Towards a Complete QED+QCD Analysis of SIDIS

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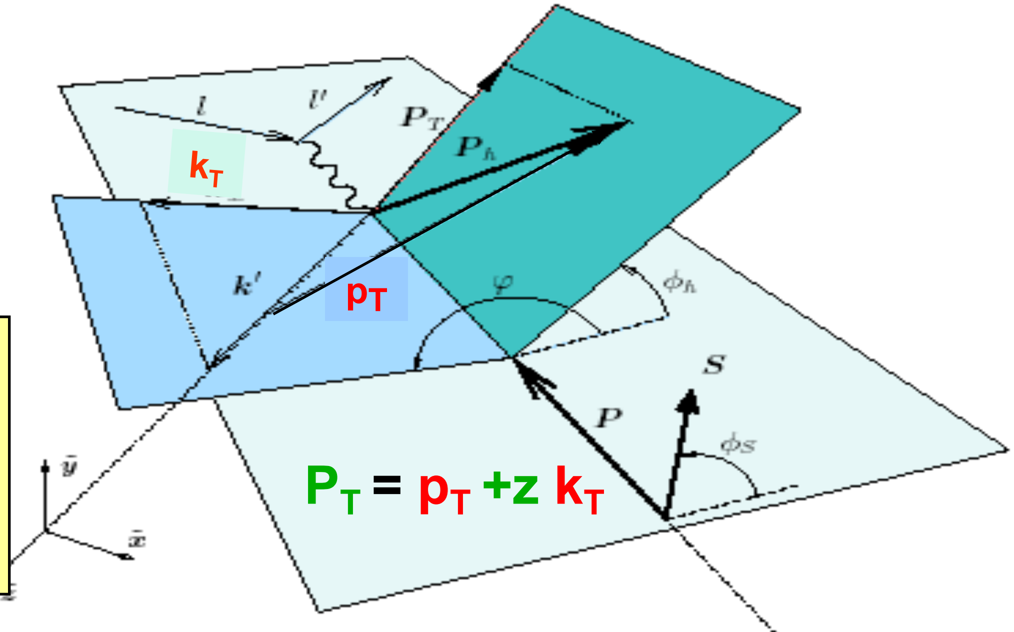
SIDIS: partonic cross sections

$$\nu = (qP)/M$$
$$Q^2 = (k-k')^2$$
$$y = (qP)/(kP)$$
$$x = Q^2/2(qP)$$
$$z = (qP_h)/(qP)$$

$$\sigma = \sigma_0(1 + c_1(y)A_{UU}^{\cos\phi} + \cdots + c_6(y)A_{UT}^{\sin\phi_S} + c_7(y)A_{UT}^{\sin(\phi-\phi_S)})$$



Azimuthal moments in hadron production in SIDIS provide access to different structure functions and underlying transverse momentum dependent distribution and fragmentation functions.

$$\int d^2\vec{k}_T d^2\vec{p}_T \delta^{(2)}(\vec{k}_T + \vec{p}_T - \vec{P}_T/z)$$

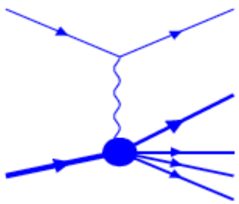
Ji,Ma,Yuan Phys.Rev.D71:034005,2005

$$F_{XY}^h(P_T) \propto \sum e_q^2 H \times f^q(x, k_T, ..) \otimes D^{q\to h}(z, p_T, ..)$$

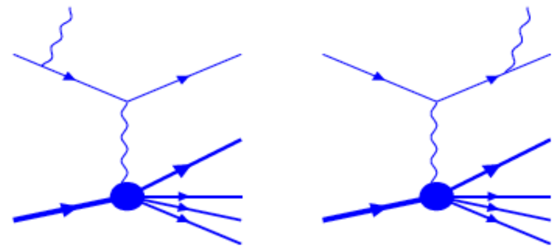
beam polarization   target polarization

Radiative corrections in SIDIS

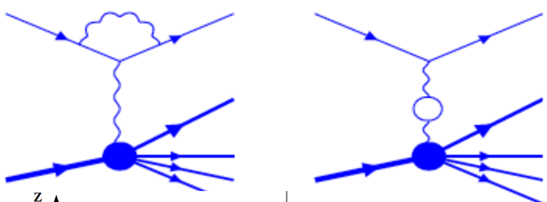
The Born cross section



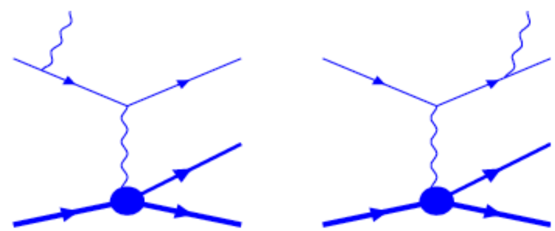
Emission of a radiated photon (semi-inclusive processes)

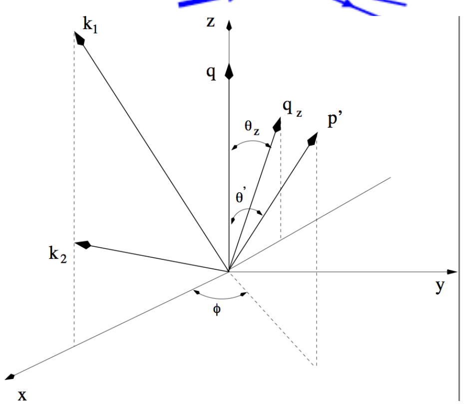


Loop diagrams



Emission of a radiated photon (exclusive pro- cesses)





The real polar angle of virtual photon is changing due to radiation of the real photon, introducing azimuthal dependence, coupling to ϕ-dependence of the x-section
Akushevich, Ilyichev, Osipenko, PL B672 (2009) 35

Measuring cross sections and asymmetries

Due to radiative corrections, coupling of shifted γ^* angle with ϕ-dependent x-section

$$\sigma_{Rad}^{ehX}(x,y,z,P_{hT},\phi,\phi_S) \rightarrow$$
$$\sigma_0^{ehX}(x,y,z,P_{hT},\phi_h,\phi_S) \times R(x,y,z,P_{hT},\phi_h) + R_A(x,y,z,P_{hT},\phi_h,\phi_S)$$

Even neglecting the virtual photon angle with polarization vector, radiative effects can contribute to all moments, in particular transverse asymmetries

$$Y_{\phi,\phi_S} \sim \quad +S_T\Big[\sin(\phi_h-\phi_S)\big(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\, F_{UT,L}^{\sin(\phi_h-\phi_S)}\big) + \varepsilon\sin(\phi_h+\phi_S) F_{UT}^{\sin(\phi_h+\phi_S)} + \varepsilon \sin(3\phi_h-\phi_S) F_{UT}^{\sin(3\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1+\varepsilon)}\sin\phi_S F_{UT}^{\sin\phi_S} + \sqrt{2\,\varepsilon(1+\varepsilon)}\sin(2\phi_h-\phi_S) F_{UT}^{\sin(2\phi_h-\phi_S)}\Big]$$

