# TMD fragmentation functions

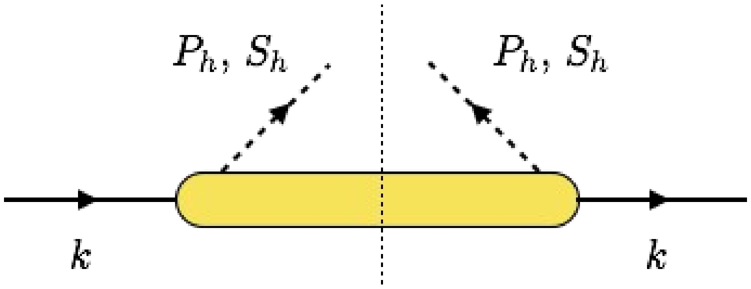
**Andrea Signori**

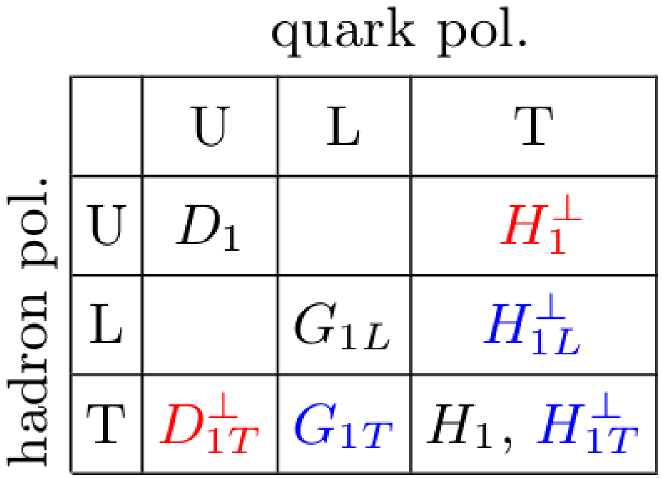
Workshop on Novel Probes of the Nucleon Structure in SIDIS, $e^+e^-$ and pp

**FF2019 - Duke U.**

March 14-16 2019

# Outline of the talk





| | | quark pol. | | |
|---|---|---|---|---|
| | | U | L | T |
| hadron pol. | U | $D_1$ | | $H_1^\perp$ |
| | L | | $G_{1L}$ | $H_{1L}^\perp$ |
| | T | $D_{1T}^\perp$ | $G_{1T}$ | $H_1$, $H_{1T}^\perp$ |

- phenomenology of the TMD $D_1$

- investigations of $e^+e^-$ annihilation into two hadrons

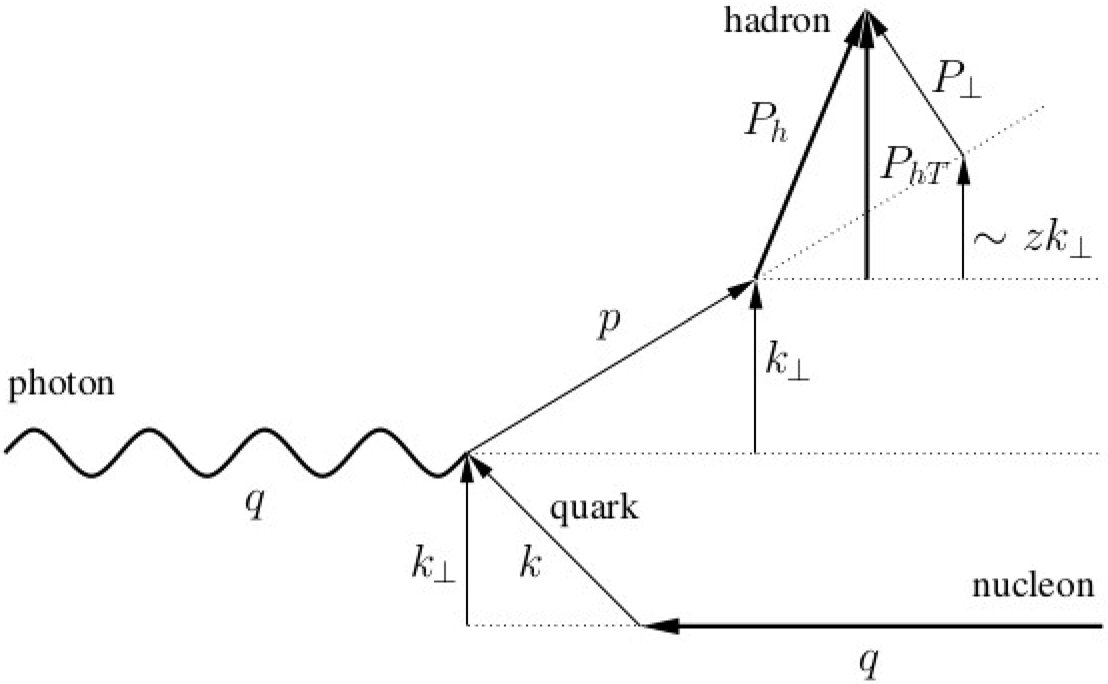
- twist-3 FF and the dynamical generation of mass/momentum

