Challenges and recent progress in SIDIS

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ODU/JLab

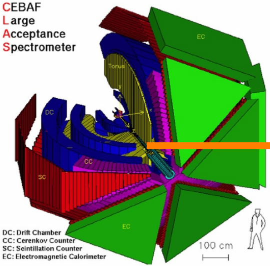
Workshop on Novel Probes of the
Nucleon Structure in SIDIS, e+e- and pp (FF2019)
Duke University
Durham, 2019

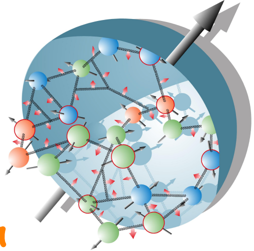
Outline

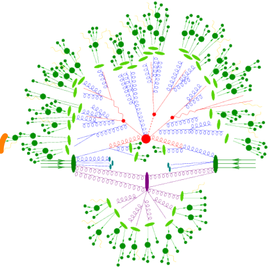
- **Recent progress in SIDIS large p_T**
  - Gonzalez-Hernandez, Rogers, NS, Wang
    (PRD98 **2018**)
  - Wang, Gonzalez-Hernandez, Rogers, NS
    (arXiv:1903.01529 **2019**)

- **New developments to identify SIDIS regions**
  - Boglione, Collins, Gamberg, Gonzalez-Hernandez, Rogers, NS
    (PLB 766 **2017**)
  - Boglione, Gamberg, Gordon, Gonzalez-Hernandez, Prokudin, Rogers, NS
    (in preparation)

Zooming in at the femtometer scale using JLab12







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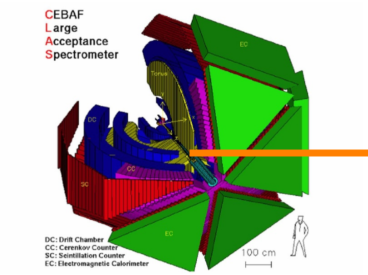
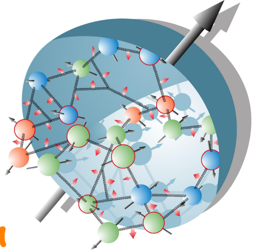
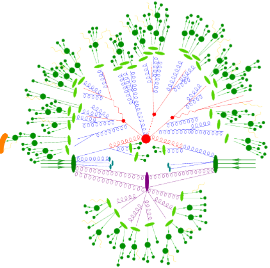


QED
corrections





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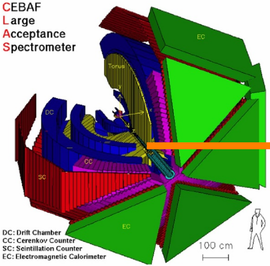
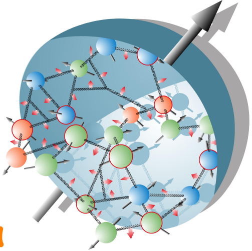
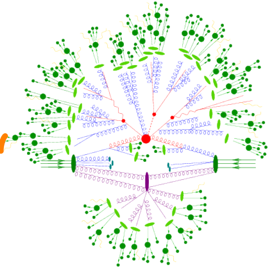


QED corrections

QCD factorization





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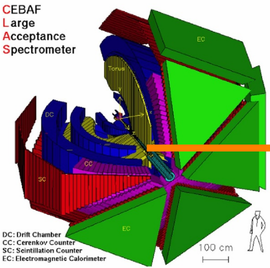
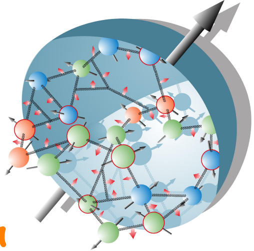
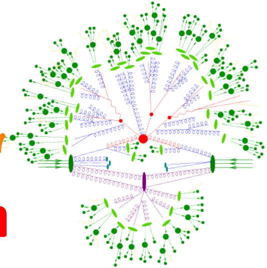


