# Workshop on Novel Probes of the Nucleon Structure in SIDIS, $e^+e^-$ and $pp$ (FF2019)

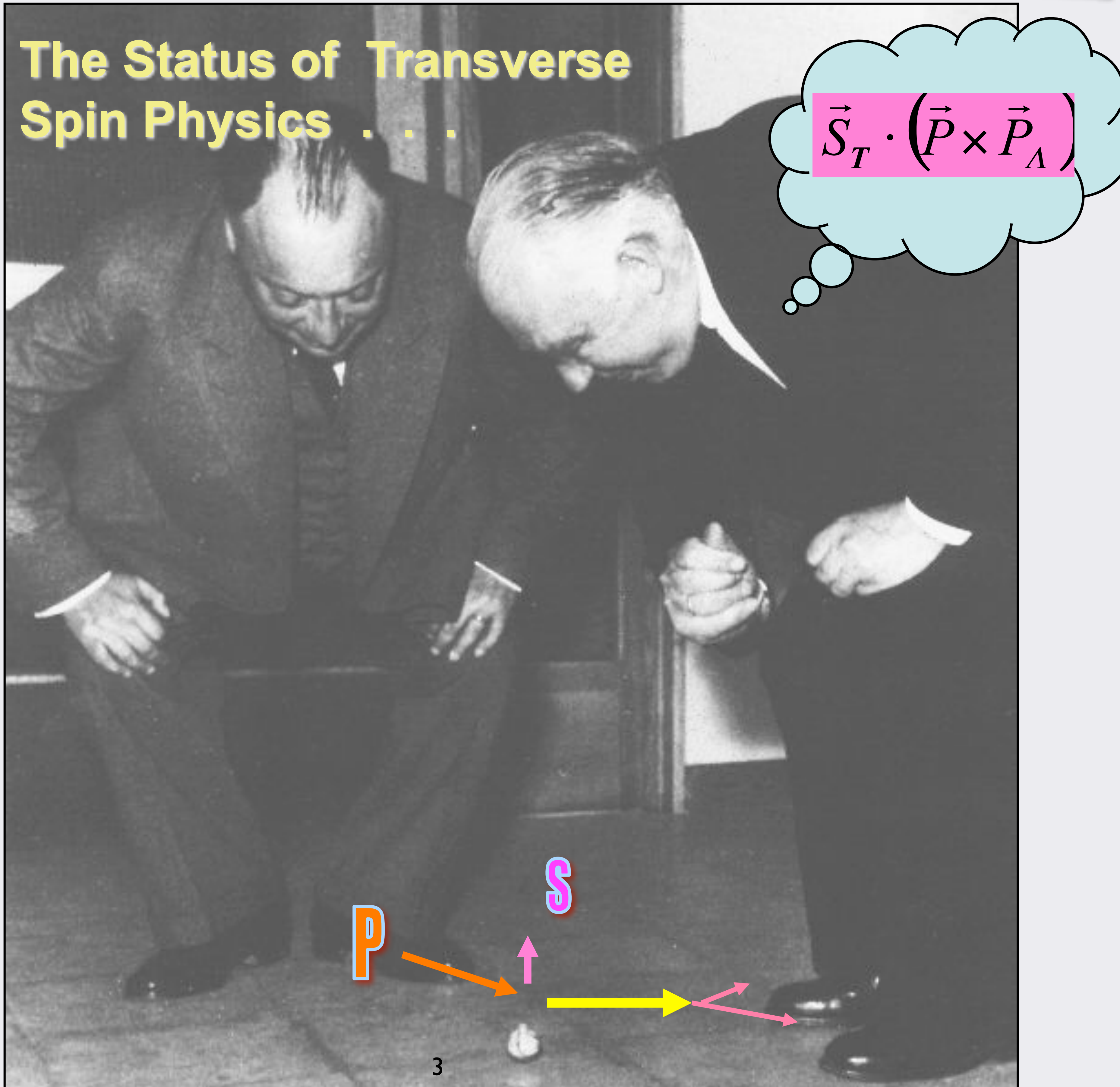
## Lambda production
## Collinear formalism for $\Lambda^\uparrow$ polarization

**Leonard Gamberg Penn State Berks, March 17th , 2019**

# Outline

- Motivation to study on $\Lambda^{\uparrow}$ physics
- Review inclusive scattering for fragmentation processes "twist three" analysis
- Why the simplest process $e^{+}e^{-} \rightarrow \Lambda^{\uparrow}X$ is most interesting to process to study;
- Test of time reversal in QCD
- Study twist 3 factorisation at NLO

# Kinematics of parity conserving Transverse Spin



## Kane, Pumplin, and Repko predict zero from QCD

VOLUME 41, NUMBER 25 PHYSICAL REVIEW LETTERS 18 DECEMBER 1978

# Transverse Quark Polarization in Large-$p_T$ Reactions, $e^+e^-$ Jets, and Leptoproduction: A Test of Quantum Chromodynamics

G. L. Kane
*Physics Department, University of Michigan, Ann Arbor, Michigan 48109*

and

J. Pumplin and W. Repko
*Physics Department, Michigan State University, East Lansing, Michigan 48823*

(Received 5 July 1978)

We point out that the polarization $P$ of a scattered or produced quark is calculable perturbatively in quantum chromodynamics for $e^+e^- \to q\bar{q}$, large-$p_T$ hadron reactions, and large-$Q^2$ leptoproduction, and is infrared finite. The quantum-chromodynamics prediction is that $P=0$ in the scaling limit. Experimental tests are or will soon be possible in $pp \to \Lambda X$ [where presently $P(\Lambda) \simeq 25\%$ for $p_T > 2$ GeV/$c$] and in $e^+e^- \to$ quark jets.

At least for the cases when $P$ is small, tests should be available soon in large-$p_T$ production [where currently $P(\Lambda) = 25\%$ for $p_T \gtrsim 2$ GeV/$c$], and $e^+e^-$ reactions. While fragmentation effects could dilute polarizations, they cannot (by parity considerations) induce polarization. Consequently, observation of significant polarizations in the above reactions would contradict either QCD or its applicability.

# Measurement of Lambda-spin through weak decay $\Lambda^0 \to p\,\pi^-$

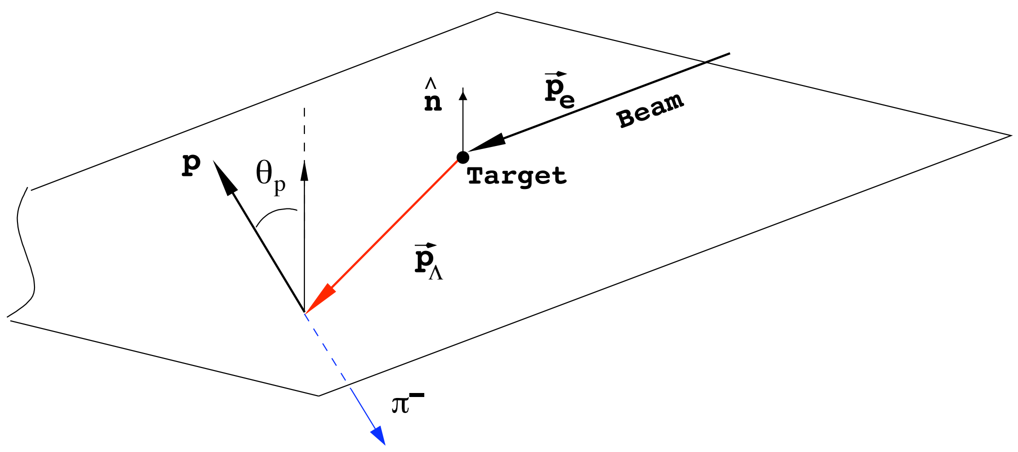



FIG. 1: Schematic diagram of inclusive $\Lambda$ production and decay. The angle $\theta_p$ of the decay proton with respect to the normal $\hat{n}$ to the production plane is defined in the $\Lambda$ rest frame.

- Proton preferentially emitted along $\Lambda$-spin
- In $\Lambda$ rest frame: pol. decay distribution

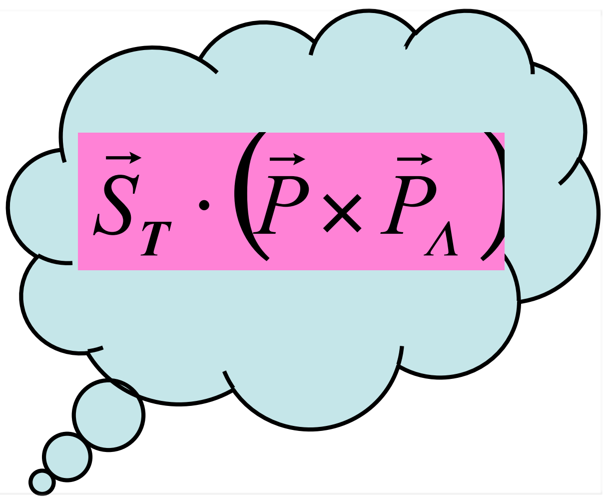
$$\left(\frac{dN}{d\Omega_p}\right)_{\mathrm{pol}} = \left(\frac{dN}{d\Omega_p}\right)_{\mathrm{unpol}} \left(1 + \alpha\, P_n^{\Lambda} \cos(\theta_p)\right)$$

$P^{\Lambda}$: Transverse Lambda Polarization

**QCD is Parity conserving so any final state hadron must be polarised perpendicular to the production plane**

**Weak decay of Lambda; proton is preferentially along the hyperon spin direction can reconstruct the Lambda spin**

**The Status of Transverse Spin Physics . . .**



$$\vec{S}_T \cdot \left(\vec{P} \times \vec{P}_\Lambda\right)$$

# What does QCD Predict ...

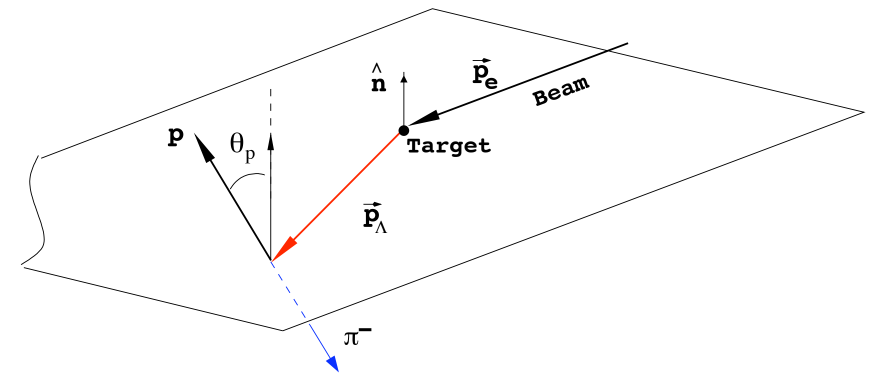
$$pA \to \Lambda^{\uparrow} X$$

$$pp \to \Lambda^{\uparrow} X$$

$$\nu N \to \Lambda^{\uparrow} X$$

$$\gamma^* N \to \Lambda^{\uparrow} X$$

$$e^+ e^- \to \Lambda^{\uparrow} X$$

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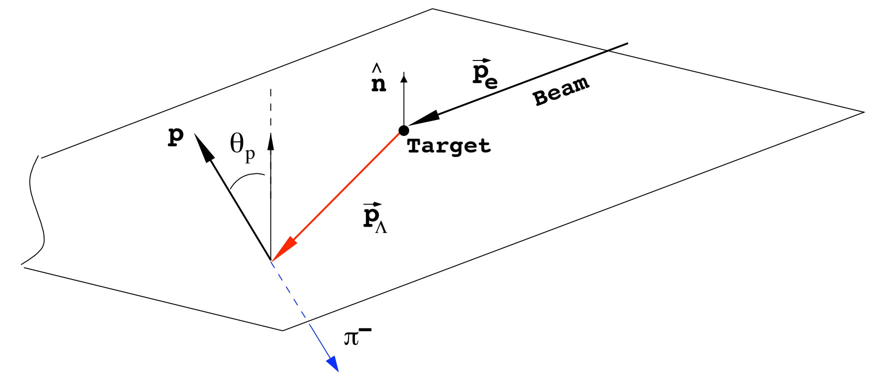


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$$\nu N \to \Lambda^{\uparrow} X$$

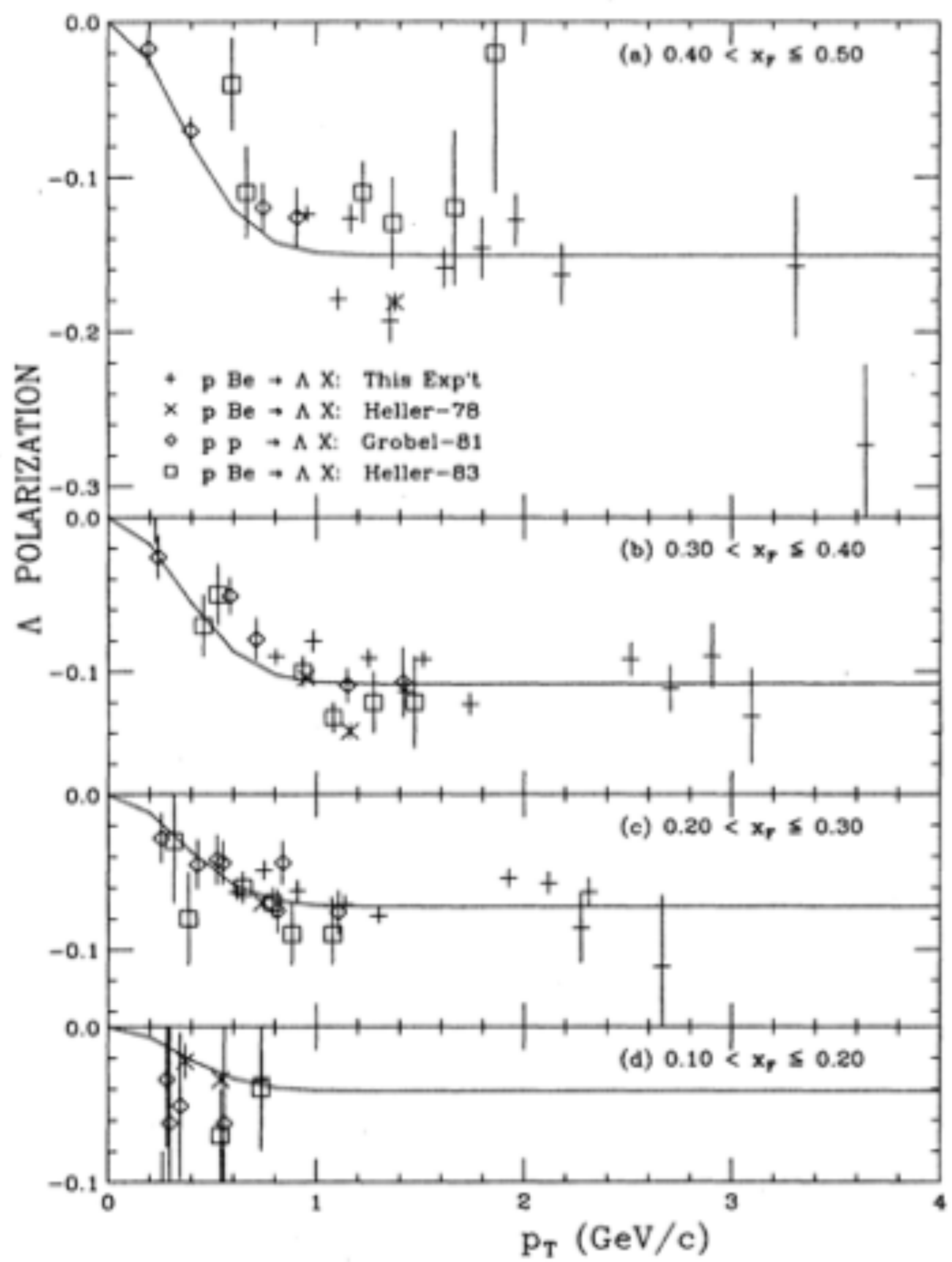
$$\gamma^{*} N \to \Lambda^{\uparrow} X$$

$$e^{+}e^{-} \to \Lambda^{\uparrow} X$$

# Transverse $\Lambda$ polarisation a long history

**One of, if not the first TSSA measurement**

$$p + Be \rightarrow \Lambda^{\uparrow} + X$$




Transverse $\Lambda$ polarization in pA:

One of the first transverse spin effects at Fermilab (1976):

p+Be$\longrightarrow \Lambda^0$ +X and many more follow-up measurements, also at CERN SPS (NA48), HERA-B

# Proton-Nuclei cont ...

$$pA \to \Lambda^{\uparrow} X$$

V. Fanti et al.: NA 48   450 GeV proton energy

Eur. Phys. J. C 6, 265–269 (1999) **CERN SPS**

Lundberg et al prd40 (1989) 400 GeV

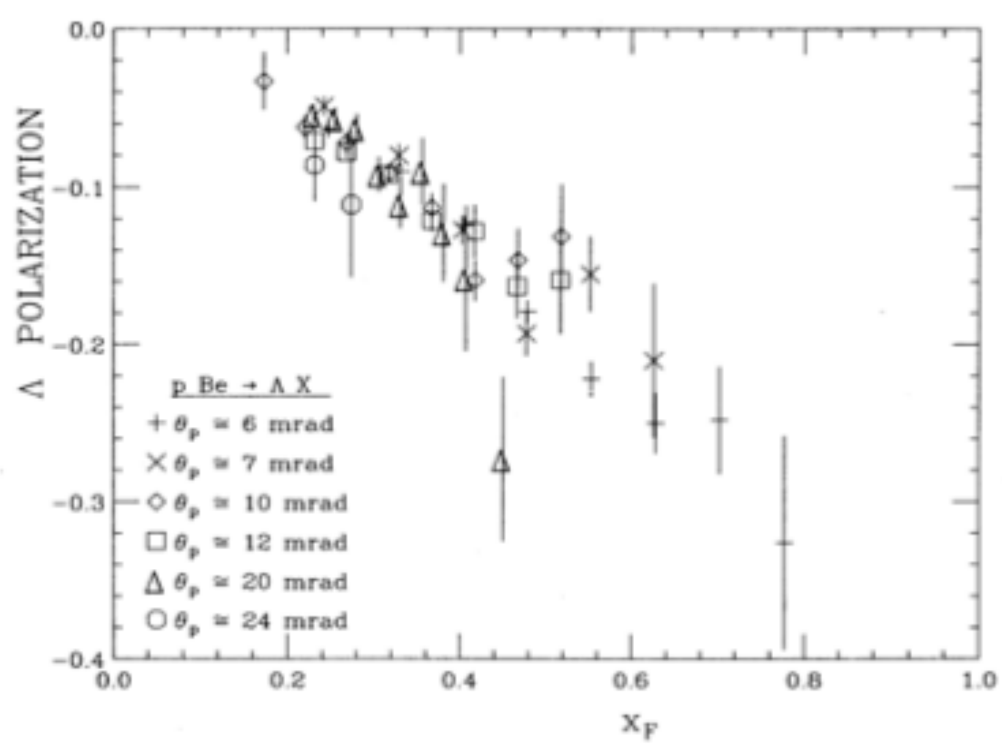
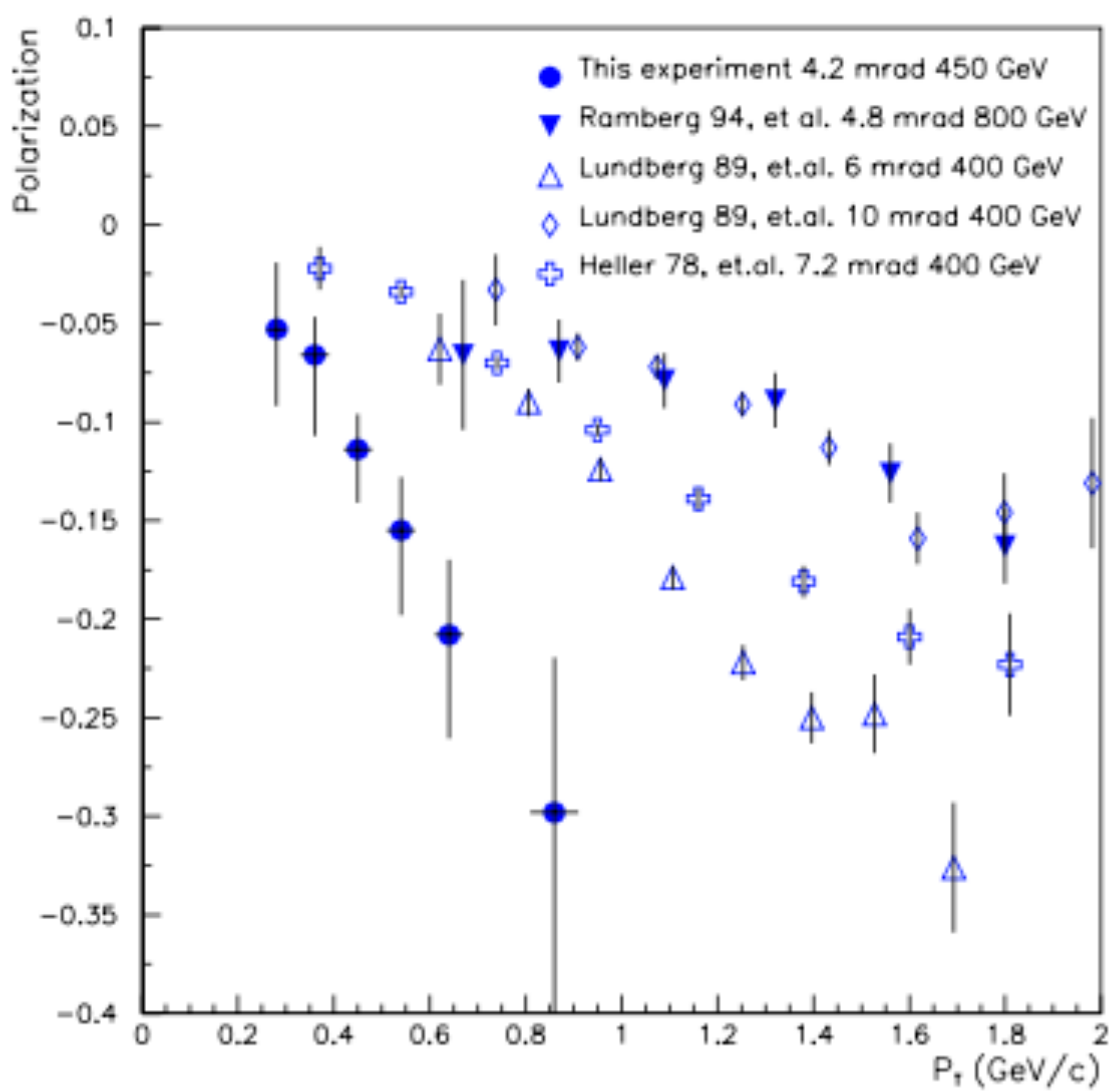



FIG. 4. The Λ polarization is shown as a function of $x_F$ for all production angles. Over this range of production angles and within experimental uncertainties, the polarization is angle (or $p_T$) independent.



