Radiative Corrections in SIDIS: New Approaches and Recent Developments

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Outline

- From Mo and Tsai Methods to Current RC Calculations: Brief Historic Review
  - Mo and Tsai approach, Δ-dependence, peaking approximation
  - Covariant approaches of Bardin and Shumeiko
  - POLRAD 2.0: Properties and opportunities
  - Other codes and other research groups dealing with RC

- Radiative Correction in SIDIS
  - Contributions to RC calculation and available codes
  - Hadronic tensor and exact calculation for SIDIS with polarized particles
  - Structure functions and RC procedure of experimental data
  - Unfolding procedure

Mo and Tsai Approach for Elastic and Inelastic Processes

- Mo and Tsai firstly elaborated a systemic approach to calculate radiative corrections in elastic and inelastic electron scattering.
- For inelastic and deep inelastic processes they showed that actual Q^2 and W^2 going to hadronic part cover a wide kinematic region including the resonance region.
- Also they proved that elastic processes with the radiated photon (so-called radiative tail from elastic peak or simply elastic radiative tail) has to be added as a contribution to the total RC.
- One assumption (and limitation) in their calculations was the approximate way to consider the soft-photon contribution
  - Specifically, they introduced a parameter Δ such that $\Delta \ll m_e, E, E'$.
  - Then they considered the region over photon energy ω and kept only the leading term $1/\omega$. This allowed them to calculate the term with the soft photons analytically (even with a photon mass λ) and extract infrared divergence in the form of $\log(m_e/\lambda)$. The infrared divergence is canceled with respective term obtained when calculating loop diagrams (i.e., the vertex function).
  - The correction from the region above Δ is evaluated numerically.

Mo and Tsai Approach: Δ-Dependence

- An artificial parameter Δ had to be introduced to divide the integration region over the photon energy into two parts, the soft and the hard energy regions.
  - The hard energy region $\sigma_{in}(\Delta)$ can be calculated without any approximations.
  - The soft photon part is calculated for photon energies approaching zero.

$$\sigma_{rad} = \int_0^{\omega_{max}} d\omega\ \sigma_{rad}(\omega) = \int_0^{\Delta} d\omega\ \sigma_{rad}(\omega) + \int_{\Delta}^{\omega_{max}} d\omega\ \sigma_{rad}(\omega) = \int_0^{\Delta} \frac{d\omega}{\omega} \tilde{\sigma}_{rad}(0) + \int_{\Delta}^{\omega_{max}} d\omega\ \sigma_{rad}(\omega)$$

Then the first term is regularized (e.g., using the photon mass λ) because of the infrared divergence that cancel with loop contributions.

- How to choose Δ : not too small (because of numeric instabilities in the second term) and not too high (because of approximations in the first integral)
- The best way: is to calculate for several Δ and identify the region where the dependence of the Δ is close to flat.
- Accuracy of the approximations in Mo and Tsai approach can be evaluated by comparing to the formulas not involving Δ

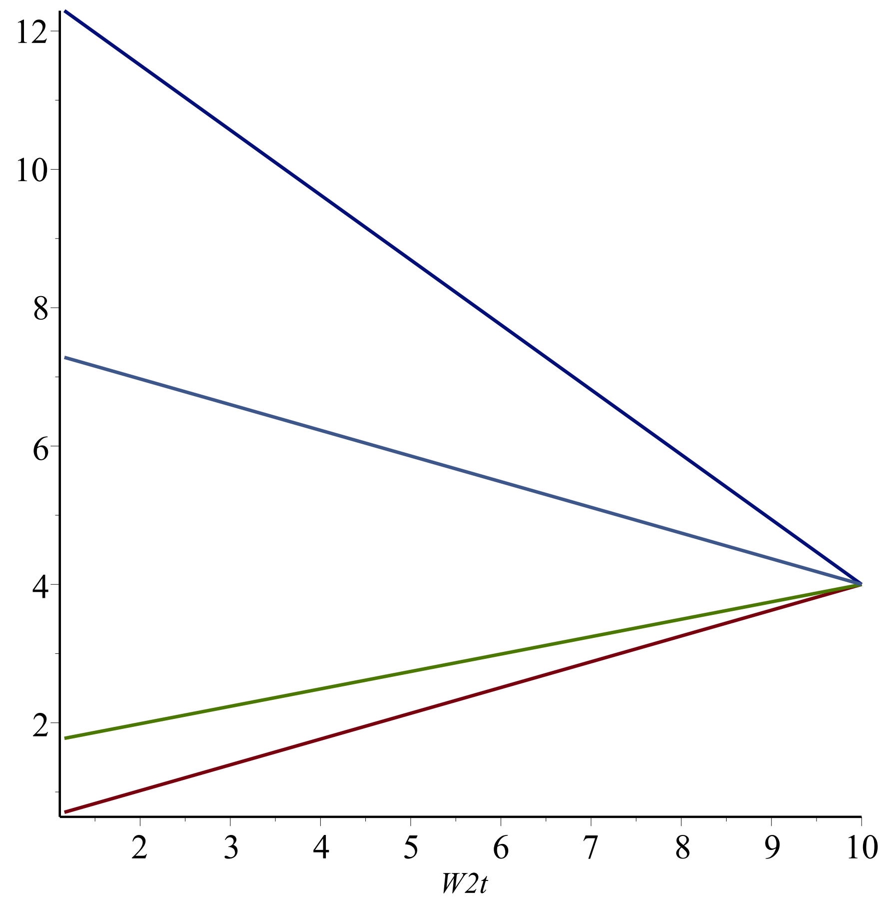
Peaking Approximation

$$\sigma_{rad}(Q^2, W^2) = \alpha C \int\limits_{(M+m_\pi)^2}^{W^2} dW_{tr}^2 \int\limits_{t_{min}}^{t_{max}} dQ_{tr}^2 \int\limits_{-\pi}^{\pi} \phi_{rad} \sum_i K_i(W^2, Q^2, W_{tr}^2, Q_{tr}^2, \phi_{rad}) F_i(W_{tr}^2, Q_{tr}^2)$$

- The cross section containes terms $1/kk_1$, $1/kk_1^2$, $1/kk_2$, $1/kk_2^2$, and terms independent on k. Peaking approximation assumes
  - Terms containing $1/kk_1$ and $1/kk_1^2$ contribute to s-peak
  - Terms containing $1/kk_2$ and $1/kk_2^2$ contribute to p-peak
  - Terms independent on k contribute to both s- and p-peak (a half to each peak)
  - Q_{tr}^2 are taken in the respective peaks: $Q_{tr}^2 \to Q_s^2$ and $Q_{tr}^2 \to Q_p^2$ in arguments of SFs, i.e., $F_i(W_{tr}^2, Q_{tr}^2) \to F_i(W_{tr}^2, Q_s^2)$ and $F_i(W_{tr}^2, Q_{tr}^2) \to F_i(W_{tr}^2, Q_p^2)$.
- Then integrals in respect to Q_{tr}^2 and ϕ_{rad} can be calculated.
- Quality of the approximation cannot be predicted a priory and therefore needs to be estimated by comparison with exact calculations.
- Accuracy will be inappropriate if Q_{tr}^2-dependence is not close to flat.
- Peaking approximation does not work for elastic, quasielastic radiative tals.