# Phenomenology of $D_1$ TMD

“Global” fit of quark TMDs – Bacchetta et al. : 1703.10157

# Extractions – SIDIS

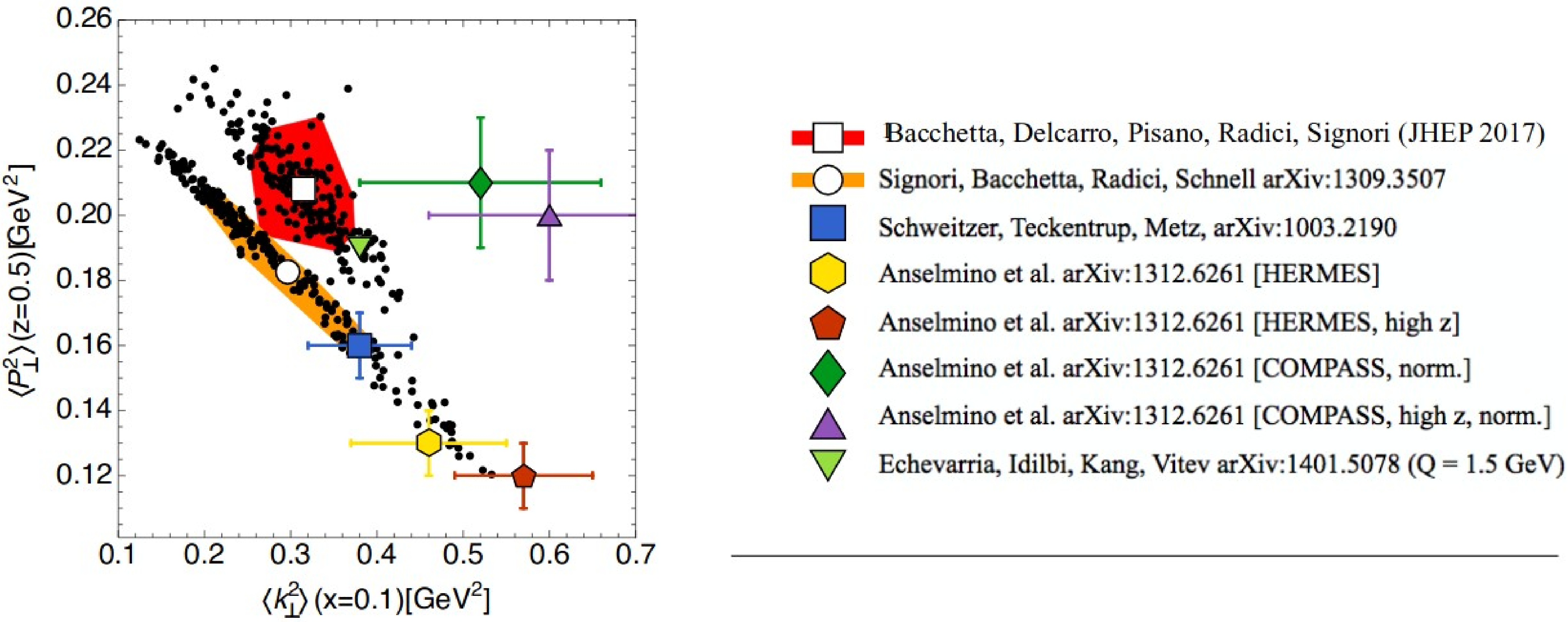
The **only available fits** are from **SIDIS** data, no e+e- data yet



The **priority** is to get **independent information from e+e-**

In SIDIS the information on FFs is always **convoluted** with (TMD) PDFs

# Extractions – SIDIS



Red/orange regions : 68% CL from replica method

Inclusion of Compass dara increases the average t.m. generated in fragmentation

Inclusion of DY/Z diminishes the correlation

# Extractions - SIDIS

1703.10157

$$D_{1NP}^{a\to h}(z,P_\perp^2) = \frac{1}{\pi}\frac{1}{g_{3a\to h}+(\lambda_F/z^2)g_{4a\to h}^2}\left(e^{-\frac{P_\perp^2}{g_{3a\to h}}}+\lambda_F\frac{P_\perp^2}{z^2}\,e^{-\frac{P_\perp^2}{g_{4a\to h}}}\right)$$

Model for the large bT part of the TMD FF : - 6 parameters
- 1 for evolution

TMD evolution included at NLL

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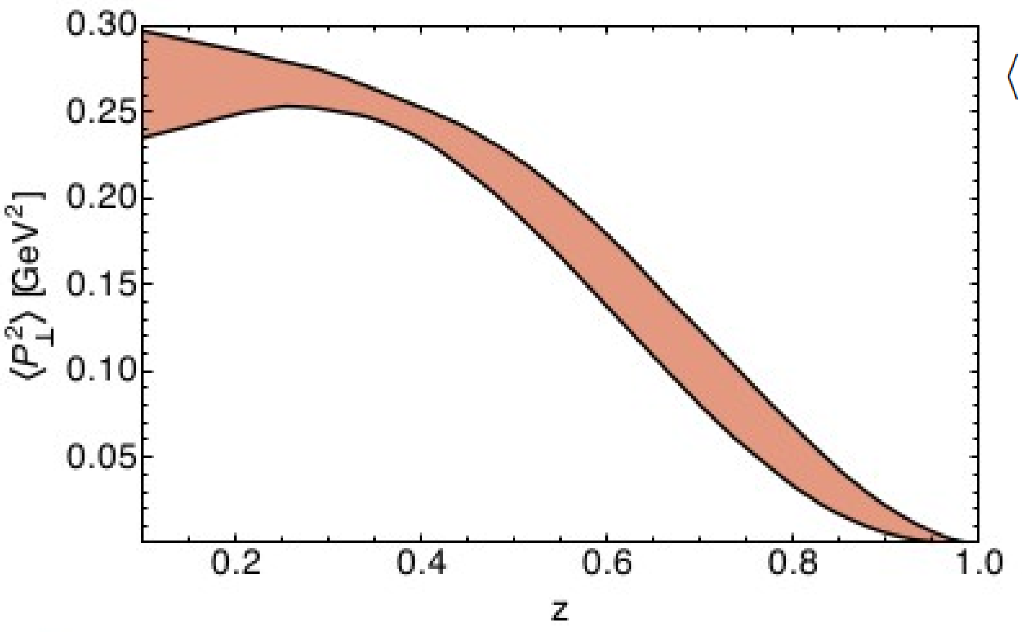
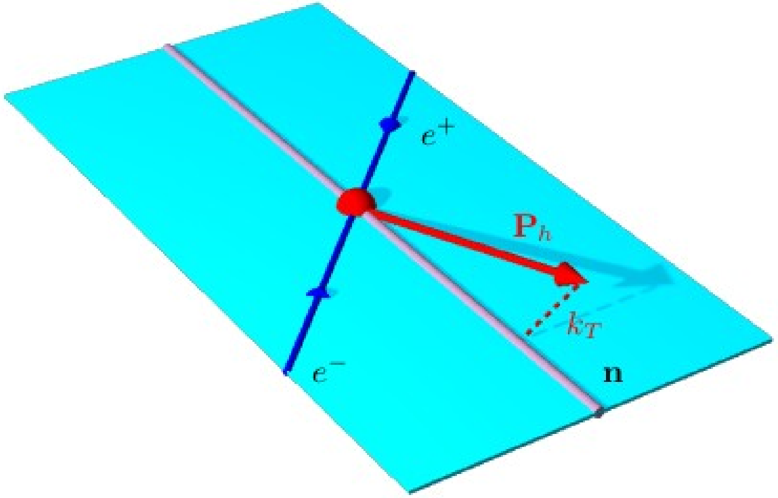


$$\langle P_\perp^2 \rangle(z) = \frac{\int d^2 P_\perp \; P_\perp^2 \; D_1^{a\to h}(z, P_\perp^2, Q = 1\,\mathrm{GeV})}{\int d^2 P_\perp \; D_1^{a\to h}(z, P_\perp^2, Q = 1\,\mathrm{GeV})}$$

