Hadron mass corrections in SIDIS and DIS

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FF 2019

Duke U. – Mar 15th, 2019

Overview

- **Hadron Mass Corrections in SIDIS**
  - Collinear factorization with non-zero masses
  - Kaons (and pions) at HERMES vs COMPASS (vs JLab)
- **Testing HMCs in a spectator model**
  - DIS case, to begin with
- **Fragmentation w/o fragments: "Inclusive jet" mass effects**
  - Dressed vs. perturbative quark
  - Jet mass as χ-simmetry order parameter
  - Observability:
    - Non-perturbative FF sum rule for Etilde
    - g_2 in DIS; e+e– collisions

Hadron mass corrections in SIDIS

Guerrero, Accardi, PRD 97 (2018) 114012

Guerrero, Ethier, Accardi, Melnitchouk, Casper, JHEP 1509 (2015) 169

Accardi, Hobbs, Melnitchouk, JHEP 0911 (2009) 084

Strange quark parton distribution function (PDF)

LHC

$$p + p \rightarrow W^{+/-},\ Z$$

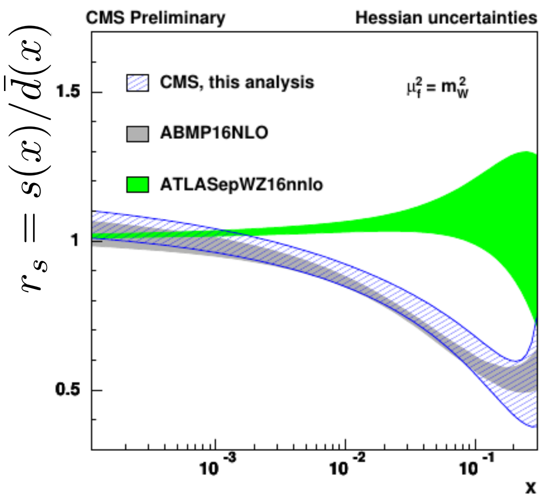
$$p + p \rightarrow W + c$$

Charged current DIS

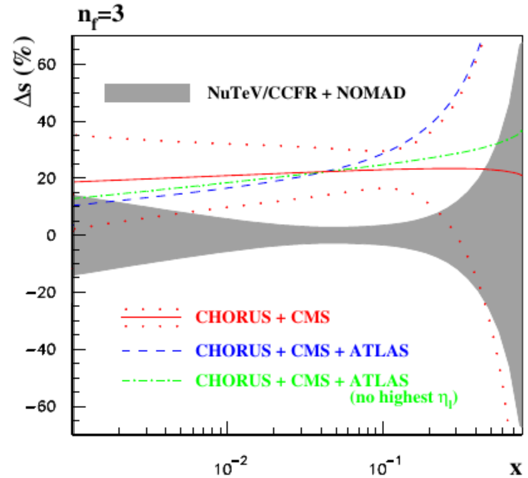
$$\nu + A \rightarrow l + c + X$$

- ATLAS: no suppression
- CMS: suppression
- νA : suppression

→ **Need another measurement**



Svenja Pflitsch, DIS 2018



$$g\, s_p \rightarrow W\, c$$

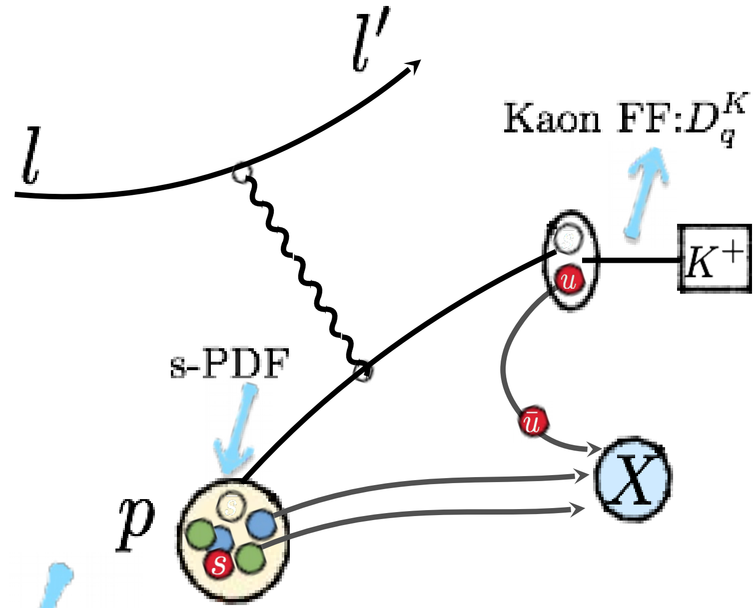
$$\nu\, s_A \rightarrow \mu^- \mu^+ \nu_\mu\, s$$

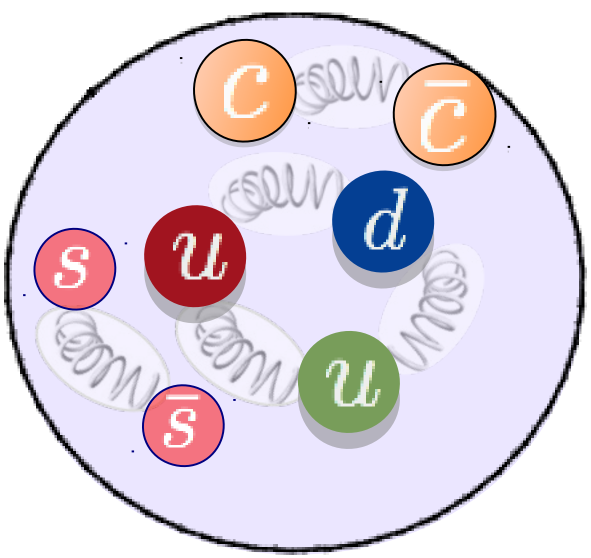
Alekhin et al., arXiv:1404.6469

s-PDF from SIDIS

Measuring a Kaon in Semi inclusive Deep inelastic scattering (SIDIS)

$$e^- + p \rightarrow e^- + K + X$$

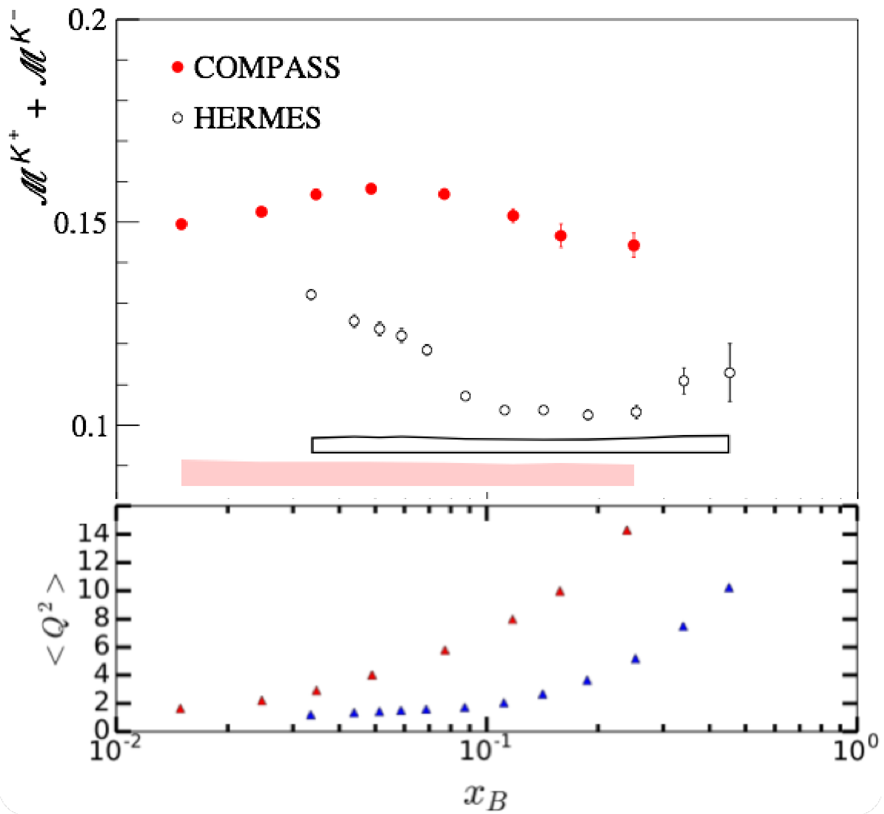




- Kaons contain one s-quark in their valence structure.
- Detect a Kaon: good proxy for a strange quark in proton

Integrated Kaon Multiplicities: SIDIS on Deuteron

- **HERMES:**
  - Claim very different s-quark shape compared to CTEQ6L.
  - → strange PDF may not be what we think!
- **But COMPASS:**
  - Different x_B dependence
  - Overall values higher



Where does this difference come from?

Is it real or apparent?

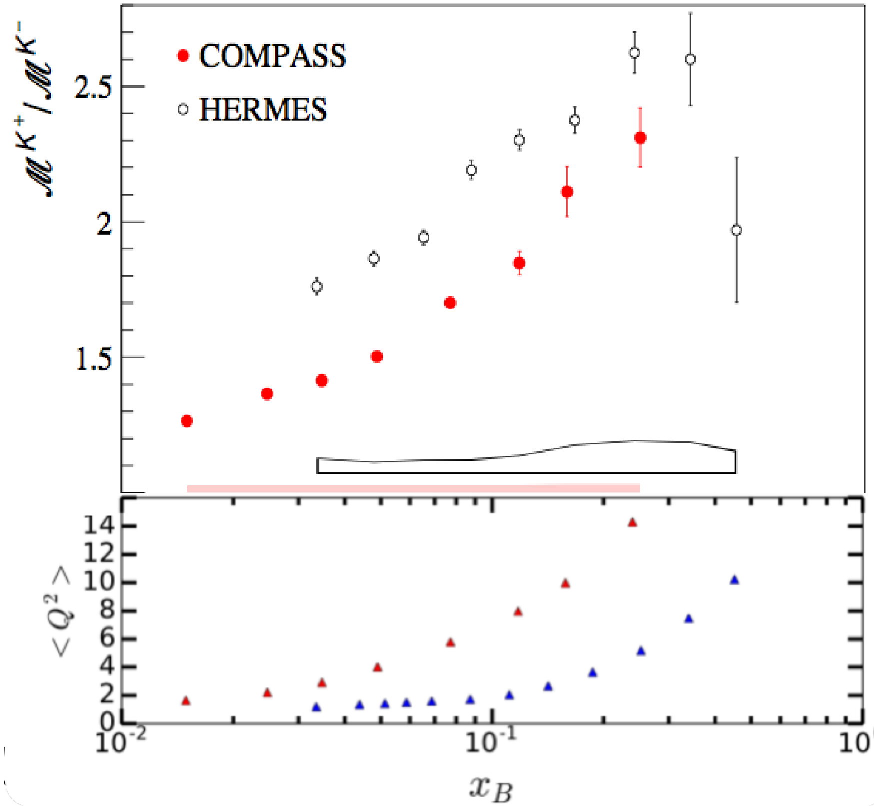
Integrated Kaon Multiplicities: SIDIS on Deuteron

HERMES:

- Claim very different s-quark shape compared to CTEQ6L.
  → strange PDF may not be what we think!