Integration Area in the variables Q^2_{tr} and W^2_{tr}

- Kinematic point:
  - $E_{beam} = 10GeV$
  - $Q^2 = 4GeV^2$
  - $W^2 = 10GeV^2$
  - x=0.3
  - y=0.66



Covariant Approach of Bardin and Shumeiko

- Bardin and Shumeiko improved the calculation approach in 5 aspects:
- They developed an approach for extraction and cancellation of the infrared divergence which is free of the artificial parameter Δ.
- They presented all results in the covariant form, so the formulas can be directly applied in any coordinate system.
- They developed a code TERAD that calculates RC for unpolarized target including nuclear targets; in this case radiative tail from quasielastic peak have to be considered and added.
- They suggested the radiative correction procedure of experimental data or unfolding procedure.
- With collaboration with Kukhto they obtained the formulas for RC on polarized protons.

RC to DIS of Polarized Particles: POLRAD 2.0

- We (IA, Shumeiko) essentially improved the calculation of RC to polarized targets.
  - The most essential improvement was the idea of using the basis in the four-dimensional space and of expansion polarization vectors over momenta such as momenta of initial and final electrons and initial proton. This allowed us to avoid a tedious and intricate procedure of tensor integration used before.
- We (IA, Ilyichev, Soroko, Shumeiko, Tolkachev) created the code POLRAD 2.0 that allows to calculations for:
  - RC in DIS on polarized targets of spin of $1/2$ and 1. All contributions including quasielastic radiative tail were implemented.
  - RC to quadruple asymmetry for spin-one targets.
  - RC to semi-inclusive DIS (including polarized targets) in the simple quark-parton model i.e., for the three-dimensional cross section $d\sigma/dxdydz$.
  - Approximate contribution of double bremsstrahlung
  - Electroweak effects
  - The iterative procedure of RC of experimental data

Monte Carlo Generator RADGEN

We constructed the Monte Carlo generator RADGEN using POLRAD 2.0 (IA, Boettcher, Ryckbosch, hep-ph/9906408)

The cross section is represented in the sum of two positively definite contributions

$$\sigma_{obs} = \sigma_{non-rad} + \sigma_{rad}$$

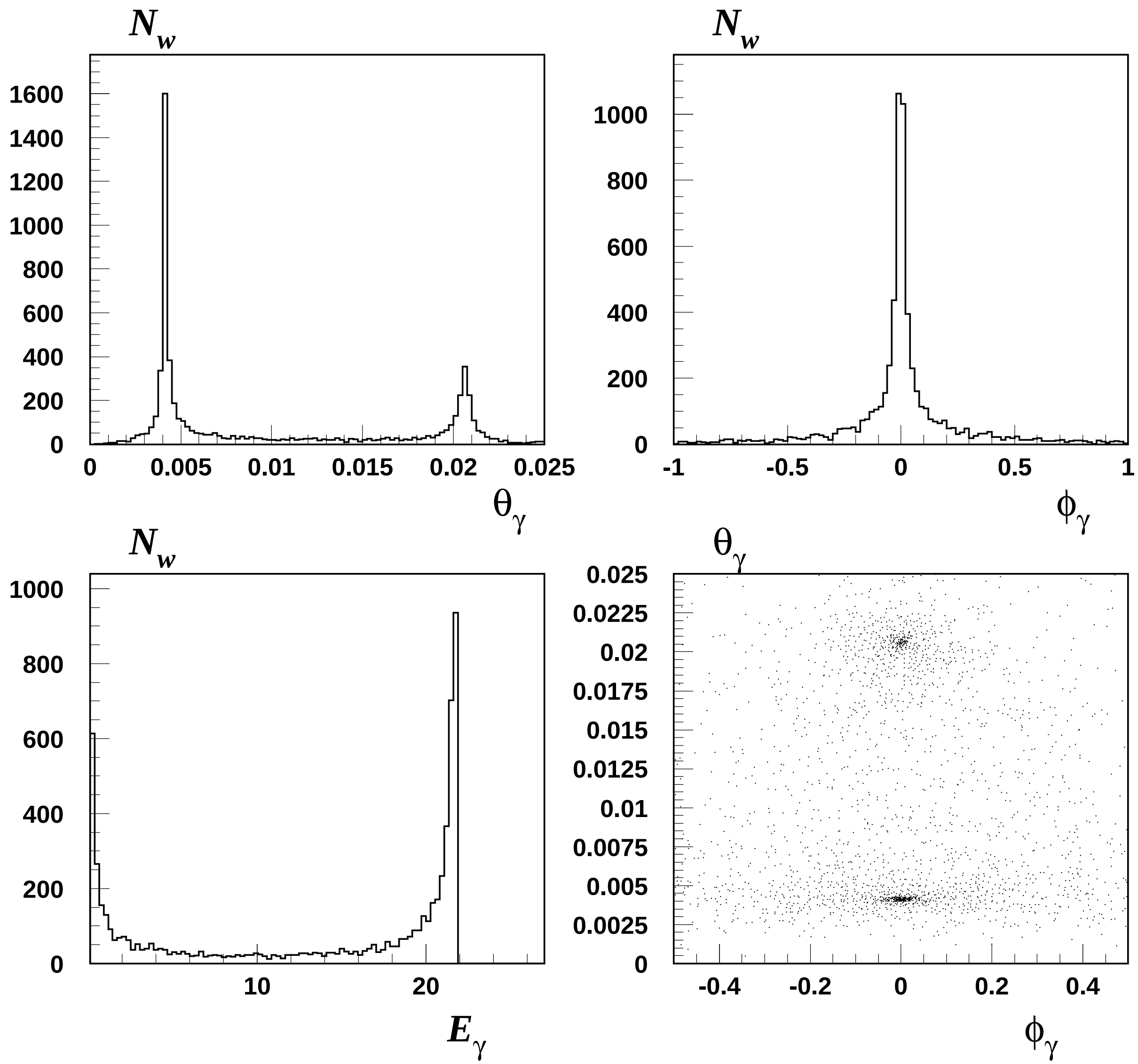
where $\sigma_{non-rad}$ contains loop diagrams and soft photon emission and σ_{rad} is the contribution of additional hard photon emission with energy larger than a minimal photon energy ϵ_{min} associated with resolution in calorimeter.

In spite of introducing the artificial parameter ϵ_{min} there is no loosing an accuracy and no acquired dependence of the cross section of this parameter

Two modes for generator operation

- integrals are calculated for each event and grid for a simulation is stored
- look-up table calculated in advance is used for interpolation of the grid

Monte Carlo Generator RADGEN



RC Procedure of Experimental Data for polarized DIS

- Sometimes experimentalists calculate RC to spin asymmetry using a RC code (e.g., POLRAD 2.0) and then add the calculated effect to observed asymmetry in a bin. This is incorrect procedure because of bias resulted from the difference between the values of g_1 finally extracted in the bin and used for RC calculation.
- Example of correct procedure is in using

$$A_{extr}(x,Q^2) = A_{meas} - \alpha C \int dx_{true} dQ^2_{true} K A_{teor}(x_{true}, Q^2_{true})$$

and solving this equation with the iteration procedure that provides equality of $A_{extr}(x,Q^2)$ and $A_{teor}(x,Q^2)$

- The similar procedure can be defined (and it is implemented in POLRAD 2.0) for the case when both g_1 and g_2 are measured in the same experiment and solving respective equations are required.
- These ideas have to be kept in mind when we define the procedure of RC of experimental data for other asymmetries, e.g., those measured in SIDIS.

