News from NNPDF: collinear fragmentation functions from e^+e^- and pp

Workshop on Novel Probes of the Nucleon Structure in SIDIS, e^+e^- and pp

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Nikhef - Amsterdam

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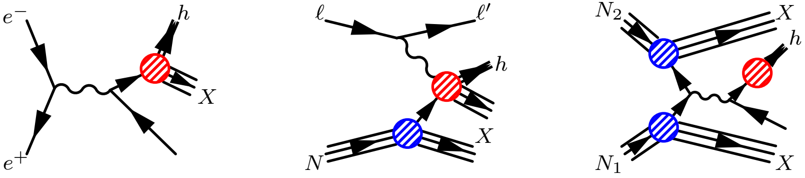
Foreword

Underlying principle of a global fit: (collinear) factorisation of physical observables

$$\mathcal{O}_I = \sum_{i=q,\bar{q},g} C_{Ii}(y, \alpha_s(\mu^2)) \otimes D_i(y, \mu^2) + \text{p.s. corrections} \qquad f \otimes g = \int_x^1 \frac{dy}{y} f\left(\frac{x}{y}\right) g(y)$$



$e^+ + e^- \to h + X$ single-inclusive annihilation (SIA)

$\ell + N \to \ell' + h + X$ semi-inclusive deep-inelastic scattering (SIDIS)

$N_1 + N_2 \to h + X$ high-p_T hadron production in proton-proton collisions (PP)

	DHESS	JAM	NNFF
SIA	☑	☑	☑
SIDIS	☑	☑	☒
PP	☑	☒	☑ ($h^\pm$)
statistical treatment	Iterative Hessian 68% - 90%	Monte Carlo	Monte Carlo
parametrisation	standard	standard	neural network
pert. order	(N)NLO	NLO	up to NNLO

DEHSS (see talk by Rodolfo)
$\pi^\pm$ [PRD 91 (2015) 014035]
$K^\pm$ [PRD 95 (2017) 094019]

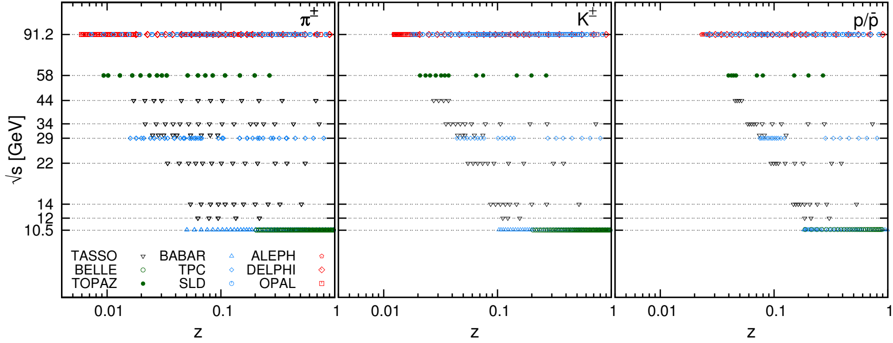
JAM (see talk by Wally)
$\pi^\pm$, $K^\pm$ [PRD 94 (2016) 114004]
Simultaneous fit [PRL 119 (2017) 132001]

NNFF (this talk)
$\pi^\pm$, $K^\pm$, $p/\bar{p}$ [EPJ C77 (2017) 516]
$h^\pm$ [EPJ C78 (2018) 651]

1. Fragmentation Functions from lepton collisions

[EPJ C77 (2017) 516]

The dataset



CERN-LEP: ALEPH [ZP C66 (1995) 353] DELPHI [EPJ C18 (2000) 203] OPAL [ZP C63 (1994) 181]

KEK: BELLE ($n_f = 4$) [PRL 111 (2013) 062002] TOPAZ [PL B345 (1995) 335]

DESY-PETRA: TASSO [PL B94 (1980) 444, ZP C17 (1983) 5, ZP C42 (1989) 189]

SLAC: BABAR ($n_f = 4$) [PR D88 (2013) 032011] SLD [PR D58 (1999) 052001] TPC [PRL 61 (1988) 1263]

$$\frac{d\sigma^h}{dz} = \frac{4\pi\alpha^2(Q^2)}{Q^2}\mathcal{F}_2^h(z,Q^2) \quad h = \pi^+ + \pi^-,\ K^+ + K^-,\ p + \bar{p} \quad \text{possibly normalised to } \sigma_{\rm tot}$$

$N_{\rm dat}^{\pi^\pm} = 428$ $\qquad$ $N_{\rm dat}^{K^\pm} = 385$ $\qquad$ $N_{\rm dat}^{p/\bar{p}} = 360$

From observables to fragmentation functions

$$\boxed{\mathcal{F}_2^h = \langle e^2 \rangle \left\{ C_{2,q}^{\rm S} \otimes D_\Sigma^h + n_f \mathcal{C}_{2,g}^{\rm S} \otimes D_g^h + \mathcal{C}_{2,q}^{\rm NS} \otimes D_{\rm NS}^h \right\}}$$

$$\langle e^2 \rangle = \frac{1}{n_f} \sum_{q=1}^{n_f} \hat{e}_q^2 \qquad D_\Sigma^h = \sum_{q=1}^{n_f} D_{q^+}^h \qquad D_{\rm NS}^h = \sum_{q=1}^{n_f} \left(\frac{\hat{e}_q^2}{\langle e^2 \rangle} - 1 \right) D_{q^+}^h \qquad D_{q^+}^h = D_q^h + D_{\bar{q}}^h$$

Coefficient functions and splitting functions known up to NNLO

[NPB 751 (2006) 18; NPB 749 (2006) 1; PLB 638 (2006) 61; NPB 845 (2012) 133]

$$\begin{aligned} F_2^{h,n_f=5} =& \frac{1}{5} \left[\left(2\hat{e}_u^2 + 3\hat{e}_d^2 \right) C_{2,q}^{\rm S} + 3\left(\hat{e}_u^2 - \hat{e}_d^2 \right) C_{2,q}^{\rm NS} \right] \otimes \left(D_{u^+}^h + D_{c^+}^h \right) \\ &+ \frac{1}{5} \left[\left(2\hat{e}_u^2 + 3\hat{e}_d^2 \right) C_{2,q}^{\rm S} - 2\left(\hat{e}_u^2 - \hat{e}_d^2 \right) C_{2,q}^{\rm NS} \right] \otimes \left(D_{d^+}^h + D_{s^+}^h + D_{b^+}^h \right) \\ &+ \left(2\hat{e}_u^2 + 3\hat{e}_d^2 \right) C_{2,g}^{\rm S} \otimes D_g^h \end{aligned}$$

No sensitivity to individual quark and antiquark FFs

Limited sensitivity to flavour separation via the variation of $\hat{e}_q$ with Q^2

$\hat{e}_u^2/\hat{e}_d^2(Q^2 = 10\,{\rm GeV}) \sim 4 \Rightarrow D_{u^+}^h,\, D_{d^+}^h + D_{s^+}^h$; $\quad \hat{e}_u^2/\hat{e}_d^2(Q^2 = M_Z) \sim 0.8 \Rightarrow D_\Sigma^h$

Flavor separation between uds and c, b quarks achieved thanks to tagged data

Direct sensitivity to D_g^h only beyond LO, as $C_{2,g}^{\rm S}$ is $\mathcal{O}(\alpha_s^2)$, and tenous

Indirect sensitivity to D_g^h via scale violations in the time-like DGLAP evolution

Fit settings

Physical parameters: consistent with the `NNPDF3.1` PDF set [EPJ C77 (2017) 663]

$$\alpha_s(M_Z) = 0.118,\ \alpha(M_Z) = 1/127,\ m_c = 1.51\ \text{GeV},\ m_b = 4.92\ \text{GeV}$$

Solution of DGLAP equations: numerical solution in z-space as implemented in `APFEL`

extensive benchmark performed up to NNLO [JHEP 1503 (2015) 046]

Parametrisation: each FF is parametrised with a feed-forward neural network (2-5-3-1)

$$D_i^h(Q_0, z) = \text{NN}(x) - \text{NN}(1),\quad Q_0 = 5\ \text{GeV}$$

$$h = \pi^+ + \pi^-,\ h = K^+ + K^-,\ h = p + \bar{p} \qquad i = u^+, d^+ + s^+, c^+, b^+, g$$

we assume charge conjugation, from which $D_{q^+}^{\pi^+} = D_{q^+}^{\pi^-}$

initial scale above m_b, but below the lowest c.m. energy of the data, avoid threshold crossing

