

# Fragmentation in Jets

Felix Ringer

Lawrence Berkeley National Laboratory

Duke University, 03/15/19

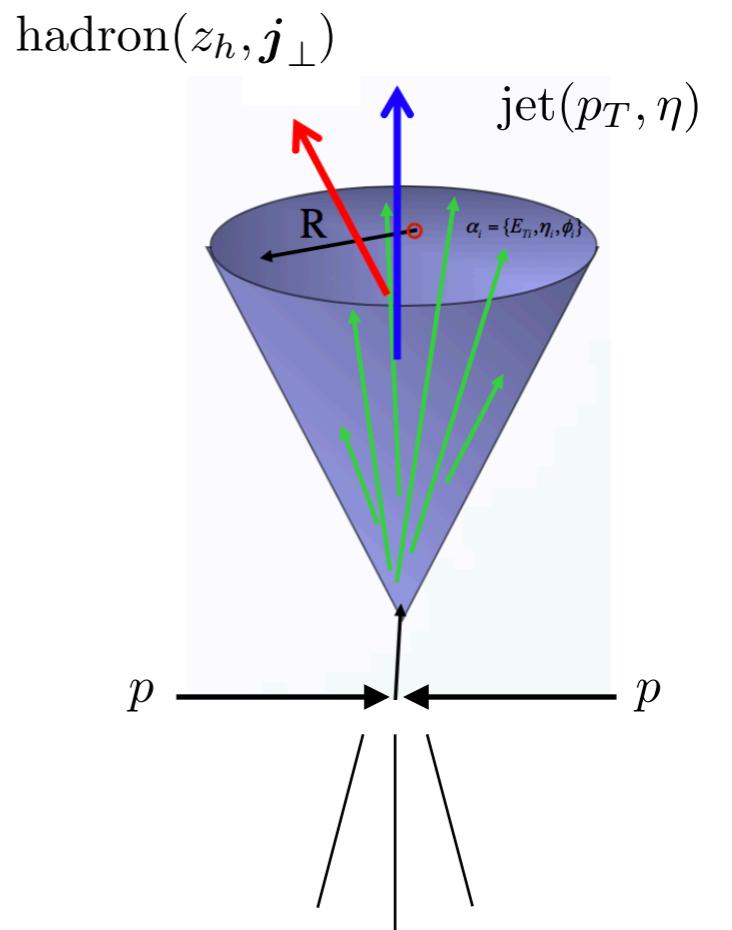


# Hadron in jet fragmentation

Inclusive production of jets  $p_T, \eta$

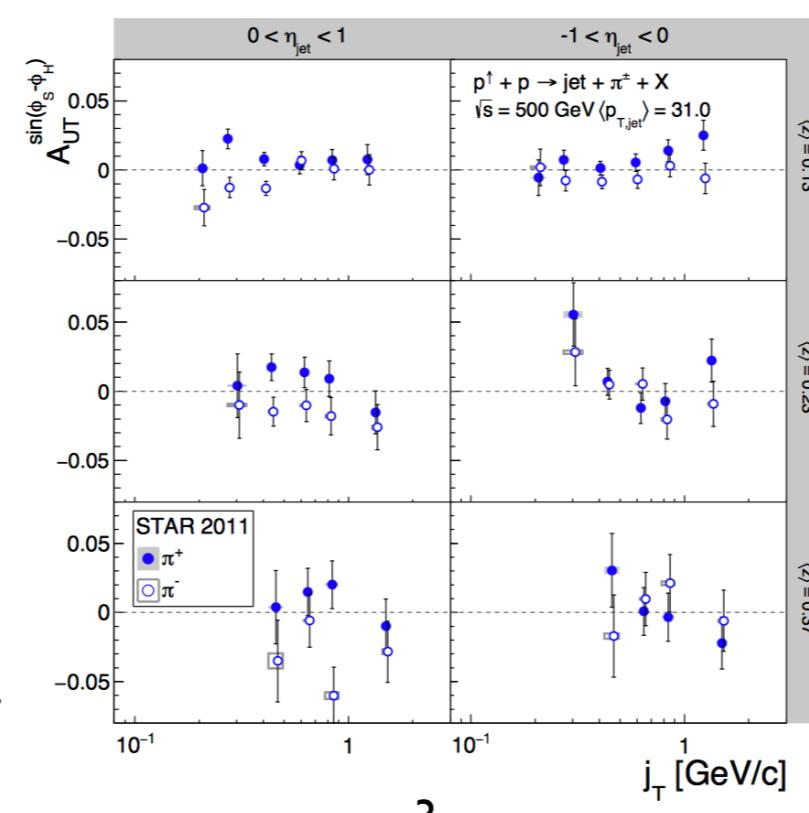
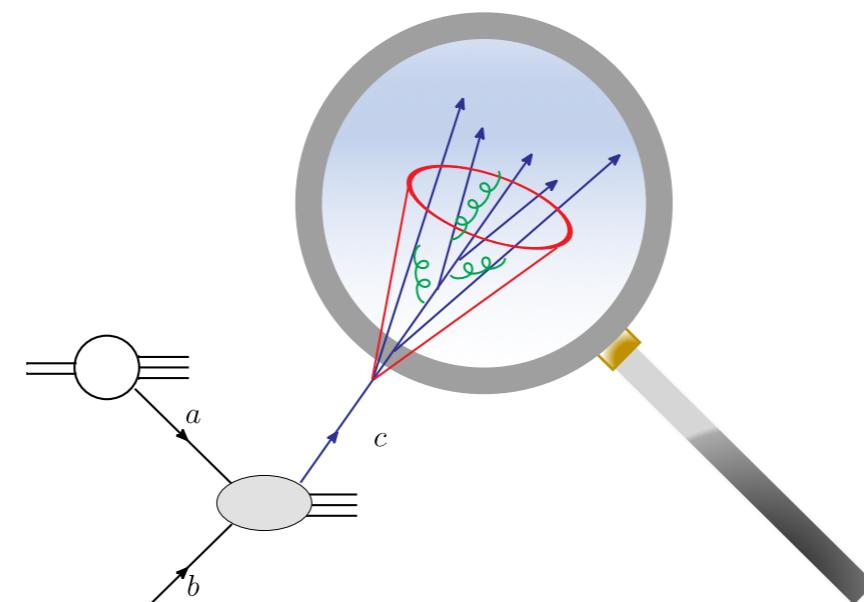
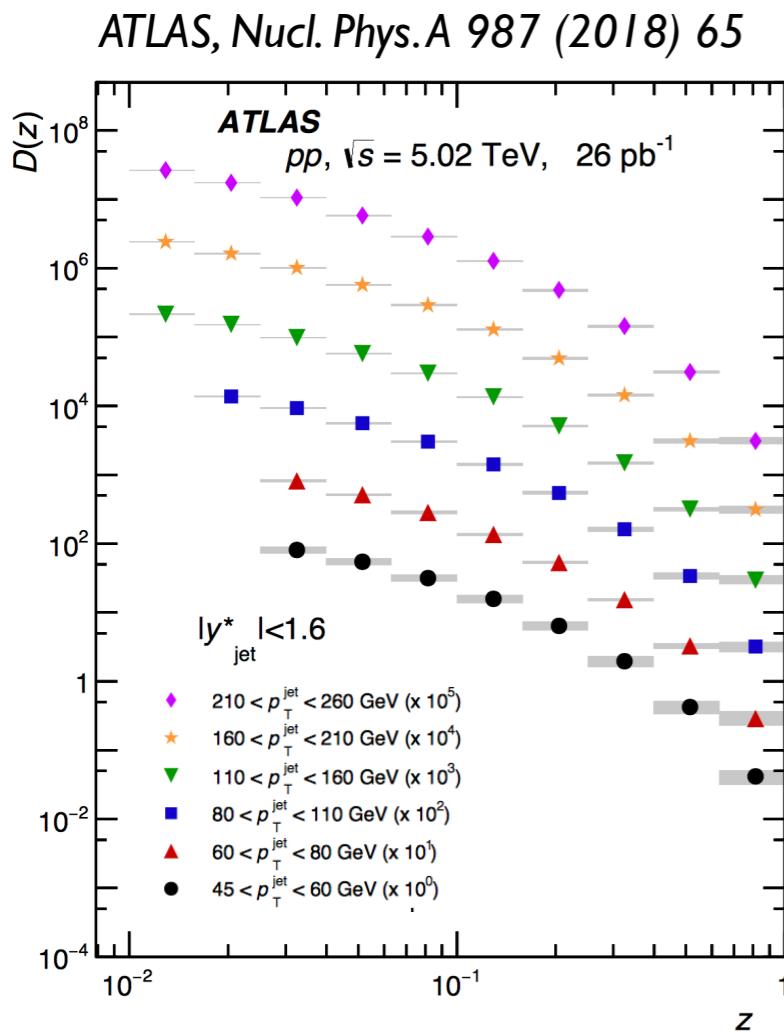
- Identify the hadrons in the jet and measure additional two variables:
  - Longitudinal momentum fraction  $z_h = p_T^h / p_T$
  - Relative transverse momentum wrt. to a predetermined axis  $\mathbf{j}_\perp$

$$F(z_h, \mathbf{j}_\perp; \eta, p_T, R) = \frac{d\sigma^{pp \rightarrow (\text{jet } h)X}}{dp_T d\eta dz_h d^2 \mathbf{j}_\perp} \Bigg/ \frac{d\sigma^{pp \rightarrow \text{jet} X}}{dp_T d\eta}$$

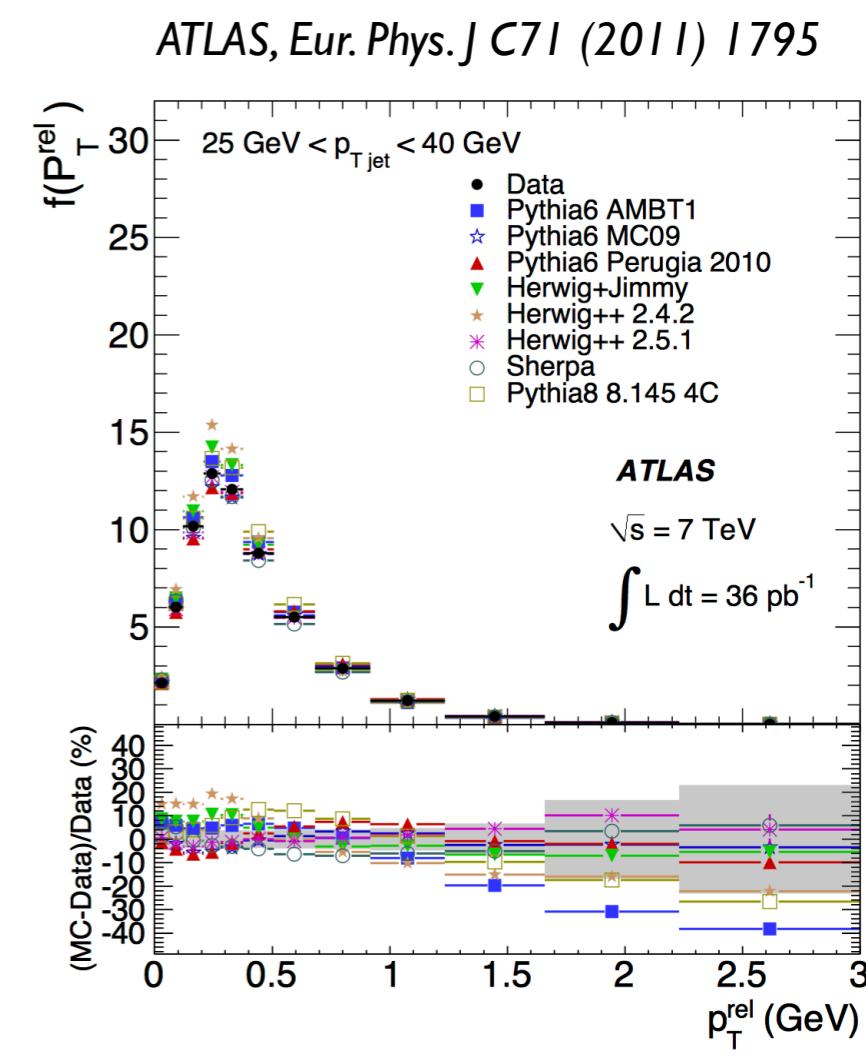


- Constrain (gluon) fragmentation function
- Test of universality and (TMD) evolution

