Hadronization in Jets: Part II

Yiannis Makris

Los Alamos National Laboratory
EST. 1943

Based on… in collaboration with…

- Fragmentation with in groomed jets (light hadrons)

  (arXiv:1712.07653: YM, D. Neill, and V. Vaidya)

- Fragmentation with in groomed jets (heavy mesons)

  (arXiv:1807.09805: YM and V. Vaidya)

- Dijet decorrelation (lepton colliders)/ jet-lepton decorrelation (DIS)

  (In progress: YM, V. Vaida, L. Zoppi, D. G.-Reyes, and I. Scimemi)

In this talk

- Grooming procedure (mMDT/soft-drop)

- Dijet de-correlation (e+e-)

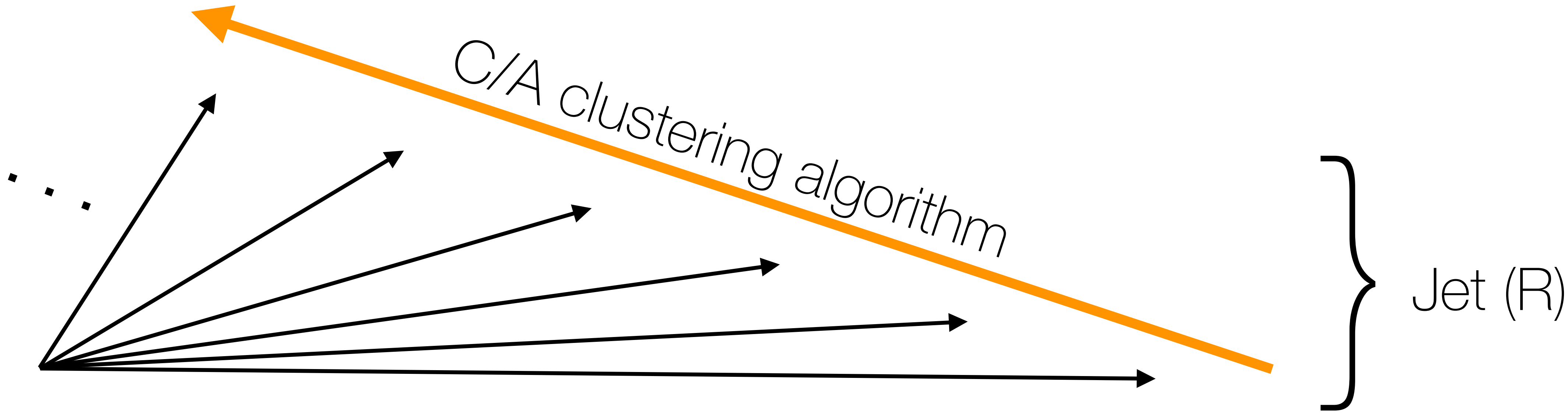
  Hadronization effects - Pythia simulations

  Factorization and resummation @ NNLL

- TMD spectra of heavy mesons in groomed jets:

  New and interesting hadronization effects

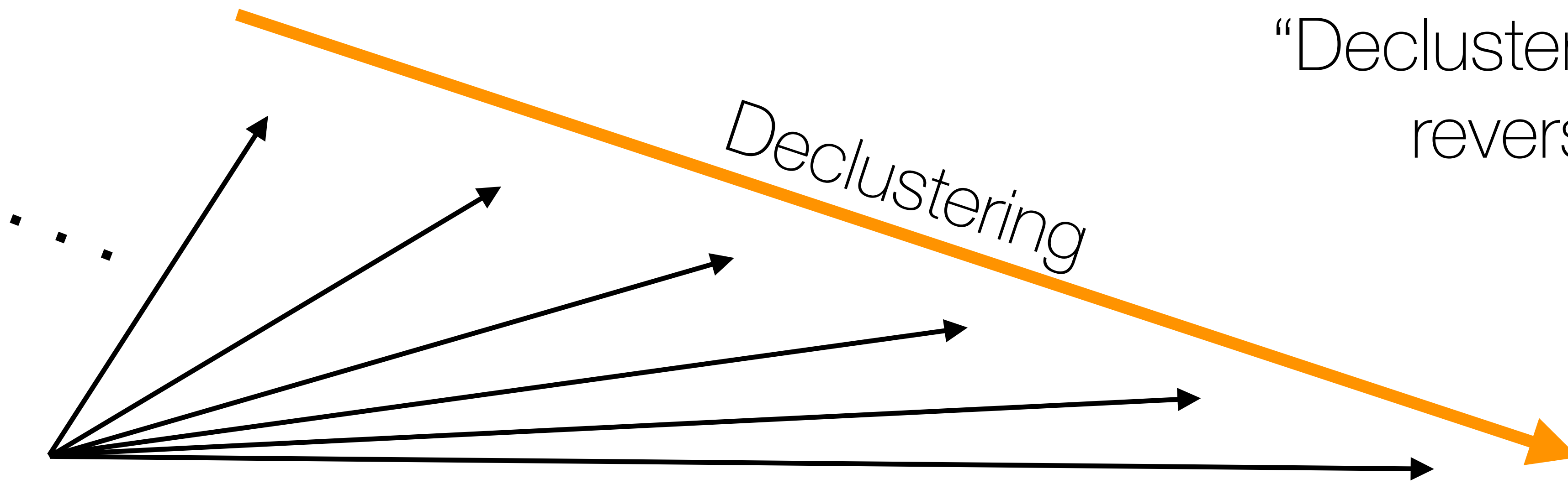
Grooming algorithm: mMDT/soft-drop $(\beta = 0)$



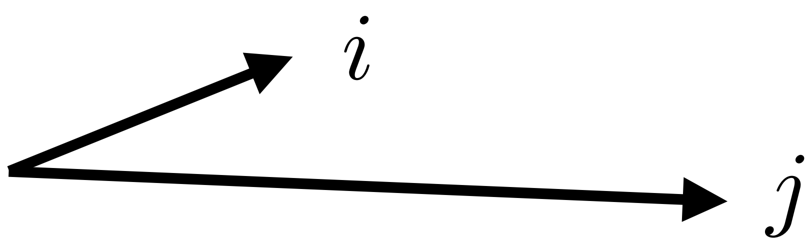
- The algorithm is imposed **only** on the jet constituents
- Record clustering history in each step
- Particles closer in angle get clustered first

For details on soft-drop see: **arXiv:1402.2657** A. J. Larkoski, S. Marzani, G. Soyez, and J. Thaler

Grooming algorithm: mMDT/soft-drop ($\beta = 0$)



“Declustering” with the reverse order:

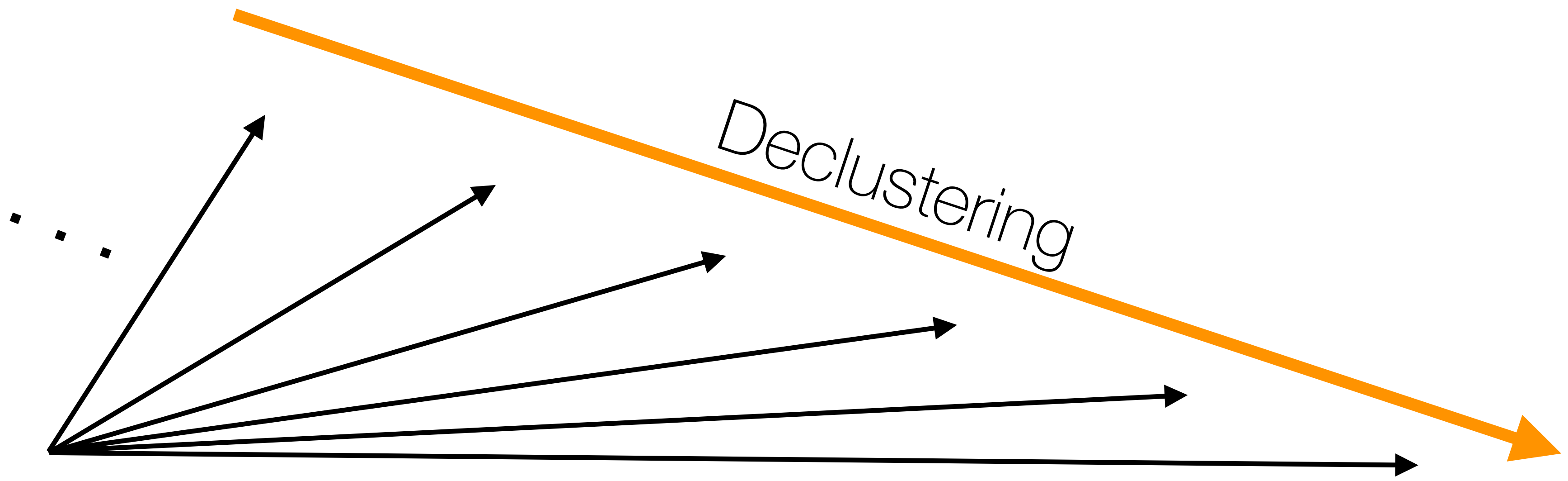


$$\frac{\min\{E_i, E_j\}}{E_i + E_j} > z_{\rm cut}$$

True
Stop. Remaining particles consist groomed jet

False
Drop the softer of the two branchings

Grooming algorithm: mMDT/soft-drop ($\beta = 0$)

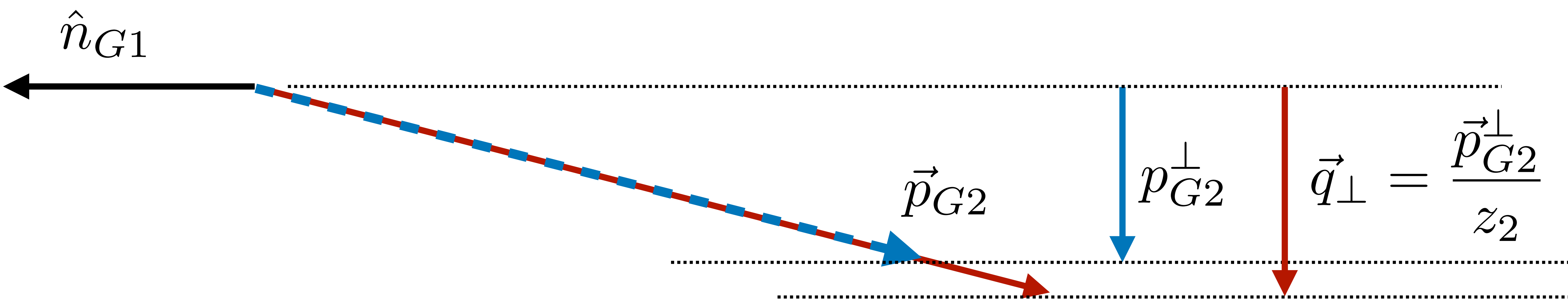


- Removes soft wide angle radiation sensitive to the cone/boundary and non-global effects (NGLs)
- Isolates collinear-energetic radiation near the center of the jet
- Smaller sensitivity to underlying event

Dijet de-correlation (work in progress)

jet + jet
jet + hadron (out)

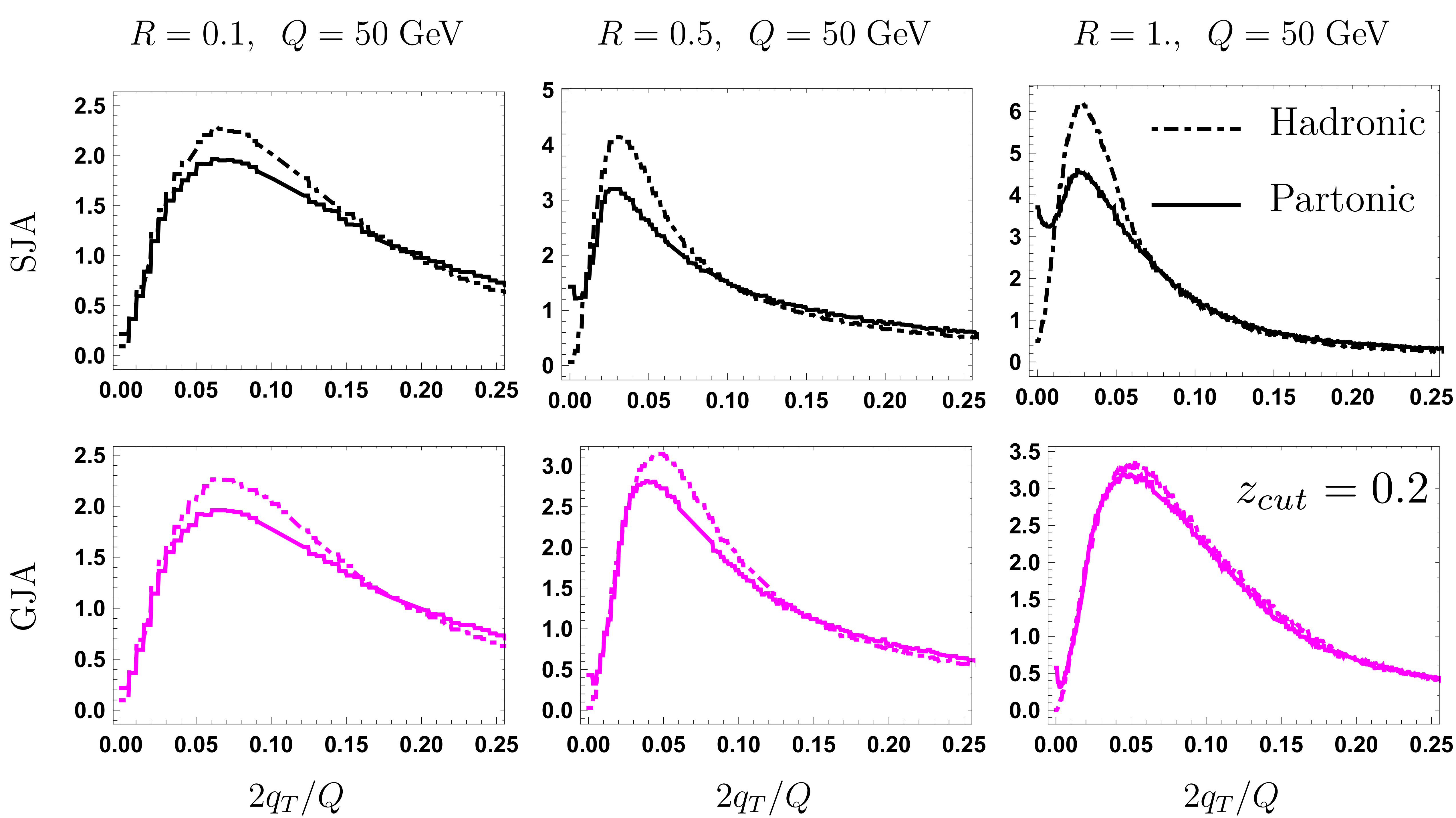
$$Q \gg Q z_{cut} \gg q_\perp , \quad R \sim 1$$



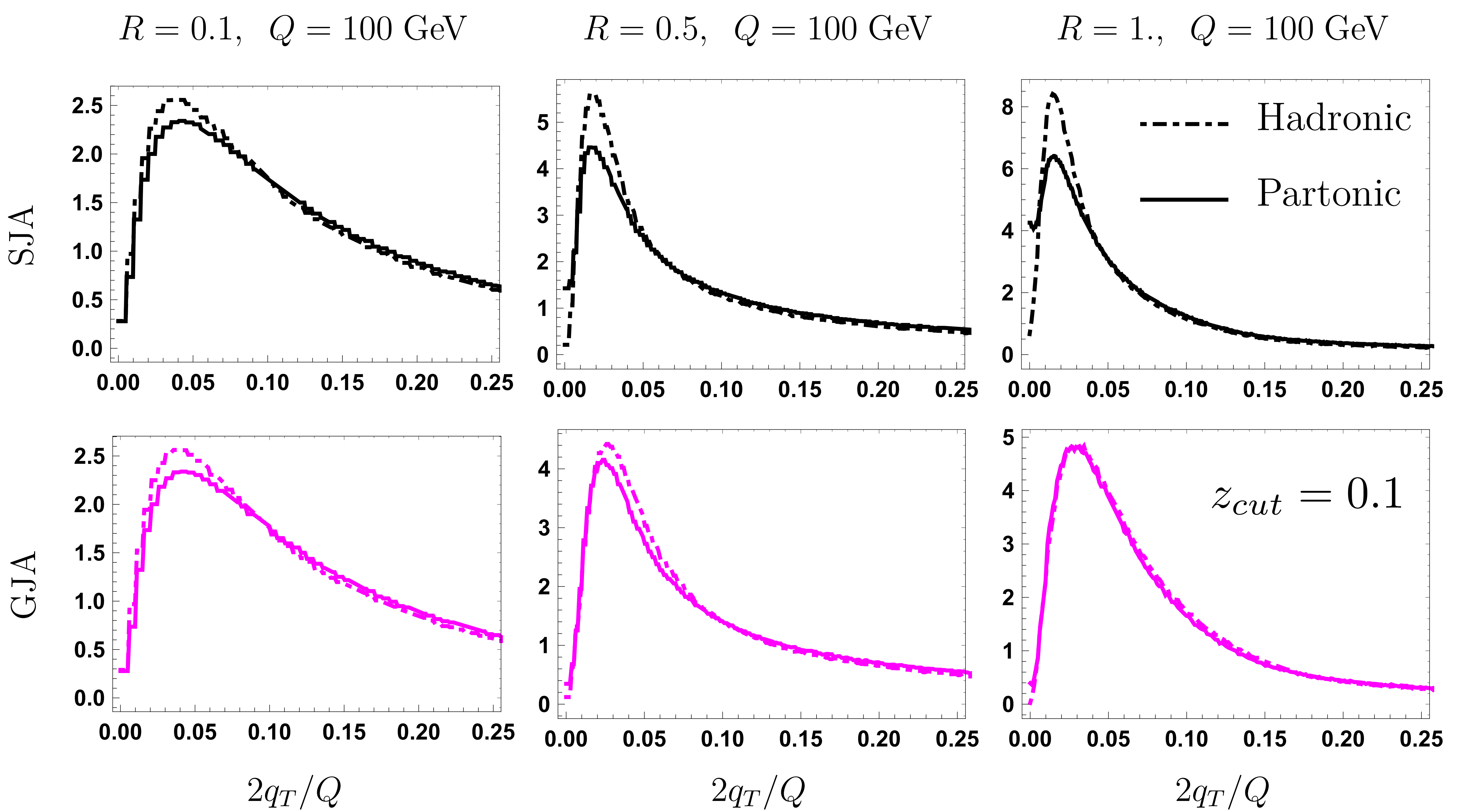
$$\theta = \frac{2q_\perp}{Q} \qquad z_i = \frac{2E_{Gi}}{Q}$$

