

MICHAEL PAOLONE

TEMPLE UNIVERSITY

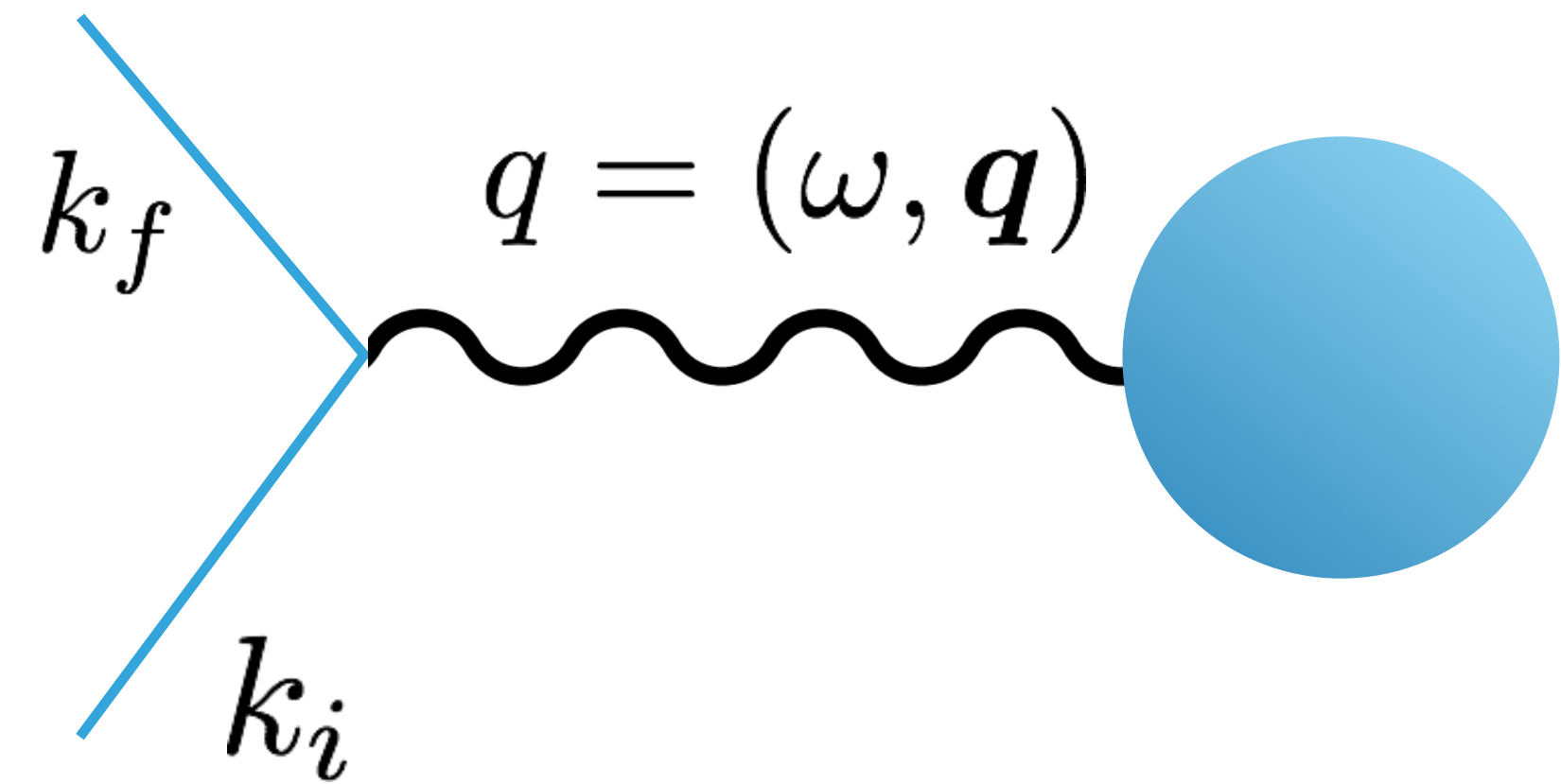
FOR THE E05-110 COLLABORATION.

# THE COULOMB SUM RULE IN NUCLEI

# COULOMB SUM RULE

Inclusive electron scattering cross-section:

$$\frac{d^2\sigma}{d\Omega d\omega} = \sigma_{\text{Mott}} \left[ \frac{q^4}{|\mathbf{q}|^4} R_L(\omega, |\mathbf{q}|) + \left( \frac{q^2}{2|\mathbf{q}|^2} + \tan^2 \frac{\theta}{2} \right) R_T(\omega, |\mathbf{q}|) \right]$$



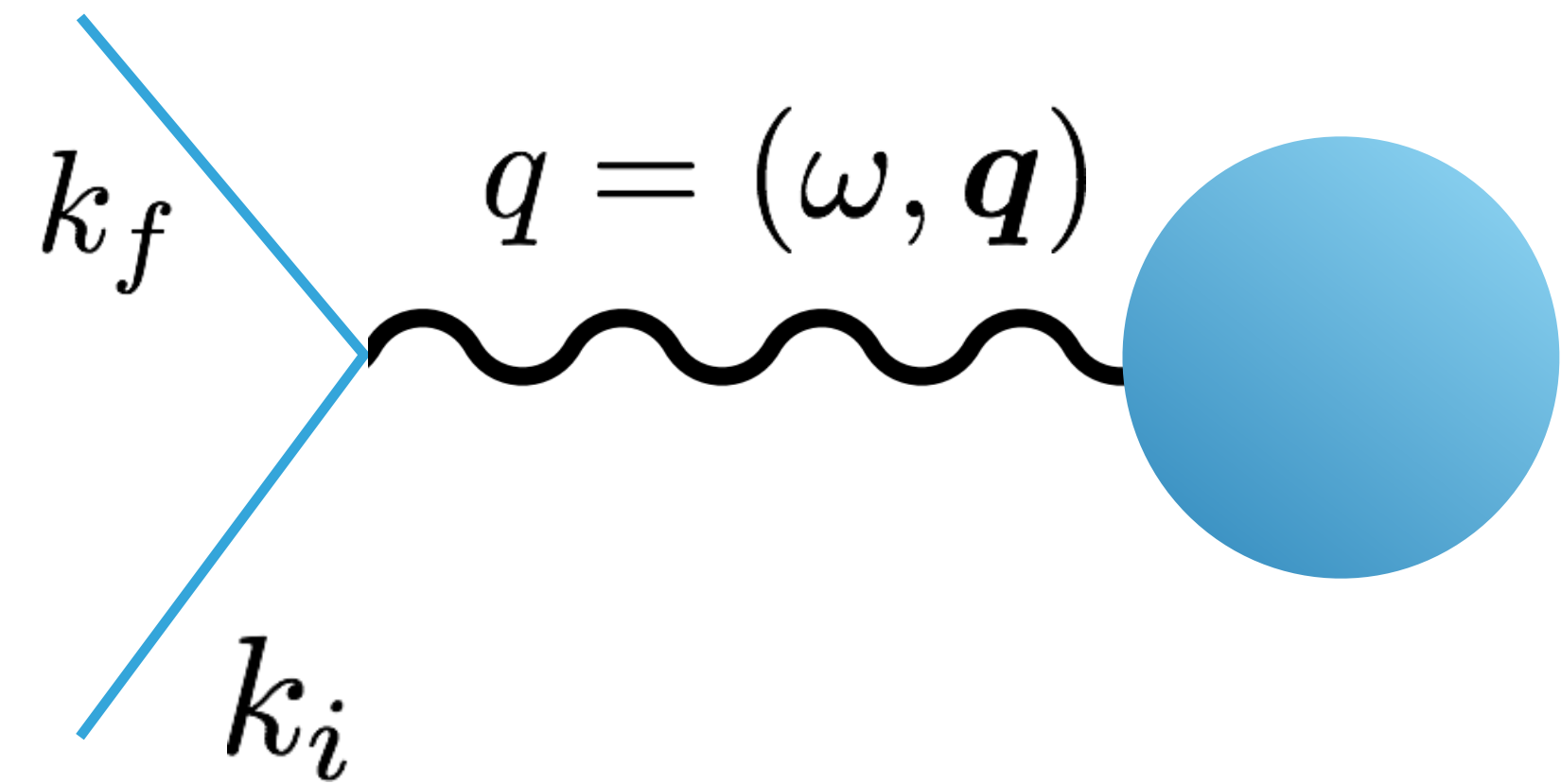
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due to **charge** properties

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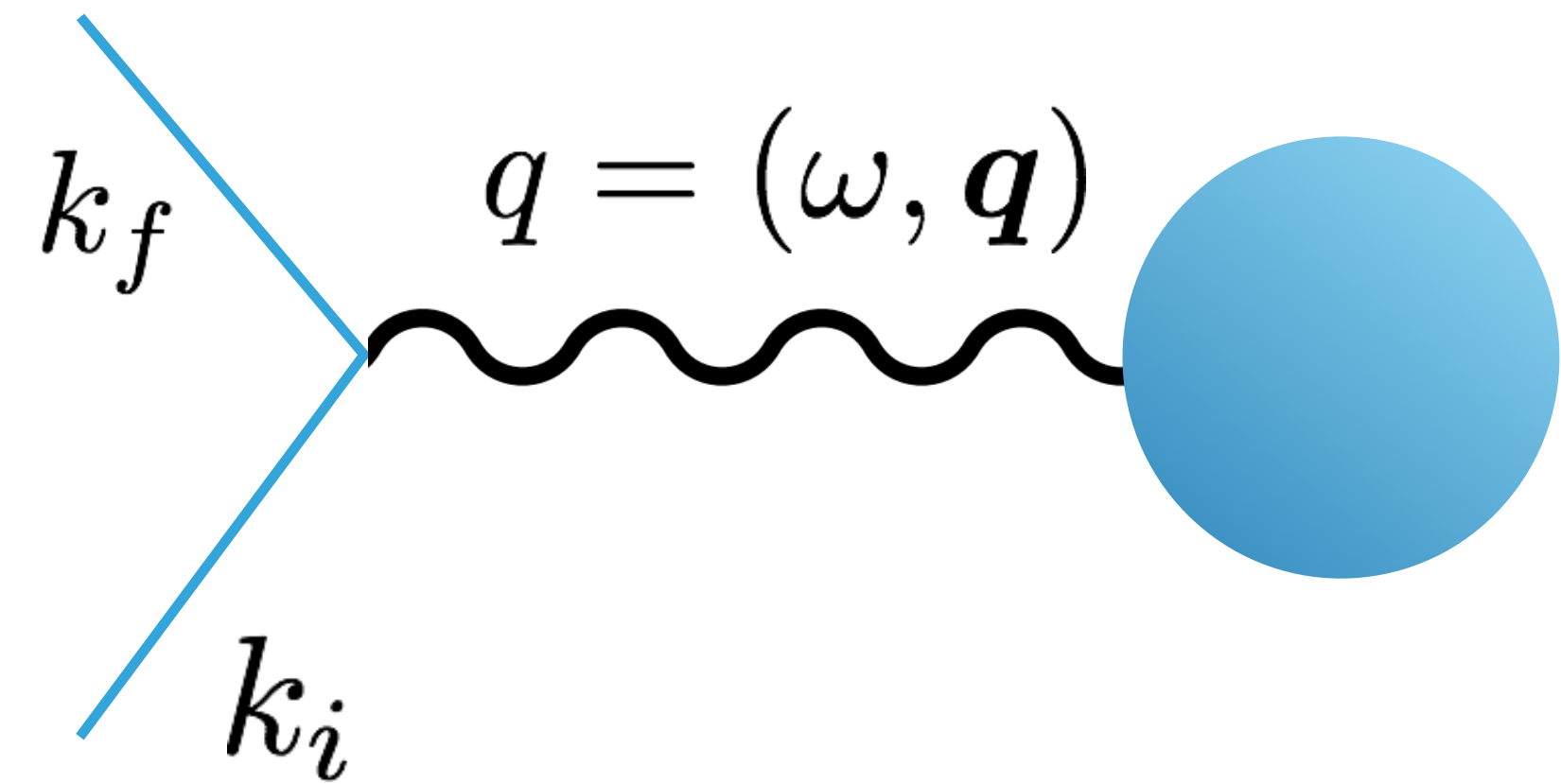
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If one integrates the charge response divided by the total charge form factor over all available virtual photon energies, naively one might expect the integral to go to unity.



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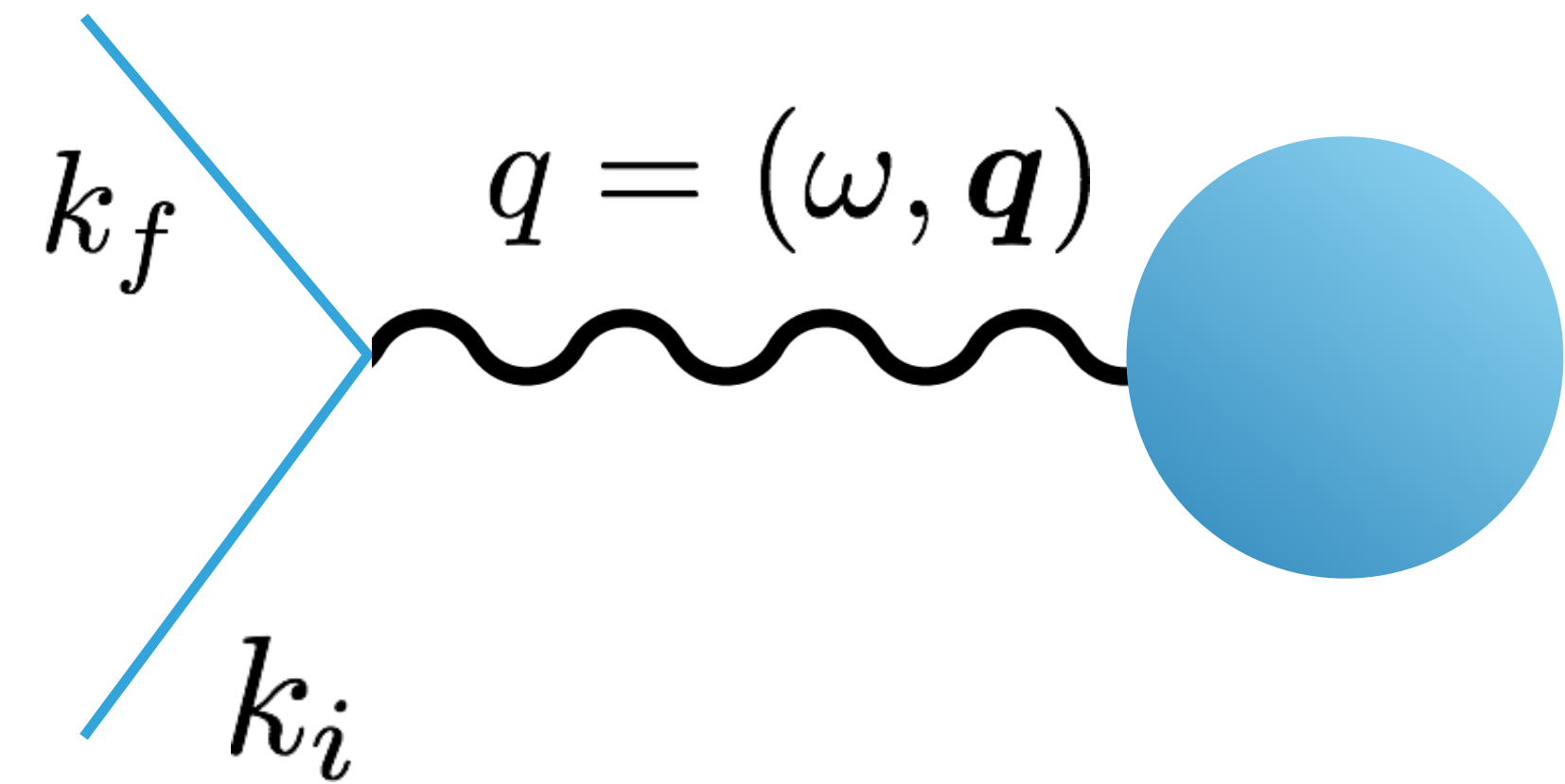
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due to long range nuclear effects, Pauli blocking.  
(directly calculable, well understood).