# The longitudinal and transverse structure of jets



STAR, Phys. Rev. D 97 (2018) 032004



# Outline

- Introduction
- Collinear FFs in jets
- TMD FFs in jets
- The jet shape
- Conclusions

# The jet fragmentation function $pp \rightarrow (\text{jeth})X$

Kang, FR, Vitev '16

- First reconstruct a jet and then identify the hadrons inside the jet

$$F(z_h, p_T) = \frac{d\sigma^{pp \rightarrow (\text{jeth})X}}{dp_T d\eta dz_h} / \frac{d\sigma^{pp \rightarrow \text{jet}X}}{dp_T d\eta} \quad \text{where} \quad z_h = p_T^h / p_T$$

- Factorization for inclusive jet production

$$\frac{d\sigma^{pp \rightarrow (\text{jet } h) + X}}{d\eta dp_T dz_h} = \sum_{abc} f_a \otimes f_b \otimes H_{ab}^c \otimes \mathcal{G}_c^h$$

where  $\mathcal{G}_q^h(z, \cancel{z}_h, p_T R, \mu) = \sum_j \mathcal{J}_{ij}(z, \cancel{z}_h, p_T R, \mu) \otimes D_j^h(\cancel{z}_h, \mu)$



see also: Procura, Stewart '10, Jain, Procura, Waalewijn '11, Arleo et al. '14,  
Kaufmann, Mukherjee, Vogelsang '15

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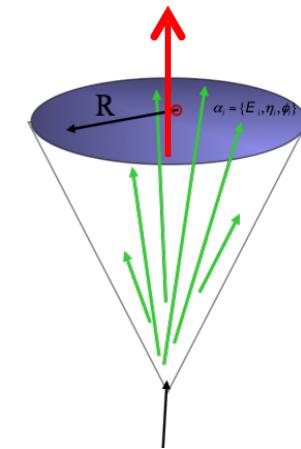
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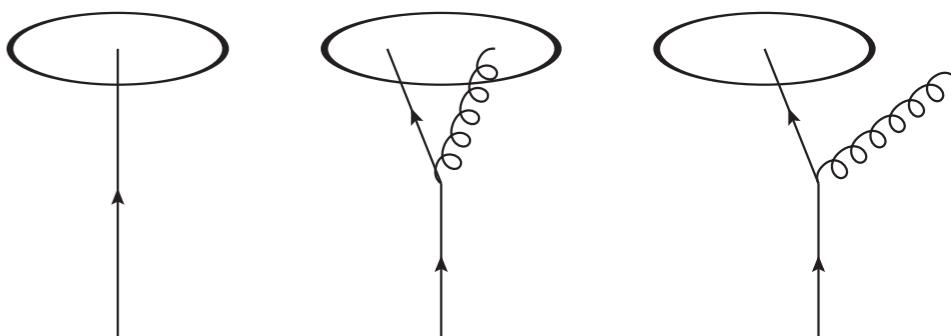
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$$= \sum_{jk} j_{ij}(z, p_T R, \mu) \times \tilde{j}_{jk}(z_h, p_T R, \mu) \otimes D_k^h(z_h, \mu) + \mathcal{O}(\alpha_s^2) \quad z_h \neq 1 \quad z \neq 1$$



see also: Procura, Stewart '10, Jain, Procura, Waalewijn '11, Arleo et al. '14,  
Kaufmann, Mukherjee, Vogelsang '15



# The jet fragmentation function $pp \rightarrow (\text{jeth})X$

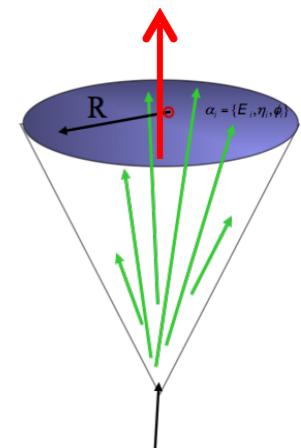
Kang, FR, Vitev '16

- Factorization for inclusive jet production

$$\frac{d\sigma^{pp \rightarrow (\text{jeth})X}}{d\eta dp_T dz_h} = \sum_{abc} f_a \otimes f_b \otimes H_{ab}^c \otimes \mathcal{G}_c^h$$

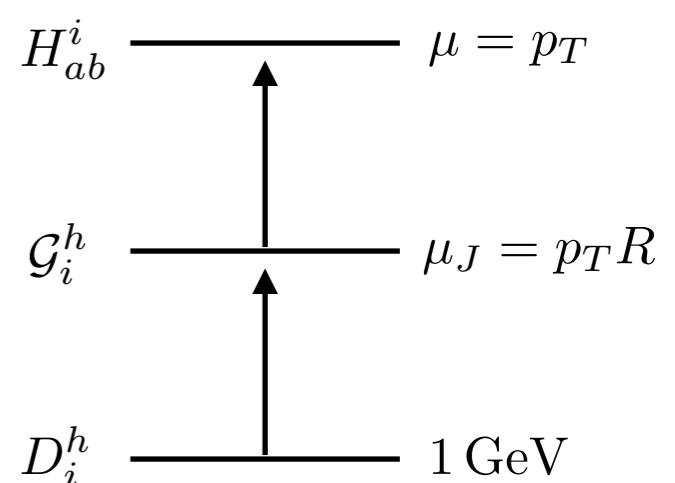
where  $\mathcal{G}_q^h(z, z_h, p_T R, \mu) = \sum_j \mathcal{J}_{ij}(z, z_h, p_T R, \mu) \otimes D_j^h(z_h, \mu)$

matching coefficients  
standard collinear FFs



- $\alpha_s^n \ln^n R$  resummation again via DGLAP

$$\mu \frac{d}{d\mu} \mathcal{G}_i^h(z, z_h, p_T R, \mu) = \sum_j P_{ji}(z) \otimes \mathcal{G}_j^h(z, z_h, p_T R, \mu)$$



see also: Procura, Stewart '10, Jain, Procura, Waalewijn '11, Arleo et al. '14,  
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2x DGLAP

# Phenomenology

- Light charged hadrons

Arleo, Fontannaz, Guillet, Nguyen '14  
 Kaufmann, Mukherjee, Vogelsang '15  
 Kang, FR, Vitev '16  
 Neill, Scimemi, Waalewijn '16  
 Makris, Neill, Vaidya '17

- Heavy flavor mesons

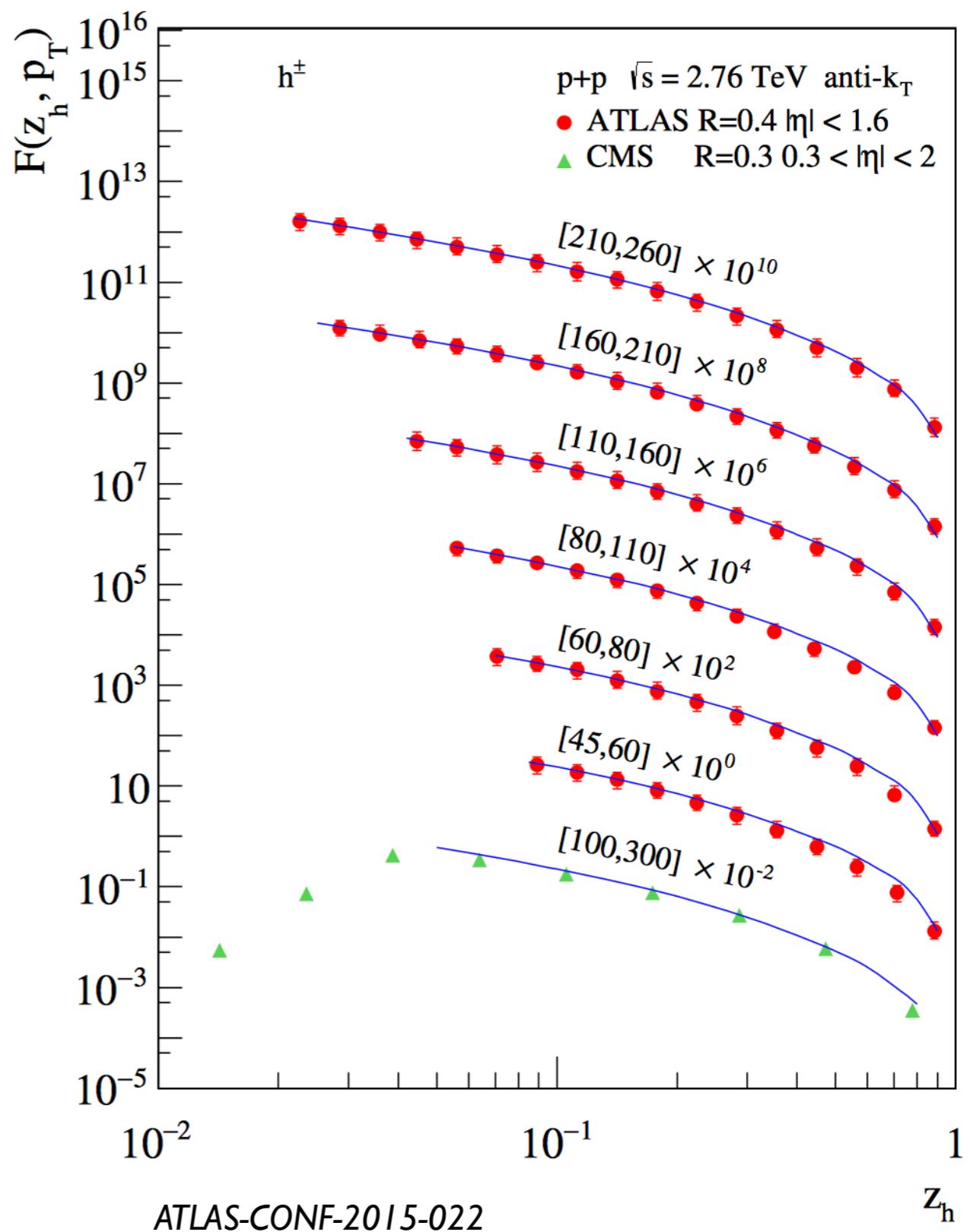
Chien, Kang, FR, Vitev, Xing '15  
 Bain, Dai, Hornig, Leibovich, Makris, Mehen '16  
 Anderle, Kaufmann, Stratmann, FR, Vitev '17

- Quarkonia

Baumgart, Leibovich, Mehen, Rothstein '14  
 Bain, Dai, Hornig, Leibovich, Makris, Mehen '16  
 Kang, Qiu, FR, Xing, Zhang '17  
 Bain, Dai, Leibovich, Makris, Mehen '17

- Photons

Kaufmann, Mukherjee, Vogelsang '16



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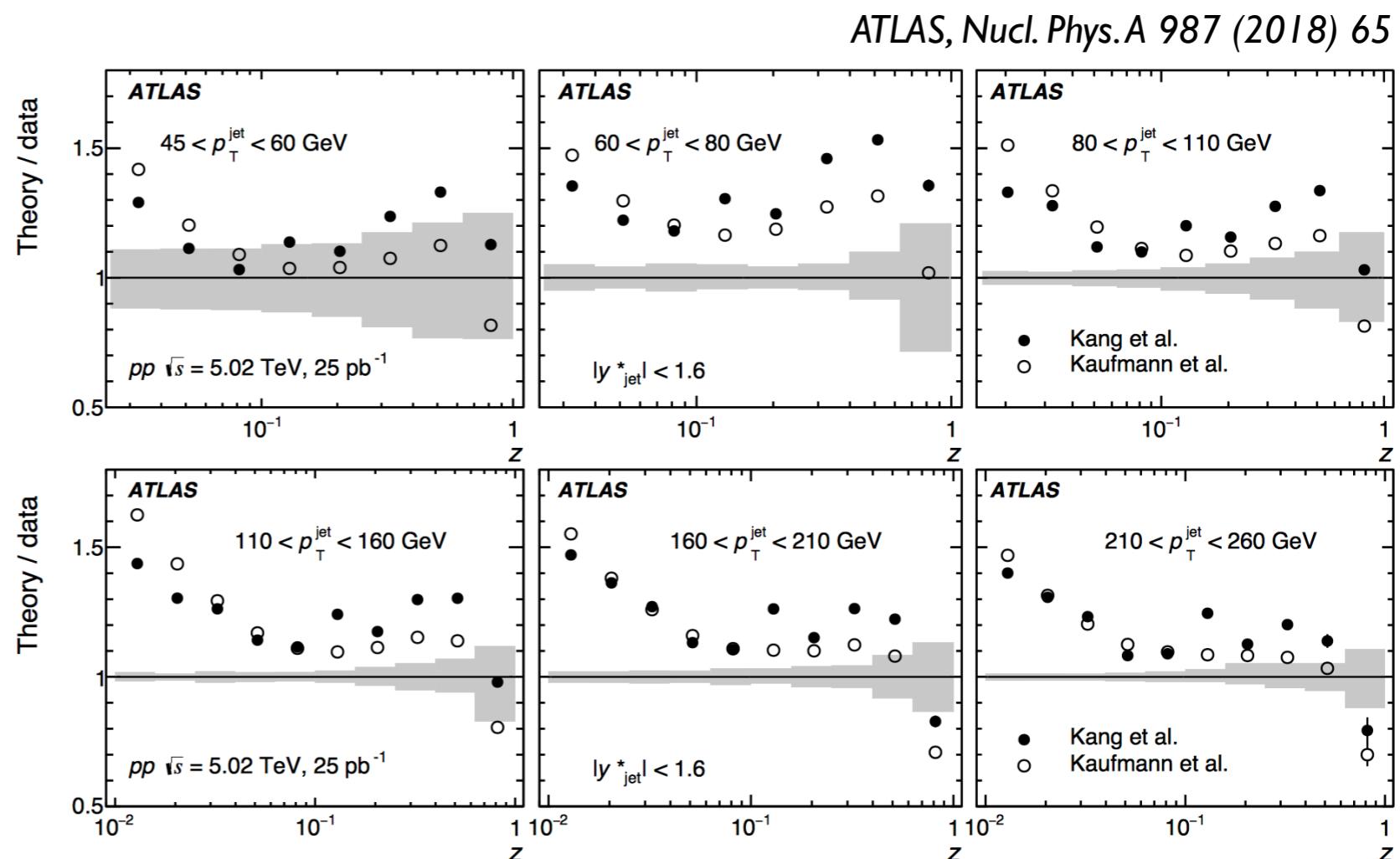
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# In-jet TMD distributions