Kinematic dependence
of the average square transverse momentum
generated during hadronization

# New experimental information



New data from Belle:
1902.01552
see Ralf's talk

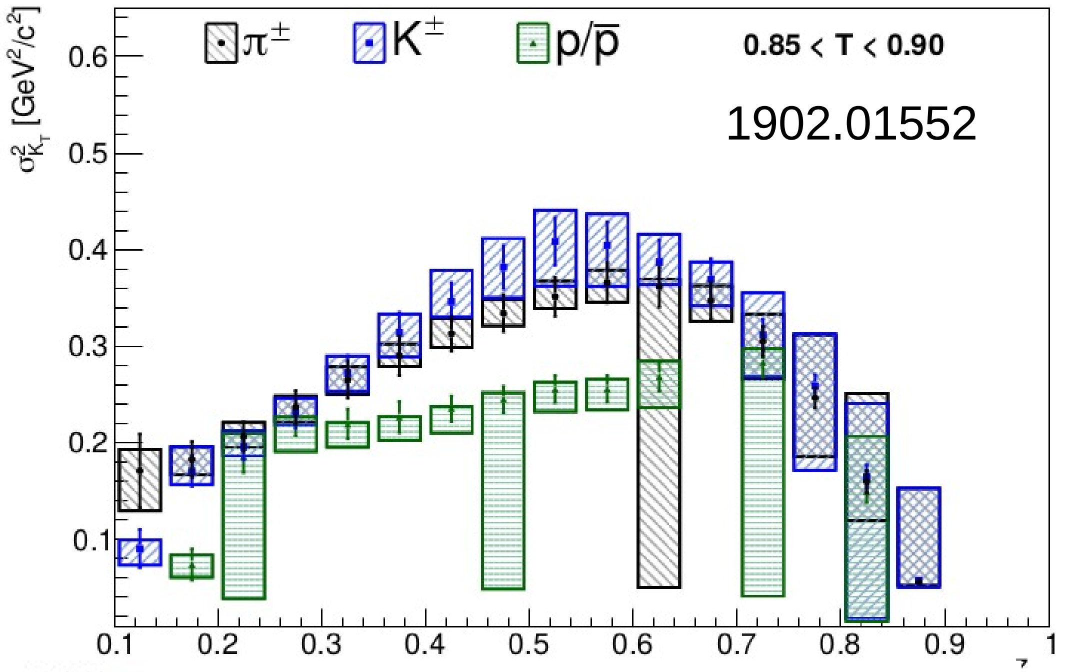
**Homework for the theorists**:
factorize the cross section
in terms of a TMD FF and
something else (?)

But also:

- information from **hadronization in jet** (see Jim and Felix's talks)

- 1704.08882: TMD shapes at low transverse momentum
  from TASSO/PLUTO/MARK-II data
  (but no factorization and integrated over z)

# Cross sections and z-dep. widths



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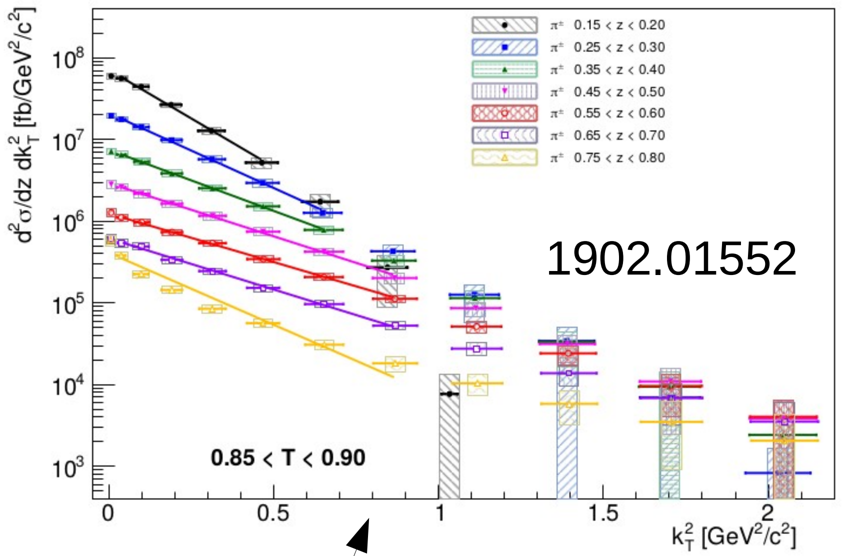

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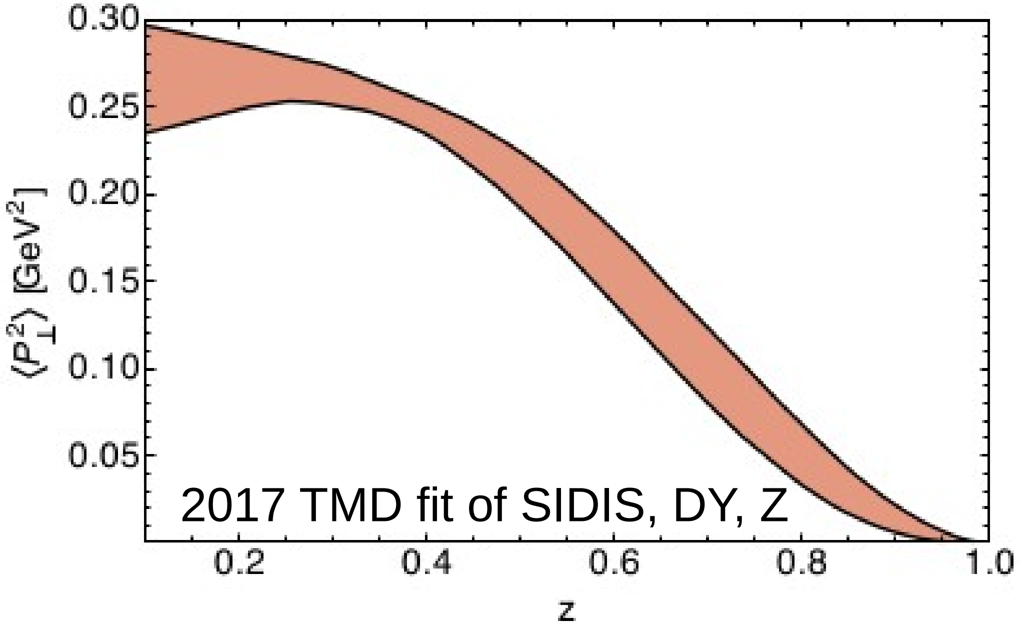