The only information used from the groomed jets is their directions

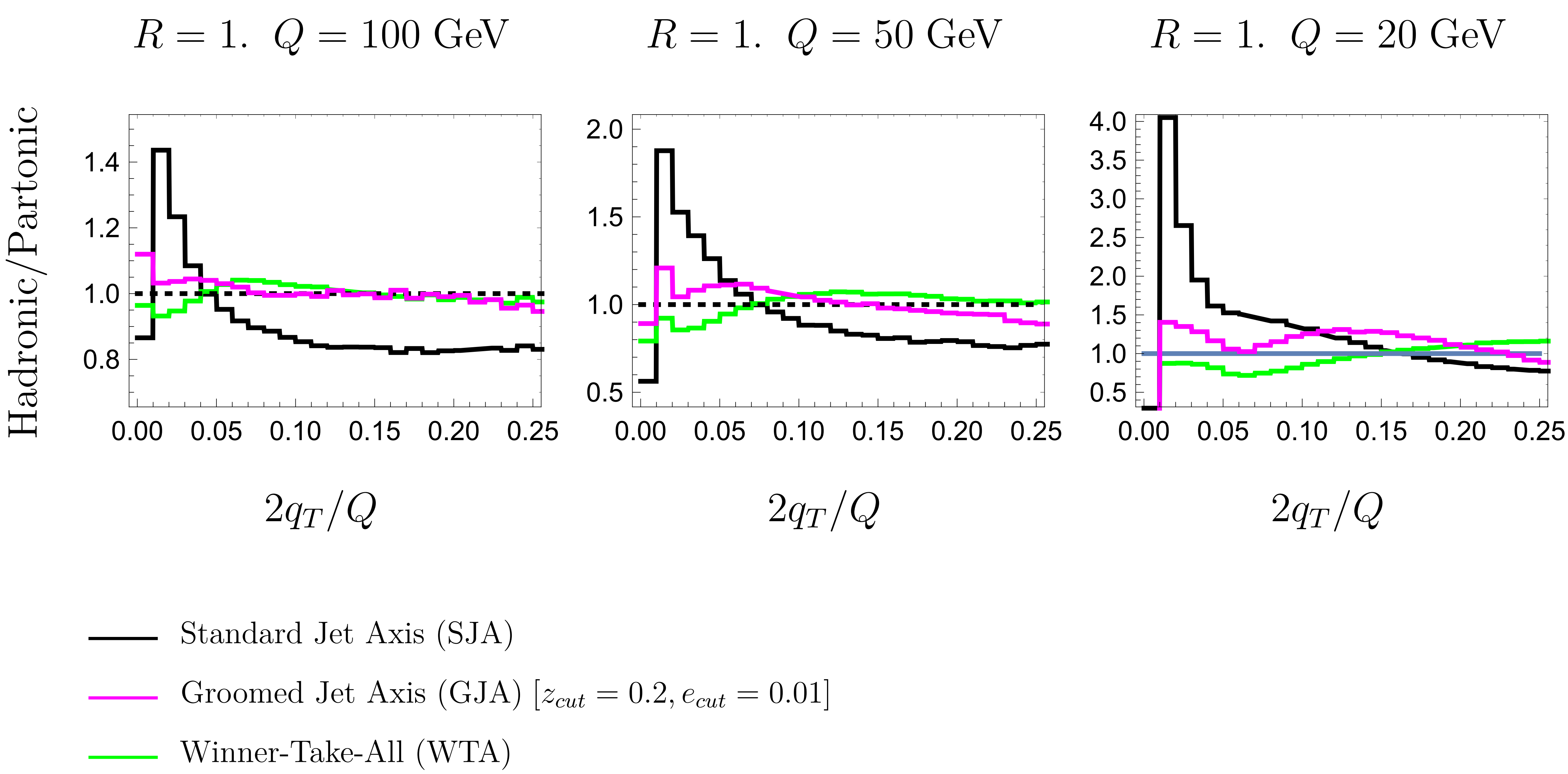
Groomed Jet Axis (GJA) - Hadronization Effects



Groomed Jet Axis (GJA) - Hadronization Effects

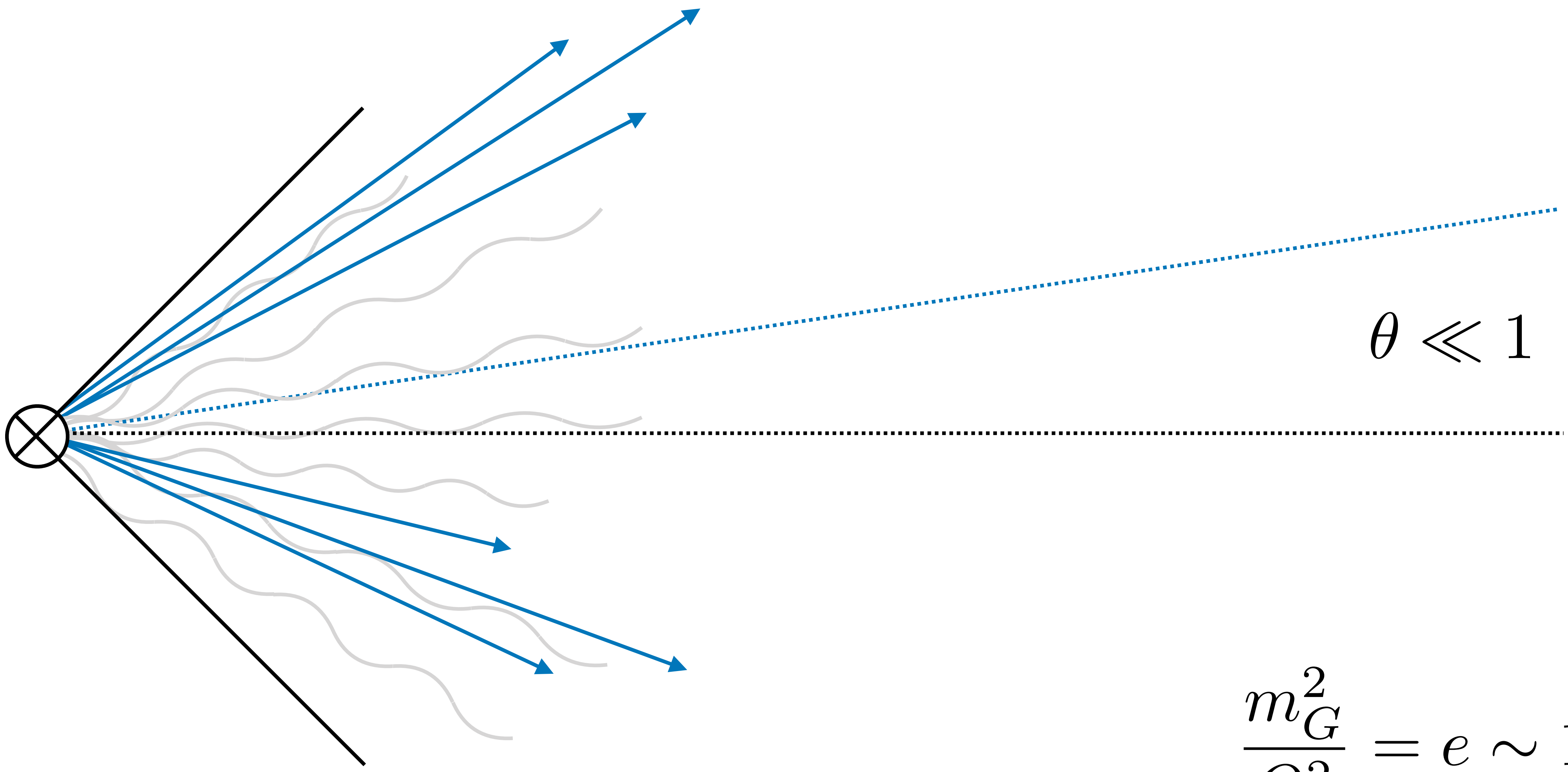


Groomed Jet Axis (GJA) - Hadronization Effects



The jet mass cutoff

Improve our observable by imposing groomed jet mass cutoff. This will eliminate hard splittings which could induce large NGLs

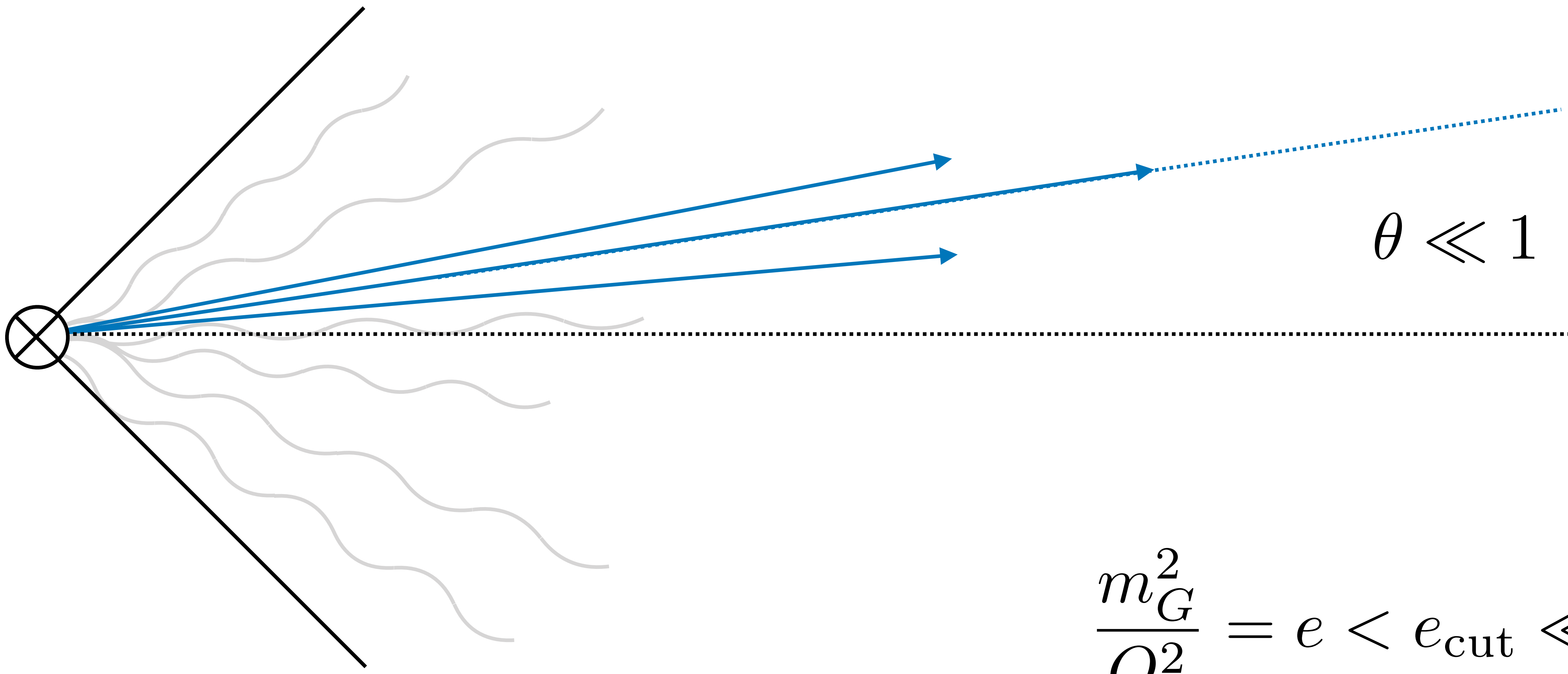


$$\theta \ll 1$$

$$\frac{m_G^2}{Q^2} = e \sim 1$$

The jet mass cutoff

Improve our observable by imposing groomed jet mass cutoff. This will eliminate hard splittings which could induce large NGLs

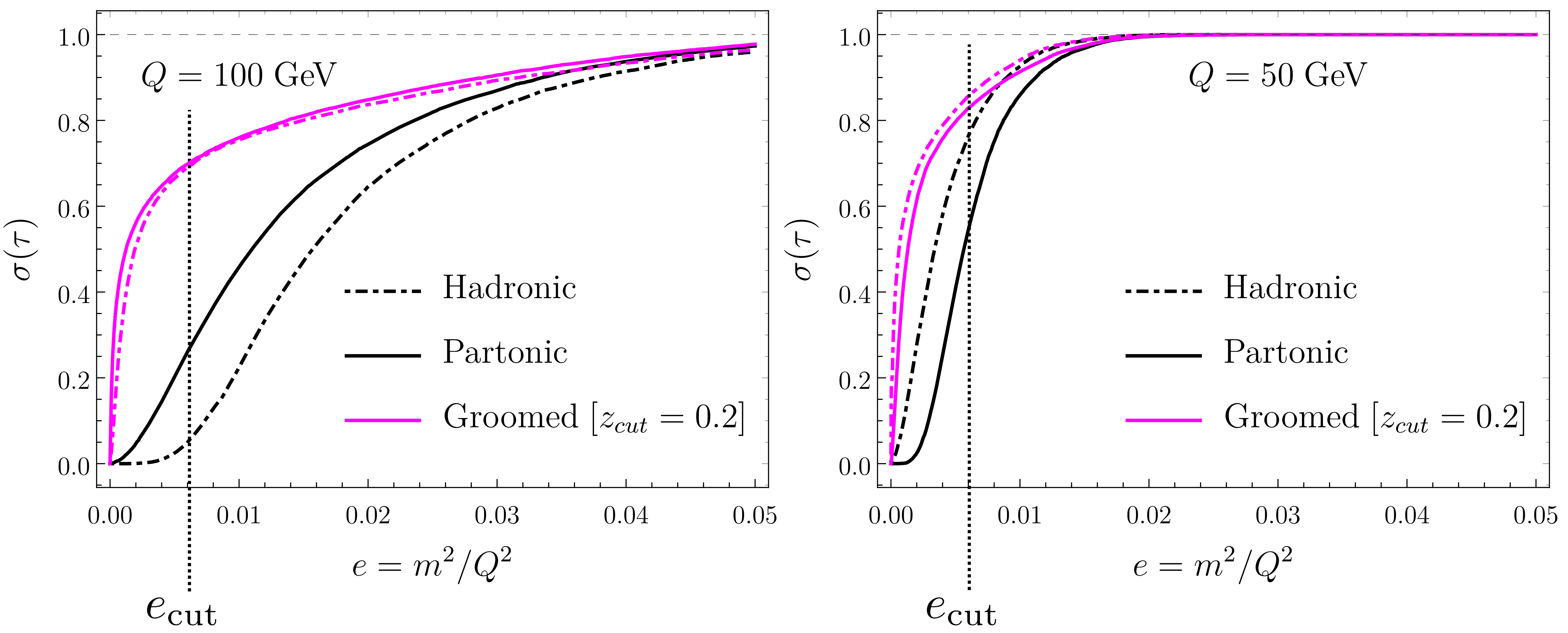


$$\frac{m_G^2}{Q^2} = e < e_{\mathrm{cut}} \ll 1$$

The jet mass cutoff

Improve our observable by imposing groomed jet mass cutoff. This will eliminate hard splittings which could induce large NGLs

$$\frac{m_G^2}{Q^2} = e < e_{\text{cut}} \ll 1$$



Hierarchies and Factorization

$$Q \gg Q z_{\rm cut} \gg q_\perp \sim Q\sqrt{e_{\rm cut}}$$

electron-positron annihilation

$$\frac{d\sigma}{d^2\mathbf{q}_T} = H_2^{ij}(Q;\mu) \times S_2^\perp(\mu,\nu) \otimes \mathcal{J}_i^\perp(e_{\rm cut}, Q, z_{\rm cut};\mu,\nu) \otimes \mathcal{J}_j^\perp(e_{\rm cut}, Q, z_{\rm cut};\mu,\nu)$$

$$\mathcal{J}_j^\perp(e_{\rm cut}, Q, z_{\rm cut};\mu,\nu) = \int_0^{e_{\rm cut}} de\, \mathcal{J}_j^\perp(e, Q, z_{\rm cut};\mu,\nu)$$

$q_\perp \sim Q\sqrt{e}$ / $q_\perp \gg Q\sqrt{e}$

soft-collinear: $p_{sc}^\mu \sim Q z_{\rm cut}(\lambda_{sc}^2, 1, \lambda_{sc}),\ \lambda_{sc} = q_\perp/(Q z_{\rm cut})$

collinear: $p_c^\mu \sim Q(\lambda_c^2, 1, \lambda_c),\ \lambda_c = \sqrt{e}$

collinear-soft: $p_{cs}^\mu \sim Q z_{\rm cut}(\lambda_{cs}^2, 1, \lambda_{cs}),\ \lambda_{cs} = \sqrt{e/z_{\rm cut}}$

<u>TMD Jet function re-factorization:</u>

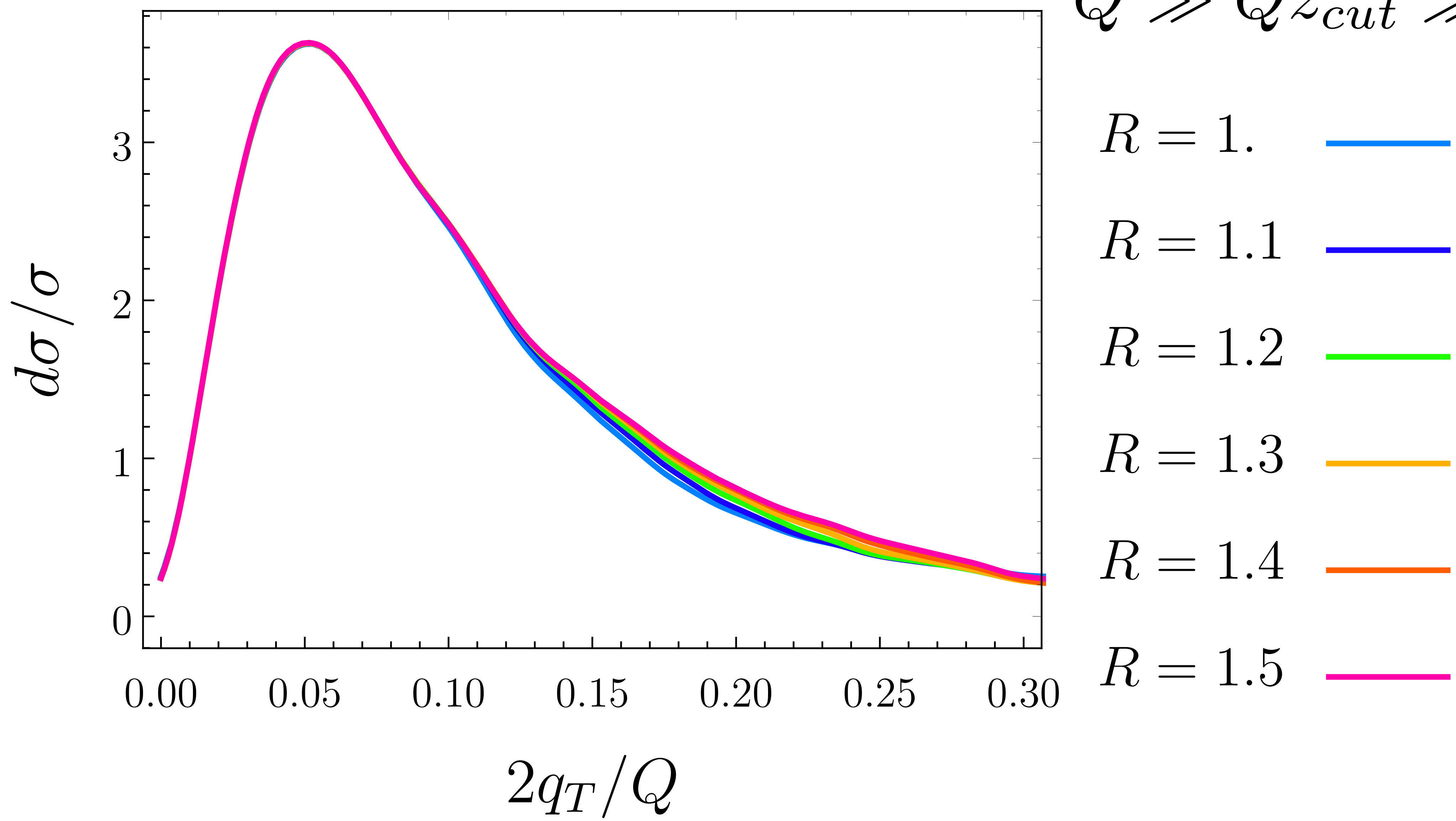
$$\mathcal{J}_i^\perp(e, Q, z_{\rm cut};\mu,\nu) = S_{sc,i}^\perp(Q z_{\rm cut};\mu,\nu) \times \int de'\, S_{cs,i}(e-e', Q z_{\rm cut};\mu) J_i(e', Q;\mu)$$