- Overview of in-jet TMD distributions with respect to a given axis

Standard jet axis

*Bain, Makris, Mehen '16*

*Kang, Liu, FR, Xing '17*

*Kang, Lee, Liu, Neill, FR  
- in preparation*

Recoil free axis  
e.g. Winner-take-all

*Neill, Scimemi, Waalewijn '17*  
*Neill, Papaefstathiou, Waalewijn, Zoppi '18*

Standard jet axis

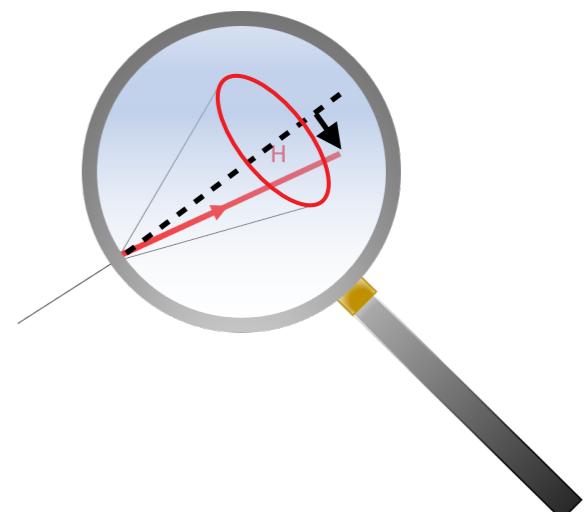
*Makris, Neill, Vaidya '17*

→ see Yianni's talk

Soft sensitivity,  
related to standard TMDs

Collinear factorization only

Grooming (soft drop)



# TMD in jet fragmentation

Kang, Liu, FR, Xing '17  
Kang, Prokudin, FR, Yuan '17

- Measure the relative transverse momentum of the hadron wrt. to the jet axis

$$F(z_h, \mathbf{j}_\perp; \eta, p_T, R) = \frac{d\sigma^{pp \rightarrow (\text{jet } h)X}}{dp_T d\eta dz_h d^2 \mathbf{j}_\perp} \Bigg/ \frac{d\sigma^{pp \rightarrow \text{jet } X}}{dp_T d\eta}$$

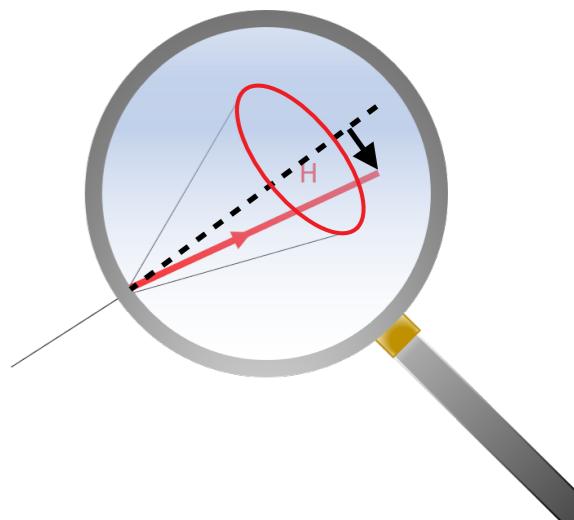
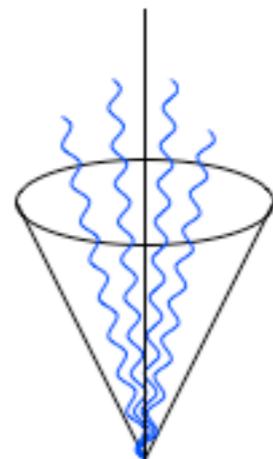
longitudinal and transverse momentum  $z_h, \mathbf{j}_\perp$

$$0 \ll z_h \ll 1$$

$$\mathcal{G}_c^h(z, z_h, p_T R, \mathbf{j}_\perp, \mu) = \mathcal{H}_{c \rightarrow i}(z, p_T R, \mu) \times D_{h/i}(z_h, \mathbf{j}_\perp, \mu) \otimes S_i(\mathbf{j}_\perp, R, \mu)$$

Out-of-jet radiation  
cf.  $j_{ij}(z, p_T R, \mu)$   
Jet algorithm dependent

Standard TMD  
fragmentation functions  
as for SIDIS and  $e^+e^-$



- Test of universality and TMD evolution
- Azimuthal asymmetries at RHIC - Collins effect ————— see James' talk

see also: Bain, Makris, Mehen '16, Neill, Scimemi, Waalewijn '17  
Makris, Neill, Vaidya '17

# The soft function

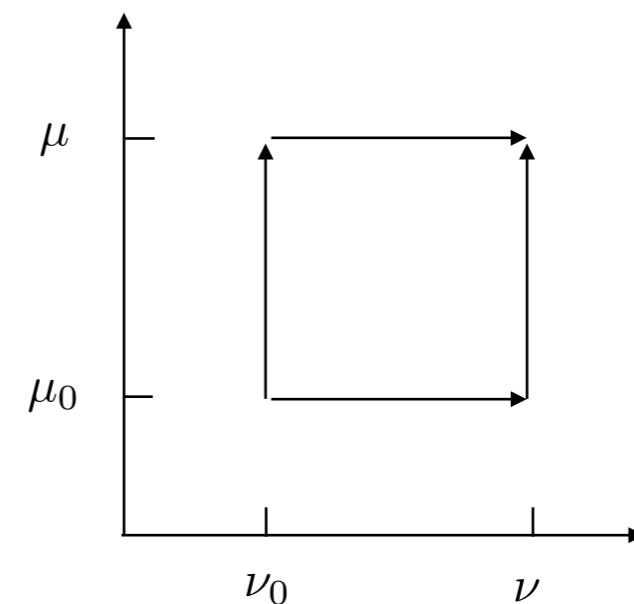
- Global soft function

$$\downarrow \times \frac{1}{2}, \nu \rightarrow \nu R \tan(R/2)$$

$$\hat{S}_i(\mathbf{b}, \mu, \nu) = 1 + \frac{\alpha_s}{2\pi} C_i \left[ \frac{4}{\eta} \left( -\frac{1}{\epsilon} - \ln \left( \frac{\mu^2}{\mu_b^2} \right) \right) + \frac{2}{\epsilon^2} - \frac{2}{\epsilon} \ln \left( \frac{\nu^2}{\mu^2} \right) \right. \\ \left. - 2 \ln \left( \frac{\mu^2}{\mu_b^2} \right) \ln \left( \frac{\nu^2}{\mu_b^2} \right) + \ln^2 \left( \frac{\mu^2}{\mu_b^2} \right) - \frac{\pi^2}{6} \right].$$

- Soft function in the jet (b-space):

$$S_i(\mathbf{b}, \mu, \nu R) = 1 + \frac{\alpha_s}{2\pi} C_i \left[ \frac{2}{\eta} \left( -\frac{1}{\epsilon} - \ln \left( \frac{\mu^2}{\mu_b^2} \right) \right) + \frac{1}{\epsilon^2} - \frac{1}{\epsilon} \ln \left( \frac{\nu^2 \tan^2(R/2)}{\mu^2} \right) \right. \\ \left. - \ln \left( \frac{\mu^2}{\mu_b^2} \right) \ln \left( \frac{\nu^2 \tan^2(R/2)}{\mu_b^2} \right) + \frac{1}{2} \ln^2 \left( \frac{\mu^2}{\mu_b^2} \right) - \frac{\pi^2}{12} \right]$$



# The soft function

- Global soft function

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- Proper TMD definition

- SIDIS and  $e^+e^-$ :  $\hat{\mathcal{D}}_{h/i}(z_h, \mathbf{b}; \mu_b) \equiv D_{h/i}(z_h, \mathbf{b}, \mu_D, \nu_D) \sqrt{\hat{S}_i(\mathbf{b}, \mu_{\hat{S}}, \nu_{\hat{S}})}$

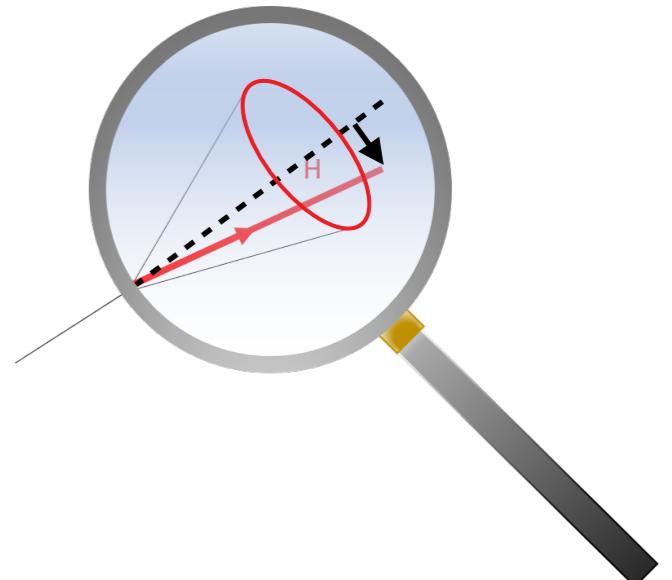
Canceled by  
evolution of  $\mathcal{H}_{c \rightarrow i}$

- In-jet TMD  $\mathcal{D}_{h/i}^R(z_h, \mathbf{b}; \mu_b) \equiv D_{h/i}(z_h, \mathbf{b}, \mu_D, \nu_D) S_i(\mathbf{b}, \mu_S, \nu_S R)$

$$\mathcal{D}_{h/i}^R(z_h, \mathbf{b}; \mu) = \hat{\mathcal{D}}_{h/i}(z_h, \mathbf{b}; \mu_J) \exp \left[ - \int_{\mu_J}^{\mu} \frac{d\mu'}{\mu'} \left( \Gamma_{\text{cusp}}^i \ln \left( \frac{\mu_J^2}{\mu'^2} \right) + \gamma^i \right) \right]$$

# TMD in jet fragmentation

$$g_c^h(z, z_h, p_T R, \mathbf{j}_\perp, \mu) = \mathcal{H}_{c \rightarrow i}(z, p_T R, \mu) \times D_{h/i}(z_h, \mathbf{j}_\perp, \mu) \otimes S_i(\mathbf{j}_\perp, R, \mu)$$