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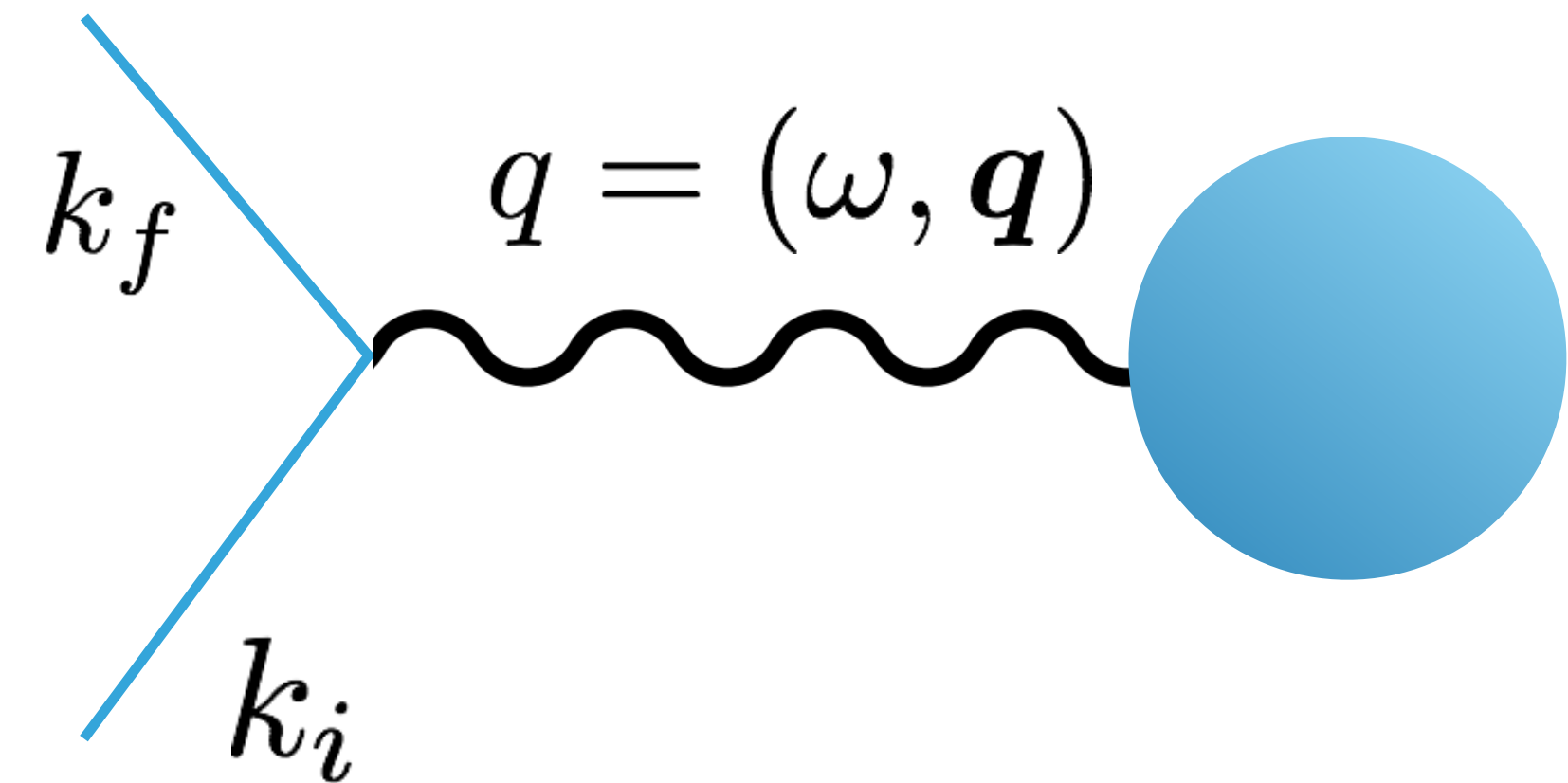
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**At large  $|\mathbf{q}| \gg 2k_f$ ,  $S_L$  should go to 1. Any significant\* deviation from this  
would be an indication of relativistic or medium effects distorting the nucleon form factor!**

\*Short range correlations will also quench  $S_L$ , but only by  $< 10\%$



## THE COULOMB SUM RULE IN NUCLEI

## COULOMB SUM RULE

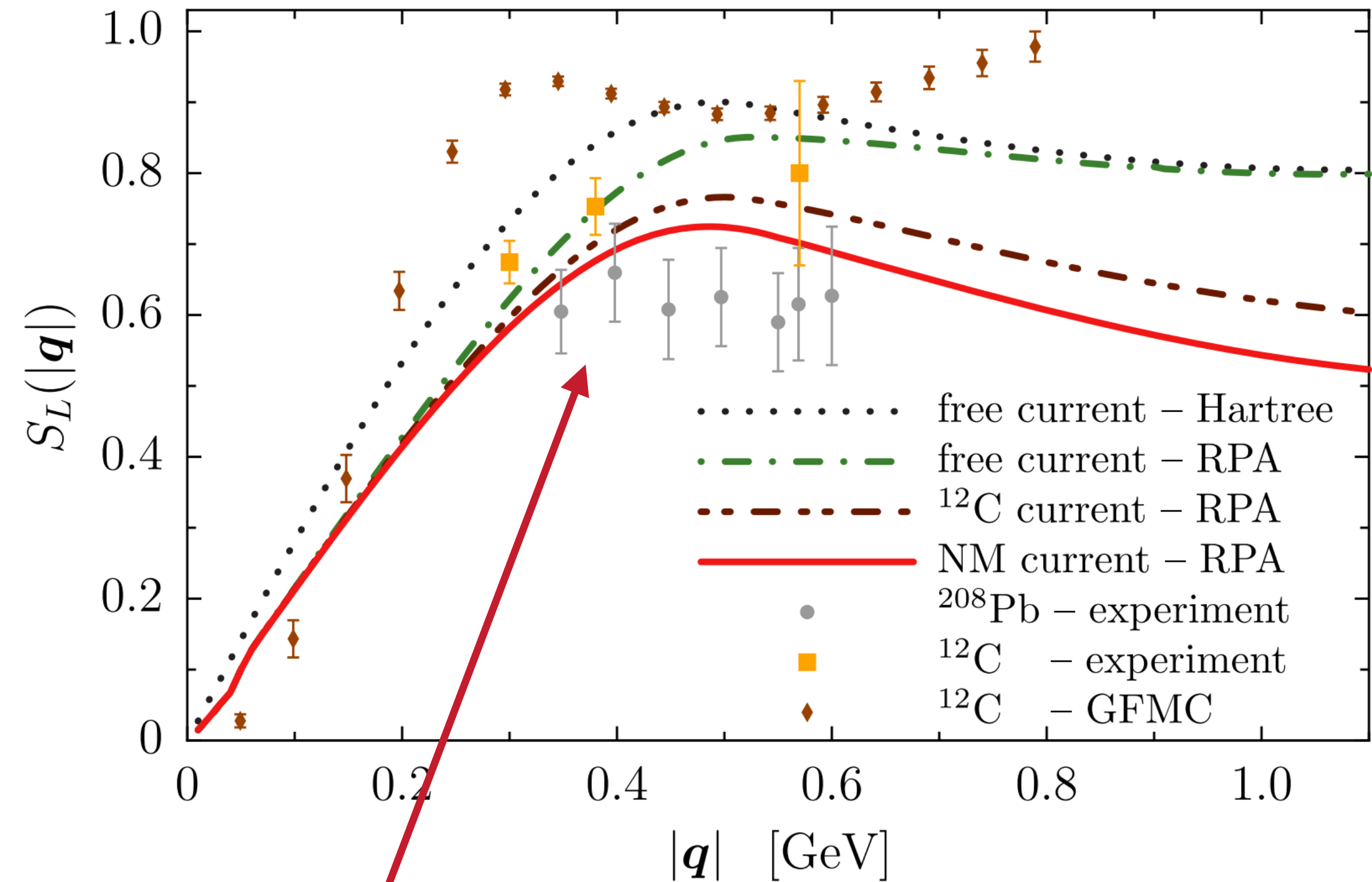
- ▶ Long standing issue with many years of theoretical interest.
- ▶ Even most state-of-the-art models cannot predict existing data.
- ▶ New precise data at larger  $|q|$  would provide crucial insight and constraints to modern calculations.

$$S_L(|\mathbf{q}|) = \int_{\omega^+}^{\infty} d\omega \frac{R_L(\omega, |\mathbf{q}|)}{Z\tilde{G}_{Ep}^2(Q^2) + N\tilde{G}_{En}^2(Q^2)}$$

## Relativistic and Nuclear Medium Effects on the Coulomb Sum Rule

Ian C. Cloët,<sup>1</sup> Wolfgang Bentz,<sup>2</sup> and Anthony W. Thomas<sup>3</sup><sup>1</sup>Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA<sup>2</sup>Department of Physics, School of Science, Tokai University, Hiratsuka-shi, Kanagawa 259-1292, Japan<sup>3</sup>CSSM and ARC Centre of Excellence for Particle Physics at the Terascale, Department of Physics, University of Adelaide, Adelaide South Australia 5005, Australia

(Received 23 June 2015; published 19 January 2016)



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# QUASI-ELASTIC SCATTERING

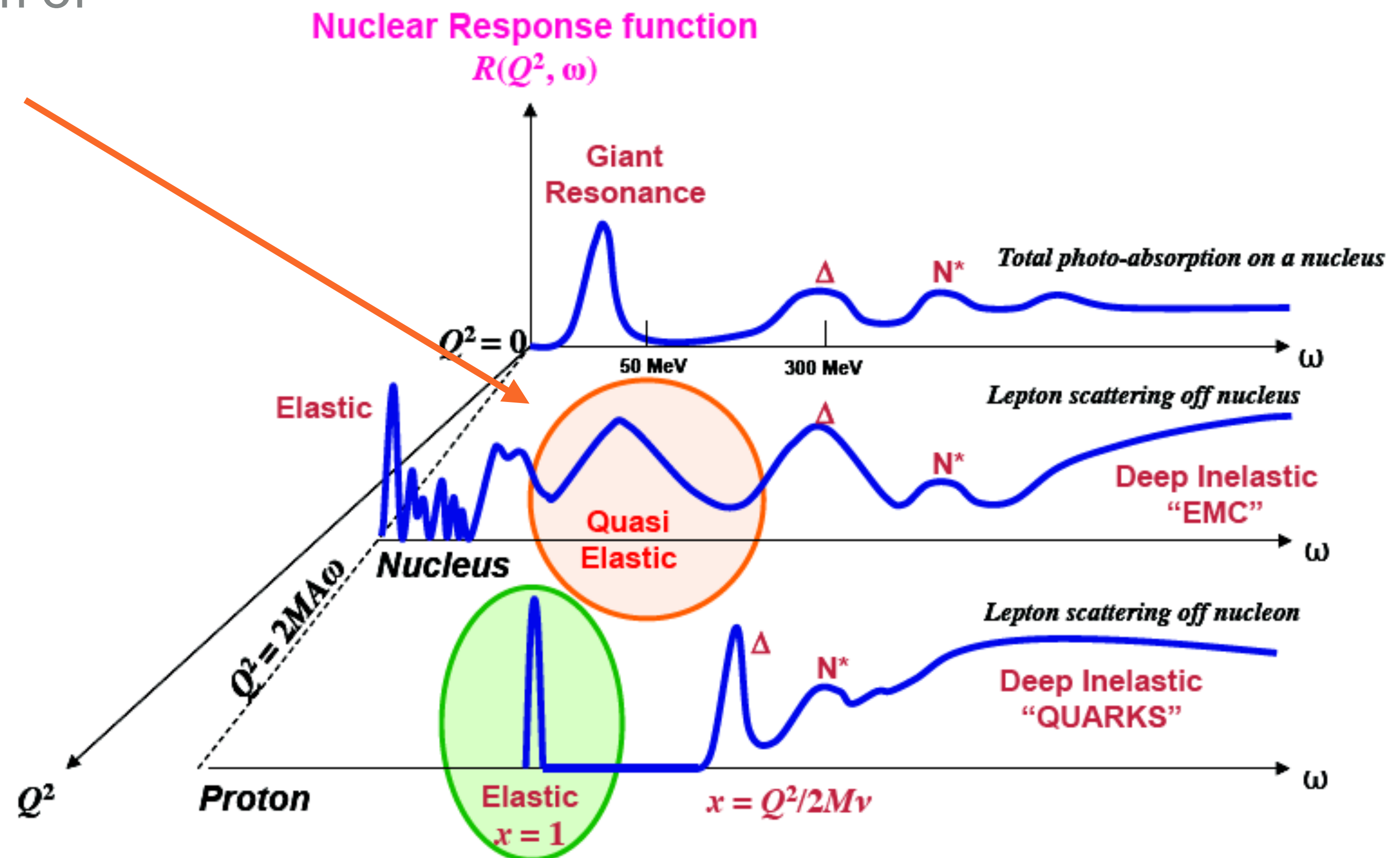
- ▶ Quasi-elastic scattering at intermediate  $Q^2$  is the region of interest for our experiment:

- ▶ Nuclei investigated:

- ▶  $^4\text{He}$
- ▶  $^{12}\text{C}$
- ▶  $^{56}\text{Fe}$
- ▶  $^{208}\text{Pb}$

$$S_L(|\mathbf{q}|) = \int_{\omega_+}^{|\mathbf{q}|} d\omega \frac{R_L(\omega, |\mathbf{q}|)}{Z\tilde{G}_{Ep}^2(Q^2) + N\tilde{G}_{En}^2(Q^2)}$$

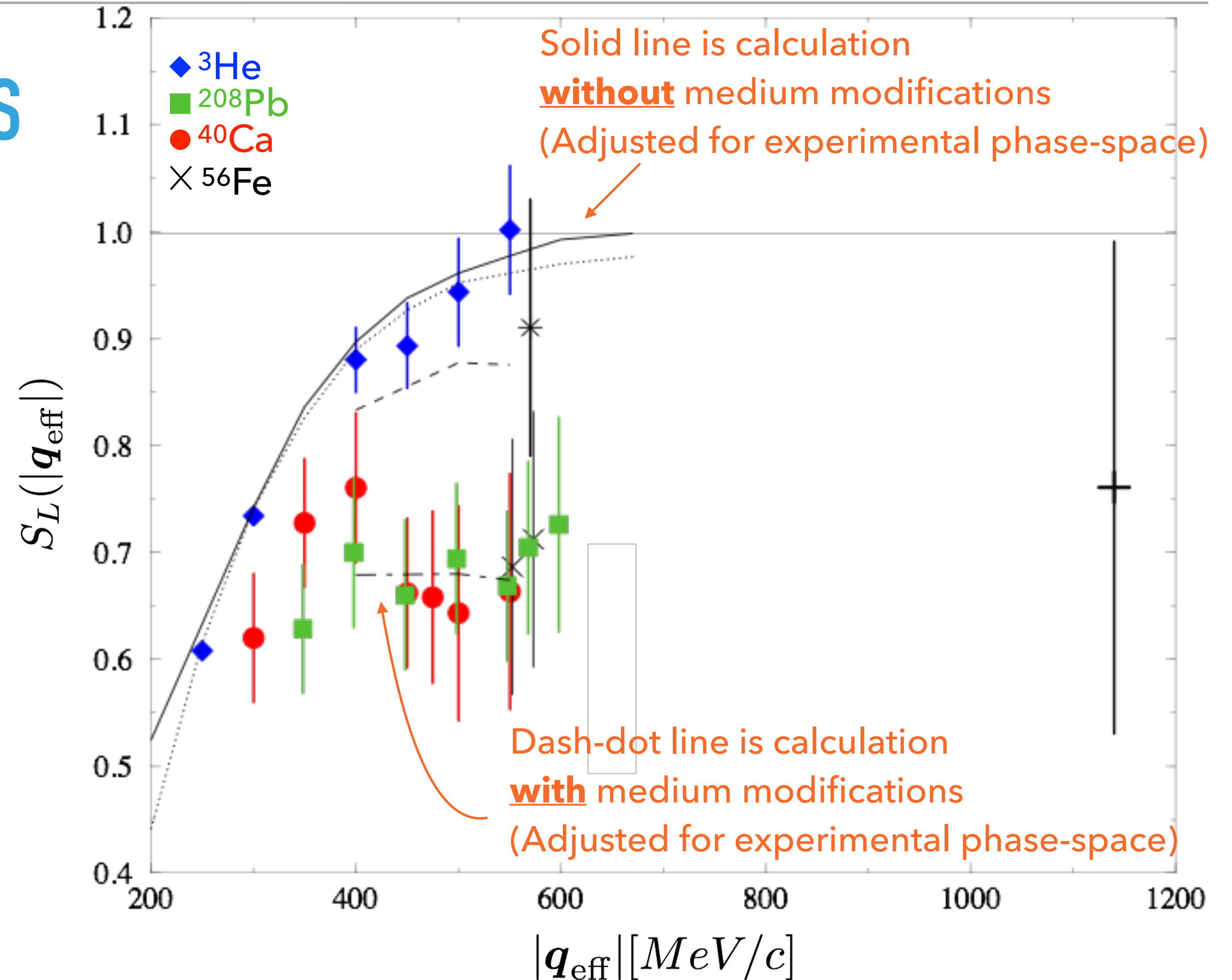
We want to integrate above the coherent elastic peak:  
Quasi-elastic is “elastic” scattering on constituent nucleons inside nucleus.



## PUBLISHED EXPERIMENTAL RESULTS

- First group of experiments from Saclay, Bates, and SLAC show a quenching of  $S_L$  consistent with medium modified form-factors.

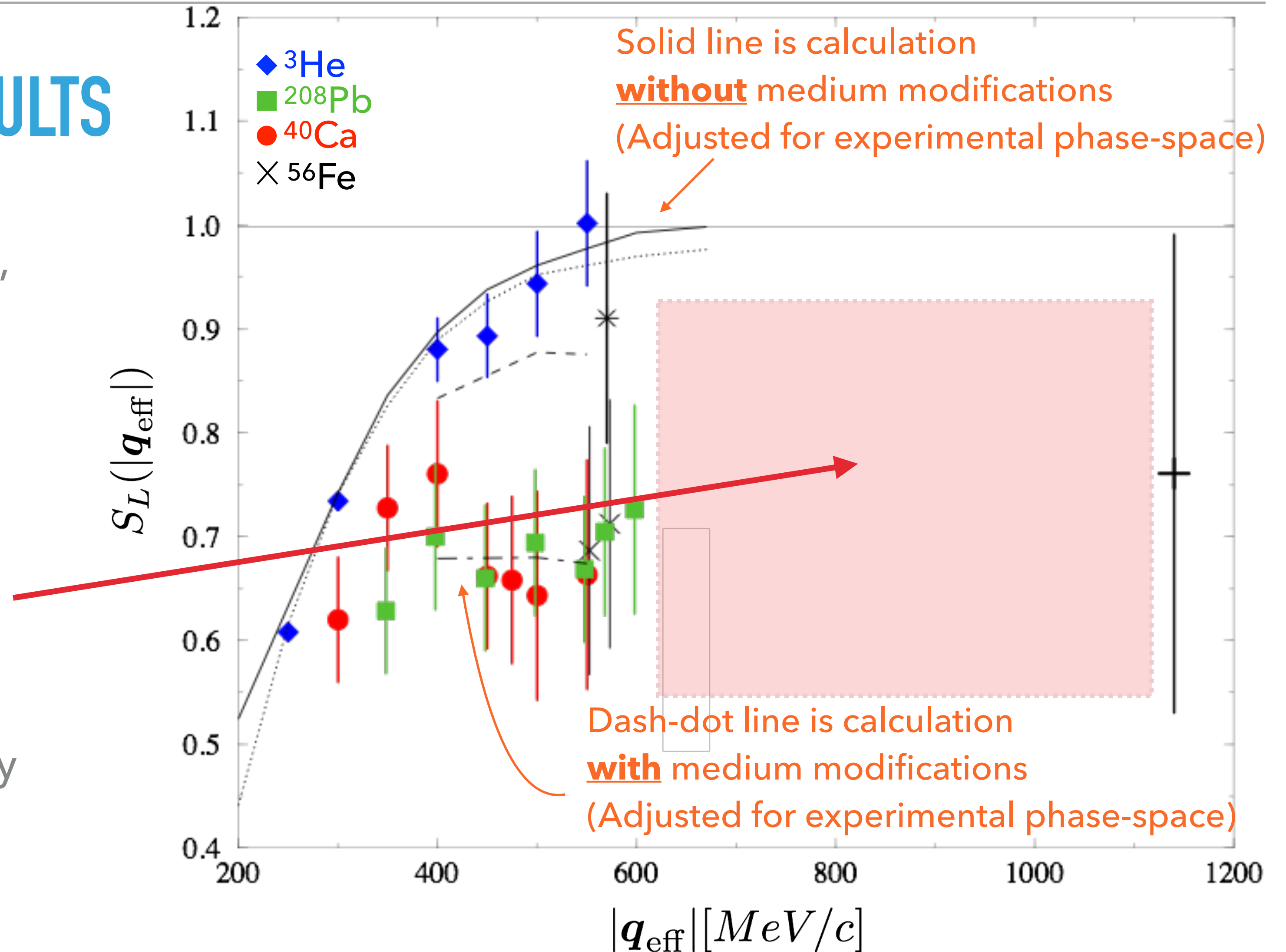
$$S_L(|\mathbf{q}|) = \int_{\omega+}^{|\mathbf{q}|} d\omega \frac{R_L(\omega, |\mathbf{q}|)}{Z\tilde{G}_{Ep}^2(Q^2) + N\tilde{G}_{En}^2(Q^2)}$$



$|\mathbf{q}_{\text{eff}}|$  is  $|\mathbf{q}|$  corrected for a nuclei dependent mean coulomb potential.  
Methodology agreed on by Andreas Aste, Steve Wallace and John Tjon.

## PUBLISHED EXPERIMENTAL RESULTS

- ▶ First group of experiments from Saclay, Bates, and SLAC show a quenching of  $S_L$  consistent with medium modified form-factors.
- ▶ Very little data above  $|\mathbf{q}|$  of 600 MeV/c, where the cleanest signal of medium effects should exist!
- ▶ Saclay, Bates limited in beam energy reach up to 800 MeV.
- ▶ SLAC limited in kinematic coverage of scattered electron at  $|\mathbf{q}|$  below 1150 MeV/c.



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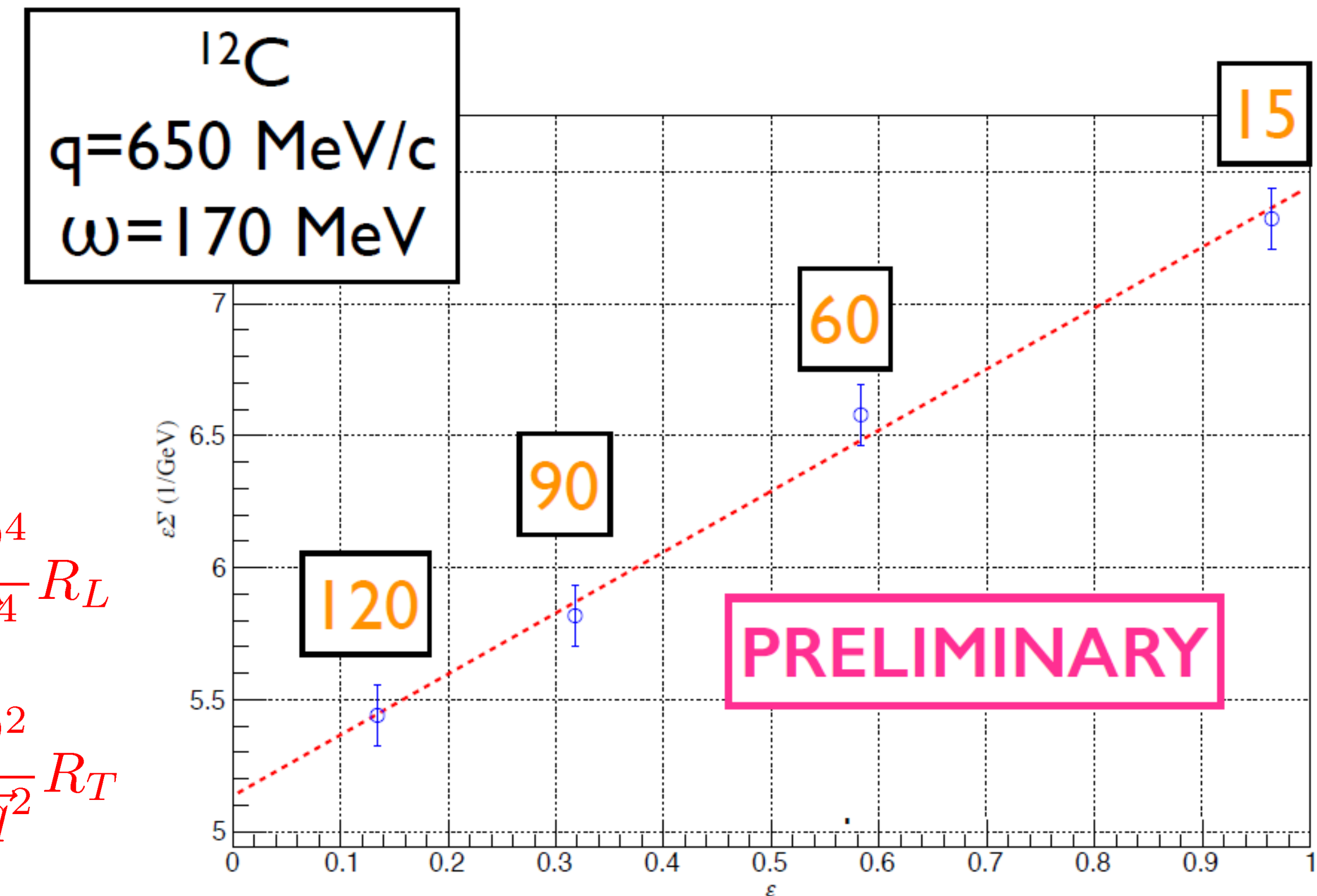
# EXPERIMENTAL DESIGN

- ▶ Need  $R_L$  → Use Rosenbluth separation!