But COMPASS ratio:

- (Almost) same shape
- Overall lower
  → still different from HERMES

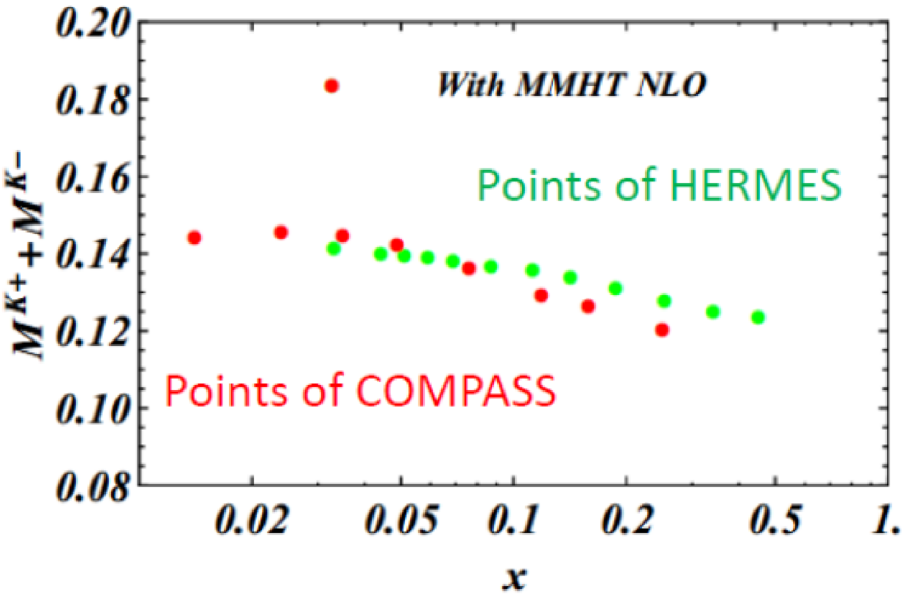


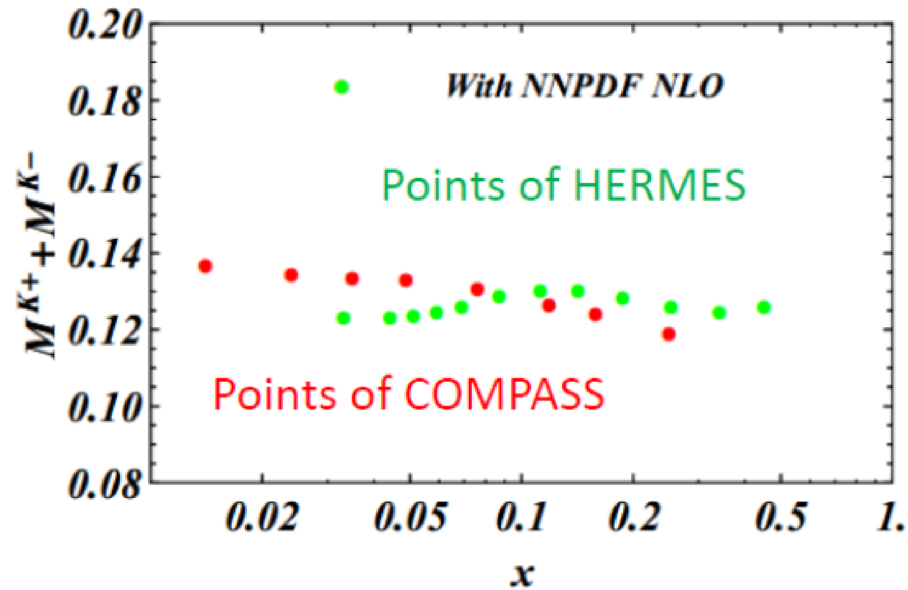
Where does this difference come from?

Is it real or apparent?

Because of NLO, Q^2 evolution?

NLO calculations by Chung-Wen Kao, talk at DIS 2018



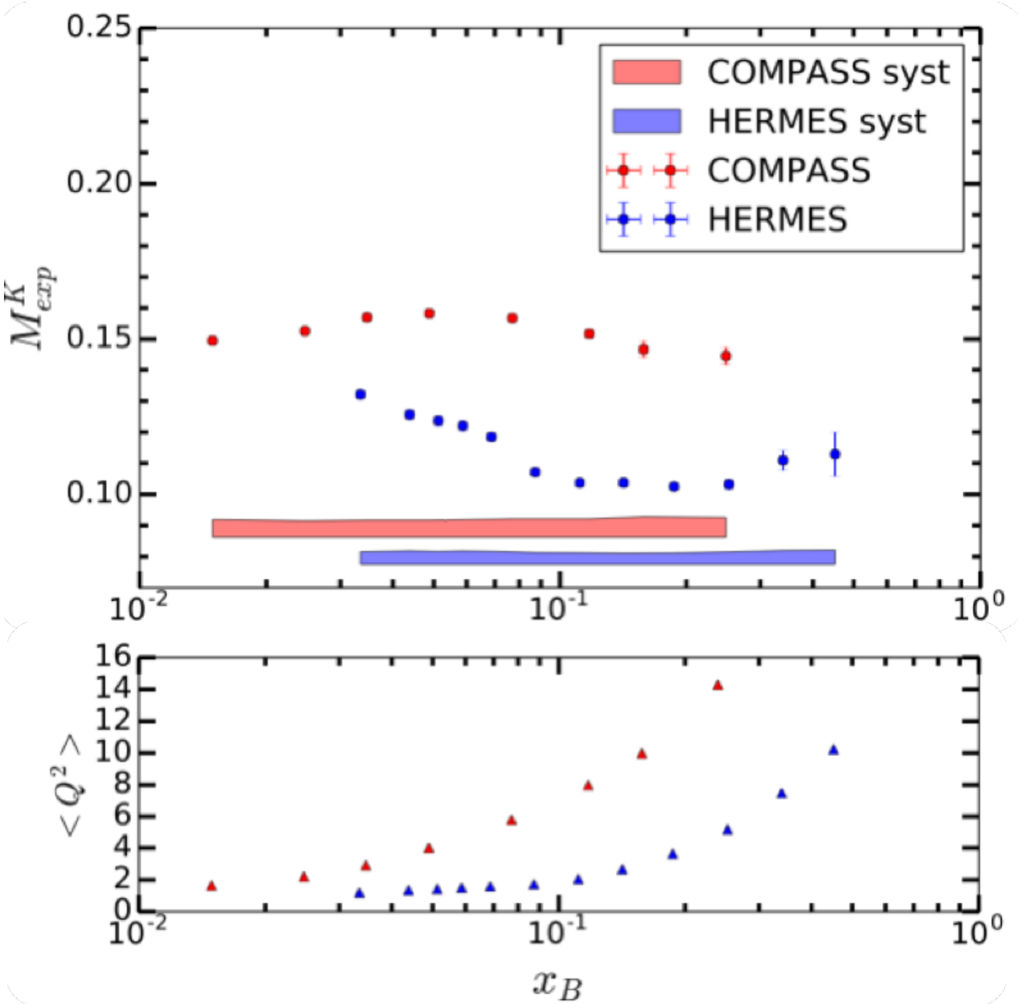


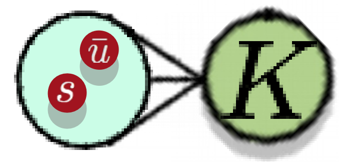
- MMHT+DSS17
- NNPDF +DSS17

- H & C should be close!
- Small Q^2 evolution
- Theory shapes =/= data
- Other effects?

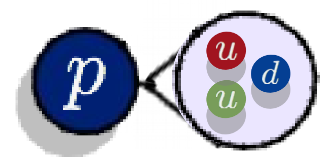
Because of Hadron Mass Effects?

Usually in pQCD, the masses of proton and detected hadron are neglected





$m_K \simeq 0.5\ \mathrm{GeV}$

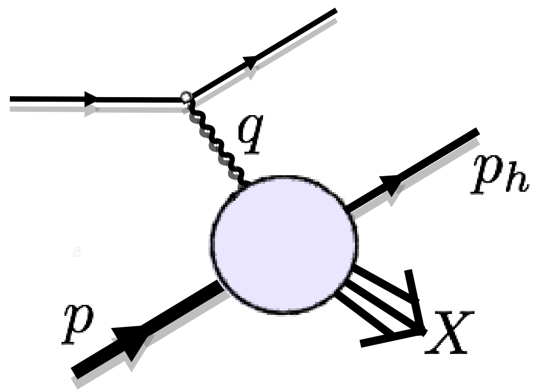


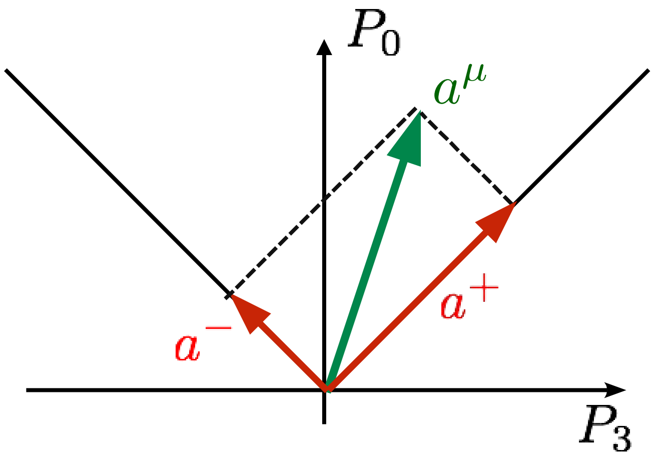
$m_p \simeq 1\ \mathrm{GeV}$

$$\overline{Q^2}_{\mathrm{C}} \gtrsim \overline{Q^2}_{\mathrm{H}} \simeq 1 - 10\ \mathrm{GeV}^2$$

Maybe masses are not so negligible!

Massive scaling variables





$$a^{\pm} \equiv \frac{a^0 \pm a^3}{\sqrt{2}}$$

Scaling Variables

Nachtmann:

$$\xi \equiv -\frac{q^+}{p^+} = \frac{2x_B}{1+\sqrt{1+4x_B^2 M^2/Q^2}}$$

Bjorken limit: $\xi \xrightarrow[Q^2\to\infty]{} x_B$

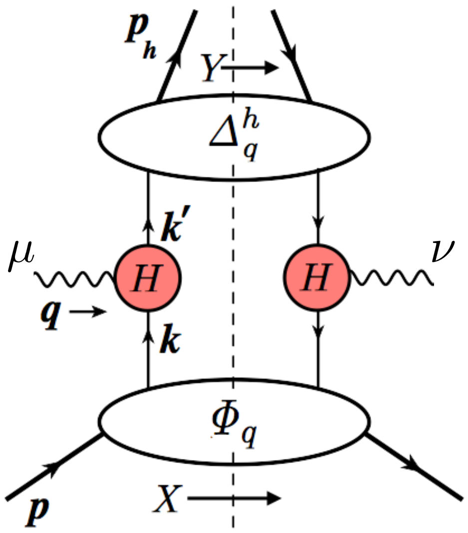
Fragmentation:

$$\zeta_h \equiv \frac{p_h^-}{q^-} = \frac{z_h}{2}\frac{\xi}{x_B}\left(1+\sqrt{1-\frac{4x_B^2 M^2 m_{h\perp}^2}{z_h^2 Q^4}}\right)$$

Bjorken limit: $\zeta_h \xrightarrow[Q^2\to\infty]{} z_h$

Collinear factorization with masses

Guerrero, Accardi, PRD 2018
(see also Collins, Rogers, Stasto 2007)



1 Expand the correlators

$$\Phi_q = k^+ \left[\phi_2(k) \bar{n}\!\!\!/ + \mathcal{O}(1/k^+)\right]$$

$$\Delta_q = k'^- \left[\delta_2(k') n\!\!\!/ + \mathcal{O}(1/k'^-)\right]$$

leading terms

contribute to Higher-Twist (HT) terms

2 Expand the hadronic tensor

$$2MW^{\mu\nu} = \int d^4k\, d^4k'\ \mathrm{Tr}\left[\Phi_q(p,k)\,\gamma^\mu\,\Delta_q^h(k',p_h)\,\gamma^\nu\right]\ \delta^{(4)}(k+q-k')$$

$$= \int d^4k\, d^4k'\ \phi_2(k)\delta_2(k')\mathrm{Tr}\left[k^+ \bar{n}\!\!\!/ \gamma^\mu\, k'^- n\!\!\!/ \gamma^\nu\right]\ \delta^{(4)}(k+q-k') + \mathrm{HT}$$

Note: $\boxed{q_\mu W^{\mu\nu} = 0}$

3 Approx only the (overall) 4-mom conserv.

$$k \approx \widetilde{k}\ ; \quad k' \approx \widetilde{k}'$$

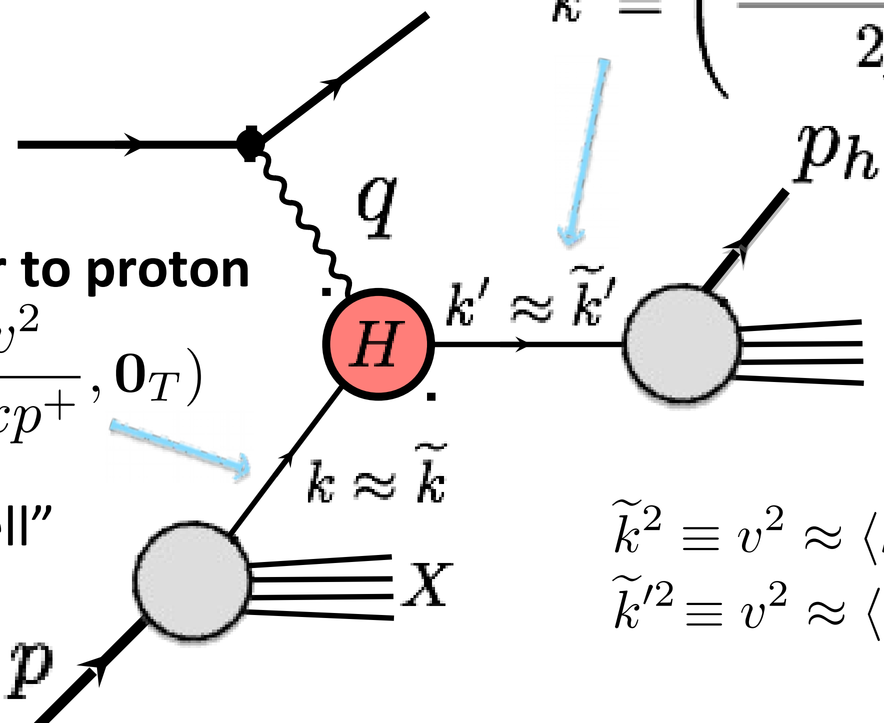
Approximation: collinear momenta

(p,q) frame: p and q collinear, 0 tr. mom.