QED corrections

QCD factorization

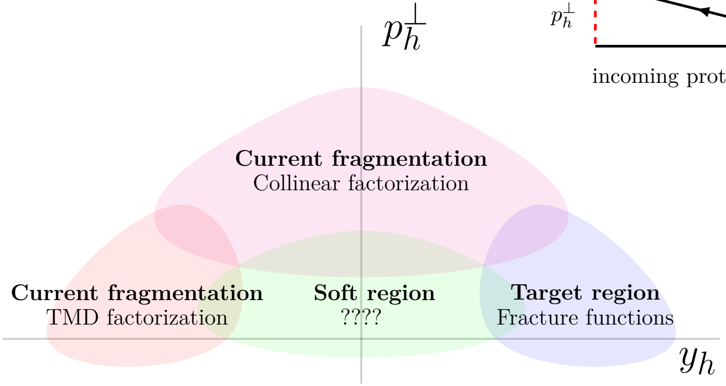




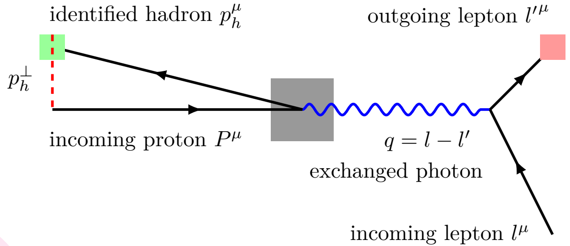
The inverse problem

Recent progress in SIDIS large p_T

SIDIS regions

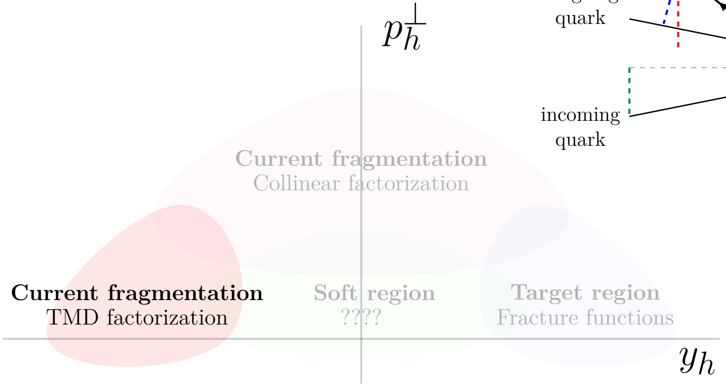


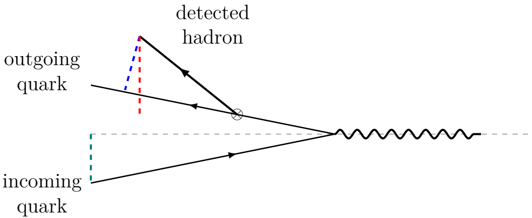
Breit frame



SIDIS regions

small transverse momentum

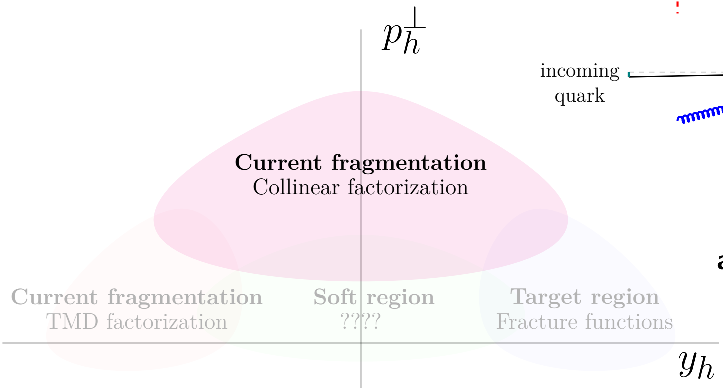


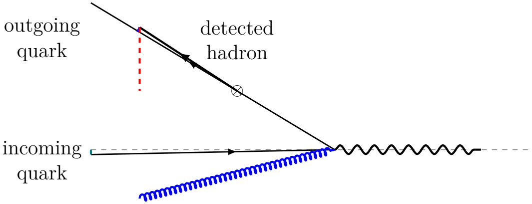


aka W

SIDIS regions

large transverse momentum



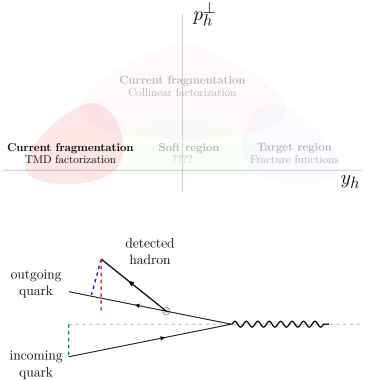


aka FO **(=fixed order)**

SIDIS regions

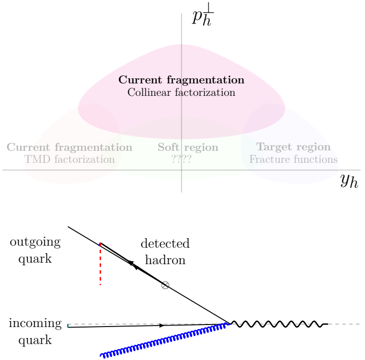
small transverse momentum

W



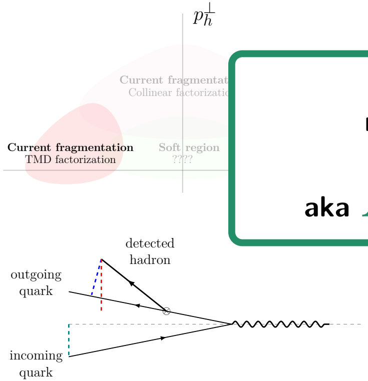
large transverse momentum

FO

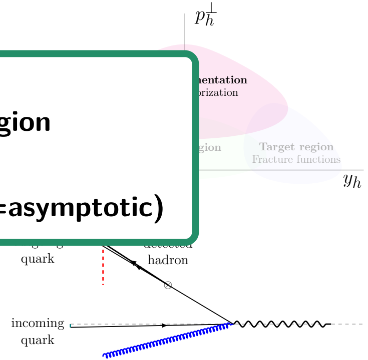


SIDIS regions

small transverse momentum W



large transverse momentum FO



matching region

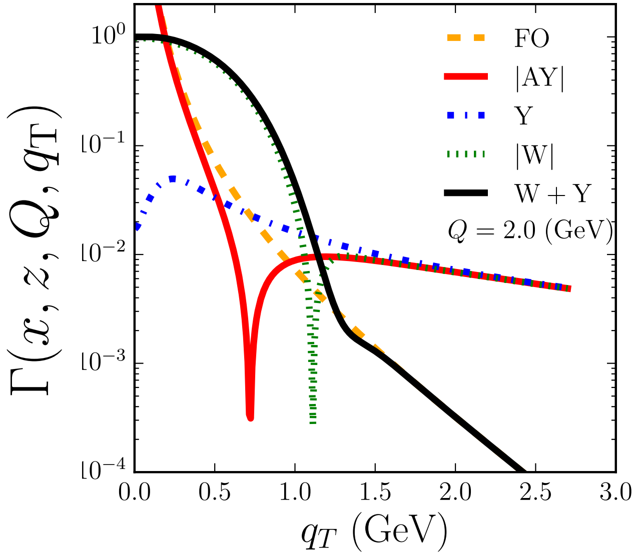
aka ASY **(=asymptotic)**

SIDIS regions

$$\frac{d\sigma}{dx\,dQ^2\,dz\,dp_h^{\perp}} = \mathrm{W} + \mathrm{FO} - \mathrm{ASY} + \mathcal{O}(m^2/Q^2)$$
$$\sim \mathrm{W} \quad \text{for } q_{\mathrm{T}} \ll Q$$
$$\sim \mathrm{FO} \quad \text{for } q_{\mathrm{T}} \sim Q$$