- Proper TMD evaluated at the jet scale

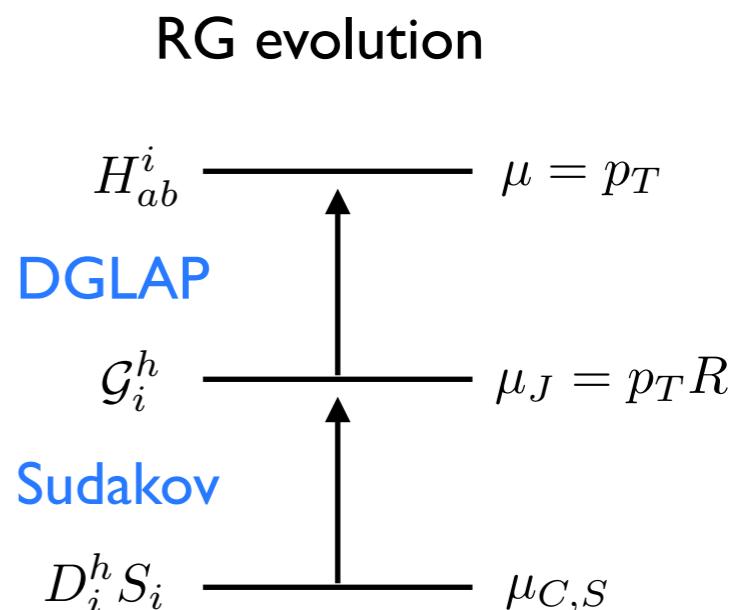
$$\hat{\mathcal{D}}_{h/i}(z_h, \mathbf{j}_\perp; \mu_J) = \frac{1}{z_h^2} \int \frac{b \, db}{2\pi} J_0(j_\perp b/z) C_{j \leftarrow i} \otimes D_{h/j}(z_h, \mu_{b_*}) e^{-S_{\text{pert}}^i(b_*, \mu_J) - S_{\text{NP}}^i(b, \mu_J)}$$

- The usual perturbative Sudakov factor

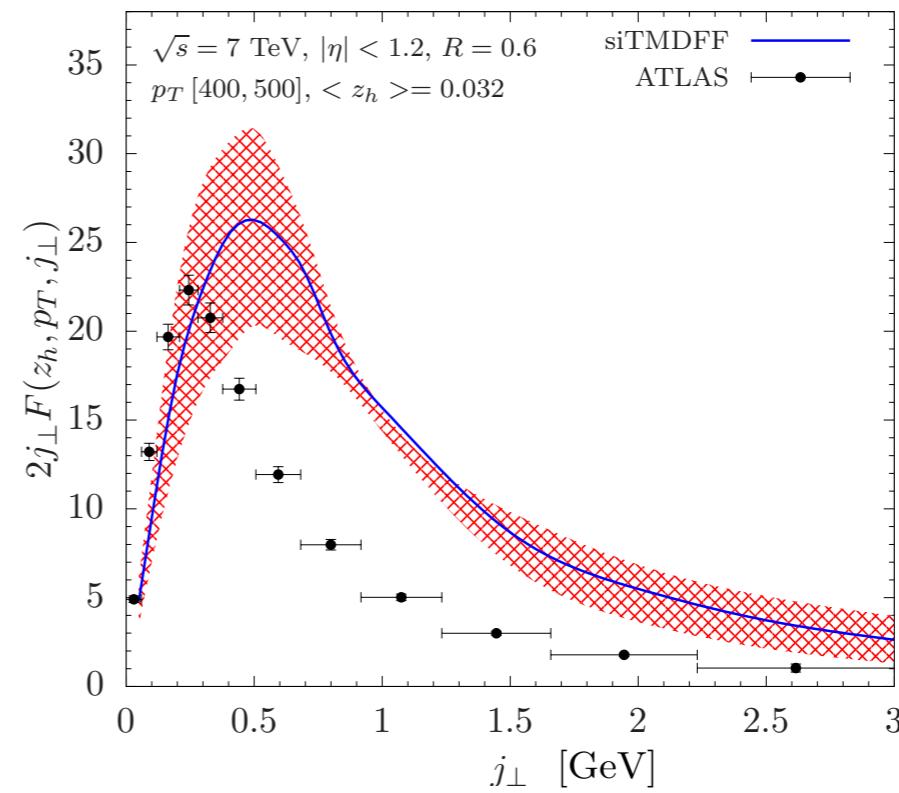
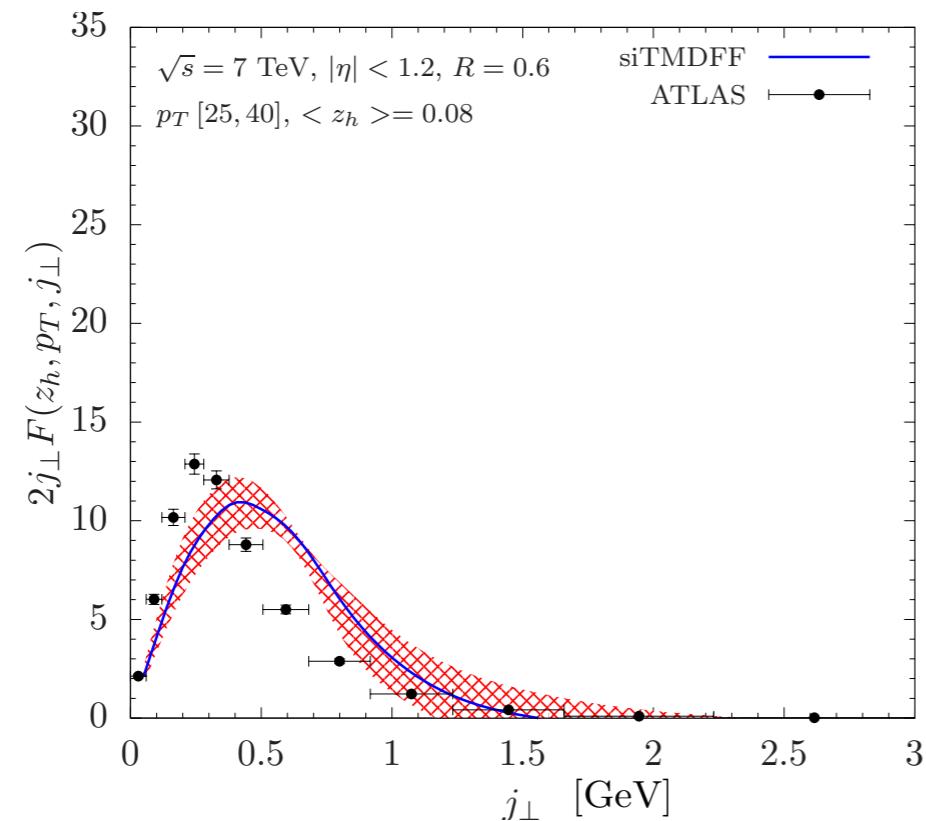
$$S_{\text{pert}}^i(b_*, \mu_J) = \int_{\mu_{b_*}}^{\mu_J} \frac{d\mu'}{\mu'} \left( \Gamma_{\text{cusp}}^i \ln \left( \frac{\mu_J^2}{\mu'^2} \right) + \gamma^i \right)$$

Collins, Soper, Sterman '85

- Non-perturbative input from Sun, Isaacson, Yuan, Yuan '14

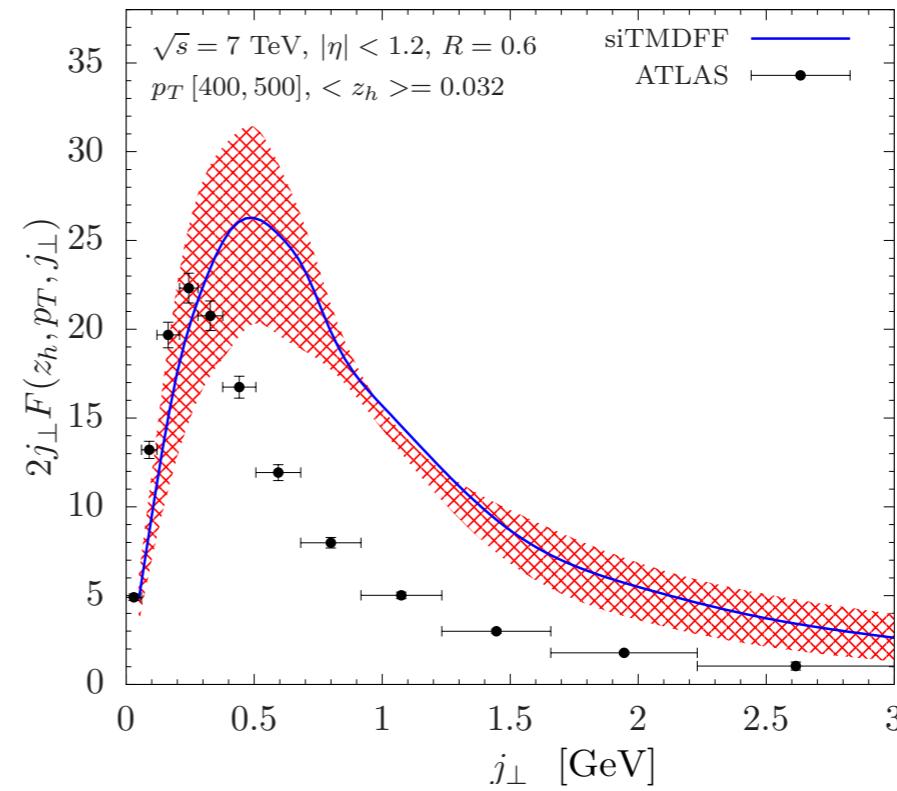
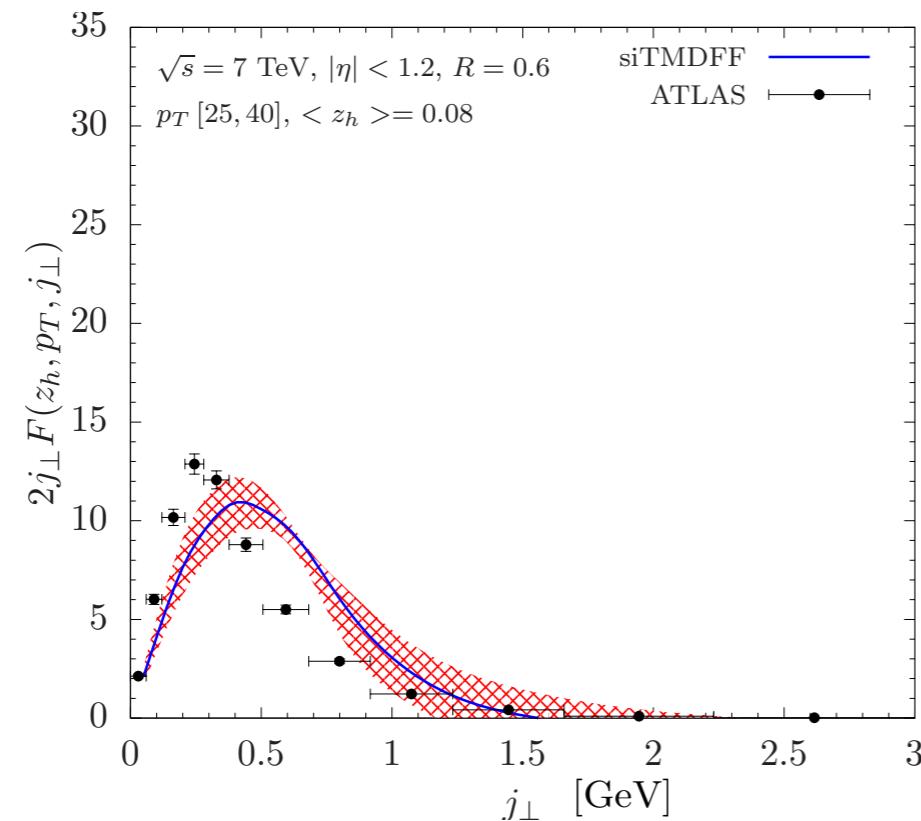


# Comparison to ATLAS data



ATLAS, Eur. Phys. J C71 (2011) 1795

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ATLAS, Eur. Phys. J C71 (2011) 1795

