect in SIDIS
  3) (Integrated) $A_{UT}^{\sin\phi_s}$ in SIDIS
  4) A_N in proton-proton collisions (fragmentation term)

  *Also will add Sivers and QS term of A_N to the analysis

- Monte Carlo (MC) sampling was used to determine error bands. For now, we use a simple Gaussian ansatz for TMDs.

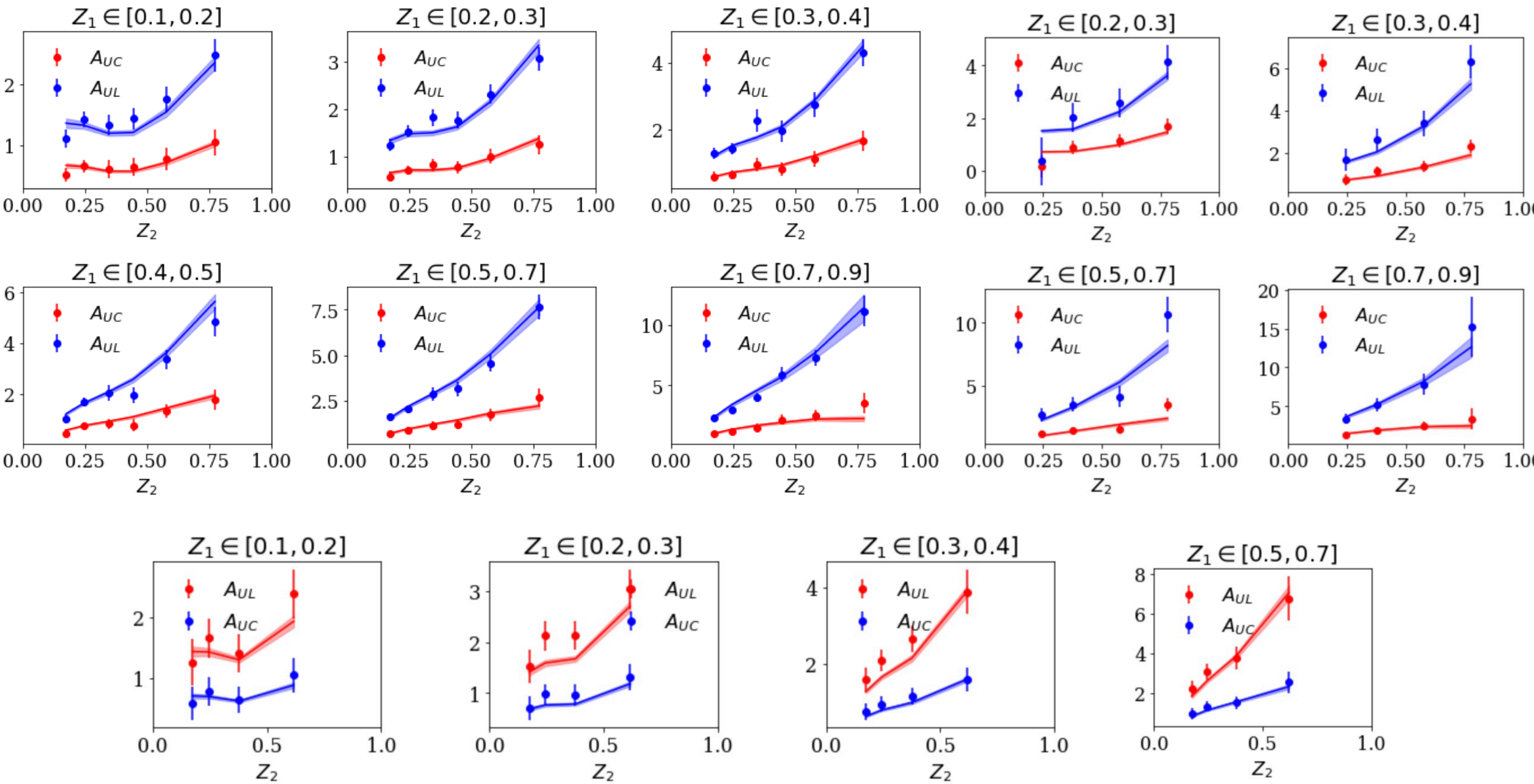
- We have found solutions for the relevant non-perturbative functions (including $\tilde{H}$!) that describe simultaneously a non-trivial amount of observables.