$$\begin{cases} p = (p^+, M^2/2p+, \mathbf{0_T}) \\ q = (-\xi p^+, Q^2/2\xi p^+, \mathbf{0_T}) \end{cases}$$

Fragmenting parton collinear to hadron

$$\widetilde{k}' = \left(\frac{v'^2 + (\boldsymbol{p_{h\perp}}/z)^2}{2p_h^-/z}, \frac{p_h^-}{z}, \frac{\boldsymbol{p_{h\perp}}}{z} \right)$$



...but fragments into a <u>**massive**</u> hadron:

$$\boxed{v'^2 = ??}$$

Parton collinear to proton

$$\widetilde{k} = (xp^+, \frac{v^2}{2xp^+}, \mathbf{0}_T)$$

... and "on-shell"

$$\boxed{v^2 = 0}$$

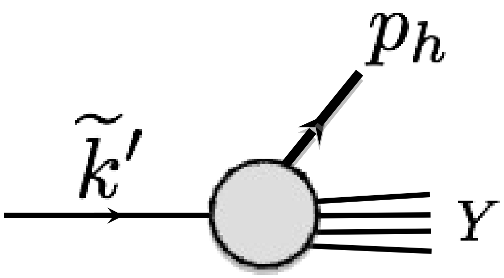
$$\left.\begin{array}{l} \widetilde{k}^2 \equiv v^2 \approx \langle k^2 + k_\perp^2 \rangle \\ \widetilde{k}'^2 \equiv v^2 \approx \langle k'^2 + k_\perp'^2 \rangle \end{array}\right\}$$ "Average virtualities"

How to match partonic and hadronic kinematics?

Matching Hadronic and Partonic Kinematics at LO

(much more detail in Guerrero et al., JHEP 2015)

Fragmenting blob: momentum conservation in + direction



$$\widetilde{k}'^{+} = p_h^+ + Y^+ \geq p_h^+$$

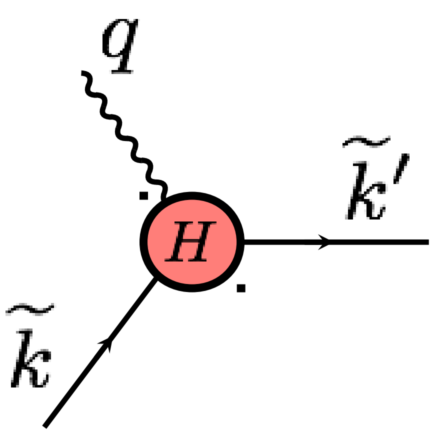
$$\boxed{v'^2 \geq \frac{m_h^2}{z} \underset{\text{LO}}{=} \frac{m_h^2}{\zeta_h}}$$

Orthodox choice:

$$v'^2 = 0$$

Albino et al. Nucl. Phys. B803 (2008) 42-104

Hard scattering: 4-momentum conservation at LO



$$x = \xi\left(1 + \frac{v'^2}{Q^2}\right)$$

$$z = \zeta_h$$

Only in Bjorken limit can one neglect v' !

Collinear factorization with masses - LO case

4 Let 3 integrations out of 4 act on correlators, obtain

$$\boxed{2MW^{\mu\nu} = \sum_q e_q^2 \int \frac{dx}{x}\frac{dz}{z}\, q(x)\, \mathcal{H}^{\mu\nu}(x,z)\, D_q(z) + \mathrm{HT}}$$

$$q(x) = \int dk^- d^2k_T \phi_2(k)$$ PDF

$$D_q(z) = (z/2)\int dk'^+ d^2k'_T \delta_2(k)$$ FF

$$\mathcal{H}^{\mu\nu}(x,z) = \frac{1}{2z}\mathrm{Tr}\left[\not{k}_0\gamma^\mu \not{k}'_0\gamma^\nu\right]$$ Hard scattering coefficient

$$\times\ \delta\Big(k_0^+ + q^+ - \frac{v'^2}{2k_0'^-}\Big)\,\delta\Big(\frac{v^2}{2k_0^+} + q^- - k_0'^-\Big)\,\delta^{(2)}(\boldsymbol{k'_{0T}})$$

$$\boxed{x = \xi_h \equiv \xi\Big(1 + \frac{m_h^2}{\zeta_h Q^2}\Big)}$$

$$\boxed{z = \zeta_h}$$

$$k_0 \equiv \widetilde{k}|_{v=0}$$
$$k'_0 \equiv \widetilde{k}'|_{v=0}$$

Leading Order (LO) Multiplicities at finite Q^2

- **With Hadron Masses:**

Scale dependent Jacobian

Finite Q^2 scaling variables

$$\boxed{M^h(x_B) = \frac{\int_{exp.} dQ^2 \int_{0.2}^{0.8} dz_h\, J_h(\xi, \zeta_h, Q^2) \sum_q e_q^2\, q(\xi_h, Q^2)\, D_q^h(\zeta_h, Q^2)}{\int_{exp.} dQ^2 \sum_q e_q^2\, q(\xi, Q^2)}}$$

Note: *Theory integrated over z, Q^2 exp. bins for each x_B*

$$\xi_h \equiv \xi\Big(1 + \frac{m_h^2}{\zeta_h Q^2}\Big)$$

- **Massless limit:** $\left(\frac{M^2}{Q^2}, \frac{m_h^2}{Q^2}\right) \to 0$

$$M^{h(0)}(x_B) = \frac{\int_{exp.} dQ^2 \sum_q e_q^2\, q(x_B, Q^2)\ \int_{0.2}^{0.8} dz_h\, D_q^h(z_h, Q^2)}{\int_{exp.} dQ^2 \sum_q e_q^2\, q(x_B, Q^2)}$$

Parton model definition

HERMES & COMPASS data: direct comparison

Use suitable **"Theoretical correction ratios"**

- Produce approximate "massless" parton model multiplicities
- Make data directly comparable
- Largely insensitive to FF normalization

COMPASS:

$$M_{exp}^{h(0)} \equiv M_{exp}^{h} \times R_{HMC}^{h}$$

HMC correction ratio

HERMES:

$$M_{exp}^{h(0)} \equiv M_{exp}^{h} \times R_{HMC}^{h} \times R_{evo}^{H \to C}$$

HERMES to COMPASS evolution

HERMES & COMPASS data: direct comparison

Use suitable
"Theoretical correction ratios"

– Produce approximate "massless" parton model multiplicities

– Make data directly comparable

– Largely insensitive to FF normalization

Multiplicities in a massless world:

– mass corrected (and evolved) M^h –

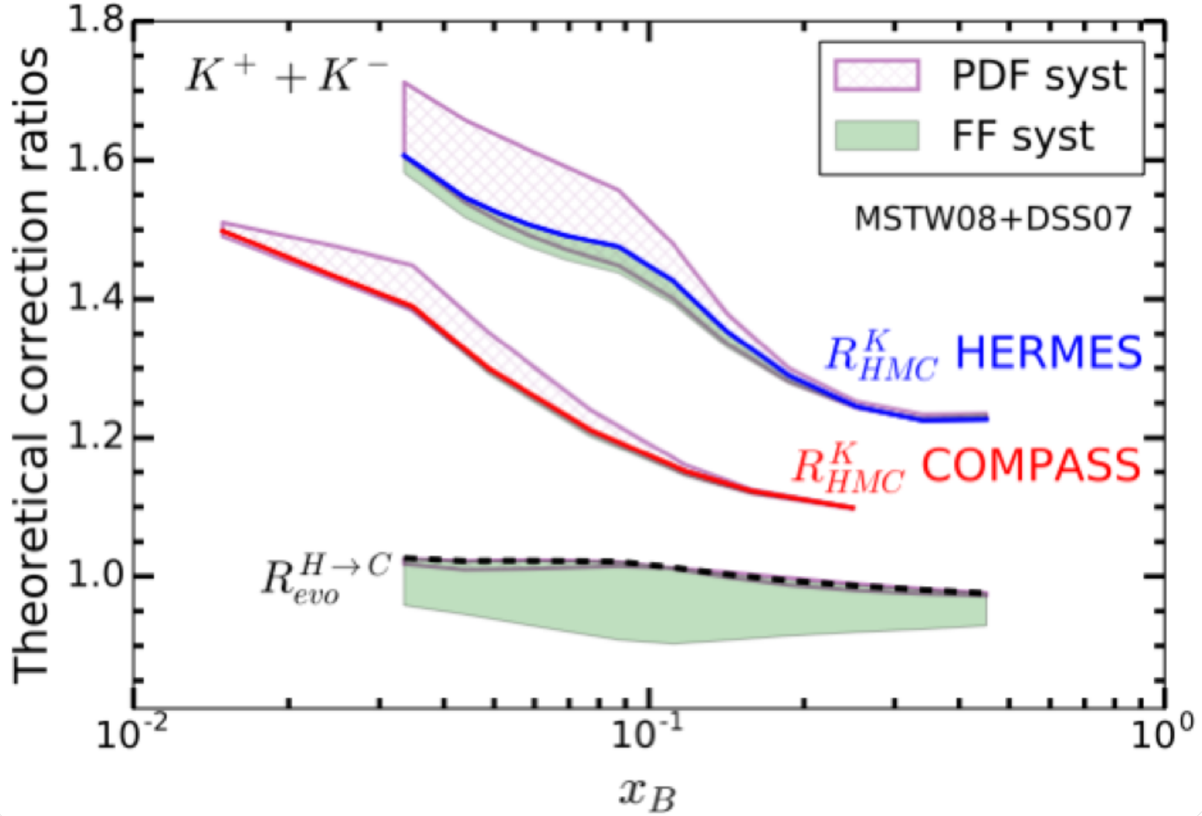
COMPASS:

$$M_{exp}^{h(0)} \equiv M_{exp}^{h} \times R_{HMC}^{h}$$

HERMES:

$$M_{exp}^{h(0)} \equiv M_{exp}^{h} \times R_{HMC}^{h} \times R_{evo}^{H \to C}$$

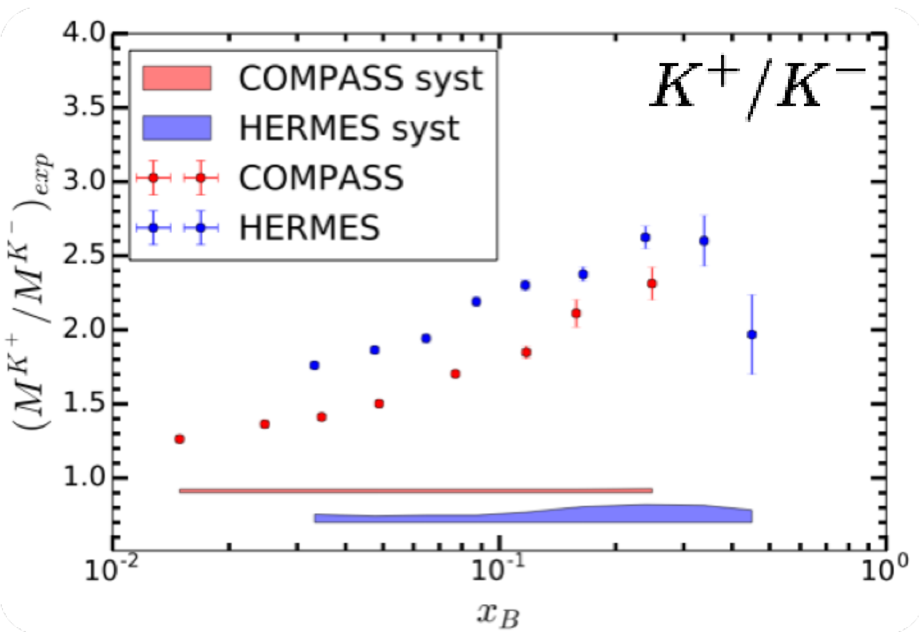
Correction ratios



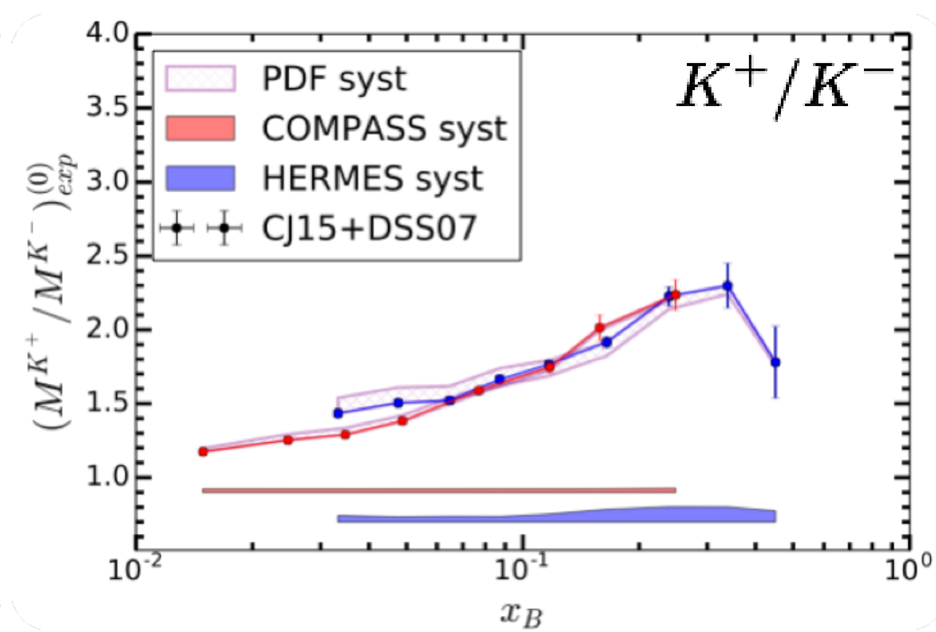
- Hadron mass effects dominant over evolution effects
- COMPASS has smaller HMCs – but non-negligible!