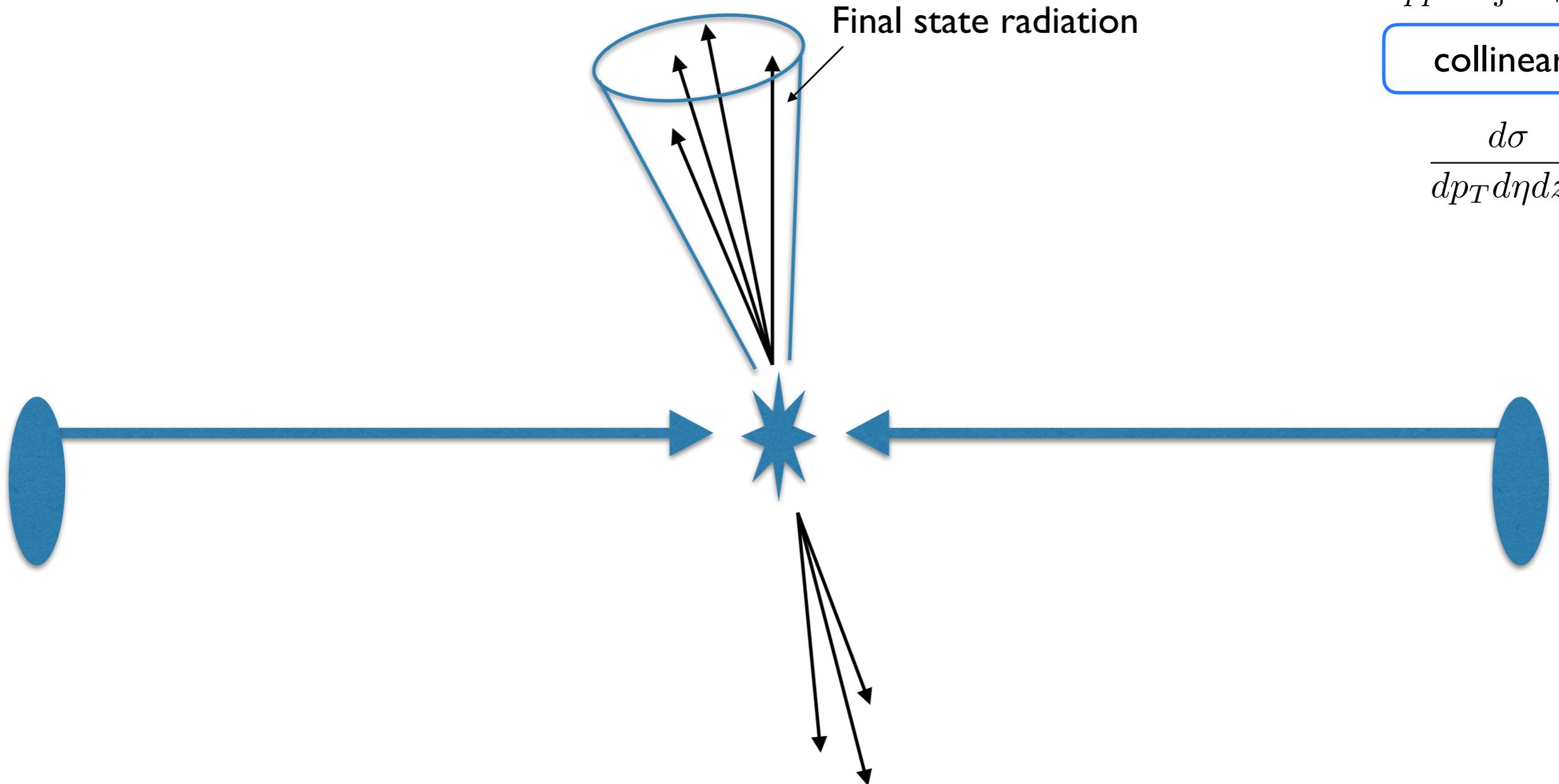
Where is the discrepancy coming from?

- $z_h$  range of the ATLAS data  $0 < z_h < 1$
- Underlying event, initial state radiation  $\rightarrow$  effective grooming  $z_h > z_h^{\text{cut}}$
- Matching at large  $j_\perp$
- NLL  $\rightarrow$  NNLL accuracy see Lee, Liu, Kang, FR '18
- Non-global logarithms
- Fit of the nonperturbative component