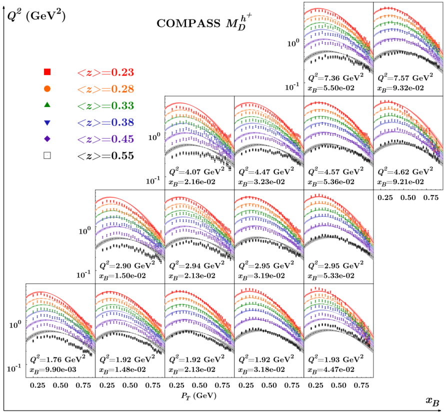
$q_{\mathrm{T}}/Q = (p_h^{\perp}/z)/Q \to$ scale separation

Toy example

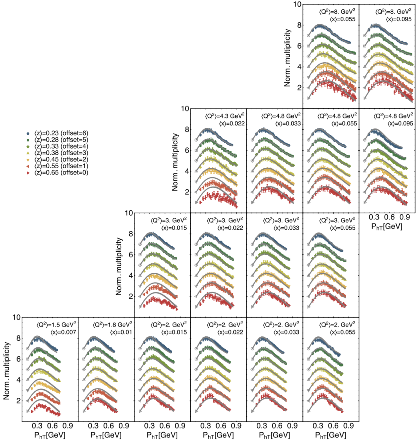


10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0.0
0.5
1.0
1.5
2.0
2.5
3.0
q_T (GeV)
$\Gamma(x, z, Q, q_T)$
FO
|AY|
Y
|W|
W + Y
$Q = 2.0$ (GeV)

Existing phenomenology



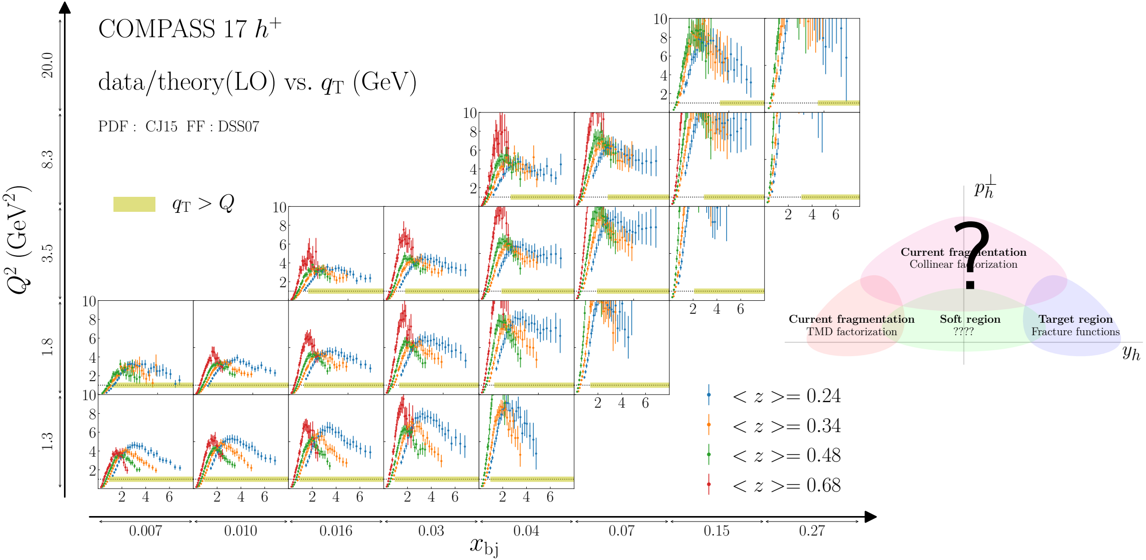
Anselmino et al



Bacchetta et al

- These analyses used only W (Gaussian, CSS) $\rightarrow$ no FO nor ASY
- Samples with $q_\mathrm{T}/Q \sim 1.63$ have been included
- **BUT TMDs are only valid for $q_\mathrm{T}/Q \ll 1$!**

FO @ LO predictions (DSS07) Gonzalez, Rogers, NS, Wang PRD98 (2018)

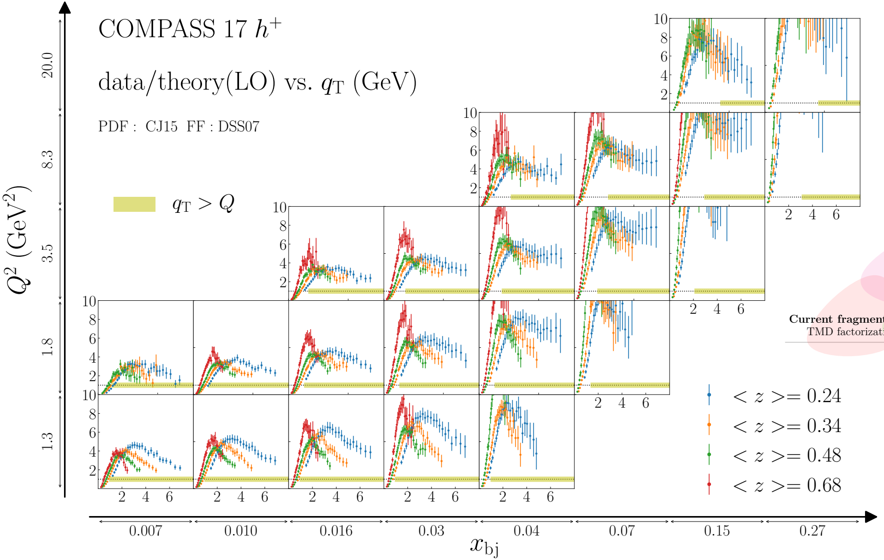


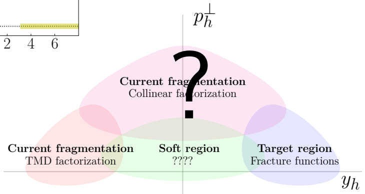
Trouble with large transverse momentum

$$\mathrm{FO} = \sum_q e_q^2 \int_{\frac{q_\mathrm{T}^2}{Q^2}\frac{xz}{1-z}+x}^{1} \frac{d\xi}{\xi - x}\; H(\xi)\; \boldsymbol{f}_q(\xi,\mu)\; \boldsymbol{d}_q(\zeta(\xi),\mu) + O(\alpha_S^2) + O(m^2/q^2)$$