# Proton-proton

$$pp \to \Lambda^{\uparrow} X$$

**PLB 1979**

$\Lambda^0$ produced in the inclusive reaction with $\sqrt{s}$ = 53 and 62 GeV at the CERN Intersecting Storage Rings (ISR) are observed to be polarized along the normal to the production plane. In the ranges of longitudinal and transverse momenta, 15-24 and 0.6-1.4 GeV/c, respectively, the mean polarization is found to be -(0.357 +/- 0.055)

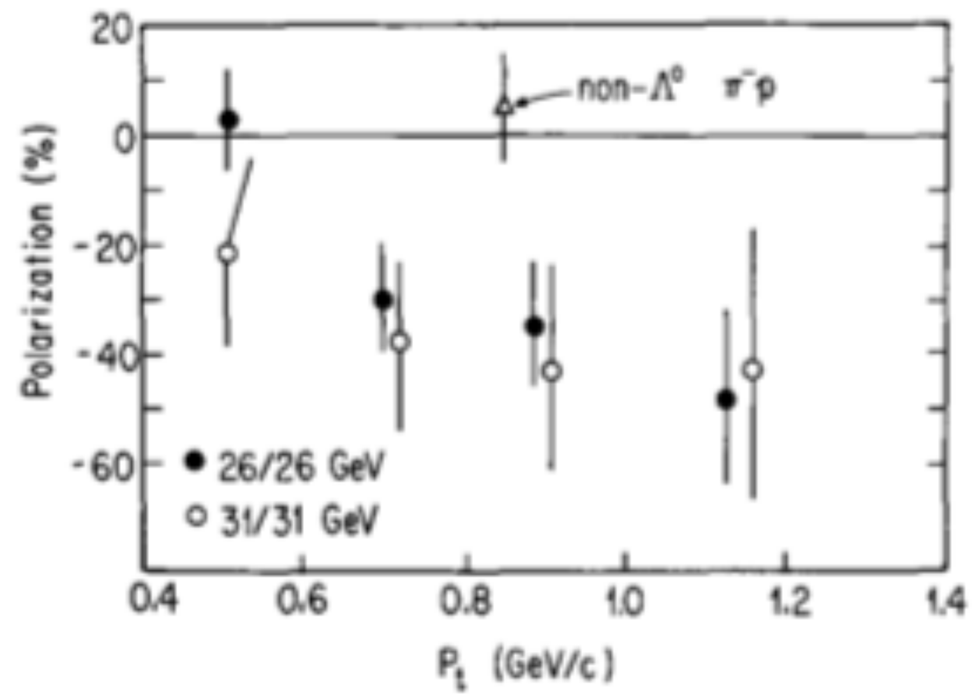



Fig. 4. Final polarization values for 26/26 and 31/31 GeV data samples as a function of $P_t$. The polarization measurement for $\pi^-p$ events which are not $\Lambda^0$ but have $P_t > 0.6$ GeV/$c$ is shown as the open triangle.

## Lepton-Nucleon

$$\nu N \to \Lambda^{\uparrow} X$$

Nuclear Physics B 588 (2000)

The Λ polarization in νμ charged current interactions measured in the **NOMAD** experiment. The dependence of the absolute value of *P*y on the Λ transverse momentum with respect to the hadronic jet direction is in qualitative agreement with the results from unpolarized hadron–hadron experiments.

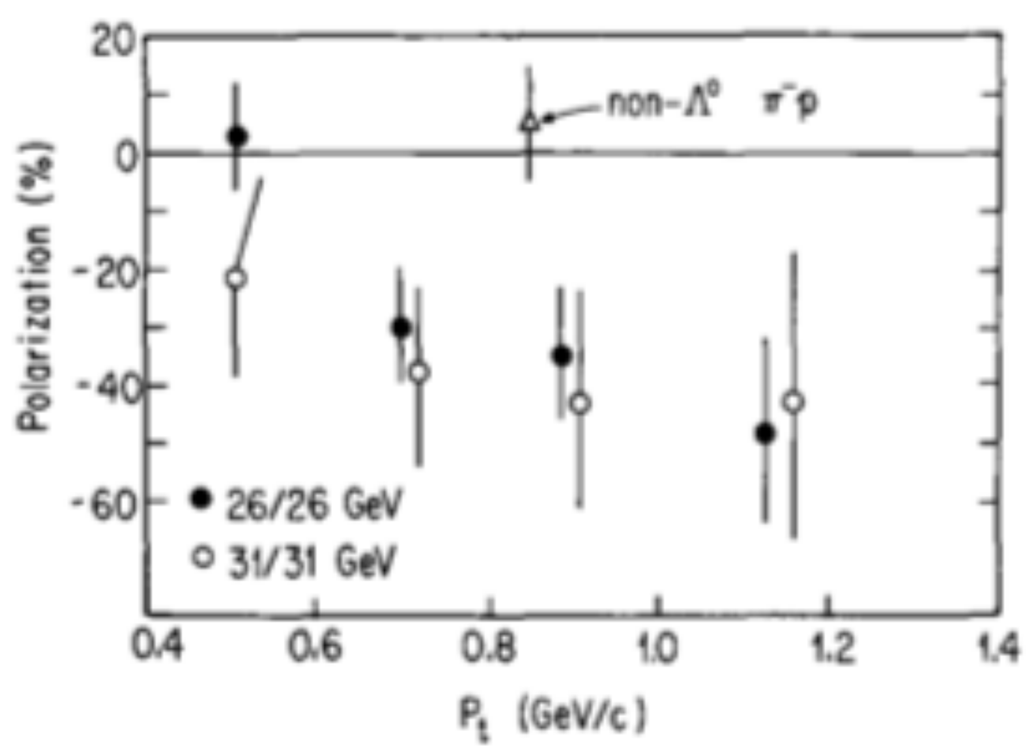
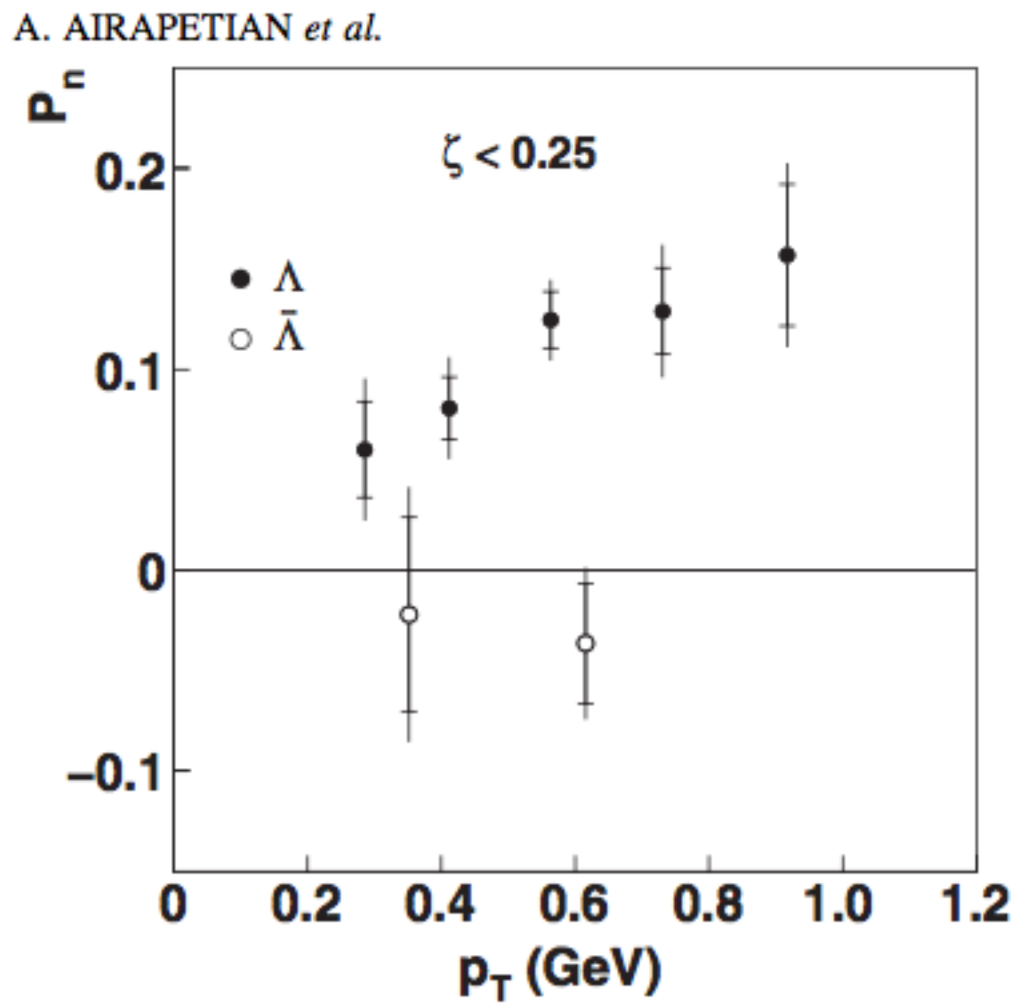



Fig. 4. Final polarization values for 26/26 and 31/31 GeV data samples as a function of $P_t$. The polarization measurement for $\pi^- p$ events which are not $\Lambda^0$ but have $P_t > 0.6$ GeV/$c$ is shown as the open triangle.

# Quasi Real + Nucleon HERMES

$$\gamma^* N \rightarrow \Lambda^{\uparrow} X$$

The HERMES experiment has measured the transverse polarization of $\Lambda$ and $\bar{\Lambda}$ hyperons produced inclusively in quasi-real photoproduction at a positron beam energy of 27.6 GeV. The transverse polarization $P_{\Lambda N}$ of the $\Lambda$ hyperon is found to be positive while the observed $\bar{\Lambda}$ polarization is compatible with zero




**PRD 76, 092008 (2007)**

# Quasi Real + Nucleon HERMES

$$\gamma^* N \rightarrow \Lambda^{\uparrow} X$$

PRD 90, 072007 (2014)

The transverse polarization of Λ hyperons was measured in inclusive quasireal photoproduction for various target nuclei ranging from hydrogen to xenon. The polarization observed is positive for light target nuclei and is compatible with zero for krypton and xenon.

TRANSVERSE POLARIZATION OF Λ HYPERONS ...

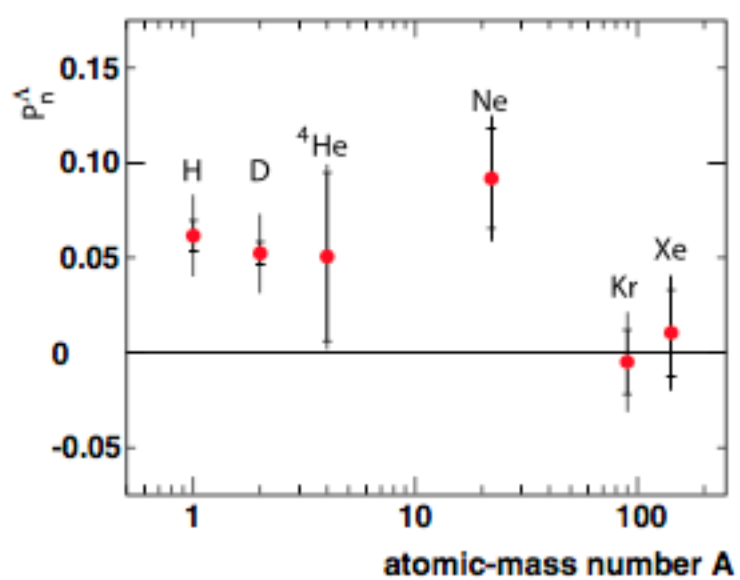
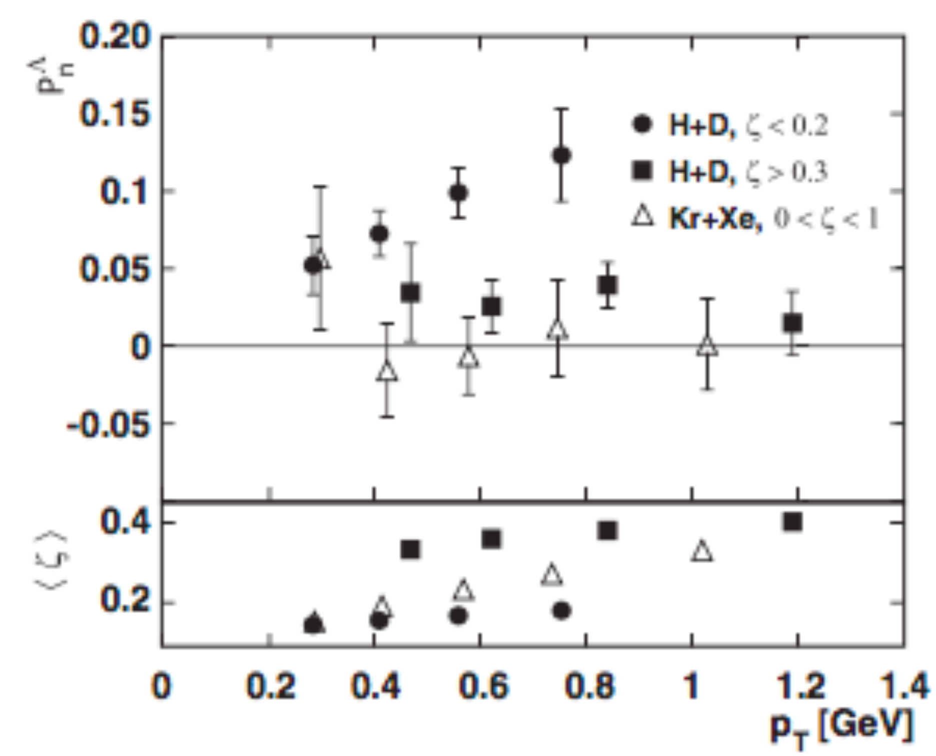
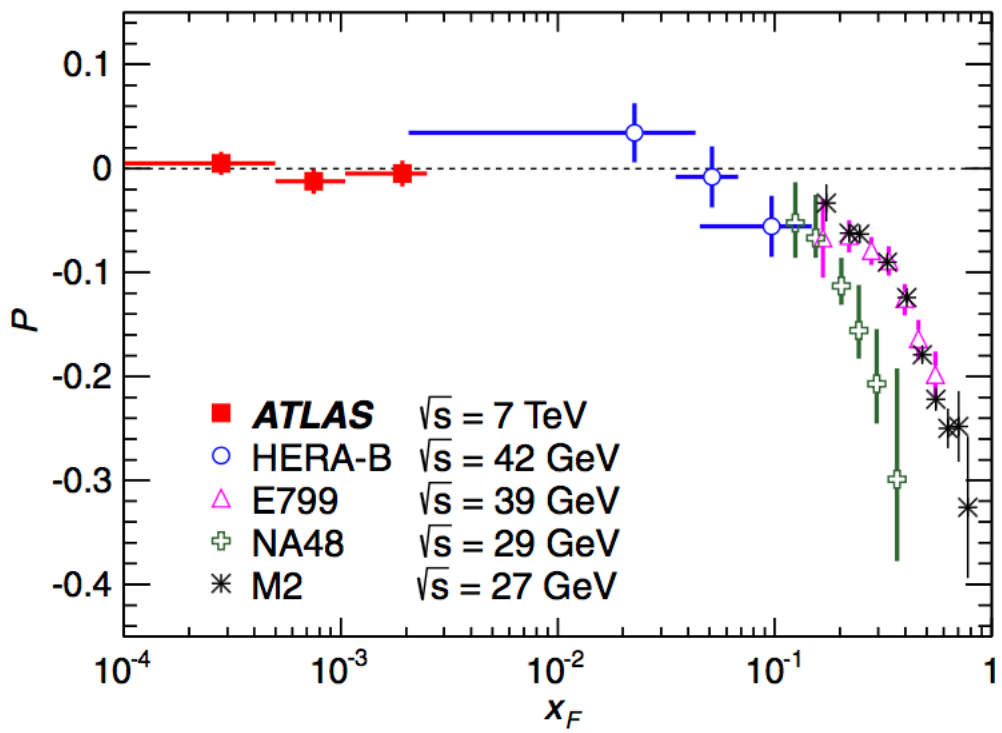



FIG. 3 (color online). Dependence of the transverse polarization $P_n^\Lambda$ on the atomic-mass number $A$ of the target nuclei. The inner error bars represent the statistical uncertainties; the full error bars represent the total uncertainties, evaluated as the sum in quadrature of statistical and systematic uncertainties.



# What about LHC?

# Is it feasible at a high energy collider?




Recent ATLAS measurement

at $\sqrt{S}$ = 7 TeV

PRD 91, 032004 (2015)

**Polarisation at mid rapidity**

## Simplest and cleanest process (like DIS): $e^+e^- \to \Lambda^\uparrow X$

OPAL at LEP on Z-pole [Eur.Phys.J C2, 49 (1998)]:

Longitudinal Polarization, but
no significant Transverse Polarization

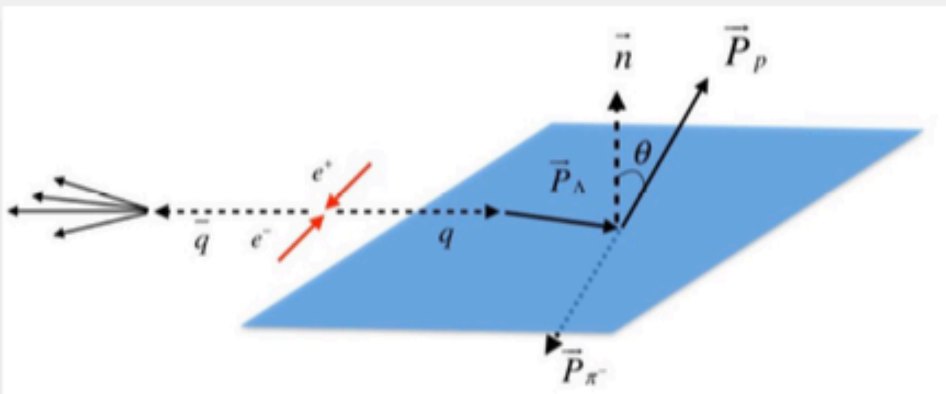
Belle data: Transverse Polarization

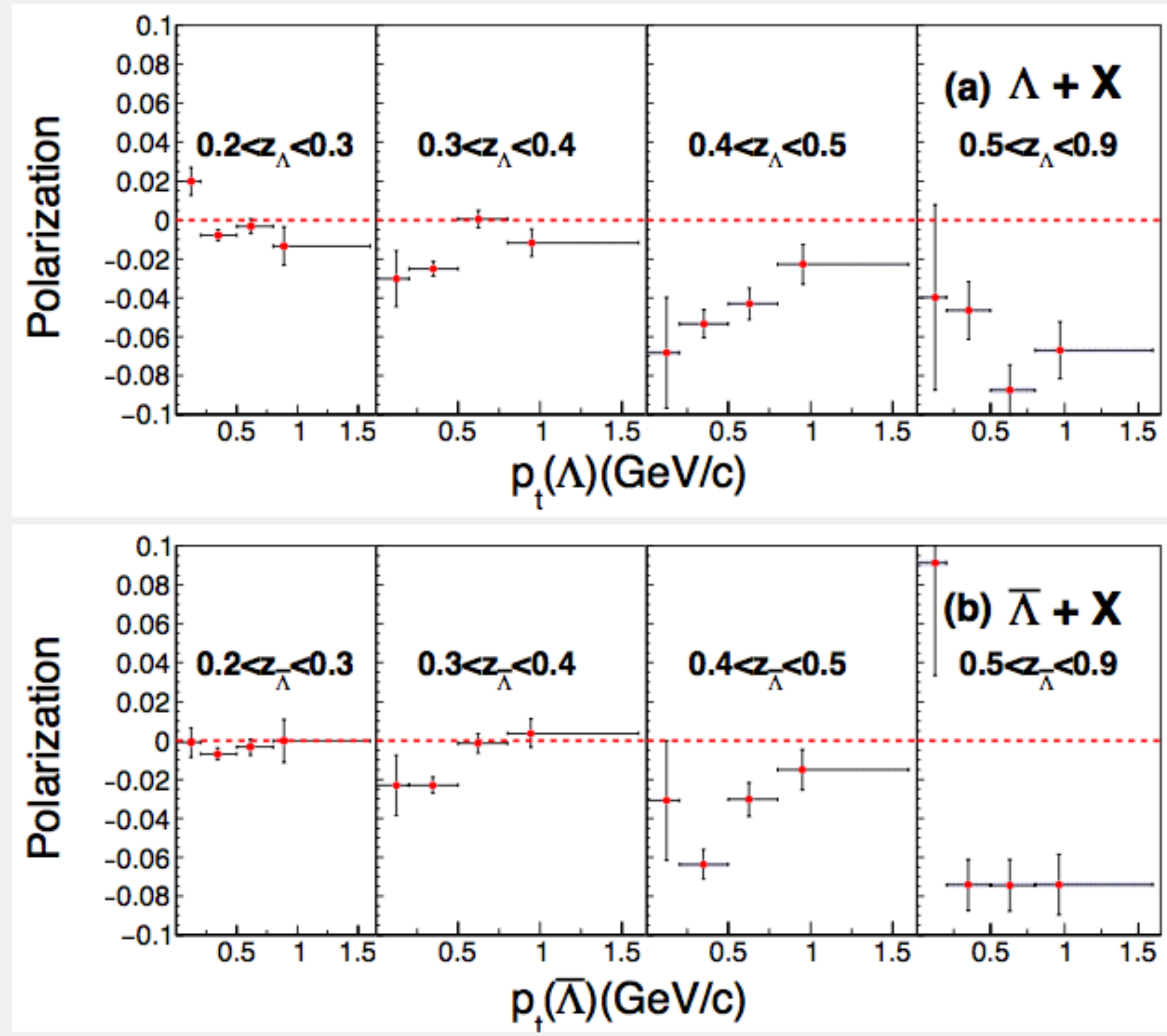
Y. Guan, et al. PRL 122 (2019) → talk by Ralf here @ FF Workshop