2017 TMD fit of SIDIS, DY, Z

Gaussian fits to Belle data
(see Ralf's talk)

There are **significant hints** about the **z-dependence** already in the Hermes and Compass **SIDIS** data!

# Studying $e^+e^- \longrightarrow h_1 h_2 X$

In collaboration with E. Moffat, T. Rogers

# $e^+e^- \to h_1\, h_2\, X$

- while we wait for the data we would like to study the cross section from the theory point of view ranging from low to high transverse momentum

- we know the perturbative part of the W term (low $q_T$) up to NNLO
- we don't know the fixed-order (F.O.) at all!
- we are calculating the F.O. at leading order and compare to pseudodata

$$\frac{d\sigma}{dz_A dz_B dq_T} = W + F.O. - A$$

See Nobuo's talk

$q_T/Q$ controls
the approximation

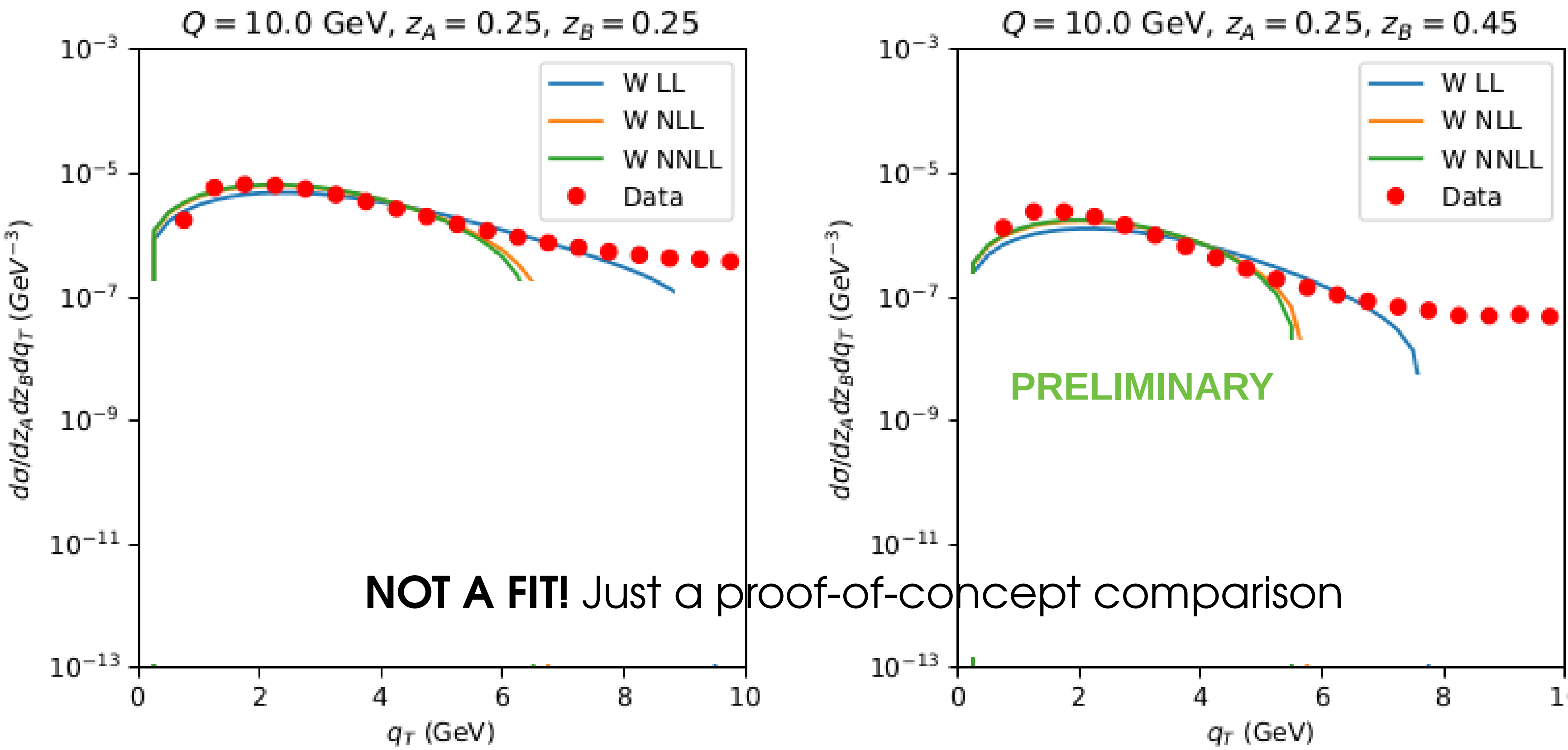
low $q_T$ approximation
(**TMD factorization and FFs**)

high $q_T$ approximation
(**collinear factorization and FFs**)