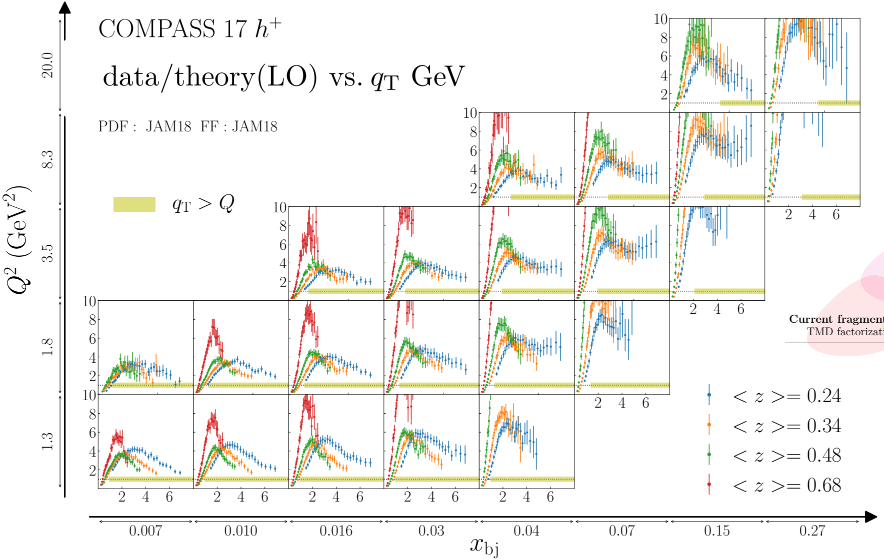
+ FFs needs to be updated?

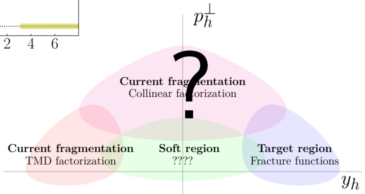
FO @ LO predictions (DSS07) Gonzalez, Rogers, NS, Wang PRD98 (2018)





FO @ LO predictions (JAM18) Gonzalez, Rogers, NS, Wang PRD98 (2018)



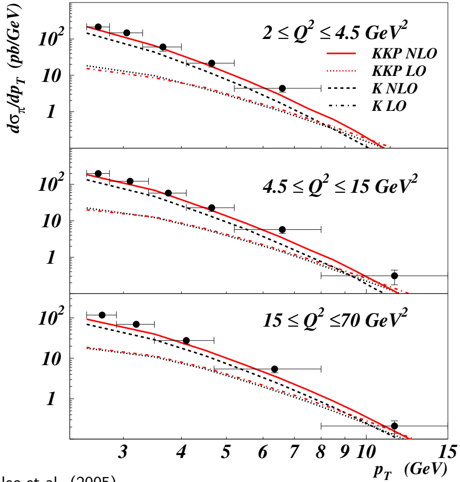


Trouble with large transverse momentum

$$\textcolor{red}{\mathrm{FO}} = \sum_q e_q^2 \int_{\frac{q_\mathrm{T}^2}{Q^2}\frac{xz}{1-z}+x}^1 \frac{d\xi}{\xi - x}\, H(\xi)\, \boldsymbol{f}_q(\xi,\mu)\, \boldsymbol{d}_q(\zeta(\xi),\mu) + O(\alpha_S^2) + O(m^2/q^2)$$

+ $O(\alpha_S^2)$ corrections might be important

order α_S^2 corrections to FO



Daleo,et al. (2005)
PRD.71.034013

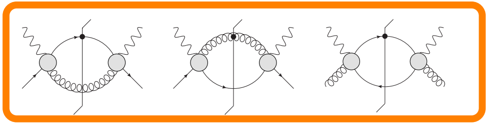
- There are strong indications that order α_S^2 corrections are very important
- An order of magnitude correction at small p_T.
- As a sanity check, we need to have an independent calculation

$O(\alpha_S^2)$ calculation (Wang, Gonzalez-Hernandes, Rogers, NS - arXiv:1903.01529)

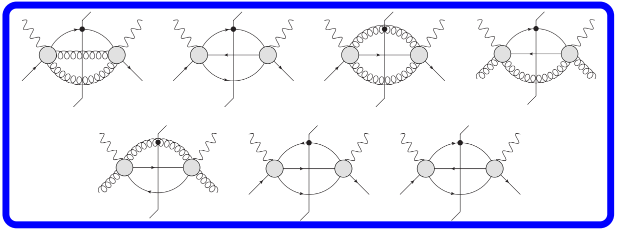
$$W^{\mu\nu}(P,q,P_H) = \int_{x-}^{1+} \frac{d\xi}{\xi} \int_{z-}^{1+} \frac{d\zeta}{\zeta^2} \hat{W}_{ij}^{\mu\nu}(q, x/\xi, z/\zeta) f_{i/P}(\xi) d_{H/j}(\zeta)$$

$$\{\mathrm{P}_g^{\mu\nu} \hat{W}_{\mu\nu}^{(N)}; \mathrm{P}_{PP}^{\mu\nu} \hat{W}_{\mu\nu}^{(N)}\} \equiv \frac{1}{(2\pi)^4} \int \{|M_g^{2\to N}|^2; |M_{pp}^{2\to N}|^2\} \, \mathrm{d}\Pi^{(N)} - \text{Subtractions}$$