- Large errors in the (transversely polarized) deuteron SIDIS data make flavor separation subject to significant correlations which can only be estimated by MC – an EIC can hopefully deliver more accurate data.

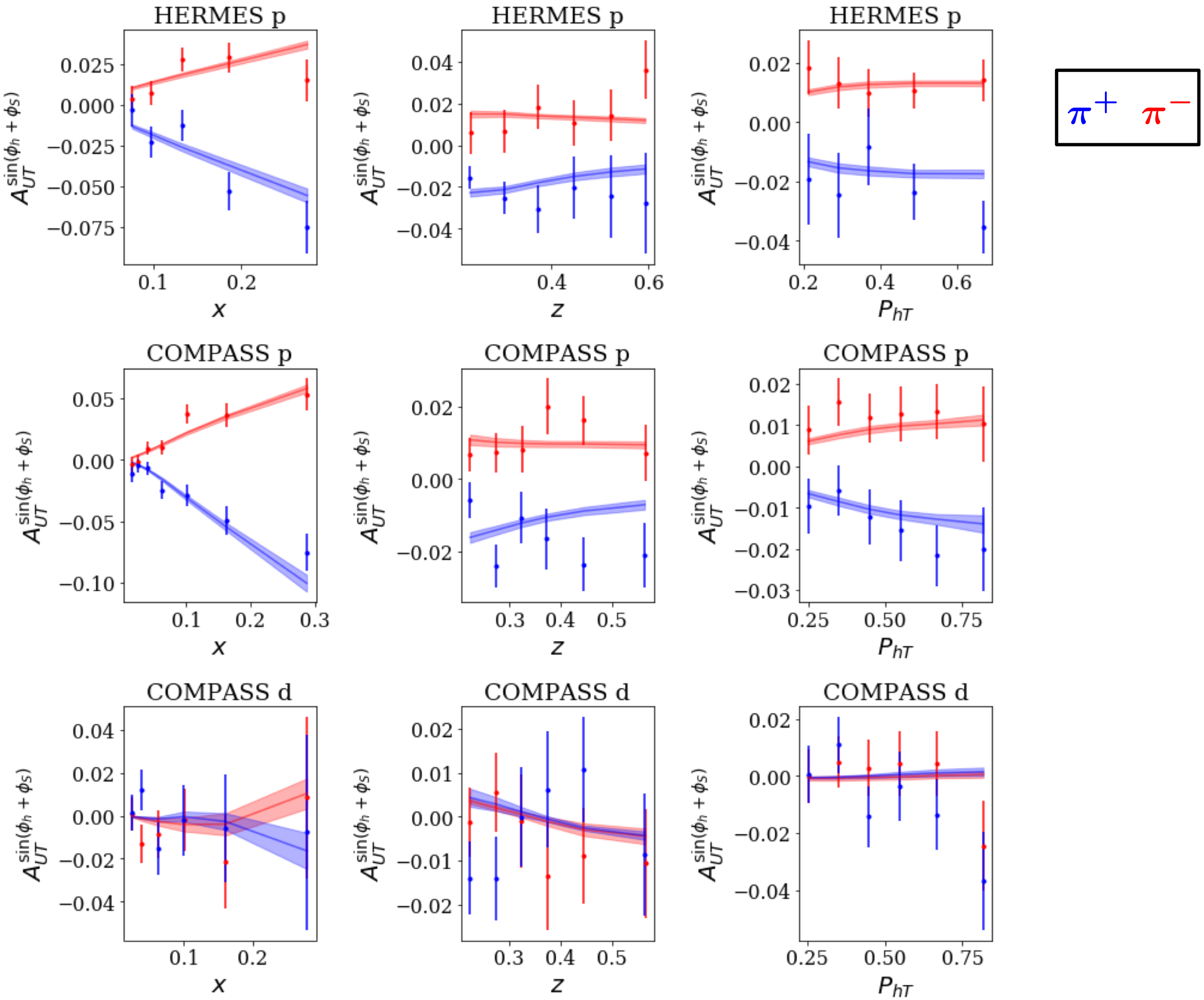
(Gamberg, Kang, DP, Prokudin, Sato, …, on-going work)