Groomed Jet Axis (GJA) - Hadronization Effects

Direction of the groomed jet axis insensitive to jet Radius (for R ~ 1):

In contrast to the SJA where the shift is greater as we approach the hemisphere limit.

$$Q \gg Q z_{cut} \gg q_\perp, \quad R \sim 1$$



Renormalization Group and Resummation

Rapidity divergences in global soft and soft-collinear using:
J.-Y. Chiu, A. Jain, D. Neill and I. Z. Rothstein **arXiv:1202.0814**

$$\tilde{F}^{\perp}(b;\mu,\zeta) \equiv \sqrt{\tilde{S}_2^{\perp}(\mu,\nu_s)}\ \tilde{S}_{sc}^{\perp}(Qz_{\rm cut};\mu,\nu_{sc}) \qquad \nu_s = \sqrt{\zeta} \quad \nu_{sc} = Qz_{\rm cut}$$

$$\mu^2 \frac{d}{d\mu^2}\tilde{F}^{\perp}(b;\mu,\zeta) = \frac{1}{2}\gamma_F(\mu,\zeta)\tilde{F}^{\perp}(b;\mu,\zeta)\ ,$$

$$\zeta \frac{d}{d\zeta}\tilde{F}^{\perp}(b;\mu,\zeta) = -\mathcal{D}(\mu)\tilde{F}^{\perp}(b;\mu,\zeta)$$

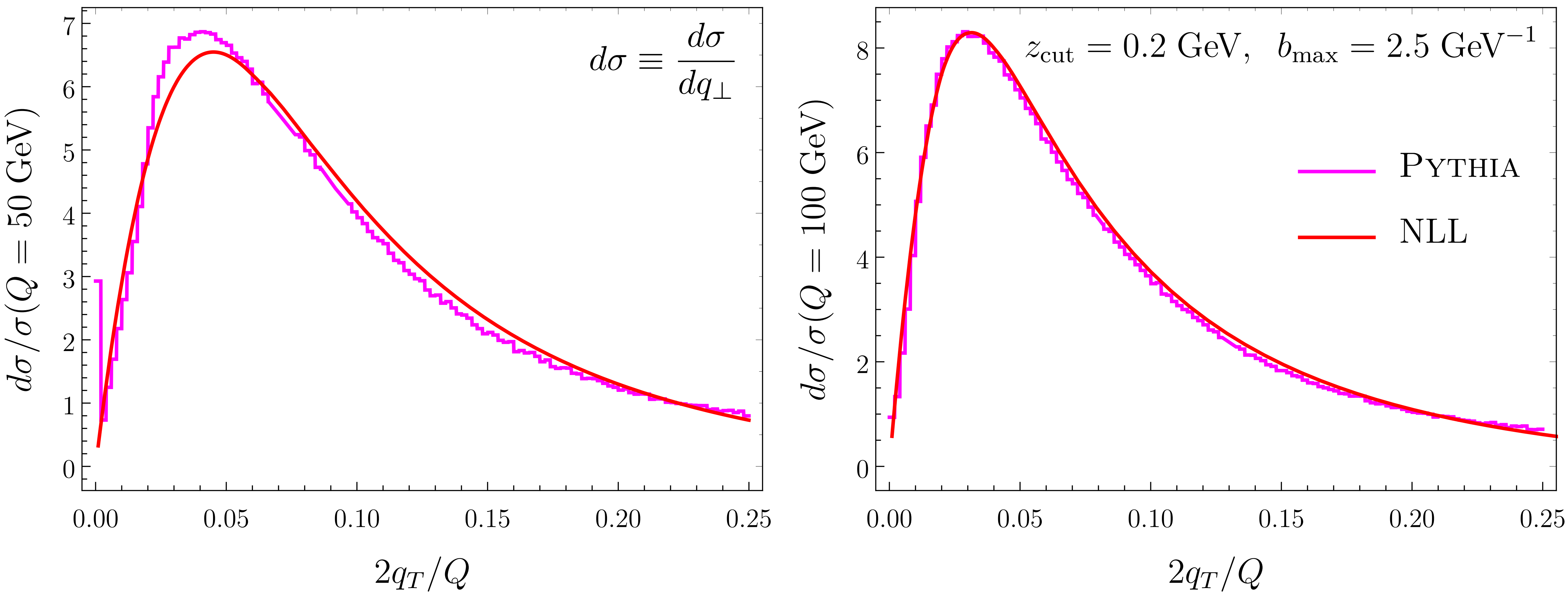
One loop results calculated explicitly. Two loop extracted from consistency

$$\tilde{F}^{\perp}(b;\mu,\zeta) = 1 + \frac{\alpha_s(\mu)C_i}{\pi}\Big\{2\ln\left(\frac{\mu_E}{\mu}\right)\ln\left(\frac{\zeta}{\mu^2}\right) - 2\ln^2\left(\frac{\mu_E}{\mu}\right) - \frac{\pi^2}{12} + \mathcal{O}(\alpha_s^2)\Big\}$$

Perturbative @NLL vs Monte-Carlo

Consistency check (effect of power corrections) against Pythia simulations.

NLL cross section in good agreement with partonic shower of MC.



Resummation in b-space @ NNLL

$$\mu^2 \frac{d}{d\mu^2} \tilde{F}^{\perp}(b;\mu,\zeta) = \frac{1}{2}\gamma_F(\mu,\zeta)\tilde{F}^{\perp}(b;\mu,\zeta) \ ,$$

We only need to figure the non-cusp part.
Use consistency of factorization

$$\gamma_F(\mu,\zeta = Q z_{\rm cut}) = \gamma_S(\mu)$$

see arXiv:1603.09338

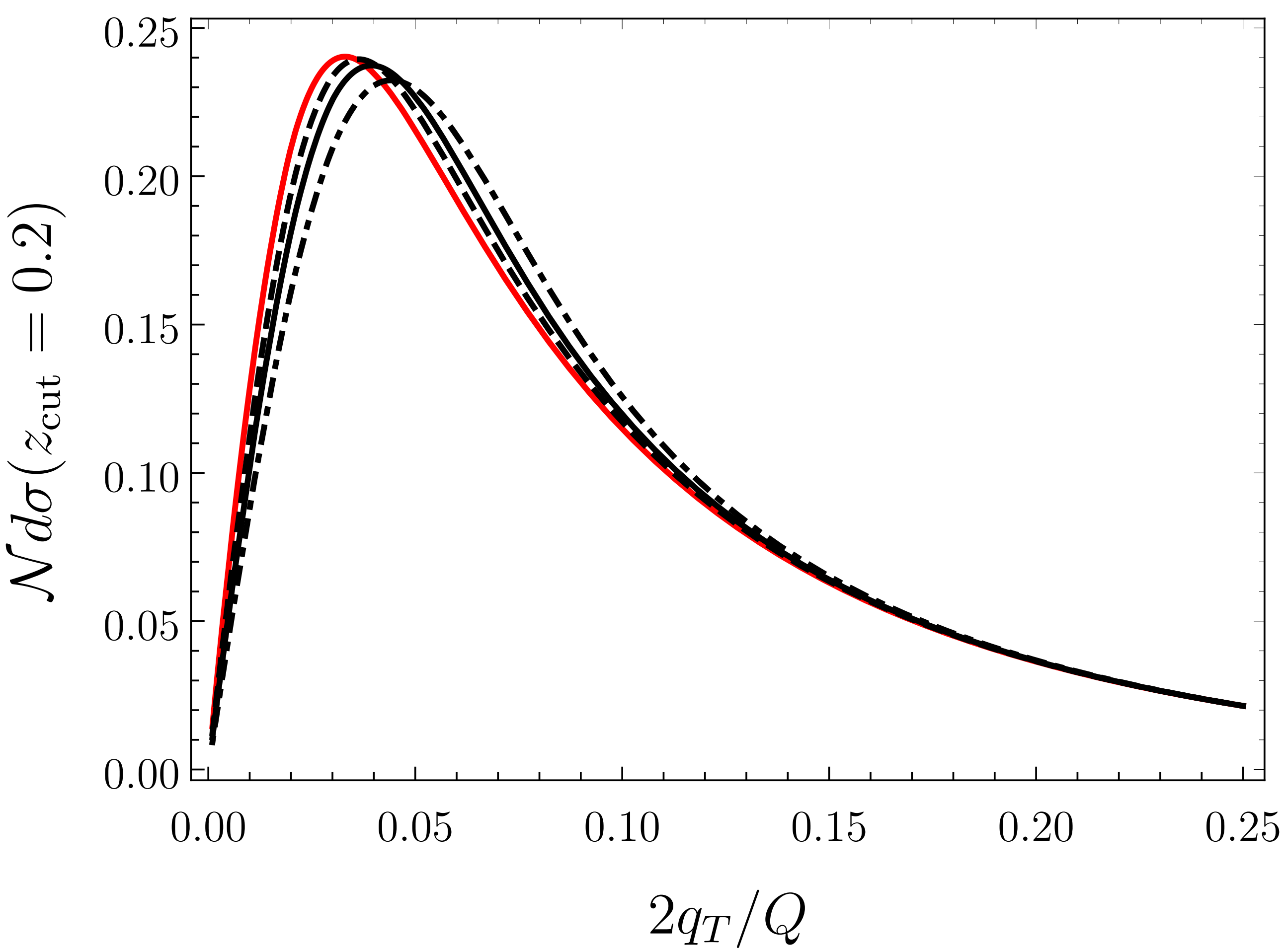
$$\zeta \frac{d}{d\zeta} \tilde{F}^{\perp}(b;\mu,\zeta) = -\mathcal{D}(\mu)\tilde{F}^{\perp}(b;\mu,\zeta)$$

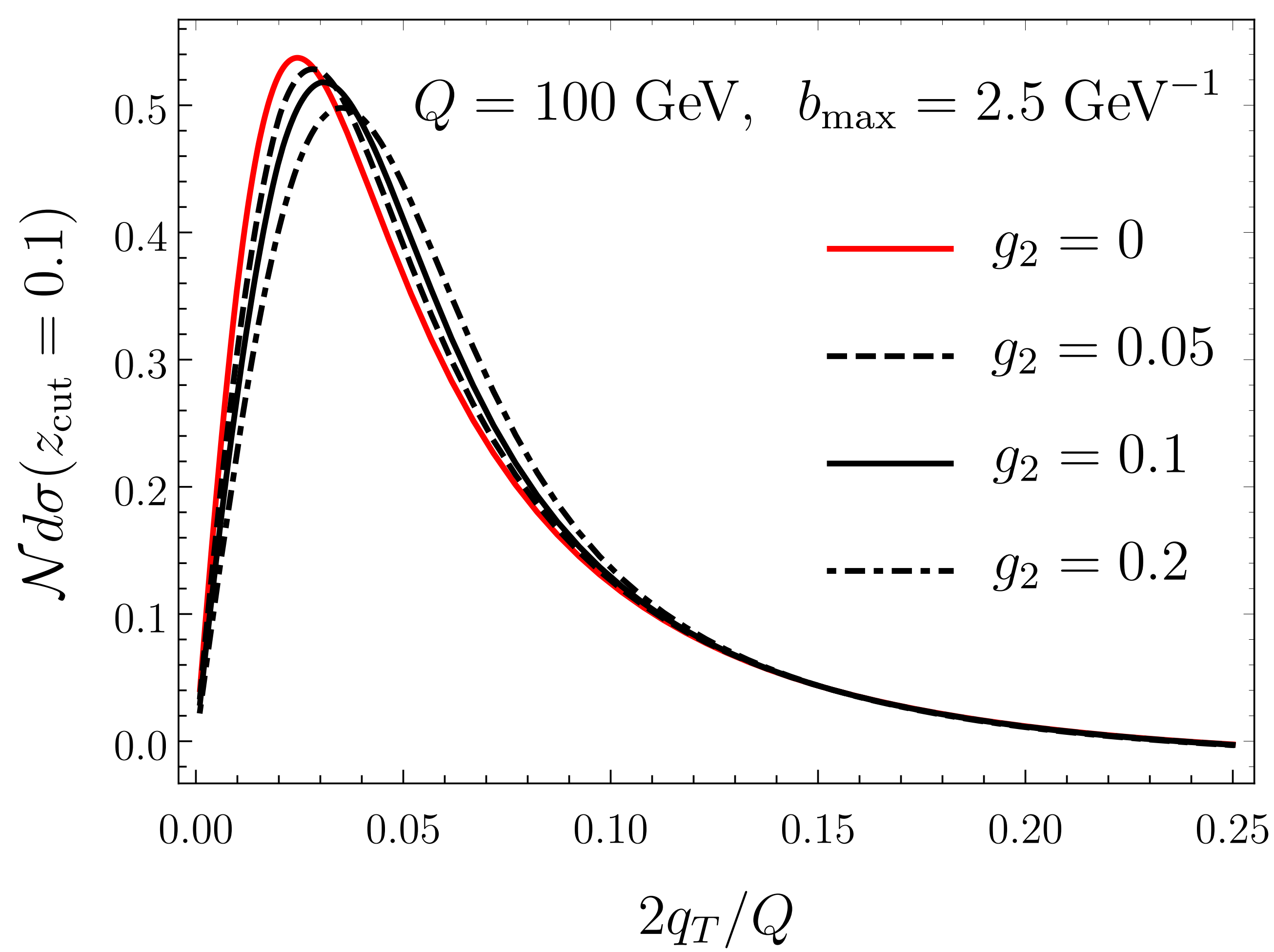
Universal well known up to N(3)LL

Resummation in b-space @ NNLL

$$\gamma^{S}_{\nu,i}(\mu) \to \gamma^{S}_{\nu,i}(\mu)\Big|_{b\to b_*} - g_K(b;b_{\max}) \qquad g_K(b;b_{\max}) = \frac{1}{2}g_2(b_{\max})b^2$$

$$b_* = \frac{b}{\sqrt{1+(b/b_{\max})^2}}$$

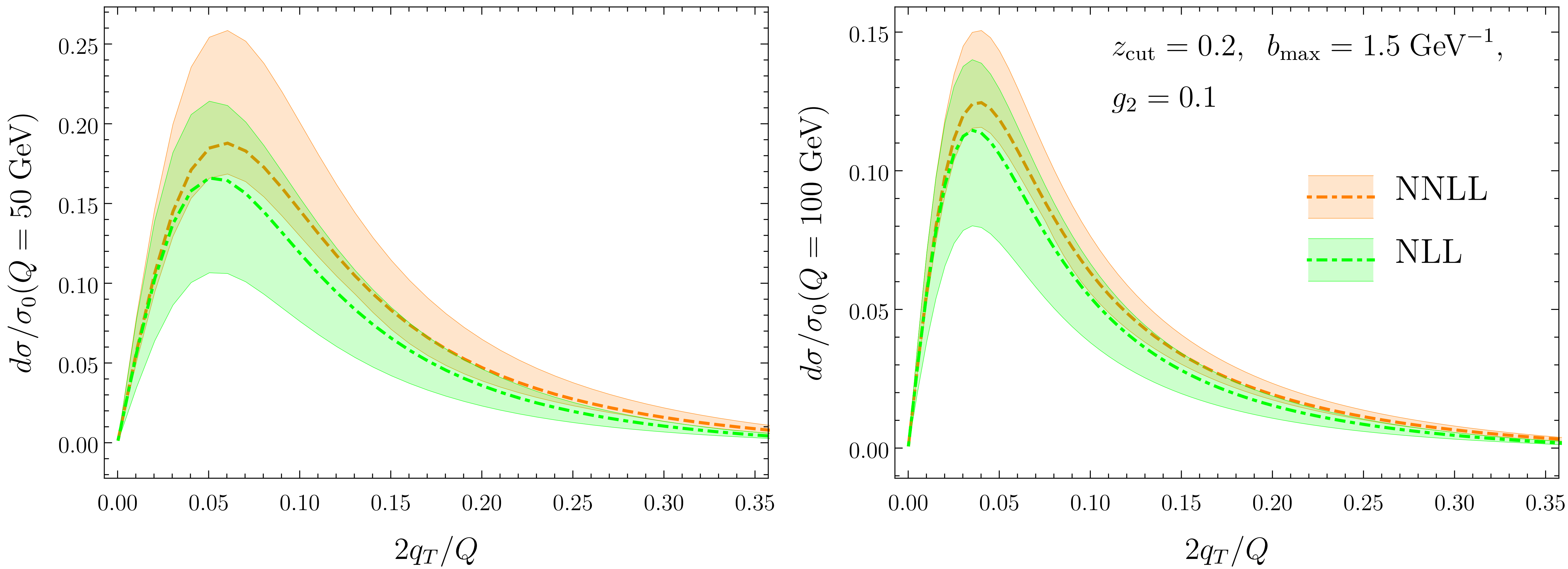




Resummation in b-space @ NNLL

For jet energies < 25 GeV, the theoretical uncertainty is relatively large for NNLL.