Direct Data Comparison: K^+/K^-

Experimental Data



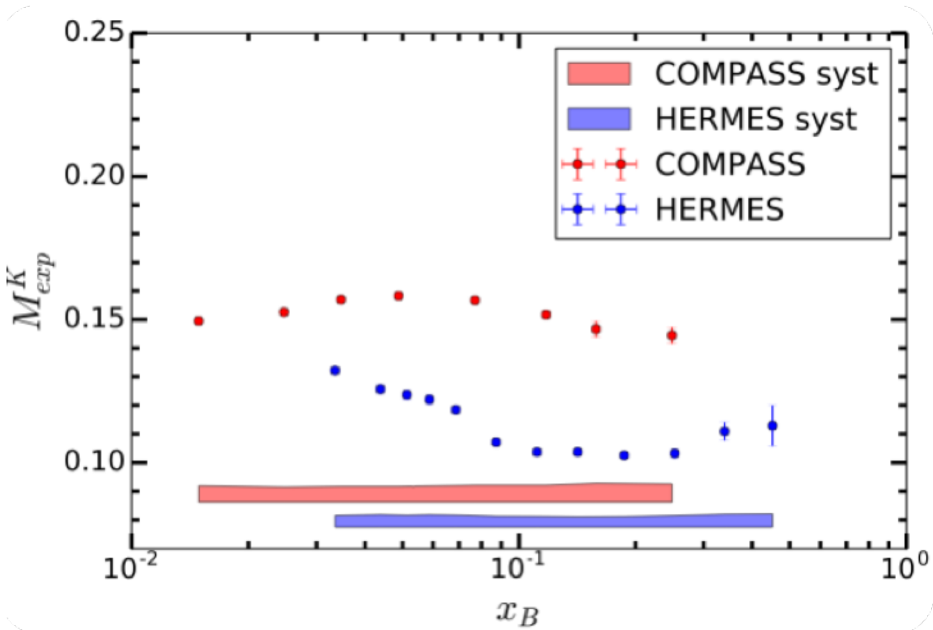
"Massless data" at same Q^2



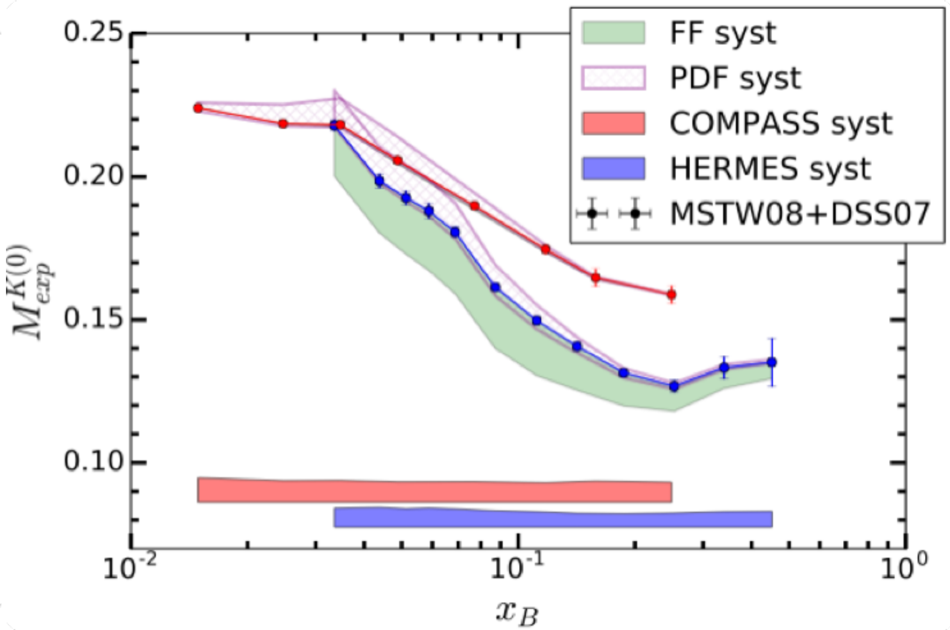
- **HERMES & COMPASS fully compatible.**
- large x downturn at HERMES ??

Direct Data Comparison: $K^+ + K^-$

Experimental Data



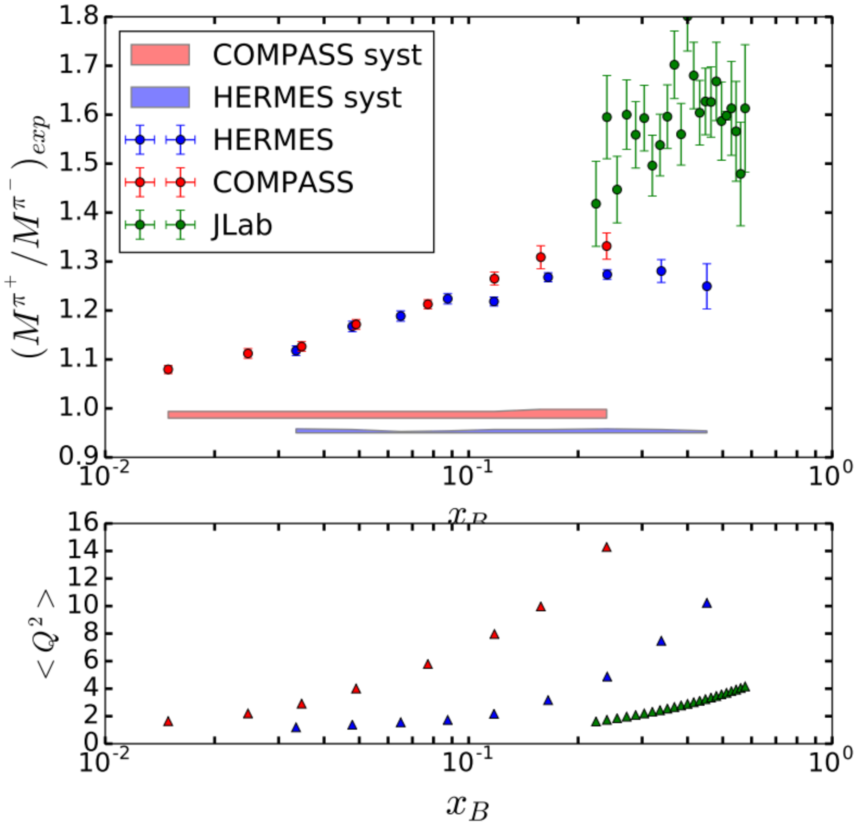
"Massless data" at same Q^2



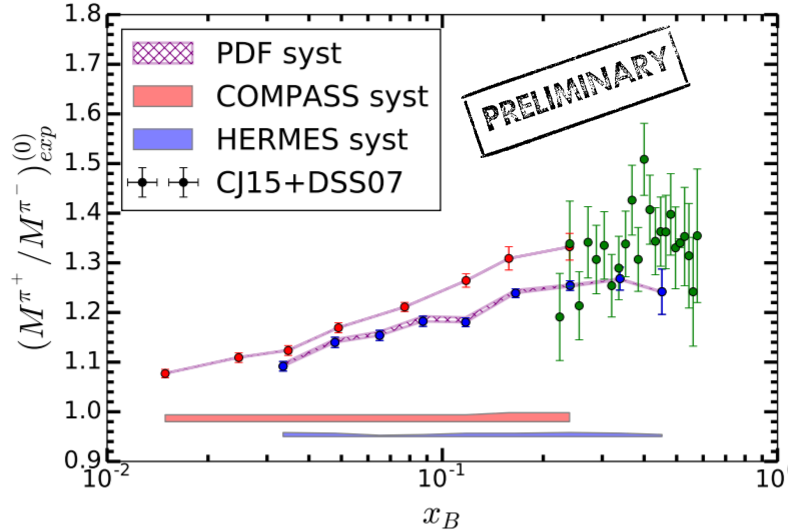
- **After HMCs:**
  - **> almost compatible in size**
  - **> negative slope, as it should**
    (but hockey stick at HERMES)
- **Residual slope difference:** needs NLO, FF refit

Pion ratios vs. JLab

Experimental Data



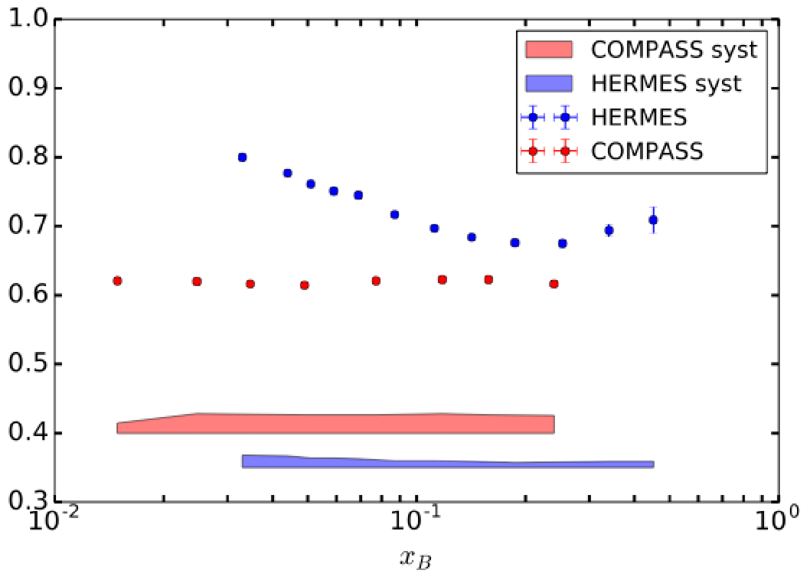
"Massless data" at same Q^2

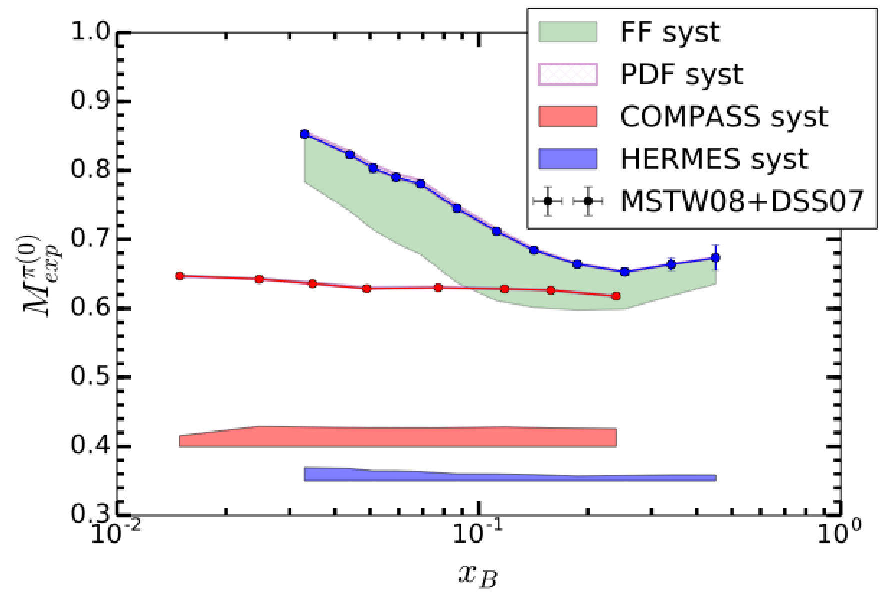


Pion ratios after HMCs:

- all approximately compatible
- JLab pions slightly prefer COMPASS
  ...but large stat. uncertainties
- small differences could be solved by:
  NLO effects, pion FF refit with HMCs

Direct Data Comparison: $\pi^+ + \pi^-$





- Shapes still incompatible
- "Hockey stick" at HERMES:
  but u, d quarks well known, not like s for Kaons!