Simple approximation used to extract Collins and Sivers effects $A_C(A_S)$ will be affected (Y →normalized yield)

$$A(\phi_h,\phi_S) = \frac{1}{P}\frac{Y_{\phi_h,\phi_S} - Y_{\phi_h,\phi_S+\pi}}{Y_{\phi_h,\phi_S} + Y_{\phi_h,\phi_S+\pi}} \approx A_C \sin(\phi_h+\phi_S) + A_S\sin(\phi_h-\phi_S), \quad ($$

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Extracting the moments with rad corrections

Moments mix in experimental azimuthal distributions

Simplest rad. correction $$R(x, z, \phi_h) = R_0(1 + r\cos\phi_h)$$

Correction to normalization

$$\sigma_0(1 + \alpha\cos\phi_h)R_0(1 + r\cos\phi_h) \to \sigma_0 R_0(1 + \alpha r/2)$$

Correction to SSA

$$\sigma_0(1 + sS_T\sin\phi_S)R_0(1 + r\cos\phi_h) \to \sigma_0 R_0(1 + sr/2S_T\sin(\phi_h - \phi_S) + sr/2S_T\sin(\phi_h + \phi_S))$$

Correction to DSA

$$\sigma_0(1 + g\lambda\Lambda + f\lambda\Lambda\cos\phi_h)R_0(1 + r\cos\phi_h) \to \sigma_0 R_0(1 + (g + fr/2)\lambda\Lambda)$$

Generate fake DSA moments (cos)

$$\sigma_0(1 + g\lambda\Lambda)R_0(1 + r\cos\phi_h) \to \sigma_0 R_0 g r\cos\phi_h$$

Simultaneous extraction of all moments is important also because of correlations!

Requirements for consistent RC corrections in SIDIS

- Preliminary studies show that RC can strongly depend on models for SFs
  - RC are particularly sensitive to P_T model choice.
- Rad corrections to polarized structure functions are important

$$\Delta A = \frac{\sigma_0^p + \sigma_{RC}^p}{\sigma_0^u + \sigma_{RC}^u} - \frac{\sigma_0^p}{\sigma_0^u} = \frac{\sigma_{RC}^p \sigma_0^u - \sigma_{RC}^u \sigma_0^p}{\sigma_0^u(\sigma_0^u + \sigma_{RC}^u)}$$

- We need the full set of SFs as continuous functions of all four variables in all kinematical regions for RC calculation in and beyond the region of an experiment on SIDIS measurements
  - The RC procedure of experimental data should involve an iteration procedure in which the fits of SFs of interest are re-estimated at each step of this iteration procedure.
  - Use experimental data or theoretical models to construct the models in the regions of softer processes, resonance region, and exclusive scattering
- Need all constructed models provide correct asymptotic behavior when we go to the kinematical bounds (Regge limit, QCD limit)

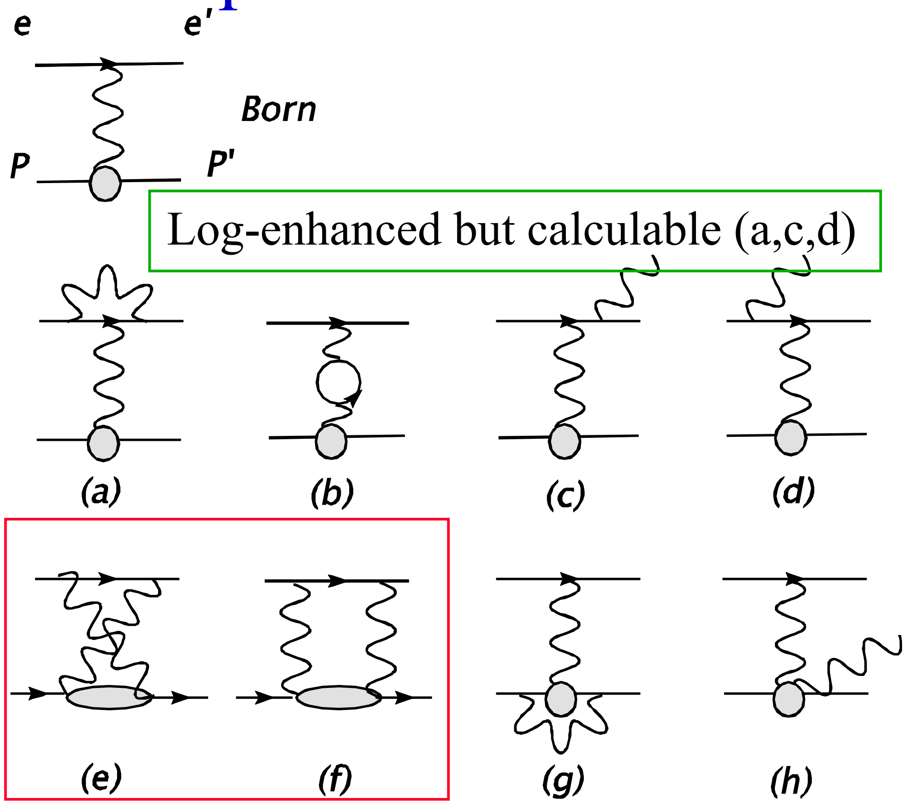
Two-Photon Exchange

Two-photon exchange effects in elastic ep-scattering

Two-photon exchange effects in inclusive DIS

Two-photon exchange effects in exclusive and semi-inclusive electroproduction of pions

Complete radiative correction in O(α_{em})



Radiative Corrections:

- Electron vertex correction (a)
- Vacuum polarization (b)
- Electron bremsstrahlung (c,d)
- Two-photon exchange (e,f)
- Proton vertex and VCS (g,h)
- Corrections (e-h) depend on the nucleon structure

•Meister&Yennie; Mo&Tsai

•Further work by Bardin&Shumeiko; Maximon&Tjon; AA, Akushevich, Merenkov;

•Guichon&Vanderhaeghen'03:
Can (e-f) account for the Rosenbluth vs. polarization experimental discrepancy? Look for ~3% ...