Collins effect e^+e^-

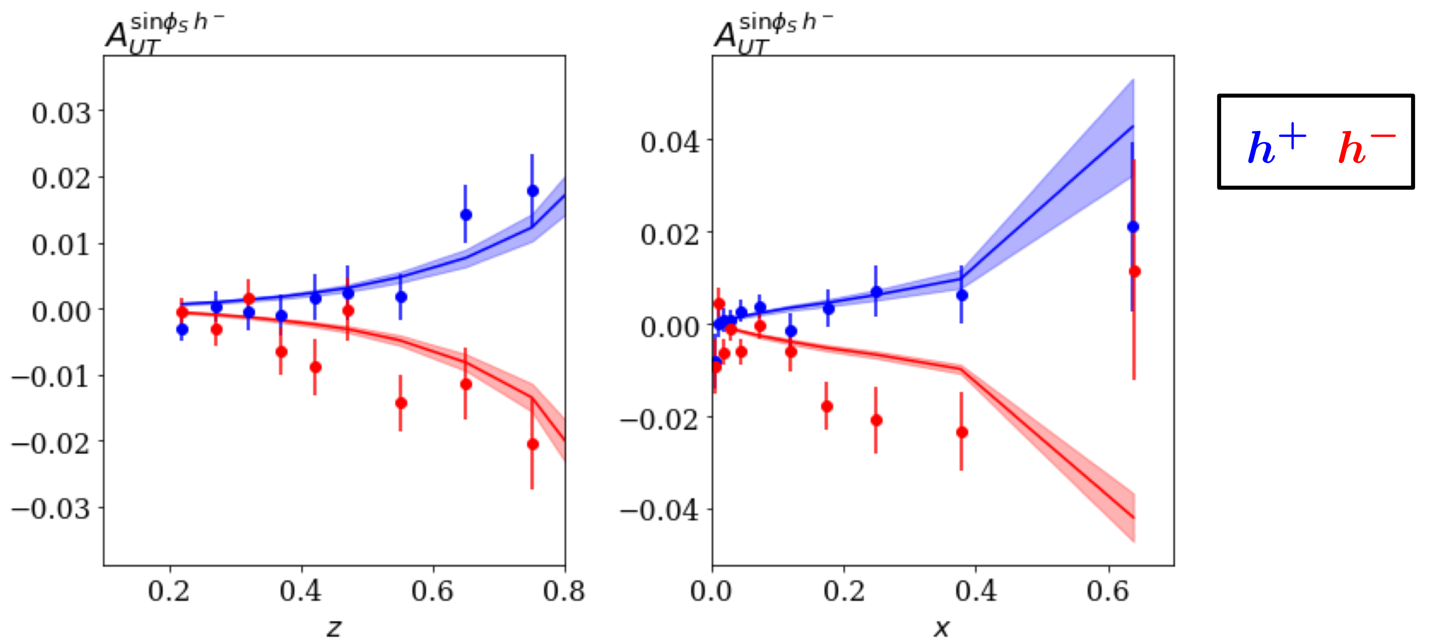
A_{UC} A_{UL}



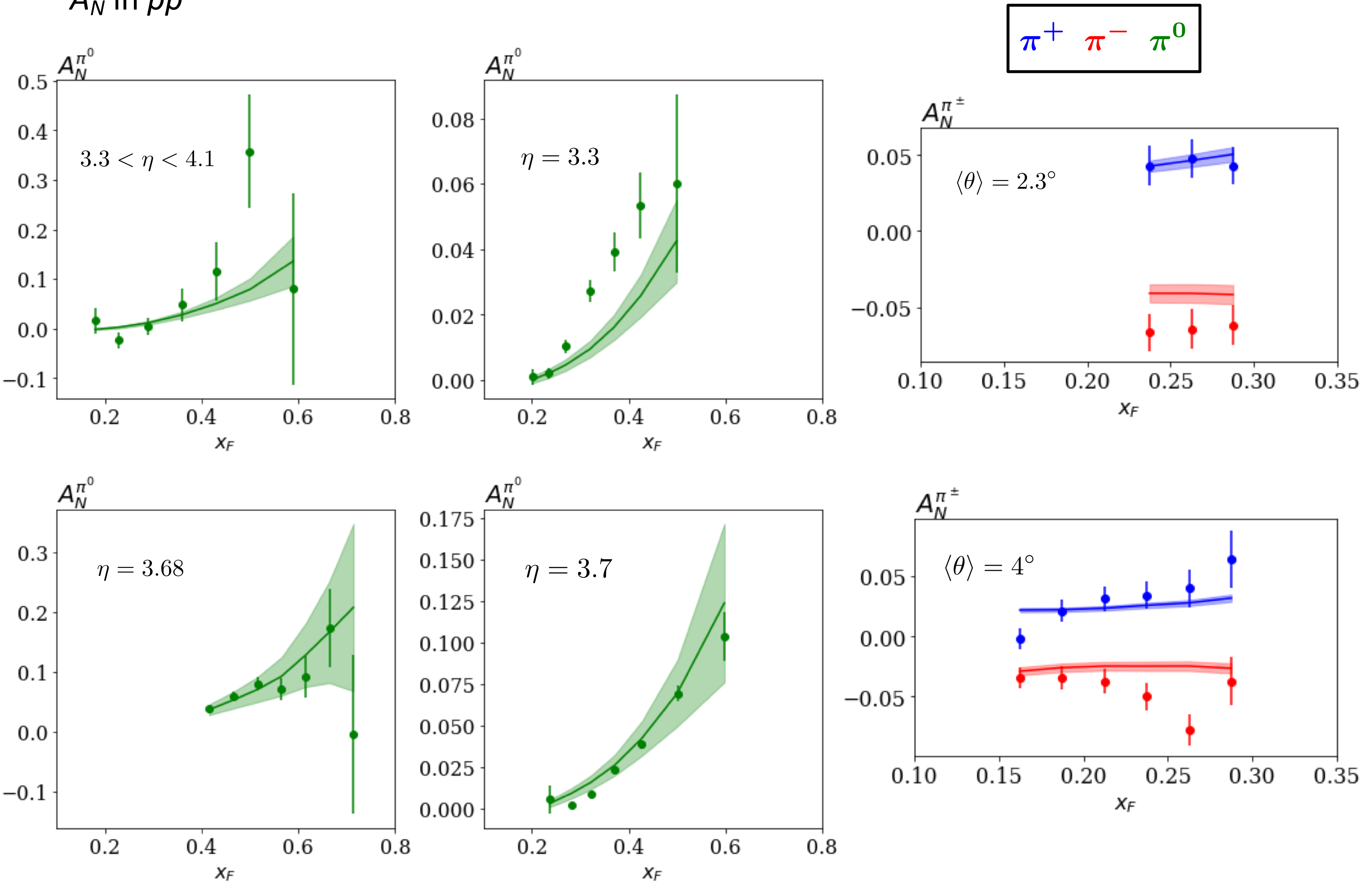
Collins effect SIDIS

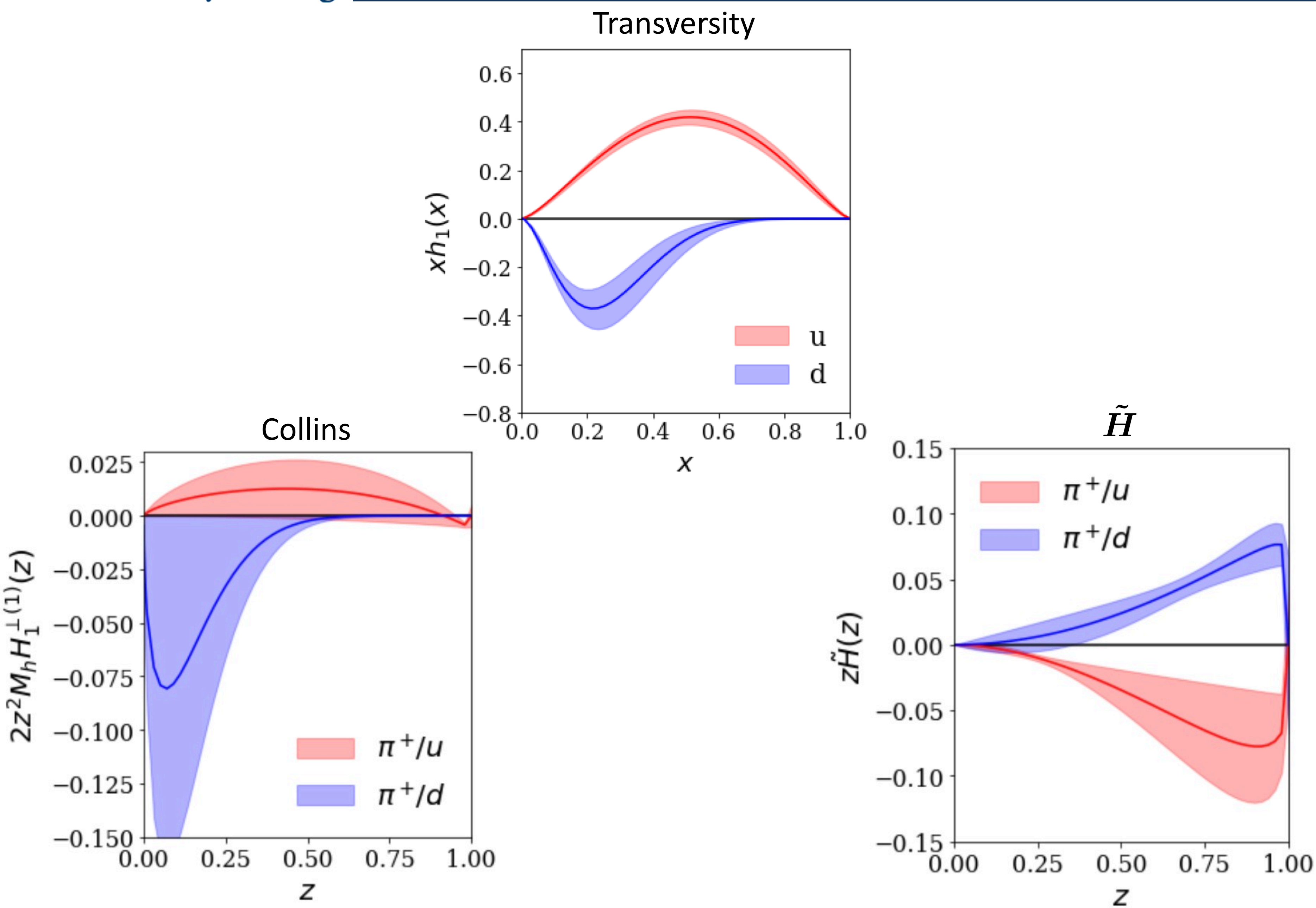


$A_{UT}^{\sin\phi_S}$ in SIDIS



A_N in *pp*


π^+ π^- π^0
$A_N^{\pi^0}$
$3.3 < \eta < 4.1$
$\eta = 3.3$
$A_N^{\pi^\pm}$
$\langle\theta\rangle = 2.3°$
$\eta = 3.68$
$\eta = 3.7$
$\langle\theta\rangle = 4°$
x_F


Transversity
$xh_1(x)$
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
0.0 0.2 0.4 0.6 0.8 1.0
x
u
d
Collins
$2z^2 M_h H_1^{\perp(1)}(z)$
0.025
0.000
−0.025
−0.050
−0.075
−0.100
−0.125
−0.150
0.00 0.25 0.50 0.75 1.00
z
π^+/u
π^+/d
$\tilde{H}$
$z\tilde{H}(z)$
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
0.00 0.25 0.50 0.75 1.00
z
π^+/u
π^+/d