Testing the HMC scheme

Guerrero, Accardi, in preparation

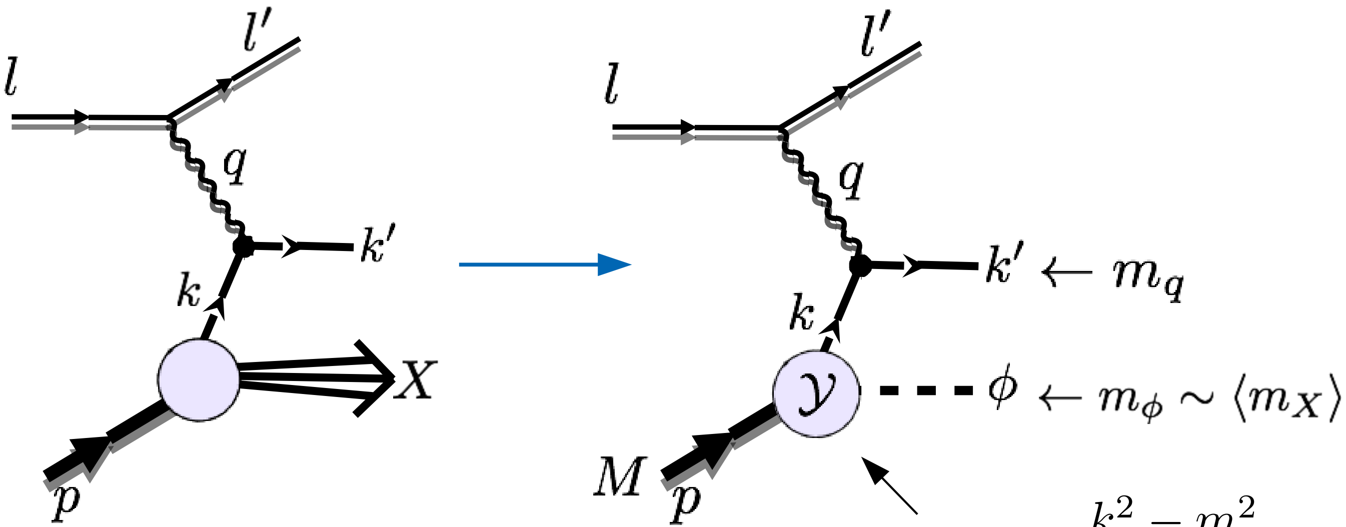
+

Accardi, Alcalá, Guerrero, in progress

Factorization with masses in a spectator model

Guerrero, AA – in prep.

- **Use spectator model:**
  - Known parameters, analytical calculations
  - Full vs. factorized cross section; PDFs: calculated vs. fitted
- **Start simple: DIS**
  - Then SIA (3-body phase space)
  - Then SIDIS (cimplex interplay of IS and FS kinematics)



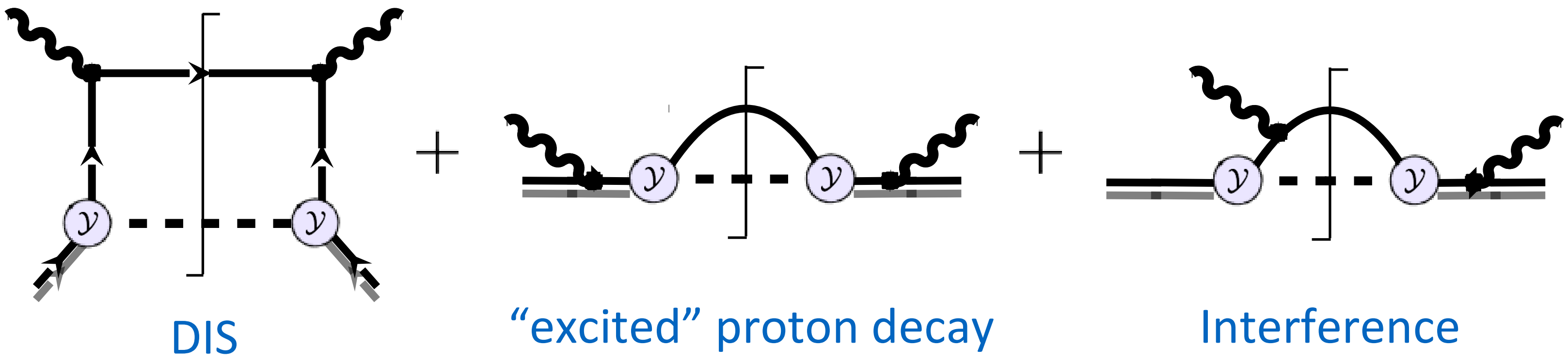
$$\mathcal{Y} = ig\frac{k^2 - m_q^2}{(k^2-\Lambda^2)^2}\mathbb{I}$$

simulates confinement

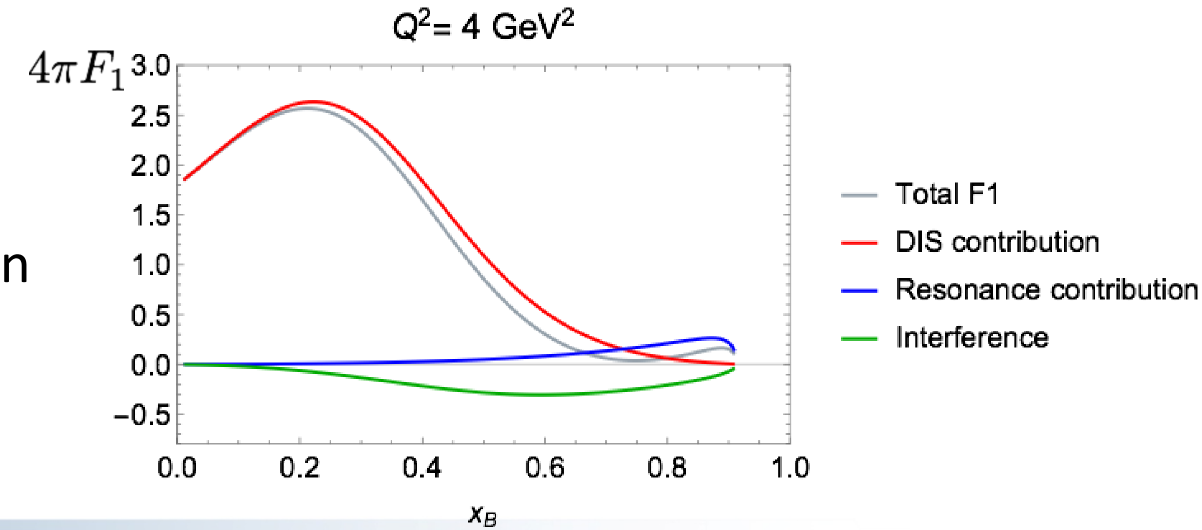
Factorization with masses in a spectator model

- **Gauge invariance:** need also quasi-elastic photon-proton scattering

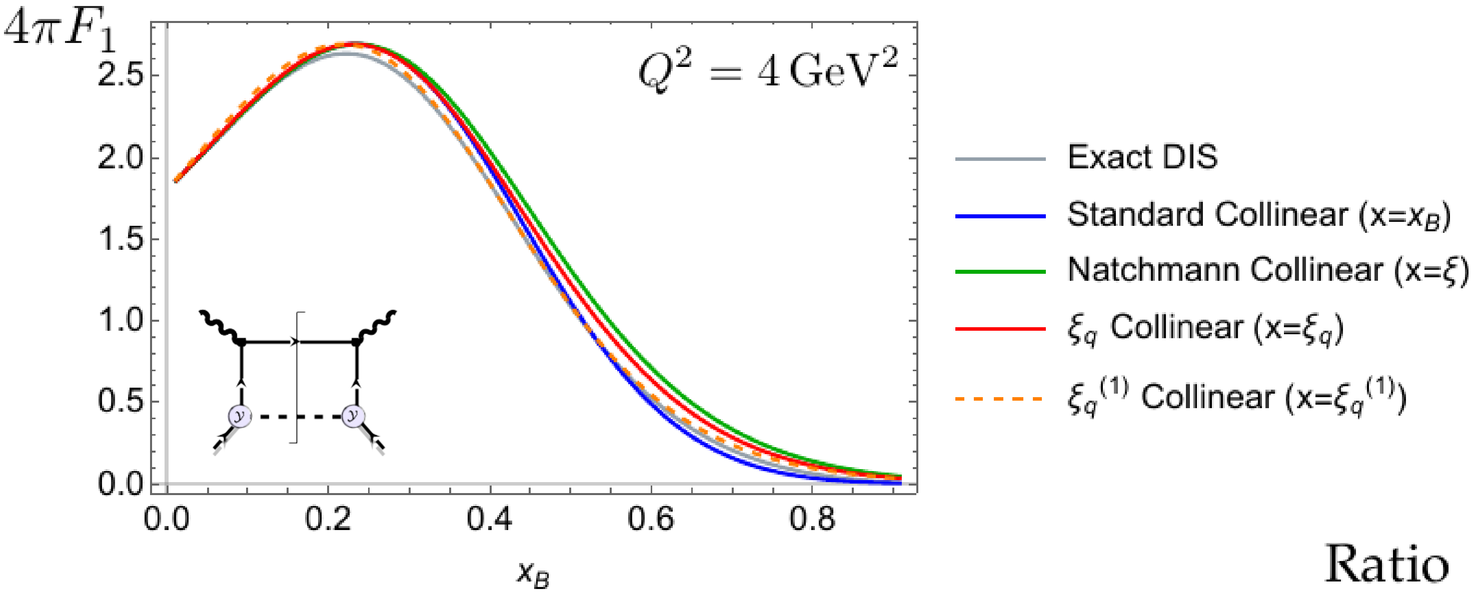
Moffat et al, PRD 95 (2017)

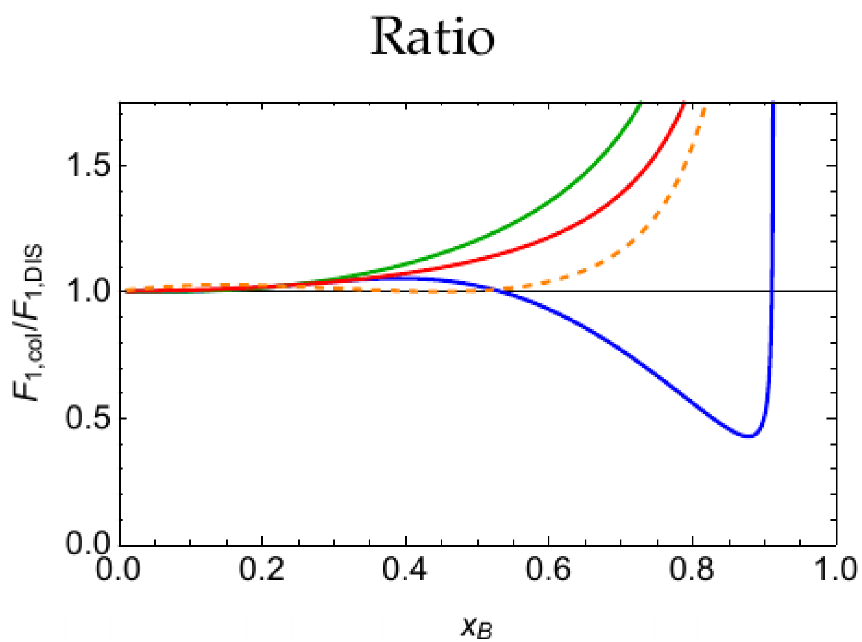


- **Gauge invariant individual contributions** *Guerrero, AA – in prep.*
  - Use $P_T^{\mu,\nu}$, $P_L^{\mu,\nu}$ projectors
  - Non-negligible interf. contribution even at small x_B



Structure Function: DIS vs. Collinear





- **Highlights:**
- Considering $\langle k_t^2 \rangle$ the remaining gap is largely filled.
- k_T^2/Q^2 fills remaining gap: controlled in collinear factorization at higher-twist.
- Factorization breaks above $x_B \sim 0.7$

“Inclusive jet” mass effects: fragmentation without fragments

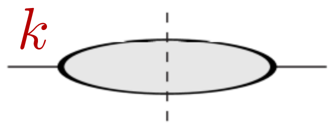
Accardi, Signori, arXiv:1903.04458
Accardi, Bacchetta, PLB 773 (2017) 632