Born/Virtual

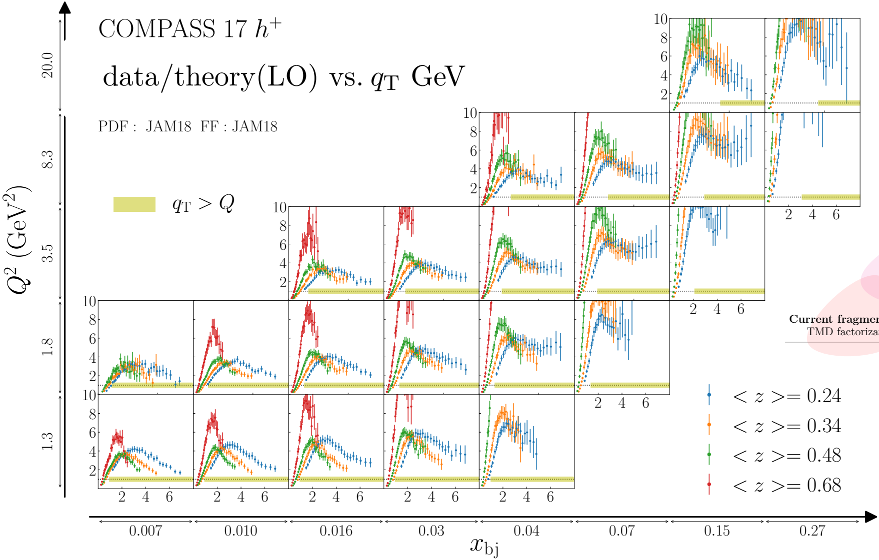


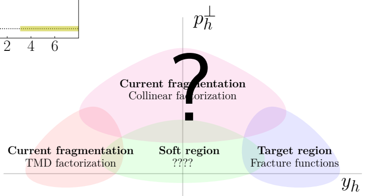
Real



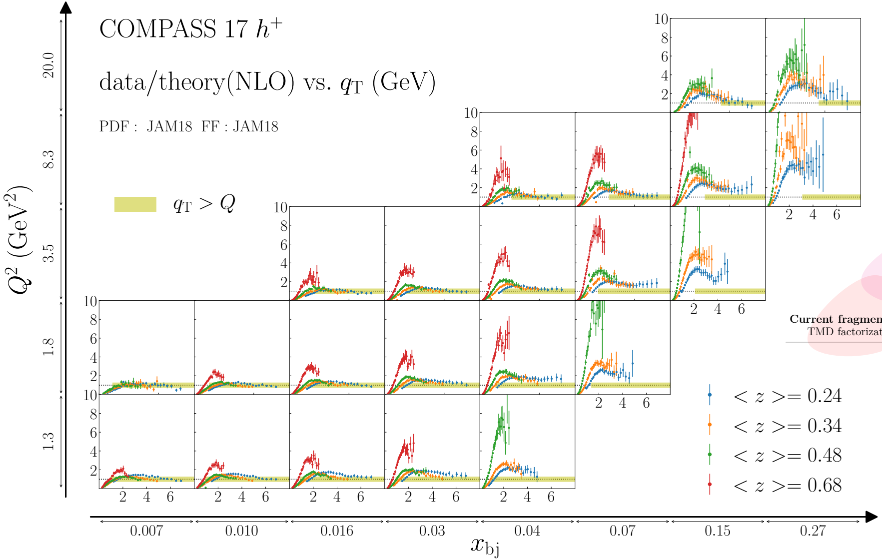
- ✓ Generate all $2 \to 2$ and $2 \to 3$ squared amplitudes
- ✓ Evaluate $2 \to 2$ virtual graphs (Passarino-Veltman)
- ✓ Integrate 3-body PS analytically
- ✓ Check cancellation of IR poles

FO @ LO predictions (JAM18)

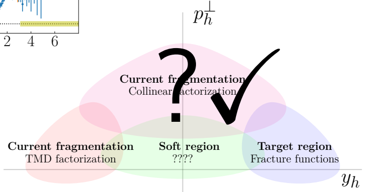




FO @ NLO (JAM18)

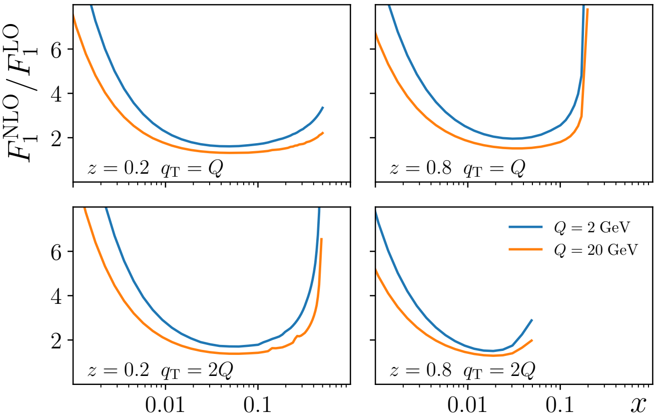


COMPASS 17 h^+
data/theory(NLO) vs. q_T (GeV)
PDF : JAM18 FF : JAM18
$q_T > Q$
Q^2 (GeV^2)
20.0
8.3
3.5
1.8
1.3
x_{bj}
0.007
0.010
0.016
0.03
0.04
0.07
0.15
0.27
$< z >= 0.24$
$< z >= 0.34$
$< z >= 0.48$
$< z >= 0.68$



$p_h^\perp$
Current fragmentation
Collinear factorization
Current fragmentation
TMD factorization
Soft region
????
Target region
Fracture functions
y_h
?
✓

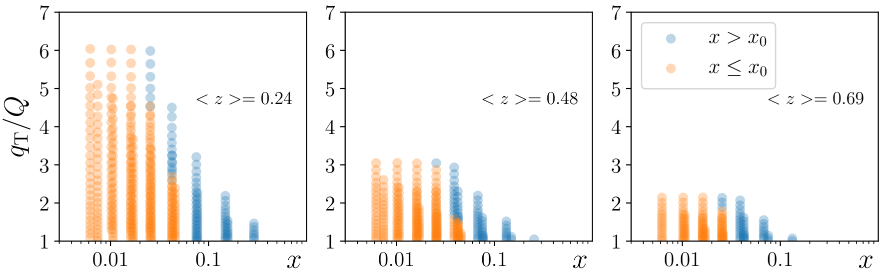
Understanding the large x (Wang, Gonzalez-Hernandes, Rogers, NS - arXiv:1903.01529)