$$S_L(|\mathbf{q}|) = \int_{\omega^+}^{|\mathbf{q}|} d\omega \frac{R_L(\omega, |\mathbf{q}|)}{Z\tilde{G}_{Ep}^2(Q^2) + N\tilde{G}_{En}^2(Q^2)}$$

$$\text{Slope} = \frac{Q^4}{\vec{q}^4} R_L$$

$$\text{Intercept} = \frac{Q^2}{2\vec{q}^2} R_T$$



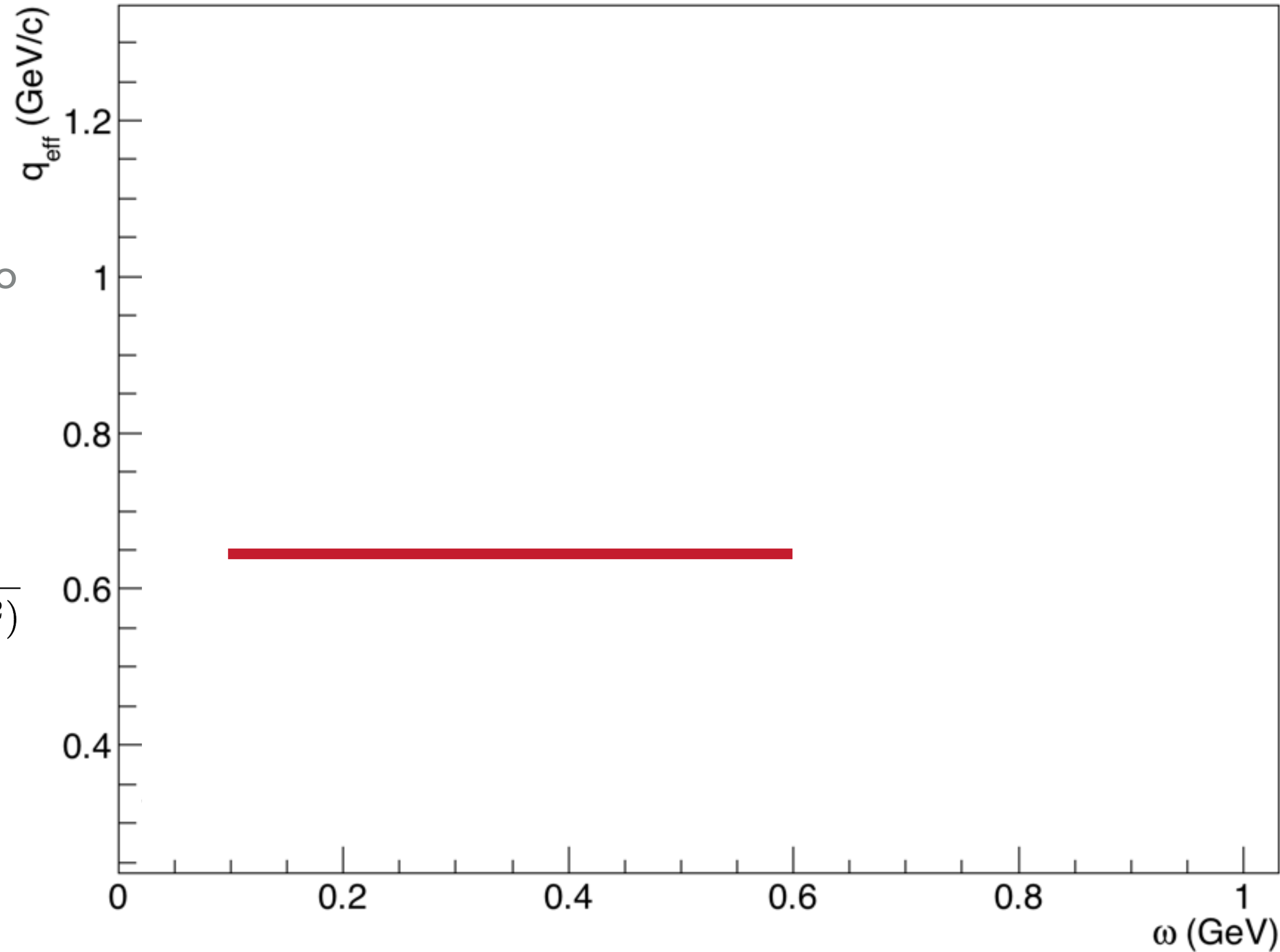
- ▶ Experiment run at 4 angles per target: 15, 60, 90, 120 degs. Very large lever arm for precise calculation of  $R_L$ !
- ▶ Need data for each angle at a constant  $|\mathbf{q}|$  over an  $\omega$  range starting above the elastic peak up to  $|\mathbf{q}|$ .
  - ▶ When running a single arm experiment with fixed beam energy and scattering angle,  $|\mathbf{q}|$  is NOT constant over your momentum acceptance.
    - ▶ Need to take data at varying beam energies, and “map-out”  $|\mathbf{q}|$  and  $\omega$  space.

## EXPERIMENTAL DESIGN

- ▶ If one wants to measure from 100 to 600 MeV  $\omega$  at constant  $|\mathbf{q}| = 650$  MeV/c

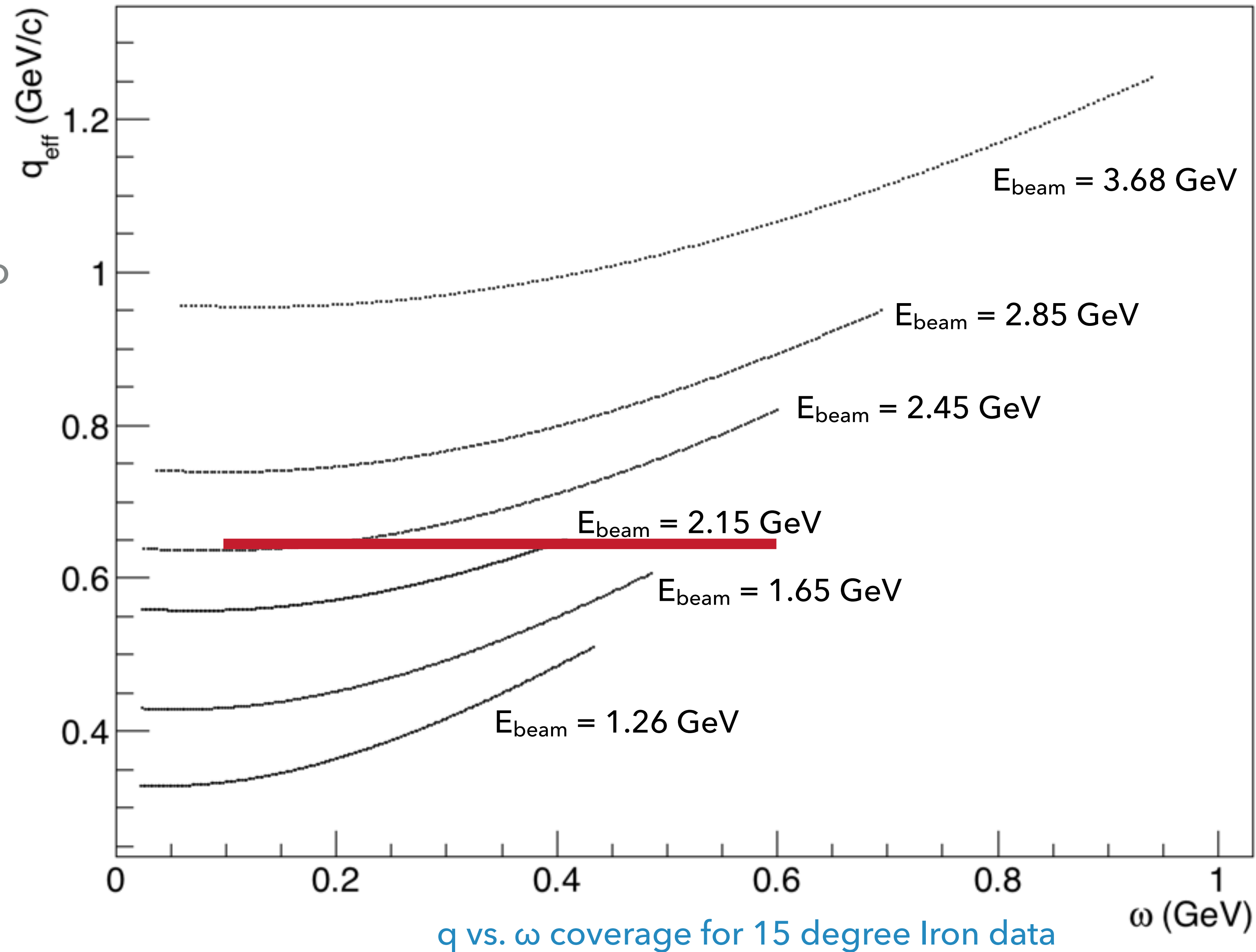
CSR calculated at constant  $|\mathbf{q}|$  !!

$$S_L(|\mathbf{q}|) = \int_{\omega_+}^{|\mathbf{q}|} d\omega \frac{R_L(\omega, |\mathbf{q}|)}{Z\tilde{G}_{Ep}^2(Q^2) + N\tilde{G}_{En}^2(Q^2)}$$



## EXPERIMENTAL DESIGN

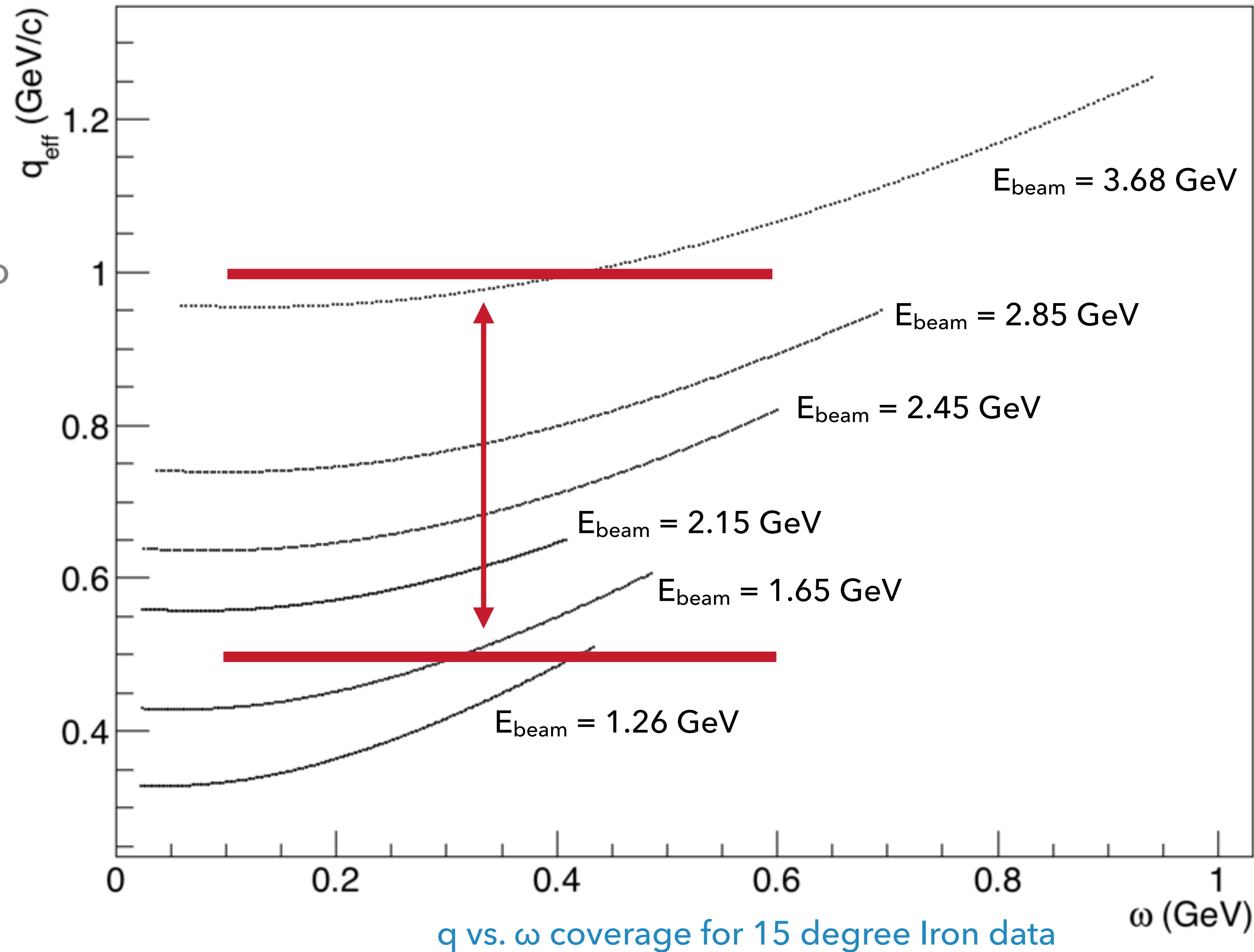
- ▶ If one wants to measure from 100 to 600 MeV  $\omega$  at constant  $|q| = 650$  MeV/c
- ▶ Take data at different beam energies, and interpolate to determine cross-section at constant  $|q|$ .