Exact Calculation of the RC of the lowest order

The complete RC of the lowest order (and multiple soft photon contributions) calculated suing the covariant technique (i.e., that used to create POLRAD, DIFFRAD, EXCLURAD, and other codes) is represented in the form

$$\sigma_{obs} = \sigma_0 \exp(\delta_{inf})(1 + \delta_{VR} + \delta_{vac}) + \sigma_F$$

Here the corrections δ_{inf} and δ_{vac} come from the radiation of soft photons and the effects of vacuum polarization, the correction δ_{VR} is infrared-free sum of factorized parts of real and virtual photon radiation, and σ_F is an infrared free contribution from the process of emission of an additional real photon.

The contribution of hard photons σ_F is represented in the form of three-dimensional integral over kinematic variables of an unobserved photon.

$$\sigma_F = \alpha^4 C_{kin} \int d\Omega_k \int\limits_0^{v_m} dv \sum_n \left[\frac{v f_{kin}}{\tilde{Q}^4} L^{(n)}_{\mu\nu,\mu'} T^{(n)}_{\mu\nu,\mu'} - \frac{f^0_{kin}}{v Q^4} L^{0(n)}_{\mu\nu,\mu'} T^{0(n)}_{\mu\nu,\mu'} \right]$$

The integrals need to be calculated numerically. This integral is finite for $v \to 0$ and not positively definite.

Leading, Next-to-Leading, and Exact Contributions to RC

By "exactly" calculated RC we understand the estimation of the lowest order RC contribution with any predetermined accuracy.

The structure of the dependence on the electron mass in RC cross section:

$$\sigma_{RC} = A \log \frac{Q^2}{m^2} + B + O(m^2/Q^2)$$

where A and B do not depend on the electron mass.

$\log(\frac{Q^2}{m^2}) \sim 15$ for $Q^2 \sim$ 1GeV^2

- If only A is kept, this is the leading log approximation.
- If both contributions are kept (i.e., contained A and B), this is the calculation with the next-to-leading accuracy, practically equivalent to exact calculation.

Other Codes for RC in ep-scattering

POLRAD 2.0 *FORTRAN code for the RC procedure of experimental data in polarized inclusive and semi-inclusive DIS. The iteration procedure based on MINUIT fitting the data is included. Estimation of higher order and electroweak corrections is done.*

RADGEN *Monte Carlo generator of radiative events in the DIS on polarized and unpolarized targets. Can be applied for RC generation in inclusive, semi-inclusive and exclusive DIS processes. This version uses a look-up table for photonic angles which provides for fast event generation.*

DIFFRAD *FORTRAN code for RC calculation in the processes of electroproduction of vector mesons. Versions with Monte Carlo and numerical integrations are available. Monte Carlo code allows to estimate RC to the quasi-real photoproduction case (i.e., the final electron is not detected)*

HAPRAD *FORTRAN code for RC calculation in the processes of semi-inclusive hadron electroproduction. Versions with Monte Carlo and numerical integrations are available.*

ESFRAD *(Authors: A. Afanasev, I. Akushevich, N. Merenkov) FORTRAN code for RC calculation in the processes of elastic, inelastic and deep inelastic scattering using the method of electron structure functions.*

ELARADGEN *(Authors: A. Afanasev, I. Akushevich, A. Ilychev, B. Niczyporuk) Monte Carlo generator of radiative events in the kinematics of elastic ep-scattering measurements.*

MASCARAD *FORTRAN code for RC calculation in elastic electron-nucleon scattering with a polarized target and/or recoil polarization. The experimental acceptances are accounted for.*

EXCLURAD *(Authors: A. Afanasev, I. Akushevich, V. Burkert, K. Joo) FORTRAN code for RC calculation in the process of exclusive pi electroproduction on a nucleon.*

All these codes can be downloaded from https://www.jlab.org/RC/

Other Groups Contributed to RC in ep-scattering

Bohm, Hollik, Spiesberger: *One-loop correction in electroweak physics; general theory of renormalization in electroweak theory. We compared POLRAD 2.0 with code HERACLES produced by Hubert Spiesberger.*

Bardin et al.: *We worked in parallel using the same approach of covariant calculation of RC. We compared POLRAD 2.0 with the code HECTOR produced by Dima Bardin with collaborators.*

Eduard Kuraev: *Produced multiple brilliant results in quantum electrodynamics. I was involved in his group. In our last conversations he advised me how to construct hadronic tensor for SIDIS. We directly used the results of his group in our work on RC to DVCS such as*

- *Asymptotic expressions for loop integrals in non–collinear kinematics, JINR E2-98-53, hep-ph/0703048*
- *Approach for the calculation in leading log approximation, shifted kinematics, Phys.Rev. C77, 055206 (2008)*

Nikolai Merenkov: Andrei Afanasev and me worked a lot together for calculating RC for elastic ep scattering and producing the codes MASCARAD and ESFRAD

Marc Vanderhaeghen: One-loop correction and soft photon emission in DVCS, Phys.Rev. C62(2000)025501.

Profs. Maximon and Tjon: Phys.Rev. C62 (2000) 054320; Contributions of the box and crossed-box (two-photon exchange) diagrams.

POLRAD/RADGEN vs. HERACLES/DJANGOH

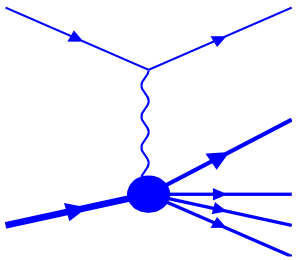
	POLRAD/RADGEN	HERACLES/DJANGOH
Exact calculation	Yes	Mo/Tsai Δ-approximation
Spin-dependent part	Yes	No
Semi-inclusive part	Yes	No
Valid for fixed targets	Yes	No
Elastic, Quasielastic Rad. Tails	Yes	No
Exclusive Rad. Tail	Yes	No
Resonance Region SFs	Yes	No
Valid for collider	Yes, but some tuning is needed	Yes
Electroweak effects	Yes, but some tuning is needed	Yes

Part II:

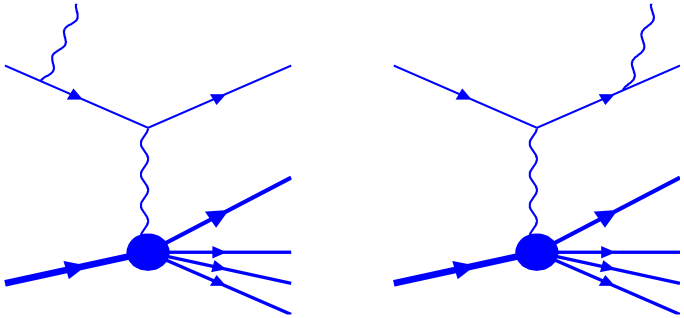
RC to SIDIS

Contribution to RC in SIDIS

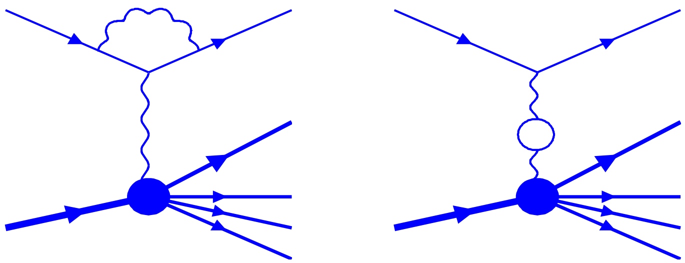
The Born cross section



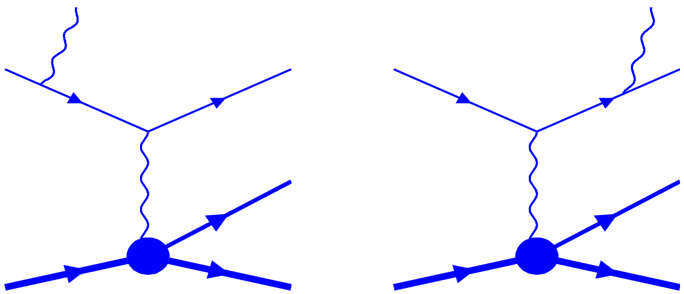
Emission of a radiated photon (semi-inclusive processes)