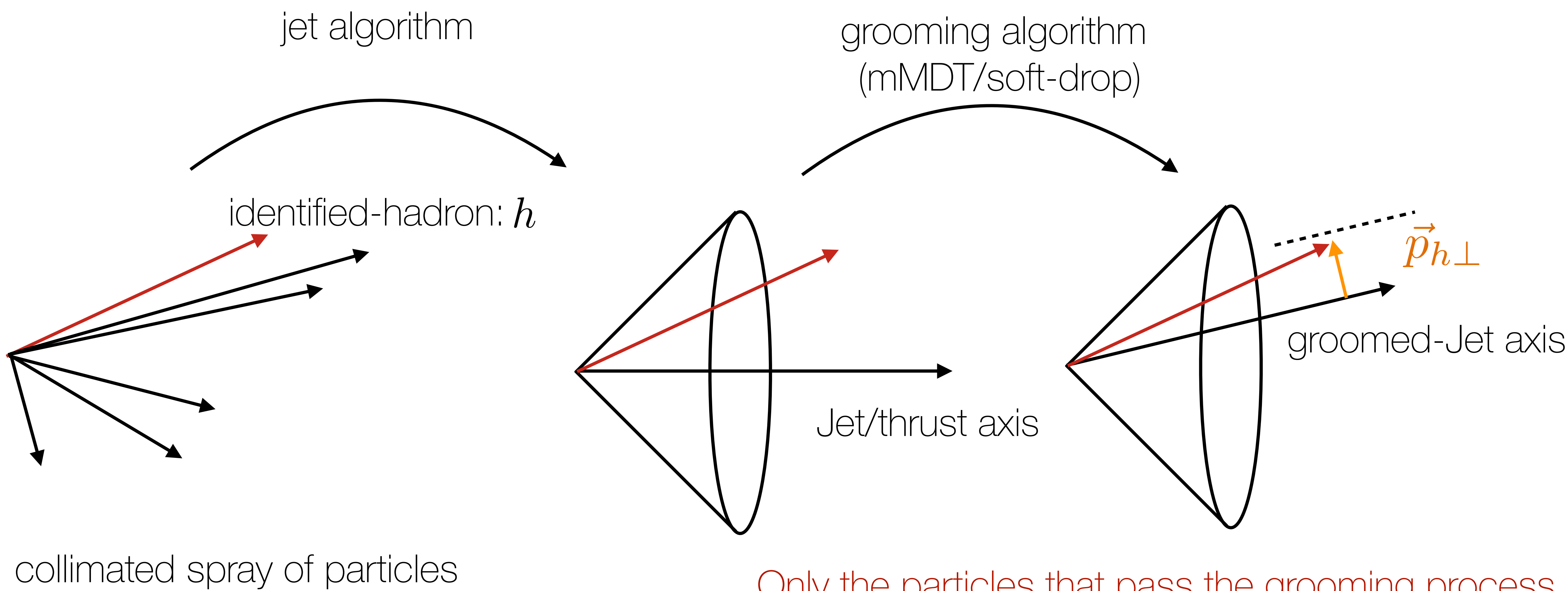
<u>Work in progress:</u>
Implement other resummation schemes, e.g. \zeta-prescription.
Get cross section for DIS



Dijet de-correlation summary

- Groomed jet axis much more stable during hadronization, independent of jet radius (R ~ 1) or jet algorithms.
- Factorization involves universal soft function.
- Additional measurement needs to be imposed to avoid NGLs (Not necessary for Winner-Take-All, see arXiv:1807.07573)
- All ingredients for NNLL resummation are available
- In progress: Groomed jets in DIS give access to TMDPDFs with minimum additional input
- In progress: \zeta-prescription using the arTiMiDe package.

Identified hadrons within groomed jet



Only the particles that pass the grooming process will determine the direction of the groomed-jet axis

Measurements: $\vec{p}_{h\perp}, \; z = \dfrac{E_h}{E_J}$

At the small transverse momentum limit no additional measurement is required to ensure collinear configurations

Factorization with groomed jet(h)

$$\frac{d\sigma}{d\vec{p}_J d\vec{q}_\perp dz_h} = F_i(\mathcal{M}, \vec{p}_J, R, z_{cut}) J_{i/h}(z_{cut}, q^+, \vec{q}_\perp, m)$$

Fraction of quark and gluon-initiated jets Independent of the measurement within the jet

groomed TMD Fragmenting Jet Function (TMDFJF)

$\vec{q}_\perp$ is the transverse momentum of jet with respect to hadron

$$\vec{q}_\perp = -\frac{\vec{p}_{\perp h}}{z_h}$$

See also:

- TMDFJF (measurement along the jet axis)

  arXiv:1610.06508 (Reggie Bain, YM, Thomas Mehen)

- JTMDFF (measurement along the winner-take-all axis)

  arXiv:1612.04817 (Duff Neill, Ignazio Scimemi, Wouter J. Waalewijn)

- siTMDFJF (semi-inclusive)

  arXiv:1705.08443 (Zhong-Bo Kang, Xiaohui Liu, Felix Ringer, Hongxi Xing)

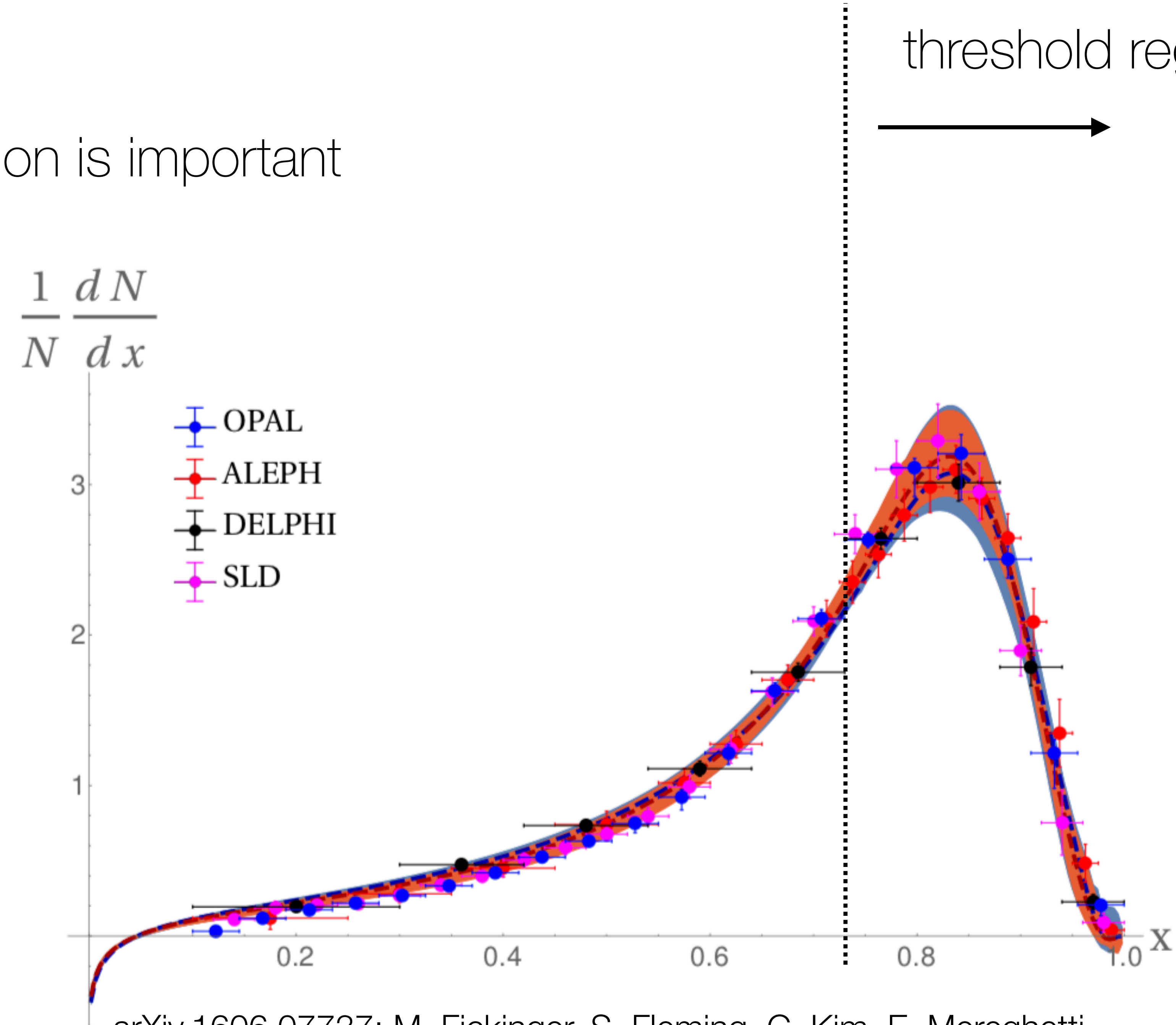
- gTMDFJF (groomed light hadrons)

  arXiv:1712.07653 (YM, D. Neill, and V. Vaidya)

Re-Factorization for heavy quark

- mass effects
- threshold region is important

threshold region

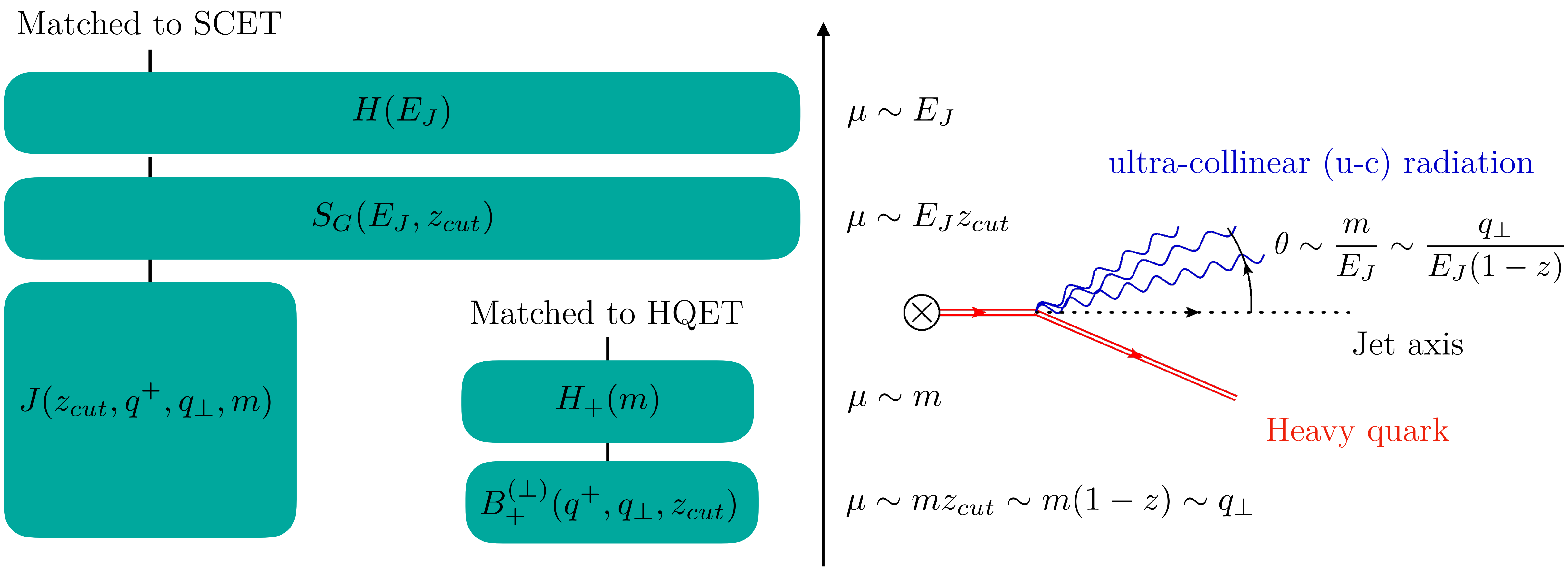


arXiv.1606.07737: M. Fickinger, S. Fleming, C. Kim, E. Mereghetti

Re-Factorization for heavy quark

Region 1
Small transverse momentum: $\theta \sim \theta_{min}$

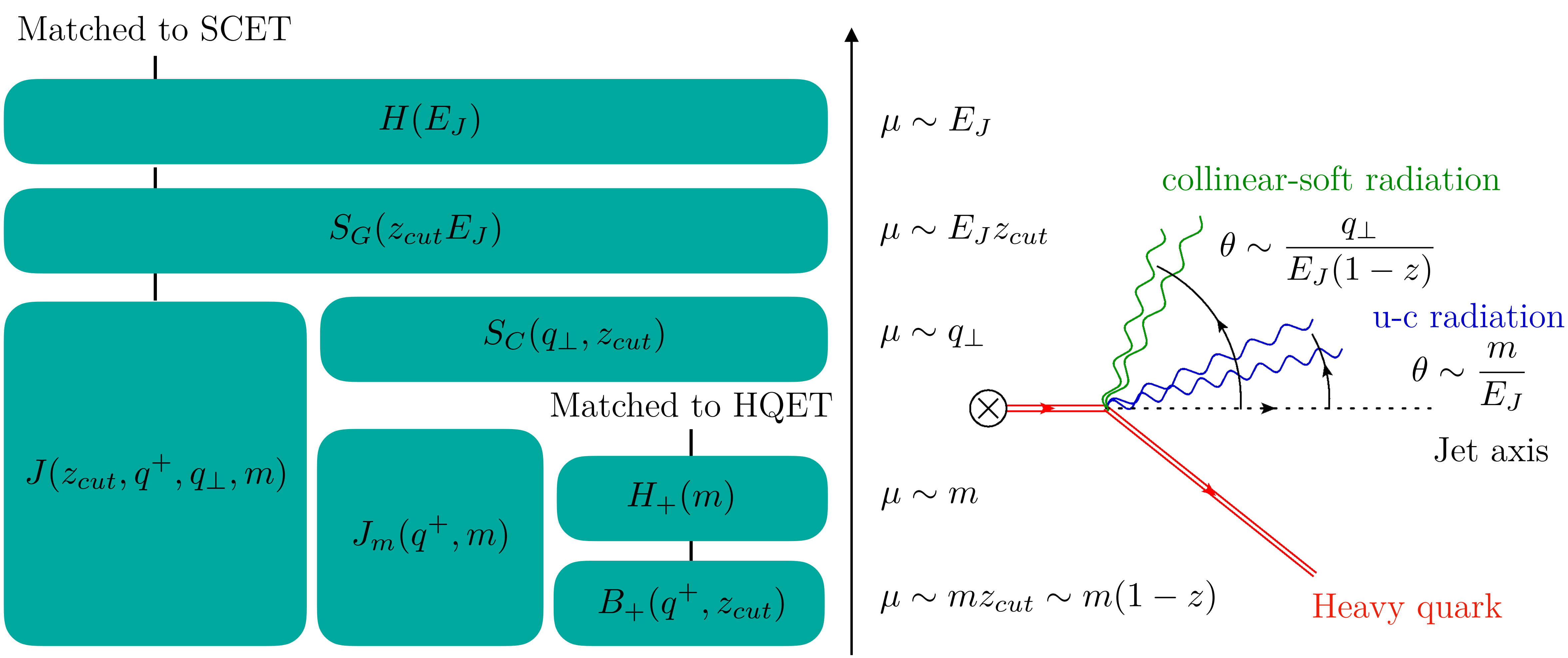
We work in the threshold region: $q^+ = Q(1-z)$



$$J_{q/h}(q_\perp, E_J, z_{cut}, m) = H(m) \times B_+^{(\perp)}(q_\perp, E_J(1-z), E_J z_{cut}, m z_{cut})$$

Re-Factorization for heavy quark

Region 2
"Large" transverse
momentum: $\theta \gg \theta_{min}$



$$J_{q/h}(q_\perp, E_J, z_{cut}, m) = H(m) \times S_C(E_J z_{cut}, q^+, q_\perp) \otimes_z B_+(q^+, E_J z_{cut}, m z_{cut})$$

Merging Region (1) and Region (2)

Additive matching:

$$d\sigma = \mathrm{W} + \mathrm{FO} - \mathrm{ASY}$$

- captures power corrections
- numerically unstable

Multiplicative matching:

$$d\sigma = \frac{\mathrm{W} \times \mathrm{FO}}{\mathrm{ASY}}$$

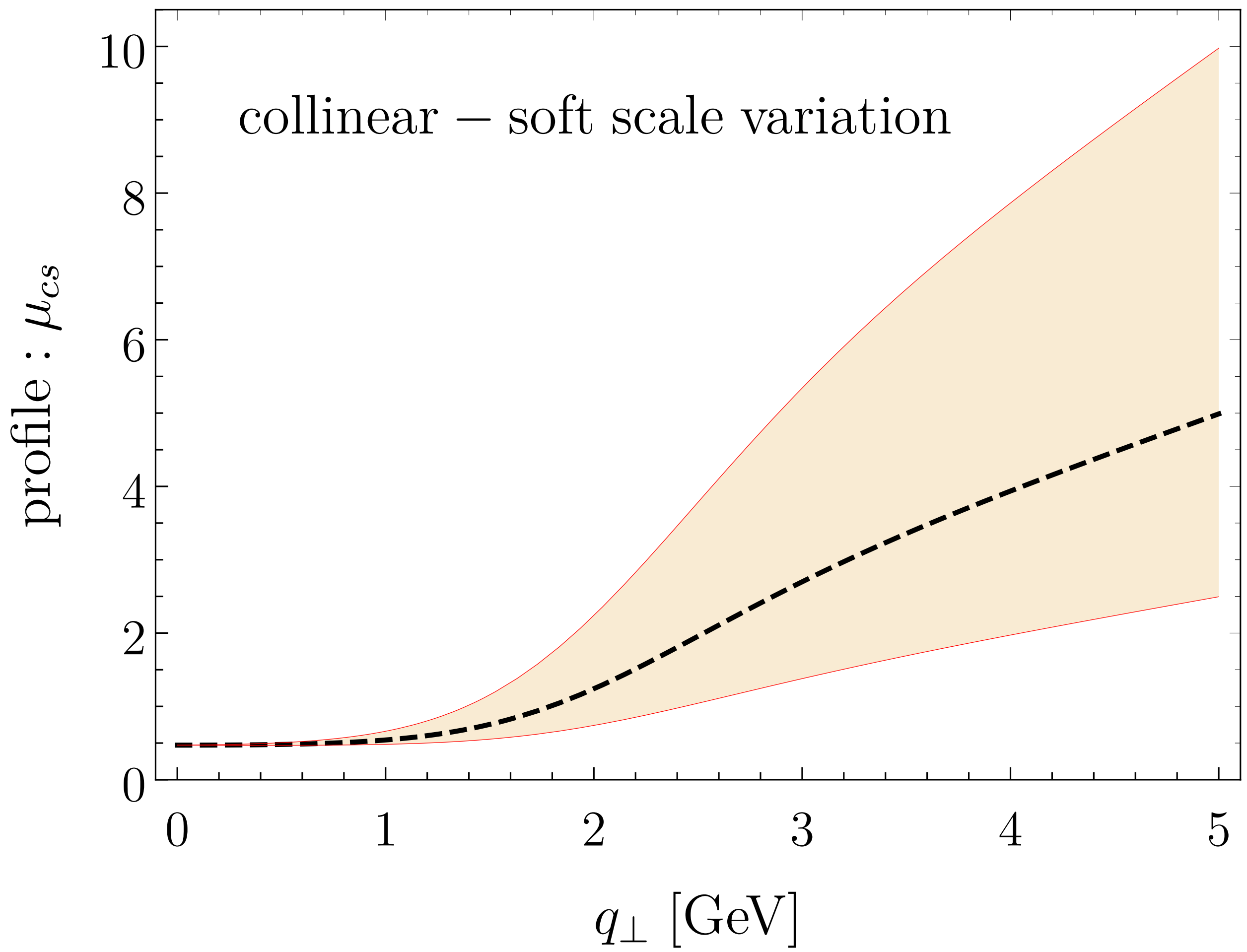
Small pT

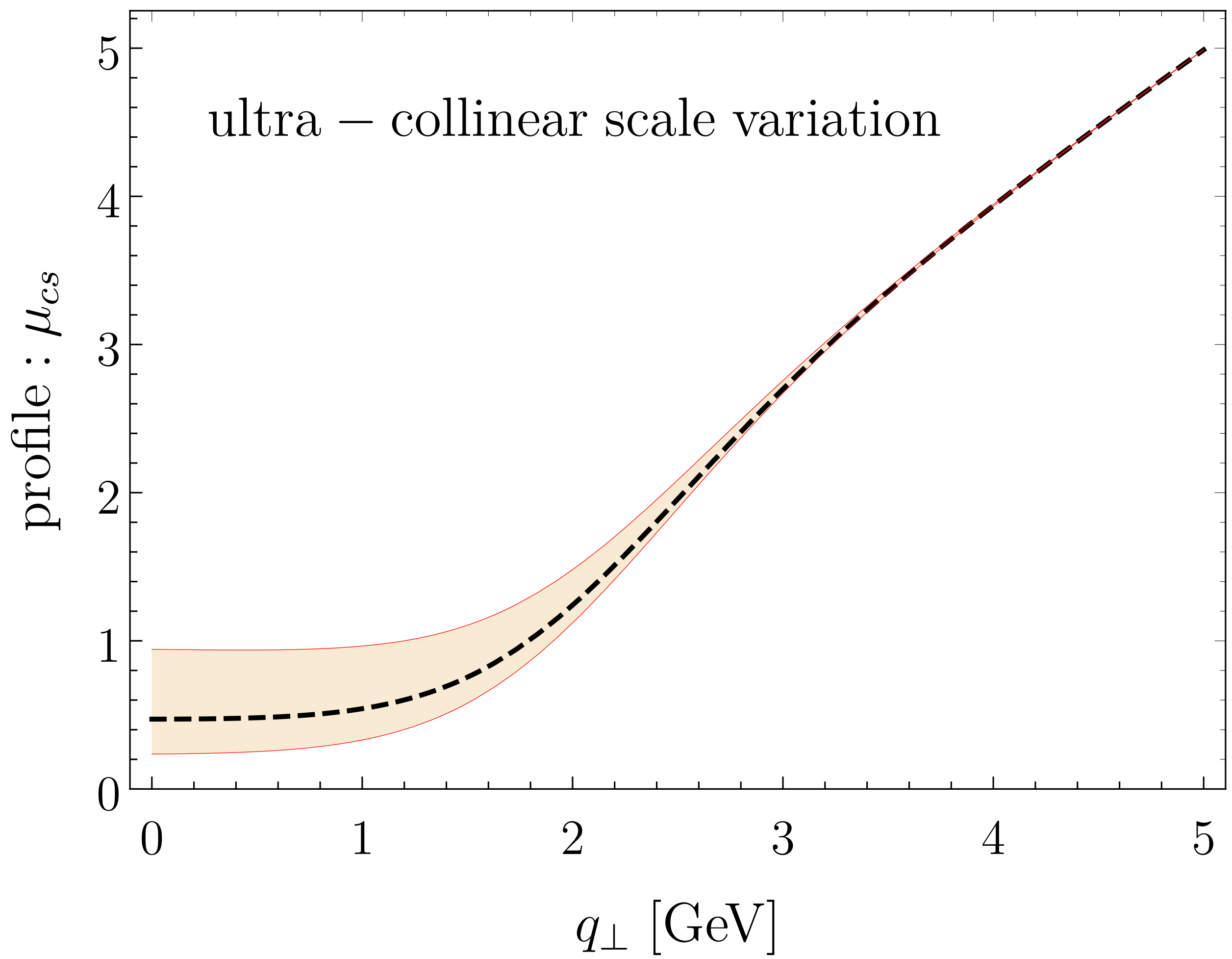
“Large” pT

Merging Region (1) and Region (2)

Multiplicative matching: $\frac{d\sigma^{(1+2)}}{dz d^2\vec{q}_\perp} = \frac{d\sigma^{(1)}}{dz d^2\vec{q}_\perp} \times \frac{d\sigma^{(2)}}{dz d^2\vec{q}_\perp} \Big/ \frac{d\sigma^{(2\text{-FO})}}{dz d^2\vec{q}_\perp}$

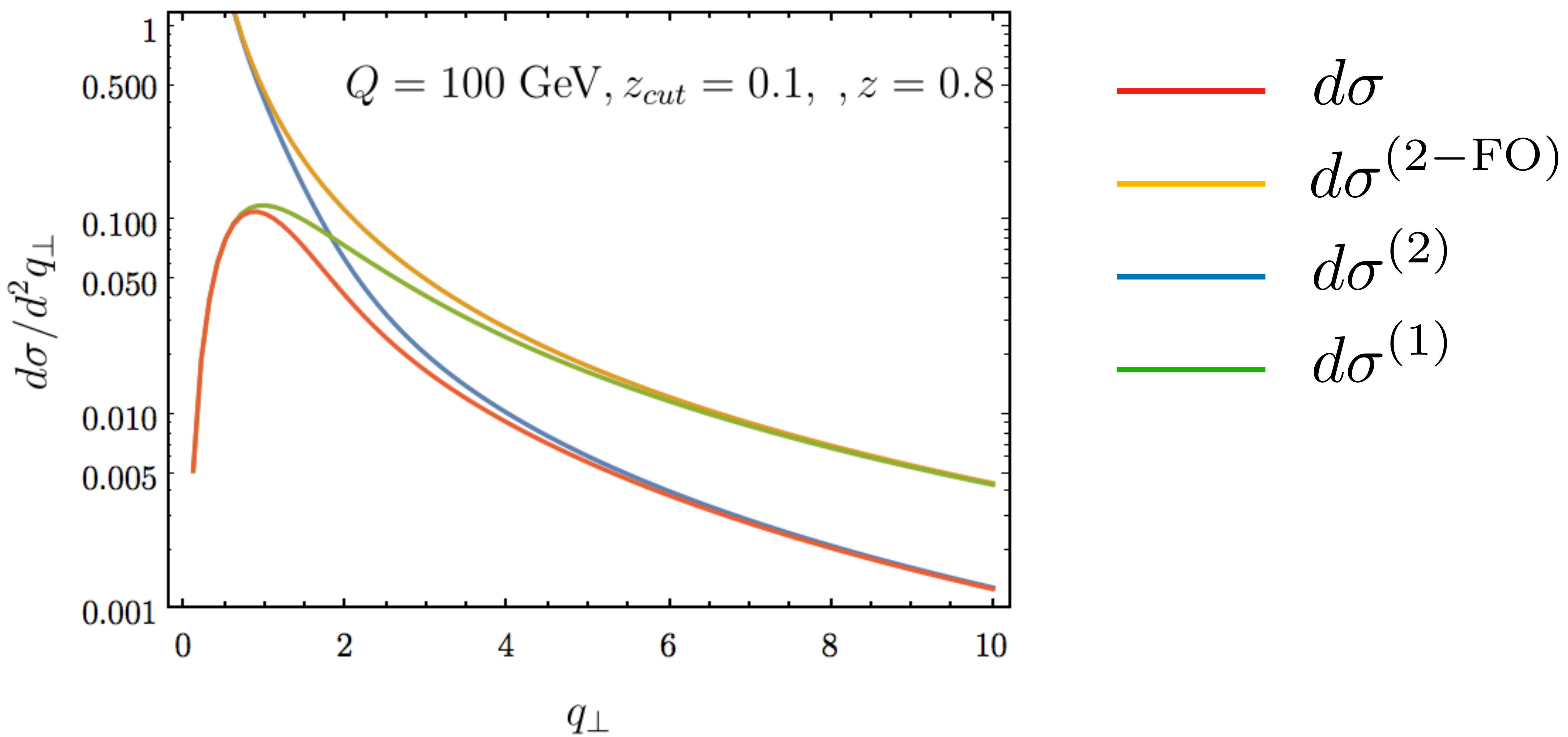
Use “profile functions” for switching scales:





Merging Region (1) and Region (2)

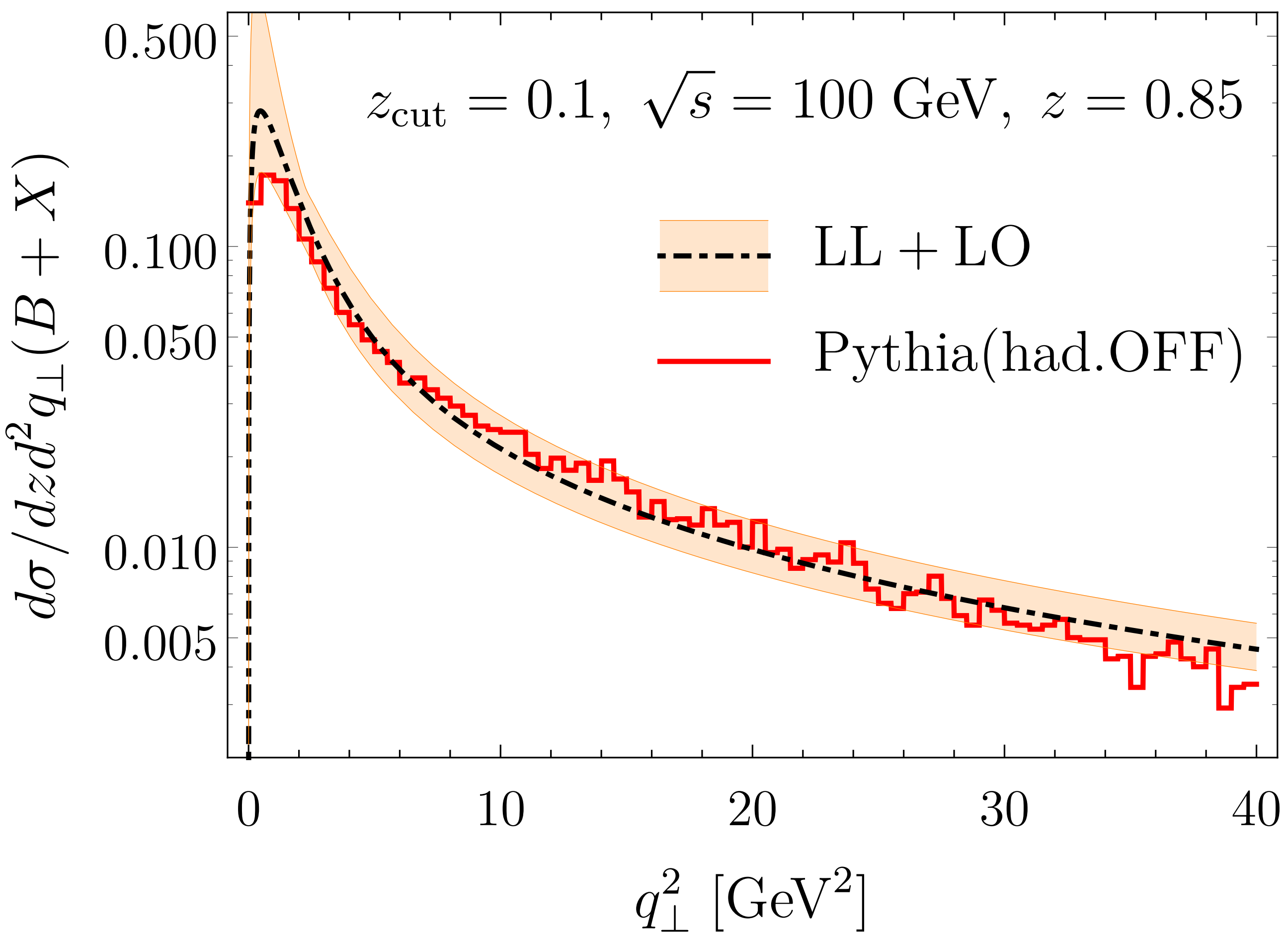
Multiplicative matching: $\frac{d\sigma^{(1+2)}}{dzd^2\vec{q}_\perp} = \frac{d\sigma^{(1)}}{dzd^2\vec{q}_\perp} \times \frac{d\sigma^{(2)}}{dzd^2\vec{q}_\perp} \Big/ \frac{d\sigma^{(2\text{-FO})}}{dzd^2\vec{q}_\perp}$

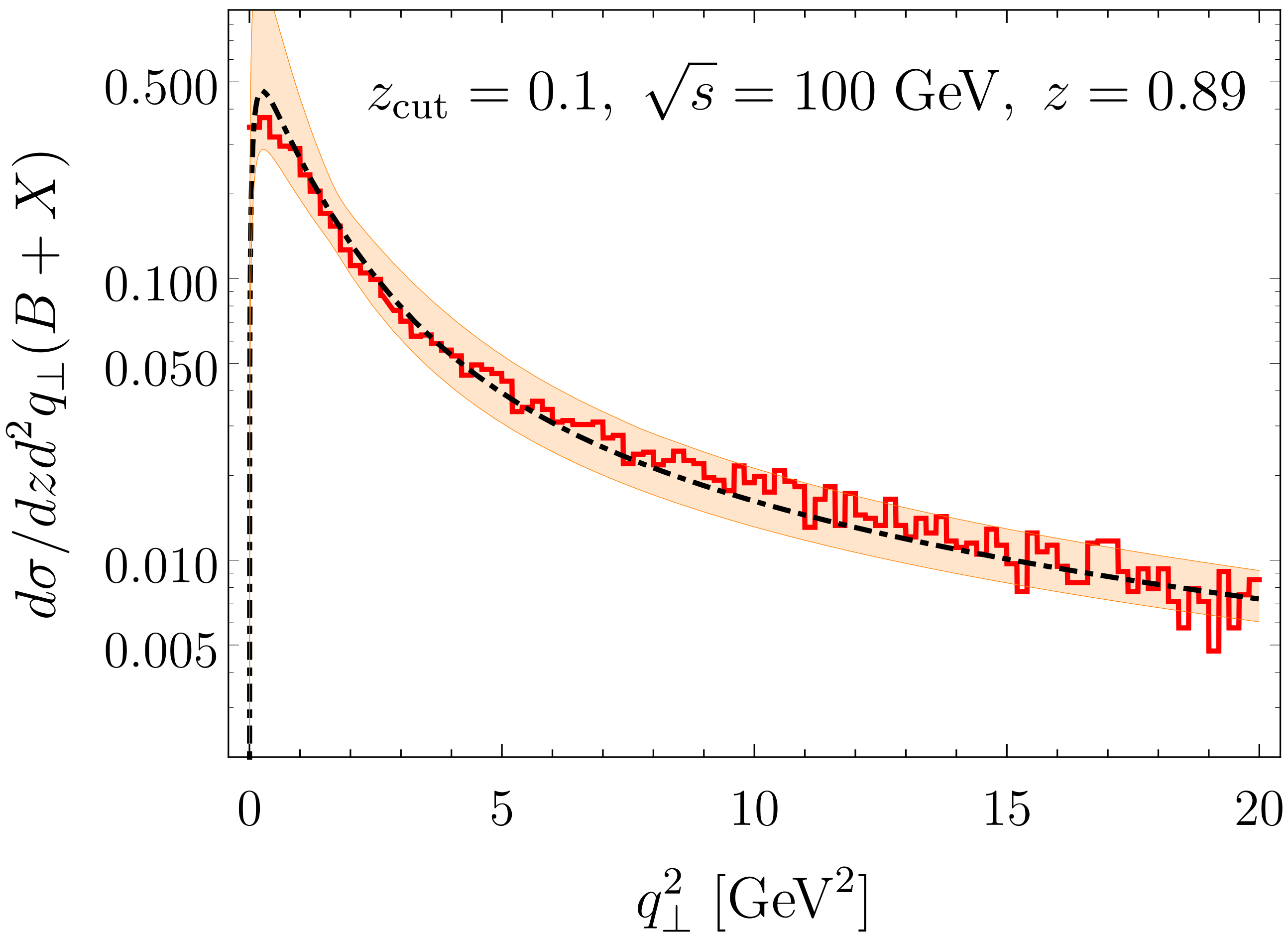


Merging Region (1) and Region (2)

Multiplicative matching: $\frac{d\sigma^{(1+2)}}{dz d^2\vec{q}_\perp} = \frac{d\sigma^{(1)}}{dz d^2\vec{q}_\perp} \times \frac{d\sigma^{(2)}}{dz d^2\vec{q}_\perp} \Big/ \frac{d\sigma^{(2\text{-FO})}}{dz d^2\vec{q}_\perp}$