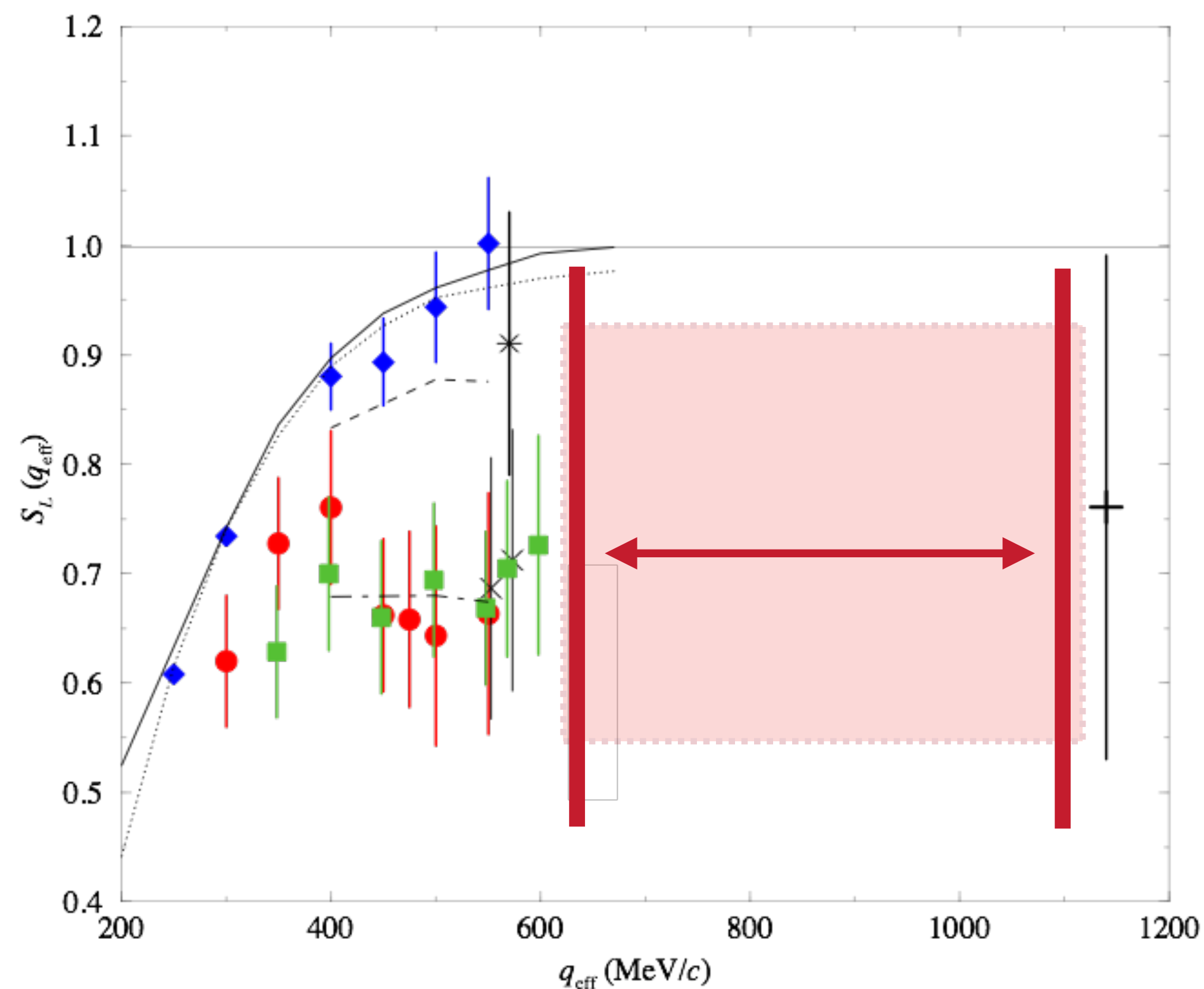
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- ▶ If one wants to measure from 100 to 600 MeV  $\omega$  at constant  $|q| = 650$  MeV/c
- ▶ Take data at different beam energies, and interpolate to determine cross-section at constant  $|q|$ .
- ▶  $|q|$  can be selected between 550 and 1000 MeV/c

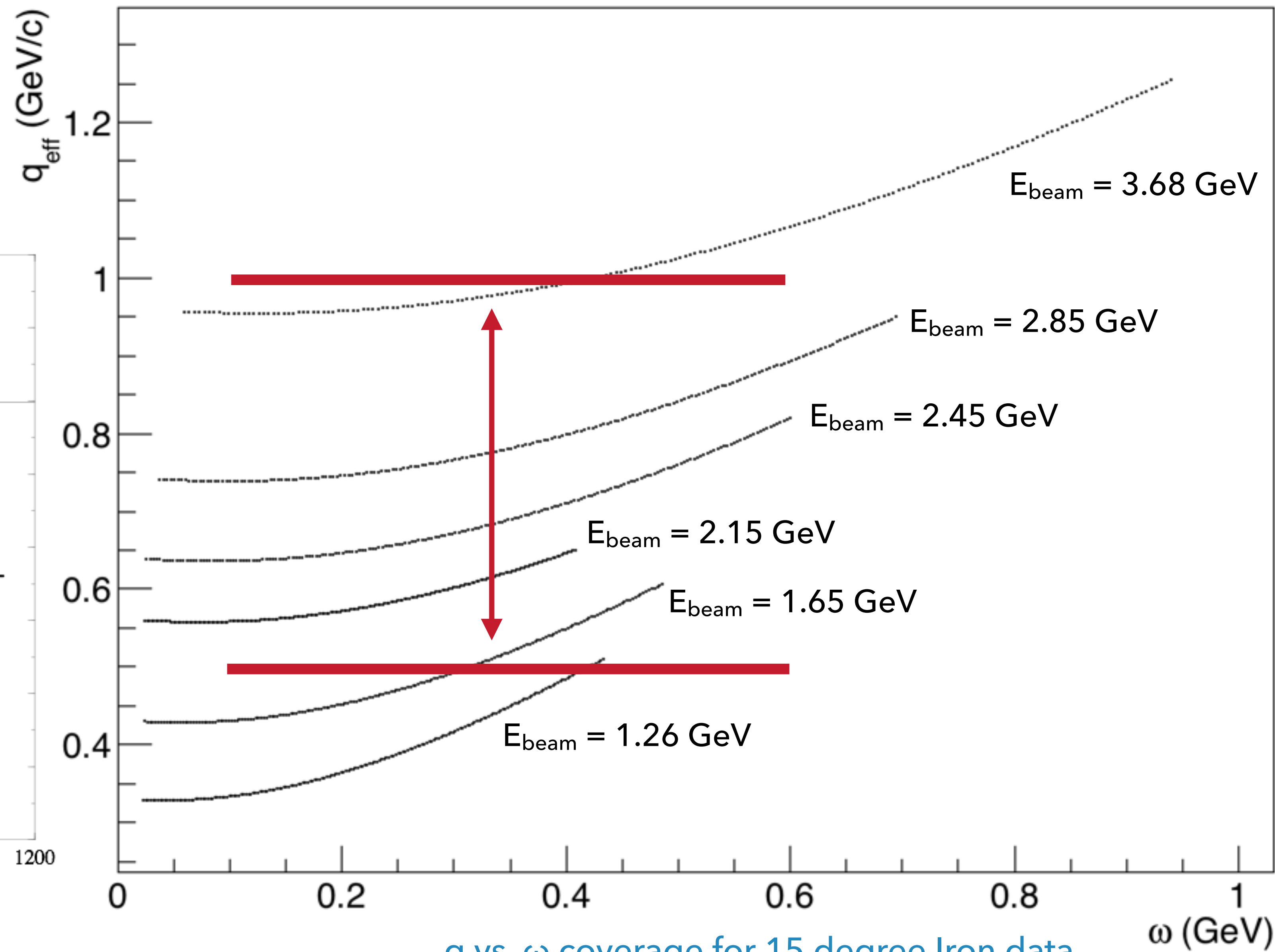
Repeat this “mapping” for 60, 90, and 120 degree spectrometer central angles.



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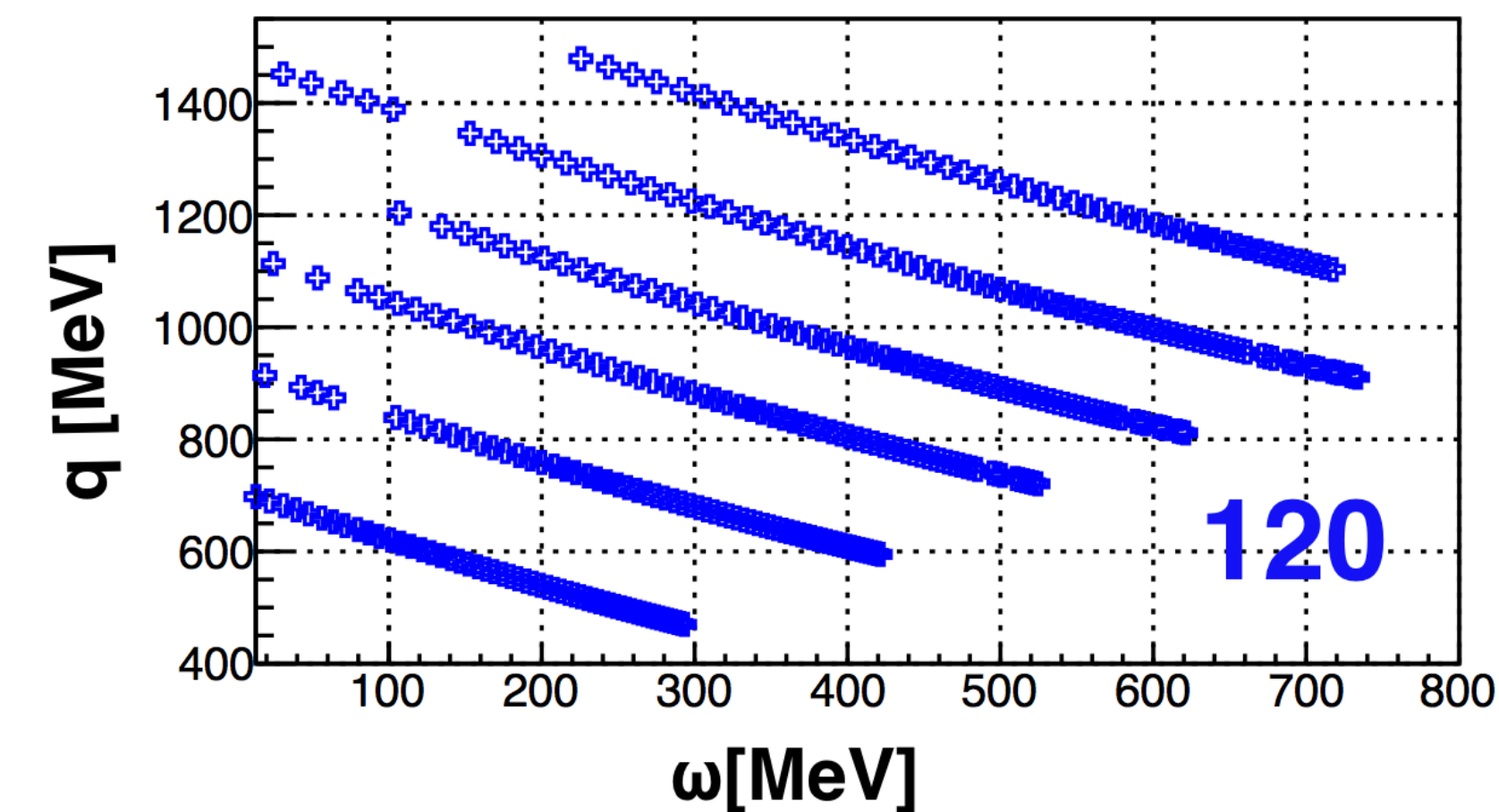
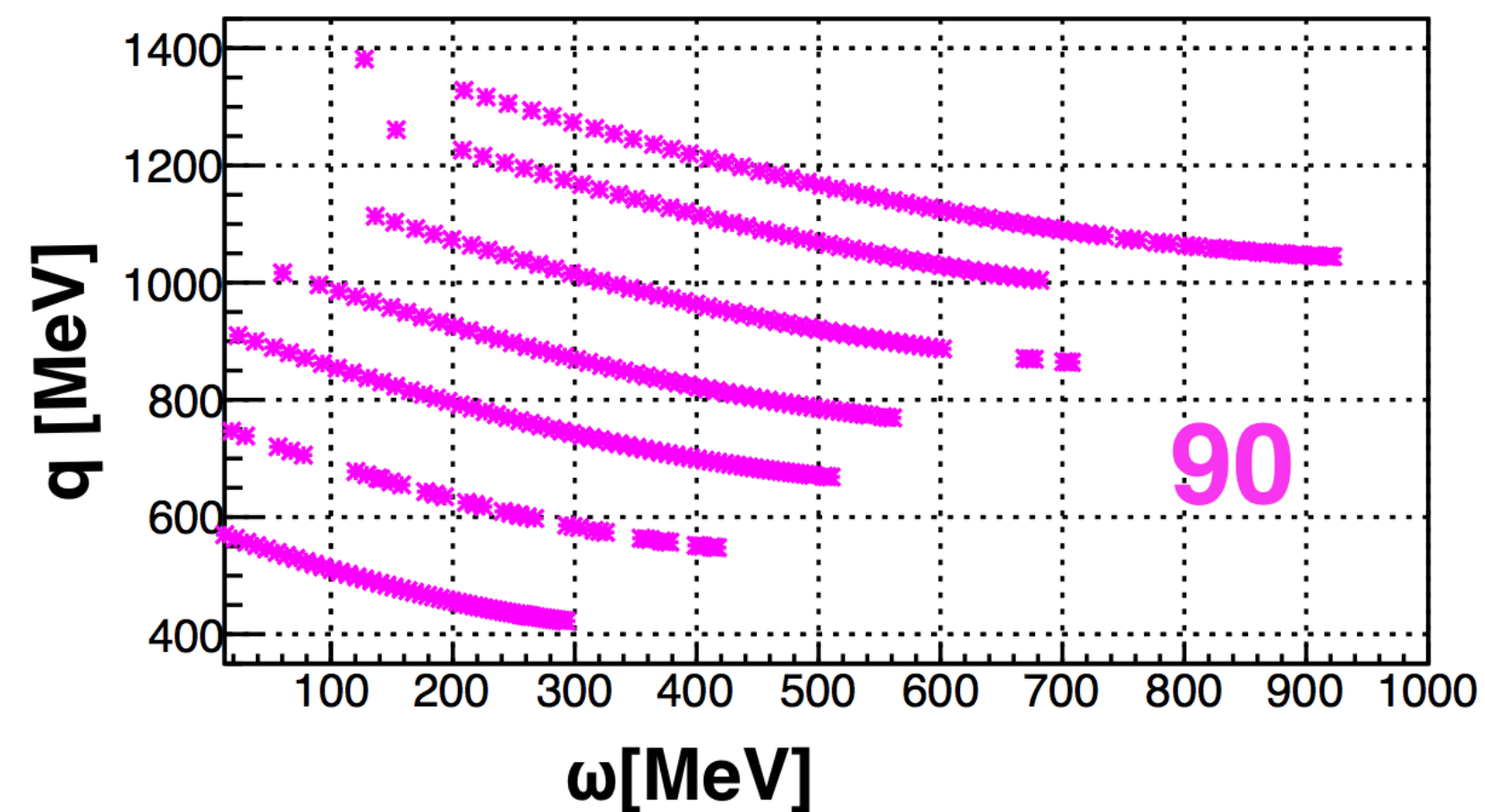
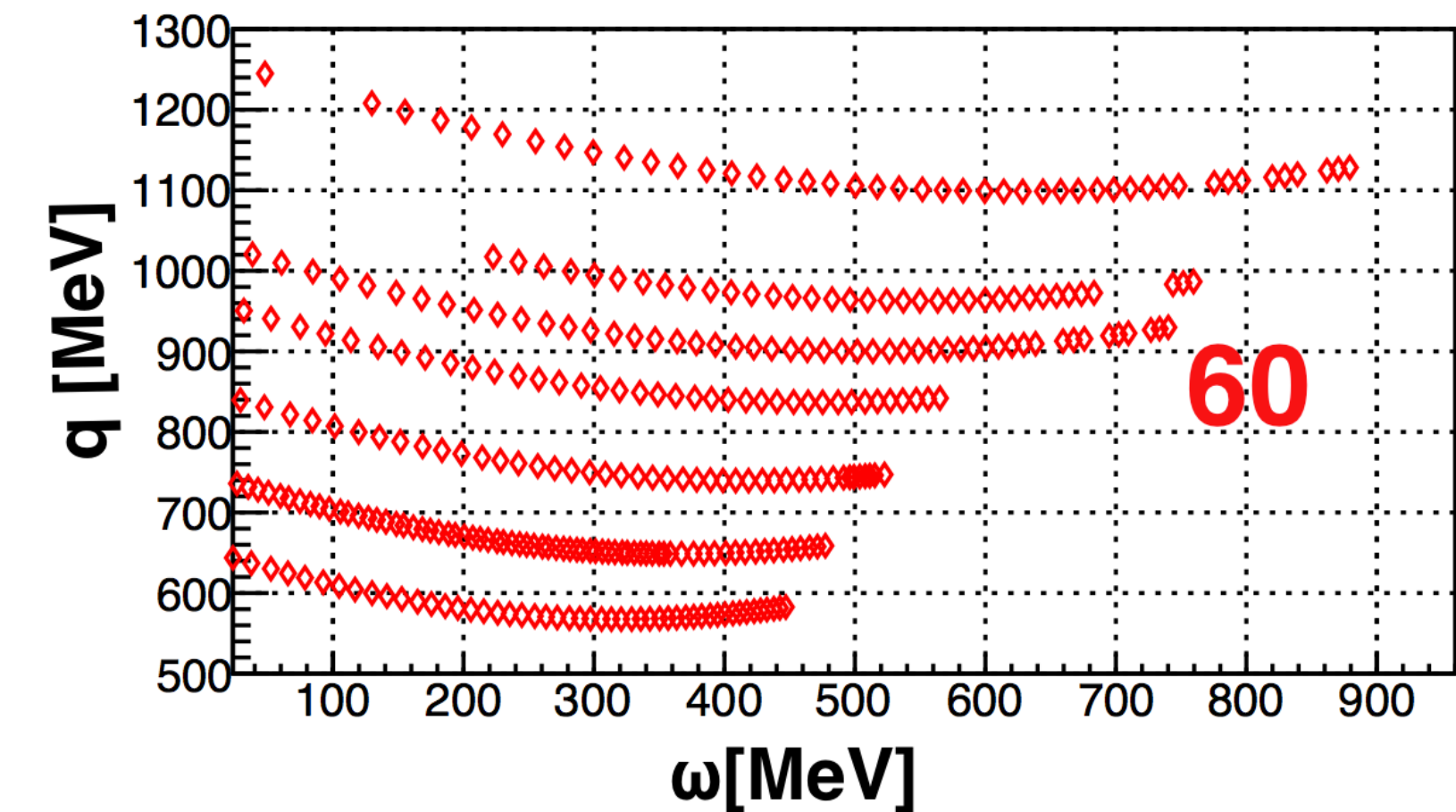
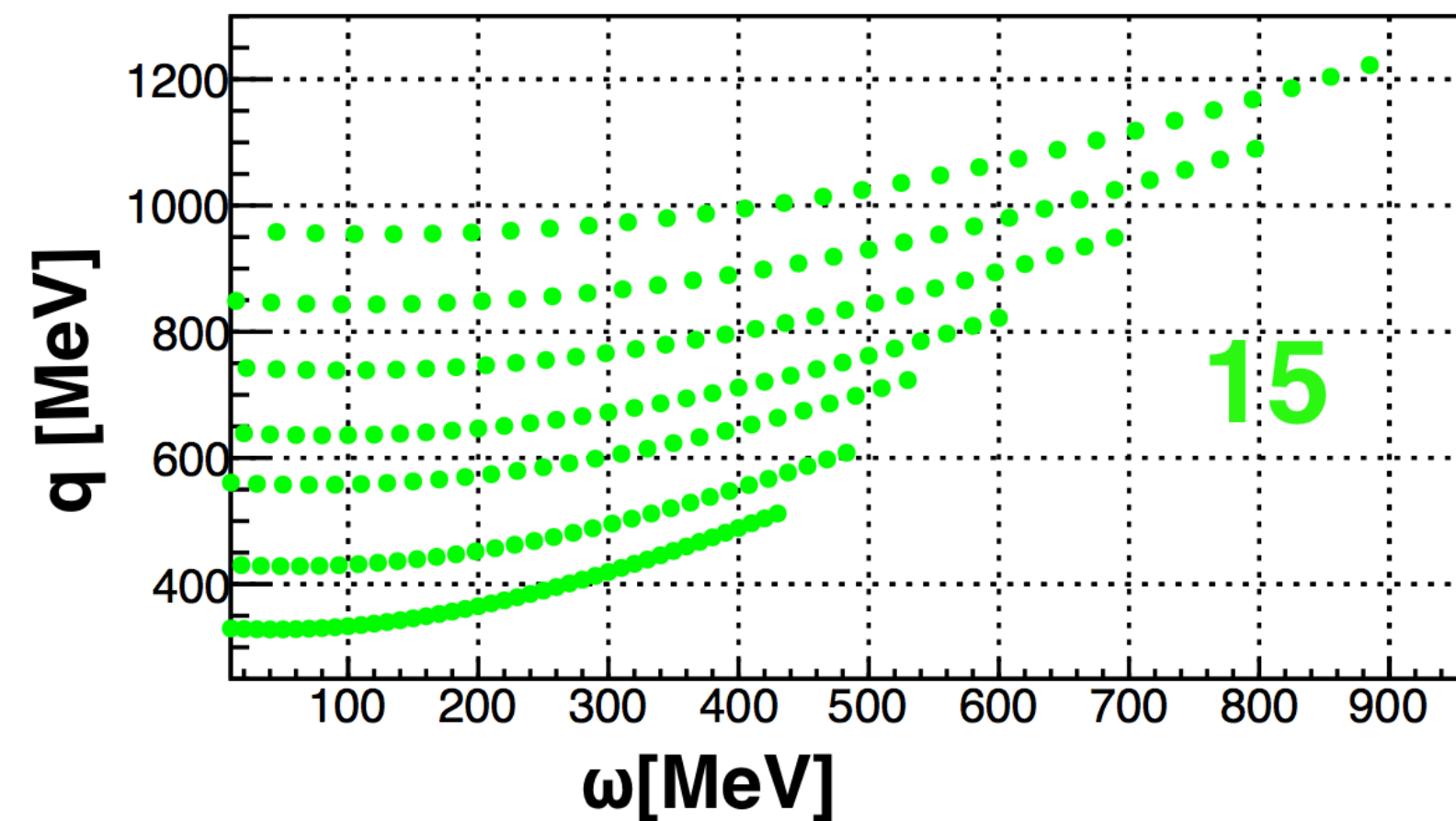
$q$  vs.  $\omega$  coverage for 15 degree Iron data