⇒ significant transverse polarization

(measured w.r.t. thrust axis ) TMD formalism …

# FIRST OBSERVATION BY BELLE




PHYSICAL REVIEW LETTERS 122, 042001 (2019)

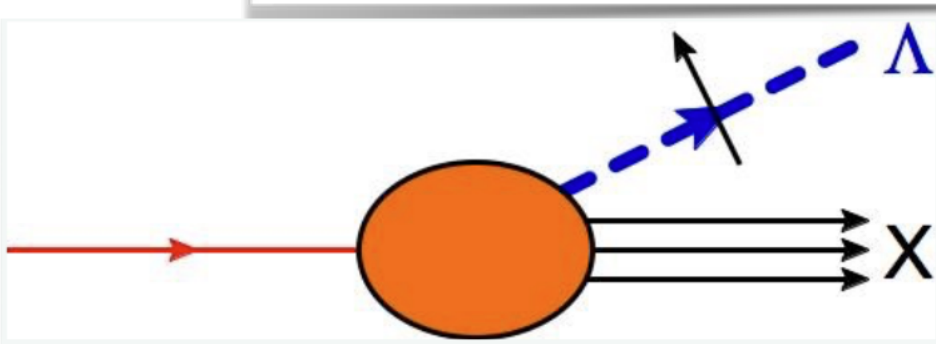
**From Anselm's INT talk**

$$e^+e^- \to \Lambda^\uparrow X$$

# $\Lambda^{\uparrow}$ “fragmentation” in pQCD

## **Partonic picture of fragmentation,**

Collins & Soper, Nucl. PhysB 1982, Jaffe & Ji Nucl. Phys B 1992, Ji PRD49 1994




$$\langle P_\Lambda, S_\Lambda; X|\, \bar{q}(0)\, |0\rangle$$

- Partonic interpretation of the fragmentation process a matrix element that describes the hadronization of a parton into a jet of hadrons
- $q$ stands for a generic partonic field (quark, anti-quark, or gluon) and $|X\rangle$ is an arbitrary hadronic multi-particle state which forms an (unobserved) jet
- one hadrons of the jet is detected and its four-momentum $P_\mathrm{h}$ and four-spin $S_\mathrm{h}$ are measured
- Soft fragmentation process in pQCD formula is the "square" as a cut forward transition amplitude where sum over all possible unobserved hadron states.

$$\Delta^q_{ij}(z) = \frac{1}{N_c} \sum\!\!\!\!\!\!\!\int_X \int_{-\infty}^{\infty} \frac{\mathrm{d}\lambda}{2\pi}\, \mathrm{e}^{-i\frac{\lambda}{z}} \langle 0|[\infty;0]\, q_i(0)\, |P_h S_h; X\rangle\langle P_h S_h; X|\, \bar{q}_j(\lambda n)\, [\lambda;\infty]|0\rangle$$

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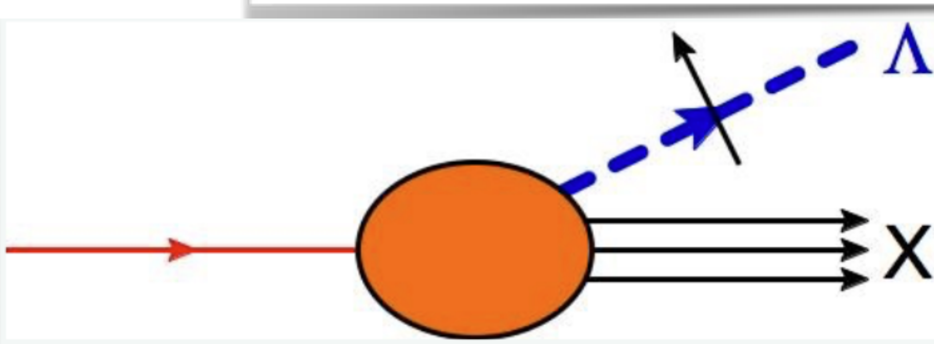
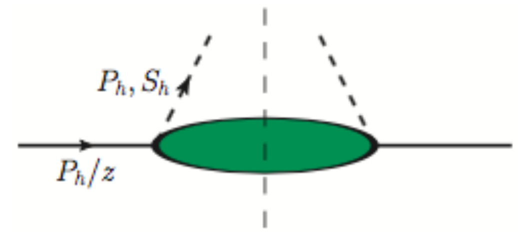
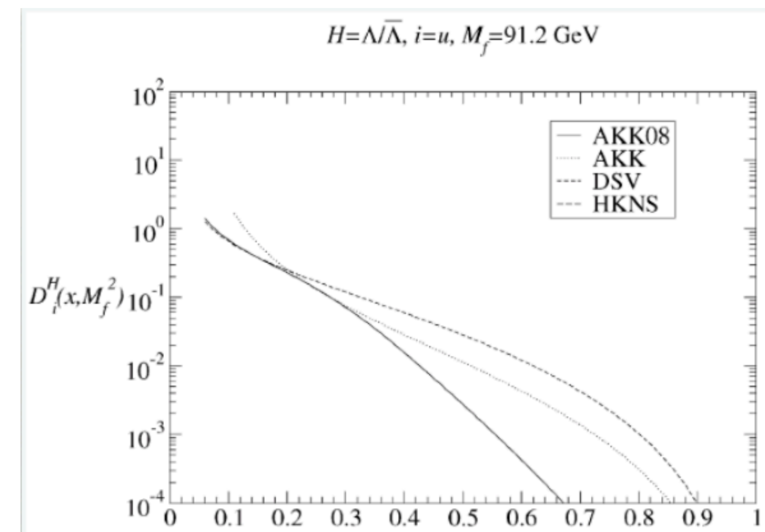
Collins & Soper, Nucl. PhysB 1982, Jaffe & Ji Nucl. Phys B 1992, Ji PRD49 1994




$$\langle P_\Lambda, S_\Lambda; X |\, \bar{q}(0)\, |0\rangle$$




$$\Delta_{ij}(z) = \frac{1}{N_c}\sum_X \int \frac{d\lambda}{2\pi}\, e^{-i\frac{\lambda}{z}} \langle 0|\, [\infty m, 0]\, q_i(0) | P_\Lambda, S_\Lambda; X\rangle \langle P_\Lambda, S_\Lambda; X |\, \bar{q}_j(\lambda m) [\lambda m, \infty m]\, |0\rangle$$




**$\Lambda$ Fragmentation at leading Twist**

$D_1^{\Lambda/q}(z)$ FF of unpolarized q $\longrightarrow \Lambda$

fairly known [fits by AKK08, DSV, ...]

$G_1^{\Lambda/q}(z)$ FF of longitudinally pol. q $\longrightarrow \Lambda$

poorly known [attempts by DSV to fit LEP data]

$H_1^{\Lambda/q}(z)$ FF of transversely pol. q $\longrightarrow \Lambda$

unknown, chiral-odd, hard to extract from single-inclusive processes Candidate to explain large transverse $\Lambda$ polarization?

K. Chen, G.R. Goldstein, R.L. Jaffe and X.-D. Ji, NPB 1995

**Collinear PDFs ($x$)**

| q pol. / H pol. | U | L | T |
|---|---|---|---|
| U | $f_1$ unpolarized | | |
| L | | $g_1$ helicity | |
| T | | | $h_1$ transversity |

**Collinear FFs ($z$)**

| q pol. / H pol. | U | L | T |
|---|---|---|---|
| U | $D_1$ | | |
| L | | $G_1$ | |
| T | | | $H_1$ |

**Courtesy of Daniel Pitonyak**

$$\Delta^q_{ij}(z) = \frac{1}{N_c}\sum_X \int_{-\infty}^{\infty} \frac{\mathrm{d}\lambda}{2\pi}\, \mathrm{e}^{-i\frac{\lambda}{z}} \langle 0|[\infty;0]\, q_i(0)\, |P_h S_h; X\rangle\langle P_h S_h; X|\, \bar{q}_j(\lambda n)\, [\lambda;\infty]|0\rangle$$

$$= \frac{1}{z}\left( \not{P}_h\, D_1^q(z) - S_{hL}\, \not{P}_h \gamma_5\, G_1^q(z) - \tfrac{1}{2}[\not{P}_h, \not{S}_h]\gamma_5\, H_1^q(z) \right)$$

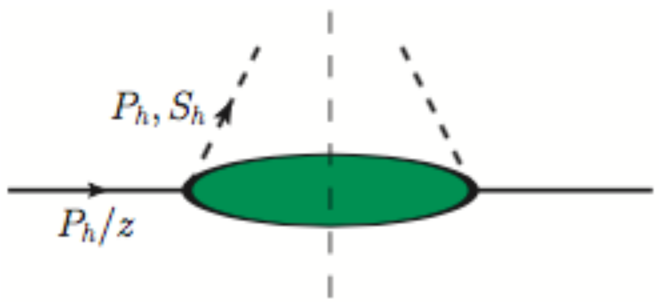
JAM like analysis for Lambda fragmentation functions ??

$$D_1^{\Lambda/q}(z)$$

See talks of Wally and Nobuo

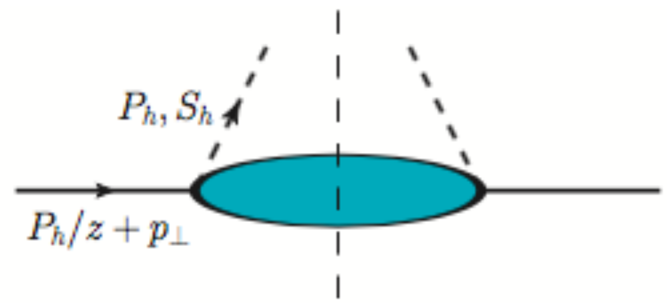
# Transverse spin $\Lambda^{\uparrow}$ Collinear Tw-3 Formalism

**Intrinsic**



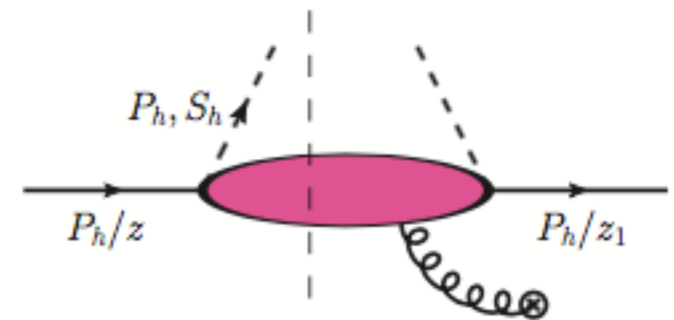

$\gamma^{\alpha}$

**Kinematic**




$k_T^{\alpha}$

**Dynamical**




$A_T^{\alpha}$

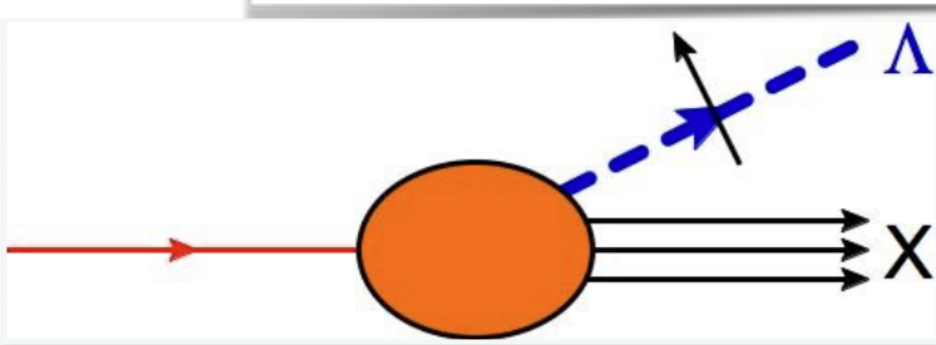
**See Daniel Pitonyak's talk**
**I will review for Lambdas**

**+**

- **Gauge Invariance**
- **Equation of motions relations (EoMs)**
- **Lorentz Invariance relations (LIRs)**

## **Partonic picture of fragmentation,**

Collins & Soper, Nucl. PhysB 1982, Jaffe & Ji Nucl. Phys B 1992, Ji PRD49 1994

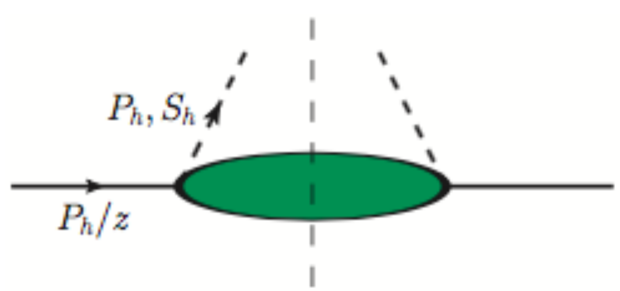



$$\langle P_\Lambda, S_\Lambda; X|\, \bar{q}(0)\, |0\rangle$$

$$\Delta_{ij}(z) = \frac{1}{N_c}\sum_X \int \frac{d\lambda}{2\pi}\, \mathrm{e}^{-i\frac{\lambda}{z}}\, \langle 0|\, [\infty m, 0]\, q_i(0)|P_\Lambda, S_\Lambda; X\rangle\langle P_\Lambda, S_\Lambda; X|\, \bar{q}_j(\lambda m)[\lambda m, \infty m]\, |0\rangle$$

Fragmentation at Twist-3 Intrinsic twist 3 with transverse spin $\Lambda$

$$p^\mu \simeq \frac{1}{z}P_h^\mu.$$




$$D_T^{\Lambda/q}(z)$$

$$G_T^{\Lambda/q}(z)$$

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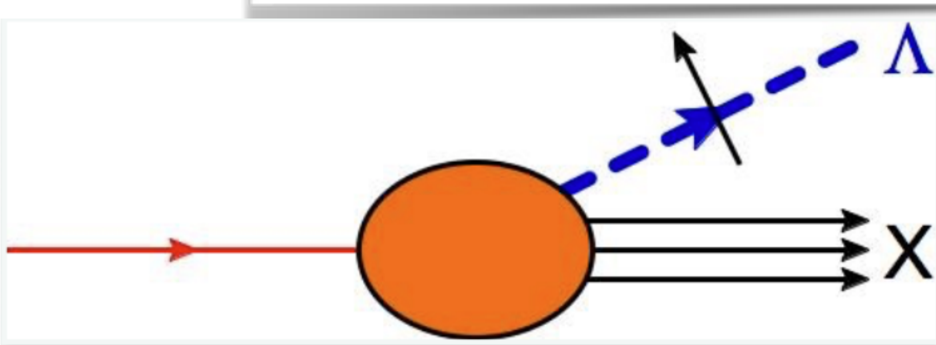


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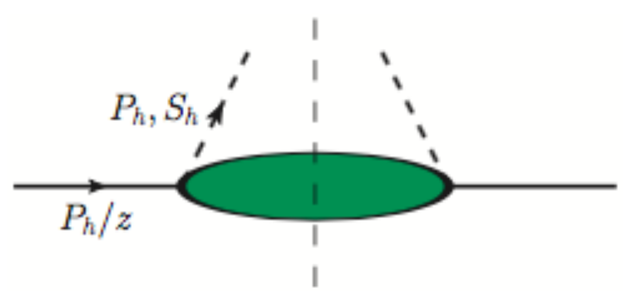


$D_T^{\Lambda/q}(z)$

$G_T^{\Lambda/q}(z)$

$$\Delta(z) = \frac{1}{z}\left( \cdots - M_h \epsilon^{P_h n \alpha S_h} \gamma_\alpha\, D_T^q(z) - M_h \not{S}_{hT} \gamma_5\, G_T^q(z) \ldots \right)$$

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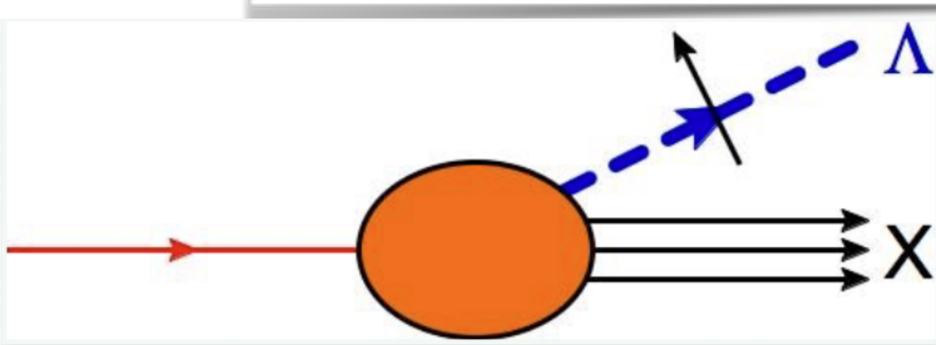
Collins & Soper, Nucl. PhysB 1982, Jaffe & Ji Nucl. Phys B 1992, Ji PRD49 1994

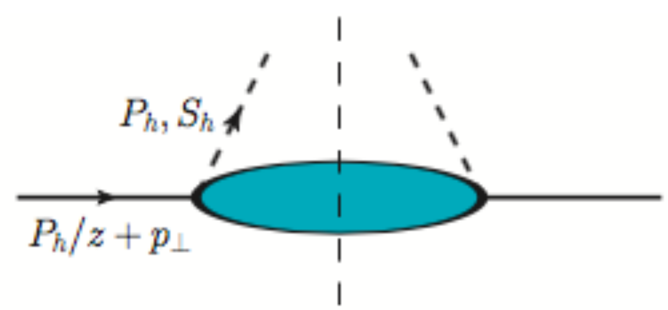



$$\langle P_\Lambda, S_\Lambda; X|\, \bar{q}(0)\, |0\rangle$$

$$\Delta^\rho_\partial(z) = \int d^2p_T\, p_T^\rho \Delta(z, zp_T)$$

Fragmentation at Twist-3 Kinematic twist 3 with transverse spin $\Lambda$

$$p^\mu \simeq \frac{1}{z}P_h^\mu + p_T^\mu .$$




$$D_{1T}^{\perp(1),\Lambda/q}(z)$$

$$G_{1T}^{\perp(1),\Lambda/q}(z)$$

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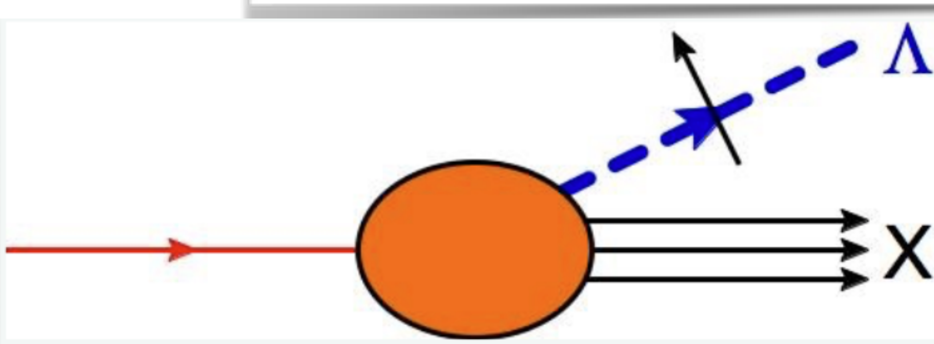