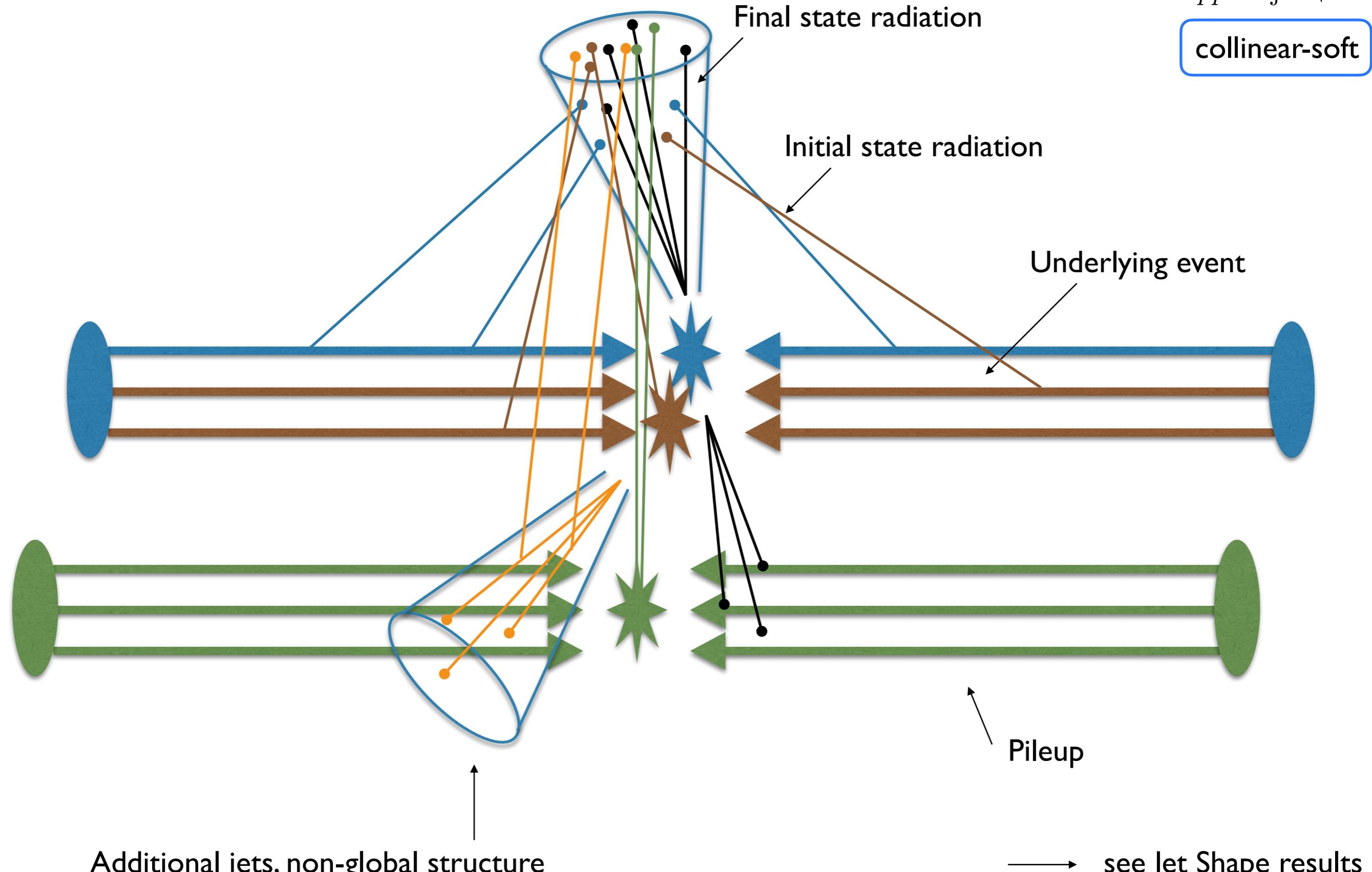
$pp \rightarrow \text{jet} + X$ 

collinear

$$\frac{d\sigma}{dp_T d\eta dz_h}$$

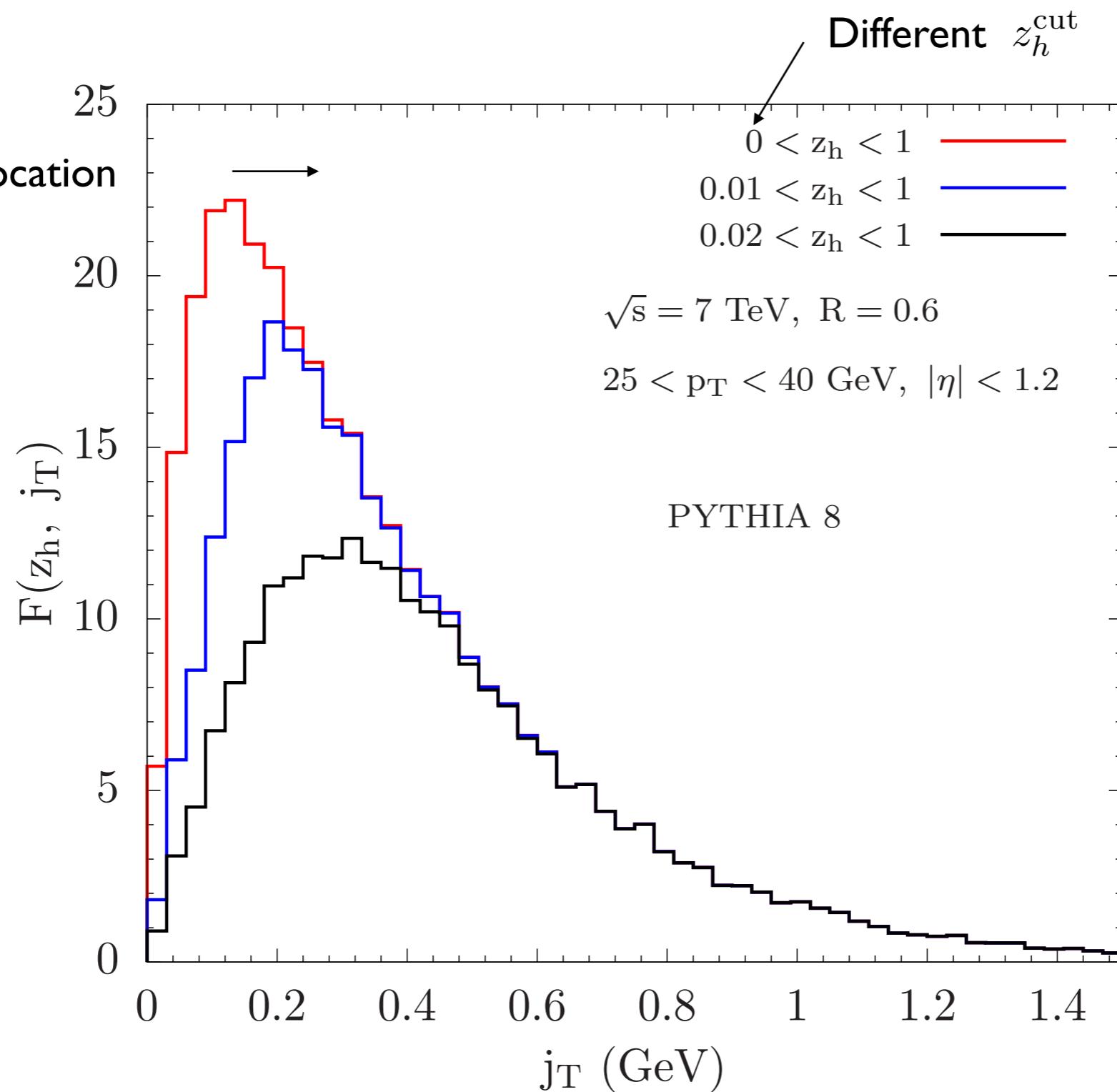


$pp \rightarrow \text{jet} + X$   
collinear-soft



# Dependence on the longitudinal momentum fraction

Shift of the peak location



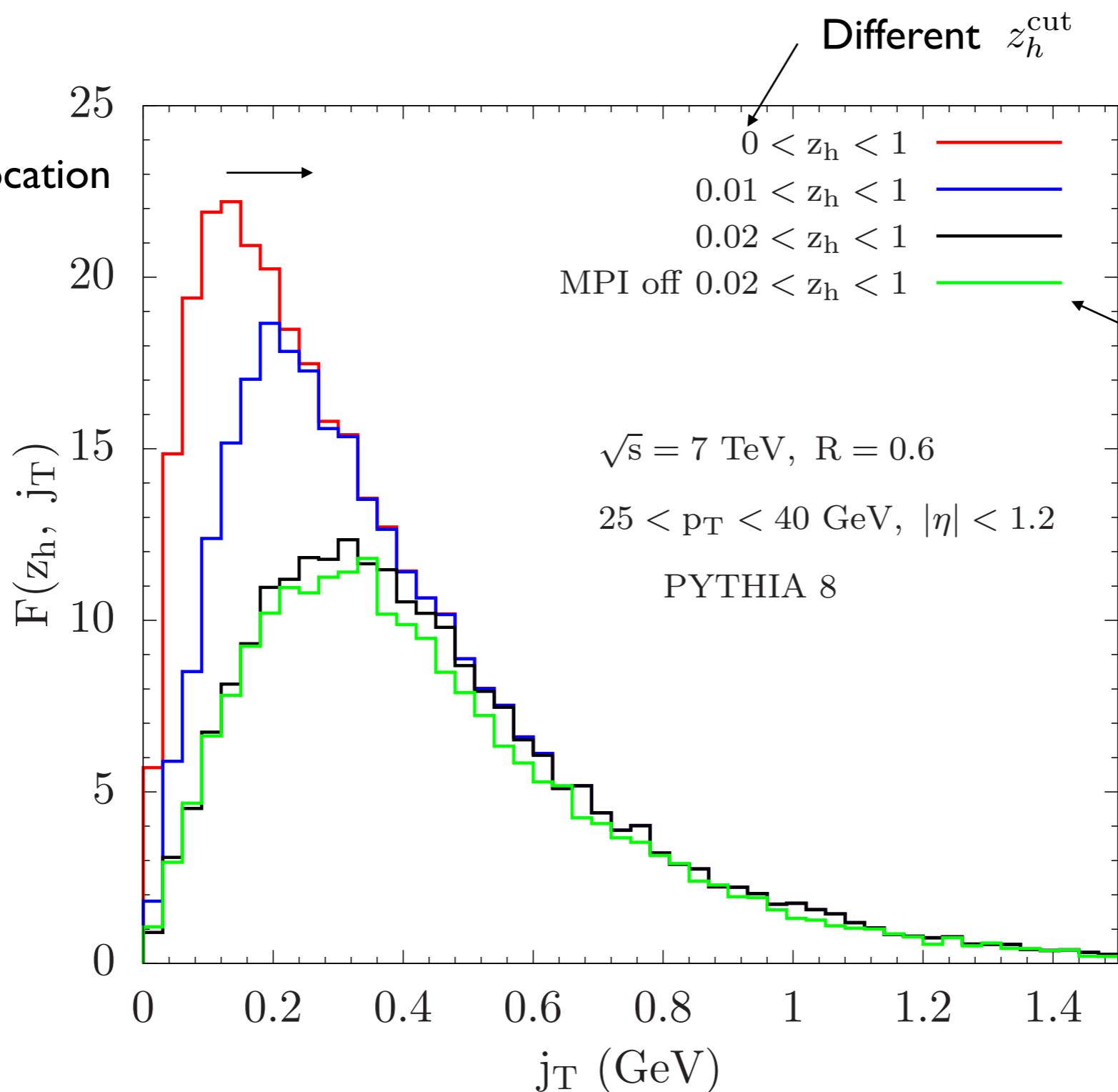
Preliminary

Effectively grooming the jet like trimming

Krohn, Thaler, Wang '10

# Dependence on the longitudinal momentum fraction

Shift of the peak location



Almost no sensitivity  
to the underlying  
event for  
 $z_h^{\text{cut}} = 0.02$

Preliminary

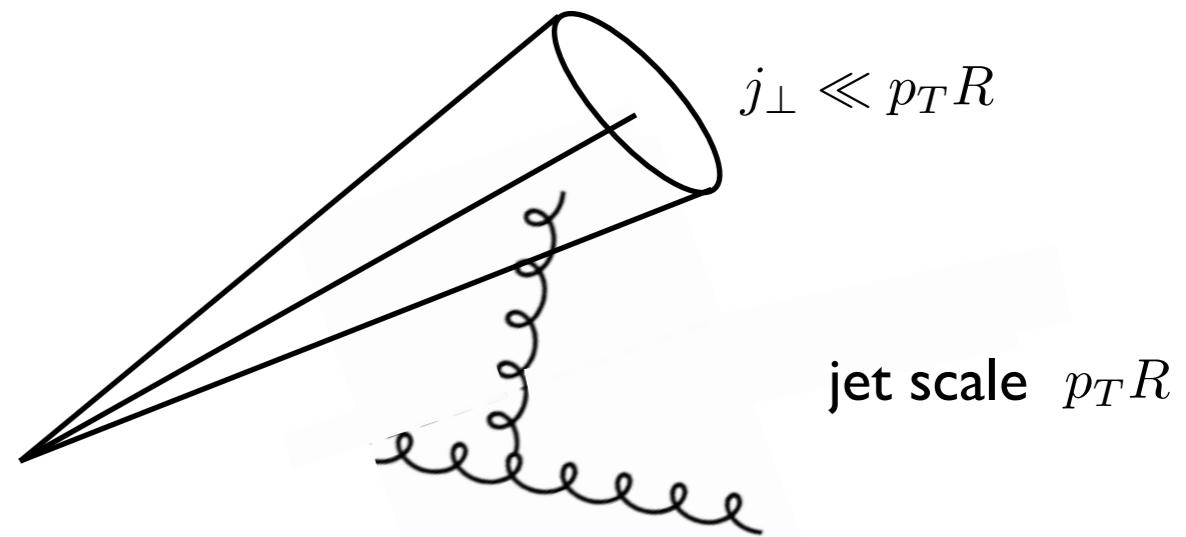
# Non-global logarithms

- $pp \rightarrow \text{jet} + X$  at small jet radii

Banfi, Dasgupta '04

$\alpha_s^2 \ln^2(j_\perp/(p_T R))$  contribution obtained  
in the strongly ordered limit

- Include higher order corrections  $\alpha_s^n \ln^n(j_\perp/(p_T R))$   
Leading logarithmic, leading color accuracy



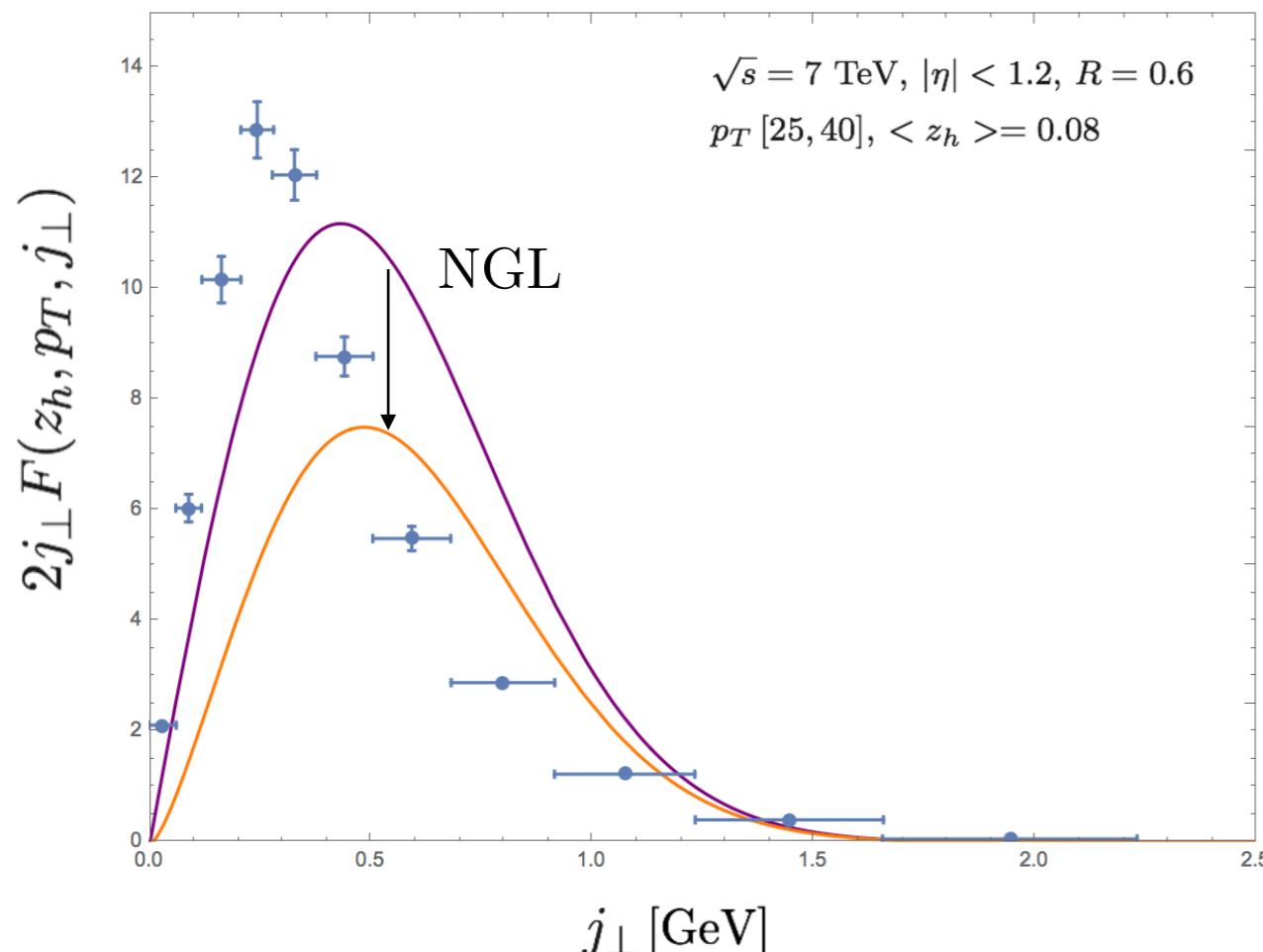
- Monte-Carlo Dasgupta, Salam '01
- BMS equation Banfi, Marchesini, Smye '02
- Fixed order expansions Schwartz, Zhu '14
- Beyond leading color Hatta, Ueda '13

boosted version of the  
 $e^+ e^-$  hemisphere jet mass case  
Dasgupta, Salam '01

$$d\sigma = \sum_{abcd} f_a f_b H_{ab}^c \mathcal{H}_{cd} \hat{\mathcal{D}}_d \times S_{d,\text{NGL}}$$

# Non-global logarithms

ATLAS, Eur. Phys. J C71 (2011) 1795

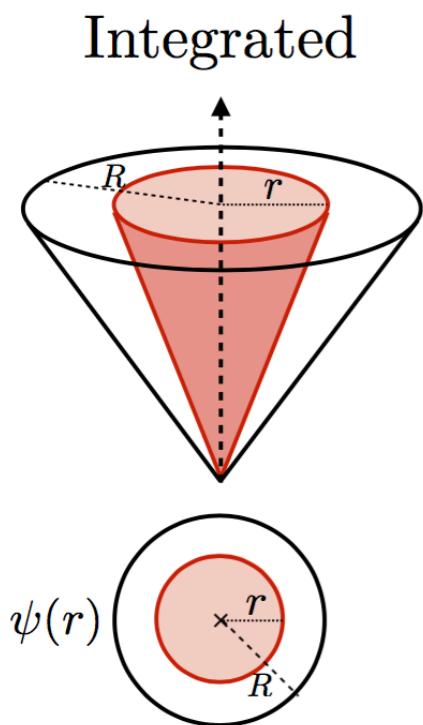


- NGL Monte-Carlo  
Dasgupta, Salam '01

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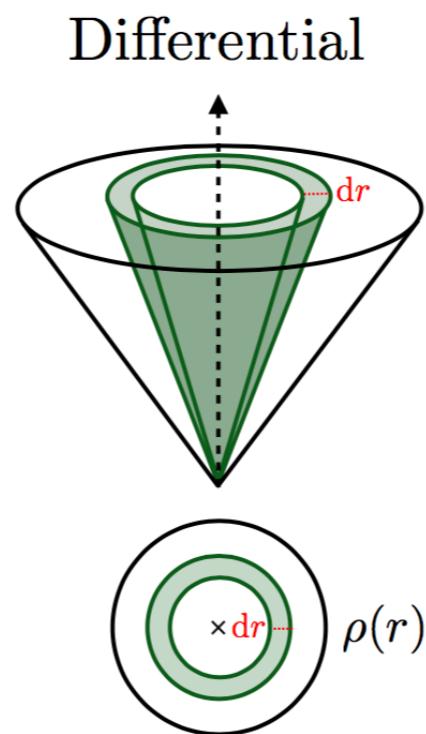
# The jet shape