Loop diagrams



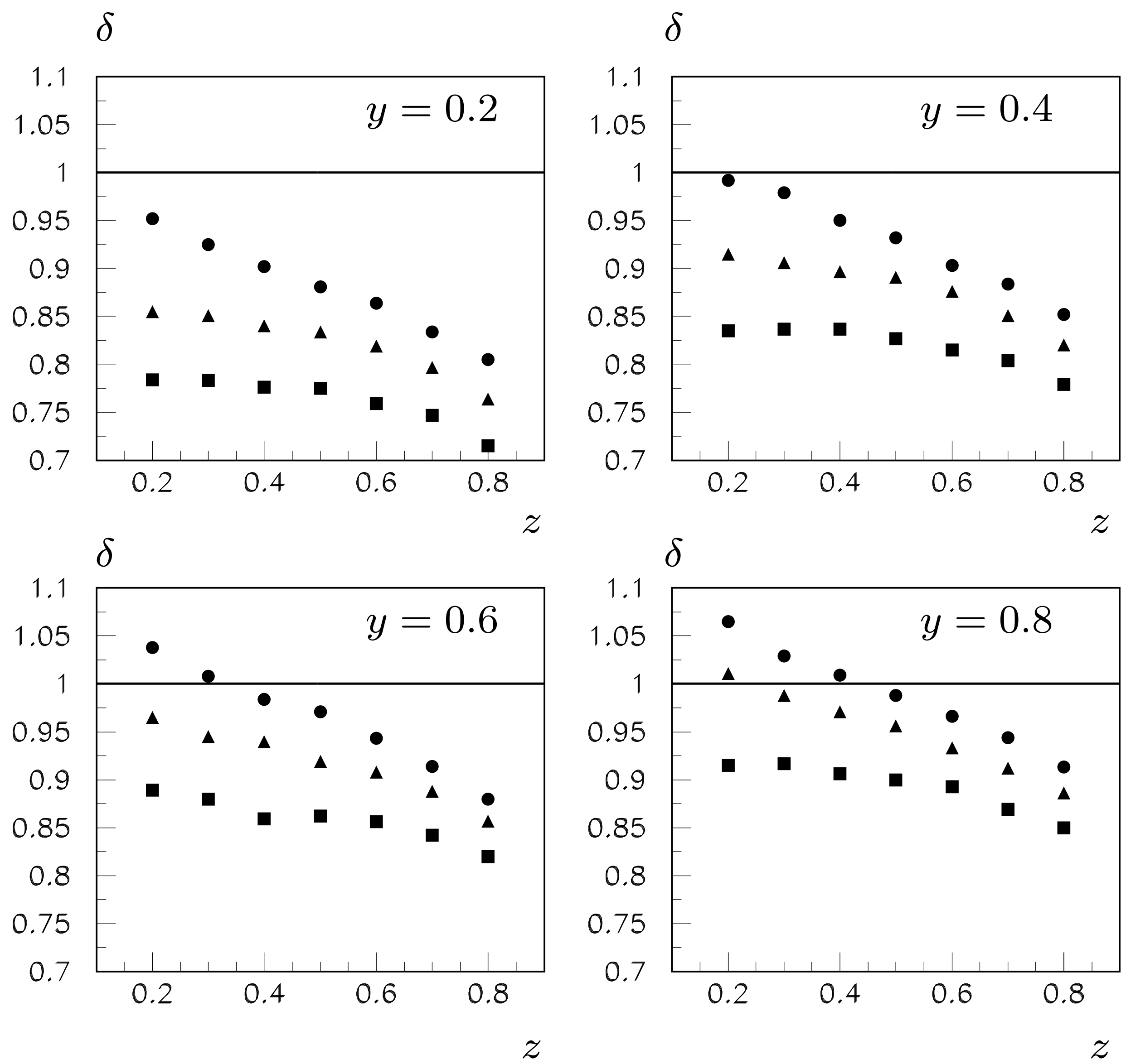
Emission of a radiated photon (exclusive processes)



Calculation of RC to SIDIS using POLRAD 2.0

- Option SIRAD has to be used.
- Simple QPM is assumed. The RC cross section is calculated in terms of parton distributions $f(x)$ and fragmentation functions $D(z)$.
- RC for unpolarized cross section and polarized part of the cross section are calculated.
- No exclusive radiative tail is separately calculated.
- Several models for parton distributions and fragmentation functions are implemented.
- RC has traditional form: factorized part representing the contributions of loops and soft photon emission and the term in the form of two-dimensional integral representing the hard photon emission.
- Integration over p_t and ϕ_h is assumed, i.e., the RC to the three-fold cross section is calculated: $d\sigma/dxdydz$.