- Large corrections threshold corrections are observed
- The x at the minimum can be used as an indicator of where such corrections are expected to be large

Understanding the large x (Wang, Gonzalez-Hernandes, Rogers, NS - arXiv:1903.01529)

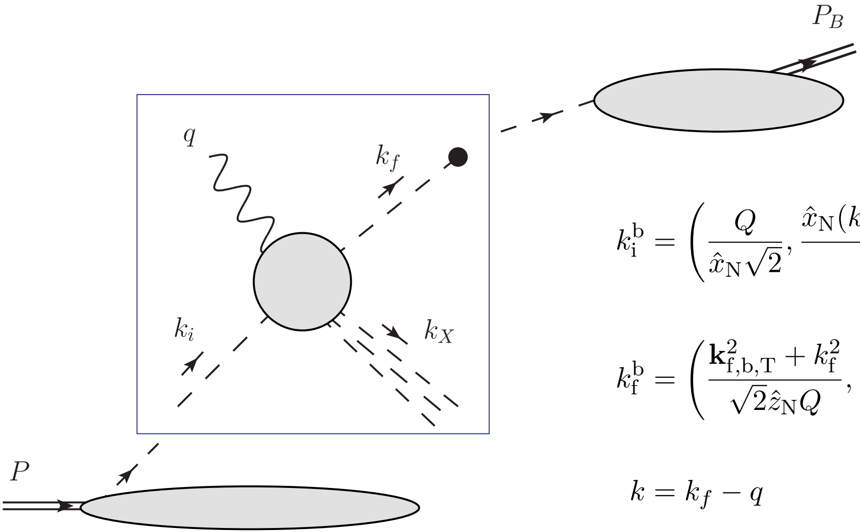
COMPASS kinematics



- The blue region might receive large threshold corrections
- This can potential explain why the $O(\alpha_S^2)$ fail to describe the data at large x

New developments to identify SIDIS regions

SIDIS region indicators

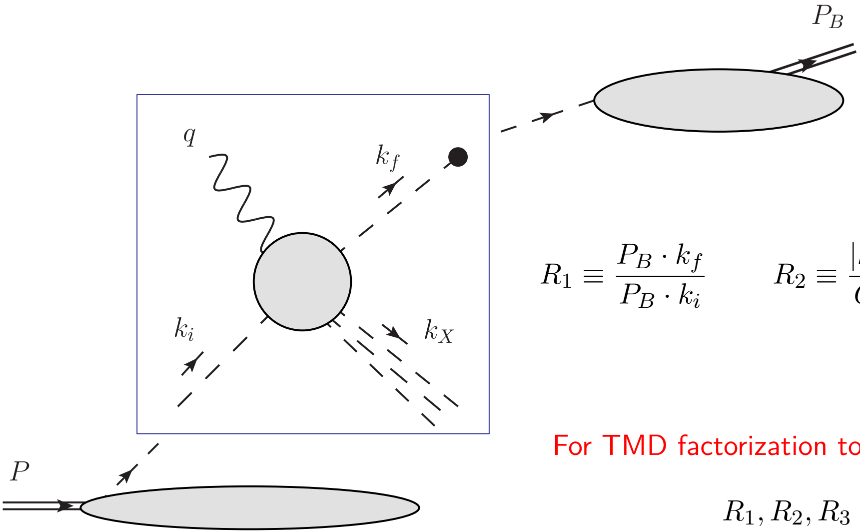


$$k_{\mathrm{i}}^{\mathrm{b}} = \left(\frac{Q}{\hat{x}_{\mathrm{N}}\sqrt{2}}, \frac{\hat{x}_{\mathrm{N}}(k_{\mathrm{i}}^2 + \mathbf{k}_{\mathrm{i,b,T}}^2)}{\sqrt{2}Q}, \mathbf{k}_{\mathrm{i,b,T}} \right)$$

$$k_{\mathrm{f}}^{\mathrm{b}} = \left(\frac{\mathbf{k}_{\mathrm{f,b,T}}^2 + k_{\mathrm{f}}^2}{\sqrt{2}\hat{z}_{\mathrm{N}}Q}, \frac{\hat{z}_{\mathrm{N}}Q}{\sqrt{2}}, \mathbf{k}_{\mathrm{f,b,T}} \right)$$

$$k = k_f - q$$

SIDIS region indicators



$$R_1 \equiv \frac{P_B \cdot k_f}{P_B \cdot k_i} \qquad R_2 \equiv \frac{|k^2|}{Q^2} \qquad R_3 \equiv \frac{|k_X^2|}{Q^2}$$

For TMD factorization to hold one needs

$$R_1, R_2, R_3 \ll 1$$

SIDIS region indicators

- **Web app is available**
  - https://sidis.herokuapp.com/
  - use chrom (slow in safary)
  - feedback/questions are welcomed
  - it might take few seconds to load be patient

SIDIS regions analysis tool

About: Numerical evaluation of ratios described at arxiv:...

Select available apps below:

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app1(3D): R_i vs. (x_b, z_h)

app2(3D): W2_(SIDIS) vs. (x_b, z_h)

app3(3D): y_h vs. (xb, zh)

app4(2D): W2_SIDIS vs. (x_b, Q)

app5(2D): x_N/x_bj vs. x_b

app6(2D): z_N/z_h vs. z_h

app7(2D): R_i vs. (x_b, Q)

app8(2D): rat_exp vs. (x_b, Q)

app9(2D): qT/Q vs. rap

Authors:

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- S. Gordon (ODU)
- T. Rogers (ODU)
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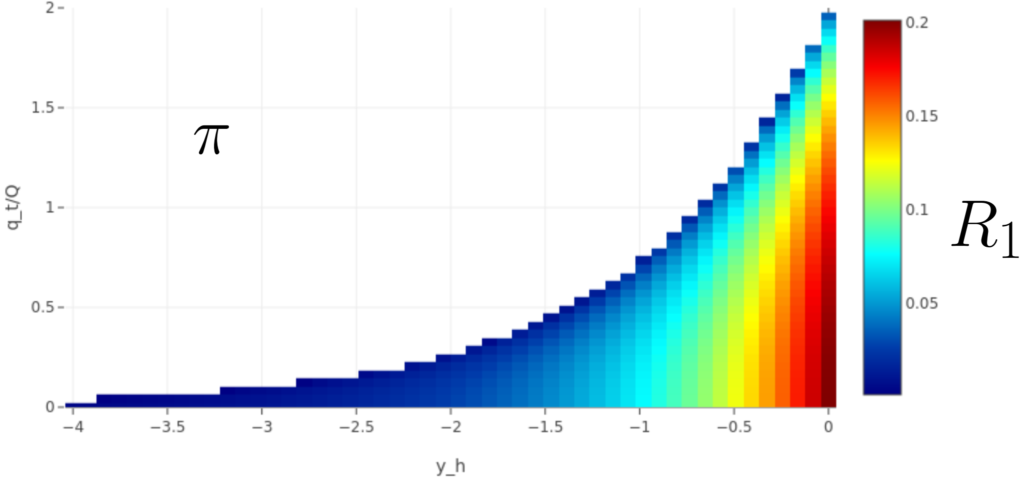
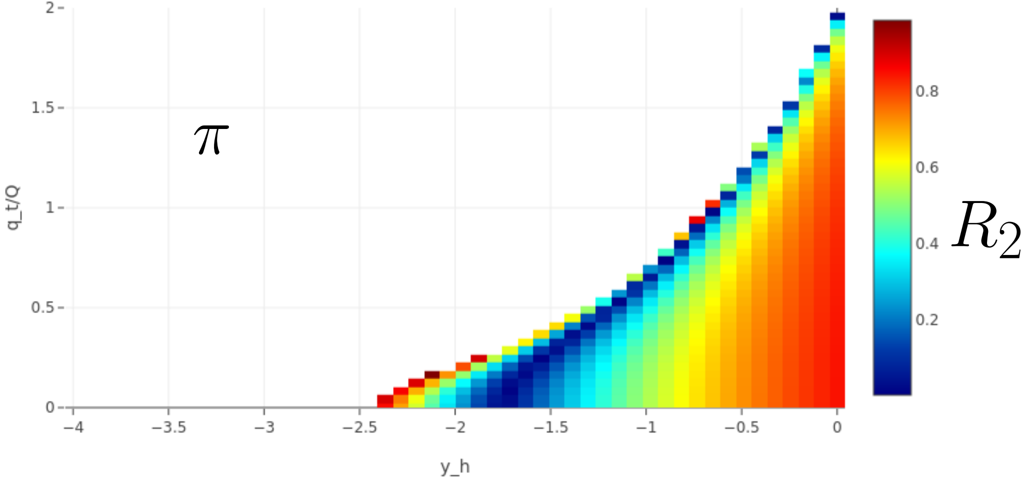
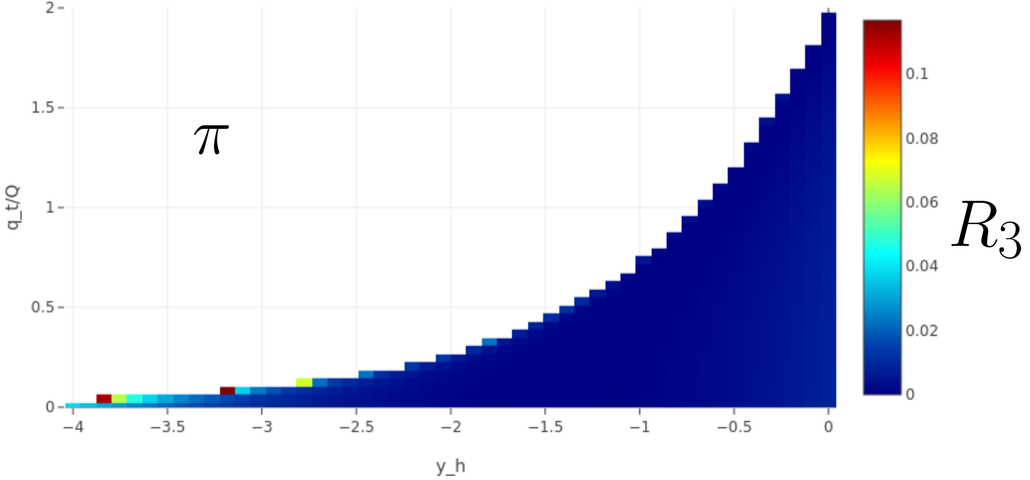
SIDIS region indicators - example