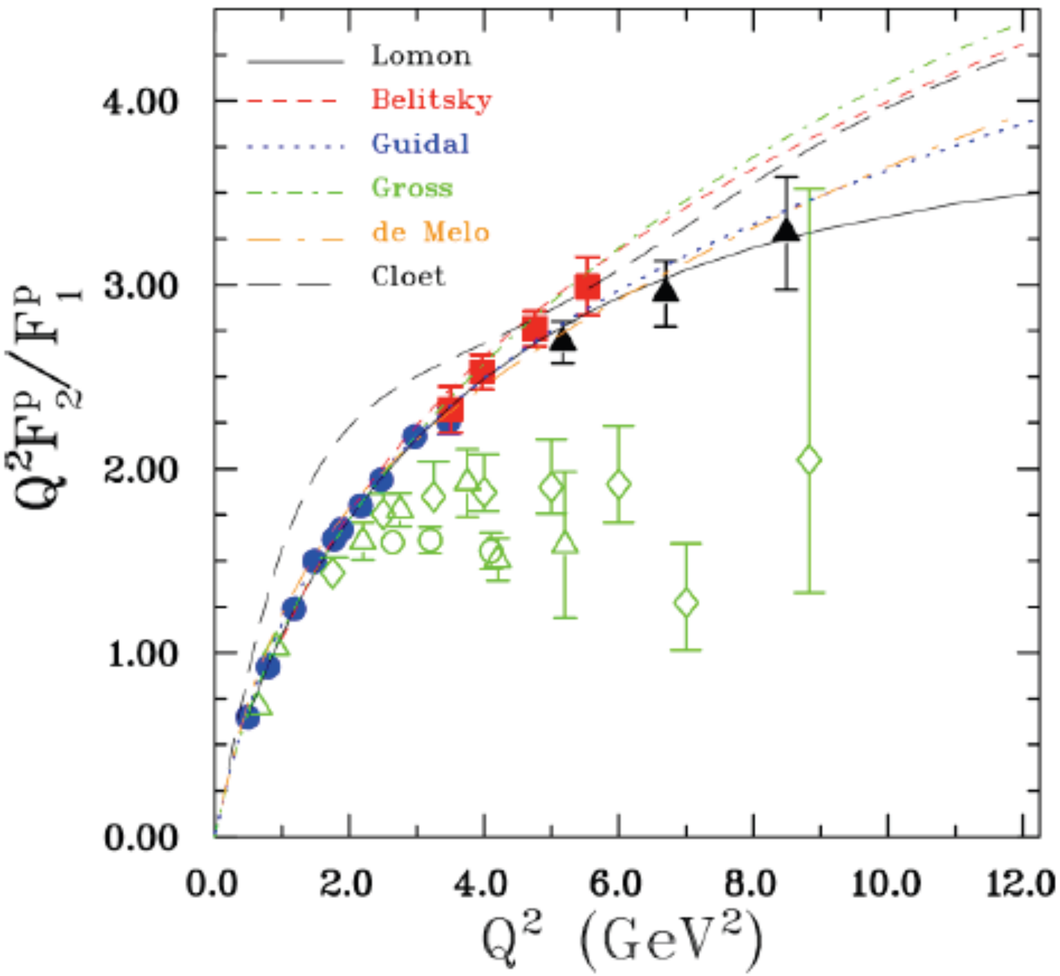
Main issue: Corrections dependent on nucleon structure

Model calculations:

•Blunden, Melnitchouk,Tjon, Phys.Rev.Lett.**91**:142304,2003

•Chen, AA, Brodsky, Carlson, Vanderhaeghen, Phys.Rev.Lett.**93**:122301,2004

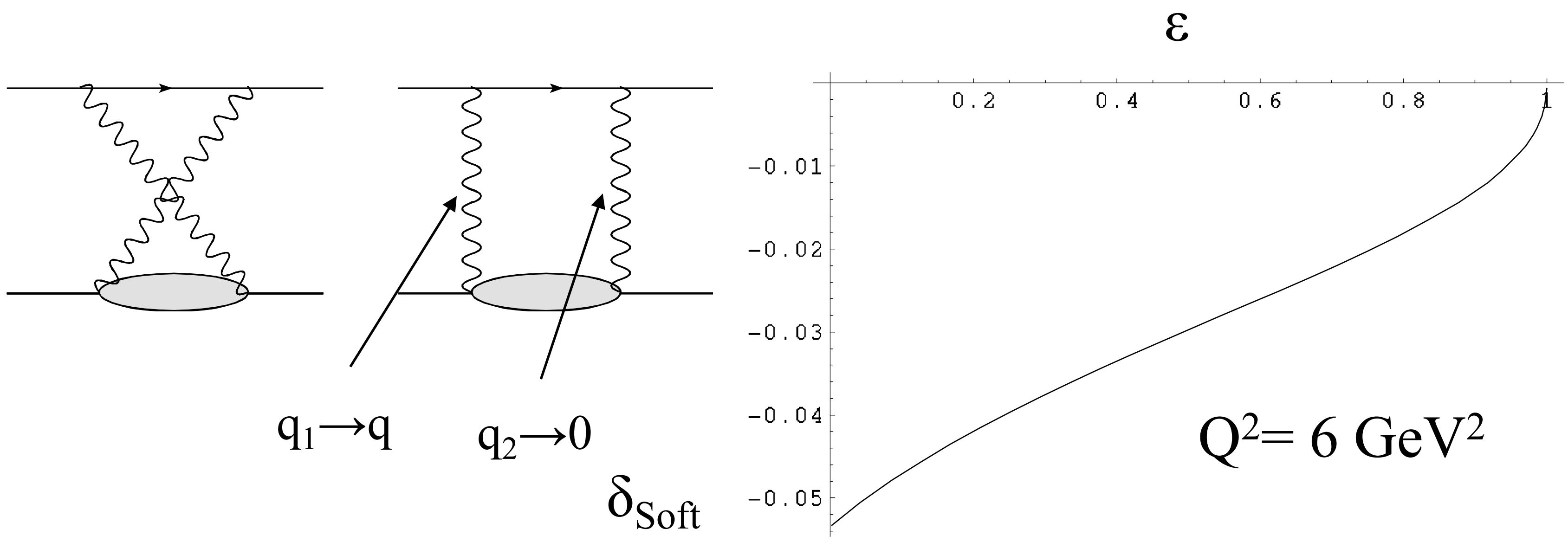
Proton Form Factors: Experiment vs Theory



- **Theory curves:**
- Lomon 2002, 2006 (VMD)
- Belitsky 2003 (pQCD scaling)
- Guidal 2005 (GPD)
- Gross, Ramalho, Pena 2008 (covariant spectator model)
- de Melo 2009 (Bethe-Salpeter Amplitude)
- Cloet 2009 (Dyson-Schwinger/Faddeev/quark-diquark)

Separating *soft* 2-photon exchange

- Tsai; Maximon & Tjon (k→0); similar to Coulomb corrections at low Q^2
- Grammer &Yennie prescription PRD 8, 4332 (1973) (also applied in QCD calculations)
- Shown is the resulting (soft) QED correction to cross section
- **Already included in experimental data analysis**
- NB: Corresponding effect to polarization transfer and/or asymmetry is zero



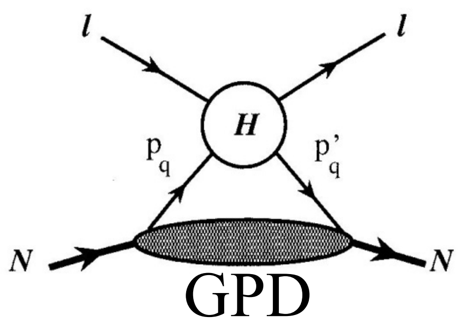
What is missing in the calculation?