Here $q_T$ is the transverse momentum of the intermediate photon

# $e^+e^- \rightarrow h_1\, h_2\, X$ and W-term

**PRELIMINARY**



**NOT A FIT!** Just a proof-of-concept comparison

Pythia: 10M events generated using the Belle settings

W-term calculated at LO + Sudakov at LL, NLL, NNLL

The **pseudodata follow the W most probably outside of TMD region** …

# FFs and a dynamical quark mass

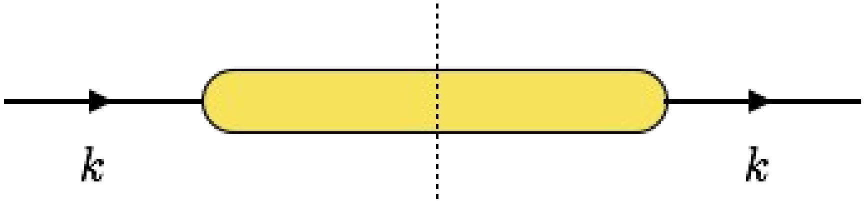
In collaboration with A. Accardi : 1903.04458

# The cut quark propagator

Sterman - NPB 281, 310 (`87)

$$\Xi_{ij}(k;v) = \mathrm{Disc}\int \frac{d^4\xi}{(2\pi)^4} e^{\imath k\cdot\xi}\,\frac{Tr_c}{N_c}\langle\Omega|\mathcal{T}W_1(\infty,\xi;v)\psi_i(\xi)\overline{\psi}_j(0)W_2(0,\infty;v)|\Omega\rangle$$

**Partonic**
picture



- this is the object that enters the **inclusive DIS cross section**
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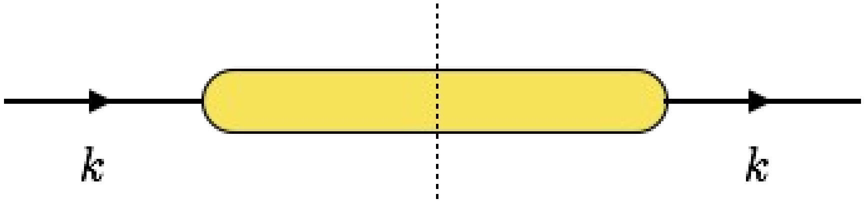
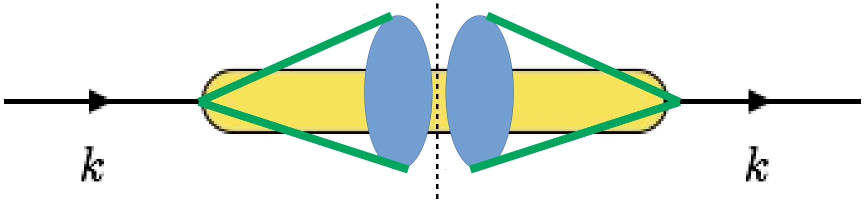


- this is the object that enters the **inclusive DIS cross section**

- the **color is neutralized** : there is a cut

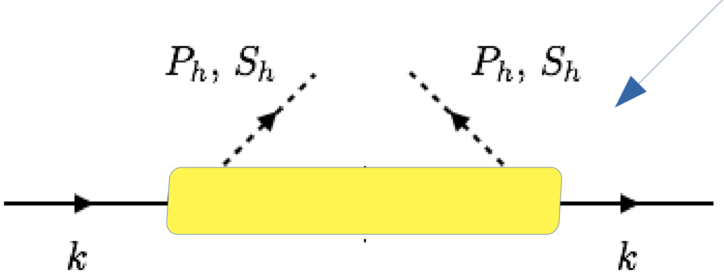
- because of the cut, we can also interpret this as the **”inclusive” hadronization of the quark** (“inclusive jet”) - no cone/jet axis

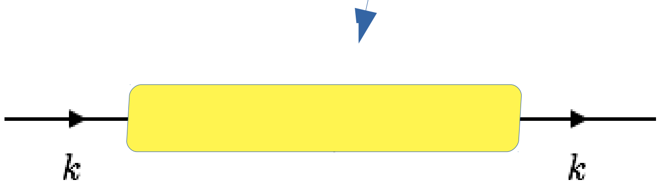
**Hadronic**
picture



# Momentum sum rules - operator level

$$\sum_{h,\,S_h}\int \frac{d^4P_h}{(2\pi)^3}\,\delta(P_h^2-M_h^2)P_h^\mu\,\Delta^h(k,P_h,S_h) = k^\mu\,\Xi^{uncut}(k)$$

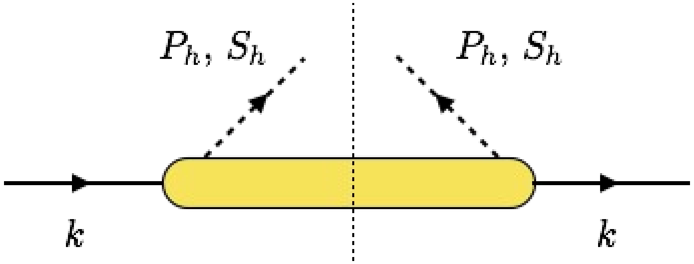




We can obtain the **dressed quark propagator** from calculating the **“average” on-shell four momentum produced by hadronization**, weighted by the 1h fragmentation correlator

Calculating Dirac projections we can give momentum sum rules for TMD FFs !

# Dirac structures



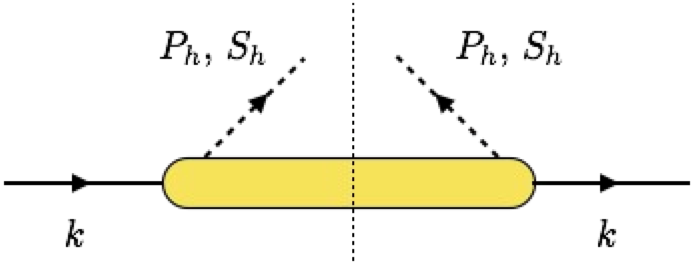
Integration over suppressed momentum components allows to define the TMD correlators:

$$\Delta^h_{ij}(z, P_{h\perp}) = \frac{\gamma^+}{2} D_1^h + \frac{M_h}{2P_h^-} E^h + \frac{P_{h\perp}}{2zP_h^-} D^{\perp h}$$

+ quark-polarized terms

TMD FFs

# Dirac structures



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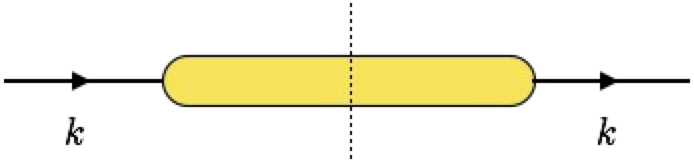
TMD FFs