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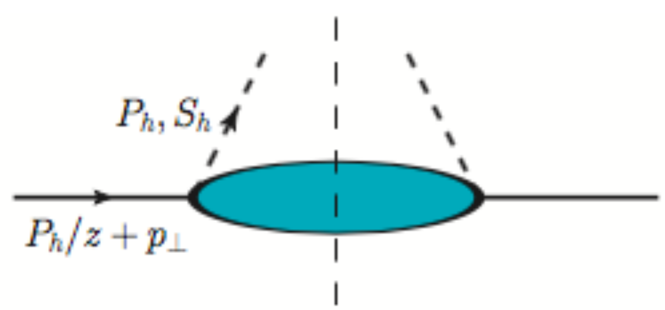


$$D_{1T}^{\perp(1),\Lambda/q}(z)$$

$$G_{1T}^{\perp(1),\Lambda/q}(z)$$

$$\Delta^\rho_\partial(z) = \frac{1}{z} M_h \left( \epsilon^{P_h n \rho S_{hT}}\, \not{P}_h\, D_{1T}^{\perp(1),q}(z) - S_{hT}^\rho\, \not{P}_h \gamma_5\, G_{1T}^{\perp(1),q}(z) + \ldots \right)$$

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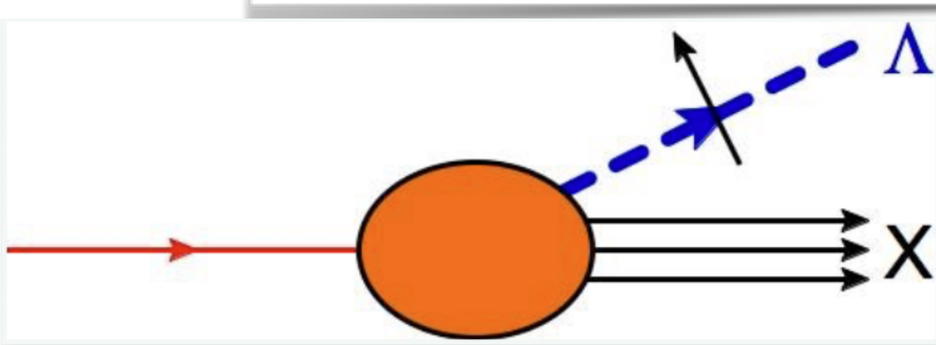
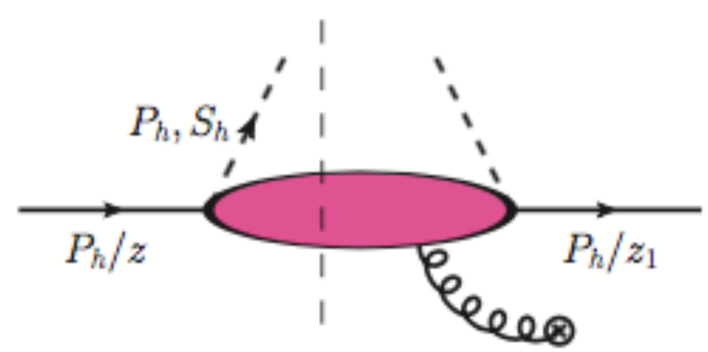



$$\langle P_\Lambda, S_\Lambda; X|\, \bar{q}(0)\, |0\rangle$$

$$\Delta_F^\alpha(z,z') \sim \langle 0|\, q(\lambda m)\, gF^{m\alpha}(\mu m)|P_\Lambda, S_\Lambda; X\rangle\langle P_\Lambda, S_\Lambda; X|\, \bar{q}(0)\, |0\rangle$$

Fragmentation at Twist-3 "Dynamical twist 3 with transverse spin $\Lambda$



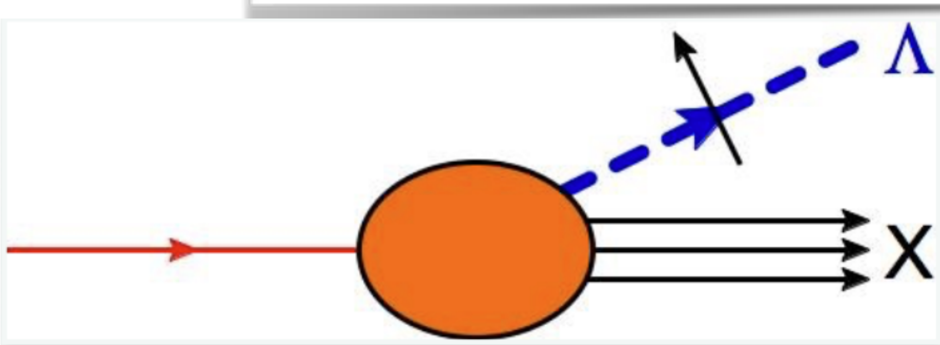

$\hat{D}_{FT}^{\Lambda/q}(z, z')$

$\hat{G}_{FT}^{\Lambda/q}(z, z')$

$A_T^\alpha$

## Partonic picture of fragmentation,

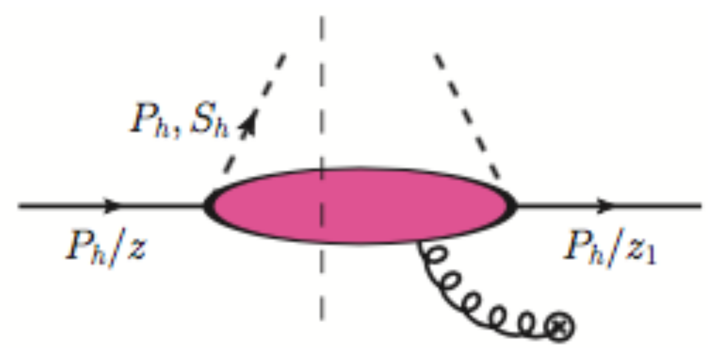
Collins & Soper, Nucl. PhysB 1982, Jaffe & Ji Nucl. Phys B 1992, Ji PRD49 1994




$$\langle P_\Lambda, S_\Lambda; X|\, \bar{q}(0)\, |0\rangle$$

$$\Delta_F^\alpha(z,z') \sim \langle 0|\, q(\lambda m)\, gF^{m\alpha}(\mu m)|P_\Lambda, S_\Lambda; X\rangle\langle P_\Lambda, S_\Lambda; X|\, \bar{q}(0)\, |0\rangle$$

Fragmentation at Twist-3 “Dynamical twist 3 with transverse spin $\Lambda$




$\hat{D}_{FT}^{\Lambda/q}(z,z')$

$\hat{G}_{FT}^{\Lambda/q}(z,z')$

$$A_T^\alpha$$

$$\Delta^\alpha(z,z') = \frac{M_h}{z}\left(\epsilon^{P_h n\alpha S_h}\, \not{P}_h\, i\hat{D}_{FT}^{qg}(z,z') + S_{hT}^\alpha\, \not{P}_h\gamma_5\, \hat{G}_{FT}^{qg}(z,z')\dots\right)$$

## Partonic picture of fragmentation,

Collins & Soper, Nucl. PhysB 1982, Jaffe & Ji Nucl. Phys B 1992, Ji PRD49 1994

$$\Delta_F^{\alpha}(z,z') \sim \langle 0|\, q(\lambda m)\, gF^{m\alpha}(\mu m)|P_\Lambda, S_\Lambda; X\rangle\langle P_\Lambda, S_\Lambda; X|\,\bar{q}(0)\,|0\rangle$$

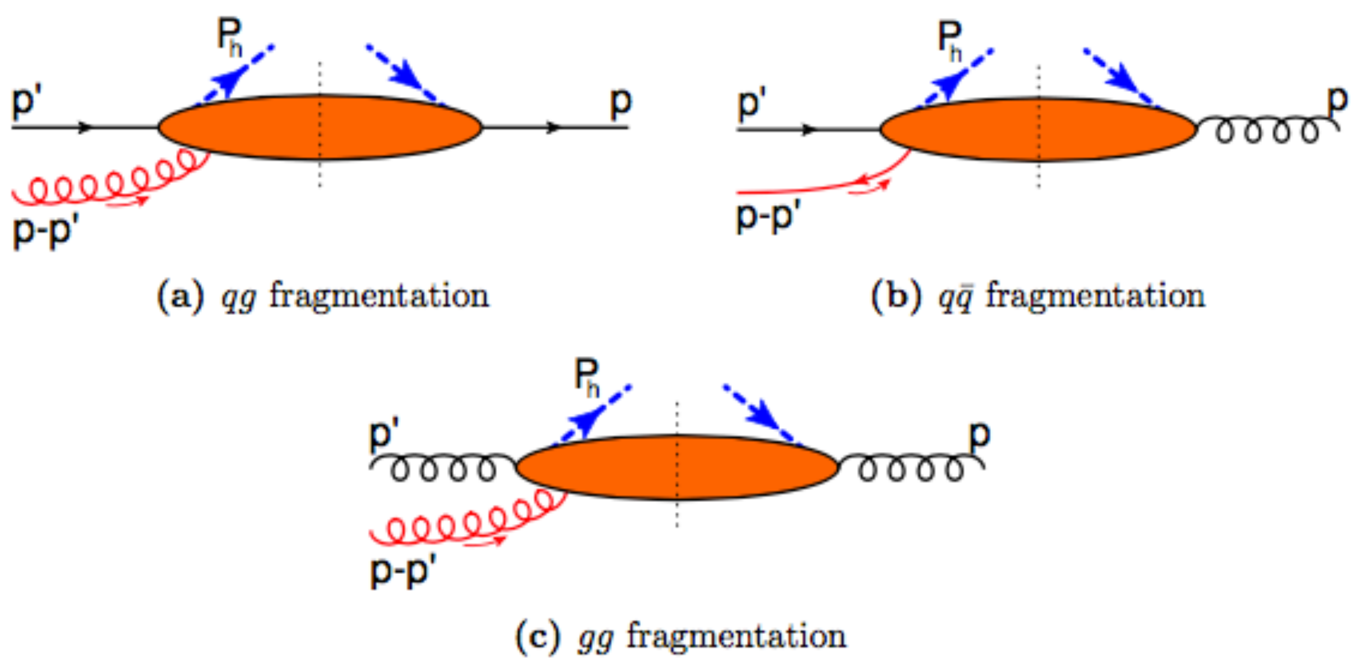
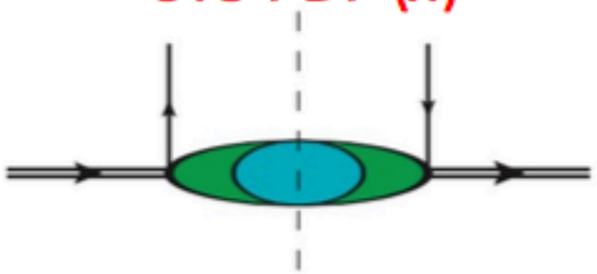
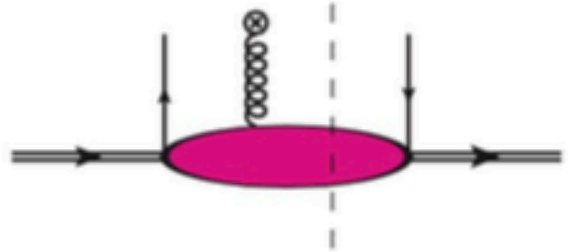
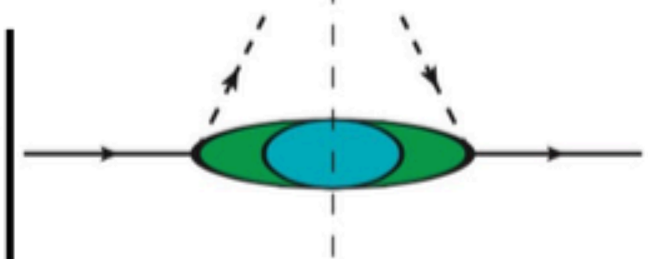
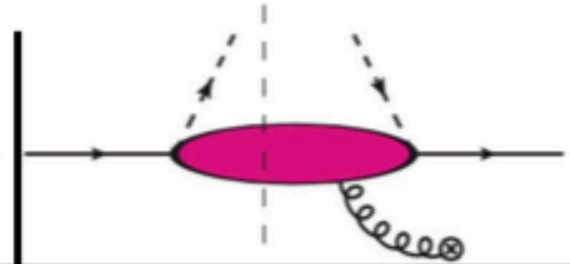



**Figure 2.** Diagrammatic representation of three-parton fragmentation correlations.

**Can have interference w/ qg & q, qq & g, gg & g**

Gluonic *poles* of the dynamical FF *vanish* (Metz, 2003, Meissner & Metz (2009), Gamberg, Mukherjee, Mulders 2008, 2011, Boer, Kang, Vogelsang, Yuan Phys.Rev.Lett. 105 (2010) .

This makes the calculation of twist-3 fragmentation effects different from the calculation of soft-gluon and soft-fermion poles on the PDF side.

| Hadron Pol. | CT3 PDF ($x$)  | | CT3 PDF ($x$, $x_1$)  | CT3 FF ($z$)  | | CT3 FF ($z$, $z_1$)  |
|---|---|---|---|---|---|---|
| | intrinsic | kinematical | dynamical | intrinsic | kinematical | dynamical |
| **U** | $e$ | $h_1^{\perp(1)}$ | $H_{FU}$ | $E, H$ | $H_1^{\perp(1)}$ | $\hat{H}_{FU}^{\Re,\Im}$ |
| **L** | $h_L$ | $h_{1L}^{\perp(1)}$ | $H_{FL}$ | $H_L, E_L$ | $H_{1L}^{\perp(1)}$ | $\hat{H}_{FL}^{\Re,\Im}$ |
| **T** | $g_T$ | $f_{1T}^{\perp(1)}$, $g_{1T}^{\perp(1)}$ | $F_{FT}, G_{FT}$ | $D_T, G_T$ | $D_{1T}^{\perp(1)}$, $G_{1T}^{\perp(1)}$ | $\hat{D}_{FT}^{\Re,\Im}, \hat{G}_{FT}^{\Re,\Im}$ |

**Courtesy of Daniel Pitonyak**

# Relations: Equation of Motion & Lorentz-Invariance

## See talk of Daniel Pitonyak

Kanazawa, Koike, Metz, Pitonyak, MS, PRD 93, 054024 (2016) …

**EOM**

$$D_{1T}^{\perp(1)}(z) + \frac{D_T(z)}{z} = \int_0^1 d\beta \frac{\Im[\hat{D}_{FT}(z,z/\beta)] - \Im[\hat{G}_{FT}(z,z/\beta)]}{1-\beta}$$

$$G_{1T}^{\perp(1)}(z) - \frac{G_T(z)}{z} = \int_0^1 d\beta \frac{\Re[\hat{D}_{FT}(z,z/\beta)] - \Re[\hat{G}_{FT}(z,z/\beta)]}{1-\beta}$$

**LIR**

$$\frac{D_T(z)}{z} = -\left(1 - z\frac{d}{dz}\right) D_{1T}^{\perp(1)}(z) - 2\int_0^1 d\beta \frac{\Im[\hat{D}_{FT}(z,z/\beta)]}{(1-\beta)^2}$$

$$\frac{G_T(z)}{z} = \frac{G_1(z)}{z} + \left(1 - z\frac{d}{dz}\right) G_{1T}^{\perp(1)}(z) - 2\int_0^1 d\beta \frac{\Re[\hat{G}_{FT}(z,z/\beta)]}{(1-\beta)^2}$$

Two equations, three functions

→ can eliminate

‘intrinsic & kinematical twist-3’

Kanazawa, Metz, Pitonyak, Schlegel, PLB **742** (2015); Kanazawa, Metz, Pitonyak, Schlegel, PLB **744** (2015); Koike, Pitonyak, Takagi, Yoshida PLB **752** (2016); Koike, Pitonyak, Yoshida PLB **759** (2016); Kanazawa, Koike, Metz, Pitonyak, Schlegel, PRD **93** (2016); Gamberg, Kang, Pitonyak, Schlegel, Yoshida JHEP **1901** (2019))

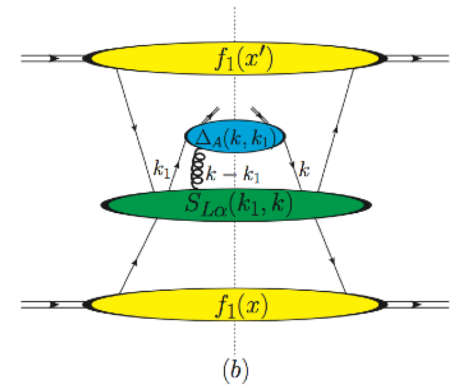
# Single inclusive hard processes for $\Lambda^{\uparrow}$ production in pQCD

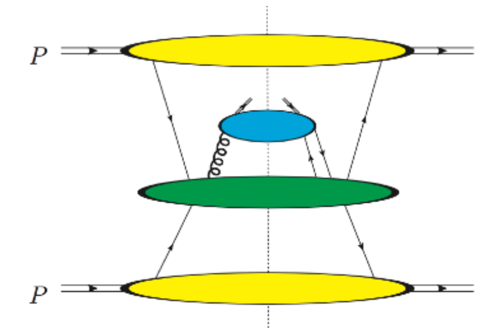
# Single inclusive hard processes for $\Lambda^{\uparrow}$ production in pQCD

$pp$ - collisions $pp \to \Lambda^{\uparrow}$ X

complete LO formulae not available, complicated

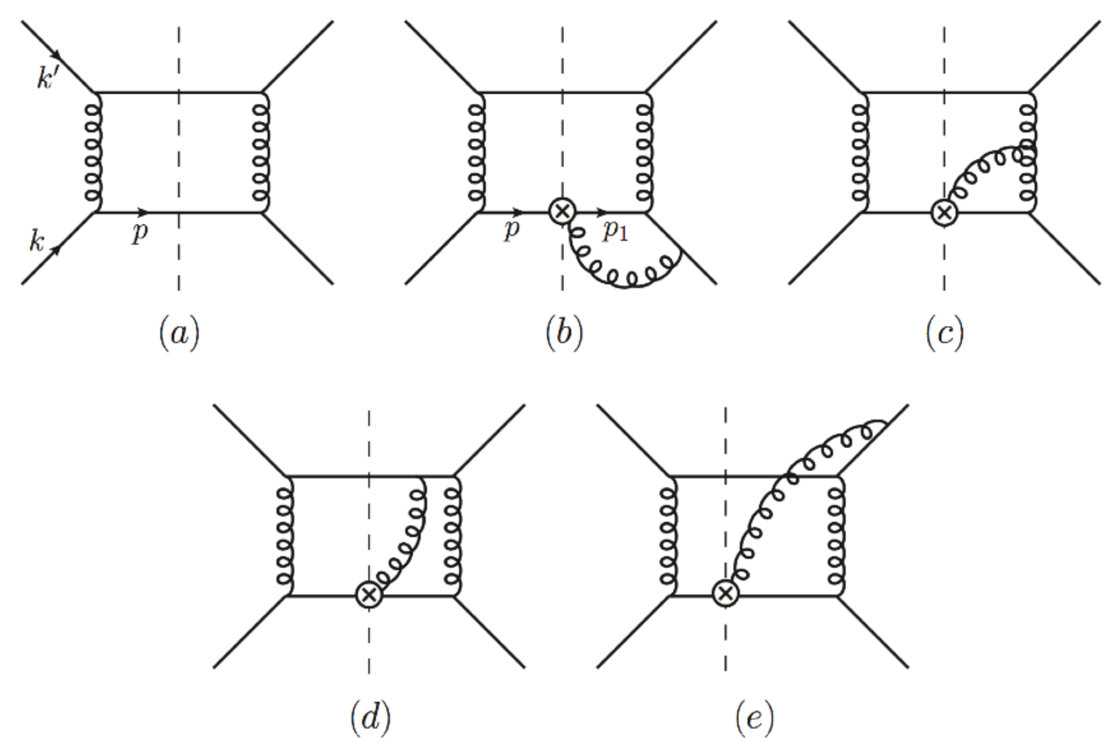
Koike, Metz, Pitonyak, Yabe, Yoshida, PRD 2017




They computed the LO terms that are related to quark-gluon-quark fragmentation correlators, while terms given by quark-antiquark-gluon correlators and pure gluon (gg and ggg) correlators have yet to be considered

**Many more diagrams …**



# Single inclusive hard processes for $\Lambda^{\uparrow}$ production in pQCD

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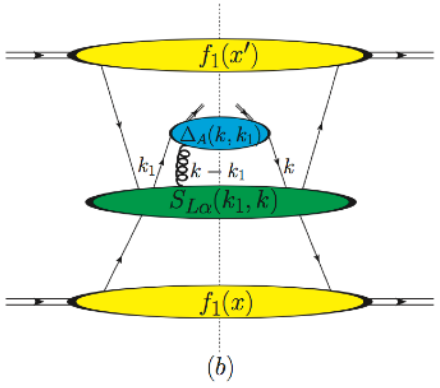
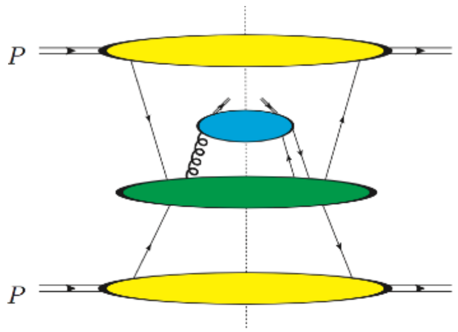
Koike, Metz, Pitonyak, Yabe, Yoshida, PRD 2017]