- 2-photon exchange contributions for non-soft intermediate photons
  - Can estimate based on a text-book example from *Berestetsky, Lifshitz, Pitaevsky: Quantum Electrodynamics*
  - Double-log asymptotics of electron-quark backward scattering

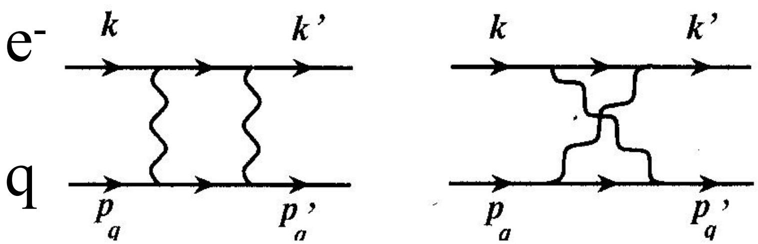
$$\delta = -\frac{e_q e}{8\pi^3} \log^2 \frac{s}{m_q^2}$$

  - Negative sign for backward ep-scattering; zero for forward scattering → Can (at least partially) mimic the electric form factor contribution to the Rosenbluth cross section
  - Numerically ~3-4% (for SLAC kinematics and m_q~300 MeV)
  - **Motivates a more detailed calculation of 2-photon exchange at quark level**

"GPD-based approach"



Model schematics:
- Hard eq-interaction
- GPDs describe quark emission/absorption
- Soft/hard separation
  - Use Grammer-Yennie prescription



Hard interaction with a quark

AA, Brodsky, Carlson, Chen, Vanderhaeghen, Phys.Rev.Lett.**93**:122301,2004; Phys.Rev.D**72**:013008,2005

Note also: "QCD factorization" approach (Kivel, Vanderhaeghen, PRL 103:092004,2009) uses pQCD for VCS amplitude calculation

Short-range effects; on-mass-shell quark (AA, Brodsky, Carlson, Chen, Vanderhaeghen)

Two-photon probe directly interacts with a (massless) quark
Emission/reabsorption of the quark is described by GPDs

$$A^{2\gamma}_{eq\to eq} = \frac{e_q^2}{t}\frac{\alpha_{em}}{2\pi}(V^e_\mu \otimes V^q_\mu \times f_V + A^e_\mu \otimes A^q_\mu \times f_A),$$

$$V^{e,q}_\mu = \bar{u}_{e,q}\gamma_\mu u_{e,q},\ A^{e,q}_\mu = \bar{u}_{e,q}\gamma_\mu\gamma_5 u_{e,q}$$

$$f_V = -2[\log(-\frac{u}{s}) + i\pi]\log(-\frac{t}{\lambda^2}) - \frac{t}{2}[\frac{1}{s}(\log(\frac{u}{t}) + i\pi) - \frac{1}{u}\log(-\frac{s}{t})] +$$

$$+\frac{(u^2 - s^2)}{4}[\frac{1}{s^2}(\log^2(\frac{u}{t}) + \pi^2) + \frac{1}{u^2}\log(-\frac{s}{t})(\log(-\frac{s}{t}) + i2\pi)] + i\pi\frac{u^2 - s^2}{2su}$$

$$f_A = -\frac{t}{2}[\frac{1}{s}(\log(\frac{u}{t}) + i\pi) + \frac{1}{u}\log(-\frac{s}{t})] +$$

$$+\frac{(u^2 - s^2)}{4}[\frac{1}{s^2}(\log^2(\frac{u}{t}) + \pi^2) - \frac{1}{u^2}\log(-\frac{s}{t})(\log(-\frac{s}{t}) + i2\pi)] + i\pi\frac{t^2}{2su}$$

Note the additional effective (axial-vector)^2 interaction; absence of mass terms;
The amplitude has a non-zero imaginary part for scattering on a free quark

Quark-level calculations for elastic ep

- Kivel, Vanderhaeghen
  - SCET, JHEP 1304 (2013) 029
- pQCD calculations, Phys.Rev.Lett. 103 (2009) 092004
  - Two photons couple to separate quarks, need one less hard gluon to transfer a large momentum to a nucleon
- See Afanasev, Blunden, Hassell, Raue, Prog. Part. Nucl. Phys. 95, 245 (2017).

Quark-Parton Calulations (cont)

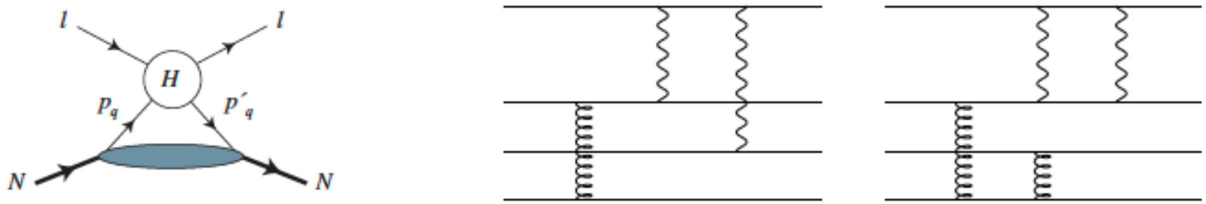


Figure 2.11: *Left:* TPE diagram in the GPD-based approach to eN scattering at high Q^2 [47, 48]. Both photons interact with the same quark, while the others are spectators. *Right:* Sample TPE diagrams in the QCD factorization approach. For the leading order term the photons interact with different quarks, with a single gluon exchange. The interaction of two photons with the same quark is of subleading order in this approach, as it involves two gluons. Figures taken from Ref. [49].