For the (fully) inclusive jet, instead, we have:

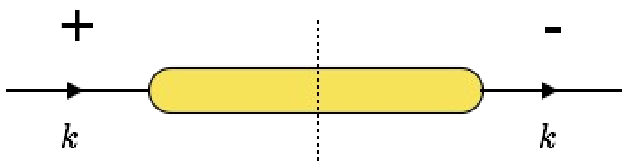


(Fully) Inclusive jet functions

$$J(k^-, k_T) = \frac{1}{2}\alpha(k^-)\left[\gamma^+ + \frac{\not{k}_T}{k^-}\right] + \frac{\Lambda}{2k^-}\zeta(k^-)\mathbb{I}$$

+ higher-twist terms

# The jet / quark mass

$Tr\Big($ 

$\Big) \sim M_j(k^-)$

# The jet / quark mass

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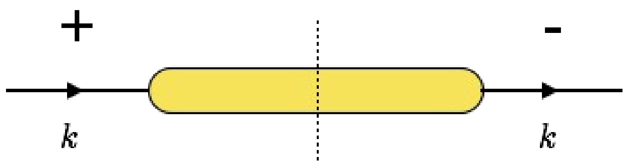


$\Big) \sim M_j(k^-)$

In the **light-cone gauge** the expression simplifies to:

$$M_j = \int_0^{+\infty} d\mu^2 \, \sqrt{\mu^2}\, \rho_1(\mu^2)$$

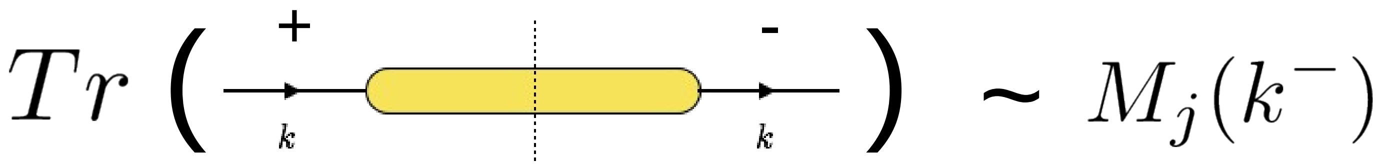
See also Accardi-Bacchetta: 1706.02000

**Mass** associated to the

**chiral-odd component**

of the amplitude squared

# The jet / quark mass



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**Mass** associated to the
**chiral-odd component**
of the amplitude squared

Provides a definition of the mass for the **color-screened dressed quark**, which is:

- **gauge invariant**
- renormalization-scale dependent
- calculable, should you know the chiral-odd specral function
- and **most importantly, it is measurable via momentum sum rules!**

# Mass sum rule

Projecting the sum rule at the operator level with the identity Dirac structure:

$$\sum_{h,\,S_h} \int dz M_h E^h(z) = M_j$$

Jet / quark mass as the
**average of the masses**
**of the produced hadrons weighted by**
**the chiral-odd E FF**

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EOM relation for E

$$E = \tilde{E} + z\frac{m_q}{M_h} D_1$$

$$\xrightarrow[\text{q-g-q correlations}]{\text{neglecting}}$$

WW approximation

$$M_j = m_q$$

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WW approximation

$$M_j = m_q$$

$$\downarrow$$

**Dynamical mass!**

**Full QCD**

$$M_j = m_q + m_q^{corr}$$

$$\longleftarrow$$

$$\sum_{h,\,S_h} \int dz M_h \tilde{E}^h(z) = m_q^{corr}$$

We expect this is not zero also in the chiral limit!

# Full set of momentum sum rules

The full set of momentum sum rules for quarks into unpolarized hadrons

- 🔴 $\sum_{h\,S_h}\int dz\, z\, D_1^h(z) = 1$
- 🟢 $\sum_{h\,S_h}\int dz\, M_h E^h(z) = M_j$
- 🟢 $\sum_{h\,S_h}\int dz\, M_h \tilde{E}^h(z) = M_j - m_{q0} = m_q^{corr}$
- 🟢 $\sum_{h\,S_h}\int dz\, M_h H^h(z) = 0$
- 🔴 $\sum_{h\,S_h}\int dz\, M_h \tilde{H}^h(z) = 0$
- 🔴 $\sum_{h\,S_h}\int dz\, z\, M_h H_1^{\perp\,(1)\,h}(z) = 0$
- 🟢 $\sum_{h\,S_h}\int dz\, M_h^2 D^{\perp\,(1)\,h}(z) = 0$
- 🟢 $\sum_{h\,S_h}\int dz\, M_h^2 \tilde{D}^{\perp\,(1)\,h}(z) = -\frac{1}{2}\langle \boldsymbol{P}_\perp^2/z\rangle$
- 🟢 $\sum_{h\,S_h}\int dz\, M_h^2 G^{\perp\,(1)\,h}(z) = 0$
- 🟢 $\sum_{h\,S_h}\int dz\, M_h^2 \tilde{G}^{\perp\,(1)\,h}(z) = 0\ .$

**Fully dynamical quantities!**

# Conclusions

**Regarding the phenomenology of TMD D1:**

- we only have information from SIDIS at the moment, we have to study data where the **information is purely from FFs**:
- **new data from Belle**, but we don't know how to interpret them yet
  **theorists have a homework**
- possibility from in-jet fragmentation

- regarding the production of **two hadrons**: study of the large qT region is ongoing
  we have to figure out what's going on

**Hadronization and dynamical generation of mass**

- we can quantitatively connect quark **fragmentation** to the **dynamical generation of mass**
- **gauge invariant** definition for jet/color-screened dressed quark **mass**, which is **observable**
- for that, we need to work in the chiral-odd sector AND at least at twist 3
  e.g. : g2, di-hadron plus one jet, ...