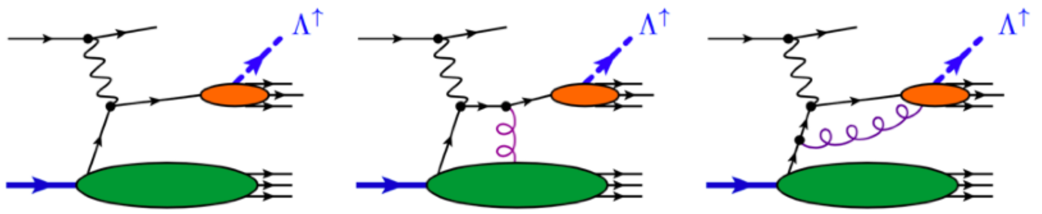



$ep \rightarrow \Lambda^{\uparrow} X$ single-inclusive $\Lambda^{\uparrow}$- production

complete LO-formula (including EoM & LIR)

Kanazawa, Koike, Metz, Pitonyak, MS, PRD 2016

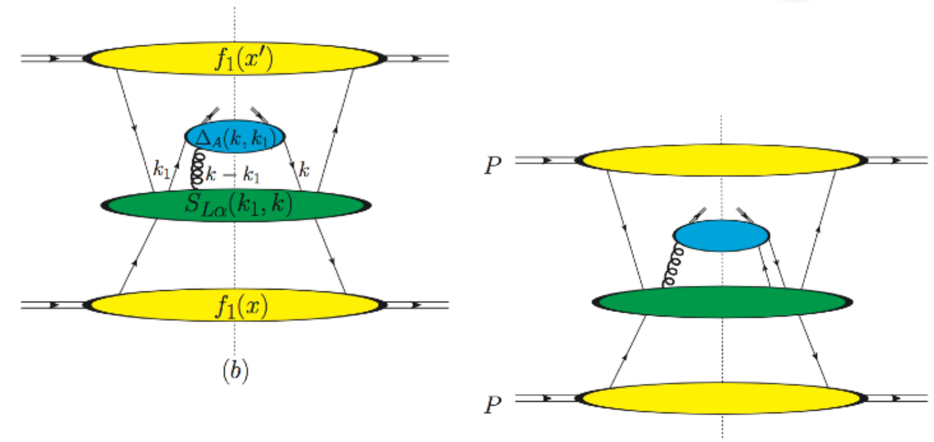


# Single inclusive hard processes for $\Lambda^{\uparrow}$ production in pQCD

*pp* - collisions $pp \to \Lambda^{\uparrow} X$

complete LO formulae not available, complicated

Koike, Metz, Pitonyak, Yabe, Yoshida, PRD 2017]

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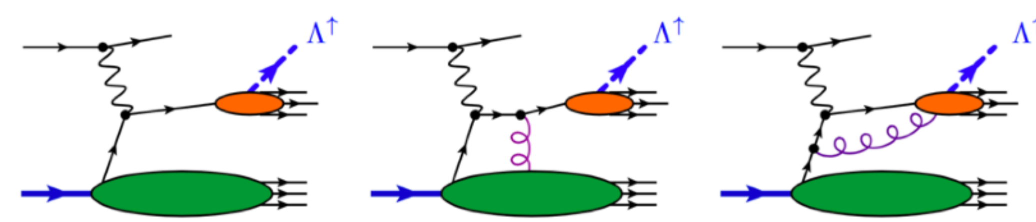
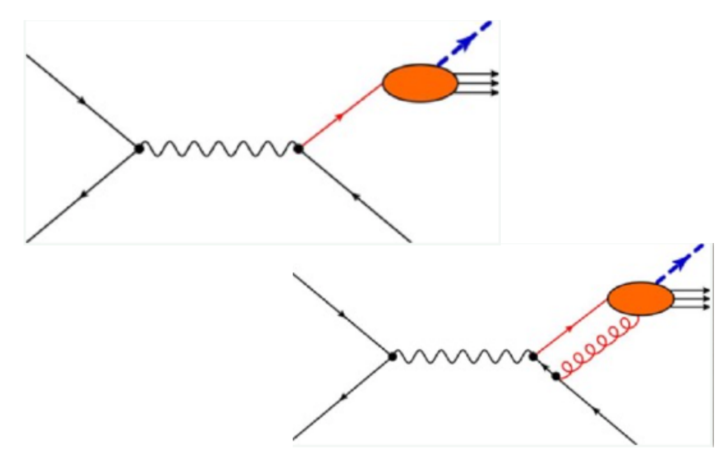
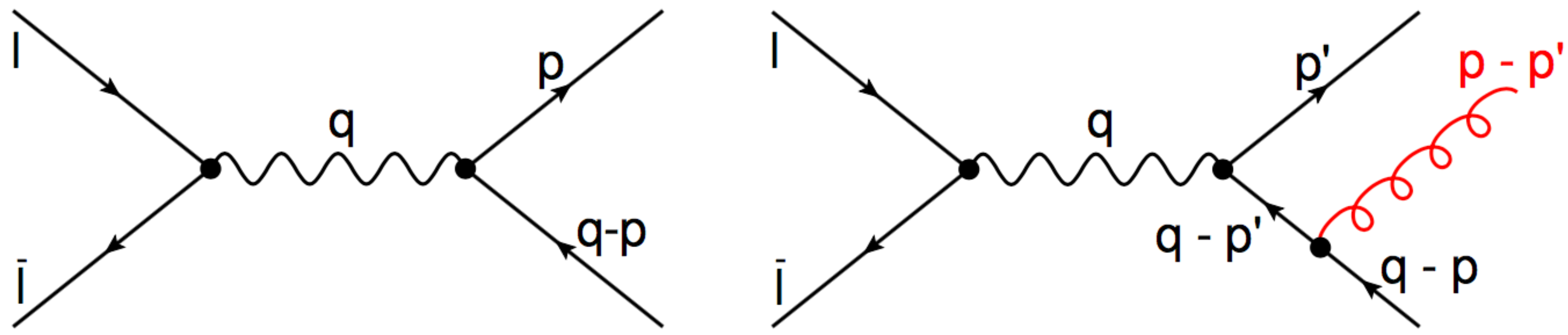



$e^+e^- \longrightarrow \Lambda^{\uparrow} X$ inclusive $\Lambda^{\uparrow}$ production

Gamberg, Kang, Pitonyak, Schlegel, Yoshida JHEP 2019, LO & NLO



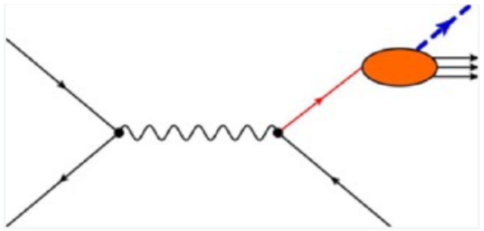
# Single inclusive hard processes for $\Lambda^{\uparrow}$ production in pQCD

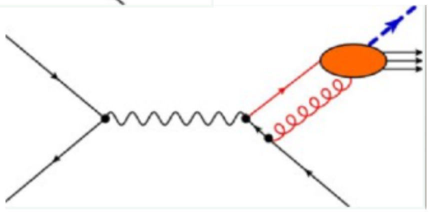



(a) LO quark fragmentation. (b) LO quark-gluon fragmentation.

$e^+e^- \longrightarrow \Lambda^{\uparrow}$ X inclusive $\Lambda^{\uparrow}$ production

Gamberg, Kang, Pitonyak, Schlegel, Yoshida JHEP 2019, LO & NLO

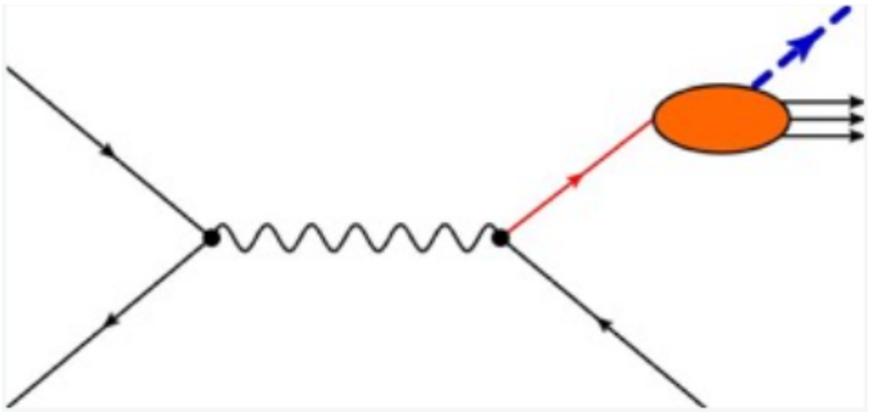




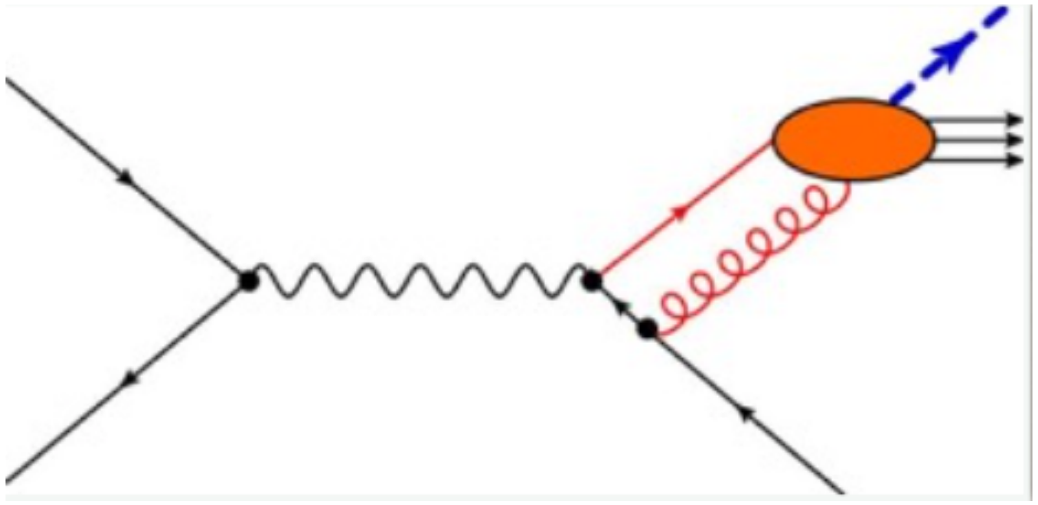
## Now Consider Transverse $\Lambda^{\uparrow}$ polarization at *LO*

There are contributions from

'Intrinsic' & 'kinematical' twist-3 FF



Dynamical' twist-3 FF:



Intrinsic  Kinematical  Dynamical

$$\frac{d\sigma(S_{\Lambda T})}{dz_h\, d\phi} = C\,|S_{\Lambda T}|\sin(\phi_S)\sum_q e_q^2\left[\frac{D_T^{\Lambda/q}(z_h)}{z_h} - D_{1T}^{\perp(1)\Lambda/q}(z_h) + \int_0^1 d\beta\,\frac{\Im[\hat{D}_{FT}-\hat{G}_{FT}]^{\Lambda/q}(z_h, z_h/\beta)}{1-\beta}\right]$$

# Now Consider Transverse $\Lambda^{\uparrow}$ polarization at *LO*

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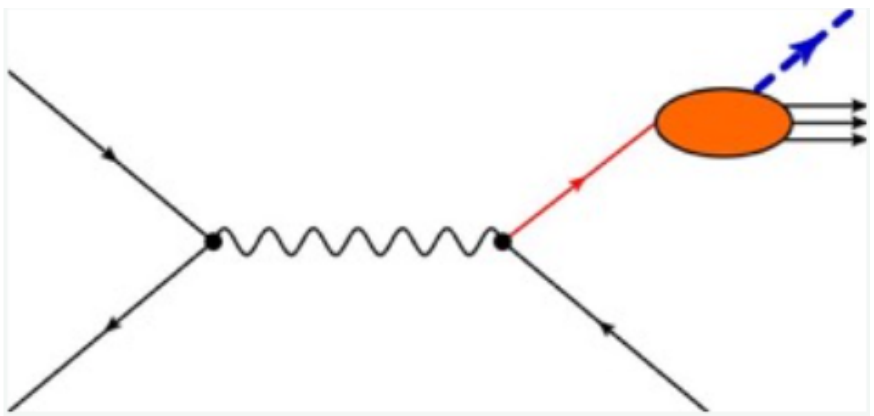
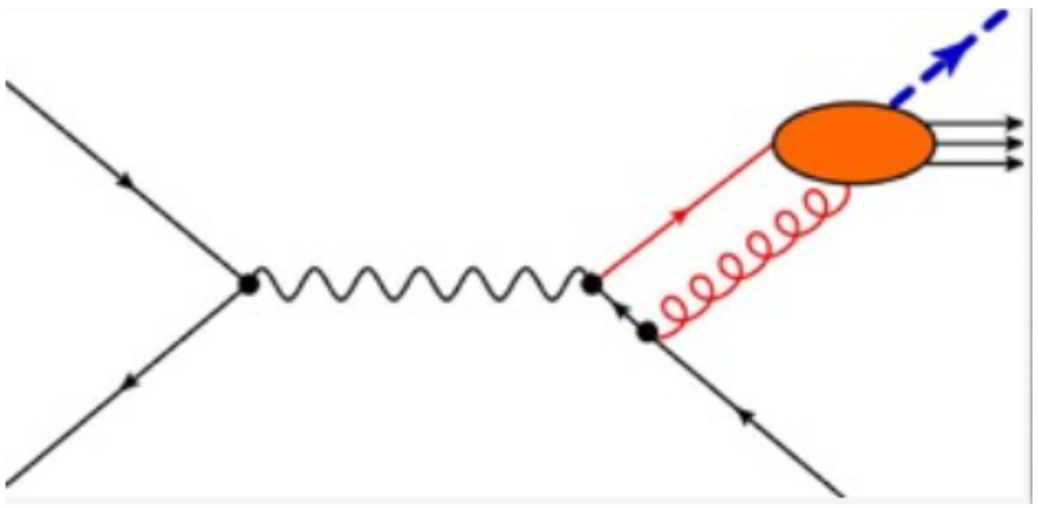


Dynamical' twist-3 FF:



Intrinsic Kinematical Dynamical

$$\frac{d\sigma(S_{\Lambda T})}{dz_h\, d\phi} = C\, |S_{\Lambda T}| \sin(\phi_S) \sum_q e_q^2 \left[ \frac{D_T^{\Lambda/q}(z_h)}{z_h} - D_{1T}^{\perp(1)\Lambda/q}(z_h) + \int_0^1 d\beta \frac{\Im[\hat{D}_{FT} - \hat{G}_{FT}]^{\Lambda/q}(z_h, z_h/\beta)}{1-\beta} \right]$$

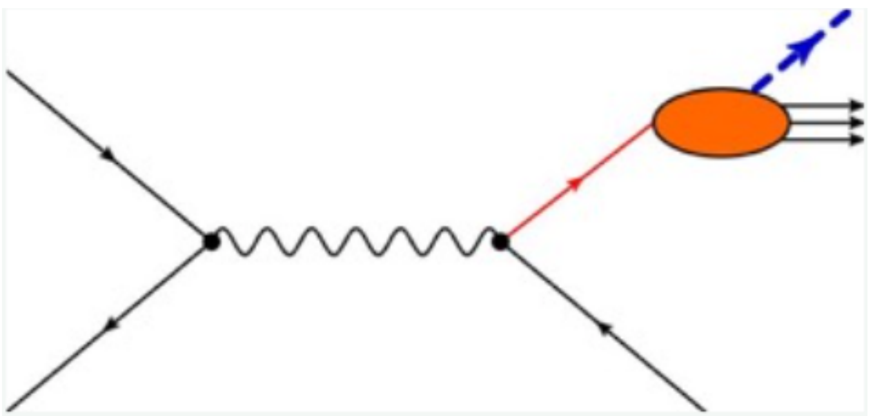
**Noting , the EOM….**

$$D_{1T}^{\perp(1)}(z) + \frac{D_T(z)}{z} = \int_0^1 d\beta \frac{\Im[\hat{D}_{FT}(z, z/\beta)] - \Im[\hat{G}_{FT}(z, z/\beta)]}{1-\beta}$$

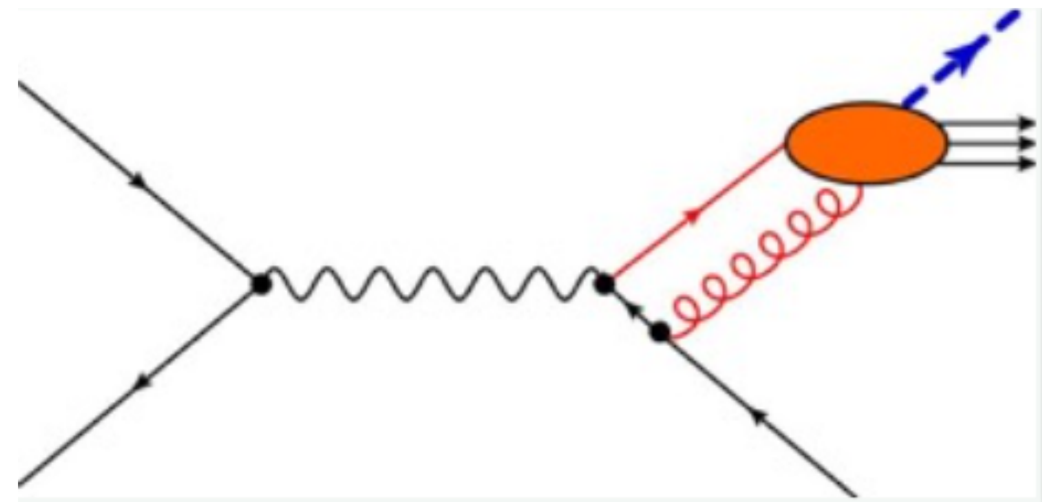
# Now Consider Transverse $\Lambda^{\uparrow}$ polarization at *LO*

There are contributions from

'Intrinsic' & 'kinematical' twist-3 FF



Dynamical' twist-3 FF:



Intrinsic Kinematical Dynamical

$$\frac{d\sigma(S_{\Lambda T})}{dz_h\, d\phi} = C\, |S_{\Lambda T}| \sin(\phi_S) \sum_q e_q^2 \left[ \frac{D_T^{\Lambda/q}(z_h)}{z_h} - D_{1T}^{\perp(1)\Lambda/q}(z_h) + \int_0^1 d\beta\, \frac{\Im[\hat{D}_{FT} - \hat{G}_{FT}]^{\Lambda/q}(z_h, z_h/\beta)}{1-\beta} \right]$$

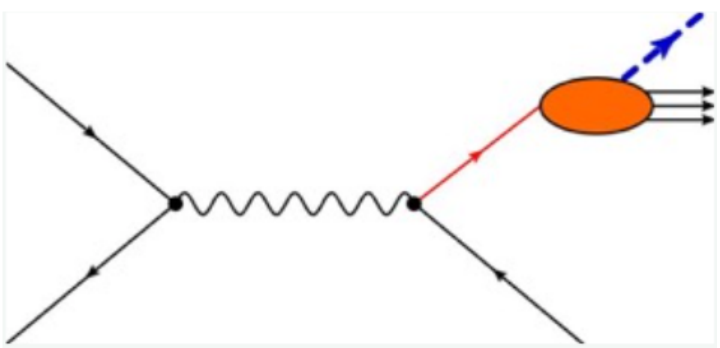
**Using the EOMs and LIRs CS can be expressed soley in terms of** $D_T^{\Lambda/q}(z)$

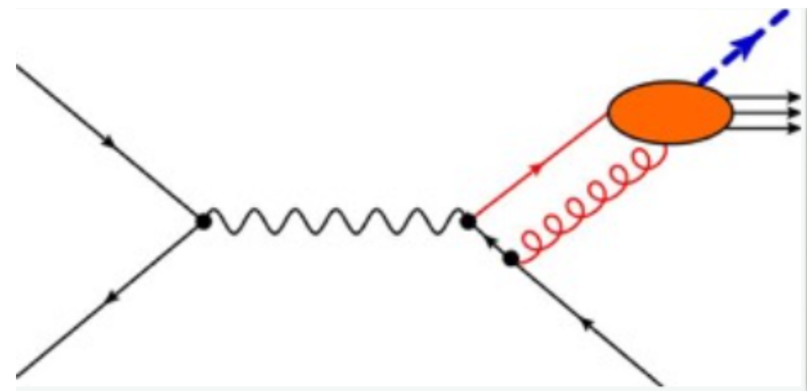
$$\frac{d\sigma(S_{\Lambda T})}{dz_h\, d\phi} = C\, |S_{\Lambda T}| \sin(\phi_S) \sum_q e_q^2 \left[ 2\, \frac{D_T^{\Lambda/q}(z_h)}{z_h} \right]$$

# Comments …

# Single-Transverse $\Lambda^\uparrow$ spin asymmetry

Unique effect driven by a single fragmentation function $D_T^{\Lambda/q}(z)$ → absent in DIS (1 $\gamma$)





Intrinsic

$$\frac{d\sigma(S_{\Lambda T})}{dz_h\, d\phi} = C\,|S_{\Lambda T}|\sin(\phi_S)\sum_q e_q^2\left[2\,\frac{D_T^{\Lambda/q}(z_h)}{z_h}\right]$$

**See also Boer, Jakob, Mulders NPB (1997)**

***n.b.*** **some intuition …**

**Consider crossing this process to inclusive DIS for transverse polarised target**

**Would have the function** $f_T^{q/\Lambda}(x)$ , $\frac{d\sigma(S_{\Lambda\, T})}{dx d\phi} \sim \sin(\phi_S)\sum_q e_q^2 f_T^{q/\Lambda}(x) = 0 \quad !!!$