+ work in progress:
AA, Signori
AA, Bacchetta, Radici, Signori

Inclusive jet correlator

AA, Signori, 1903.04458

- Quark are not asymptotic states
  - Hadronization products pass the cut
  - Define a gauge invariant quark-to-jet amplitude squared



Inclusive $q \longrightarrow X$ "jet" correlator

$$\Xi_{ij}(k;n_+) = \mathrm{Disc}\int \frac{d^4\xi}{(2\pi)^4} e^{\mathrm{i}k\cdot\xi} \frac{\mathrm{Tr}_c}{N_c} \langle\Omega|\mathcal{T}\, W^{n_+}_{(\infty,\xi)}\, \psi_i(\xi)\overline{\psi}_j(0)\, W^{n_T}_{(0,\infty)}|\Omega\rangle$$

- Integrate out the large momentum component:

TMD Inclusive jet correlator

$$J_{ij}(k^-,\boldsymbol{k}_T) \equiv \frac{1}{2}\int dk^+\, \Xi_{ij}(k)\,,$$

TMD jet correlator in full glory

AA, Signori, 1903.04458
+ work in preparation

- Expand in Dirac structures, take traces, use spectral representation:

$$J(k^-, \boldsymbol{k}_T) = \left\{ \gamma^+ + \frac{M_j}{k^-}\mathbb{I} + \frac{\not{\boldsymbol{k}}_T}{k^-} + \frac{(K_{jet}^2 + T_{jet}^2 + g.f.) + k_T^2}{2(k^-)^2}\not{n}_- \right\} \theta(k^-)$$

where, in light-cone gauge,

$$M_{jet} = \int_0^\infty d\mu^2\, \mu\, \rho_1(\mu^2)$$

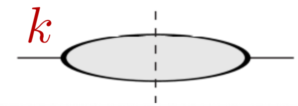
Jet "mass" ~ dressed quark mass ~ O(100 MeV)

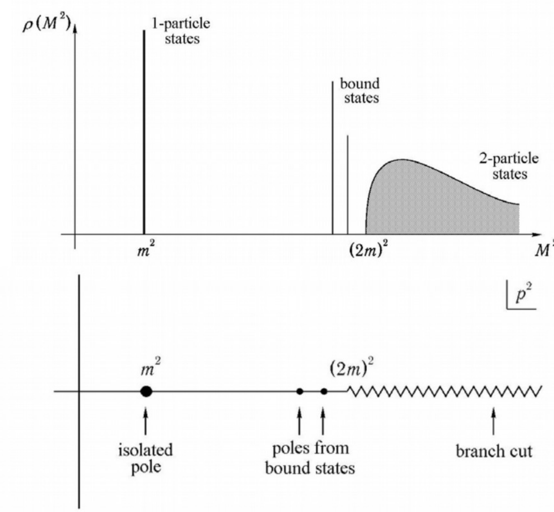
$$K_{jet}^2 = \int_0^\infty d\mu^2\, \mu^2\, \rho_3(\mu^2)$$

Jet's "virtuality"

$$T_{jet}^2 \sim \langle\langle \mu_T^2 \rangle\rangle$$

Jet's "transverse size"





- $\rho_{1,3}$ are Khallen-Lehman spectral functions:

$\longrightarrow$ strength of quark-to-multihadron coupling

TMD jet correlator in full glory

AA, Signori, 1903.04458
+ work in preparation

- Expand in Dirac structures, take traces, use spectral representation:

$$J(k^-, \boldsymbol{k}_T) = \left\{ \gamma^+ + \frac{M_j}{k^-}\mathbb{I} + \frac{\not{\boldsymbol{k}}_T}{k^-} + \frac{(K_{jet}^2 + T_{jet}^2 + g.f.) + k_T^2}{2(k^-)^2}\, \not{n}_- \right\} \theta(k^-)$$

"Perturbatively", or neglecting quark-gluon-quark correlations:

$M_{jet}^{pert} = m_q$ Current quark mass ~ O(1 MeV) << M_{jet}

${P_{jet}^2}^{pert} = m_q^2$ On-shell quark

${T_{jet}^2}^{pert} = \text{...in progress...}$

Novel FF sum rules: M_{jet} is observable!

AA, Signori 1903.04458

- General jet correlator sum rule:

$$\sum_{h,S_h} \int d^2 p_{hT} \frac{dp_h^-}{2p_h^-} p_h^\mu \, \Delta^h(l, p_h) = \begin{cases} k^- \, \Xi(l) & \mu = - \quad \text{longitudinal} \\ k_T^\mu \, \Xi(l) & \mu = 1,2 \quad \text{transverse} \end{cases}$$

- For TMDs, integrate out k+, take suitable traces

Quark-quark sum rules — **Quark-gluon-quark sum rules**

Collins-Soper

$[\Gamma = \gamma^-]$ $$\sum_{h,S_h} \int dz\, z\, D_1^h(z) = 1$$

Non-zero in χ limit:
order parameter for DχSB

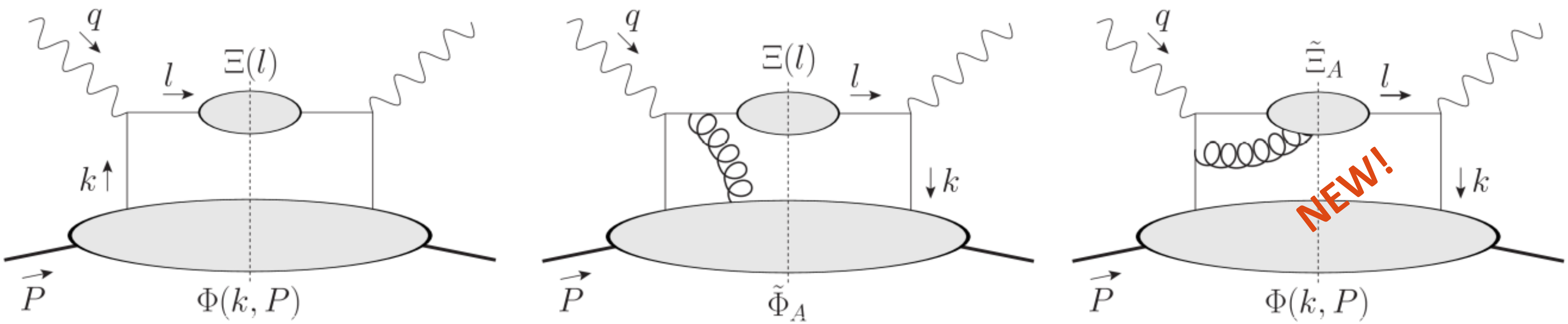
$[\Gamma = \mathbb{I}]$ $$\sum_{h,S_h} \int dz M_h E^h(z) = M_j \qquad \boxed{\sum_{h\, S_h} \int dz M_h \tilde{E}^h(z) = M_j - m_q}$$

$[\Gamma = \gamma^i]$ $$\sum_{h,S_h} \int dz M_h^2 D^{\perp(1)h}(z) = 0 \qquad \sum_{h\, S_h} \int dz M_h^2 \tilde{D}^{\perp\,(1)\,h}(z) = -\frac{1}{2} \langle \frac{\boldsymbol{P}_{h\perp}^2}{z} \rangle$$

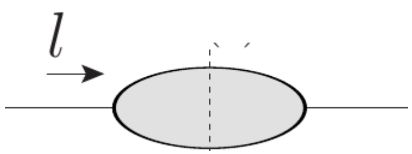
$$f^{(1)}(z) \equiv \int d^2 P_{hT} \frac{P_{hT}^2}{2\Lambda^2} f(z, P_{hT})$$

Inclusive DIS with jet correlators

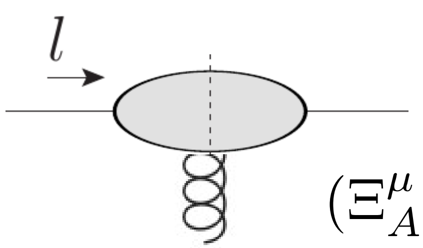
AA, Bacchetta, PLB 773 ('17) 632



Jet correlators: → non-asymptotic quark states / dressed quarks



$$\Xi_{ij}(l,n_+) = F.T. \langle\Omega|\, W^{n_+}_{(+\infty,\xi)}\, \psi_i(\xi)\bar{\psi}_j(0)\, W^{n_+}_{(0,+\infty)}\, |\Omega\rangle$$

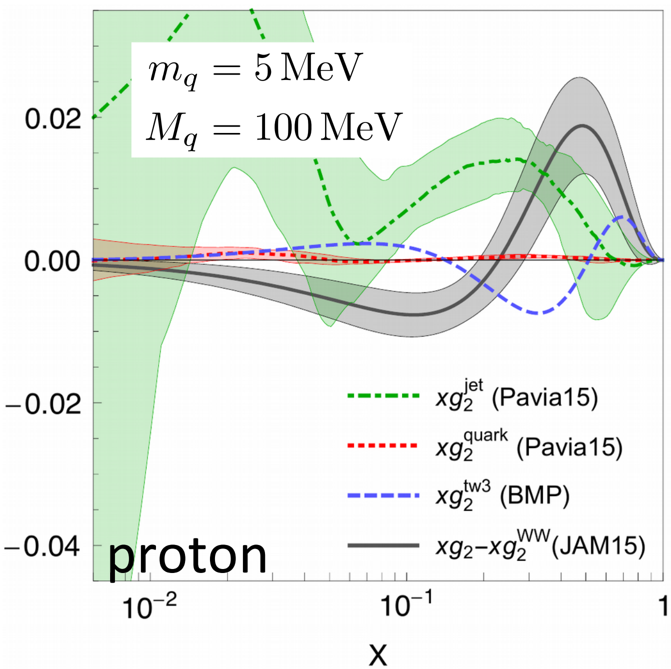


$$(\Xi^{\mu}_{A})_{ij} = F.T.\, \langle\Omega|\, W^{n_+}_{(+\infty,\xi)}\, gA^{\mu}(\xi)\ \psi_i(\xi)\, \bar{\psi}_j(0)\, W^{n_+}_{(0,+\infty)} |\Omega\rangle$$

g2 structure function revisited

- Integrating SIDIS, and using EOM, Lorentz Invariance Relations:

$$g_2(x_B) - g_2^{WW}(x_B)$$
$$= \frac{1}{2}\sum_a e_a^2 \left(g_2^{q,\mathrm{tw3}}(x_B) + \overbrace{\frac{m_q}{M}\left(\frac{h_1^q}{x}\right)^\star (x_B)}^{\equiv\, g_2^{quark}} + \overbrace{\boxed{\frac{M_j - m_q}{M}\frac{h_1^q(x_B)}{x_B}}}^{\equiv\, g_2^{jet}} \right)$$



Consequences:

- h1 accessible in inclusive DIS
  $\leftrightarrow$ Potentially large signal
- Burkardt-Cottingham sum rule broken

$$\int_0^1 g_2(x) = (M_j - m_q)\int_0^1 dx\,\frac{h_1(x)}{x}$$

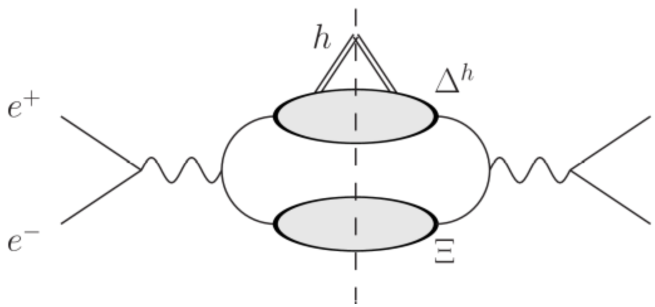
- ETL: novel way to measure tensor charge

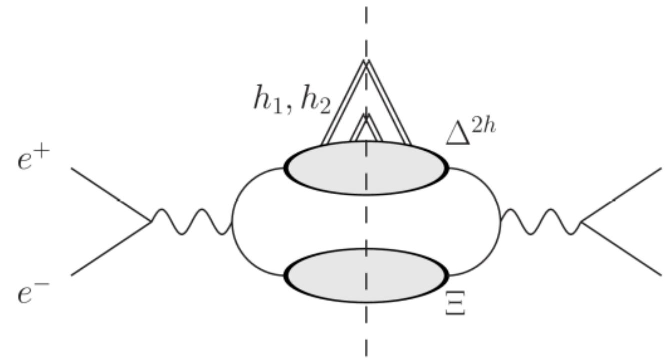
$$\int_0^1 x g_2^{q-\bar q}(x) = 2\,(M_j - m_q)\int_0^1 dx\, h_1^{q-\bar q}(x)$$

Measuring the jet correlator

Accardi, Bacchetta, Signori, Radici, in progress

- Jet mass $\mathbf{M_{jet}}$ **can be measured in polarized $e^+ + e^-$:**





  - Needs **LT asymmetry** in semi-inclusive **Lambda** production

$$\frac{d\sigma^L(e^+e^- \to \text{jet}\, h\, X)}{d\Omega dz} = \frac{3\alpha^2}{Q^2} \lambda_e \sum_a e_a^2 \left\{ \frac{C(y)}{2} \lambda_h G_1 + D(y) |\boldsymbol{S}_T| \cos(\phi_S) \frac{2M_h}{Q} \left(\frac{G_T}{z} + \frac{M_q - m_q}{M_h} H_1 \right) \right\}$$

  - Similarly a **LU asymmetry** in unpolarized **dihadron** production

Where can we measure jet correlators?