Heavy flavours: heavy-quark FFs are parametrised independently at the initial scale Q_0

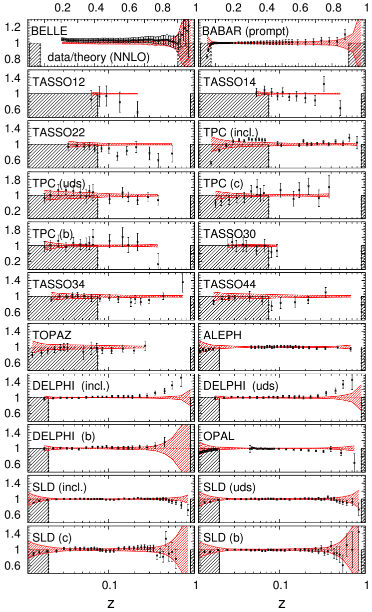
Kinematic cuts: $z \to 0$: contributions $\propto \ln z$; $z \to 1$: contributions $\propto \ln(1-z)$

$$z_{\min} = 0.075,\ z_{\min} = 0.02\ (\sqrt{s} = M_Z);\ z_{\max} = 0.90$$

Momentum sum rule: check a posteriori that

$$\sum_{h=\pi^\pm, K^\pm, p/\bar{p}} \int_{z_{\min}}^1 dz\, z D_i^h(z, Q) < N \qquad N \begin{cases} = 1 & \text{for } i = g \\ = 2 & \text{for } i = u^+, c^+, b^+ \\ = 4 & \text{for } i = d^+ + s^+ \end{cases}$$

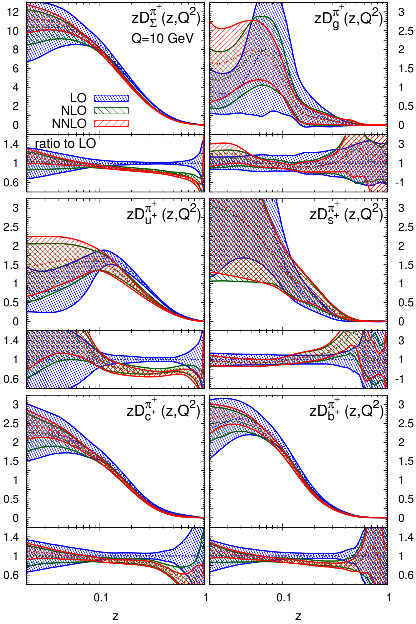
Fit quality: π^+



Exp.	$N_{\rm dat}$	NNLO theory $\chi^2/N_{\rm dat}$	remarks
BELLE	70	0.09	lack of correlations
BABAR	40	0.78	☑
TASSO12	4	0.87	small sample
TASSO14	9	1.70	} data fluctuations
TASSO22	8	1.91	
TPC	13	0.85	☑
TPC-UDS	6	0.49	☑
TPC-C	6	0.52	☑
TPC-B	6	1.43	☑
TASSO34	9	1.00	☑
TASSO44	6	2.34	data fluctuations
TOPAZ	5	0.80	☑
ALEPH	23	0.78	☑
DELPHI	21	1.86	tension with OPAL
DELPHI-UDS	21	1.54	tension with OPAL
DELPHI-B	21	0.95	☑
OPAL	24	1.84	tension with DELPHI
SLD	34	0.83	☑
SLD-UDS	34	0.52	☑
SLD-C	34	1.06	☑
SLD-B	34	0.36	☑
TOTAL	428	0.87	☑

Overall good description of the dataset
Signs of tension OPAL vs DELPHI (inclusive)
Anomalously small $\chi^2/N_{\rm dat}$ for BELLE

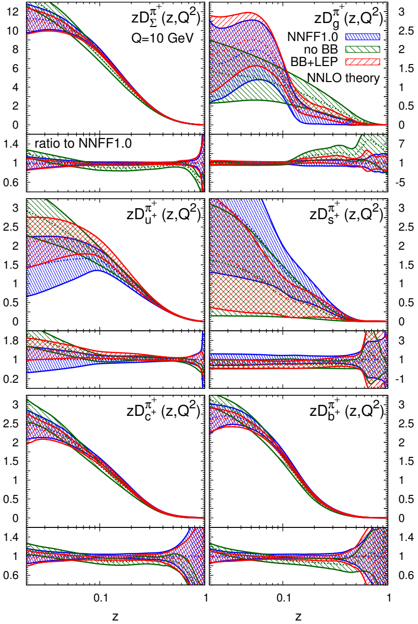
Dependence upon perturbative order: π^+



Exp.	$N_{\rm dat}$	LO $\chi^2/N_{\rm dat}$	NLO $\chi^2/N_{\rm dat}$	NNLO $\chi^2/N_{\rm dat}$
BELLE	70	0.60	0.11	0.09
BABAR	40	1.91	1.77	0.78
TASSO12	4	0.70	0.85	0.87
TASSO14	9	1.55	1.67	1.70
TASSO22	8	1.64	1.91	1.91
TPC	13	0.46	0.65	0.85
TPC-UDS	6	0.78	0.55	0.49
TPC-C	6	0.55	0.53	0.52
TPC-B	6	1.44	1.43	1.43
TASSO34	9	1.16	0.98	1.00
TASSO44	6	2.01	2.24	2.34
TOPAZ	5	1.04	0.82	0.80
ALEPH	23	1.68	0.90	0.78
DELPHI	21	1.44	1.79	1.86
DELPHI-UDS	21	1.30	1.48	1.54
DELPHI-B	21	1.21	0.99	0.95
OPAL	24	2.29	1.88	1.84
SLD	34	2.33	1.14	0.83
SLD-UDS	34	0.95	0.65	0.52
SLD-C	34	3.33	1.33	1.06
SLD-B	34	0.45	0.38	0.36
TOTAL	428	1.44	1.02	0.87

Excellent perturbative convergence
FFs almost stable from NLO to NNLO
LO FF uncertainties larger than HO

Dependence upon the dataset: π^+

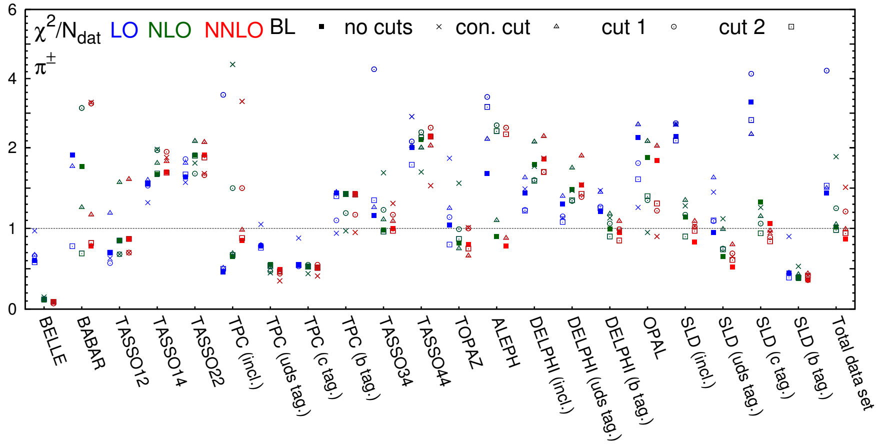


NNLO theory Exp.	$N_{\rm dat}$	NNFF1.0 $\chi^2/N_{\rm dat}$	no BB $\chi^2/N_{\rm dat}$	BB+LEP $\chi^2/N_{\rm dat}$
BELLE	70	0.09	[4.92]	0.09
BABAR	40	0.78	[144]	0.88
TASSO12	4	0.87	0.52	[0.87]
TASSO14	9	1.70	1.38	[1.71]
TASSO22	8	1.91	1.29	[2.15]
TPC	13	0.85	2.12	[2.15]
TPC-UDS	6	0.49	0.54	[0.77]
TPC-C	6	0.52	0.74	[0.58]
TPC-B	6	1.43	1.60	[1.48]
TASSO34	9	1.00	1.17	[1.38]
TASSO44	6	2.34	2.52	[1.97]
TOPAZ	5	0.80	0.92	[1.72]
ALEPH	23	0.78	0.57	0.74
DELPHI	21	1.86	1.97	1.82
DELPHI-UDS	21	1.54	1.56	1.42
DELPHI-B	21	0.95	1.01	0.95
OPAL	24	1.84	1.75	1.92
SLD	34	0.83	0.87	0.95
SLD-UDS	34	0.52	0.53	0.63
SLD-C	34	1.06	0.69	0.96
SLD-B	34	0.36	0.49	0.37
TOTAL		0.87	1.06	0.82

no BB: larger uncertainties; different gluon shape and different light flavour separation
BB+LEP: comparable uncertainties; slightly different size of gluon and light flavoured quarks