***Constraints from time reversal on quark correlation function***

***Goeke, Metz, Schlegel PLB 2006, Bacchetta et al JHEP 2007, Christ & Lee 1960***

# A unique test of time reversal in QCD: Non-zero intrinsic

Unique effect driven by a single fragmentation function $D_T^{\Lambda/q}(z)$ → absent in DIS (1 $\gamma$)

## Single-Transverse $\Lambda^{\uparrow}$ spin asymmetry

$$\frac{d\sigma(S_{\Lambda T})}{dz_h\, d\phi} = C\,|S_{\Lambda T}|\sin(\phi_S)\sum_q e_q^2\left[2\,\frac{D_T^{\Lambda/q}(z_h)}{z_h}\right]$$

$$\frac{d\sigma(S_{\Lambda\, T})}{dxd\phi} \sim \sin(\phi_S)\sum_q e_q^2 f_T^{q/\Lambda}(x) = 0 \quad !!! \qquad f_T^{q/\Lambda}(x)$$

***Constraints from time reversal on quark correlation function***
***Goeke, Metz, Schlegel PLB 2006, Bacchetta et al JHEP 2007, Christ & Lee 1960***

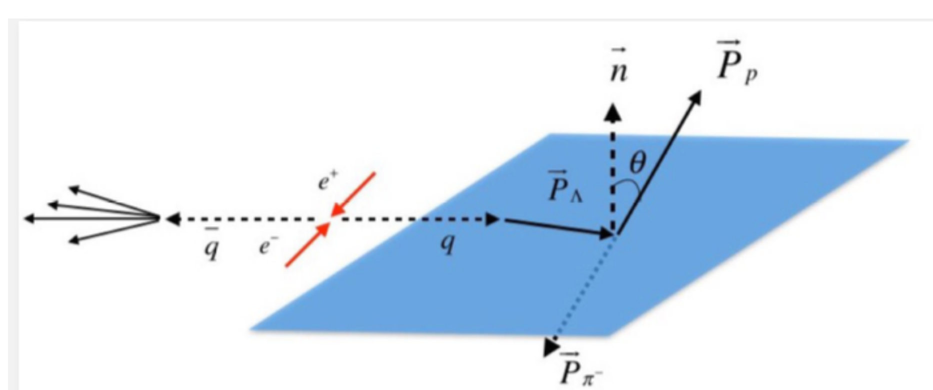
# Predicted non-zero TSSA from twist-3 ff function $D_T^{\Lambda/q}(z)$ a new result

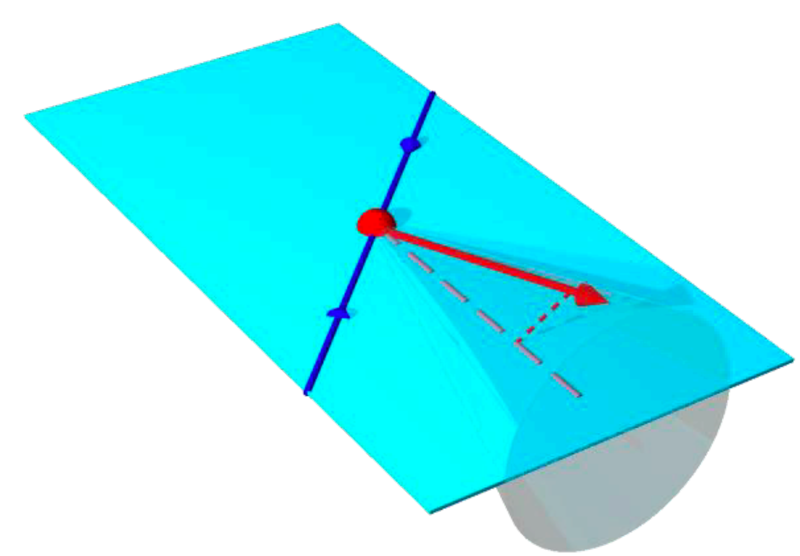
Using the EOMs and LIRs CS can be expressed soley in terms of $D_T^{\Lambda/q}(z)$

$$\frac{d\sigma(S_{\Lambda T})}{dz_h\, d\phi} = C\,|S_{\Lambda T}|\sin(\phi_S)\sum_q e_q^2\left[2\,\frac{D_T^{\Lambda/q}(z_h)}{z_h}\right]$$

Note that this observable probes the *intrinsic FF* $D_T^{\Lambda/q}(z)$ rather than the kinematic FF $D_{1T}^{\perp(1)\Lambda/q}$ of thrust axis semi-inclusive like measurement of Belle

Anselmino Boer, D'Alesio, Murgia. PRD 2001, Boer, Kang, Vogelsang, Yuan Phys.Rev.Lett. 105 (2010)



TMD PDFs ($x$, $k_T$)

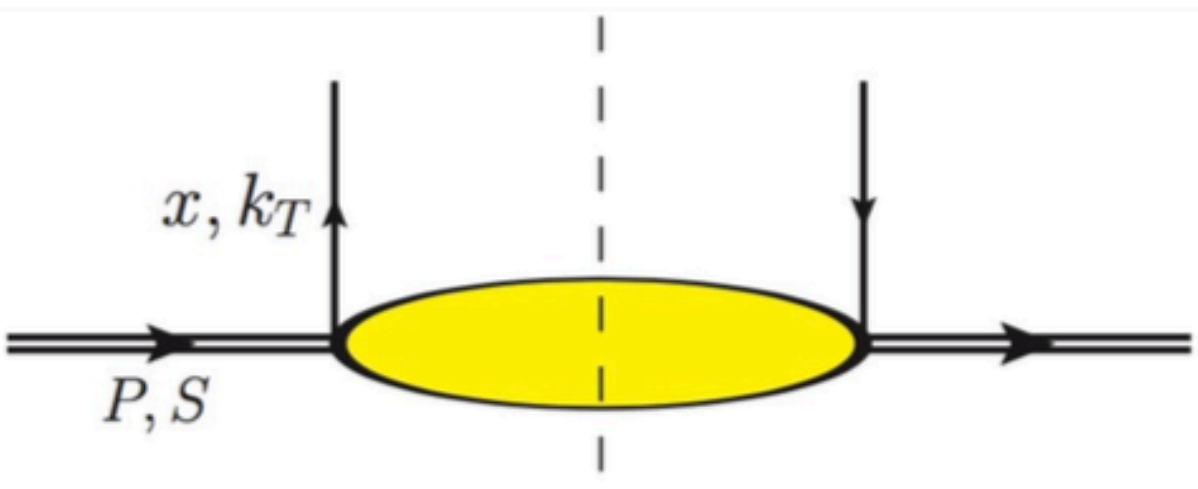
| q pol. / H pol. | U | L | T |
|---|---|---|---|
| U | $f_1$ | | $h_1^\perp$ |
| L | | $g_{1L}$ | $h_{1L}^\perp$ |
| T | $f_{1T}^\perp$ | $g_{1T}$ | $h_{1T}$ $h_{1T}^\perp$ |

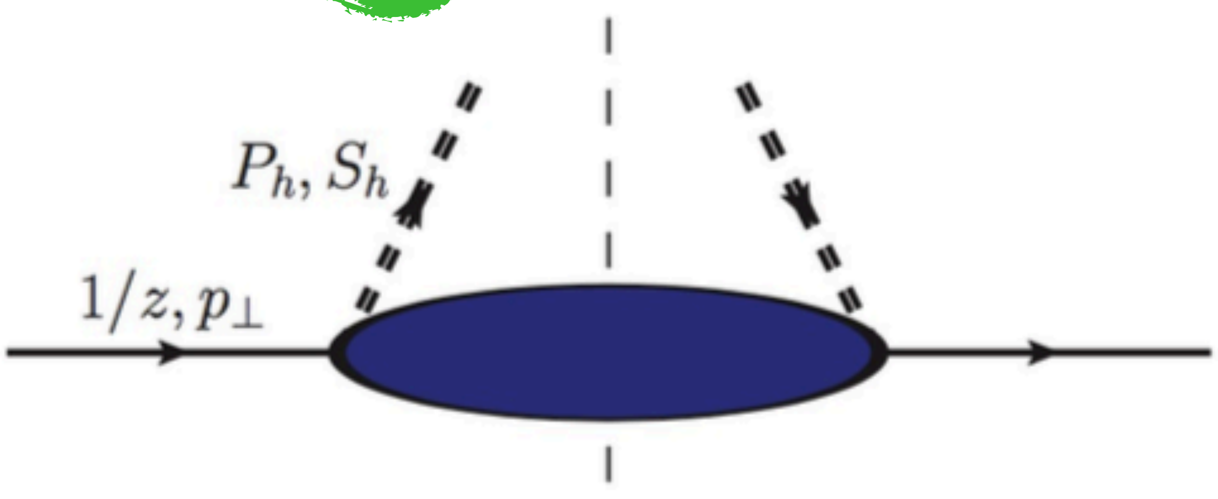
(Mulders, Tangerman (1996); Goeke, Metz, Schlegel (2005))

TMD FFs ($z, p_\perp$)

| q pol. / H pol. | U | L | T |
|---|---|---|---|
| U | $D_1$ | | $H_1^\perp$ |
| L | | $G_{1L}$ | $H_{1L}^\perp$ |
| T | $D_{1T}^\perp$ | $G_{1T}$ | $H_{1T}$ $H_{1T}^\perp$ |

(Boer, Jakob, Mulders (1997))



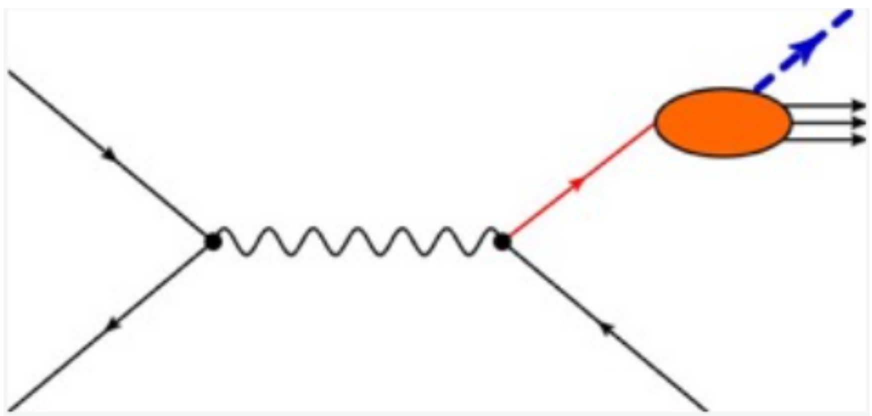




**Courtesy of Daniel Pitonyak**

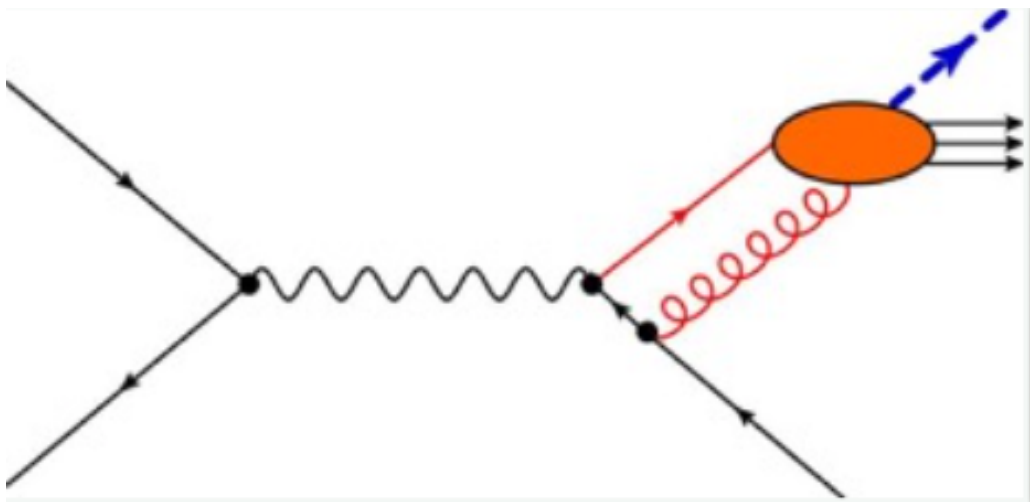
## Also can express Transverse $\Lambda^{\uparrow}$ polarization at *LO as…*

There are contributions from

’Intrinsic’ & ‘kinematical’ twist-3 FF



Dynamical’ twist-3 FF:



Intrinsic Kinematical Dynamical

$$\frac{d\sigma(S_{\Lambda T})}{dz_h\, d\phi} = C\,|S_{\Lambda T}|\sin(\phi_S)\sum_q e_q^2\left[\frac{D_T^{\Lambda/q}(z_h)}{z_h} - D_{1T}^{\perp(1)\Lambda/q}(z_h) + \int_0^1 d\beta\,\frac{\Im[\hat{D}_{FT} - \hat{G}_{FT}]^{\Lambda/q}(z_h, z_h/\beta)}{1-\beta}\right]$$

… Or can be expressed in terms of $\hat{D}_{FT}^{\Lambda/q}(z, z')$ $\hat{G}_{FT}^{\Lambda/q}(z, z')$ & $D_{1T}^{\perp(1),\Lambda/q}(z)$

$$\frac{d\sigma(S_{\Lambda T})}{dz_h\, d\phi} = C\,|S_{\Lambda T}|\sin(\phi_S)\sum_q e_q^2\left[-2\,D_{1T}^{\perp(1)\Lambda/q}(z_h) + 2\int_0^1 d\beta\,\frac{\Im[\hat{D}_{FT} - \hat{G}_{FT}]^{\Lambda/q}(z_h, z_h/\beta)}{1-\beta}\right]$$

## Now consider Transverse $\Lambda^{\uparrow}$ polarization at *NLO*

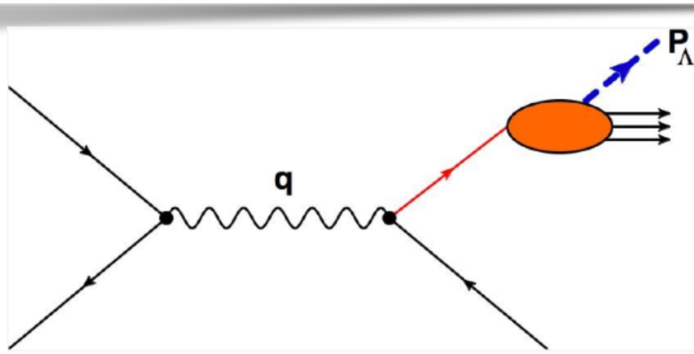
# Now consider Transverse $\Lambda^{\uparrow}$ polarization at *NLO*

- **Factorisation and scale evolution go hand and hand**

**To appreciate the challenge we consider first unpolarised case**

- **Factorisation and scale evolution go hand and hand**
- **Use of the  subtraction procedure**

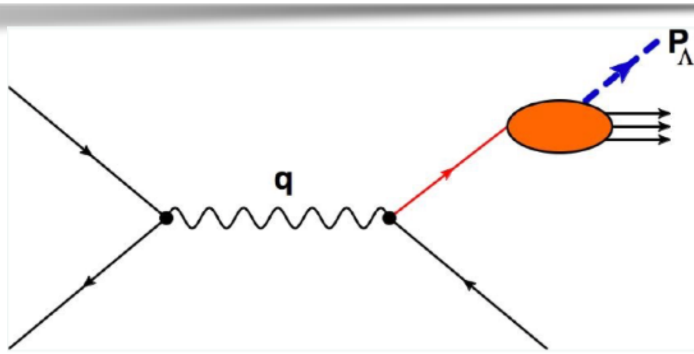
# Factorization Unpolarized $e^+e^- \to \Lambda \ X$ cross section



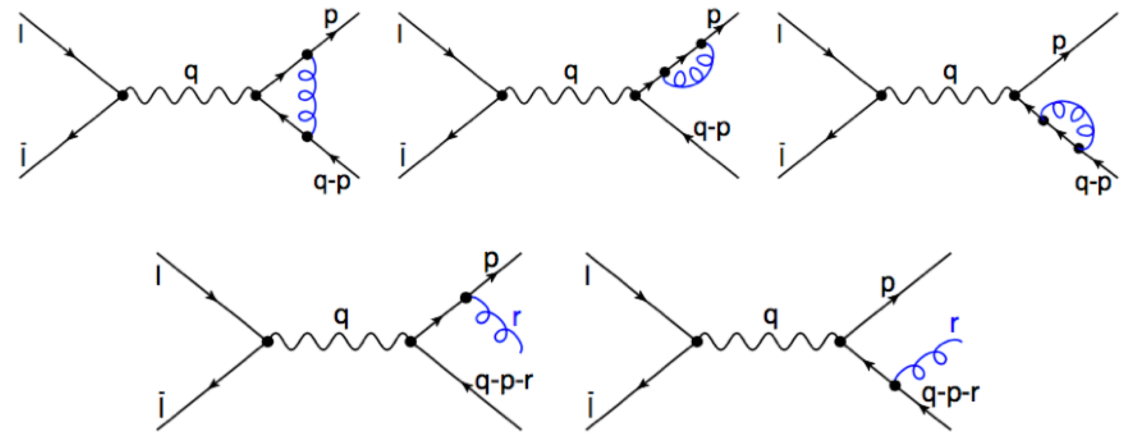

$$\sigma \sim \hat{\sigma} \otimes D_1^\Lambda$$

$$E_\Lambda \frac{d\sigma}{d^3 \vec{P}_\Lambda} \propto \sum_q e_q^2 \ D_1^{\Lambda/q}(z_h), \quad z_h = \frac{2P_\Lambda \cdot q}{q^2}$$

# Factorization Unpolarized $e^+e^- \to \Lambda \; X$ cross section



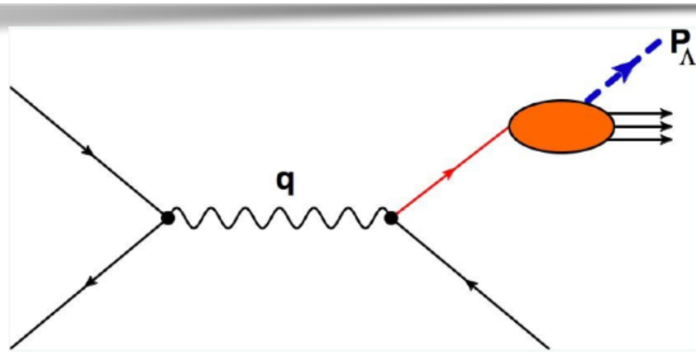

$$\sigma \sim \hat{\sigma} \otimes D_1^{\Lambda}$$



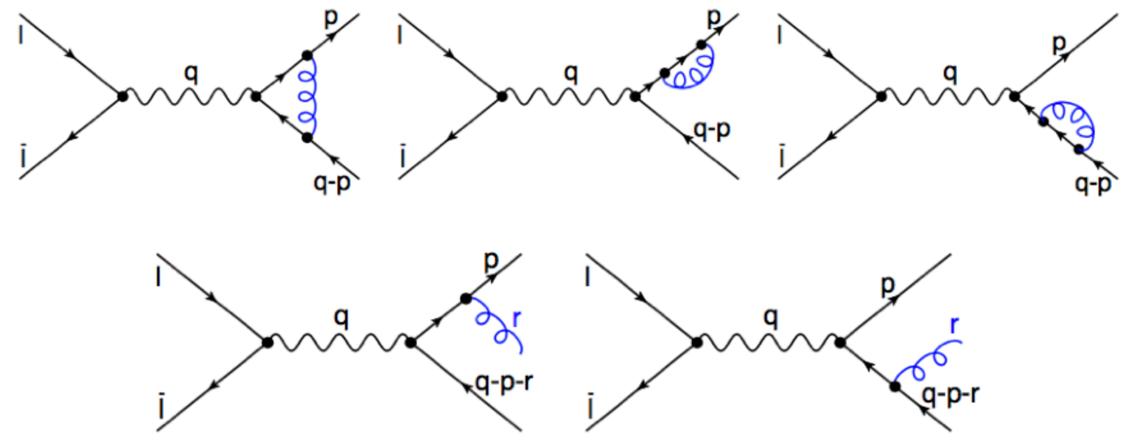

$$E_\Lambda \frac{d\sigma}{d^3\vec{P}_\Lambda} \propto \sum_q e_q^2 \; D_1^{\Lambda/q}(z_h)\,, \qquad z_h = \frac{2P_\Lambda \cdot q}{q^2}$$

$$\left(E_\Lambda \frac{d\sigma}{d^3\vec{P}_\Lambda}\right)_{\rm NLO} \propto \sum_q e_q^2 \int_{z_h}^{1} \frac{dw}{w} \Big[ \hat{\sigma}^{\overline{\rm MS},q}(w, s/\mu^2) D_1^{\Lambda/q}(z_h/w, \mu) + \hat{\sigma}^{\overline{\rm MS},g}(w, s/\mu^2) D_1^{\Lambda/g}(z_h/w, \mu) \Big]$$

# Factorization Unpolarized $e^+e^- \to \Lambda\ X$ cross section




$$\sigma \sim \hat{\sigma} \otimes D_1^\Lambda$$



$$E_\Lambda \frac{d\sigma}{d^3\vec{P}_\Lambda} \propto \sum_q e_q^2\ D_1^{\Lambda/q}(z_h)\,, \qquad z_h = \frac{2P_\Lambda \cdot q}{q^2}$$

$$\left(E_\Lambda \frac{d\sigma}{d^3\vec{P}_\Lambda}\right)_{\rm NLO} \propto \sum_q e_q^2 \int_{z_h}^1 \frac{dw}{w} \Big[\hat{\sigma}^{\overline{\rm MS},q}(w, s/\mu^2) D_1^{\Lambda/q}(z_h/w,\mu) + \hat{\sigma}^{\overline{\rm MS},g}(w, s/\mu^2) D_1^{\Lambda/g}(z_h/w,\mu)\Big]$$

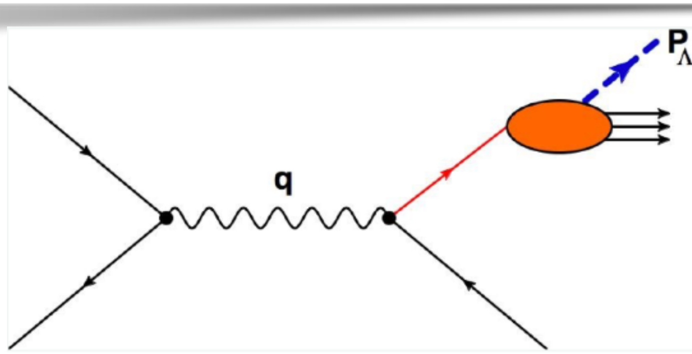
Typical NLO features:

infrared safe (cancellation of $1/\varepsilon^2$ - poles in dim. reg.)