Comparison to simulations

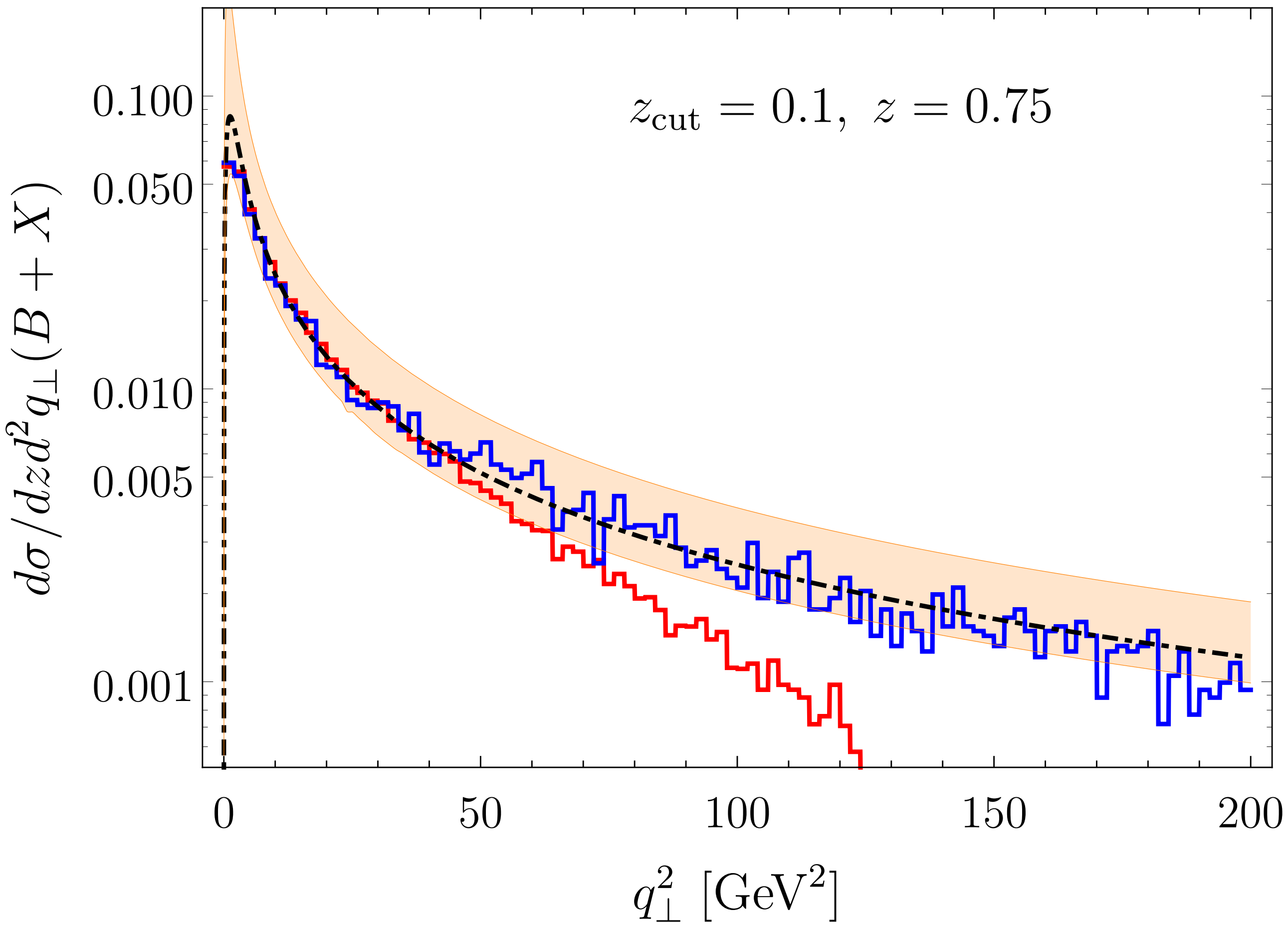


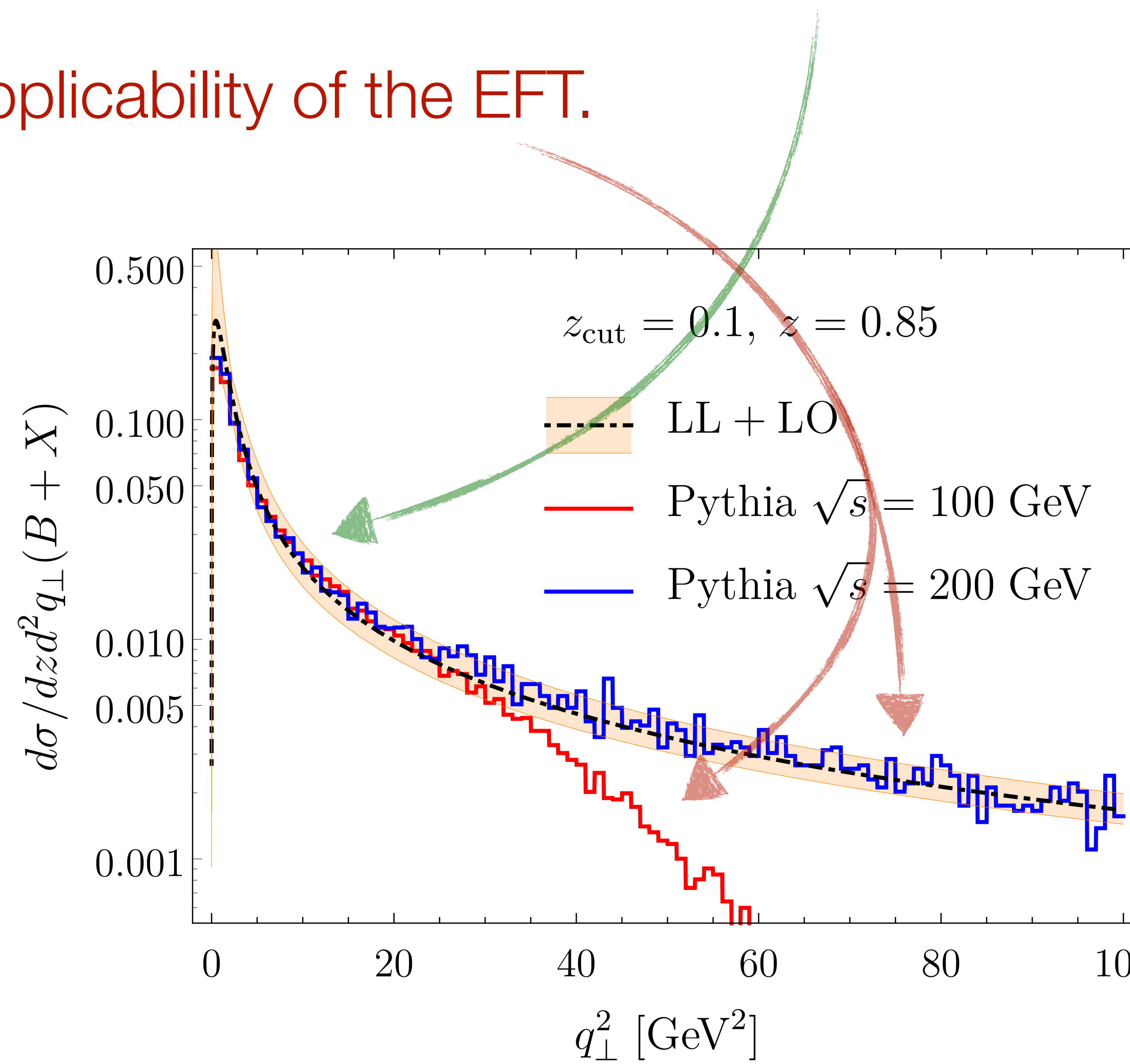


Merging Region (1) and Region (2)

The shape of the distribution does not depend on the hard scale of the process (Q).

Change of the hard scale extends the applicability of the EFT.





Pythia: Hadronization OFF

Hadronization Effects

Leading contribution comes from the shift in energy fraction.
Opposite effect from standard TMD-models.

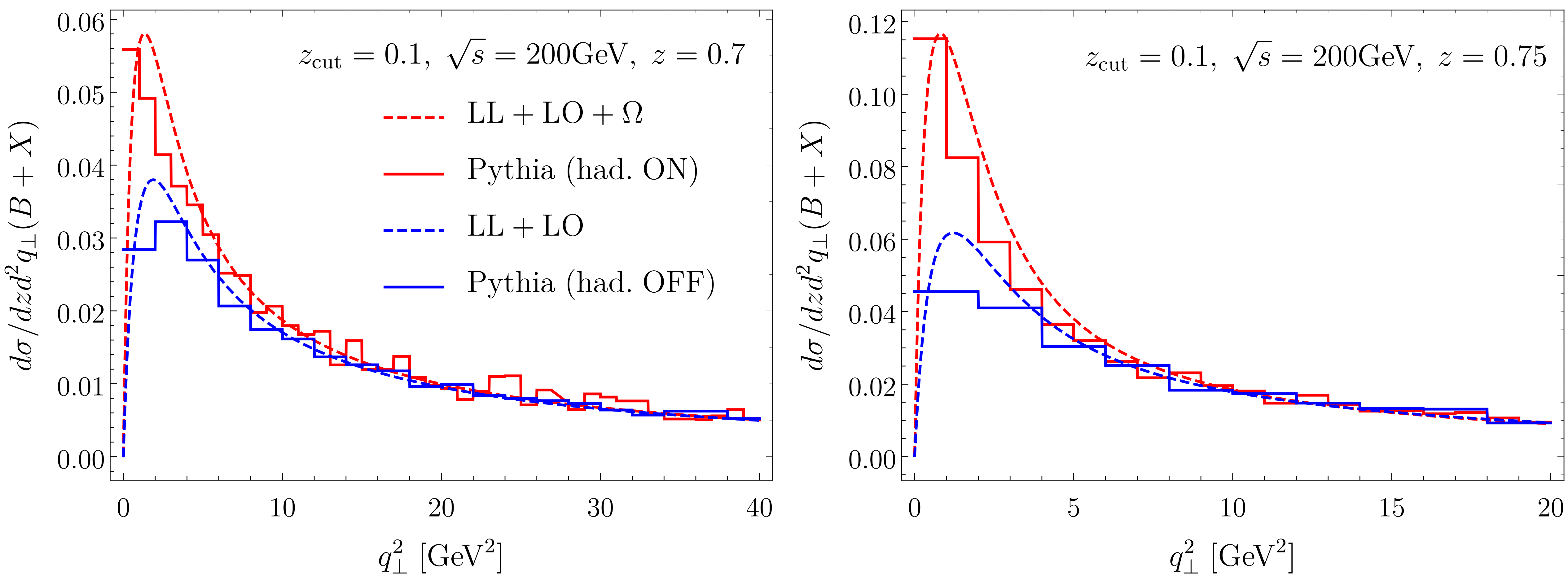
OPE leading contribution consistent with the model: $f_{\rm np}(q^+) = \delta\Big(q^+ - Q\frac{\Lambda}{m}\Big)$

$$B_+^{(\perp)}(E_J(1-z))\Big|_{\rm had.} = B_+^{(\perp)}(q^+) \otimes_z f(q^+) = B_+^{(\perp)}(E_J(1-z-\Lambda/m))$$

$$\Lambda \sim 0.2 \ \text{GeV}$$

Hadronization Effects

Leading contribution comes from the shift in energy fraction.
Opposite effect from standard TMD-models.



Hadronic distributions are simnifically narrower and shifted towards smaller pT

In jet fragmentation summary

Study fragmentation within groomed jets:

- Use EFT (SCET/HQET) for factorization and resummation of large logarithms
- No logarithmic enhancements from boundary effects (NGLs)
- Can easily extended for hadrons + jet substructure (e.g. jet mass and angularities in preparation + preliminary results for light hadrons)
- Observable easy to relate between e+ e- and DIS

In the perturbative regime:

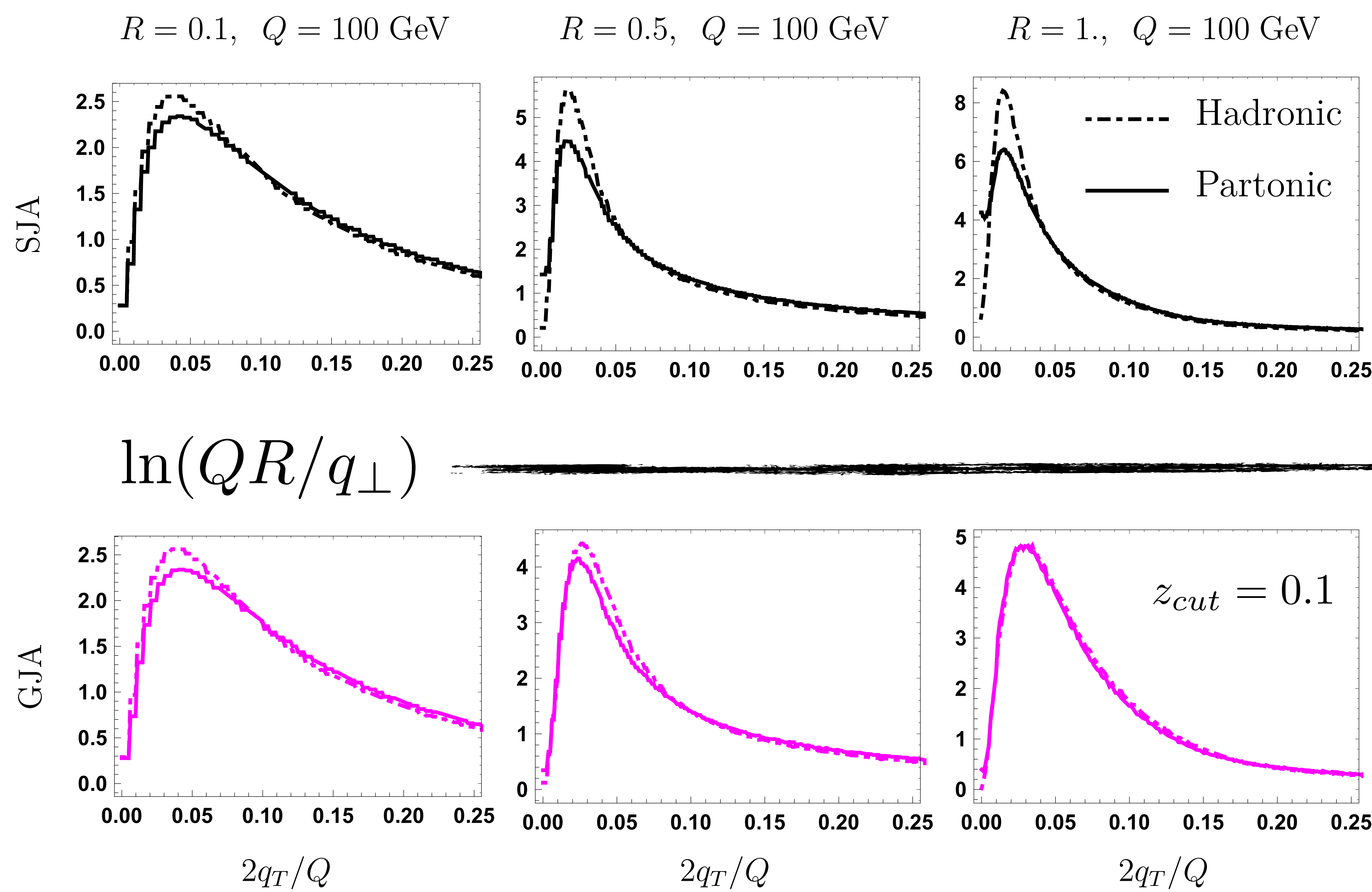
- Groomed TMD fragmentation can be studied directly in momentum space

In the non-perturbative regime:

- Good discriminating observable for extracting non-perturbative TMD evolution

Additional slides

Groomed Jet Axis (GJA) - Hadronization Effects

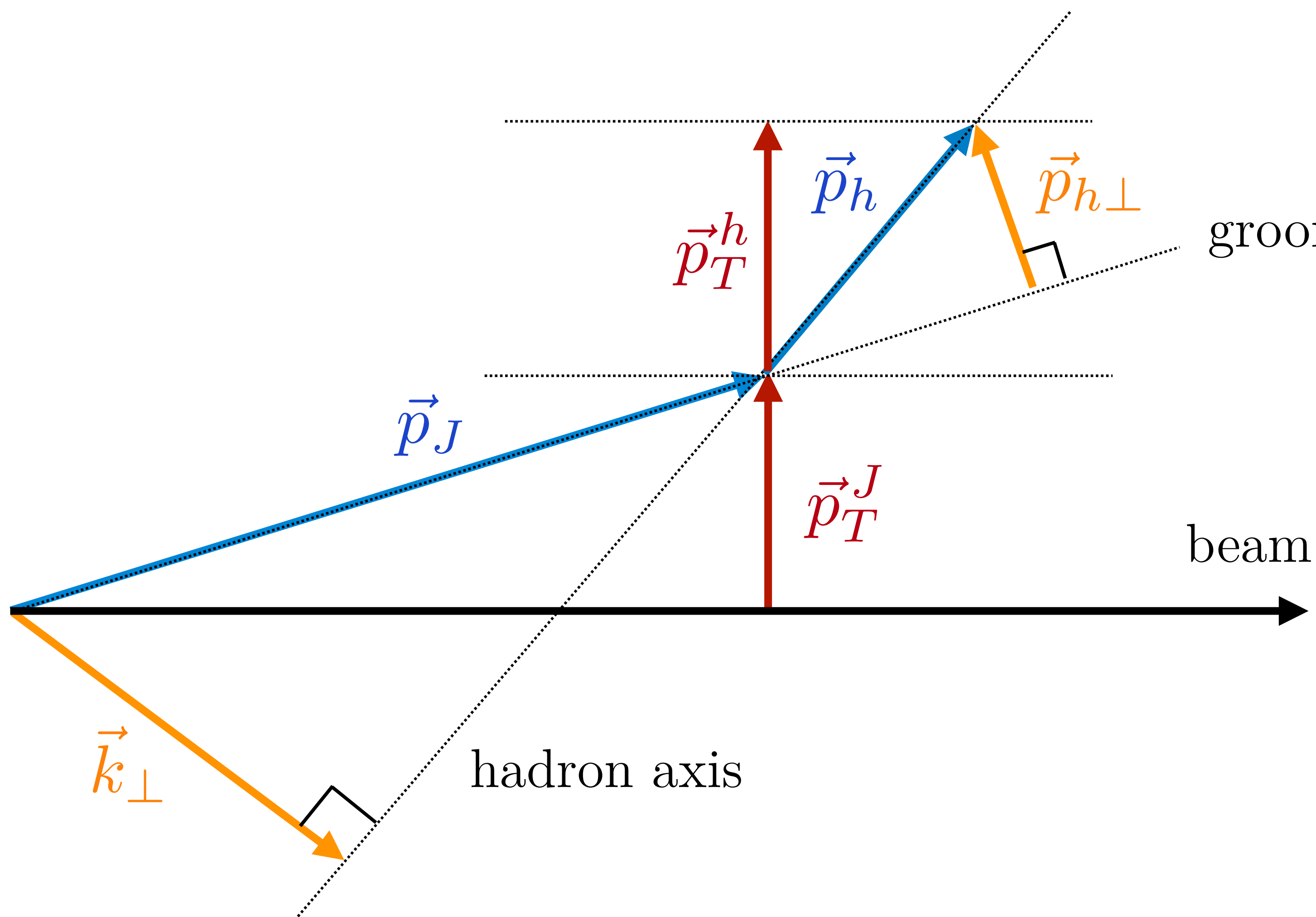


$\ln(QR/q_\perp)$ →

$\ln(QR/q_\perp)$ **?** $\ln(Qz_{cut}/q_\perp)$ →

Groomed TMD fragmenting jet function

$$\mathcal{G}_{q/h}\Big(z_h, \vec{k}_\perp, z_{\rm cut}, R, E_J\Big)$$
$$= z_h \sum_{X \in {\rm Jet}(R)} \frac{1}{2N_c} \delta(2E_J - p_X^- - p_h^-) {\rm tr}\left[\frac{\bar{n}\!\!\!/}{2} \langle 0|\delta^{(2)}(\vec{k}_\perp + \vec{\mathcal{P}}_\perp^{SD})\chi_n(0)|Xh\rangle\langle Xh|\bar{\chi}_n(0)|0\rangle\right]_{\vec{p}_{h\perp} = \vec{0}}$$