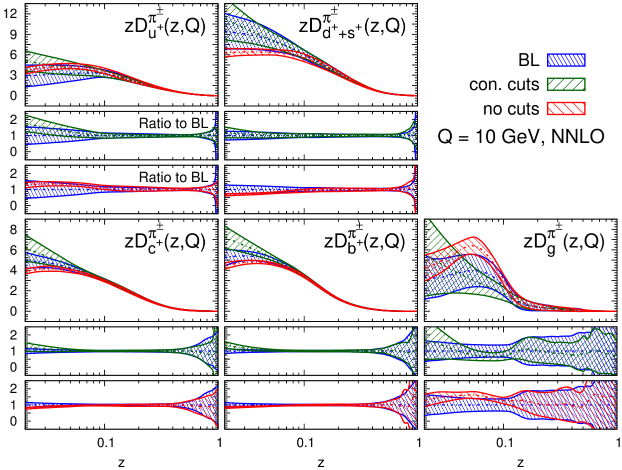
Dependence upon kinematic cuts: π^+

BL		no cuts		con. cut		cut1		cut2	
$z_{\rm min}^{(m_Z)}$	$z_{\rm min}$	$z_{\rm min}^{(m_Z)}$	$z_{\rm min}$	$z_{\rm min}^{(m_Z)}$	$z_{\rm min}$	$z_{\rm min}^{(m_Z)}$	$z_{\rm min}$	$z_{\rm min}^{(m_Z)}$	$z_{\rm min}$
0.02	0.075	0.00	0.00	0.05	0.10	0.01	0.05	0.01	0.075

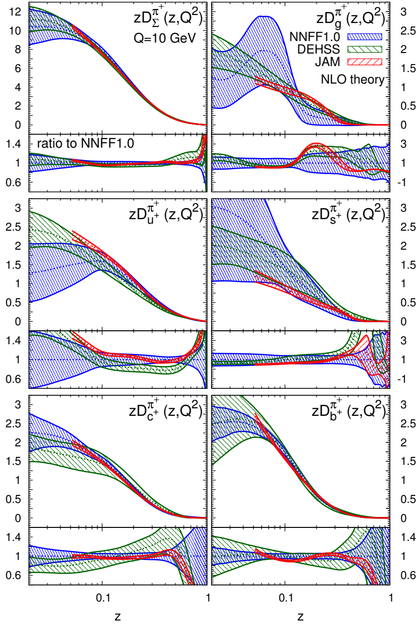


Dependence upon kinematic cuts: π^+

BL		no cuts		con. cut		cut1		cut2	
$z_{\rm min}^{(m_Z)}$	$z_{\rm min}$	$z_{\rm min}^{(m_Z)}$	$z_{\rm min}$	$z_{\rm min}^{(m_Z)}$	$z_{\rm min}$	$z_{\rm min}^{(m_Z)}$	$z_{\rm min}$	$z_{\rm min}^{(m_Z)}$	$z_{\rm min}$
0.02	0.075	0.00	0.00	0.05	0.10	0.01	0.05	0.01	0.075



Comparison with other FF determinations: π^+



DEHSS [PRD 91 (2015) 014035]
(+SIDIS +PP)

JAM [PRD 94 (2016) 114004]
(almost same dataset as NNFF1.0)

different cuts at small z

$D_\Sigma^{\pi^+}$: excellent mutual agreement
both c.v. and unc. (bulk of the dataset)

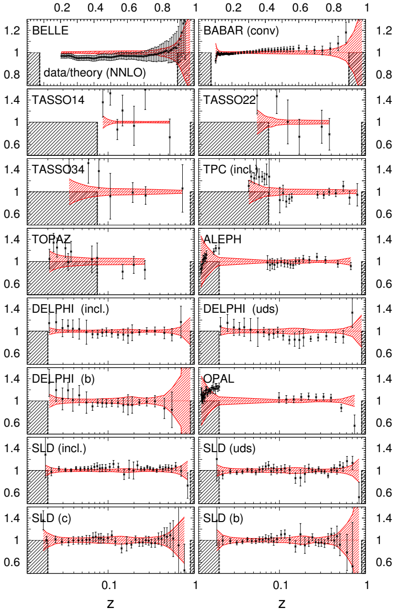
$D_g^{\pi^+}$: slight disagreement
different shapes, larger uncertainties
DEHSS: data; JAM: parametrisation

$D_{u^+}^{\pi^+}$, $D_{s^+}^{\pi^+}$: good overall agreement
excellent with JAM, though larger uncertainties
slightly different shape w.r.t. DHESS (dataset)

$D_{c^+}^{\pi^+}$, $D_{b^+}^{\pi^+}$: good overall agreement
excellent with JAM, same uncertainties
slightly different shape w.r.t. DHESS (dataset)

Fit quality: K^+

Exp.	$N_{\rm dat}$	NNLO theory $\chi^2/N_{\rm dat}$	remarks
BELLE	70	0.32	lack of correlations
BABAR	43	0.95	☑
TASSO12	3	1.02	} small sample
TASSO14	9	2.07	
TASSO22	6	2.62	
TPC	13	1.01	☑
TASSO34	5	0.36	} small sample
TOPAZ	3	0.99	
ALEPH	18	0.56	☑
DELPHI	22	0.34	☑
DELPHI-UDS	22	1.32	☑
DELPHI-B	22	0.52	☑
OPAL	10	1.66	tension with other M_Z data
SLD	35	0.57	☑
SLD-UDS	35	0.93	☑
SLD-C	34	0.38	☑
SLD-B	35	0.62	☑
TOTAL	385	0.73	☑



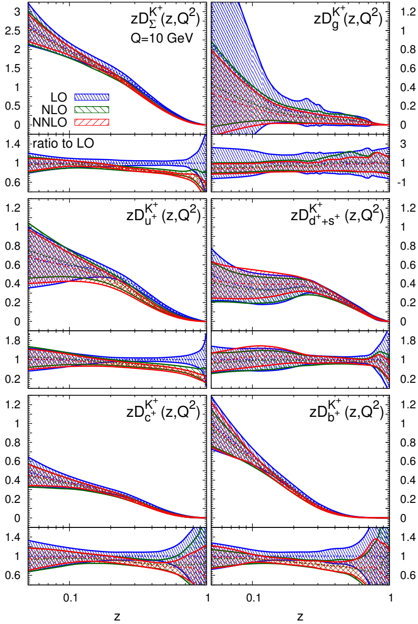
Overall good description of the dataset
Excellent BELLE/BABAR consistency
Signs of tension OPAL vs DELPHI (inclusive)
Anomalously small $\chi^2/N_{\rm dat}$ for BELLE
Dependence upon the data set and kin cuts similar to pions

Dependence upon perturbative order: K^+

Exp.	$N_{\rm dat}$	LO $\chi^2/N_{\rm dat}$	NLO $\chi^2/N_{\rm dat}$	NNLO $\chi^2/N_{\rm dat}$
BELLE	70	0.21	0.32	0.32
BABAR	43	2.86	1.11	0.95
TASSO12	3	1.10	1.03	1.02
TASSO14	9	2.17	2.13	2.07
TASSO22	6	2.14	2.77	2.62
TPC	13	0.94	1.09	1.01
TASSO34	5	0.27	0.44	0.36
TOPAZ	3	0.61	1.19	0.99
ALEPH	18	0.47	0.55	0.56
DELPHI	22	0.28	0.33	0.34
DELPHI-UDS	22	1.38	1.49	1.32
DELPHI-B	22	0.58	0.49	0.52
OPAL	10	1.67	1.57	1.66
SLD	35	0.86	0.62	0.57
SLD-UDS	35	1.31	1.02	0.93
SLD-C	34	0.92	0.47	0.38
SLD-B	35	0.59	0.67	0.62
TOTAL	385	1.02	0.78	0.73