# EXPERIMENTAL SPECIFICS

Each data line represents a constant beam-energy

## ► E05-110:

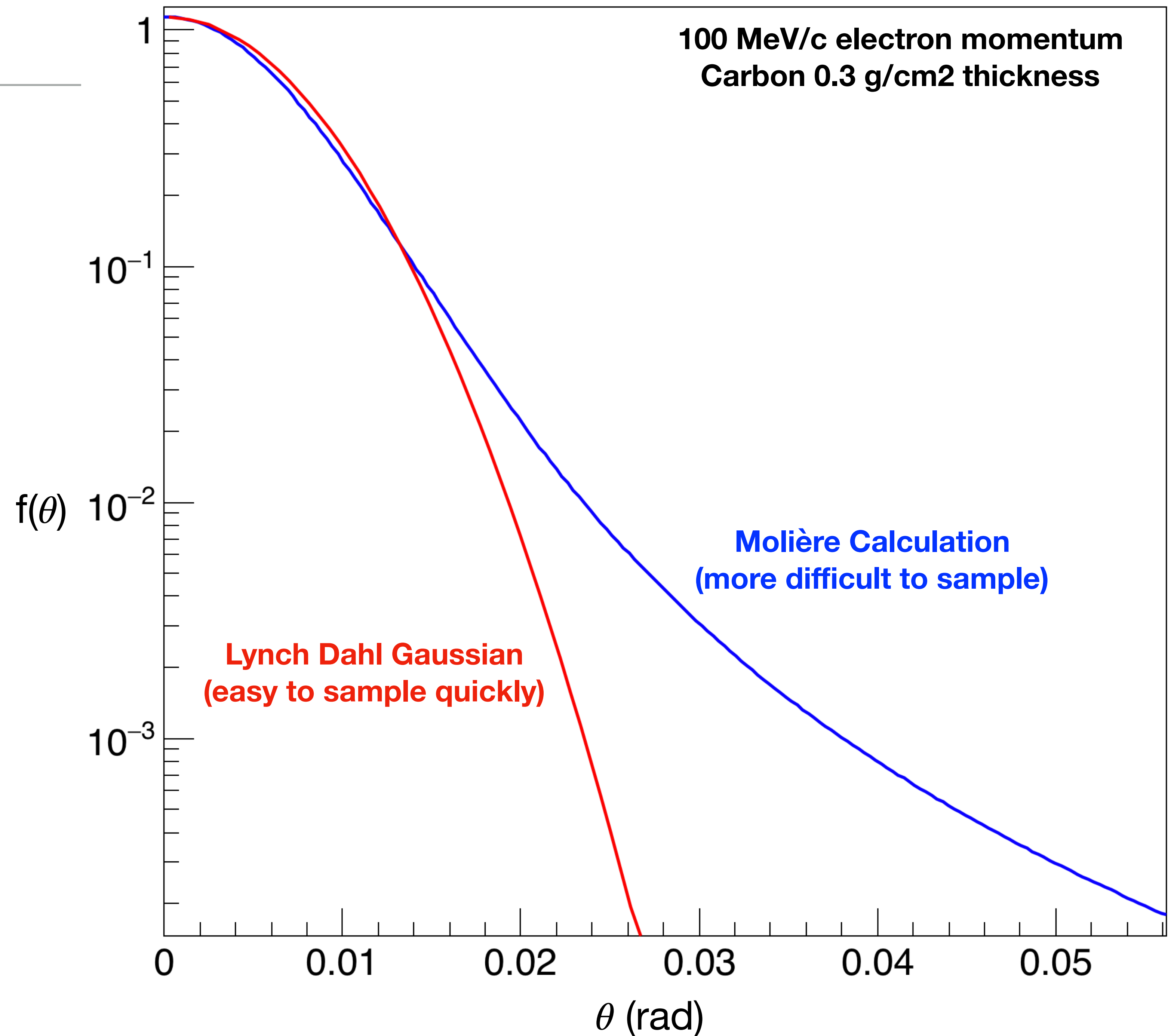
- Data taken from October 23rd 2007 to January 16th 2008
- 4 central angle settings: 15, 60, 90, 120 degs.
- Many beam energy settings: 0.4 to 4.0 GeV
- Many central momentum settings: 0.1 to 4.0 GeV
- LHRS and RHRS independent (redundant) measurements for most settings
- 4 targets:  $^4\text{He}$ ,  $^{12}\text{C}$ ,  $^{56}\text{Fe}$ ,  $^{208}\text{Pb}$ .



## RECENT UPDATES: MULTIPLE SCATTERING FOR LOW MOMENTUM ELECTRONS

- ▶ Typical handling of multiple scattering in simulation is to use a gaussian form based on Highland's (1979) approximation to Molière's (1948) and Bethe's (1953) derivation.
  - ▶ Highland's equations are the one's listed in the PDG, however they do comment that there is a (slightly) more accurate approximation from Lynch and Dahl (1990).
  - ▶ NONE of these gaussian approximations handle the tails of the Molière calculation correctly.
    - ▶ Not very important until we are at smaller energies

# MULTIPLE SCATTERING FOR LOW MOMENTUM ELECTRONS



# APPROXIMATING MOLIÈRE

- In the literature, the function is observed to follow a gaussian shape at low theta, and then turn to approximately  $1/\theta^4$ .
- I've found that using a function that is the sum of 2 gaussians and transitions to a function that is  $1/\theta^n$  can give a very close approximation.
- The parameters of the function are completely defined by the Lynch-Dahl gaussian-width and the parameter B used in the power series expansion of Molière's function.

$$g(\theta) = A \left( e^{-\frac{1}{2} \frac{\theta^2}{\sigma(B)^2}} + R(B) e^{-\frac{1}{2} \frac{\theta^2}{(2\sigma(B))^2}} \right) \quad \theta < \theta_{trans}$$

$$= b/\theta^{n(B)} \quad \theta > \theta_{trans}$$

Note:  $g(\theta)$  is not continuous at transition point, but the two functions are equal at that point

$$R(B) = r_1 + r_2 B + r_3 B^2$$

$$\theta_{trans} = t_1 \sigma(B) / \log(B)$$

$$\sigma(B) = \sigma / (a_1 + a_2 B + a_3 B^2)$$

$$n(B) = n_1 B + n_2 B^2 + n_3 B^3$$

$$b = g(\theta_{trans}) \theta_{trans}^{n(B)}$$

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**B is the input parameter to Molière's equations, and depends on A, Z, thickness of target + momentum of electron**