$R$  = Jet radius  
 $r$  = Subjet radius

$$z_r = \frac{p_T^{\text{subjet}}}{p_T^{\text{jet}}}$$

$$\psi(r) = \frac{\pi r^2}{\pi R^2}$$



$$\psi(r) = \frac{\sum_{r_i < r} p_{Ti}}{\sum_{r_i < R} p_{Ti}}$$

$$\rho(r) = \frac{d\psi(r)}{dr}$$

$$\psi(r) = \int_0^1 dz_r z_r \frac{d\sigma}{dp_T d\eta dz_r}$$



Derive factorization for a central subjet

- Data from LEP, HERA, Tevatron, LHC ...  
 $pp, pp\bar{}, e^+e^-, ep, AA$
- Constrain parton showers
- Quark/gluon discrimination
- BSM searches, heavy flavor

# Factorization

Cal, FR, Waalewijn '19

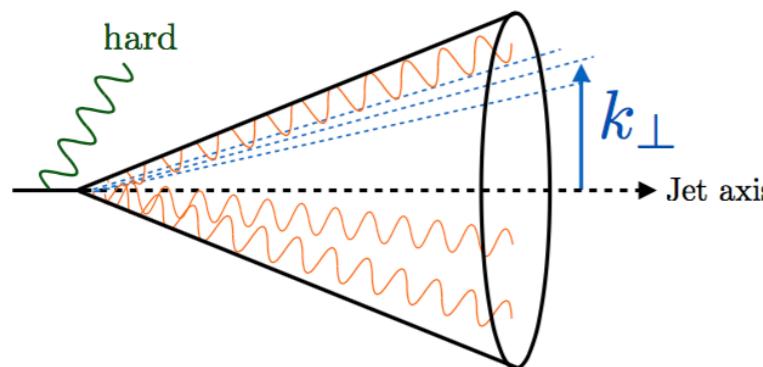
- Relevant jet function

$$\mathcal{G}_c^{\text{jet}}(z, z_r, p_T R, r/R, \mu) \stackrel{\text{NLL}'}{=} \sum_d H_{cd}(z, p_T R, \mu) \int d^2 k_\perp C_d(z_r, p_T r, k_\perp, \mu, \nu) \\ \times S_d^G(k_\perp, \mu, \nu R) S_d^{\text{NG}}\left(\frac{r}{R}\right) \left[1 + \mathcal{O}\left(\frac{r}{R}\right)\right]$$

↓  
Global

↓  
Non Global

← Same as for TMD hadron-in-jet



- First extension beyond LO + LL. Previous work, see

Ellis, Kunszt, Soper '92  
Seymour '98  
Li, Li, Yuan '11  
Chien, Vitev '14

# Factorization

Cal, FR, Waalewijn '19

- Relevant jet function

$$\begin{aligned} \mathcal{G}_c^{\text{jet}}(z, z_r, p_T R, r/R, \mu) &\stackrel{\text{NLL}'}{=} \sum_d H_{cd}(z, p_T R, \mu) \int d^2 k_\perp C_d(z_r, p_T r, k_\perp, \mu, \nu) \\ &\times S_d^G(k_\perp, \mu, \nu R) S_d^{\text{NG}}\left(\frac{r}{R}\right) \left[1 + \mathcal{O}\left(\frac{r}{R}\right)\right] \end{aligned}$$

Global      Non Global

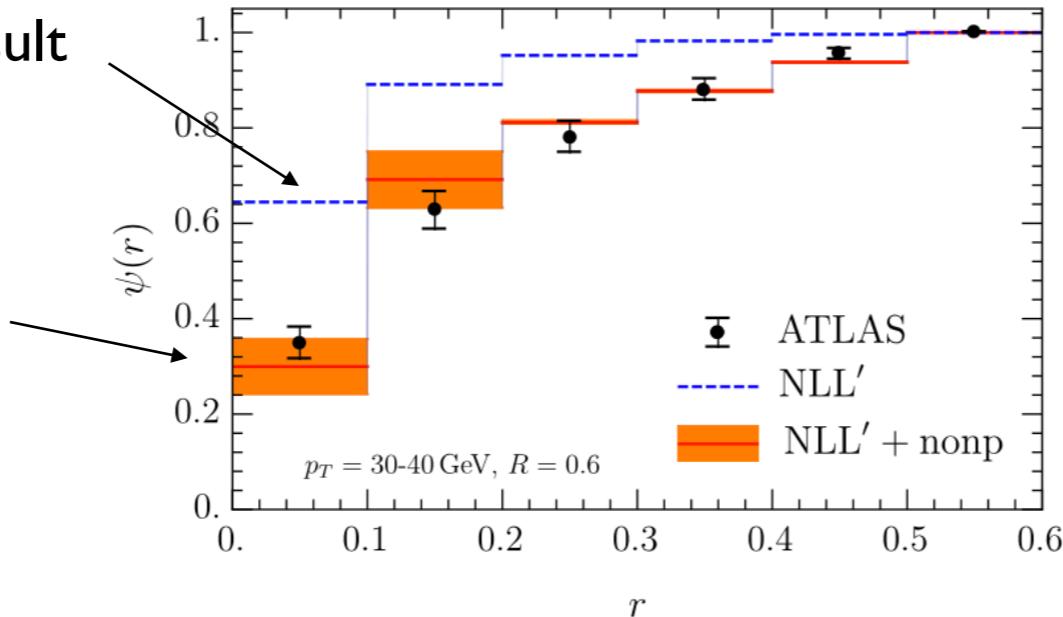
- Collinear function ...

$$\begin{aligned} C_q^{(\theta < r)} &= \frac{\alpha_s C_F}{2\pi^2} \int_0^{2\pi} d\phi \left\{ \delta(1 - z_r) \left[ \frac{1}{\eta} \left( \frac{1}{\epsilon} + L_1 \right) + \frac{1}{\epsilon} \left( L_\nu + \frac{3}{4} \right) + L_\nu L_1 + \frac{3L_1}{4} \right. \right. \\ &\quad \left. \left. - \ln^2(1 - \tilde{\beta}) + 2 \ln \tilde{\beta} \ln(1 - \tilde{\beta}) - \frac{3}{2} \ln \tilde{\beta} + 2 \text{Li}_2(1 - \tilde{\beta}) - \frac{\tilde{\beta}}{2} - \frac{\pi^2}{3} + 2 \right] \right. \\ &\quad \left. + \Theta(z_r > \tilde{\beta}) \left[ -(1 + z_r^2) \left( \frac{\ln(1 - z_r)}{1 - z_r} \right)_+ + \ln \left( \frac{z_r(1 - \tilde{\beta})}{\tilde{\beta}} \right) \frac{1 + z_r^2}{(1 - z_r)}_+ \right] \right. \\ &\quad \left. + \Theta(z_r > 1 - \tilde{\beta}) \left[ \frac{1 + (1 - z_r)^2}{z_r} \ln \left( \frac{z_r \tilde{\beta}}{(1 - z_r)(1 - \tilde{\beta})} \right) \right] \right\}, \\ C_q^{(\theta > r)} &= \frac{\alpha_s C_F}{2\pi^2} \left[ \delta(1 - z_r) \left( \frac{2}{\eta} + 2L_\nu \right) - \frac{1 + z_r^2}{(1 - z_r)_+} - \frac{1 + (1 - z_r)^2}{z_r} \right] \int_{-\phi_{\max}}^{\phi_{\max}} d\phi \ln \left( \frac{\beta_2^{\min}}{\beta_2^{\max}} \right) \end{aligned}$$

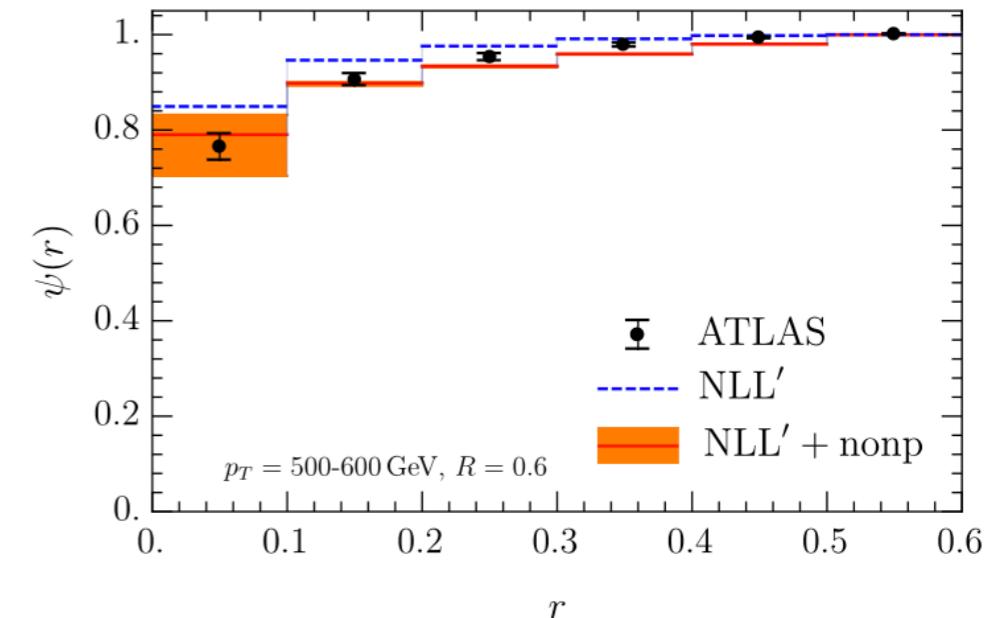
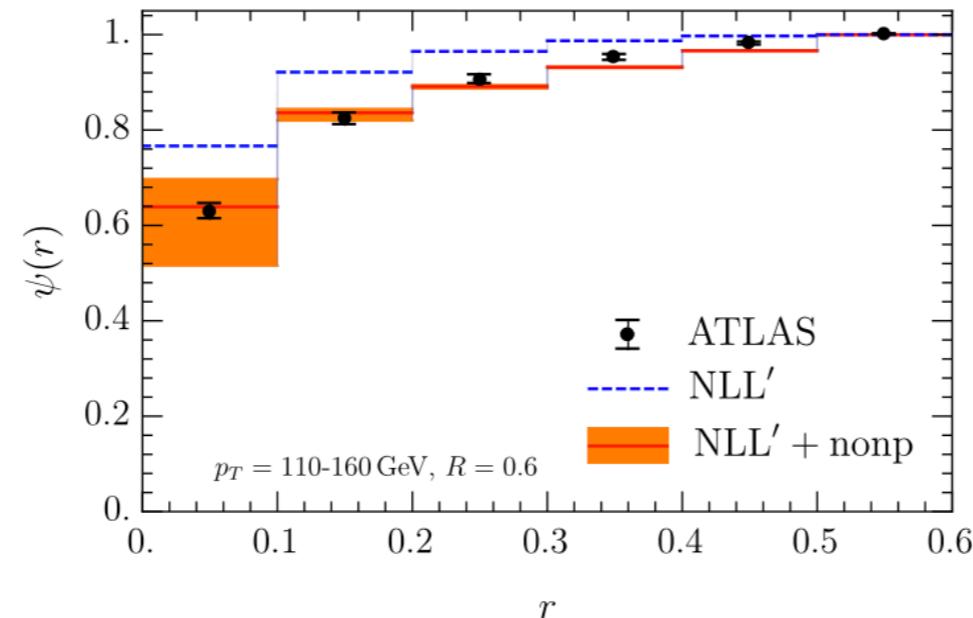
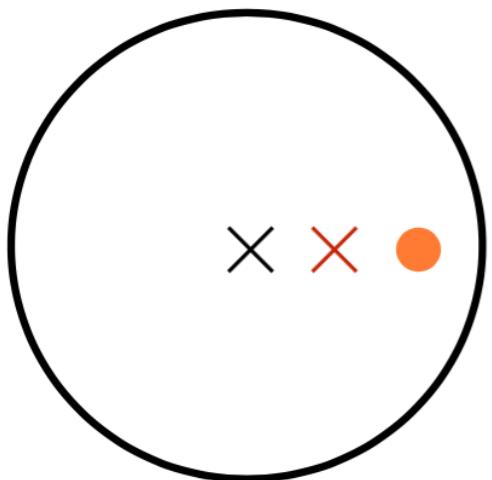
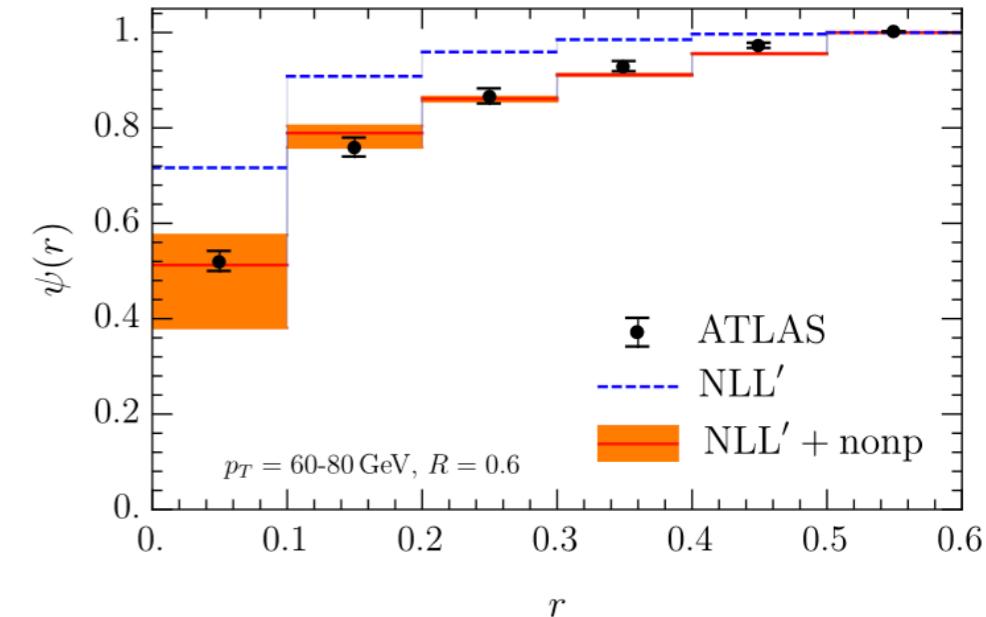
# Comparison to LHC data