Excellent perturbative convergence
FFs almost stable from NLO to NNLO
LO FF uncertainties larger than HO

i	N^{i+1}LO/N^{i}LO	D_g	D_Σ	D_{c^+}	D_{b^+}
0	NLO/LO [%]	95-300	70-80	65-80	70-85
1	NNLO/NLO [%]	70-130	90-100	90-110	95-115



Comparison with other FF determinations: K^+

DEHSS [PRD 95 (2017) 094019]
(+SIDIS +PP)

JAM [PRD 94 (2016) 114004]
(almost same dataset as NNFF1.0)

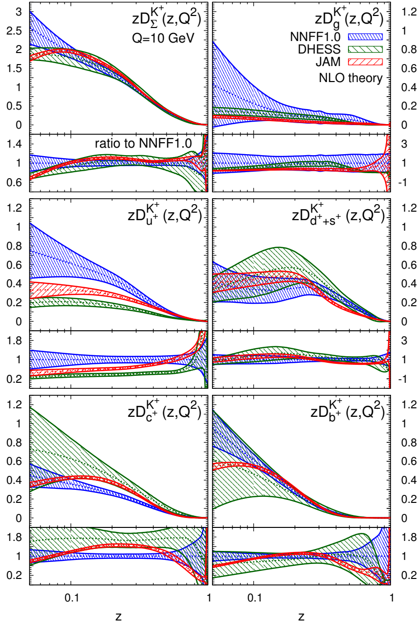
$D_\Sigma^{\pi^+}$: excellent agreement (both c.v. and unc.)
bulk of the dataset

$D_g^{\pi^+}$: good mutual agreement
similar shapes, larger uncertainties
DEHSS: data; JAM: parametrisation

$D_{u^+}^{\pi^+}$: mutual sizable disagreement
differences in dataset and parametrisation
comparable uncertainties in the data region

$D_{d^+}^{\pi^+} + D_{s^+}^{\pi^+}$: fair mutual agreement
differences in dataset and parametrisation
comparable uncertainties in the data region

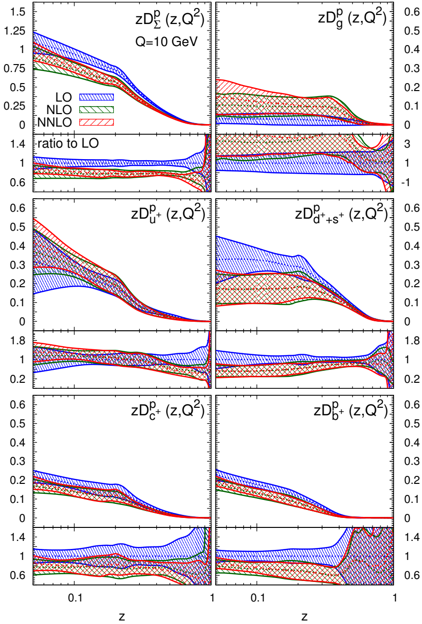
$D_{c^+}^{\pi^+}$, $D_{b^+}^{\pi^+}$: excellent mutual agreement
uncertainties similar to JAM
DHESS shows inflated uncertainties



Dependence upon perturbative order: $p/\bar{p}$

Exp.	$N_{\rm dat}$	LO $\chi^2/N_{\rm dat}$	NLO $\chi^2/N_{\rm dat}$	NNLO $\chi^2/N_{\rm dat}$
BABAR	43	0.10	0.31	0.50
BELLE	29	4.74	2.75	1.25
TASSO12	3	0.69	0.70	0.72
TASSO14	9	1.32	1.25	1.22
TASSO22	9	0.98	0.92	0.93
TPC	20	1.04	1.10	1.08
TASSO30	2	0.25	0.19	0.18
TASSO34	6	0.82	0.81	0.78
TOPAZ	4	0.79	1.21	0.19
ALEPH	26	1.36	1.43	1.28
DELPHI	22	0.48	0.49	0.49
DELPHI-UDS	22	0.47	0.46	0.45
DELPHI-B	22	0.89	0.89	0.91
SLD	36	0.66	0.65	0.64
SLD-UDS	36	0.77	0.76	0.78
SLD-C	36	1.22	1.22	1.21
SLD-B	35	1.12	1.29	1.33
TOTAL	360	1.31	1.13	0.98

Excellent perturbative convergence
FFs almost stable from NLO to NNLO
LO FF uncertainties larger than HO
Dependence upon data set and kin cuts
similar to pions and kaons

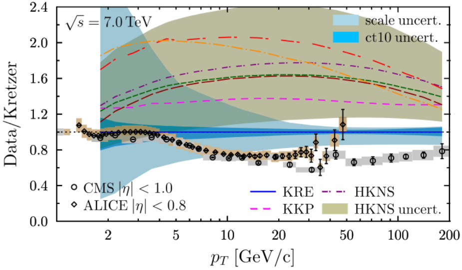


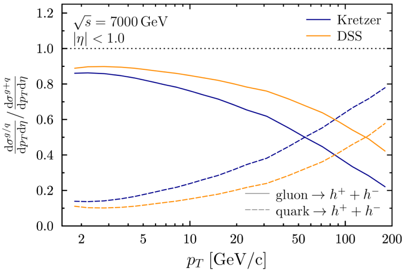
2. Fragmentation Functions from hadron collisions

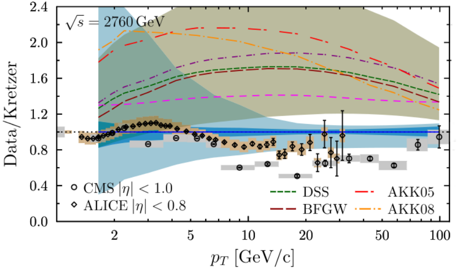
[EPJ C78 (2018) 651]

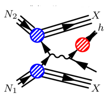
Motivation

Ratio of the inclusive charged-hadron spectra measured by CMS and ALICE









$$E_h \frac{d^3\sigma^{h^\pm}}{d^3p^h} = \sum_{a,b,c} \hat{\sigma}^c_{ab} \otimes f_a \otimes f_b \otimes D^h_c$$

Figures taken from [NPB 883 (2014) 615]

Predictions from all available FF sets are not compatible with CMS and ALICE data, not even within scale and PDF/FF uncertainties

$\longrightarrow$ Can this discrepancy be reconciled?

A strategy to determine FFs from hadronic data

Construct a prior set of Monte Carlo replicas from e^+e^- data: NNFF1.0h

Reweight NNFF1.0h with the hadronic data to obtain a posterior set of FFs: NNFF1.1h

Construction of the prior: fit quality

Repeat the NNFF1.0 analysis, but for unidentified charged hadron FFs
(measurements are also available for the longitudinal structure function F_L)

CERN-LEP: ALEPH [PLB 357 (1995) 487] DELPHI [EPJ C5 (1998) 585; C6 (1999) 19] OPAL [EPJ C7 (1999) 369]

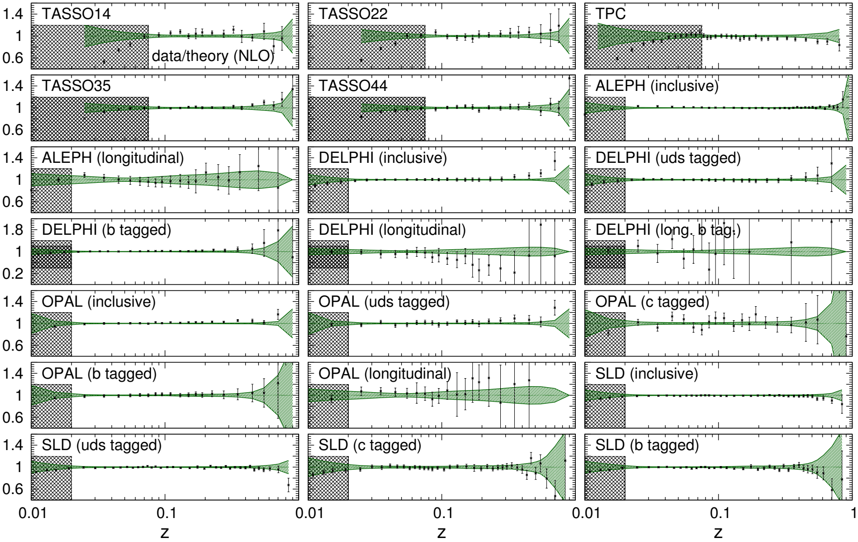
DESY-PETRA: TASSO [ZP C47 (1990) 187]