Numerical results for pions



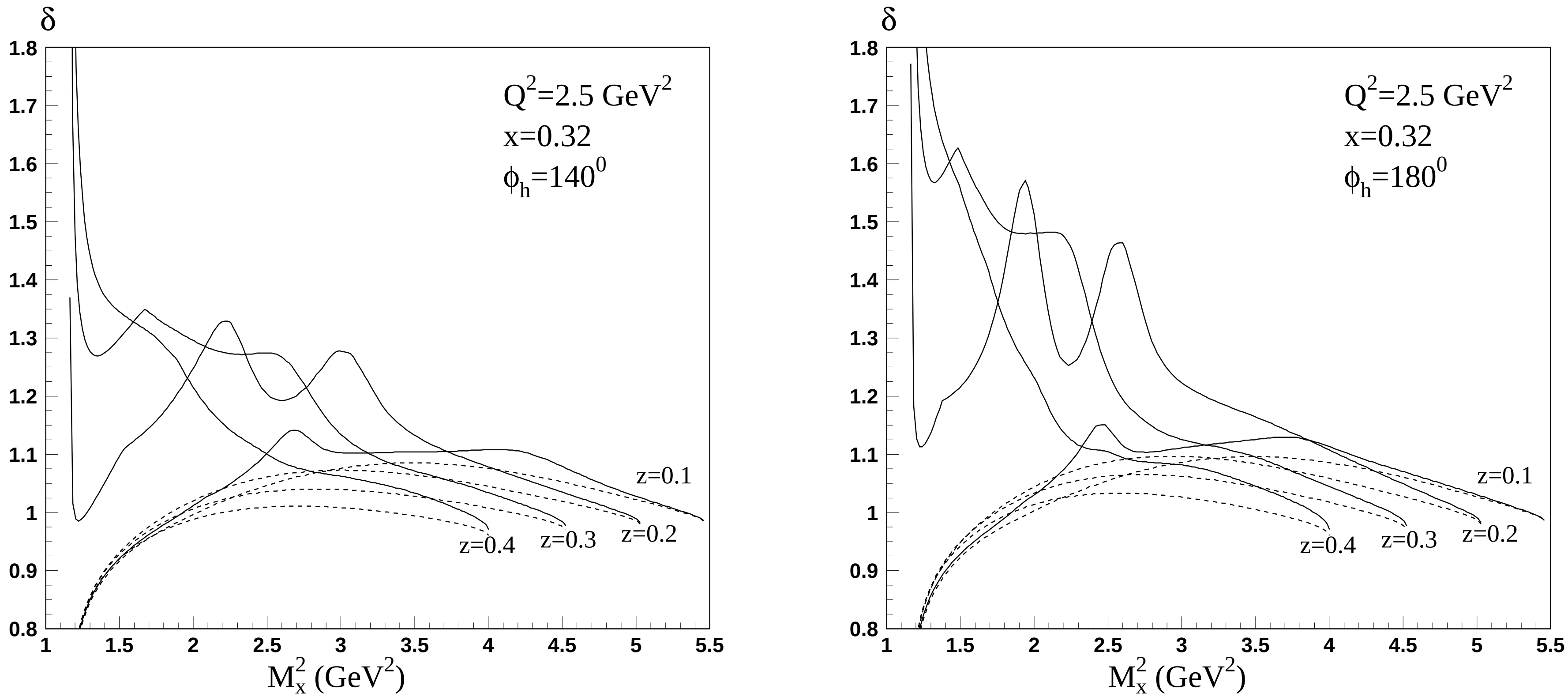
Radiative correction to the semi-inclusive cross section for kinematics of HERMES; $\sqrt{S}$=7.19 GeV. Symbols from top to bottom correspond to the x=0.05, 0.45 and 0.7. The results for x=0.15 are skipped, because they practically coincide with ones for x=0.05.

Calculation of RC to SIDIS using HAPRAD 2.0

- Original version is based on the calculation in Akushevich, Soroko, Shumeiko EPJ C10(1999)681. In this paper an approach to calculate radiative corrections to unpolarized cross section of semi-inclusive electroproduction is developed. Then the contribution of the exclusive radiative tail was calculated in (Akushevich, Ilyichev, Osipenko, Phys.Lett. B672(2009)35)
- calculate the RC to five-dimensional cross section $d^5\sigma/dxdydzdp_t^2d\theta_h$ as well as to four- and three-dimensional cross sections (i.e., $d^4\sigma/dxdydzdp_t^2d$ and $d^3\sigma/dxdydz$).
- For the three-dimensional cross section the results for RC are close to that given by POLRAD 2.0 (SIRAD)
- The correction is of standard form:

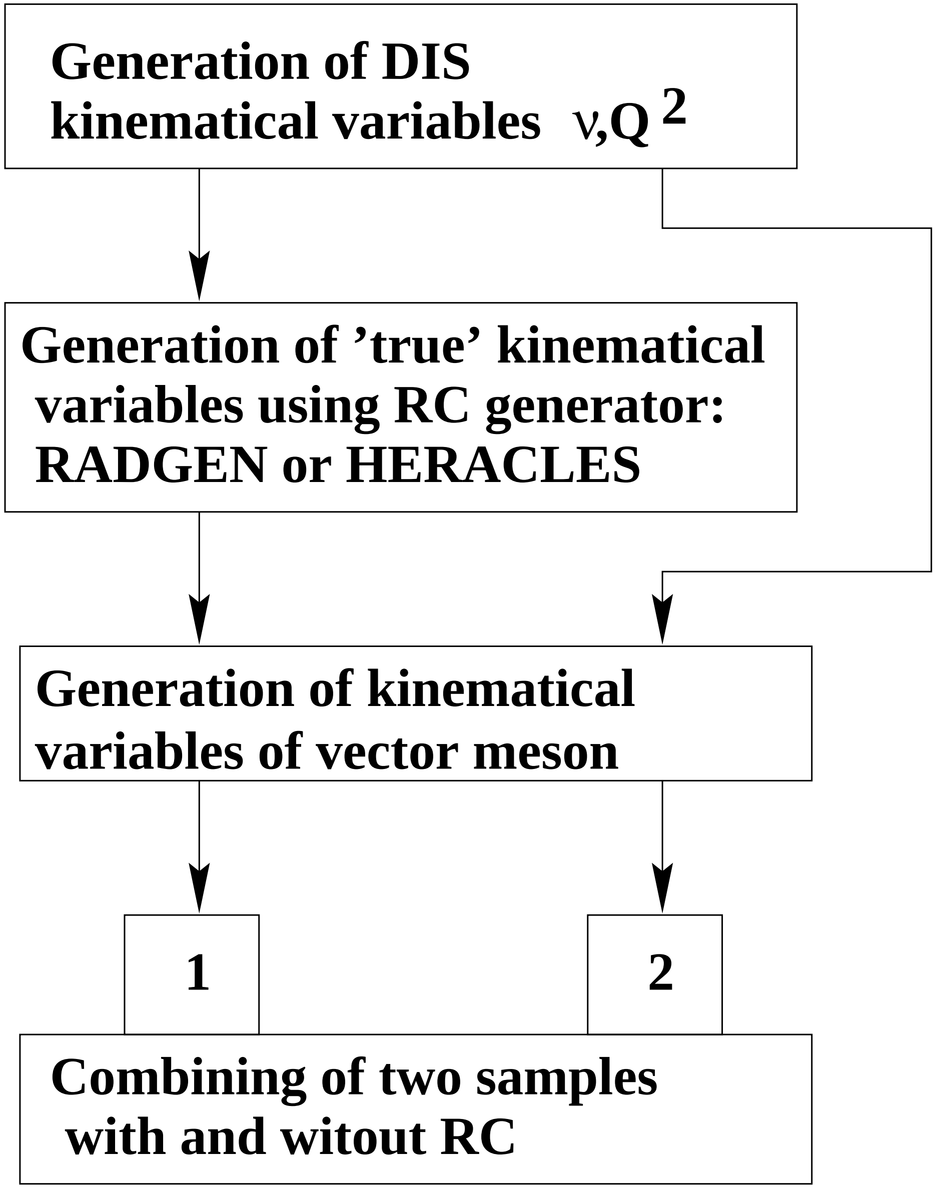
$$\sigma_{obs} = \sigma_0 e^{\delta_{inf}} (1 + \delta_{VR} + \delta_{vac}) + \sigma_F + \sigma_{exl}.$$

Importance of exclusive radiative tail



M_X^2-dependence of the RC factor for the semi-inclusive π^+ electroproduction at fixed proton for lepton beam energy 6 GeV: solid lines show the total correction, dashed lines represent the correction excluding the exclusive radiative tail (Akushevich, Ilyichev, Osipenko, Phys.Lett. B672(2009)35)

Calculation of RC to SIDIS using Monte Carlo generator



Possible scheme of Monte Carlo calculation of the RC factor (Akushevich, hep-ph/9906410)

Calculation of RC to SIDIS using Monte Carlo generator

Can we calculate RC using generators like RADGEN or DJANGOH?