Cal, FR, Waalewijn '19

Purely perturbative result



Including a NP model

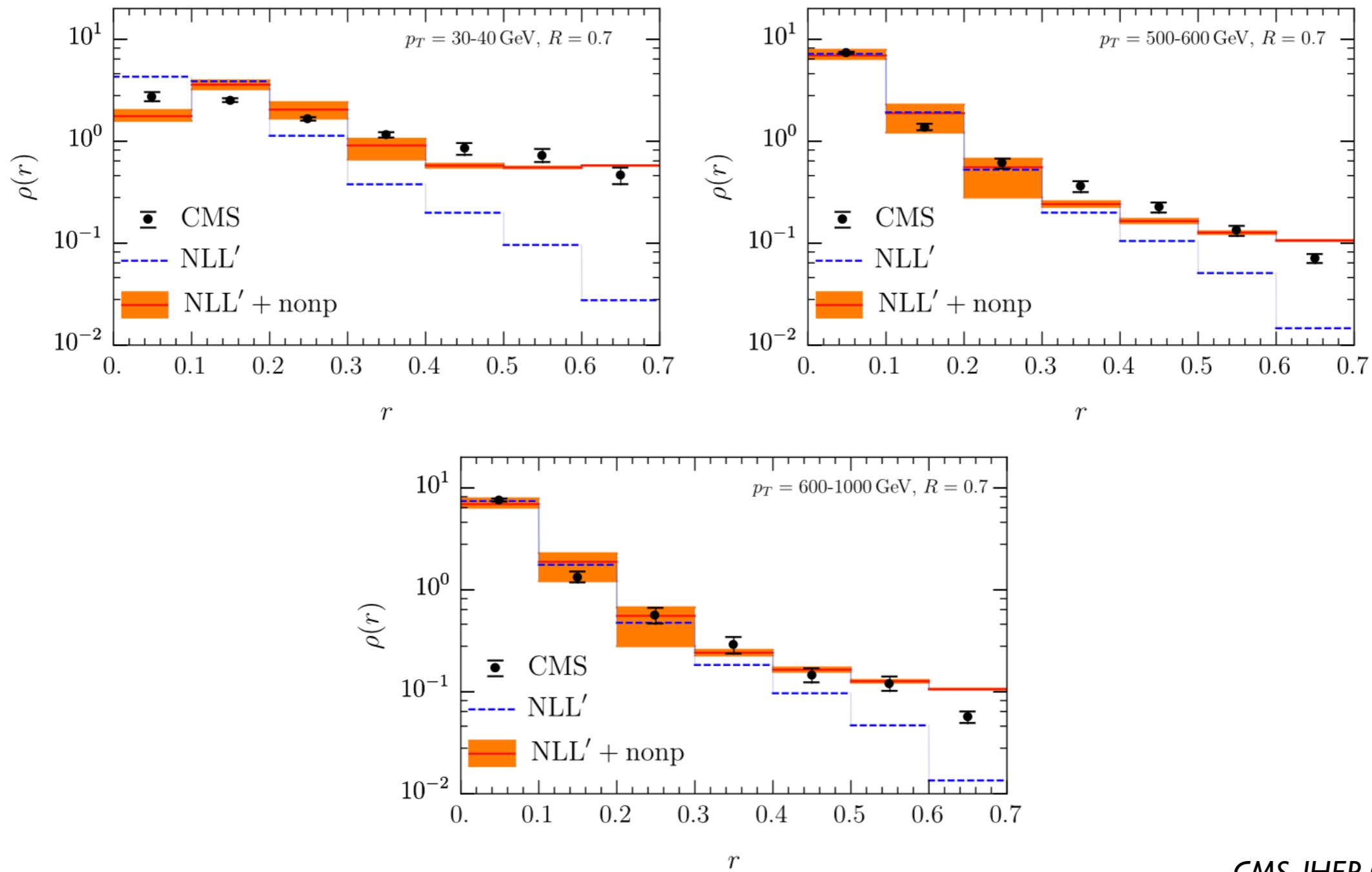


Include axis displacement

ATLAS, Phys. Rev. D 83 (2011) 052003

# Comparison to LHC data

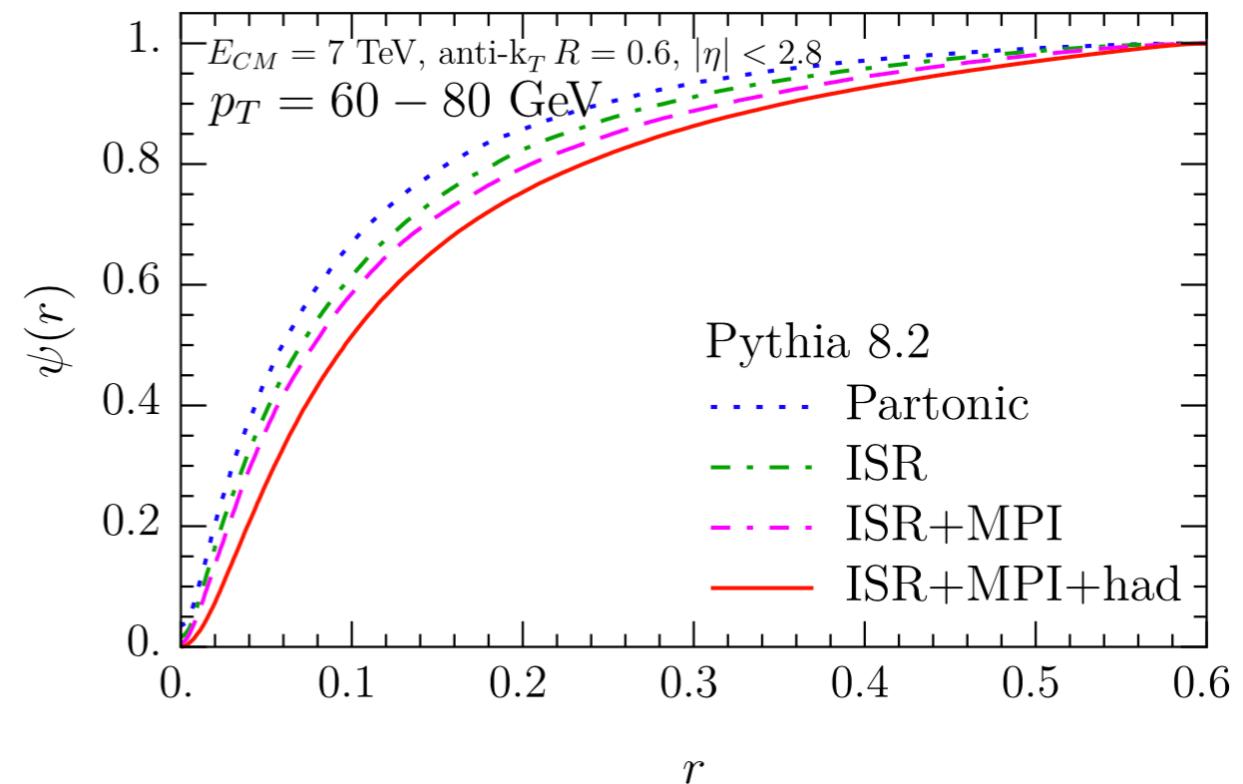
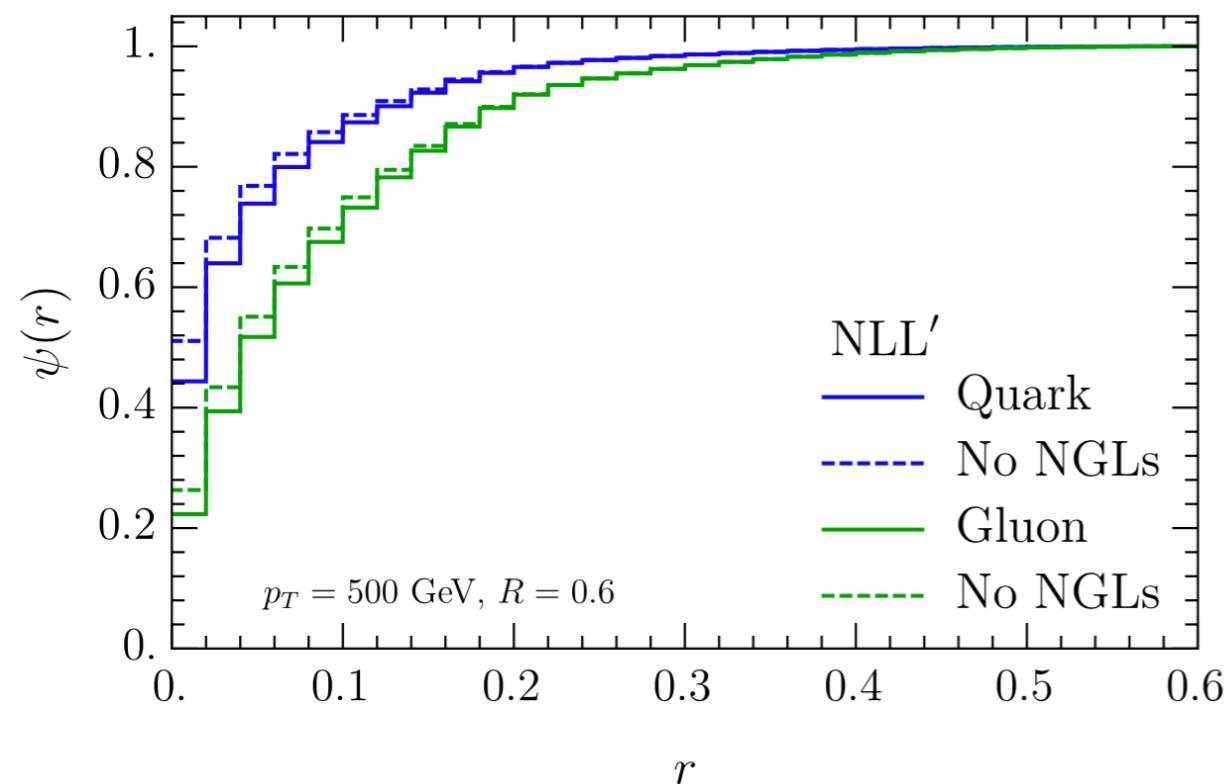
Cal, FR, Waalewijn '19



CMS, JHEP 06 (2012) 160

# Non-global logarithms and Pythia comparisons

Cal, FR, Waalewijn '19



- Relatively small hierarchy of  $r, R$
- Initial State Radiation
- Multi Parton Interactions/underlying event
- Hadronization

# Outline

- Introduction
- Collinear FFs in jets
- TMD FFs in jets
- The jet shape
- Conclusions

# Conclusions

- Longitudinal and transverse energy distribution of jets
- More work needed for one-to-one comparison
- Non-global logarithms
- Collins asymmetries in jets