with respect to g-Jet axis

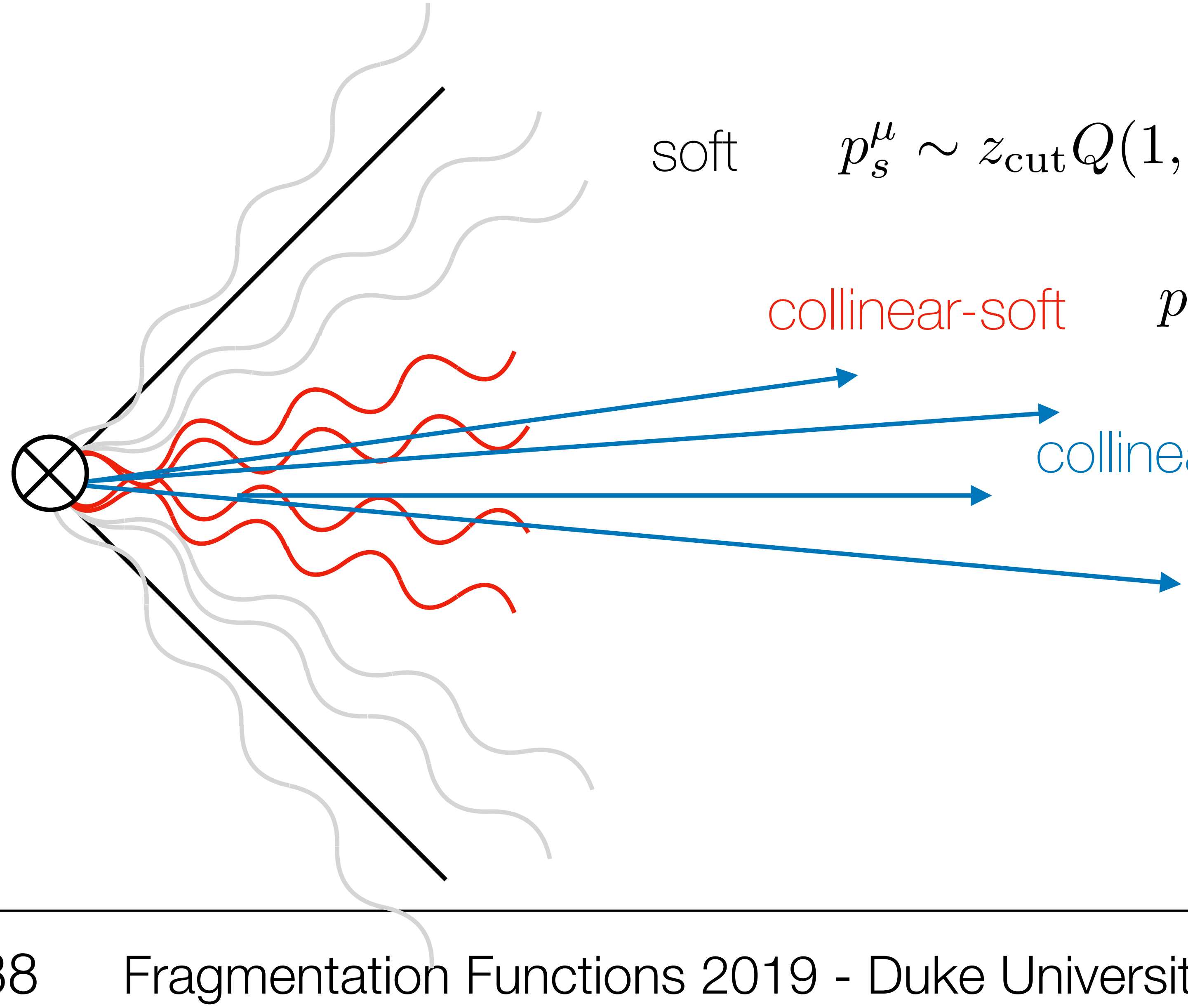
$$\vec{k}_\perp = -\frac{\vec{p}_{h\perp}}{z_h}$$

with respect to hadron axis

Re-Factorization for light hadrons

$$p_{h\perp} \ll z_{\rm cut} Q, \;\; z_{\rm cut} \ll R \sim 1$$

$$\mathcal{G}_{i/h}(z_h, \vec{k}_\perp, E_J, z_{\rm cut}; \mu_L) = \int d^2\vec{k}_{c\perp} \int d^2\vec{k}_{s\perp} \delta^2\left(\vec{k}_\perp + \vec{k}_{c\perp} + \vec{k}_{s\perp}\right) S_i^\perp\left(\vec{k}_{s\perp}, z_{cut}\right) \mathcal{D}_{i/h}^\perp\left(z_h, \vec{k}_{c\perp}\right)$$



soft $p_s^\mu \sim z_{\rm cut} Q(1,1,1)$

collinear-soft $p_{sc}^\mu \sim z_{\rm cut} Q(\lambda_{sc}^2, 1, \lambda_{sc})$ $\lambda_{sc} = \frac{p_{h\perp}}{z_{\rm cut} Q}$

collinear $p_c^\mu \sim Q(\lambda_c^2, 1, \lambda_c)$ $\lambda_c = \frac{p_{h\perp}}{Q}$

Factorization of the TMD (FJF) in SCET

$$\mathcal{G}_{i/h}(z_h, \vec{k}_\perp, E_J, z_{\rm cut}; \mu_L) = \int d^2\vec{k}_{c\perp} \int d^2\vec{k}_{s\perp} \delta^2\left(\vec{k}_\perp + \vec{k}_{c\perp} + \vec{k}_{s\perp}\right) S_i^\perp\left(\vec{k}_{s\perp}, z_{cut}\right) \mathcal{D}_{i/h}^\perp\left(z_h, \vec{k}_{c\perp}\right)$$

$$\mathcal{D}_{q/h}^\perp(z_h, \vec{k}_{c\perp}, E_J) = \sum_X \frac{z_h}{2N_c} \delta(2E_J - p^-_{Xh}) \mathrm{tr}\left[\frac{\bar{\not{n}}}{2} \langle 0|\delta^{(2)}(\vec{k}_{c\perp} - \vec{\mathcal{P}}_\perp)\chi_n(0)|Xh\rangle\langle Xh|\bar{\chi}_n(0)|0\rangle\right]_{\vec{p}_{h\perp}=0}$$

- collinear modes are energetic and always pass the grooming constraint

  → independent of the cutoff parameter $(z_{\rm cut})$

- contains the non-perturbative information of the fragmentation process

$$S_i^\perp(\vec{k}_{s\perp}, E_J, z_{\rm cut}) = \frac{1}{N_i} \mathrm{tr}\left[\langle 0|T\{S_n^i S_{\bar{n}}^i\}(0)\delta^{(2)}\left(\vec{k}_{s\perp} - \vec{\mathcal{P}}_\perp^{SD}\right)\bar{T}\{S_n^i S_{\bar{n}}^i\}(0)|0\rangle\right]$$

- describes collinear-soft radiation that can pass the grooming constraint
- universal to all light hadrons → independent of hadron's energy fraction (z_h)

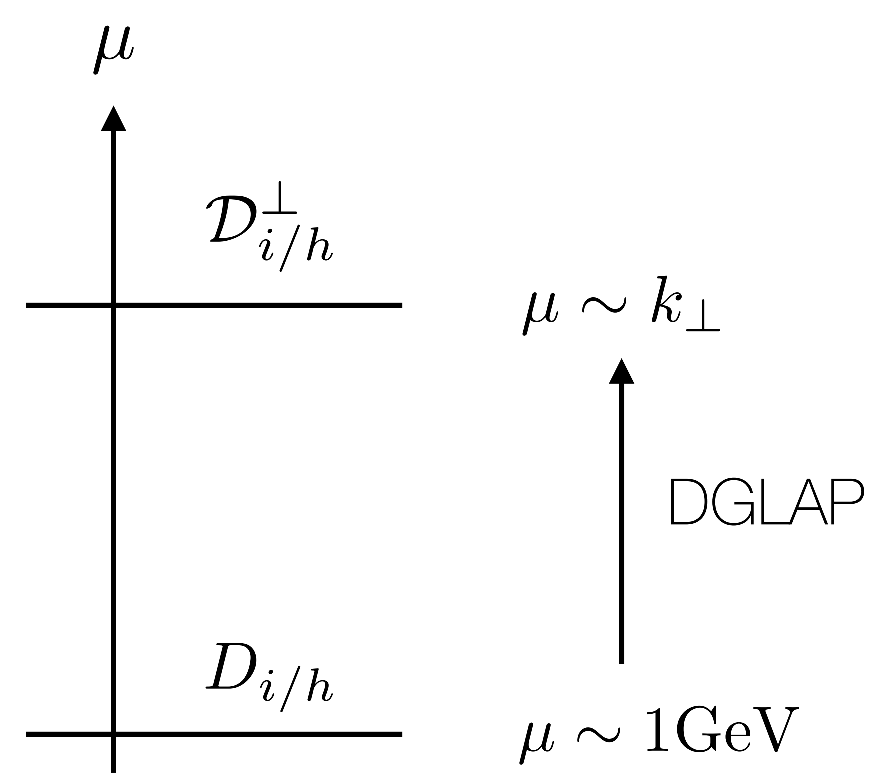
Matching onto collinear Fragmentation Functions

Although $\mathcal{D}^{\perp}_{q/h}(z_h, \vec{k}_{c\perp}, E_J)$ is a fundamentally non-perturbative object, for $k_\perp \gg \Lambda_{\mathrm{QCD}}$ can be matched onto the collinear Fragmentation Functions:

$$\mathcal{D}^{\perp}_{i/h}\left(z_h, \vec{k}_{c\perp}, E_J\right) = \int_{z_h}^{1} \frac{dx}{x} \mathcal{J}^{\perp}_{ij}\left(x, \vec{k}_{c\perp}, E_J\right) D_{j/h}\left(\frac{z_h}{x}\right)$$

short distance matching coefficients and collinear-soft calculable in perturbation theory → rapidity divergences

collinear Fragmentation Functions



NLL-Resummation in momentum space

Fourier Transform → Solve RGE → Inverse Fourier Tranform → Fix Scales

$$\omega_S = \frac{\alpha(\mu)C_i}{\pi}\ln\left(\frac{\nu_{\mathcal{D}}}{\nu_S}\right)$$

traditional TMDs

$$-\frac{\alpha(k_\perp)C_i}{\pi}\ln\left(\frac{k_\perp}{Q}\right)$$

groomed TMDFJF

$$-\frac{\alpha(k_\perp)C_i}{\pi}\ln(z_{\rm cut})$$

Solution:

Fix scales in coordinate space
and take Fourier transform numerically

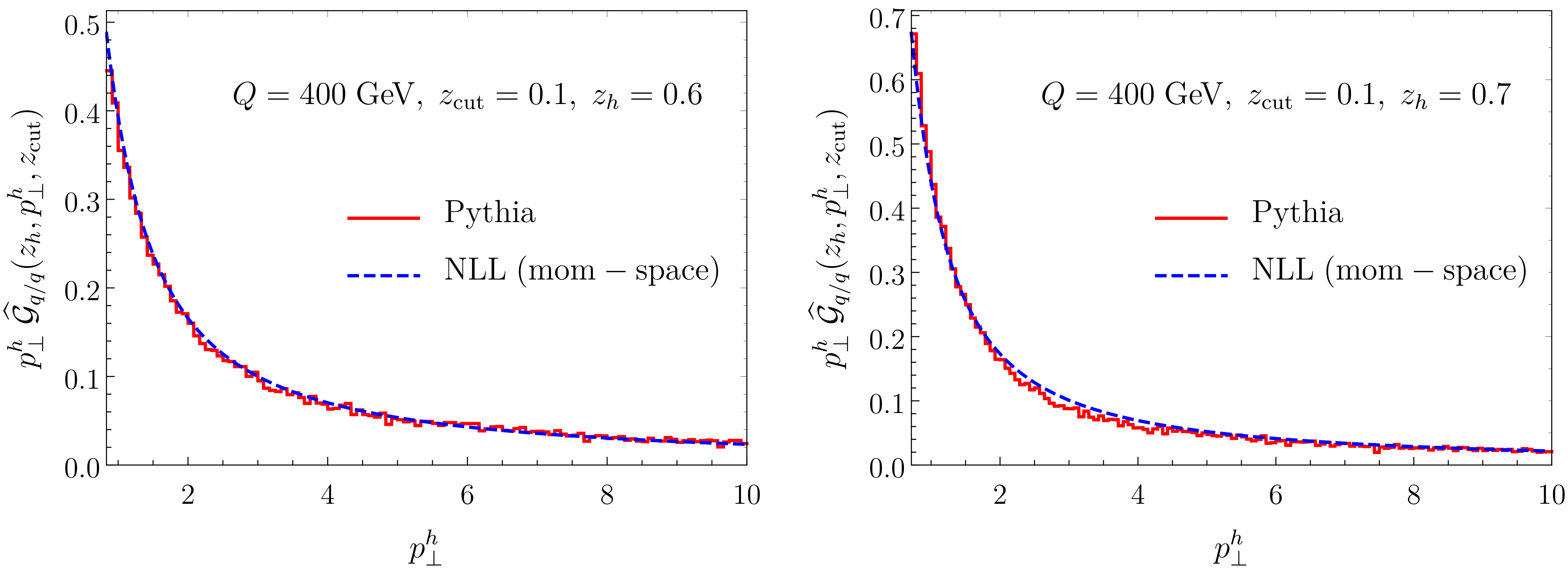
In the perturbative region ω_S is small, therefore we can fix the scales momentum space directly.

Common choice: $z_{\rm cut} = 0.1$

$\omega_S \sim 1$: Only in the non-perturbative regime

NLL-Resummation in momentum space

test against Pythia partonic shower: quark-to-quark case

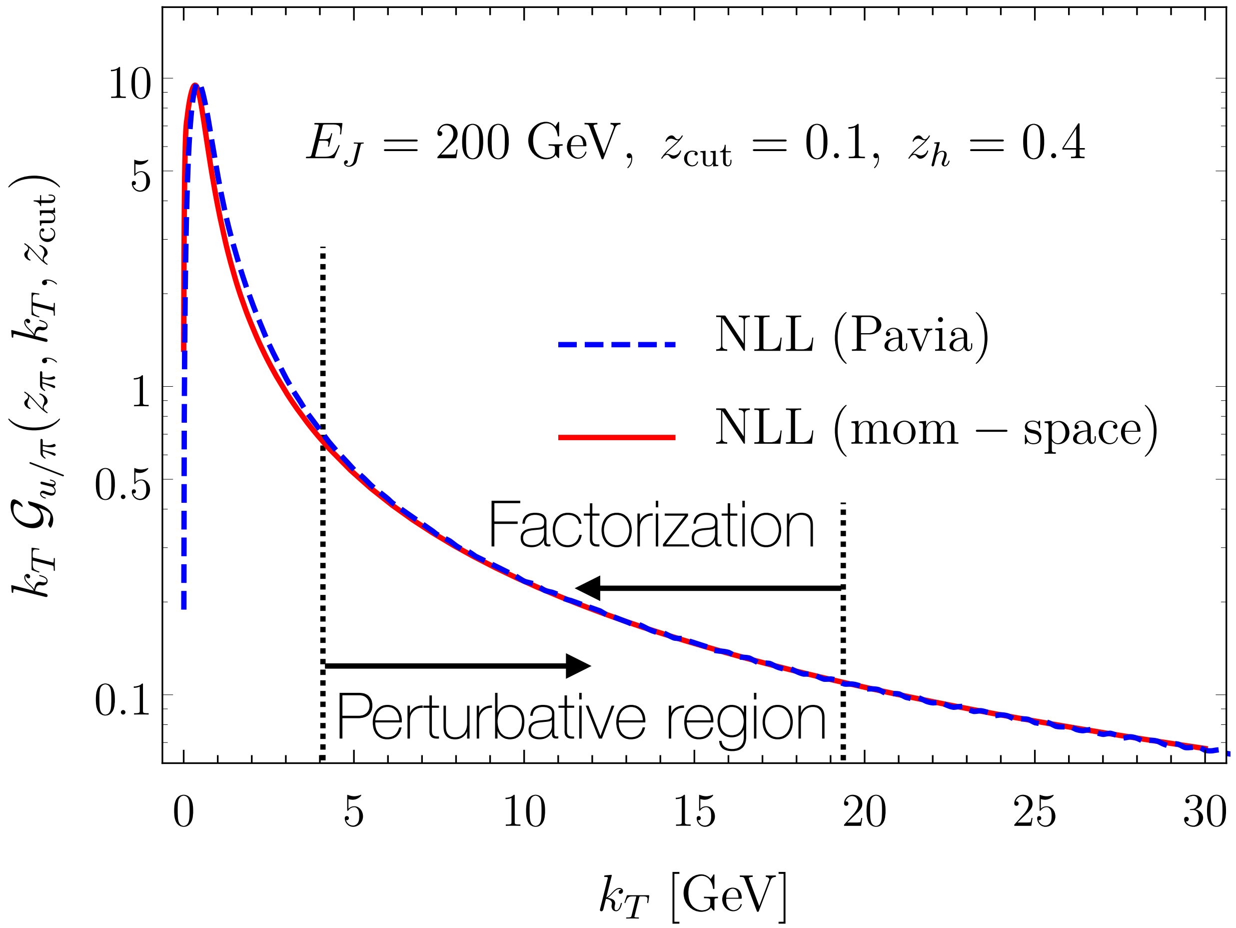


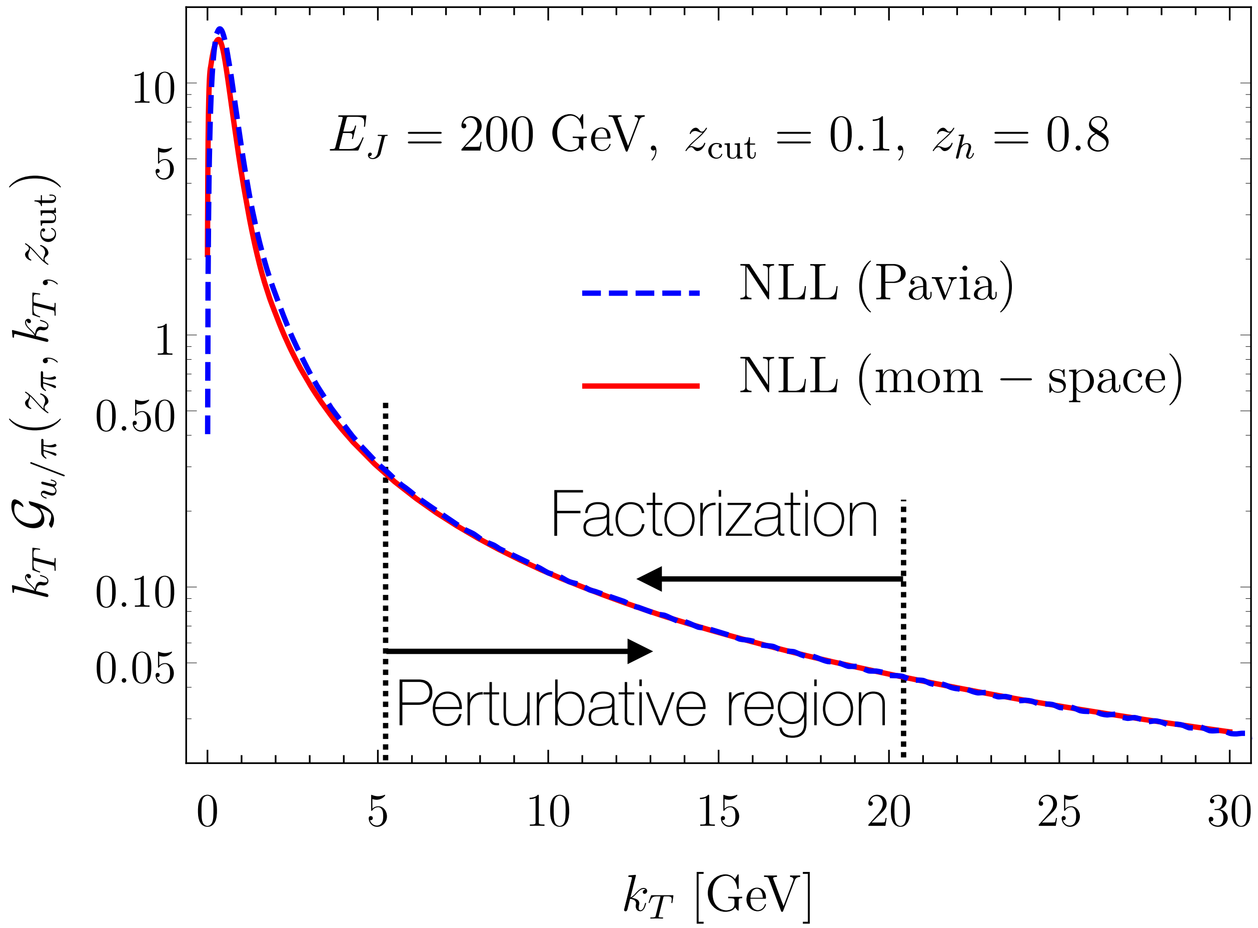
NLL: momentum space vs b-space

$$\mathcal{N}\frac{d\sigma}{dk_\perp}(e^+e^- \to jet + jet(\pi))$$

CSS:

$$g_K(b; b_{\max}) = \frac{1}{2}g_2(b_{\max})b^2$$





Pavia: **arXiv:1703.10157** A. Bacchetta, F. Delcarro, C. Pisano, M. Radici, A Signori