My answer is NO

DIS hadronic tensor (unpolarized):

$$W_{\mu\nu} = -g_{\mu\nu}F_1 + p_\mu p_\nu F_2$$

SiDIS hadronic tensor (unpolarized):

$$W_{\mu\nu} = -g_{\mu\nu}H_1 + p_\mu p_\nu H_2 + p_{h\mu}p_{h\nu}H_3 + (p_\mu p_{h\nu} + p_{h\mu}p_\nu)H_4$$

The DIS cross section:

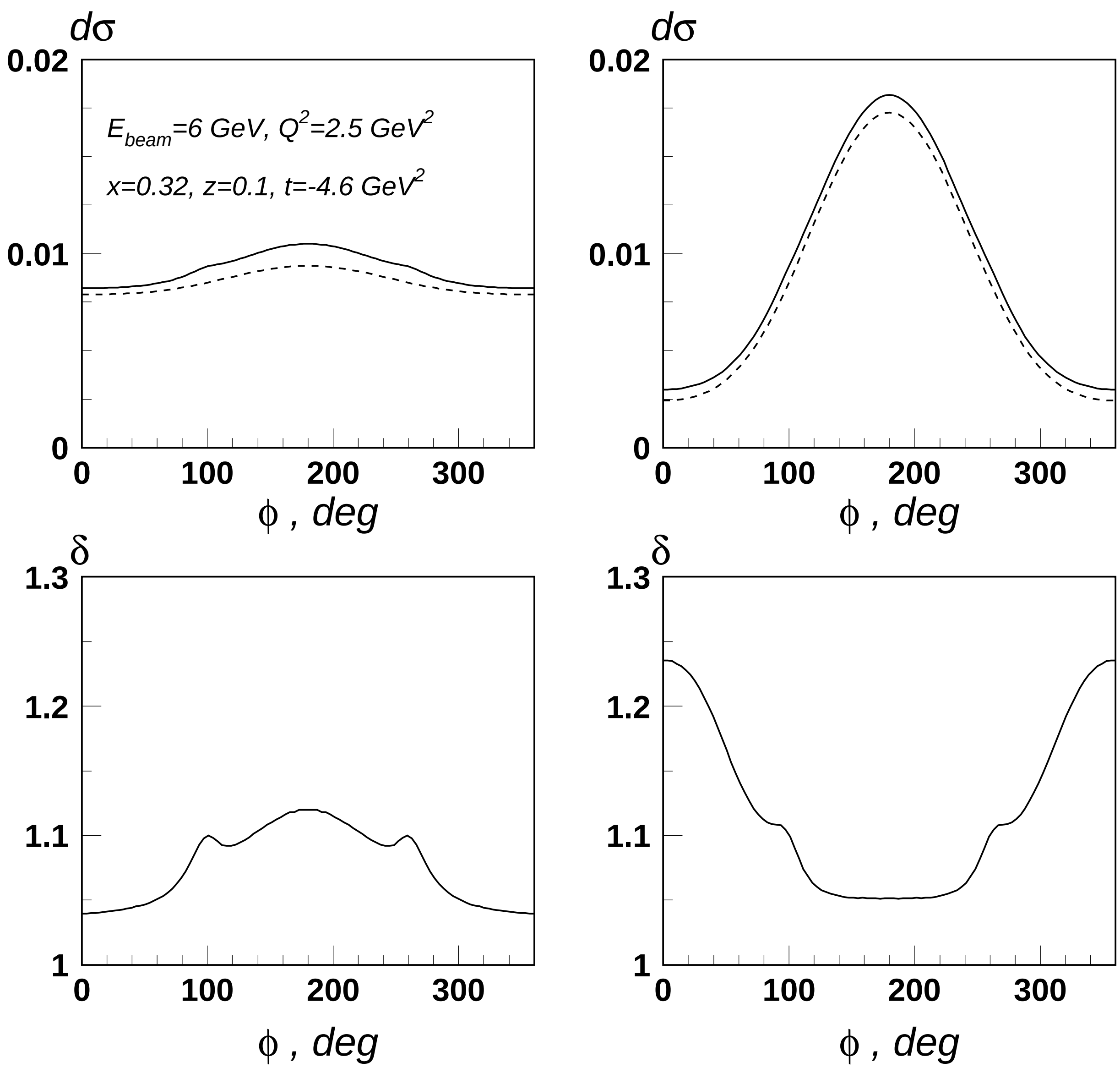
$$\sigma = K_1(x,Q^2)F_1(x,Q^2) + K_2F_2(x,Q^2)$$

The SiDIS cross section:

$$\sigma = K_1\tilde{H}_1(x,z,p_T,Q^2) + K_2\tilde{H}_2(x,z,p_T,Q^2) + K_3\tilde{H}_3(x,z,p_T,Q^2)\cos^2\phi_h + K_4\tilde{H}_4(x,z,p_T,Q^2)\cos\phi_h$$

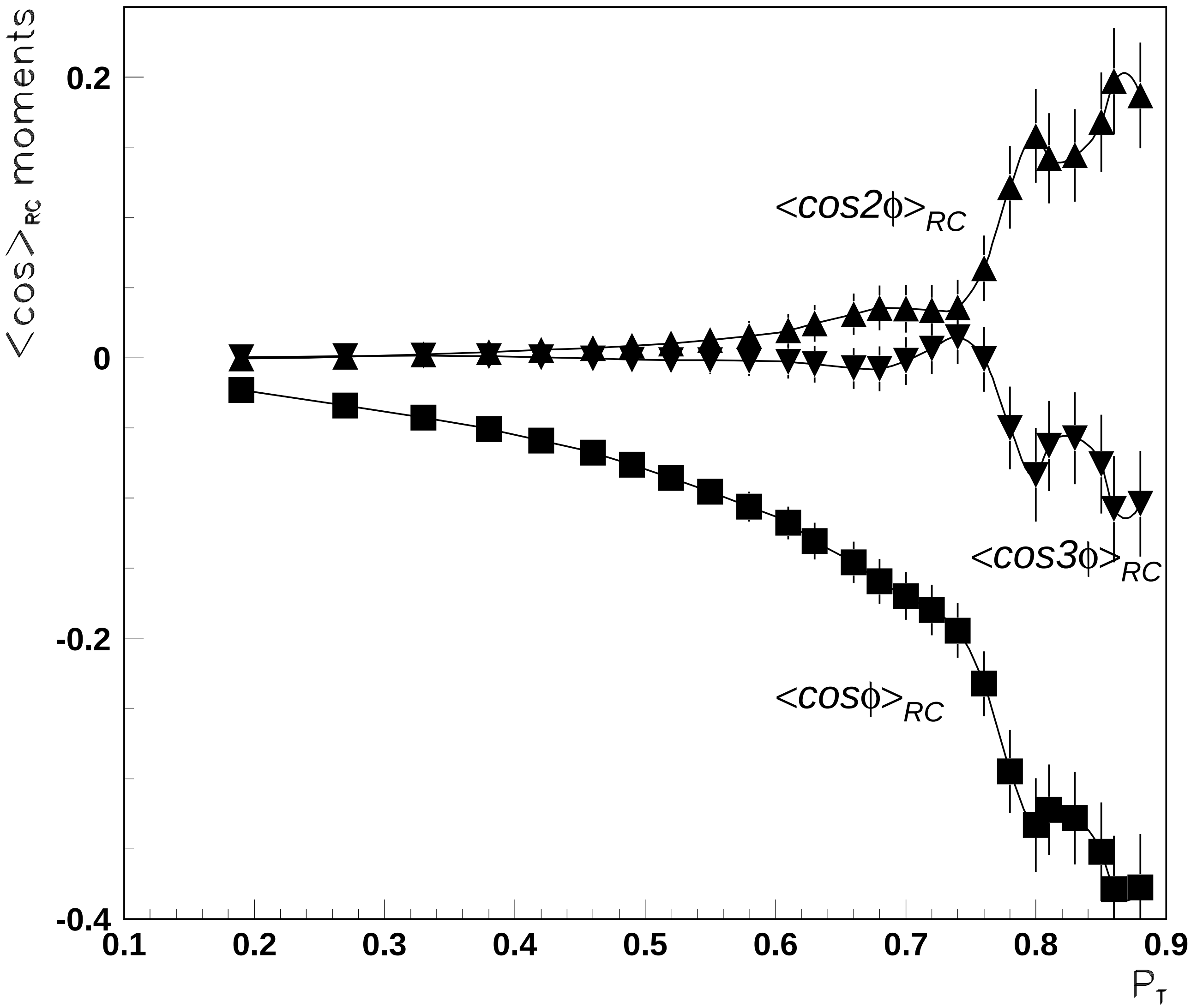
The contributions involving K_3 and K_4 cannot be reproduced using DIS generators.