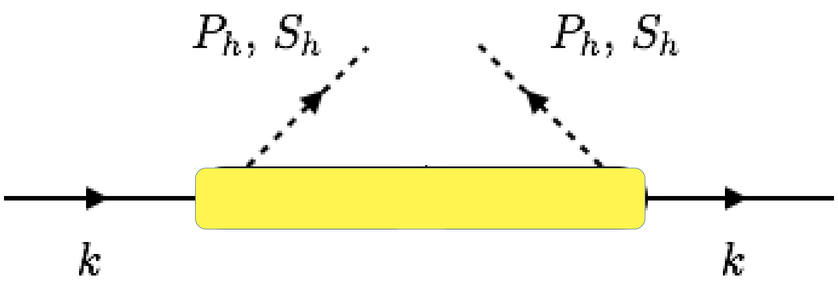
- new sum rules, which can serve as a guidance for future studies

# Backup

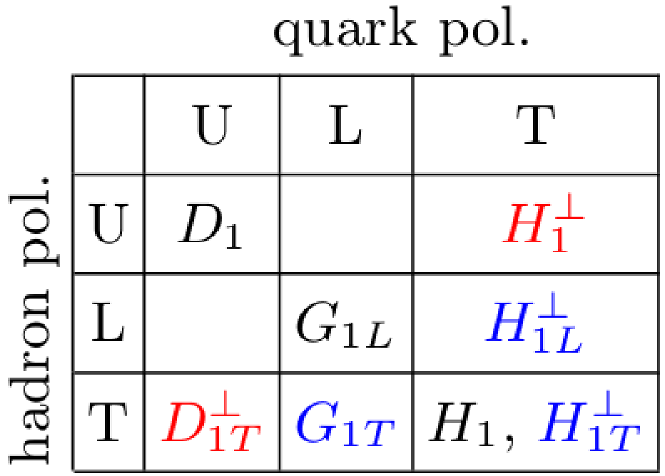
# The 1h fragmentation correlator

$$\Delta^h_{ij}(k,P_h,S_h) = \int \frac{d^4\xi}{(2\pi)^4} e^{ik\cdot\xi} \frac{Tr_c}{N_c} \langle\Omega|\mathcal{T} W_1(\infty,\xi)\psi_i(\xi)\, a_h^\dagger(P_h,S_h)a_h(P_h,S_h)\overline{\psi}_j(0)W_2(0,\infty)|\Omega\rangle$$

The correlator describing the fragmentation of one quark into an observed hadron plus other stuff



The **discontinuity** of this object integrated over the suppressed momentum component can be parametrized in terms of TMD FFs



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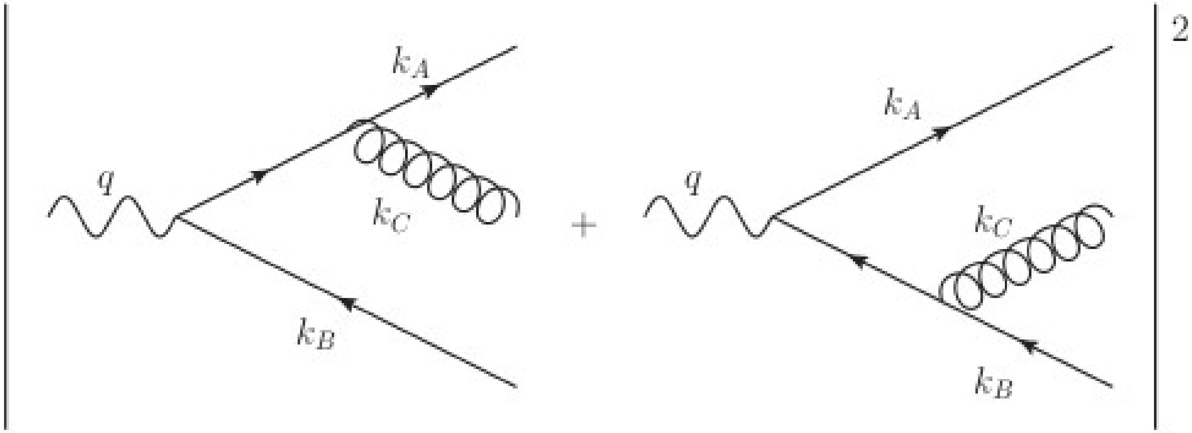
|  | T | $D_{1T}^\perp$ | $G_{1T}$ | $H_1,\ H_{1T}^\perp$ |

# $e^+e^- \to h_1\, h_2\, X$ at large $q_T$

We need partonic diagrams with the emission of a hard gluon
(the $q_T$ dependence is in the hard part of cross section and not in the TMD distributions)

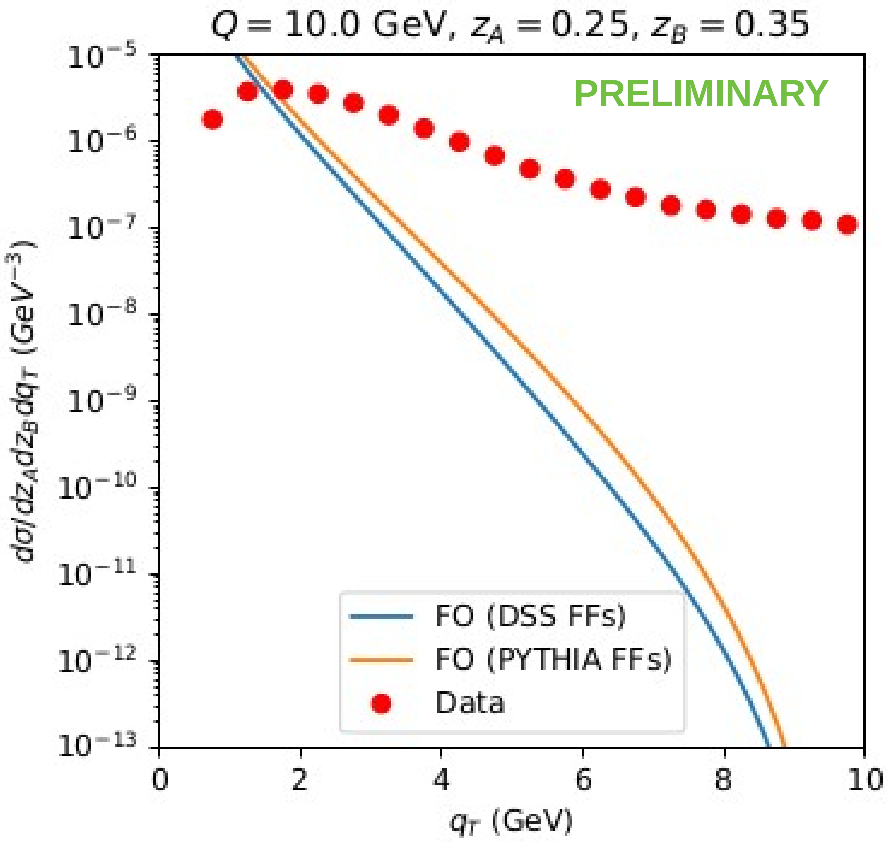


The two produced hadrons can come either from the quarks or the gluon

Cross section: hard partonic cross section convoluted with **collinear FFs**

# $e^+e^- \to h_1\ h_2\ X$ at large $q_T$

**PRELIMINARY**



There is a definitely a fundamental problem which causes the discrepancy.