SLAC: SLD [PRD 69 (2004) 072003] TPC [PRL 61 (1988) 1263]

Experiment	$\sqrt{s}$	$N_{\rm dat}$	$\chi^2_{\rm dat}$	Experiment	$\sqrt{s}$	$N_{\rm dat}$	$\chi^2_{\rm dat}$	Experiment	$\sqrt{s}$	$N_{\rm dat}$	$\chi^2_{\rm dat}$
TASSO14	14.00	15	**1.23**	TASSO22	22.00	15	**0.51**	TPC	29.00	21	**1.65**
TASSO35	35.00	15	**1.14**	TASSO44	44.00	15	**0.68**	ALEPH	91.20	32	**1.04**
ALEPH (L)	91.20	19	**0.36**	DELPHI	91.20	21	**0.65**	DELPHI (uds)	91.20	21	**0.17**
DELPHI (b)	91.20	20	**0.82**	DELPHI (L)	91.20	20	**0.72**	DELPHI (Lb)	91.20	20	**0.44**
OPAL	91.20	20	**2.41**	OPAL (uds)	91.20	20	**0.90**	OPAL (c)	91.20	20	**0.61**
OPAL (b)	91.20	20	**0.21**	OPAL (L)	91.20	20	**0.31**	SLD	91.28	34	**0.75**
SLD (uds)	91.28	34	**1.03**	SLD (c)	91.28	34	**0.62**	SLD (b)	91.28	34	**0.97**

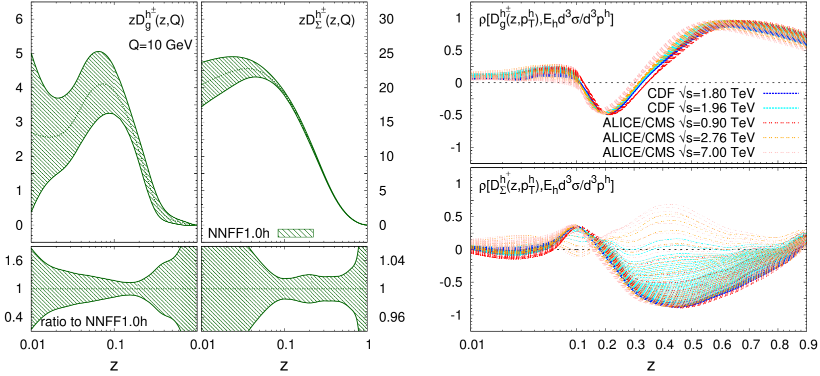
$\sqrt{s}$ in GeV

Construction of the prior: data/theory agreement



$$N_{\rm dat} = 471 \qquad \chi^2/N_{\rm dat} = 0.83 \qquad N_{\rm rep} = 2000$$

Construction of the prior: fragmentation functions



Consider all the available data from the Tevatron and the LHC

CDF [PRD 79 (2009) 112005] CMS [JHEP 08 (2011) 086;EPJ C72 (2012) 1945] ALICE [EPJ C73 (2013) 2662]

What is the expected sensitivity of these measurements to the fragmentation functions?

$$\rho[A,B] = \frac{\langle AB\rangle_{\rm rep} - \langle A\rangle_{\rm rep}\langle B\rangle_{\rm rep}}{\sigma_A\sigma_B}$$

$$D_g^{h^\pm}:\ z \gtrsim 0.4 \qquad\qquad D_\Sigma^{h^\pm}:\ 0.2 \lesssim z \lesssim 0.7$$

Reweighting: fit quality

Apply $p_T^h > p_{T,\mathrm{cut}}^h = 7$ GeV (to ensure accuracy of fixed-order theory)

Most of SppS and PHENIX data (low p_T, low $\sqrt{s}$) are excluded by this cut

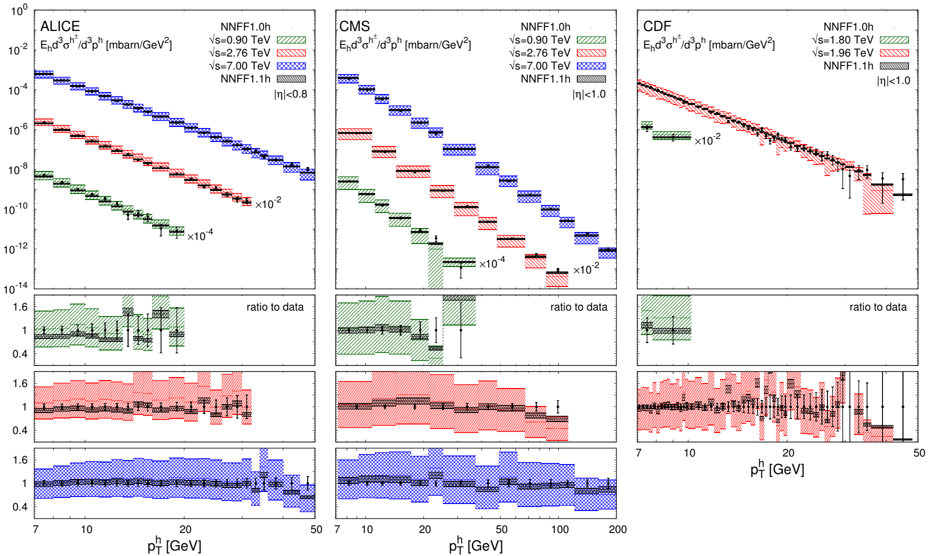
Reweight NNFF1.0h with the remaining collider data

Process	Experiment	$\sqrt{s}$ [TeV]	N_{dat}	$\chi^2_{\mathrm{in}}/N_{\mathrm{dat}}$	$\chi^2_{\mathrm{rw}}/N_{\mathrm{dat}}$	N_{eff}
SIA			471 (527)	0.83	0.83	—
pp	ALICE	0.90	11 (54)	4.94	1.88	1012
		2.76	27 (60)	13.3	0.82	574
		7.00	22 (65)	6.03	0.53	779
	CMS	0.90	7 (20)	4.20	0.70	1206
		2.76	9 (22)	10.6	1.24	579
		7.00	14 (27)	12.4	1.64	396
	CDF	1.80	2 (49)	3.32	0.20	1420
		1.96	50 (230)	2.93	1.23	735
			603 (1054)	6.54	1.11	407

The description of the data set in the prior is almost unaffected

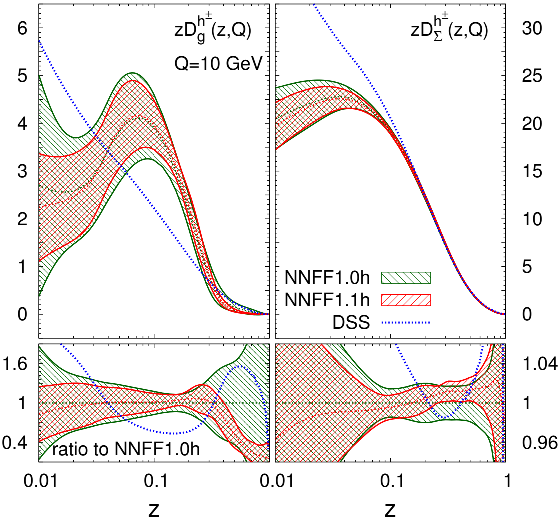
Excellent description/consistency of e^+e^- and pp data

Reweighting: observables



Significant reduction of the uncertainties for all data sets

Reweighting: fragmentation functions



Modifications in shape

$D_g^{h^\pm}$ slightly harder
for $0.1 \lesssim z \lesssim 0.3$
$D_g^{h^\pm}$ significantly softer
for $0.3 \lesssim z \lesssim 0.9$