The ϕ-dependence of the cross section



Azimuthal modulations due to RC in SIDIS: Different input of SFs produce different corrections (even by sign)

The p_T-dependence of the cross section



This plot illustrates that the RC may be very significant at large PT. Also the plot illustrates occurrence of the effects not observed at the level of the Born cross section (i.e., $< cos(3\phi) >$.

General task for RC to SIDIS

- Ultimate purpose of this task is to create the code for exact (i.e., leading+next-to-leading) calculation of RC to the SIDIS cross section of electron scattering by the proton target with both particles arbitrary polarized and to develop a Monte Carlo generator based on this code.
- Stages of the theoretical calculation and practical implementation include the steps:
  - *Elaborate the covariant hadronic tensor of SIDIS based on the developments of the theoretical groups of Aram Kotzinian (Nuclear Physics B 441 (1995) 234-256), Peter Mulders (Phys.Rev. D49 (1994) 96-113) and Alessandro Bacchetta (JHEP 0702:093,2007).*
  - *Calculate the SIDIS cross sections using the elaborated hadronic tensor, compare analytic expressions for the Born cross section between the results obtained by researchers of these groups and understand possible discrepancies.*
  - *Calculate the coefficients from contraction of leptonic tensor involving RC with tensor structures from the hadronic tensor.*
  - *Code these coefficients and complete the code for RC due to DIS radiative tail.*
  - *Obtain the hadronic tensor of exclusive process, calculate the contractions for the kinematics of exclusive process and implement this part of RC to the code.*
  - *Implement set of available four-dimensional DIS and three-dimensional exclusive SFs, investigate the model dependence of the RC, and identify the kinematical regions where uncertainty in SFs could result in significant effect on RC.*
  - *Develop a Monte Carlo generator that will generate the channel of scattering (non-radiative, radiative DIS, and radiative exclusive scattering) and the kinematical variables of the additionally radiated photon.*

Explicit Expression for SIDIS Hadronic Tensor

$$W_{\mu\nu} = n_\mu n_\nu T_1 + p_\mu p_\nu T_2 + p_{h\mu} p_{h\nu} T_3 + (p_\mu p_{h\nu} + p_{h\mu} p_\nu) T_4 + (p_\mu p_{h\nu} - p_{h\mu} p_\nu) T_5$$

$$+ (p_\mu n_\nu + n_\mu p_\nu) T_6 + (p_\mu n_\nu - n_\mu p_\nu) T_7 + (p_{h\mu} n_\nu + n_\mu p_{h\nu}) T_8 + (p_{h\mu} n_\nu - n_\mu p_{h\nu}) T_9$$

where $n_\mu = \frac{1}{M} \epsilon_{\mu p q p_h}$, $S_x = \frac{Q^2}{x}$, $\lambda_y = S_x^2 + 4M^2 Q^2$, $Z = z\lambda_Y^{1/2} - (z^2 S_x^2 - 4M^2(p_t^2 + m_h^2))^{1/2}$

$$T_1 = \frac{4M^2}{\lambda_Y p_t^2} H_{22}^{(0)} - \frac{8(nS)M^3}{\lambda_Y^{3/2} p_t^3} H_{222}^{(S)}$$

$$T_2 = \frac{1}{4M^2 \lambda_Y p_t^2} (Z^2 S_x^2 H_{11}^{(0)} - 8(ZQS_x \Re H_{01}^{(0)} - 2M^2 p_t Q^2 H_{00}^{(0)}) M^2 p_t)$$
$$+ \frac{(nS)}{2M^3 \lambda_Y^{3/2} p_t^3} (-Z^2 H_{112}^{(S)} S_x^2 + 8(ZQS_x \Re H_{012}^{(S)} - 2M^2 p_t Q^2 H_{002}^{(S)}) M^2 p_t)$$

$$T_3 = \frac{H_{11}^{(0)}}{p_t^2} - \frac{2(nS)M}{\lambda_Y^{1/2} p_t^3} H_{112}^{(S)}$$

$$T_4 = \frac{1}{2M^2 \sqrt{\lambda_Y} p_t} (-ZS_x H_{11}^{(0)} + 4M^2 p_t Q \Re H_{01}^{(0)}) + \frac{(nS)}{M \lambda_Y p_t^3} (ZS_x H_{112}^{(S)} - 4M^2 Q p_t H_{012}^{(S)})$$

$$T_5 = i \frac{2Q}{\sqrt{\lambda_Y} p_t} \Im H_{01}^{(0)} - i \frac{4(nS)MQ}{\lambda_Y p_t^2} \Im H_{012}^{(S)}$$

Explicit Expression for SIDIS Hadronic Tensor

$$W_{\mu\nu} = n_\mu n_\nu T_1 + p_\mu p_\nu T_2 + p_{h\mu} p_{h\nu} T_3 + (p_\mu p_{h\nu} + p_{h\mu} p_\nu) T_4 + (p_\mu p_{h\nu} - p_{h\mu} p_\nu) T_5$$

$$+ (p_\mu n_\nu + n_\mu p_\nu) T_6 + (p_\mu n_\nu - n_\mu p_\nu) T_7 + (p_{h\mu} n_\nu + n_\mu p_{h\nu}) T_8 + (p_{h\mu} n_\nu - n_\mu p_{h\nu}) T_9$$

where $n_\mu = \frac{1}{M} \epsilon_{\mu p q p_h}$, $S_x = \frac{Q^2}{x}$, $\lambda_y = S_x^2 + 4M^2 Q^2$, $Z = z\lambda_Y^{1/2} - (z^2 S_x^2 - 4M^2(p_t^2 + m_h^2))^{1/2}$