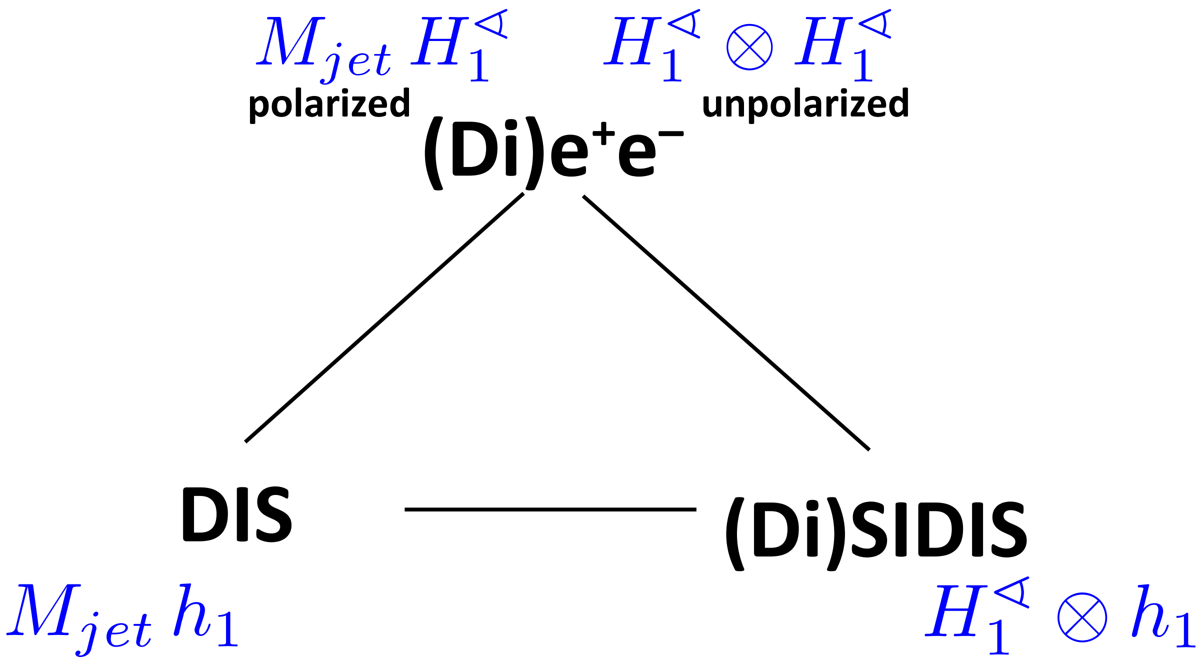
- Can we get a (polarized) **e+ e- collider at JLab / BNL?**
  - At JLab12 ? EIC + positron beam ?
- Are **existing facilities** enough?

	BEPC	super KEKB	ILC	JLab/BNL
E beam [GeV]	1.9	4 (e^-) 7 (e^-)	250	?
$\sqrt{s}$ [GeV]	3 – 5	10	500	?
polarization	?	maybe	80% e^- 60% e^+	YES!

- What else?

A new "universal" fits

- Chiral-odd collinear sector across processes:



Conclusions

Conclusions

- **Hadron mass corrections possible in collinear factorization**
  - Accounts for phase space available for hadronisation
    - with non-zero "virtuality" for fragmenting quark: $v'^2 = m_h^2/\zeta_h$
  - But needs to go beyond the usual "parton model approximation"
  - Proposed scheme phenomenologically successful!
- **HMCs non negligible**
  - Kaon ratios at H & C largely reconciled
  - Pion corrections large only at Jlab; systematic shape difference at H, C
  - Need to account for HMCs in fits
- **"Inclusive" fragmentation and jet correlator:**
  - Novel FF sum rules
  - New phenomenology
    - χSB from SIDIS, tensor charge in DIS, SIA !!
    - ...and more possible, the door is open...

Extras

Phase space limitations

Guerrero et al., JHEP 09 (2015) 169

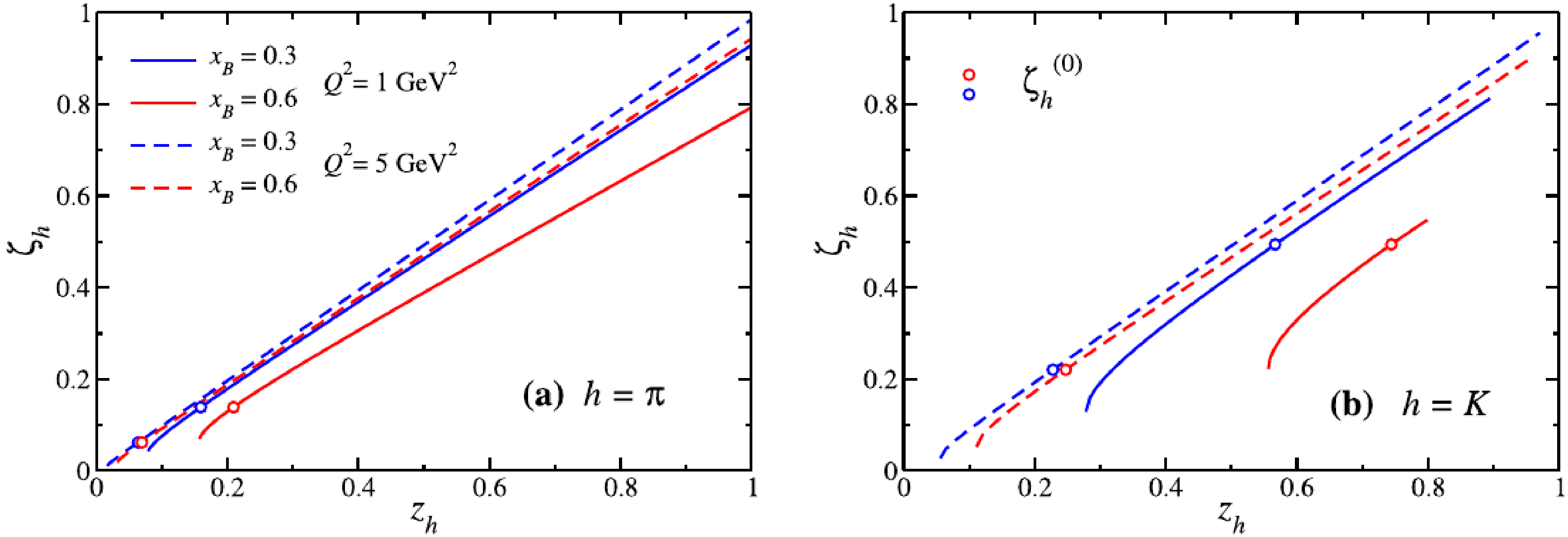


Figure 2. Finite-Q^2 fragmentation variable ζ_h versus z_h for the semi-inclusive production of (a) pions, $h = \pi$ and (b) kaons, $h = K$, at fixed values of $x_B = 0.3$ (blue curves) and 0.6 (red curves) for $Q^2 = 1$ (solid curves) and $5\,\mathrm{GeV}^2$ (dashed curves). The curves are shown only in the kinematically allowed z_h regions, and the boundaries between the current ($\zeta_h > \zeta_h^{(0)}$) and target ($\zeta_h < \zeta_h^{(0)}$) fragmentation regions are indicated by the open circles.

Current vs. target fragmentation regions

Guerrero et al., JHEP 09 (2015) 169

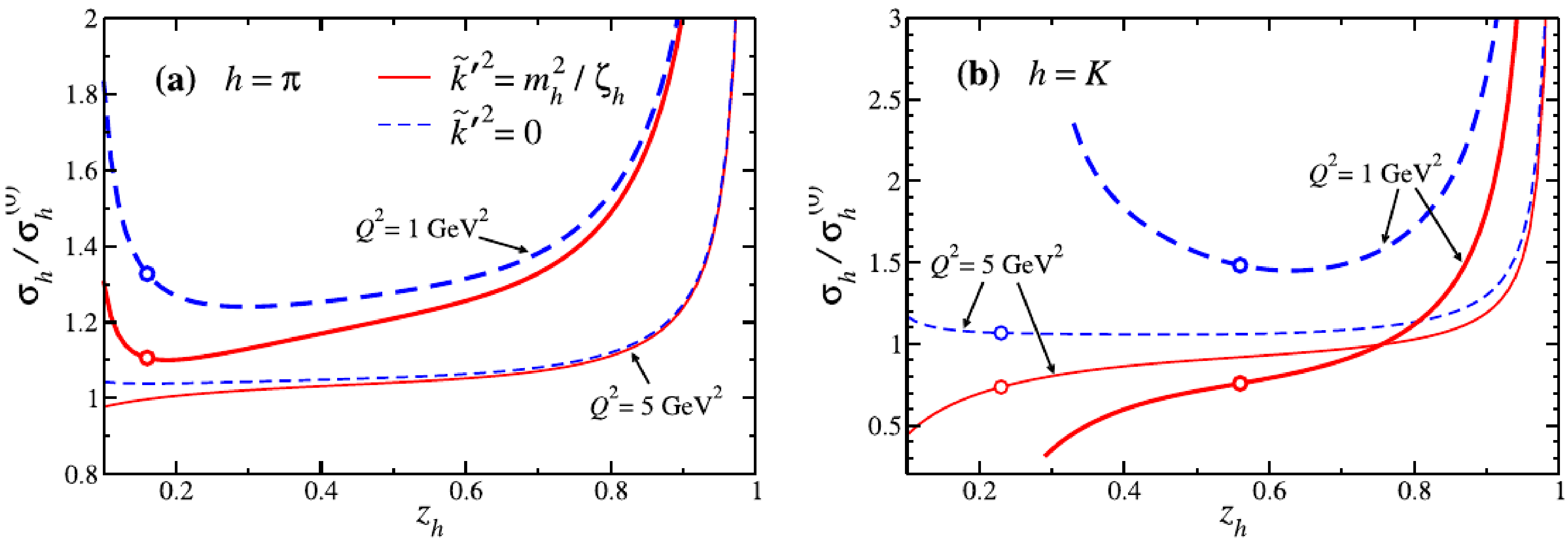
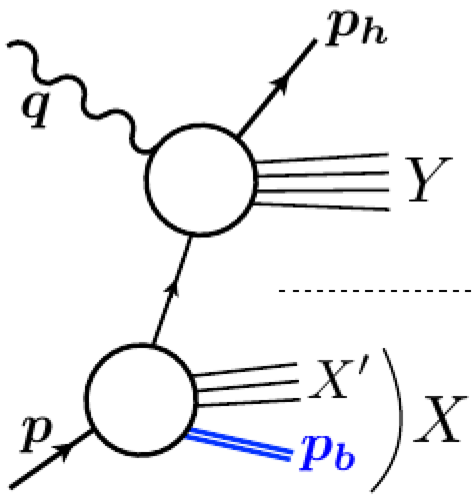


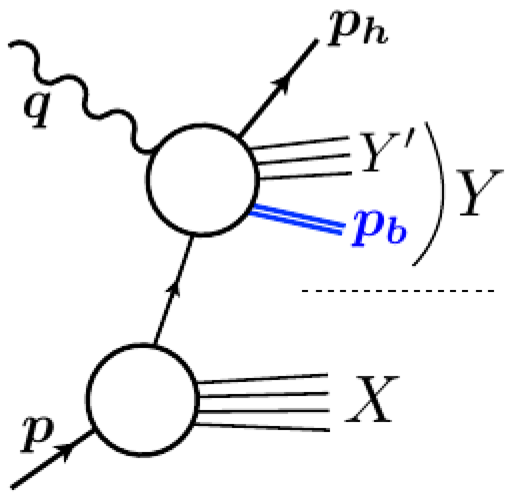
Figure 9. Ratio of spin-averaged cross sections with and without HMCs for the production of (a) pions and (b) kaons, for different choices of the scattered parton invariant mass $\widetilde{k}'^2$ at $Q^2 = 1\,\mathrm{GeV}^2$ (thick lines) and $Q^2 = 5\,\mathrm{GeV}^2$ (thin lines) for $x_B = 0.3$. The open circles denote the boundary between the target and current fragmentation regions.