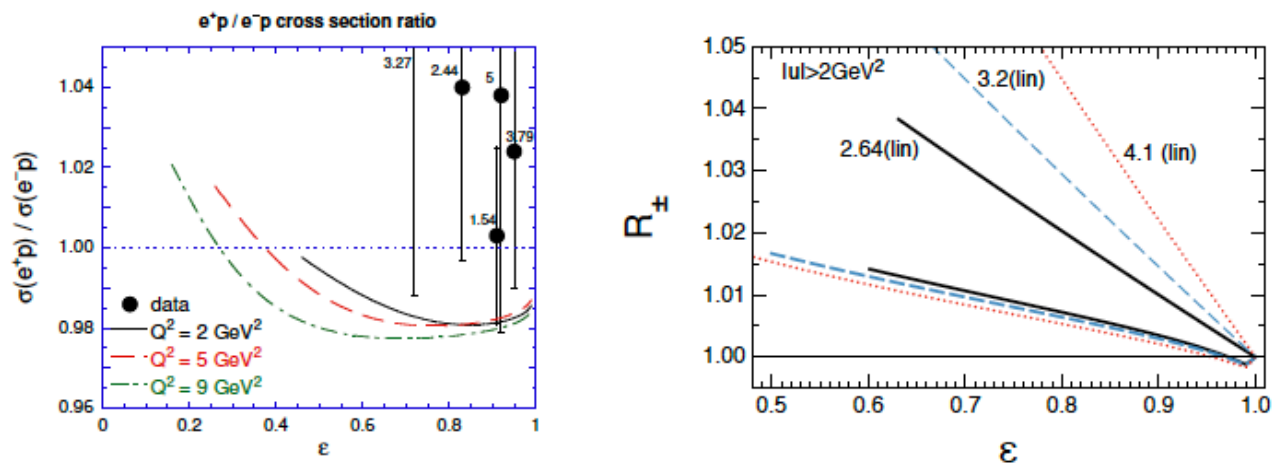
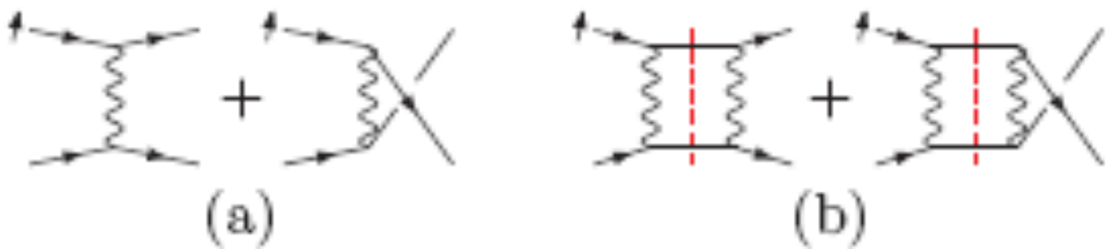


Figure 2.12: *Left:* Ratio of e^+p/e^-p elastic cross sections, taken from Ref. [48]. The GPD calculations for the TPE correction are for three fixed Q^2 values of 2, 5, and 9 GeV^2, for the kinematical range where $-u$ is above M^2. Also shown are early SLAC data [66], with Q^2 above 1.5 GeV^2. The numbers near the data give Q^2 for that point in GeV^2. *Right:* Ratio of e^+p/e^-p at high Q^2 calculated in the QCD factorization approach [65]. Also shown for comparison are the results (labelled *lin*) from the from the phenomenological fits of Ref. [67]. Figure taken from Ref. [65].

QED Spin asymmetries
Normal Beam Asymmetry in Moller Scattering

- Pure QED process, $e^-+e^-\to e^-+e^-$
  - Barut, Fronsdal , Phys.Rev.120:1871 (1960): Calculated the asymmetry in first non-vanishing order in QED O(α)
  - Dixon, Schreiber, Phys.Rev.D69:113001,2004, Erratum-ibid.D71:059903,2005: Calculated O(α) correction to the asymmetry



$$A_n \propto \frac{2M_\gamma \,\mathrm{Im}(M_{2\gamma})}{M_\gamma^2} \xrightarrow{\sqrt{s}>>m_e} \alpha \frac{m_e}{\sqrt{s}} f(\theta)$$

SLAC E158 Results (K. Kumar, private communication):
An(exp)=7.04±0.25(stat) ppm
An(theory)=6.91±0.04 ppm

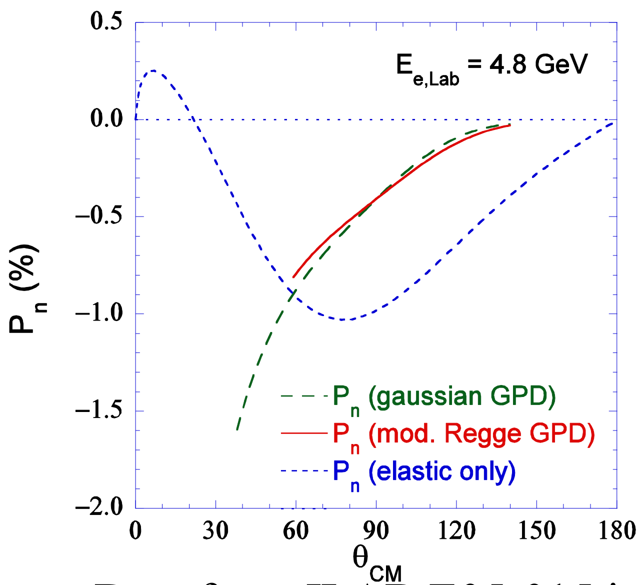
Elastic ep->ep
Quark+Nucleon Contributions to Target Asymmetry

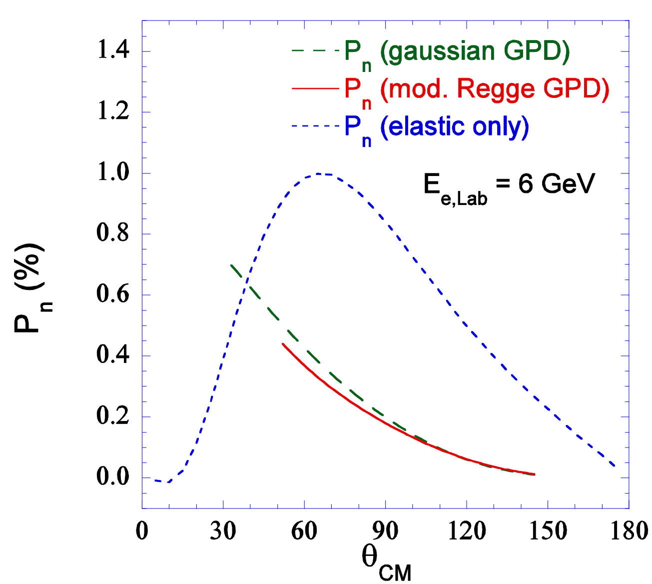
- Single-spin asymmetry or polarization normal to the scattering plane
- Handbag mechanism prediction for single-spin asymmetryof elastic eN-scattering on a polarized nucleon target (AA, Brodsky, Carlson, Chen, Vanderhaeghen)

$$A_n = \sqrt{\frac{2\varepsilon(1+\varepsilon)}{\tau}} \frac{1}{\sigma_R} \left[G_E \operatorname{Im}(A) - \sqrt{\frac{1+\varepsilon}{2\varepsilon}} G_M \operatorname{Im}(B) \right]$$