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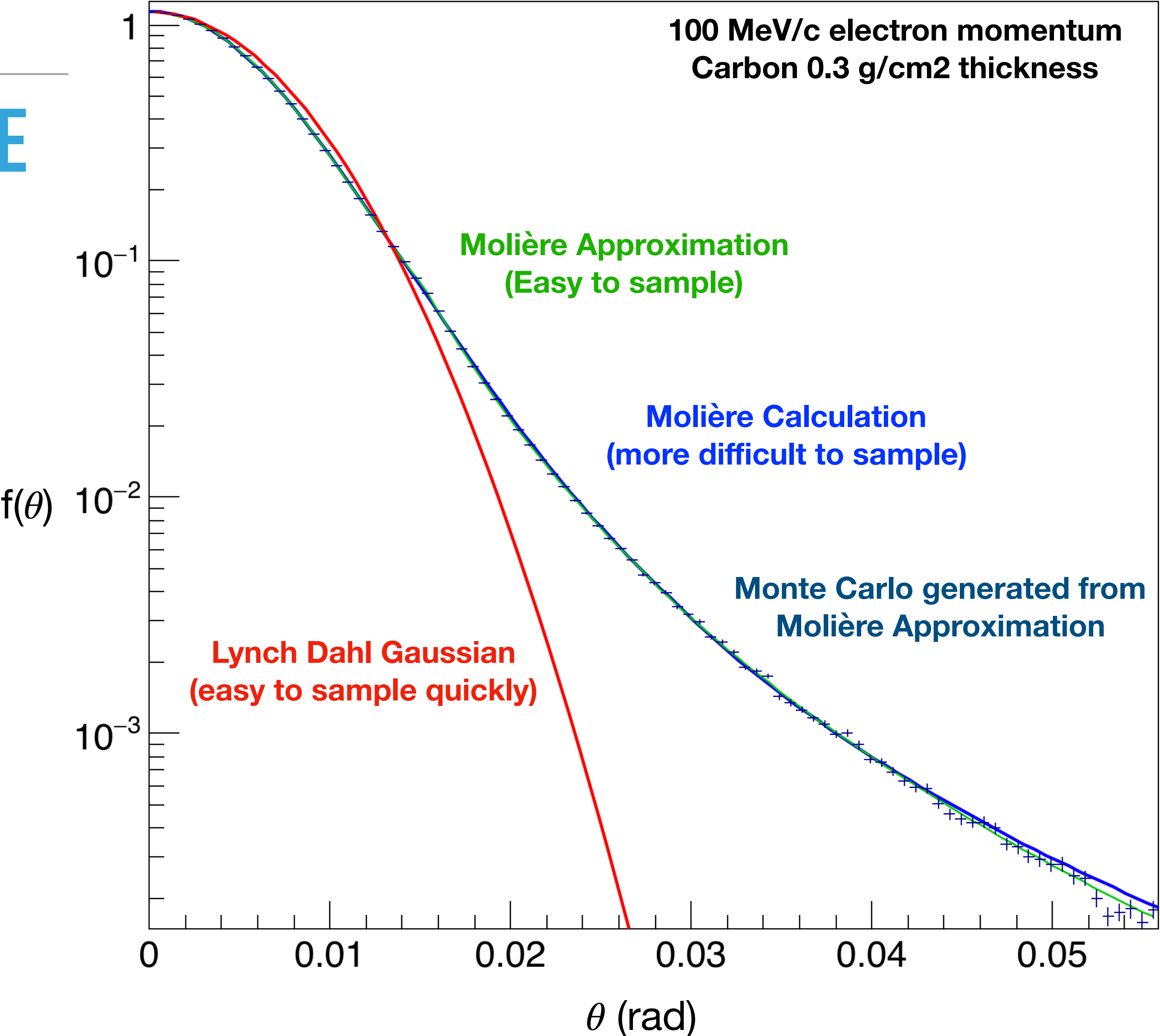
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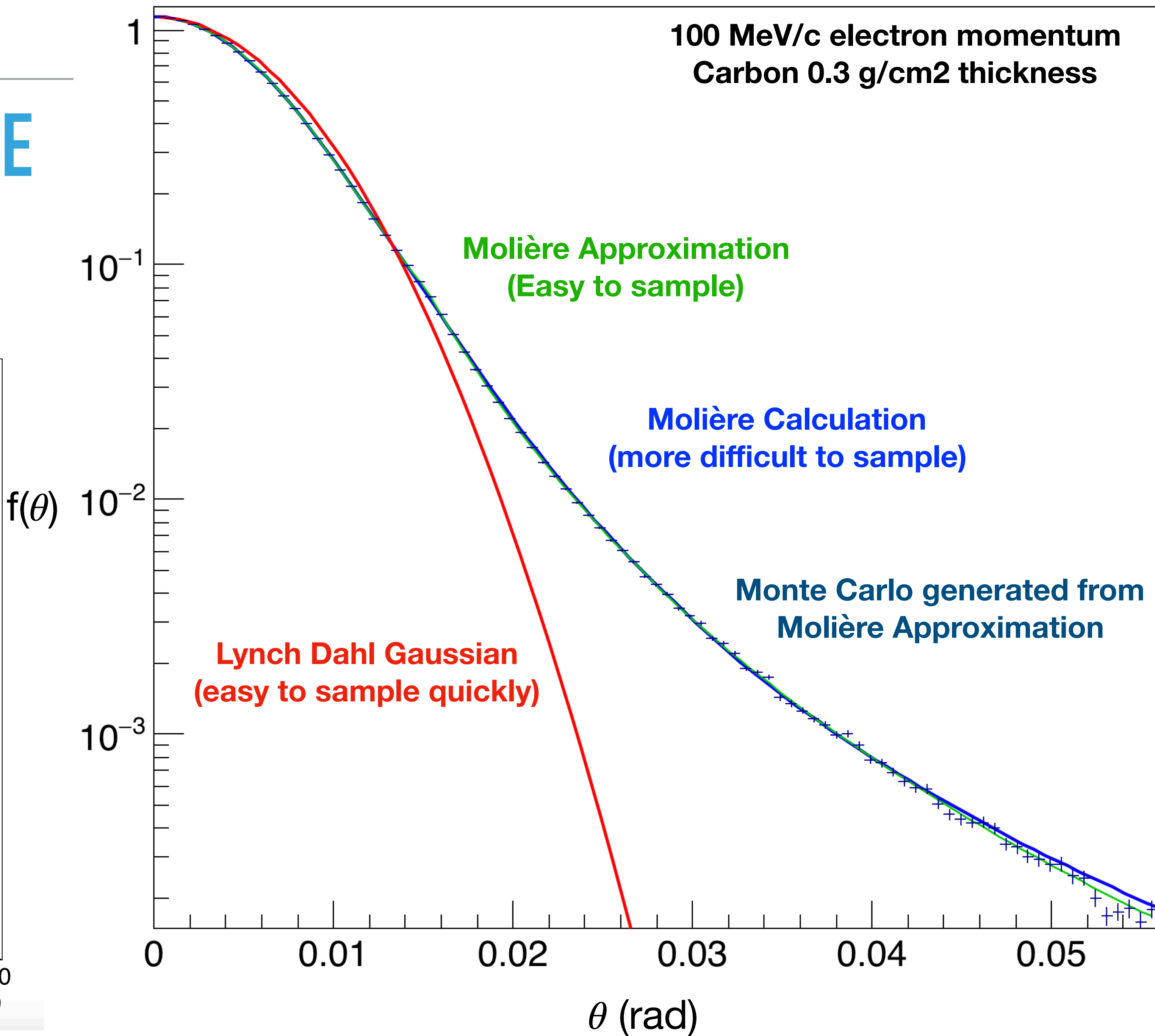
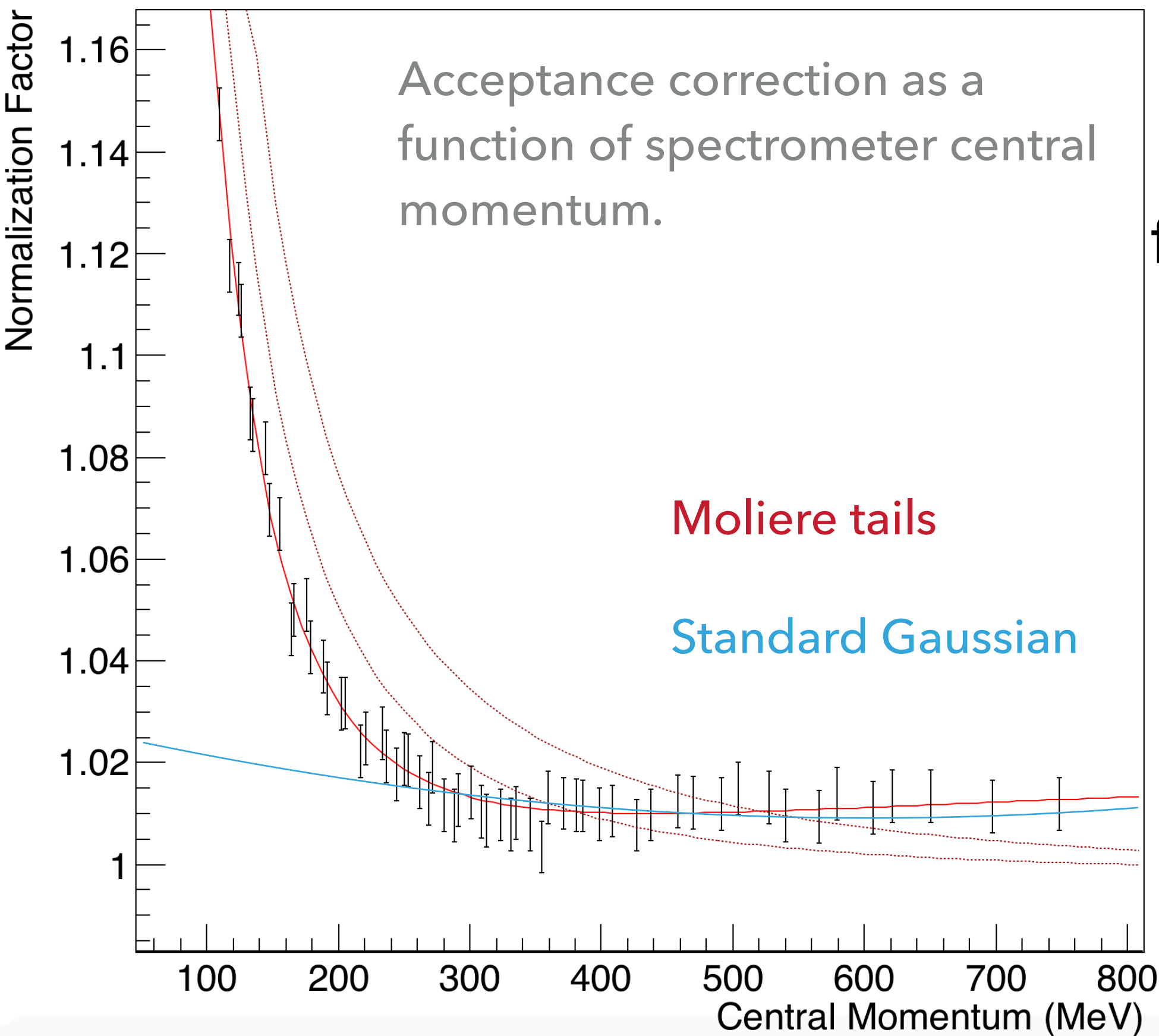
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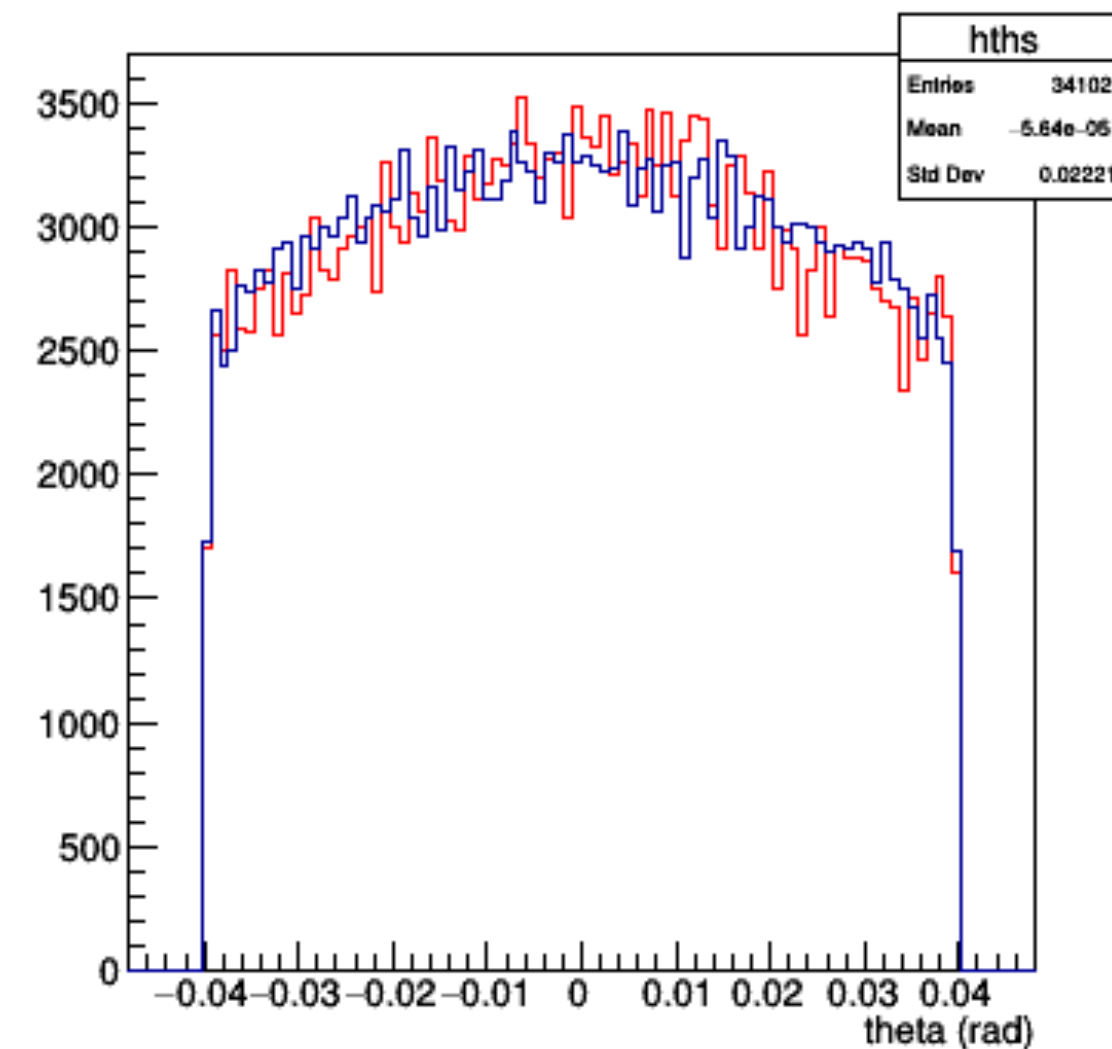
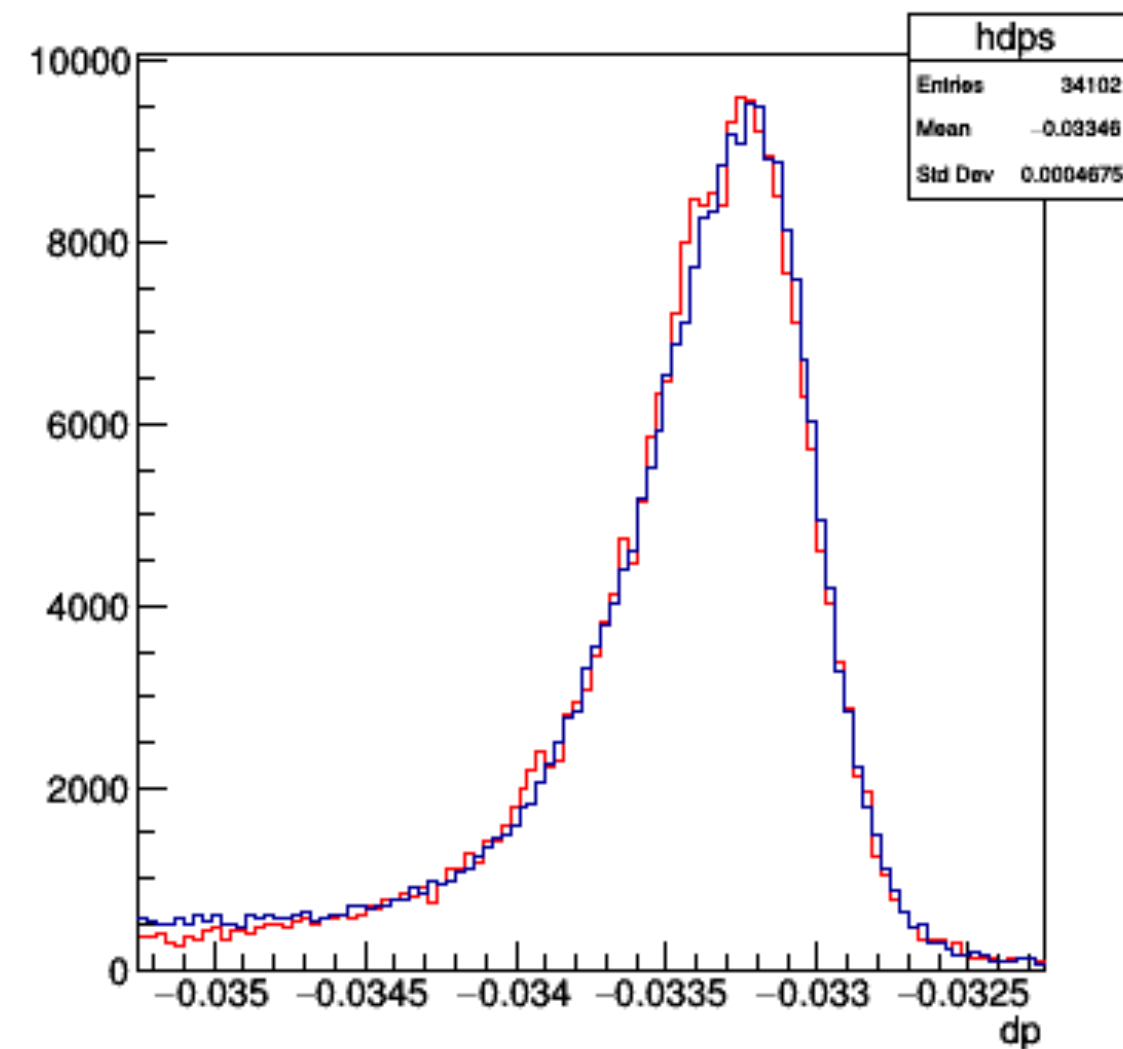
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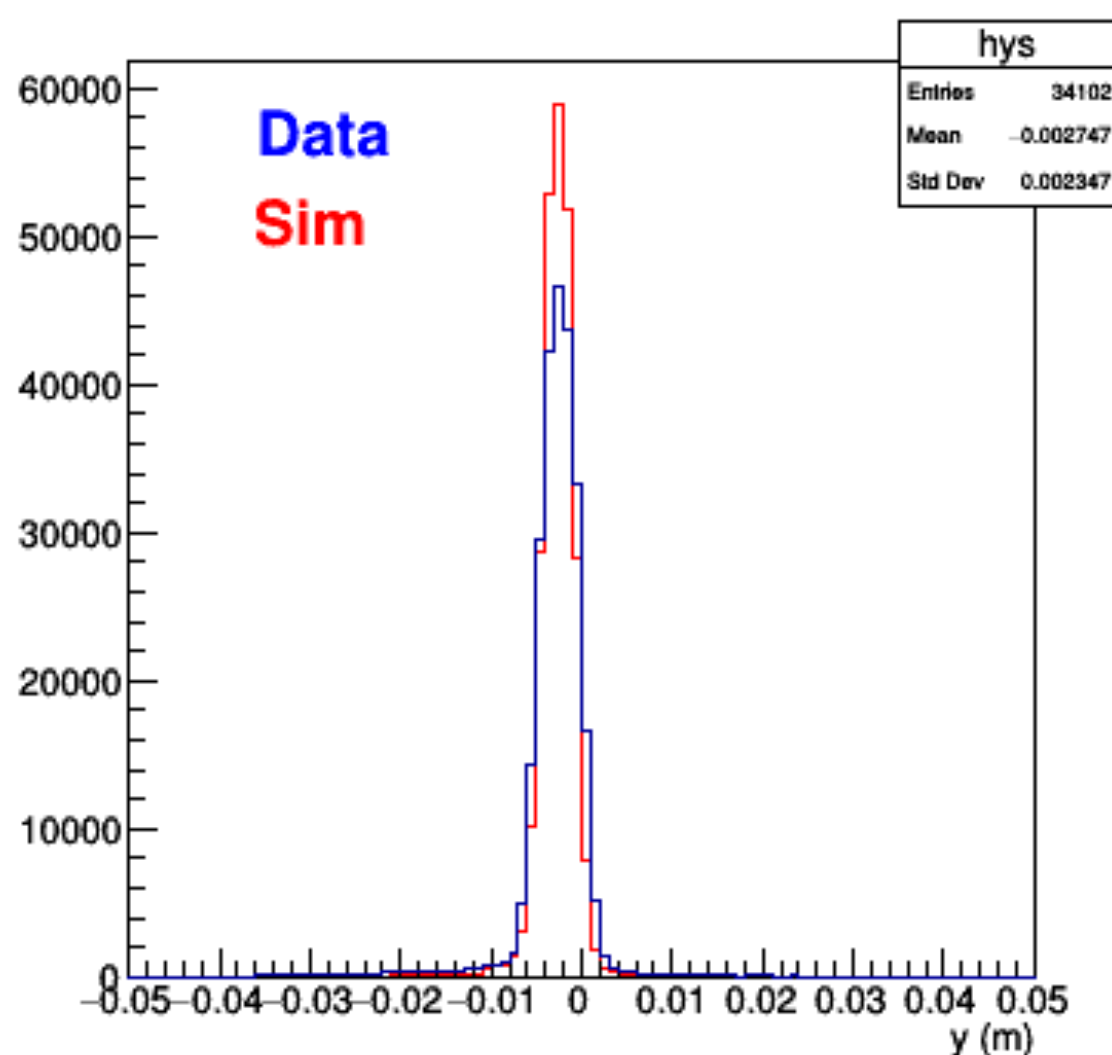
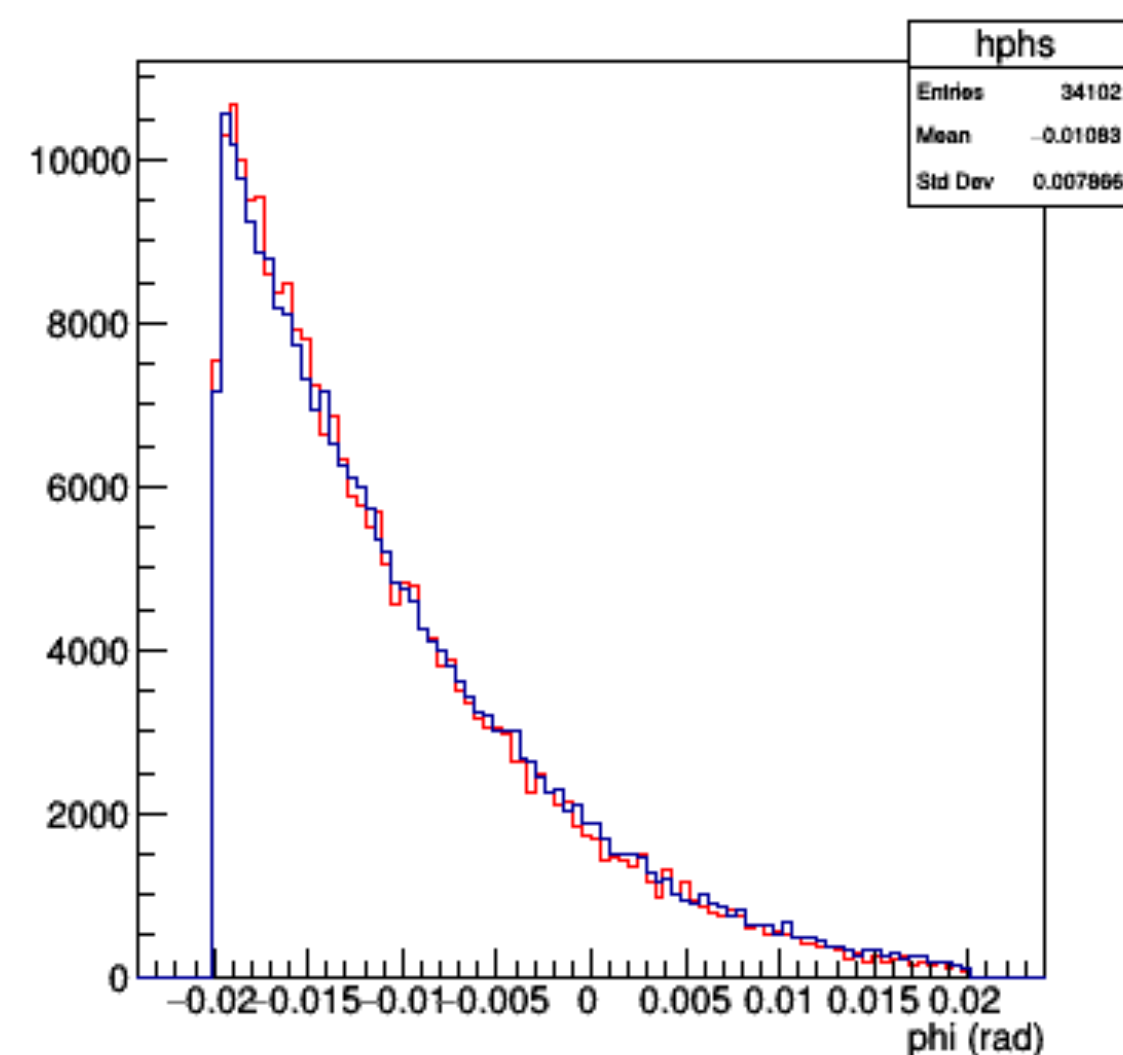
# APPROXIMATING MOLIÈRE