Current vs. target fragmentation regions

Guerrero, Accardi, PRD 97 (2018) 114012

- **Baryon in in target vs. current region:**



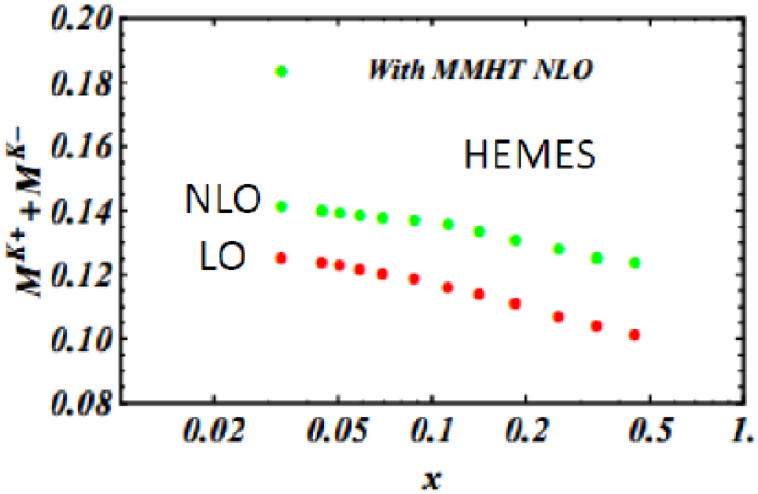


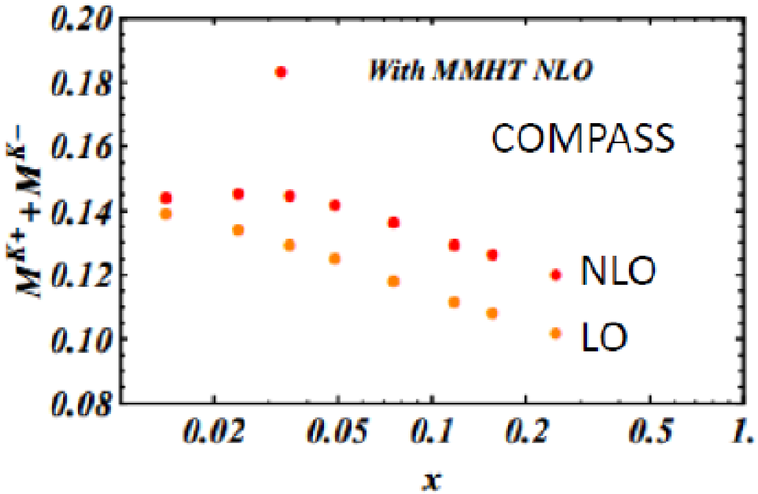
$$x = \xi \left(1 + \frac{m_h^2}{\zeta_h Q^2} \right) \qquad\qquad x = \xi \left(1 + \frac{m_h^2}{\zeta_h Q^2} + \frac{M_b^2}{(1-\zeta_h)Q^2} \right)$$

$$d\sigma_{(b)} \gg d\sigma^{(b)}$$

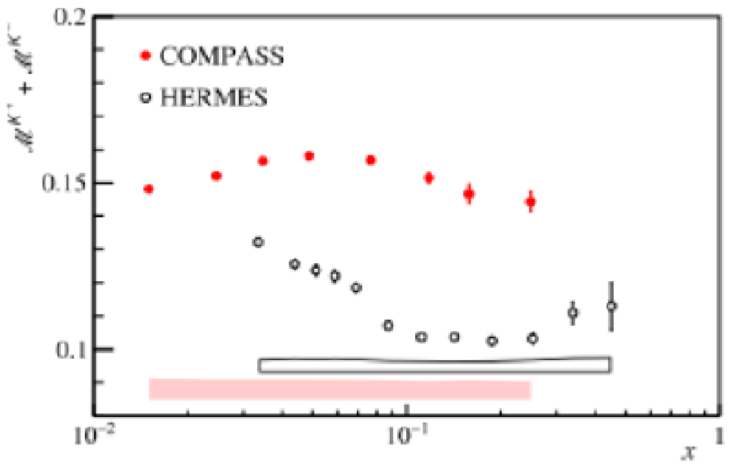
Kaon multiplicity

Chung-Wen Kao, talk at DIS 2018





Kaon multiplicity



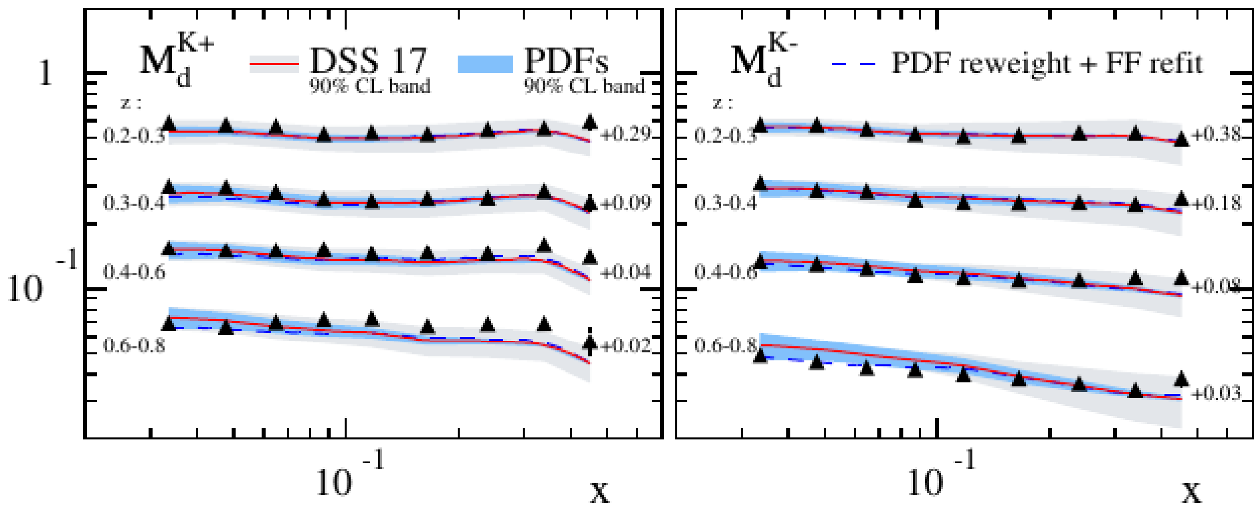
NLO vs. LO:

- ~20% higher (cancels in ratios)
- slight change of shape

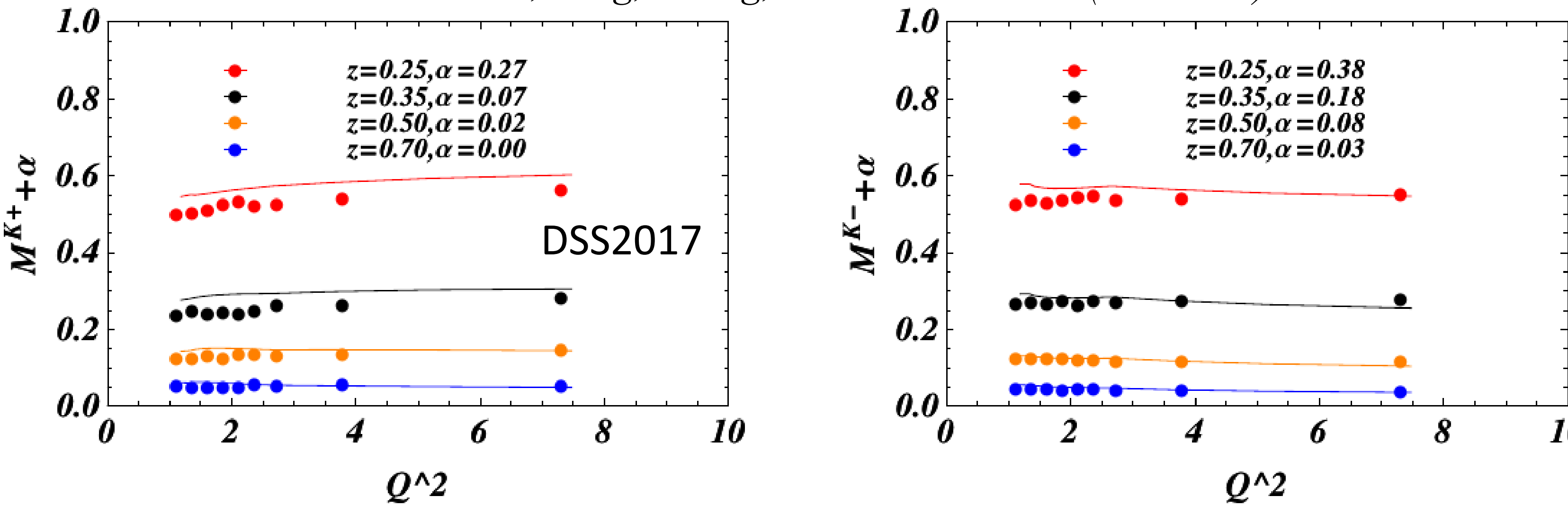
(Refitted) DSS2017 vs. HERMES

HERMES

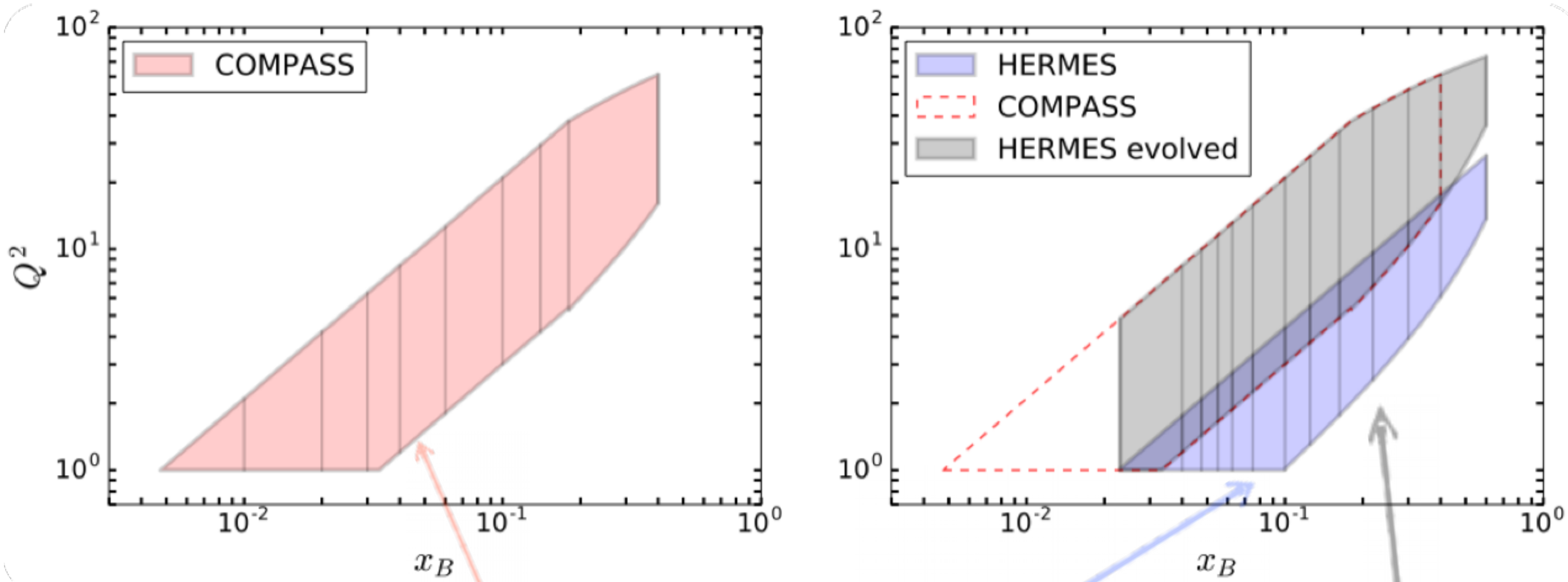
Borsa, Sassot, PRD96 (2017)



Kao, Yang, Chang, arXiv:1807.06524 (DIS 2018)



HERMES & COMPASS data: direct comparison



HMC ratio: $$R^h_{HMC} = \frac{M^{h(0)}}{M^h}$$

Evolution ratio: $$R^{H\to C}_{evo} = \frac{M^{h(0)}(x_B^{HERMES})\Big|_{\mathrm{COMPASS\ cuts}}}{M^{h(0)}(x_B^{HERMES})\Big|_{\mathrm{HERMES\ cuts}}}$$

(HERMES to COMPASS)