$D_\Sigma^{h^\pm}$ almost stable
slightly reduced for $0.1 \lesssim z \lesssim 0.4$

Reduction of uncertainties

$D_g^{h^\pm}$ reduced
from 20%-60% to 10%-15%
for $z \gtrsim 0.1$
$D_\Sigma^{h^\pm}$ reduced
from 2% to $\simeq$ 1%
for $0.1 \lesssim z \lesssim 0.4$

The NNFF1.1h shape is significantly different from the DSS [PRD 76 (2007) 074033] shape

Dependence on the value of $p^h_{T,\mathrm{cut}}$

Examine the dependence of the results upon the value of $p^h_{T,\mathrm{cut}}$

Vary $p^h_{T,\mathrm{cut}}$ by incremental steps of 1 GeV in the range 5 GeV $\leq p^h_{T,\mathrm{cut}} \leq$ 10 GeV

$p^h_{T,\mathrm{cut}}$	5 GeV		6 GeV		7 GeV		8 GeV		9 GeV		10 GeV	
Experiment	$\frac{\chi^2_{\mathrm{rw}}}{N_{\mathrm{dat}}}$	N_{dat}	$\frac{\chi^2_{\mathrm{rw}}}{N_{\mathrm{dat}}}$	N_{dat}	$\frac{\chi^2_{\mathrm{rw}}}{N_{\mathrm{dat}}}$	N_{dat}	$\frac{\chi^2_{\mathrm{rw}}}{N_{\mathrm{dat}}}$	N_{dat}	$\frac{\chi^2_{\mathrm{rw}}}{N_{\mathrm{dat}}}$	N_{dat}	$\frac{\chi^2_{\mathrm{rw}}}{N_{\mathrm{dat}}}$	N_{dat}
CDF	1.30	7	0.28	4	0.10	2	0.04	1	—	—	—	—
	1.32	60	1.26	55	1.23	50	1.20	45	1.15	40	1.15	35
CMS	0.93	10	0.67	8	0.70	7	0.71	7	0.80	6	0.80	6
	1.38	11	1.27	10	1.24	9	1.17	9	1.22	8	1.16	8
	2.01	17	1.80	15	1.64	14	1.52	14	1.47	13	1.40	13
ALICE	2.56	15	2.05	13	1.88	11	1.71	10	1.51	9	1.52	8
	0.61	21	0.72	19	0.82	17	0.89	16	0.98	15	1.08	14
	0.56	26	0.52	24	0.53	22	0.55	21	0.57	20	0.60	19
Total	1.27	167	1.14	148	1.11	132	1.09	123	1.08	111	1.08	103

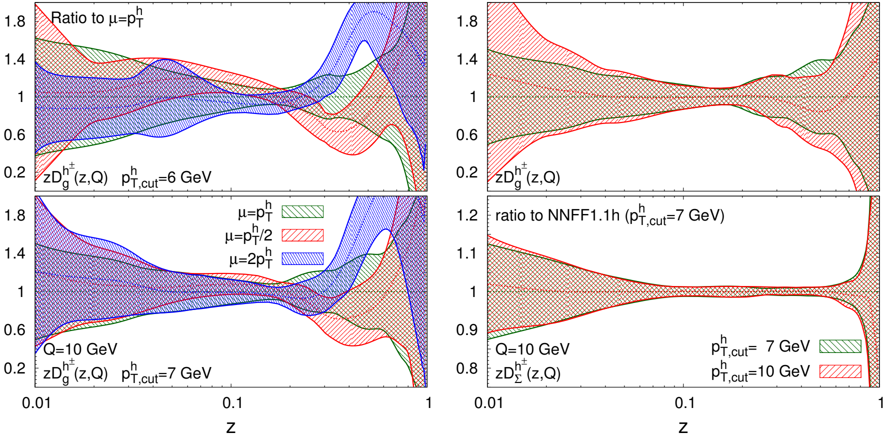
The overall fit quality turns out to be very similar for $p^h_{T,\mathrm{cut}} \geq 6$ GeV

What is the best cut in the restricted range 6 GeV $\leq p^h_{T,\mathrm{cut}} \leq$ 10 GeV?

Dependence on the value of $p^h_{T,\mathrm{cut}}$

What is the size of missing higher order uncertainties?
Repeat the reweighting exercise with different values of the scale μ

How are FFs affected by varying $p^h_{T,\mathrm{cut}}$?
Repeat the reweighting exercise with different values of $p^h_{T,\mathrm{cut}}$



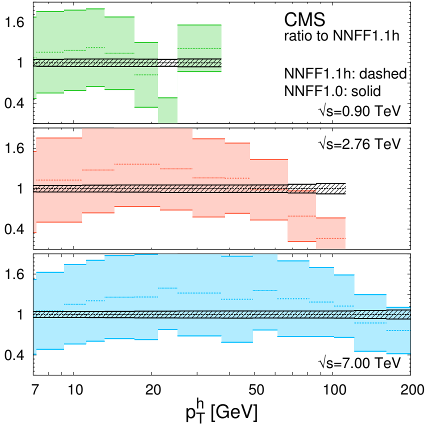
Compatibility with NNFF1.0

Fragmentation functions should satisfy "conservation of hadrons"

$$D_i^{h^\pm} = D_i^{\mathcal{H}} + D_i^{\text{res}^\pm} \qquad D_i^{\mathcal{H}} = D_i^{\pi^\pm} + D_i^{K^\pm} + D_i^{p/\bar{p}} \qquad \forall\ i = q, \bar{q}, g$$

Are the FFs for $h^\pm$ (NNFF1.1h) and the FFs for $\pi^\pm$, $K^\pm$ and $p/\bar{p}$ (NNFF1.0) compatible?

$$E_h \frac{d^3\sigma^{h^\pm}}{d^3p^h} > \sum_{\mathcal{H}=\pi^\pm,K^\pm,p/\bar{p}} E_h \frac{d^3\sigma^{\mathcal{H}}}{d^3p^h} \qquad M_i^{h^\pm}(Q) \gtrsim M_i^{\text{light}}(Q)$$



$Q = 5$ GeV i	NNFF1.1h $M_i^{h^\pm}(Q)$	NNFF1.0 $M_i^{\text{light}}(Q)$
g	0.86 ± 0.06	0.80 ± 0.18
u^+	1.24 ± 0.07	1.42 ± 0.12
$d^+ + s^+$	2.05 ± 0.08	2.07 ± 0.27
c^+	1.09 ± 0.03	1.01 ± 0.08
b^+	1.06 ± 0.02	0.98 ± 0.08

$$M_i^{h^\pm}(Q) \equiv \int_{z_{\min}}^1 dz\, z D_i^{h^\pm}(z,Q)$$

$$M_i^{\text{light}}(Q) \equiv \sum_{\mathcal{H}=\pi^\pm,K^\pm,p/\bar{p}} \int_{z_{\min}}^1 dz\, z D_i^{\mathcal{H}}(z,Q)$$

3. To conclude

Summary

1. A number of hard-scattering processes require an appropriate knowledge of FFs
   - probing nucleon momentum, spin and flavour
   - underlying spatial distributions and the dynamics of nuclear matter
2. FFs are poorly known in comparison to PDFs
   - limited set of available data, observables often require PDFs and FFs simultanously
   - troubles in describing some observables in pp and SIDIS from current FF sets
3. New analyses based on the NNPDF methodology
   - NNFF1.0 at LO, NLO and NNLO, based on SIA data for $\pi^{\pm}$, $K^{\pm}$ and $p/\bar{p}$
     detailed study of the stability of the results upon variations of the data set/kin cuts
     FF uncertainties (gluon) seem to be underestimated in previous determinations
     differences in shapes, to be further investigated
     applicability limited by insensitivity to complete flavour decomposition
   - NNFF1.1h at NLO, based on SIA and pp data for $h^{\pm}$
     significant impact of pp data on FF shape and uncertainty
     detailed study of the stability of the results upon variations of the kin cuts
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Thank you

Parametrisation: two alternative choices

$$zD_i^h(z,Q_0^2) = z^{a_i^h}\,(1-z)^{b_i^h}\,\mathscr{F}_i^h(z,\{\mathbf{c}\})$$