Only minor role of quark mass

No dependence on GPD $\widetilde{H}$

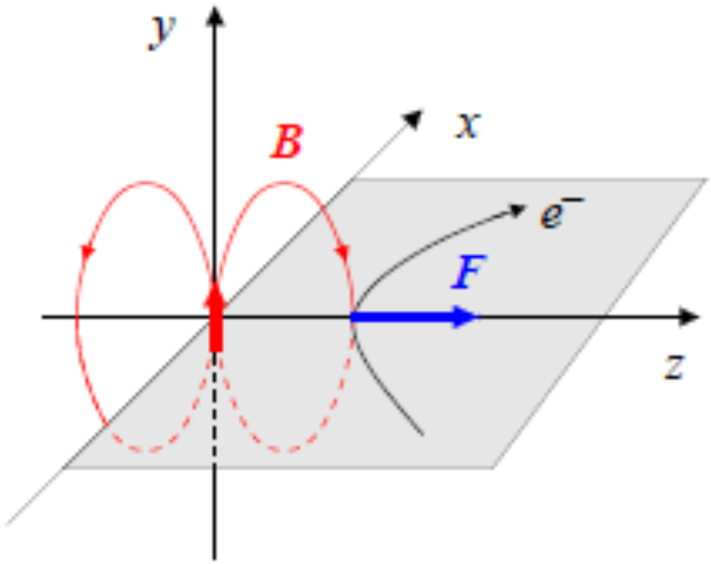




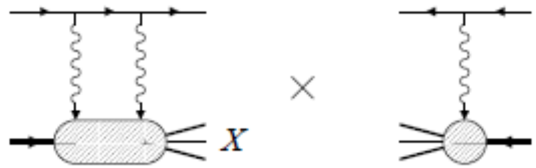
Data from JLAB E05-015 is in agreement with partonic picture.
(Inclusive scattering on normally polarized ^{3}He in Hall A)

Parity-Conserving Single-Spin Asymmetry

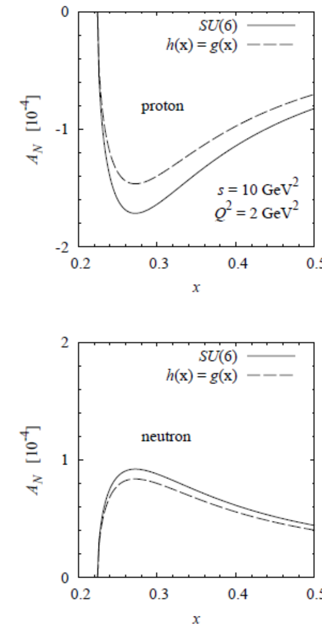
- Classical analogue: a Lorentz force **F** acting on charge moving in the magnetic field **B** of a dipole



Two-Photon Exchange in inclusive DIS



- Theory: Afanasev, Strikman, Weiss, **Phys.Rev.D77:014028,2008**
  - Asymmetry due to 2γ-exchange ~1/137 suppression
  - Addional suppression due to transversity parton density => predict asymmetry at ~10^{-4} level
  - EM gauge invariance is crucial for cancellation of collinear divergence in theory predictions
  - Hadronic non-perturbative ~1% vs partonic 10^{-4}
- Prediction consistent with HERMES measurements who set upper limits ~(0.6-0.9)x10^{-3} : **Phys.Lett.B682:351-354,2010**
- **In contradiction to JLAB observation of per-cent asymmetry** J. Katich et al. Phys. Rev. Lett. **113,** 022502 (2014).



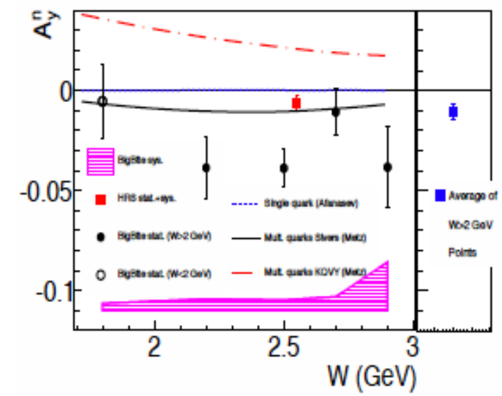
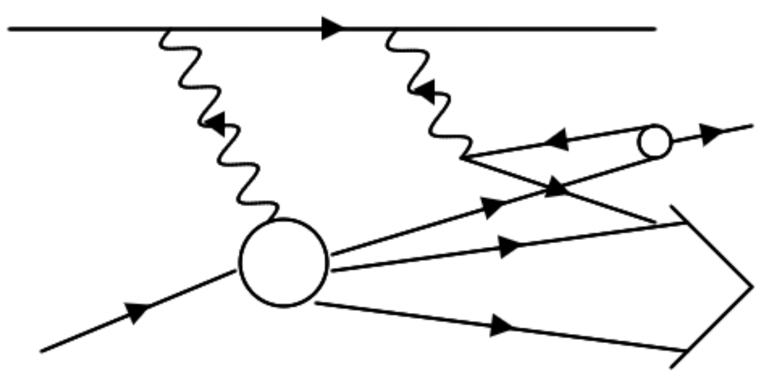


FIG. 3. Neutron asymmetry results (color online). **Left panel:** Solid black data points are DIS data ($W > 2$ GeV) from the BigBite spectrometer; open circle has $W = 1.72$ GeV. BigBite data points show statistical uncertainties with systematic uncertainties indicated by the lower solid band. The square point is the LHRS data with combined statistical and systematic uncertainties. The dotted curve near zero (positive) is the calculation by A. Afanasev *et al.* [11], The solid and dot-dashed curves are calculations by A. Metz *et al.* [12] (multiplied by −1). **Right panel:** The average measured asymmetry for the DIS data with combined systematic and statistical uncertainties.

Two-Photon Fragmentation for SIDIS

- Emission of an additional photon that converts into quark-antiquark pair leads do an additional mechanism for fragmentation
  - Produced hadron may be kinematically isolated (similar to higher-twist Berger's mechanism)



Work by Andreas Metz and collaborators

- Important: Inclusive asymmetries from TPE, coupling to the same quark vs different quarks A. Metz, D. Pitonyak, A. Schafer, M. Schlegel, W. Vogelsang, J. Zhou, Phys.Rev. D**86** (2012) 114020
- SIDIS: Metz et al, Few Body Syst. **56** (2015) 331-336
- Emphasized sin(2φ) effect for SIDIS arising from two-photon exchange

Target asymmetry:

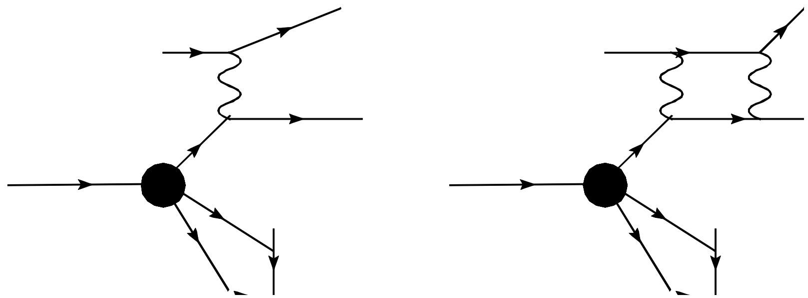
$$A_{LU}^{\sin(2\phi)} = \alpha \frac{y\left(1+\frac{2-y}{1-y}\ln y\right)}{1-y+\frac{1}{2}y^2} \sin(2\phi) \frac{\sum_q e_q^3 \mathscr{C}\left[\frac{2(\hat{h}\cdot\vec{k}_T)(\hat{h}\cdot\vec{p}_T)-\vec{k}_T\cdot\vec{p}_T}{2Mm_\pi} h_1^{\perp q} H_1^{\perp q}\right]}{\sum_q e_q^2 \mathscr{C}\left[f_1^q D_1^q\right]}$$

$$A_{UT}(x_B, y, \phi_s) = \alpha \frac{x_B M}{2Q} \frac{y(1-y)\sqrt{1-y}}{1-y+\frac{1}{2}y^2} |\vec{S}_T| \sin(\phi_s) \left(\ln\frac{Q^2}{\lambda^2} + \text{finite}\right) \frac{\sum_q e_q^3 g_T^q(x_B)}{\sum_q e_q^2 f_1^q(x_B)}$$