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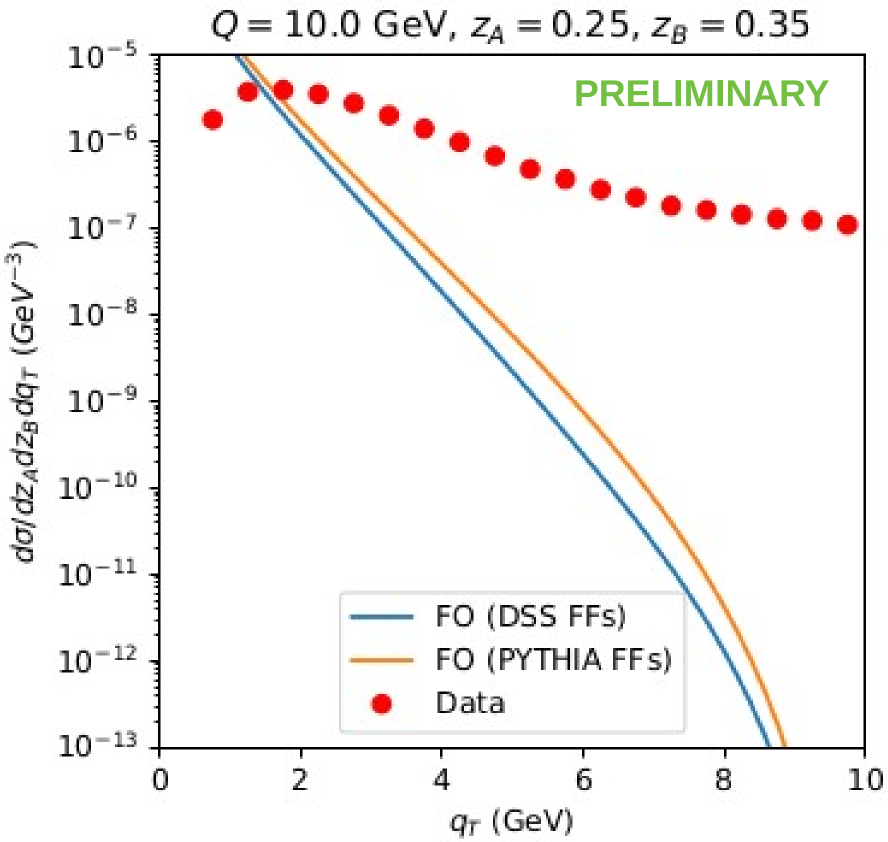

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**Some possibilities:**

1) our calculation is wrong

2) the pseudodata are not correct at large $q_T$

**Tests**:

- increase Q
- use collinear FFs extracted from Pythia

NO SIGNIFiCANT IMPROVEMENT

# $e^+e^- \to h_1 h_2 X$ at large $q_T$

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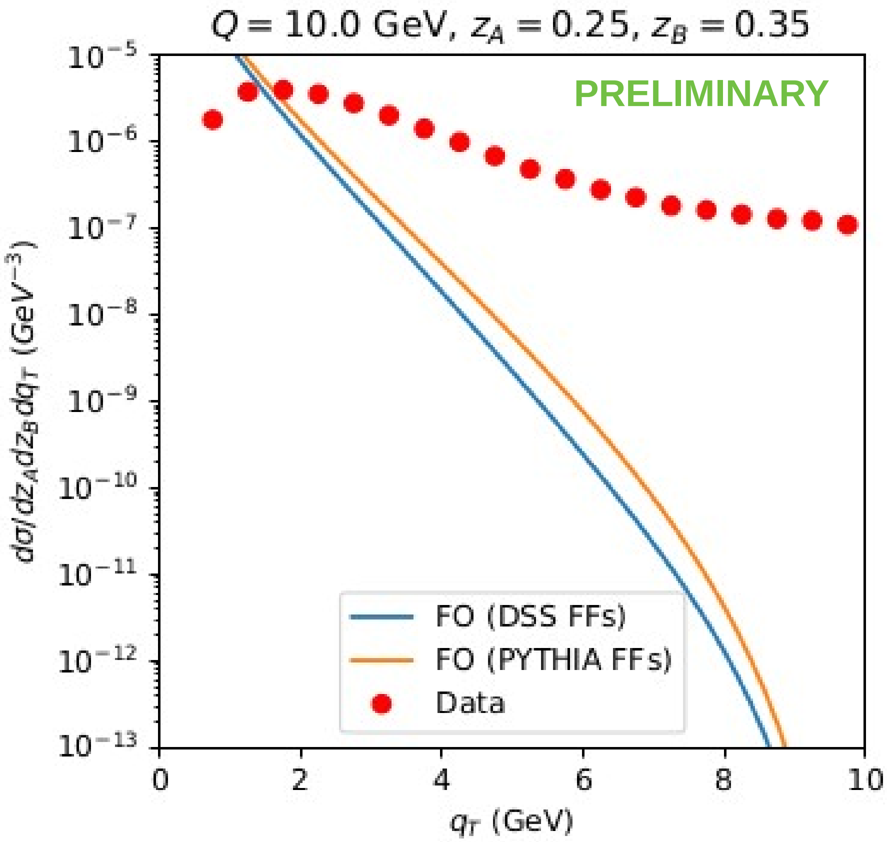

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NO SIGNIFiCANT IMPROVEMENT

The simulation is based on **Pythia**, which knows about the **tree-level** diagram only

We want to feed the high $q_T$ event into Pythia via **Madgraph** and see what happens