1. Standard parametrisation, *e.g.*

$$\mathscr{F}_i^h(z,\{\mathbf{c}\}) = \frac{1+\gamma_i^h(1-z)^{\delta_i^h}}{B[2+a_i^h,b_i^h+1]+\gamma_i^h B[2+a_i^h,b_i^h+\delta_i^h+1]}$$

in terms of a (relatively) small set of parameters ($\mathcal{O}(30)$ per PDF set)

$$\{\mathbf{a}\} = \{a_i^h, b_i^h\}\cup\{\mathbf{c}\} = \{a_i^h, b_i^h, \gamma_i^h, \delta_i^h\}$$

$\Rightarrow$ **smooth behavior (a desirable feature for a PDF)**
$\Rightarrow$ **potential source of bias if the parametrisation is too rigid**

2. Redundant parametrisation, *e.g.*

$\mathscr{F}_i^h(z,\{\mathbf{c}\})$ is a neural network

in terms of a huge set of parameters ($\mathcal{O}(200)$ per PDF set)

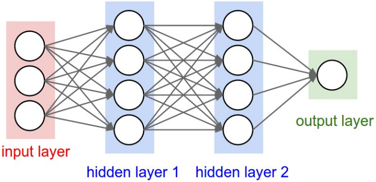
$$\{\mathbf{a}\} = \{\omega_{ij}^{(L-1),f_i^h}, \theta_i^{(L),f_i^h}\}$$

$\Rightarrow$ **potentially non-smooth**
$\Rightarrow$ **bias due to the parametrisation reduced as much as possible**

Parametrisation: what a neural network exactly is?

A convenient functional form providing a flexible parametrization
used as a generator of random functions in the FF space

EXAMPLE: MULTY-LAYER FEED-FORWARD PERCEPTRON



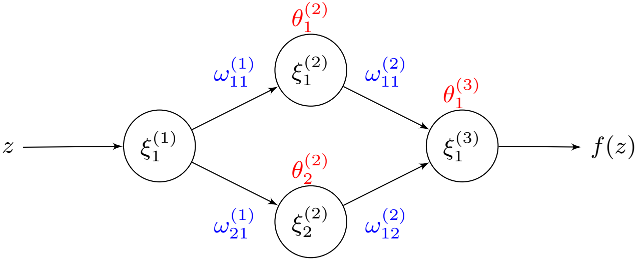
$$\xi_i^{(l)} = g\left(\sum_j^{n_l - 1} \omega_{ij}^{(l-1)} \xi_j^{(l-1)} - \theta_i^{(l)}\right)$$

$$g(y) = \frac{1}{1+e^{-y}}$$

- made of neurons grouped into layers (define the architecture)
- each neuron receives input from neurons in the preceding layer (feed-forward NN)
- activation $\xi_i^{(l)}$ determined by a set of parameters (weights and thresholds)
- activation determined according to a non-linear function (except the last layer)

Parametrisation: what a neural network exactly is?

EXAMPLE: THE SIMPLEST 1-2-1 MULTI-LAYER FEED-FORWARD PERCEPTRON

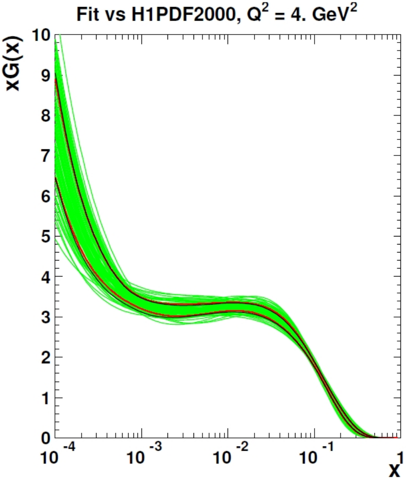


$$f(z) \equiv \xi_1^{(3)} = \left\{ 1 + \exp\left[\theta_1^{(3)} - \frac{\omega_{11}^{(2)}}{1 + e^{\theta_1^{(2)} - x\omega_{11}^{(1)}}} - \frac{\omega_{12}^{(2)}}{1 + e^{\theta_2^{(2)} - x\omega_{21}^{(1)}}} \right] \right\}^{-1}$$

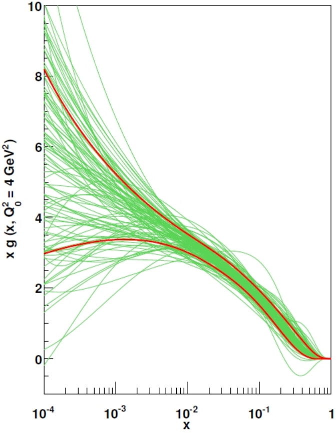
Recall: $\xi_i^{(l)} = g\left(\sum_j^{n_l - 1} \omega_{ij}^{(l-1)} \xi_j^{(l-1)} - \theta_i^{(l)} \right)$; $g(z) = \frac{1}{1 + e^{-z}}$

Parametrisation: standard *vs* redundant

HERA-LHC 2009 PDF benchmark



simple parametrization



redundant parametrization (NN)

Parameter optimisation

Optimisation usually performed by means of simple gradient descent:
compute and minimise the gradient of the fit quality with respect to the fit parameters

$$\frac{\partial\chi^2}{\partial a_i}\,, \qquad \text{for } i=1,\dots,N_{\rm par} \qquad \chi^2=\sum_{i,j}^{\rm N_{dat}}\left(T_i[\{\mathbf{a}\}]-D_i\right)\left(\mathrm{cov}^{-1}\right)_{ij}\left(T_j[\{\mathbf{a}\}]-D_j\right)$$

$$(\mathrm{cov})_{ij}=\delta_{ij}s_i^2+\left(\sum_{\alpha}^{N_c}\sigma_{i,\alpha}^{(c)}\sigma_{j,\alpha}^{(c)}+\sum_{\alpha}^{N_{\mathcal{L}}}\sigma_{i,\alpha}^{(\mathcal{L})}\sigma_{j,\alpha}^{(\mathcal{L})}\right)D_iD_j$$

Optimisation should minimise the noise in the χ^2 driven by noisy experimental data

Additional complications in case of a redundant parametrisation (huge parameter space)

1. need to explore the parameter space as uniformly as possible
   (in order to avoid stopping the fit in a local minimum)
   $\longrightarrow$ genetic algorithhms
2. need for a computationally efficient minimisation
   (non-trivial relationship between FFs and observables via convolution)
   $\longrightarrow$ adaptive algorithms
3. need to define a criterion for minimisation stopping
   (avoid learning statistical fluctuations of the data)
   $\longrightarrow$ cross-validation

The Hessian method: general strategy

1. Expand the χ^2 about its global minimum at first (nontrivial) order

$$\chi^2\{\mathbf{a}\} \approx \chi^2\{\mathbf{a_0}\} + \delta a^i H_{ij} \delta a^j, \qquad H_{ij} = \frac{1}{2} \left. \frac{\partial^2 \chi^2\{\mathbf{a}\}}{\partial a_i \partial a_j} \right|_{\{\mathbf{a}\}=\{\mathbf{a_0}\}}$$

2. Assume linear error propagation for any observable $\mathcal{O}$ depending on $\{\mathbf{a}\}$

$$\langle \mathcal{O}\{\mathbf{a}\} \rangle \approx \mathcal{O}\{\mathbf{a_0}\} + a_i \left. \frac{\partial \mathcal{O}\{\mathbf{a}\}}{\partial a_i} \right|_{\{\mathbf{a}\}=\{\mathbf{a_0}\}} \qquad \sigma_{\mathcal{O}\{\mathbf{a}\}} \approx \sigma_{ij} \left. \frac{\partial \mathcal{O}\{\mathbf{a}\}}{\partial a_i} \frac{\partial \mathcal{O}\{\mathbf{a}\}}{\partial a_j} \right|_{\{\mathbf{a}\}=\{\mathbf{a_0}\}}$$

3. Determine σ_{ij} from H_{ij} from maximum likelihood (under Gaussian hypothesis)

$$\sigma_{ij}^{-1} = \left. \frac{\partial^2 \chi^2\{\mathbf{a}\}}{\partial a_i \partial a_j} \right|_{\{\mathbf{a}\}=\{\mathbf{a_0}\}} = H_{ij}$$