Resummation in b-space

$$\gamma^S_{\nu,i}(\mu) \to \gamma^S_{\nu,i}(\mu)\Big|_{b\to b_*} - g_K(b;b_{\max})$$

$$b_* = \frac{b}{\sqrt{1+(b/b_{\max})^2}}$$

$$g_K(b;b_{\max}) \xrightarrow[b\to 0]{} 0$$

Universal component of TMD observables:

$$U_i(\mu_L,\mu_H) \equiv \mathrm{Exp}\Big[-\int_{\mu_L}^{\mu_H} d\ln\mu\,\gamma_i^F[\alpha_s(\mu)] + 2\ln(z_{\rm cut})\Big(\int_{1/b_0}^{\mu_H} d\ln\mu\,\Gamma^i_{\rm cusp}[\alpha_s(\mu)] + \gamma^r(1/b_0)\Big)\Big]$$

Rapidity anomalous dimension

variations of the cutoff parameter give as direct access to the rapidity anomalous dimension:

$$\frac{d}{d\ln z_{\rm cut}}\Big[\mathcal{N}(z_{\rm cut})\frac{d\sigma}{dp_{h\perp}}\Big]$$

Normalized cross section

Non-perturbative TMD evolution

$$\gamma^S_{\nu,i}(\mu) \to \gamma^S_{\nu,i}(\mu)\Big|_{b\to b_*} - g_K(b;b_{\max}) \qquad b_* = \frac{b}{\sqrt{1+(b/b_{\max})^2}} \qquad g_K(b;b_{\max}) \xrightarrow[b\to 0]{} 0$$

Model:Fits	g_2	$b_{\max}$ [GeV^{-1}]	b_{NP} [GeV^{-1}]
CSS:BNLY 2003	0.68	0.5	n.a.
CSS:KN 2006	0.18	1.5	n.a.
CSS:Pavia 2016	0.12	1.123	n.a.
AFGR: n.a.	0.10	0.5	2.0

CSS:

$$g_K(b;b_{\max}) = \frac{1}{2} g_2(b_{\max}) b^2$$

AFGR:

$$g_K(b;b_{\max}) = \frac{g_2(b_{\max}) b_{\text{NP}}^2}{2} \ln\left(1 + \frac{b^2}{b_{\text{NP}}^2}\right)$$

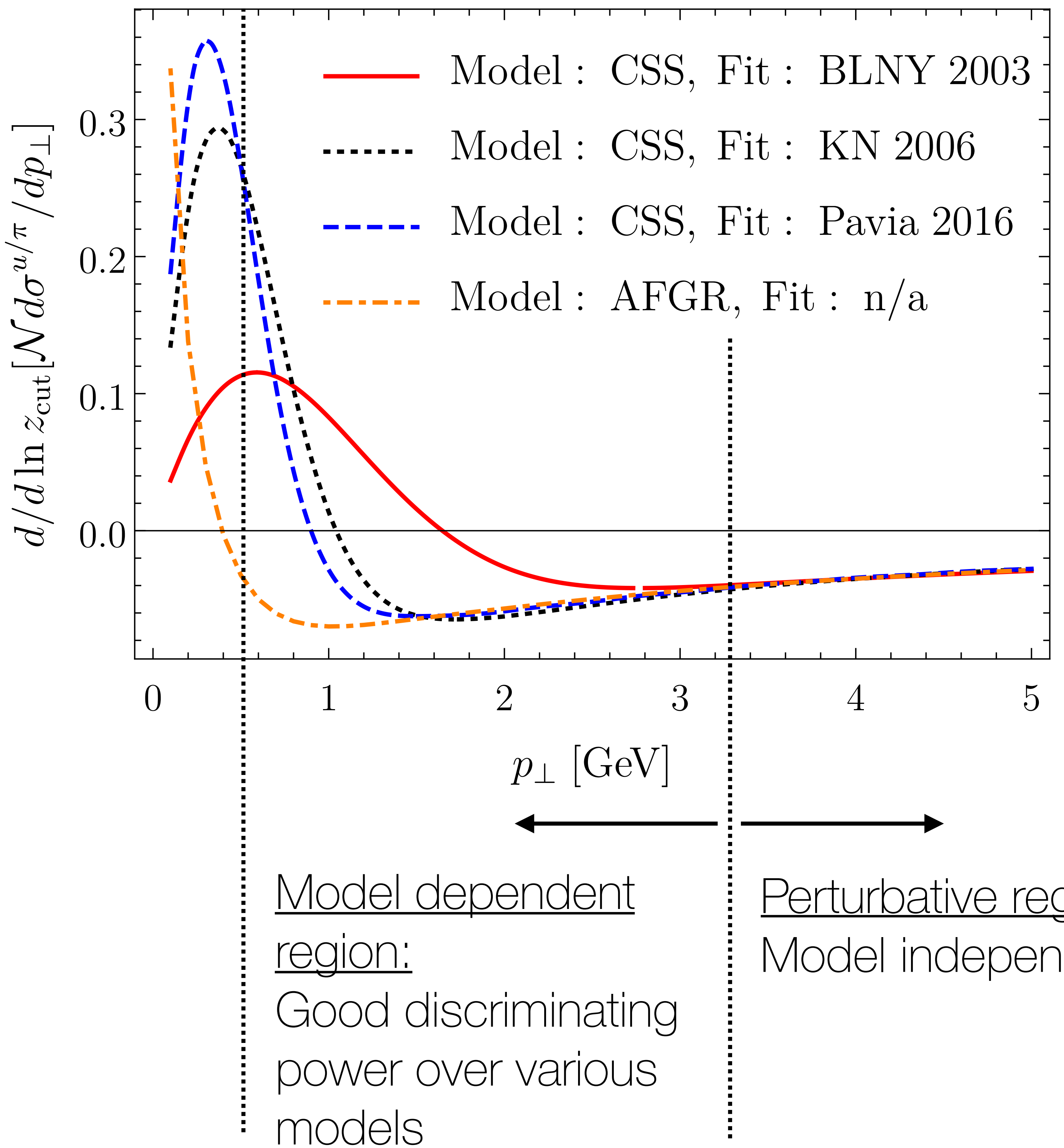
BNLY: **arXiv:0212159** F. Landry, R. Brock, P.M. Nadolsky, C.-P. Yuan

KN: **arXiv:0506225** A. V. Konychev, P. M. Nadolsky

Pavia: **arXiv:1703.10157** A. Bacchetta, F. Delcarro, C. Pisano, M. Radici, A Signori

AFGR: **arXiv:1401.2654** C. A. Aidala, B. Field, L. P. Gamberg, T. C. Rogers

Proposed observable



$$\frac{d}{d\ln z_{\mathrm{cut}}}\left[\mathcal{N}(z_{\mathrm{cut}})\frac{d\sigma}{dp_{h\perp}}\right]$$

<u>Model dependent region:</u>
Good discriminating power over various models

<u>Perturbative region:</u>
Model independent

NLL-Resummation in momentum space

Fourier Transform ⟶ Solve RGE ⟶ Inverse Fourier Tranform ⟶ Fix Scales

$$\mathcal{G}_{j/h}^{\mathrm{NLL}}(z_h, \vec{k}_\perp, z_{\mathrm{cut}}; \mu) = \mathcal{V}(\vec{k}_\perp, z_{\mathrm{cut}}, \mu_0)\mathcal{U}(\mu, \mu_0) D_{j/h}(z_h, \mu_0)\Big|_{\mu_0 = k_\perp}$$

$$\mathcal{U}(\mu, \mu_0) = \exp\left[2\pi \frac{\gamma^{D\otimes S(\mu, z_{\mathrm{cut}})}}{\beta_0\ \alpha_s(\mu)} \ln(\alpha_s(\mu_0)/\alpha_s(\mu))\right]$$

$$\mathcal{V}(\vec{k}_\perp, z_{\mathrm{cut}}, \mu) = \frac{\exp(-2\gamma_E \omega_S)}{\pi} \frac{\Gamma(1-\omega_S)}{\Gamma(\omega_S)} \frac{1}{\mu^2} \left(\frac{\mu^2}{k_\perp^2}\right)^{1-\omega_S}$$

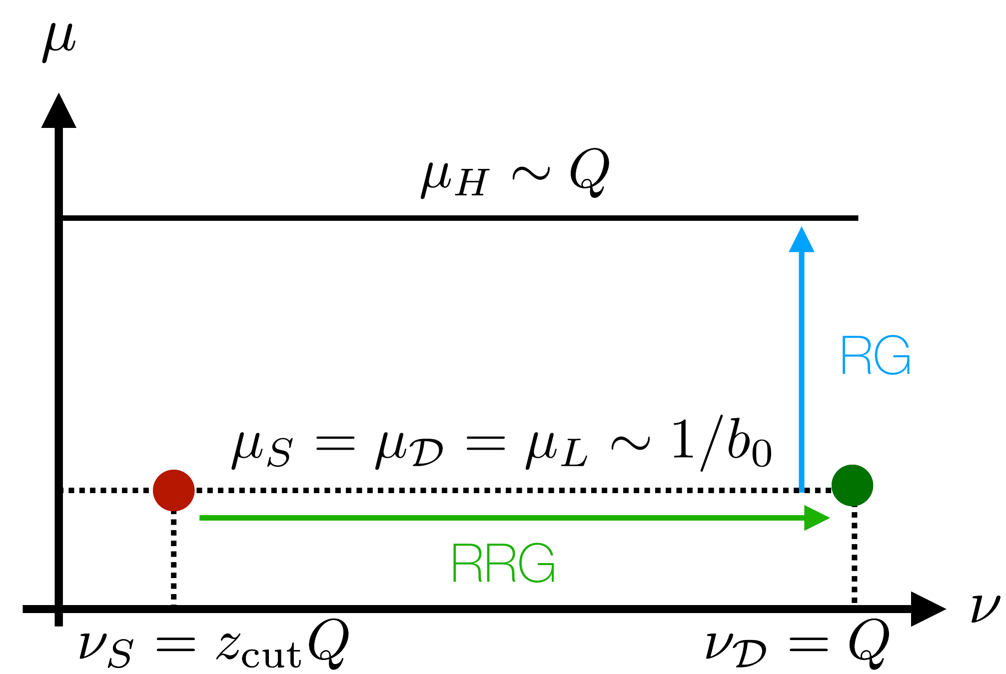
$$\omega_S = \frac{\alpha(\mu) C_i}{\pi} \ln\left(\frac{\nu_{\mathcal{D}}}{\nu_S}\right)$$

traditional TMDs ↙ ↘ groomed TMDFJF

$$-\frac{\alpha(k_\perp) C_i}{\pi} \ln\left(\frac{k_\perp}{Q}\right) \qquad -\frac{\alpha(k_\perp) C_i}{\pi} \ln(z_{\mathrm{cut}})$$

Resummation in b-space

Fourier Transform ⟶ Solve RGE ⟶ Fix Scales ⟶ Inverse Fourier Tranform



$$\mathcal{D}^{\perp}_{i/h}(\mu_H, \nu = 2E_J) S^{\perp}_i(\mu_H, \nu = 2E_J) = U_i(\mu_L, \mu_H) \times \Big[\mathcal{D}^{\perp}_{i/h}(\mu_L, \nu = 2E_J) S^{\perp}_i(\mu_L, \nu = 2E_J z_{\rm cut})\Big]$$

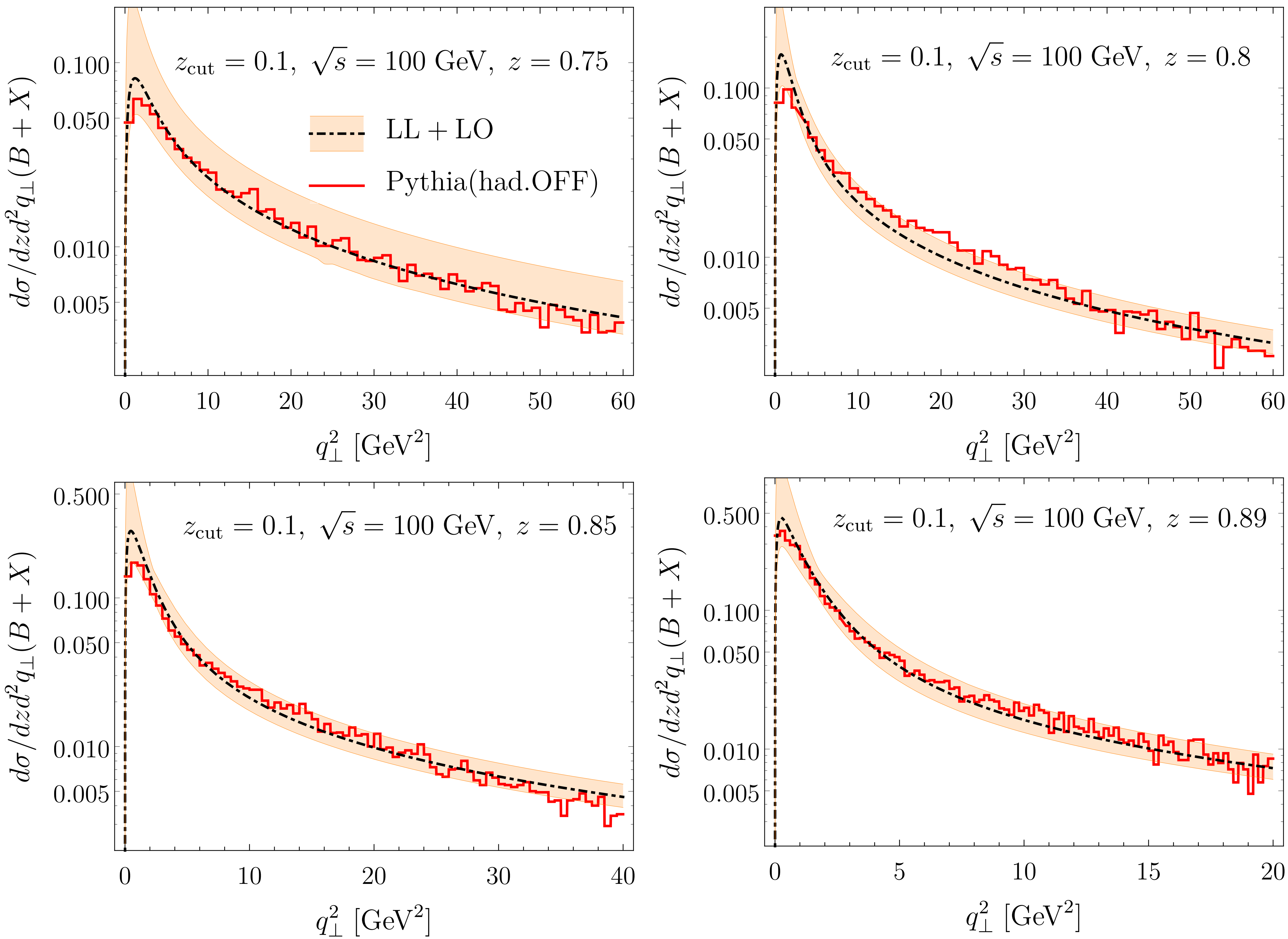
$$U_i(\mu_L, \mu_H) \equiv {\rm Exp}\Big[-\int_{\mu_L}^{\mu_H} d\ln\mu\, \gamma_i^F[\alpha_s(\mu)] + 2\ln(z_{\rm cut})\Big(\int_{1/b_0}^{\mu_H} d\ln\mu\, \Gamma^i_{\rm cusp}[\alpha_s(\mu)] + \gamma^r(1/b_0)\Big)\Big]$$

Rapidity anomalous dimension $\gamma^S_{\nu,i}(\mu) = -2\int_{1/b_0}^{\mu} d\ln\mu'\,\Gamma^i_{\rm cusp}[\alpha_s(\mu')] + \gamma^r(1/b_0)$

Results for heavy mesons

$$\frac{d\sigma^{(1)}}{dz d^2\vec{q}_\perp} = \tilde{\mathcal{N}}(E_J, z_{cut}, m) \times \Big[\frac{\alpha_s C_F}{\pi^2} \frac{q_\perp^2}{(1-z)} \frac{\theta((1-z) - z_{cut})}{((1-z)^2 m^2 + q_\perp^2)^2}\Big]$$

Results for heavy mesons



Results for heavy mesons