Beam SSA

- Beam SSA in inclusive ep-scattering
- Due to absorptive part of two-photon amplitude
- Measured at JLAB PVDIS (only upper limit in ~50ppm is set)
  - Asymmetry suppressed by a factor of electron mass/energy
  - Predicted at fraction of ppm for leading-order partonic model
  - Theory also in Metz, Schlegel, Goeke (2006)

Partonic-Level Effect



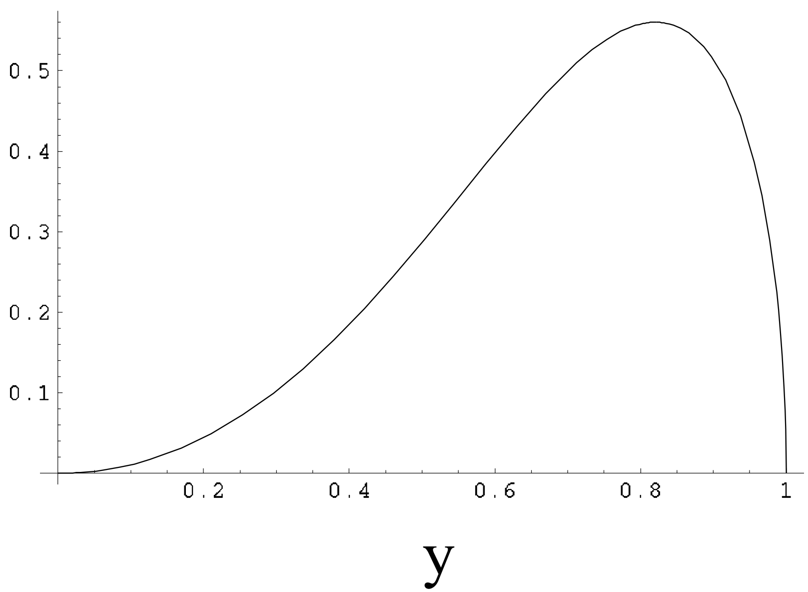
- Interference of 1-photon and 2-photon exchange is responsible for the beam single-spin normal asymmetry (SSNA)
- Adapting Barut & Fronsdal, Phys.Rev. **120** (1960) 1891, we get at the leading twist:

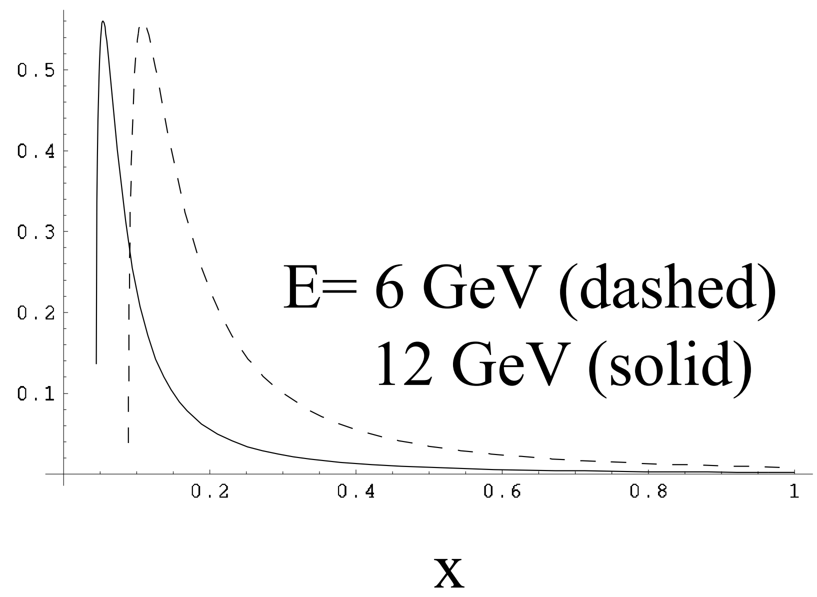
$$A_n^{Beam} = \frac{\alpha\, y^2 \sqrt{1-y^2}}{1+(1-y)^2} \frac{m_e}{Q} \sum_q (e_q)^3$$