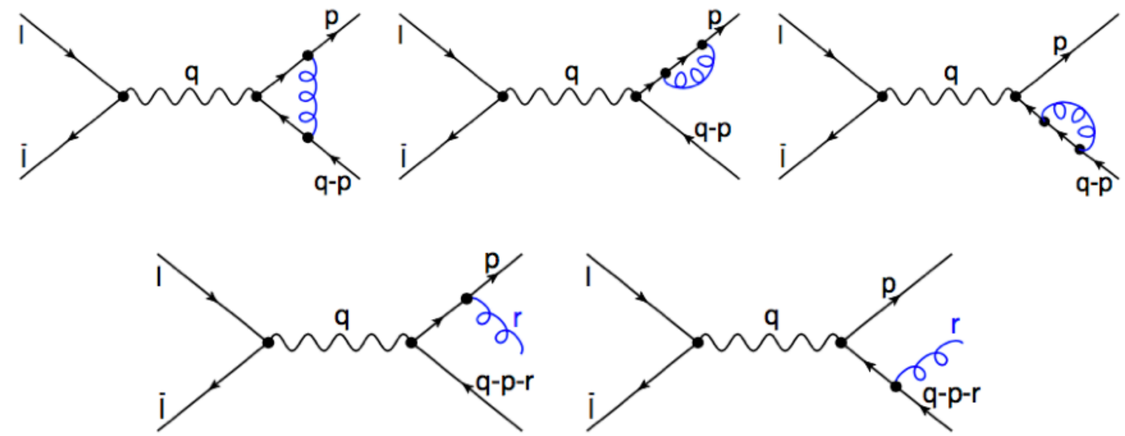
$$\hat{\sigma}_{\rm virt} + \hat{\sigma}_{\rm real} = \mathcal{O}(1/\varepsilon)$$

$$\hat{\sigma}^{q/g} \propto -\frac{1}{\varepsilon} P_{q/g\,q}(w) + \mathcal{O}(\varepsilon^0)$$

# Factorization Unpolarized $e^+e^- \to \Lambda\ X$ cross section




$$\sigma \sim \hat{\sigma} \otimes D_1^{\Lambda}$$




$$E_\Lambda \frac{d\sigma}{d^3\vec{P}_\Lambda} \propto \sum_q e_q^2\ D_1^{\Lambda/q}(z_h), \qquad z_h = \frac{2P_\Lambda \cdot q}{q^2}$$

$$\left(E_\Lambda \frac{d\sigma}{d^3\vec{P}_\Lambda}\right)_{\rm NLO} \propto \sum_q e_q^2 \int_{z_h}^1 \frac{dw}{w} \Big[\hat{\sigma}^{\overline{\rm MS},q}(w, s/\mu^2) D_1^{\Lambda/q}(z_h/w,\mu) + \hat{\sigma}^{\overline{\rm MS},g}(w, s/\mu^2) D_1^{\Lambda/g}(z_h/w,\mu)\Big]$$

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infrared safe (cancellation of $1/\varepsilon^2$ - poles in dim. reg.)

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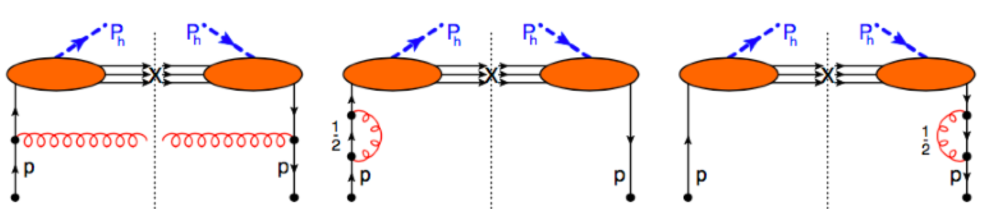
MSbar renormalization of fragmentation functions → + DGLAP evolution



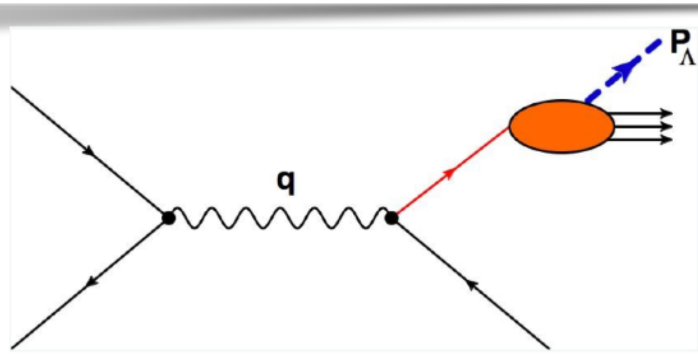

$$D_{1,\rm ren}^{\Lambda/q}(z) \sim -\frac{\alpha_s}{2\pi}\frac{S_\varepsilon^{\rm MS}}{\varepsilon} \sum_{i=q,g} \int_z^1 \frac{dw}{w} P_{iq}(w) D_{1,\rm ren}^{\Lambda/i}\left(\frac{z}{w}\right) + \mathcal{O}(\alpha_s^2)$$

Then perform subtraction with "counter cross section" which removes these 1/ε-poles

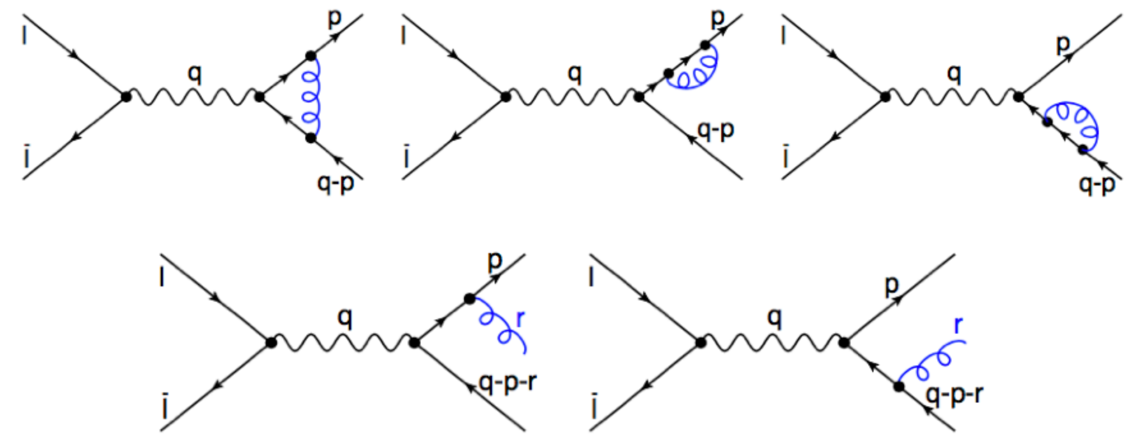
!!! necessary condition for to show process factorizes !!! Cross section is finite and DGLAP Evolution established.

Collins & Soper NPB 1982, Collins PRD58, 1998, Also Gordon and Vogelsang PRD 48 1993 & Collins 2011 Cambridge Press

# Factorization Unpolarized $e^+e^- \to \Lambda\ X$ cross section



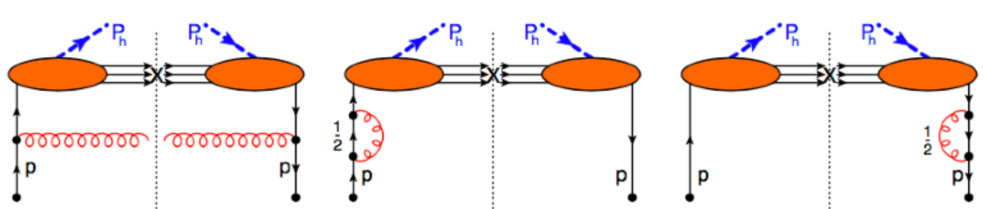

$$\sigma \sim \hat{\sigma} \otimes D_1^{\Lambda}$$




$$E_\Lambda \frac{d\sigma}{d^3\vec{P}_\Lambda} \propto \sum_q e_q^2\ D_1^{\Lambda/q}(z_h), \quad z_h = \frac{2P_\Lambda \cdot q}{q^2}$$

$$\left(E_\Lambda \frac{d\sigma}{d^3\vec{P}_\Lambda}\right)_{\rm NLO} \propto \sum_q e_q^2 \int_{z_h}^1 \frac{dw}{w} \Big[ \hat{\sigma}^{\bar{\rm MS},q}(w, s/\mu^2) D_1^{\Lambda/q}(z_h/w,\mu) + \hat{\sigma}^{\bar{\rm MS},g}(w, s/\mu^2) D_1^{\Lambda/g}(z_h/w,\mu) \Big]$$

$$- \sigma_0 \left((1-v)^2 + v^2 - \varepsilon\right) \sum_{f=q,\bar{q}} e_f^2\, D_1^{f[1]}(z_h) + O(\alpha_s^2)$$




Then perform subtraction with "counter cross section" which removes these 1/ε-poles

!!! necessary condition for to show process factorizes !!! Cross section is finite and DGLAP Evolution established.

Collins & Soper NPB 1982, Collins PRD58, 1998, Also Gordon and Vogelsang PRD 48 1993 & Collins 2011 Cambridge Press

# Postage stamp of finite unpolarised cross section

$$
\begin{aligned}
\frac{E_h\,\mathrm{d}\sigma}{\mathrm{d}^3\vec{P}_h} = \frac{2N_c\alpha_{\mathrm{em}}^2}{z_h s^2}\Bigg\{ & ((1-v)^2+v^2)\sum_{f=q,\bar{q}} e_f^2\, D_1^f(z_h,\mu) \\
& + ((1-v)^2+v^2) \\
& \times \sum_{f=q,\bar{q}} e_f^2 \int_{z_h}^1 \frac{\mathrm{d}w}{w}\left[\hat{c}_{D_1}^{1;f}(w)\, D_1^f\left(\frac{z_h}{w};\mu\right) + \hat{c}_{D_1}^{1;g}(w)\, D_1^g\left(\frac{z_h}{w};\mu\right)\right] \\
& + 4\,v\,(1-v) \\
& \times \sum_{f=q,\bar{q}} e_f^2 \int_{z_h}^1 \frac{\mathrm{d}w}{w}\left[\hat{c}_{D_1}^{2;f}(w)\, D_1^f\left(\frac{z_h}{w};\mu\right) + \hat{c}_{D_1}^{2;g}(w)\, D_1^g\left(\frac{z_h}{w};\mu\right)\right]\Bigg\}
\end{aligned}
$$

## Finte partonic cross sections

$$
\begin{aligned}
\hat{c}_{D_1}^{1;f}(w) = \frac{C_F\,\alpha_s}{2\pi}\Bigg[ & \delta(1-w)\left(\frac{3}{2}\ln\left(\frac{s}{\mu^2}\right) - \frac{9}{2} + \frac{2\pi^2}{3}\right) \\
& + \left(\frac{\ln(1-w)}{1-w}\right)_+ (1+w^2) + \frac{1+w^2}{(1-w)_+}\ln\left(w^2\frac{s}{\mu^2}\right) - \frac{3w(2-w)}{2(1-w)_+}\Bigg]
\end{aligned}
$$

$$
\begin{aligned}
\hat{c}_{D_1}^{2;f}(w) &= \frac{C_F\alpha_s}{2\pi}\,, \\
\hat{c}_{D_1}^{1;g}(w) &= \frac{C_F\alpha_s}{2\pi}\left[\frac{1+(1-w)^2}{w}\ln\left(w^2(1-w)\frac{s}{\mu^2}\right) - \frac{2(1-w)}{w}\right], \\
\hat{c}_{D_1}^{2;g}(w) &= \frac{C_F\alpha_s}{2\pi}\left[\frac{2(1-w)}{w}\right].
\end{aligned}
$$

$$
\frac{\partial}{\partial \ln\mu^2}\left(E_h\frac{\mathrm{d}\sigma}{\mathrm{d}^3\vec{P}_h}\right) = 0
$$

**The expression is the standard LO DGLAP-evolution equation.**

$$
\Longrightarrow \frac{\partial D_1^f(z;\mu)}{\partial \ln\mu^2} = \sum_{f'=f,g}\int_z^1 \frac{\mathrm{d}w}{w}\, P_{f\to f'}^{[1]}(w)\, D_1^{f'}\left(\frac{z}{w};\mu\right),
$$

**Standard LO DGLAP splitting functions**

$$
\begin{aligned}
P_{f\to f}^{[1]}(w) &= \frac{C_F\alpha_s}{2\pi}\left(\frac{1+w^2}{(1-w)_+} + \frac{3}{2}\,\delta(1-w)\right), \\
P_{f\to g}^{[1]}(w) &= \frac{C_F\alpha_s}{2\pi}\left(\frac{1+(1-w)^2}{w}\right).
\end{aligned}
$$

# Transverse $\Lambda^{\uparrow}$ polarization at *NLO*

These NLO dynamics for twist-3 fragmentation are the simplest process

Calculate the contributions from all channels qq, gg, qgq, qqg,

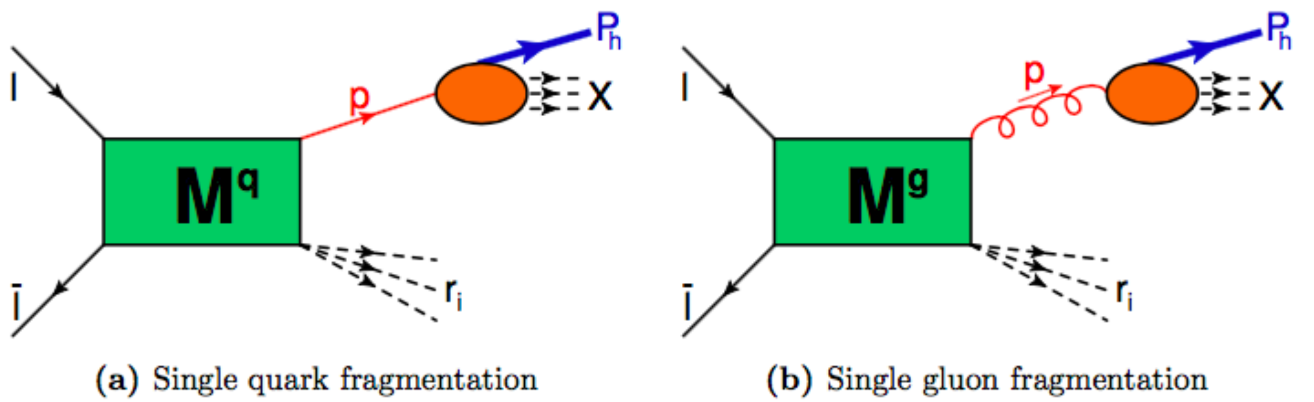



**Figure 3.** Fragmentation mechanism in $e^+e^-$ annihilation for intrinsic and kinemati contributions.

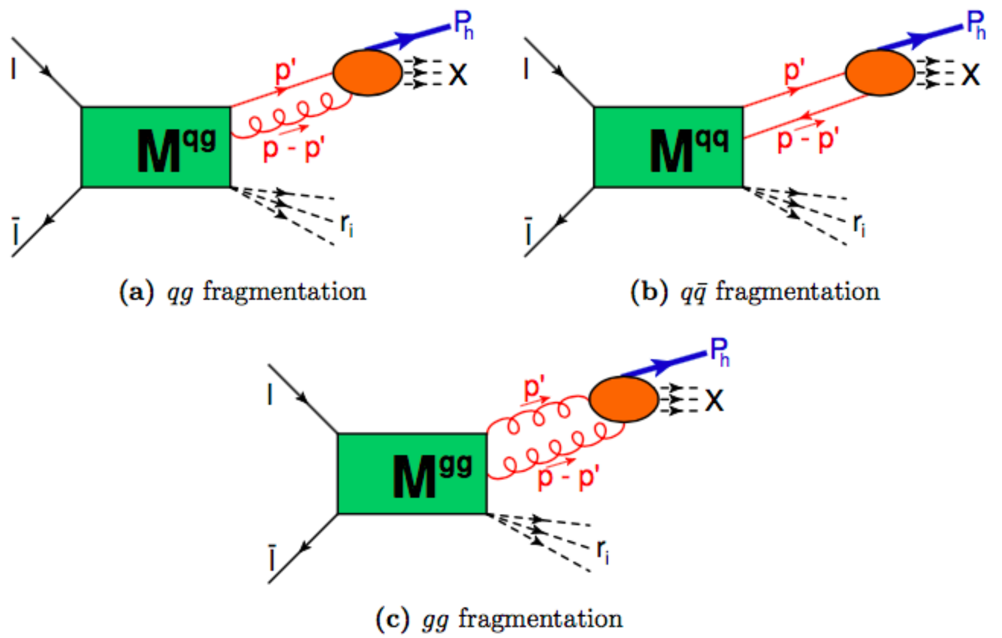
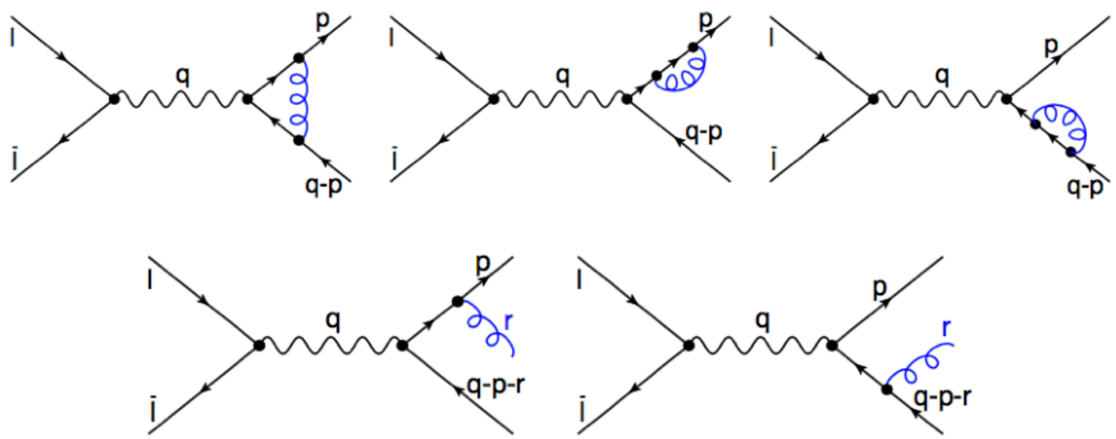
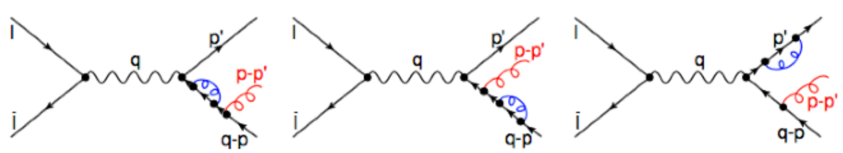



**Figure 4.** Fragmentation mechanism in $e^+e^-$ annihilation for dynamical contributions.

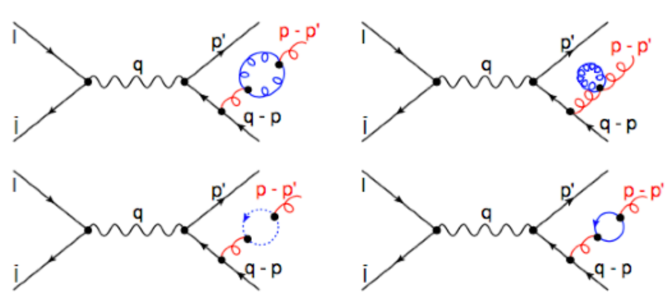
# Transverse $\Lambda^{\uparrow}$ polarization at *NLO*

Calculate intrinsic, kinematical, dynamical contributions in all channels qq, gg, qgq, qqg, ggg