$$T_6 = -\frac{1}{M{\lambda_Y}^{3/2} p_t^3} (4p_t M^2 Q(2Mp_t \Re H_{023}^{(S)} - \sqrt{\lambda_z} \Re H_{021}^{(S)}) - \sqrt{\lambda_z} S_x (2Mp_t \Re H_{123}^{(S)} - \sqrt{\lambda_z} \Re H_{121}^{(S)}))(qS) + (4M^2 Q p_t \Re H_{021}^{(S)} - S_x \sqrt{\lambda_z} \Re H_{121}^{(S)}) \sqrt{\lambda_Y} (p_h S))$$

$$T_7 = -i\frac{1}{M{\lambda_Y}^{3/2} p_t^3} (4p_t M^2 Q(2Mp_t \Im H_{023}^{(S)} - \sqrt{\lambda_z} \Im H_{021}^{(S)}) - \sqrt{\lambda_z} S_x (2Mp_t \Im H_{123}^{(S)} - \sqrt{\lambda_z} \Im H_{121}^{(S)}))(qS) + (4M^2 Q p_t \Im H_{021}^{(S)} - S_x \sqrt{\lambda_z} \Im H_{121}^{(S)}) \sqrt{\lambda_Y} (p_h S))$$

$$T_8 = -\frac{2M}{\lambda_Y p_t^3} ((qS)(-\sqrt{\lambda_z} \Re H_{121}^{(S)} + 2Mp_t \Re H_{123}^{(S)}) - (p_h S) \sqrt{\lambda_Y} \Re H_{121}^{(S)})$$

$$T_9 = i\frac{2M}{\lambda_Y p_t^3} ((qS)(\sqrt{\lambda_z} \Im H_{121}^{(S)} - 2Mp_t \Im H_{123}^{(S)}) - (p_h S) \sqrt{\lambda_Y} \Im H_{121}^{(S)})$$

where $\lambda_z = z^2 S_x^2 - 4M^2(p_t^2 + m_h^2)$, so $Z = z\lambda_Y^{1/2} - \lambda_z^{1/2}$.

Current Status and Further steps

- We calculated the born cross section using the hadronic tensor

$$\frac{d^5\sigma}{dxdydzdp_t^2d\phi_h} = \frac{\alpha^2\pi y}{4zQ^2} L_{\mu\nu} 2MW_{\mu\nu} = \frac{\alpha^2\pi y}{4zQ^2} \sum_i K_i^0 H_i(x, Q^2, z, p_t^2)$$

  and found that this expression exactly reproduces the cross section obtained by Alessandro Bacchetta (JHEP 0702:093,2007). We believe that this is complete the test of our analytical expressions for the hadronic tensor in SIDIS.

- We calculated the cross section with an additional photon radiated.

$$\frac{d^5\sigma_r ad}{dxdydzdp_t^2d\phi_h} = \frac{\alpha^3 y}{8zQ^2} L_{\mu\nu}^{rad} 2MW_{\mu\nu} = \frac{\alpha^3 y}{8zQ^2} \int \frac{d^3k}{k_0} \sum_i K_i^{rad} H_i(x_{tr}, Q_{tr}^2, z_{tr}, p_{t,tr}^2)$$

- Thus we are able to write total (observed) cross section in the standard form:

$$\sigma_{obs} = \sigma_0 e^{\delta_{inf}} (1 + \delta_{VR} + \delta_{vac}) + \sigma_F + \sigma_{exl}.$$

- One interesting effect we found for H_5. The theorem states that terms 1/squared propagator cannot appear in expression for the contractions K_i^{rad} without m^2 or terms canceling one propagator. The theorem is satisfied for H_5 but the ways of cancellation is a little exotic. We will need to further investigate the term numerically.

Monte Carlo generator of the Radiated Photon for SIDIS

- Generation in DIS using RADGEN
  - Generate final electron kinematics (i.e., Q^2 and x)
  - Calcuate RC in the generated point using RADGEN or POLRAD 2.0
  - Generate a channel: non-radiated or radiated, i.e., inelatic, elastic, or quasielastic (for nuclear targets)
  - Generation of the radiated photon kinematics

- Generation in SiDIS using a new Monte Carlo generator
  - Generate final electron kinematics (i.e., Q^2 and x) and final hadron kinematics (i.e., z, p_T, and ϕ_h)
  - Calculate RC in the generated point using HAPRAD 2.0
  - Generate a channel: non-radiated or radiated, i.e., inelatic, exclusive, or coherent (for nuclear targets)
  - Generation of the radiated photon kinematics

RC procedure of experimental data in SIDIS

The possible (successful) strategy of RC could be developed using our experience in the modeling for DIS. The RC procedure of experimental data should involve an iteration procedure in which the fits of SFs of interest are re-estimated at each step of this iteration procedure.

- *The fit of SFs are constructed to have the model in the region covered by the experiment*
- *Use experimental data or theoretical models to construct the models in the regions of softer processes, resonance region, and exclusive scattering*
- *Check that the constructed models provide correct asymptotic behavior when we go to the kinematical bounds (Regge limit, QCD limit)*
- *Joint all the models to have continuous function of all four variables in all kinematical regions necessary for RC calculation*
- *Implement this scheme in a computer code and define the iteration procedure*
- *If several SFs are measured in an experiment, implement the procedure of their separation in data and model each of them.*
- *If other SFs are necessary (e.g., unpolarized SFs when spin asymmetries are measured), construct the models for them as well.*
- *Pay specific attention to exclusive SFs, because the radiative tail from exclusive peak is important (or even dominate) in certain kinematical regions.*
- *Pay specific attention to p_T dependence because RC is too sensitive for p_T model choice.*

RC procedure of experimental for SIDIS

- Assume that we have measurements in N bins: i.e., we measure $\beta_i = \{x_i, Q_i^2, z_i, p_{Ti}, \phi_{hi}\}$
- We have the integral equation for each bin:

$$\sigma_{extr} = \sigma_{meas} + \alpha C \int dx_{tr} dz_{tr} dQ_{tr}^2 \sum K(\beta, \beta_{tr}) H(x_{tr}, Q_{tr}^2, z_{tr}, p_{T,tr})$$

- We fit σ_{meas} using measurements in all bins,
- We construct the overall fit jointing this fit and a fit from the region beyond the region of the experiment,
- We use this overall fit to calculate RC,
- We use the equation to obtain σ_{extr} in each bin,
- We fit σ_{extr} using measurements in all bins,
- ....
- Continue till fiting parameters (or σ_{extr}) do not change.

Conclusion

- Newly achieved accuracies in Jlab and new physics studied at Jlab require paying renewed attention to RC calculations and their implementation in data analysis software.
- For SIDIS RC theoretical efforts are needed both for calculation of SIDIS RC in a bin and for generation of radiated events:
  - *Hadronic tensor for the SIDIS cross sections in the covariant form is constructed and tested.*
  - *Exact calculation of RC for the complete SIDIS cross section containing 18 SFs is completed and coding is in progress.*
  - *We expect sensitivity of the results for RC to specific assumptions used for constructing SIDIS SFs:*
  - *Broad discussion and efforts of theoreticians and experimentalists are required to complete the evaluation of all SIDIS SFs as well as SFs in resonance region and exclusive SFs.*
  - *Iteration procedure with fitting of measured SFs and joining with models beyond SIDIS measurements at each iteration step has to be involved in data analysis.*
- Tools for generation of the radiated photon in DIS cannot provide valid generation of the radiated events in SIDIS .
  - *Such generator can be constructed based on a code for RC in SIDIS in the same way of how RADGEN is constructed based on POLRAD 2.0*