4. A C.L. about the best fit is obtained as the volume (in parameter space) about $\chi^2\{\mathbf{a_0}\}$ that corresponds to a fixed increase of the χ^2; for Gaussian uncertainties:

$$68\% \text{ C.L.} \Longleftrightarrow \Delta\chi^2 = \chi^2\{\mathbf{a}\} - \chi^2\{\mathbf{a_0}\} = 1$$

The Hessian method: some remarks

1. Parameters can always be adjusted so that all eigenvalues of H_{ij} are equal to one (diagonalise H_{ij} and rescale the eigenvectors by their eigenvalues)

$$\delta a_i H_{ij} \delta a_j = \sum_{i=1}^{N_{\rm par}} \left[a'_i(a_i)\right]^2 \Longleftrightarrow \sigma_{\mathcal{O}\{\mathbf{a}'\}} = \left|\nabla' \mathcal{O}\{\mathbf{a}'\}\right|$$

2. Compact representation and computation of observables and their uncertainties

$$\langle \mathcal{O}[D(x,Q^2)] \rangle = \mathcal{O}[D_0(x,Q^2)]$$

$$\sigma_{\mathcal{O}}[D(x,Q^2)] = \left[\sum_{i=1}^{N_{\rm par}} \left(\mathcal{O}[D_i(x,Q^2)] - \mathcal{O}[D_0(x,Q^2)]\right)^2\right]^{1/2}$$

3. Uncertainties obtained with $\Delta\chi^2 = 1$ might be unrealistically small (inadequacy of the linear approximation)
4. Rescale to the $\Delta\chi^2 = T$ interval such that correct C.L.s are reproduced (no statistically rigorous interpretation of T (tolerance)
5. Unmanageable Hessian matrix if the numer of parameters is huge

The Monte Carlo method: general strategy

1. Generate (art) replicas of (exp) data according to the distribution

$$\mathcal{O}_i^{(art)(k)} = \mathcal{O}_i^{(exp)} + r_i^{(k)} \sigma_{\mathcal{O}_i} \,, \qquad i = 1, \ldots N_{\rm dat} \,, \qquad k = 1, \ldots, N_{\rm rep}$$

   where $r_i^{(k)}$ are (Gaussianly distributed) random numbers for each k-th replica ($r_i^{(k)}$ can be generated with any distribution, not neccesarily Gaussian)

2. Validate the Monte Carlo sample size against experimental data
3. Perform a fit for each replica $k = 1, \ldots, N_{\rm rep}$
4. Compact computation of observables and their uncertainties
   (PDF replicas are equally probable members of a statistical ensemble)

$$\langle \mathcal{O}[D(x,Q^2)] \rangle = \frac{1}{N_{\rm rep}} \sum_{k=1}^{N_{\rm rep}} \mathcal{O}[D^{(k)}(x,Q^2)]$$

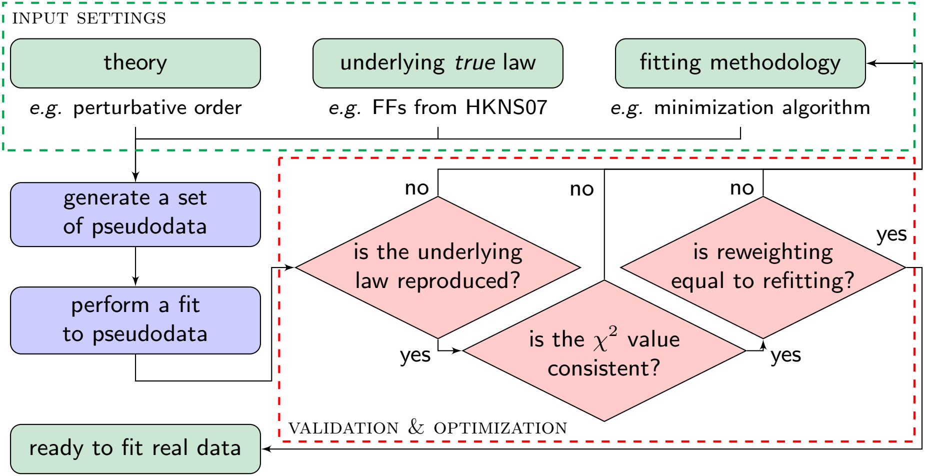
$$\sigma_{\mathcal{O}}[D(x,Q^2)] = \left[\frac{1}{N_{\rm rep}-1} \sum_{k=1}^{N_{\rm rep}} \left(\mathcal{O}[D^{(k)}(x,Q^2)] - \langle \mathcal{O}[D(x,Q^2)] \rangle \right)^2 \right]^{1/2}$$

$\Rightarrow$ **no need to rely on linear approximation**

$\Rightarrow$ **computational expensive: need to perform $N_{\rm rep}$ fits instead of one**

Methodology validation: closure tests [JHEP 1504 (2015) 040]

Validation and optimization of the fitting strategy with known underlying physical law



Full control of procedural uncertainties

Methodology validation: closure tests [JHEP 1504 (2015) 040]

1. Level 0: generate pseudodata D_i^0 with zero uncertainty
   (but $(\mathrm{cov})_{ij}$ in the χ^2 is the data covariance matrix)
   → fit quality can be arbitrarily good, if the fitting methodology is efficient: $\chi^2/N_{\mathrm{dat}} \sim 0$
   → validate fitting methodology (parametrisation, minimisation)
   → interpolation and extrapolation uncertainty

2. Level 1: generate pseudodata D_i^1 with stochastic fluctuations (no replicas)

$$D_i^1 = (1 + r_i^{\mathrm{nor}}\sigma_i^{\mathrm{nor}})\left(D_i^0 + \sum_p^{N_{\mathrm{sys}}} r_{i,p}^{\mathrm{sys}}\sigma_{i,p}^{\mathrm{sys}} + r_i^{\mathrm{stat}}\sigma_i^{\mathrm{stat}}\right)$$

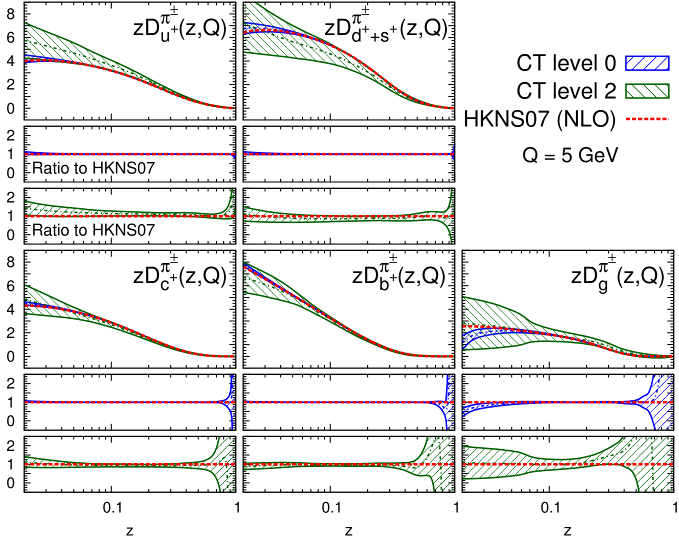
   → experimental uncertainties are not propagated into FFs: $\chi^2/N_{\mathrm{dat}} \sim 1$
   → functional uncertainty (a large number of functional forms with equally good χ^2)

3. Level 2: generate N_{rep} Monte Carlo pseudodata replicas $D_i^{2,k}$ on top of Level 1

$$D_i^{2,k} = (1 + r_i^{\mathrm{nor},k}\sigma_i^{\mathrm{nor}})\left(D_i^1 + \sum_p^{N_{\mathrm{sys}}} r_{i,p}^{\mathrm{sys},k}\sigma_{i,p}^{\mathrm{sys}} + r_i^{\mathrm{stat},k}\sigma_i^{\mathrm{stat}}\right)$$

   → propagate the fluctuations due to experimental uncertainties into FFs: $\chi^2/N_{\mathrm{dat}} \sim 1$
   → input FFs lie within the one-sigma band of the fitted FFs with a probability of ∼ 68%
   → data uncertainty

Closure testing NNFF1.0



$\chi^2/N_{\rm dat} = 0.0001$ (L0) $\chi^2/N_{\rm dat} = 1.0262$ (L1)