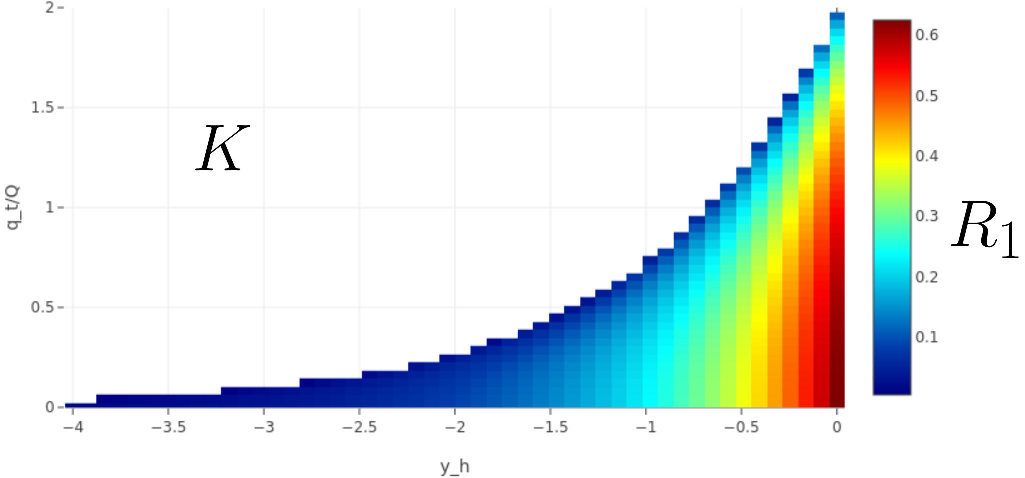
SIDIS region indicators - example



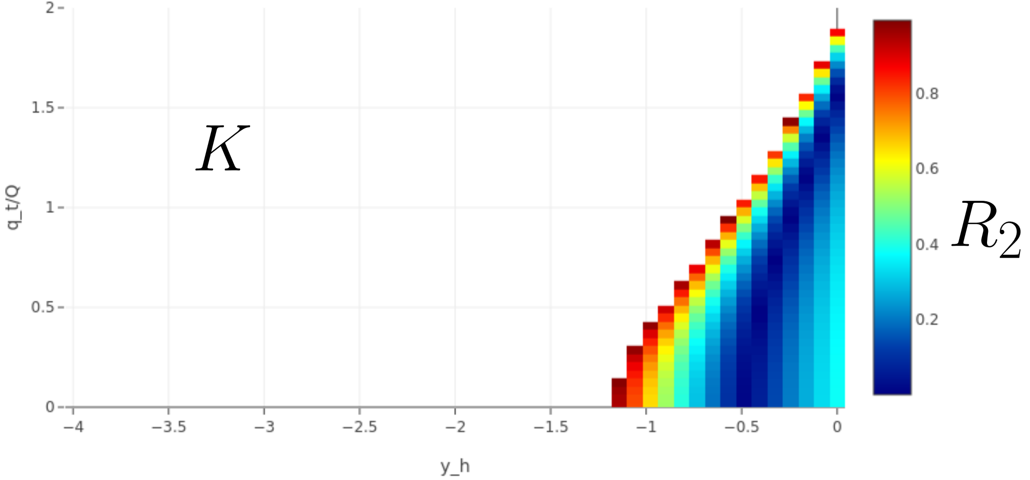
SIDIS region indicators - example



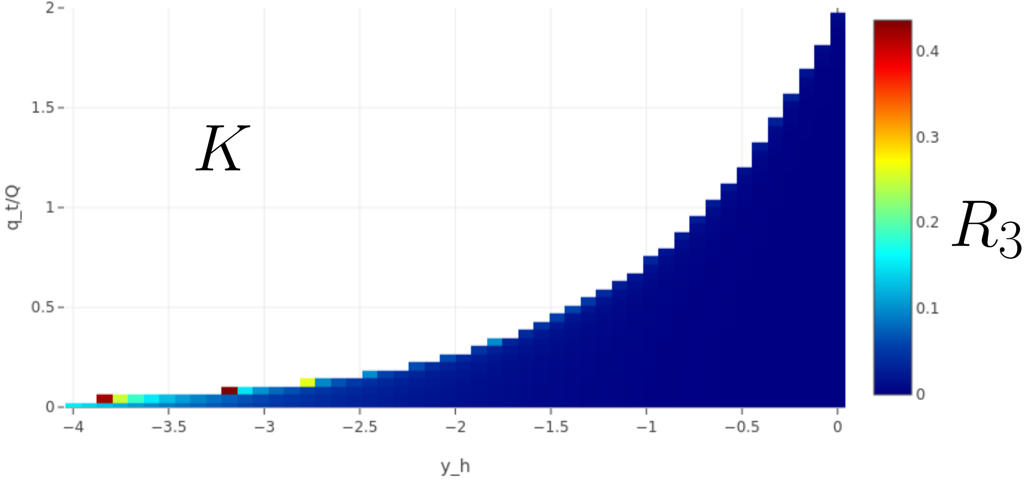
SIDIS region indicators - example



SIDIS region indicators - example



SIDIS region indicators - example



Using the ratios in pheno

- **Recall the Bayesian regression paradigm**

$$\mathcal{P}(\mathbf{a}|\text{data}) = \mathcal{L}(\mathbf{a}, \text{data})\pi(\mathbf{a})$$

$$\mathrm{E}[\mathcal{O}] = \int d^n a \; \mathcal{P}(\mathbf{a}|\text{data})\mathcal{O}(\mathbf{a})\,,$$

$$\mathrm{V}[\mathcal{O}] = \int d^n a \; \mathcal{P}(\mathbf{a}|\text{data})\left(\mathcal{O}(\mathbf{a}) - \mathrm{E}[\mathcal{O}]\right)^2$$

- **The likelihood**

$$\mathcal{L}(\mathbf{a}, \text{data}) = \exp\left(-\frac{1}{2}\sum_i \left(\frac{\text{data}_i - \text{theory}_\text{i}(\mathbf{a})}{\delta\text{data}_i}\right)^2\right)$$

- **The priors**

$$\pi(\mathbf{a}) \propto \Pi_i \theta(a_i^{\text{min}} < a_i < a_i^{\text{max}})$$

Using the ratios in pheno

- **IDEA: use R_i as priors**

$$\pi(R_k) \propto \exp\left(-|R_k|^p\right)$$

- **The full prior becomes**

$$\pi(\mathbf{a}) \propto \Pi_i \theta(a_i^{\min} < a_i < a_i^{\max}) \times \Pi_j \exp\left(-\sum_{k=1,2,3} |R_k(\mathbf{a}, \mathbf{b}, \Omega_j)|^p\right) \times \pi(\mathbf{b})$$

  - parameters $\mathbf{a}$ enter directly in TMD factorization
  - parameters $\mathbf{b}$ are other parameters that characterizes additional partonic d.o.f. (i.e. virtualities)

Summary and outlook

- **SIDIS at large p_T**
  - $O(\alpha_S^2)$ corrections are important to describe SIDIS at COMPASS
  - The large x region receives large threshold corrections which can explain the difficulty to describe the data
  - Inclusion of SIDIS large p_T data in PDFs/FFs analysis is required
- **SIDIS region indicators**
  - New tools to map SIDIS regions (web-app)
  - The indicators can be used as Bayesian priors for the regression in TMD phenomenology