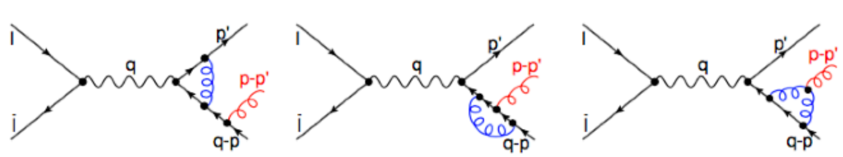


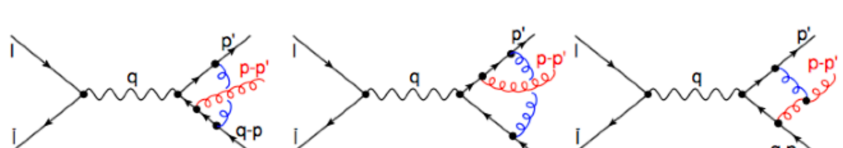

(a) Self-energy corrections to the quark lines




(b) Self-energy corrections on the gluon line




(c) Vertex corrections




(d) Box diagram

**Figure 7.** Virtual one-loop diagrams.

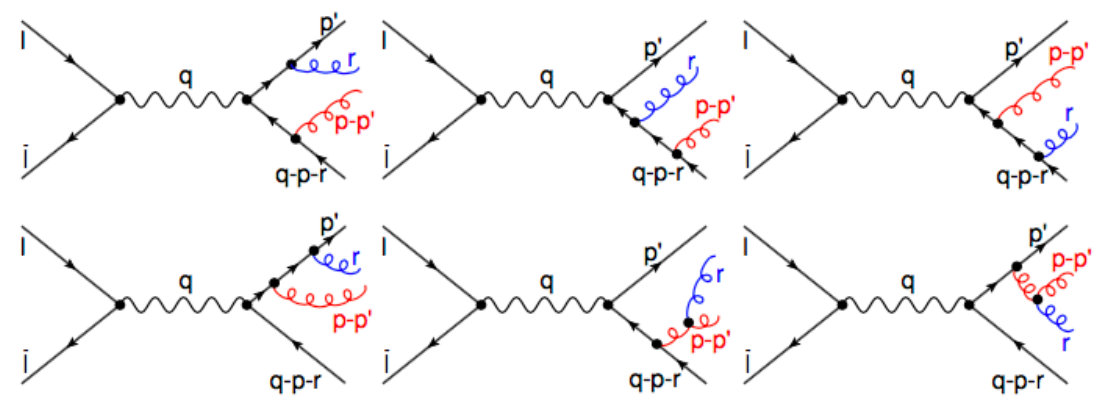



**Figure 8.** NLO corrections from real gluon radiation to quark-gluon fragmentation.

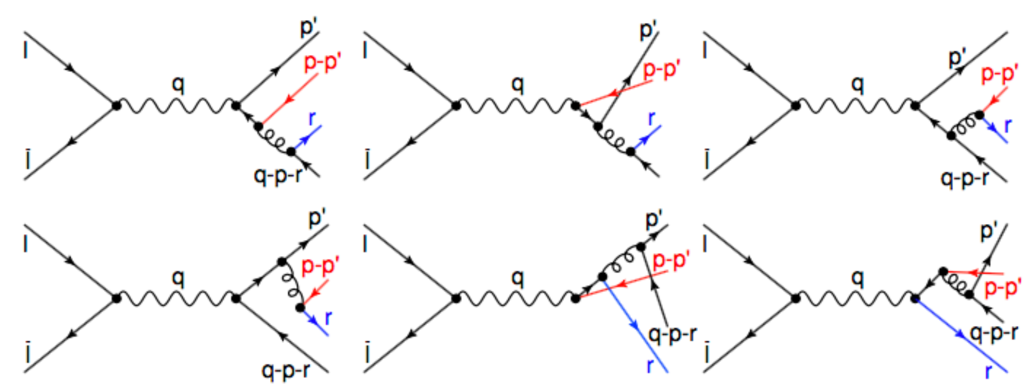



**Figure 9.** NLO corrections from quark radiation to quark/anti-quark fragmentation.

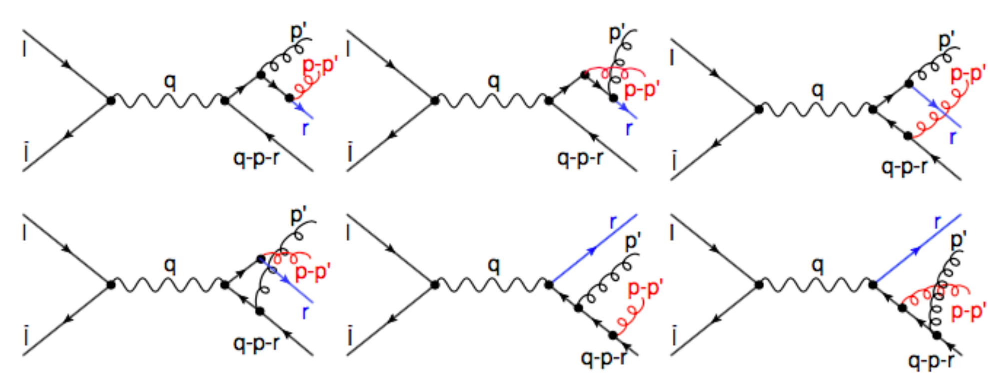



**Figure 10.** NLO corrections from quark radiation to gluon-gluon fragmentation.

# Comments

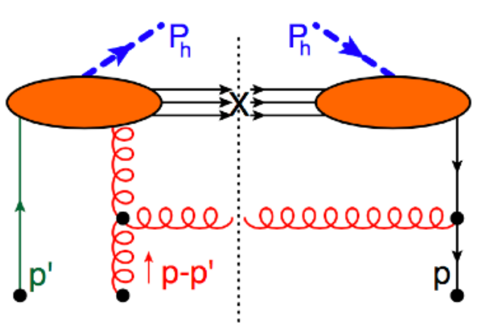
- All partonic factors calculated in Feynman gauge & Light-cone gauge, @NLO both calculations *agree*!
- E.o.M. - relations are crucial:
- Eliminating ‘intrinsic’ twist-3 contributions: can show color gauge invariance at NLO! ✔
- Infrared $1/\varepsilon^2$ - poles cancel ✔
- $1/\varepsilon$ - poles of imaginary parts of loops cancel through E.o.M. ✔
- $1/\varepsilon$ collinear poles of real parts of loops through MSbar - renormalization (?????)

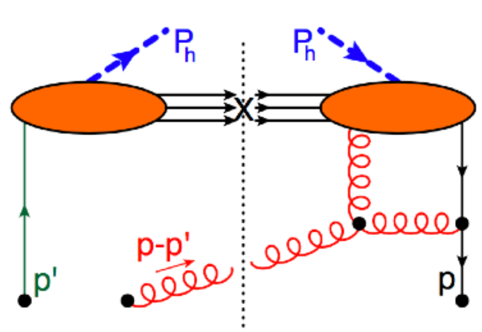
## Do the $1/\varepsilon$ - collinear poles of real parts of loops through MSbar - renormalization (???)

$$\frac{E_h\, \mathrm{d}\sigma(S_h)}{\mathrm{d}^{d-1}\vec{P}_h} = \frac{E_h\, \mathrm{d}\sigma^{\mathrm{qq\&qgq}}(S_h)}{\mathrm{d}^{d-1}\vec{P}_h} + \frac{E_h\, \mathrm{d}\sigma^{\mathrm{q\bar{q}g}}(S_h)}{\mathrm{d}^{d-1}\vec{P}_h} + \frac{E_h\, \mathrm{d}\sigma^{\mathrm{gg\&ggg}}(S_h)}{\mathrm{d}^{d-1}\vec{P}_h}$$

$$- \sigma_0\, (1-2v)\, \frac{4M_h}{z_h\, s^2}\epsilon^{ll'P_hS_h} \sum_{f=q,\bar{q}} e_f^2\, \frac{2D_T^{f[1]}(z_h)}{z_h} + O(\alpha_s^2)\,;$$

**Subtraction term "counter cross section"**



# Complete structure of the NLO result w/o intrinsic twist-3

$$E_h \frac{\mathrm{d}\sigma_U^{\mathrm{EoM},2}}{\mathrm{d}^{d-1}\vec{P}_h}(S_h) = (4\pi^2 z_h^2)^{\varepsilon} \frac{2\alpha_{\mathrm{em}}^2 N_c}{z_h s^2} \frac{2M_h \epsilon^{P_h m l S_h}}{s} (2v-1) \times$$

$$\sum_{q=u,\overline{u},\ldots} e_q^2 \Big[ -2\, D_{1T}^{\perp(1),q}(z_h) + 2 \int_0^1 \mathrm{d}\beta \frac{\Im[\hat{D}_{FT}^q - \hat{G}_{FT}^q](z_h, \frac{z_h}{\beta})}{1-\beta}$$

$$+ \frac{\alpha_s}{2\pi} S_\varepsilon \int_{z_h}^1 \frac{\mathrm{d}w}{w^2} \int_0^1 \mathrm{d}\beta \Big\{ \hat{\sigma}_{D_{1T}^{\perp(1)}}^{q;\mathrm{EoM}}(w)\, D_{1T}^{\perp(1),q}(\tfrac{z_h}{w})$$

LO

# Complete structure of the NLO result w/o intrinsic twist-3

$$
\begin{aligned}
E_h \frac{\mathrm{d}\sigma_U^{\mathrm{EoM},2}}{\mathrm{d}^{d-1}\vec{P}_h}(S_h) = {} & (4\pi^2 z_h^2)^{\varepsilon}\, \frac{2\alpha_{\mathrm{em}}^2 N_c}{z_h s^2}\, \frac{2M_h \epsilon^{P_h m l S_h}}{s}\, (2v-1) \times \\
& \sum_{q=u,\bar{u},\ldots} e_q^2 \Bigg[ -2\, D_{1T}^{\perp(1),q}(z_h) + 2 \int_0^1 \mathrm{d}\beta\, \frac{\Im[\hat{D}_{FT}^{q} - \hat{G}_{FT}^{q}](z_h, \frac{z_h}{\beta})}{1-\beta} \\
& + \frac{\alpha_s}{2\pi} S_\varepsilon \int_{z_h}^1 \frac{\mathrm{d}w}{w^2} \int_0^1 \mathrm{d}\beta \Bigg\{ \hat{\sigma}^{q;\mathrm{EoM}}_{D_{1T}^{\perp(1)}}(w)\, D_{1T}^{\perp(1),q}(\tfrac{z_h}{w}) \\
& + \hat{\sigma}^{g;\mathrm{EoM}}_{D_{1T}^{\perp(1)}}(w)\, D_{1T}^{\perp(1),g}(\tfrac{z_h}{w}) + \hat{\sigma}^{g;\mathrm{EoM}}_{H_1^{(1)}}(w)\, H_1^{(1)g}(\tfrac{z_h}{w}) \\
& + \hat{\sigma}^{qg;\mathrm{EoM}}_{\Im D_{FT}}(w,\beta)\, \frac{\Im[\hat{D}_{FT}^{q}](\frac{z_h}{w}, \frac{z_h}{w\beta})}{1-\beta} + \hat{\sigma}^{qg;\mathrm{EoM}}_{\Im G_{FT}}(w,\beta)\, \frac{\Im[\hat{G}_{FT}^{q}](\frac{z_h}{w}, \frac{z_h}{w\beta})}{1-\beta} \\
& + \hat{\sigma}_1^{gg;\mathrm{EoM}}(w,\beta)\, \Im[\hat{D}_{FT}^{gg} - \hat{G}_{FT}^{gg} + (1-\varepsilon)\, \hat{H}_{FT}^{gg}](\tfrac{z_h}{w}, \tfrac{z_h}{w\beta}) \\
& + \hat{\sigma}_3^{gg;\mathrm{EoM}}(w,\beta)\, \Im[(1-\varepsilon)\, \hat{D}_{FT}^{gg} + \hat{G}_{FT}^{gg} + \frac{\varepsilon}{2}\, \hat{H}_{FT}^{gg}](\tfrac{z_h}{w}, \tfrac{z_h}{w\beta}) \\
& + \hat{\sigma}^{q\bar{q};\mathrm{EoM}}_{D_{FT}}(w) \left( \sum_{q=u,d,\ldots} \Im[\hat{D}_{FT}^{q\bar{q}}](\tfrac{z_h}{w}, \tfrac{z_h}{w\beta}) \right) \\
& + \hat{\sigma}^{q}_{\Re}(w,\beta)\, \frac{\Re[\hat{D}_{FT}^{q} - \hat{G}_{FT}^{q}](\frac{z_h}{w}, \frac{z_h}{w\beta})}{1-\beta} \Bigg\} \Bigg] + \mathcal{O}(\Lambda^2/s),
\end{aligned}
$$

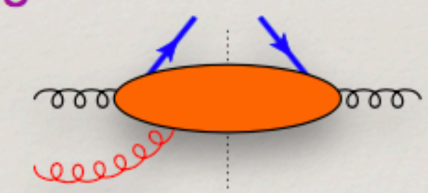
LO

NLO
2-quark correlation w/ EoM

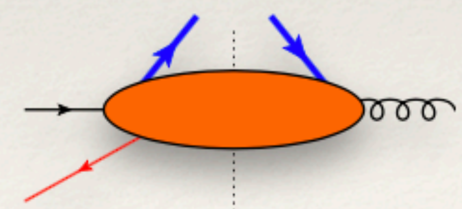
NLO
2-gluon correlation w/ EoM

NLO
q-gluon-q correlation w/ EoM

NLO
triple-gluon correlation w/ EoM



NLO
qq-gluon correlation w/ EoM



NLO
imaginary parts of loops

All partonic factors calculated in Feynman gauge & Light-cone gauge, both calculations *agree*!

*If we assume that twist-3 factorization holds…*

*read off evolution equations from collinear divergences for quark twist-3 FF* $D_T^{\Lambda/q}$

$$\frac{\partial}{\partial \ln \mu^2}\Big(D_T^f(z;\mu)\Big) = \frac{z}{2}\int_z^1 \frac{dw}{w^2}\int_0^1 d\beta \Bigg[ P^{[1]}_{1,f\to f}(w)\, D_{1T}^{\perp(1),f}(\tfrac{z}{w};\mu) + P^{[1]}_{1,f\to g}(w)\, D_{1T}^{\perp(1),g}(\tfrac{z}{w};\mu)$$

$$+ P^{[1]}_{2,f\to fg}(w,\beta)\,\frac{\Im[\hat{D}^{fg}_{FT} - \hat{G}^{fg}_{FT}](\frac{z}{w},\beta;\mu)}{1-\beta} + P^{[1]}_{3,f\to fg}(w,\beta)\,\frac{2\,\Im[\hat{D}^{fg}_{FT}](\frac{z}{w},\beta;\mu)}{(1-\beta)^2}$$

$$+ \sum_{f'=q',\bar{q}'} P^{[1]}_{4,f\to f'\bar{f}'}(w,\beta)\,\Im[\hat{D}^{f'\bar{f}'}_{FT}(\tfrac{z}{w},\beta;\mu)] + \sum_{f'=q',\bar{q}'} P^{[1]}_{5,f\to f'\bar{f}'}(w,\beta)\,\Im[\hat{G}^{f'\bar{f}'}_{FT}(\tfrac{z}{w},\beta;\mu)]$$

$$+ P^{[1]}_{6,f\to gg}(w,\beta)\,\frac{\Im[\hat{N}_2^s(\frac{z}{w},\beta;\mu)]}{\beta^2(1-\beta)^2} + P^{[1]}_{7,f\to gg}(w,\beta)\,\frac{\Im[\hat{N}_2^a(\frac{z}{w},\beta;\mu)]}{\beta^2(1-\beta)^2} + P^{[1]}_{8,f\to gg}(w,\beta)\,\frac{\Im[\hat{N}_1(\frac{z}{w},\beta;\mu)]}{\beta^2(1-\beta)^2}\Bigg]$$

ordinary DGLAP splitting functions

$$P^{[1]}_{1,f\to f}(w) = -2\frac{C_F\alpha_s}{2\pi}\left(\frac{1+w^2}{(1-w)_+} + \frac{3}{2}\,\delta(1-w)\right)$$

$$P^{[1]}_{1,f\to g}(w) = 4\frac{C_F\alpha_s}{2\pi}\left(\frac{1+(1-w)^2}{w}\right)$$

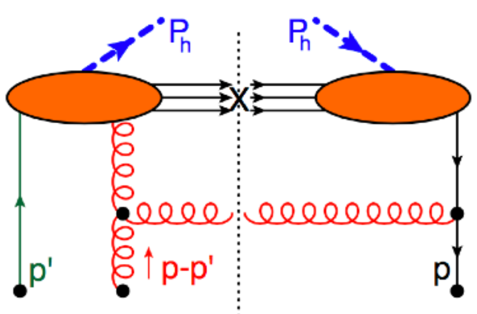
Others: more complicated

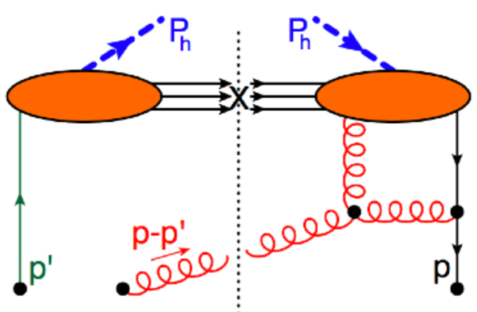
Do the $1/\varepsilon$ - collinear poles of real parts of loops through MSbar - renormalization (???)

$$\frac{E_h\, \mathrm{d}\sigma(S_h)}{\mathrm{d}^{d-1}\vec{P}_h} = \frac{E_h\, \mathrm{d}\sigma^{\mathrm{qq\&qgq}}(S_h)}{\mathrm{d}^{d-1}\vec{P}_h} + \frac{E_h\, \mathrm{d}\sigma^{\mathrm{q\bar{q}g}}(S_h)}{\mathrm{d}^{d-1}\vec{P}_h} + \frac{E_h\, \mathrm{d}\sigma^{\mathrm{gg\&ggg}}(S_h)}{\mathrm{d}^{d-1}\vec{P}_h}$$

$$-\sigma_0\,(1-2v)\,\frac{4M_h}{z_h\, s^2}\epsilon^{ll'P_h S_h}\sum_{f=q,\bar{q}} e_f^2\,\frac{2D_T^{f[1]}(z_h)}{z_h} + O(\alpha_s^2)$$

**Subtraction term "counter cross section"**




**etc ....**

Final proof of one-loop factorization:

Need to derive evolution equation directly from correlator!

Previous work on *unpolarized* chiral-odd twist-3 fragmentation:

Belitsky, Kuraev, NPB 1996; Ma, Zhang, PLB 2017

# Take aways

- Non-zero $e^+e^- \to \Lambda^\uparrow X$ inclusive result is an indication that there are no gluonic poles in ffs, ie time reversal is not a constraint on FFs: the simplest process is an interesting a test of time reversal in QCD, $D_T^{\Lambda/q} \neq 0$

- We are performing a test of twist-3 factorisation at NLO in $e^+e^- \to \Lambda^\uparrow X$

- Would be great if Belle carried out a fully inclusive measurement to directly test $D_T^{\Lambda/q} \neq 0$

$1/\varepsilon$ - collinear poles of real parts of loops through MSbar - renormalization (?????)

$$\frac{E_h\,\mathrm{d}\sigma(S_h)}{\mathrm{d}^{d-1}\vec{P}_h} = \frac{E_h\,\mathrm{d}\sigma^{\mathrm{qq\&qgq}}(S_h)}{\mathrm{d}^{d-1}\vec{P}_h} + \frac{E_h\,\mathrm{d}\sigma^{\mathrm{q\bar{q}g}}(S_h)}{\mathrm{d}^{d-1}\vec{P}_h} + \frac{E_h\,\mathrm{d}\sigma^{\mathrm{gg\&ggg}}(S_h)}{\mathrm{d}^{d-1}\vec{P}_h}$$

$$-\,\sigma_0\,(1-2v)\,\frac{4M_h}{z_h\,s^2}\epsilon^{ll'P_hS_h}\sum_{f=q,\bar{q}} e_f^2\,\frac{2D_T^{f[1]}(z_h)}{z_h} + O(\alpha_s^2)\,,$$

Subtraction/counter cross section term

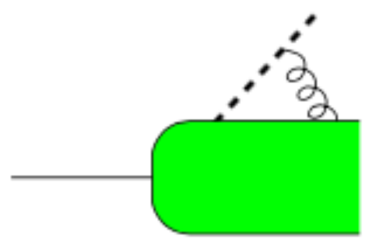
# Using EoM and LIR eliminate the intrinsic for kinematic and dynamical

$$D_T^f(z) = -z\left(D_{1T}^{\perp(1),f}(z) - \int_0^1 \mathrm{d}\beta\,\frac{\Im[\hat{D}_{FT}^{fg} - \hat{G}_{FT}^{fg}](z,\beta)}{1-\beta}\right)$$
$$= z\int_z^1 \frac{\mathrm{d}w}{w}\int_0^1 \mathrm{d}\beta\left[\frac{(1+\delta(1-w))\Im[\hat{D}_{FT}^{fg} - \hat{G}_{FT}^{fg}](\frac{z}{w},\beta)}{1-\beta} + \frac{2\,\Im[\hat{D}_{FT}^{fg}](\frac{z}{w})}{(1-\beta)^2}\right].$$

# Comment on time reversal constraint on ffs versus pdfs

The non-zero effect in the one-photon approximation in the annihilation process caused by the intrinsic twist-3 fragmentation function $D_T^{\Lambda/q}(z)$ can be attributed to the fact that fragmentation processes are not constrained by time-reversal

Boer, Mulders Pijlman, NPB [illegible]



Due due to non- perturbative interactions in the in and out states in the definition of $D_T^{\Lambda/q}(z)$ On the other hand, a corresponding intrinsic twist-3 parton correlation function in the nucleon, $f_T^{q/\Lambda}(x)$ is forbidden by time-reversal.

Goeke, Metz, Schlegel plb 2006, Bacchetta et al. JHEP 2007