Magnitude of Beam SSA in Inclusive DIS

$Q^2 = 1\ GeV^2$

Beam Asymmetry, ppm

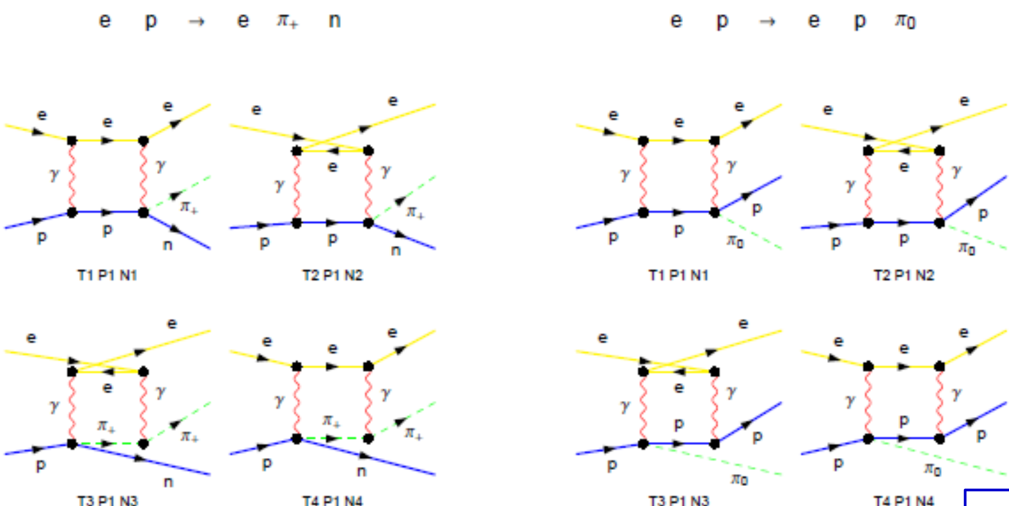


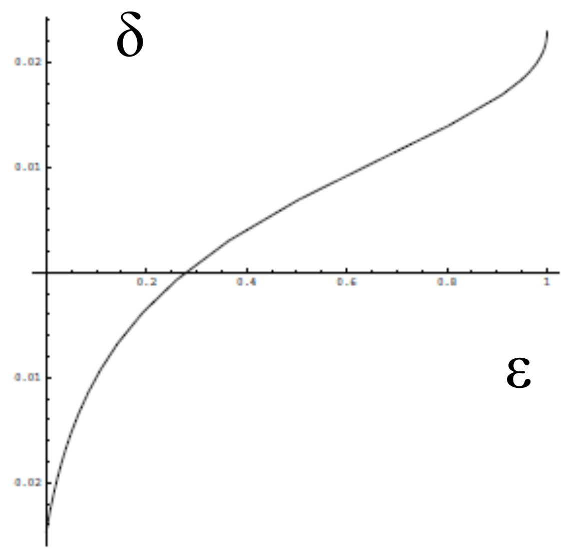


The leading-twist calculation predicts the effect around ½ ppm
QED+non-perturbative QCD: 10-100ppm
May be observed in next-generation PVDIS experiments

Two-Photon Exchange in Exclusive Electroproduction of Pions

- Standard contributions considered, e.g., AA, Akushevich, Burkert, Joo, **Phys.Rev.D66:074004,2002** (Code EXCLURAD used for data analysis)
- *Additional contributions due to two-photon exchange*, calculated by AA, Aleksejevs, Barkanova, **Phys.Rev. D88: 053008, 2013** Calculated in soft-photon approximation





Calculated ε-dependence of TPE correction. Q^2=6 GeV^2, W=3.2 GeV, E_e=5.5 GeV . Shows ±2% variation with ε.

Results for Exclusive Pion Production Phys.Rev. D88 (2013) 053008

- Soft photon exchange
- Dependence on IR photon separation
- Obtained model-independent corrections, applicable to SIDIS
- Soft-photon contributions expressed in terms of Passarino-Veltman integrals
- Can be added to HAPRAD and studied for specific experimental conditions (AA, Barkanova, Aleksejevs; Akushevich, Ilychev, Avakian)
- Equally applicable to muon scattering (important for DVMP at COMPASS)

Angular dependence of "soft" corrections
Phys.Rev. D88 (2013) 053008

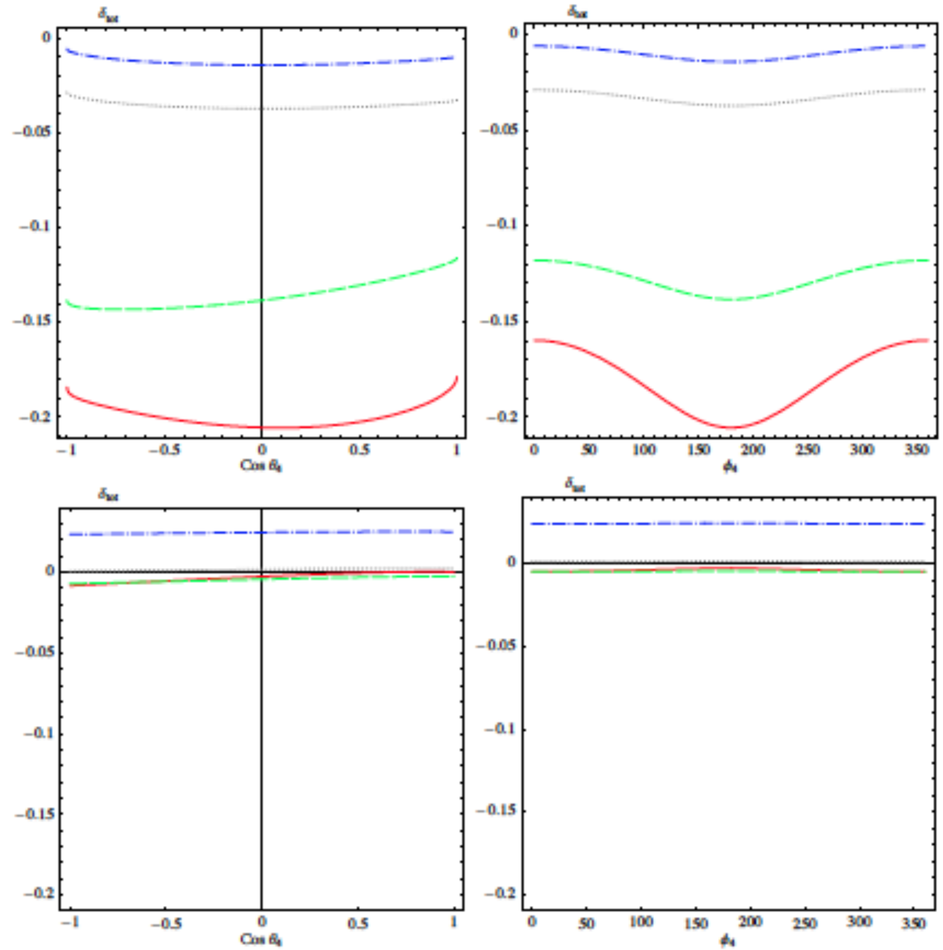


Figure 3: π^0 electroproduction two-photon box correction angular dependencies for the high $Q^2 = 6.36\,GeV^2$ (top row) and low $Q^2 = 0.4\,GeV^2$(bottom row) momentum transfers, $W = 1.232\,GeV$ and $E_{lab} = 5.75\,GeV$. Left column: dependence on $\cos\theta_4$ with $\phi_4 = 180°$. Right column: dependence on ϕ_4 with $\theta_4 = 90°$. Dot-dashed curve - SPT, dotted curve - SPT with $\alpha\pi$ subtracted, dashed curve - SPMT, solid curve - FM approach.

"Soft" two-photon corrections significantly affect angular dependences

Summary on QED loops

- Two-photon exchange
  - "Soft" photon corrections essential for cross section measurements, do not change spin asymmetries, model-independent
  - "Hard" photon corrections, alter spin structure of the amplitude, generate single-spin asymmetries, alter double-spin asymmetries
  - SSA may come with large logs (beam) or not (target)
  - SSA due to 2-photon exchange have distinctly different features from, eg. Collins and Sivers effects (would not integrate to zero wrt azimuthal angle) but need to be included in analysis
- *JLAB experiments on SSA indicate QED loop effects of the same order as SSA from strong interactions*
- Experimentally can be, e.g, extracted from sin(2φ) helicity asymmetries due to both QED loops and bremsstrahlung
- Or by comparing SIDIS with electron and positron beams

Strategy for SIDIS

- Model development of QED loop effects at partonic level
  - Soft/hard scale separation
- Integration with self-consistent covariant approaches to soft+hard radiation
- Inclusion into Monte-Carlo and/or semi-analytic approaches for SIDIS analysis