## ONGOING STUDIES: ELASTIC CROSS-SECTIONS AND LOW MOMENTUM NORMALIZATION

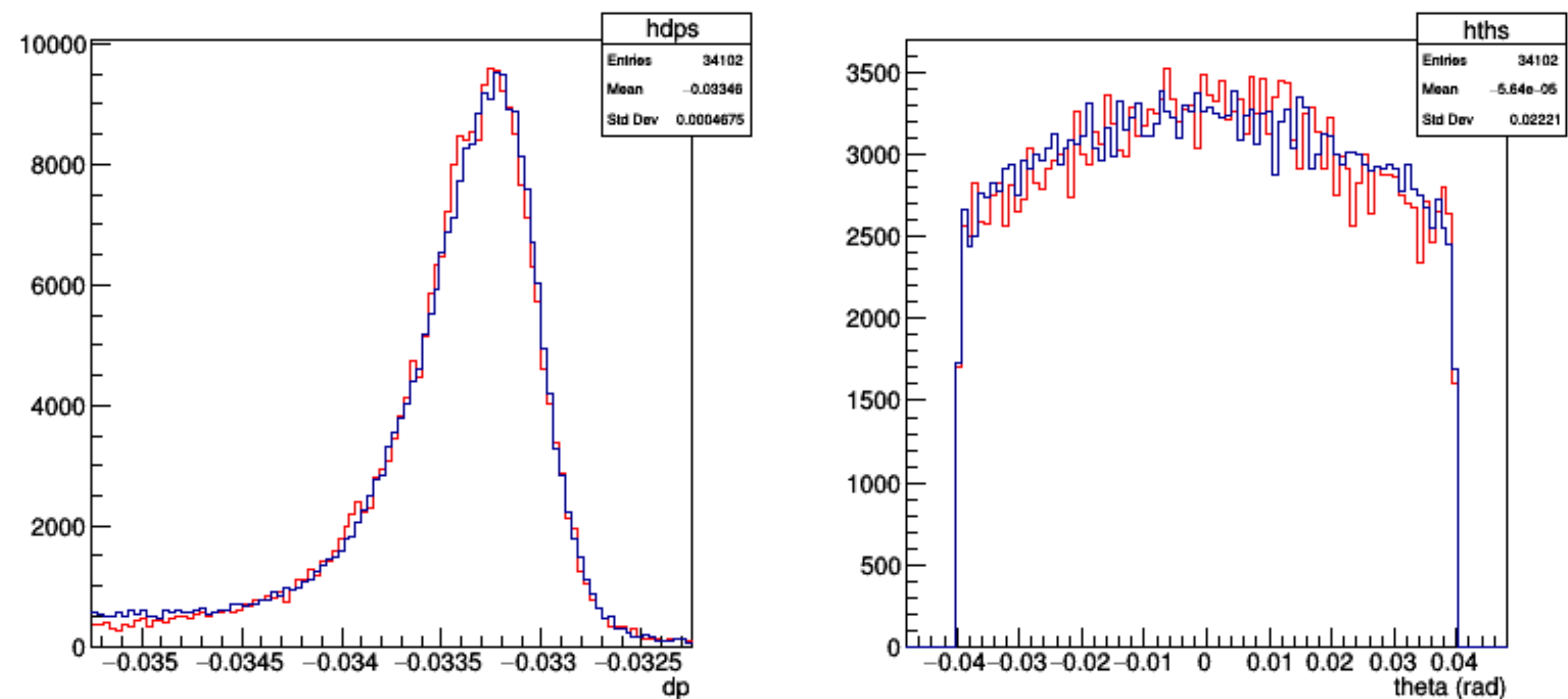


**$^{12}\text{C}$  elastic XS at 1260 MeV, 15 degrees**

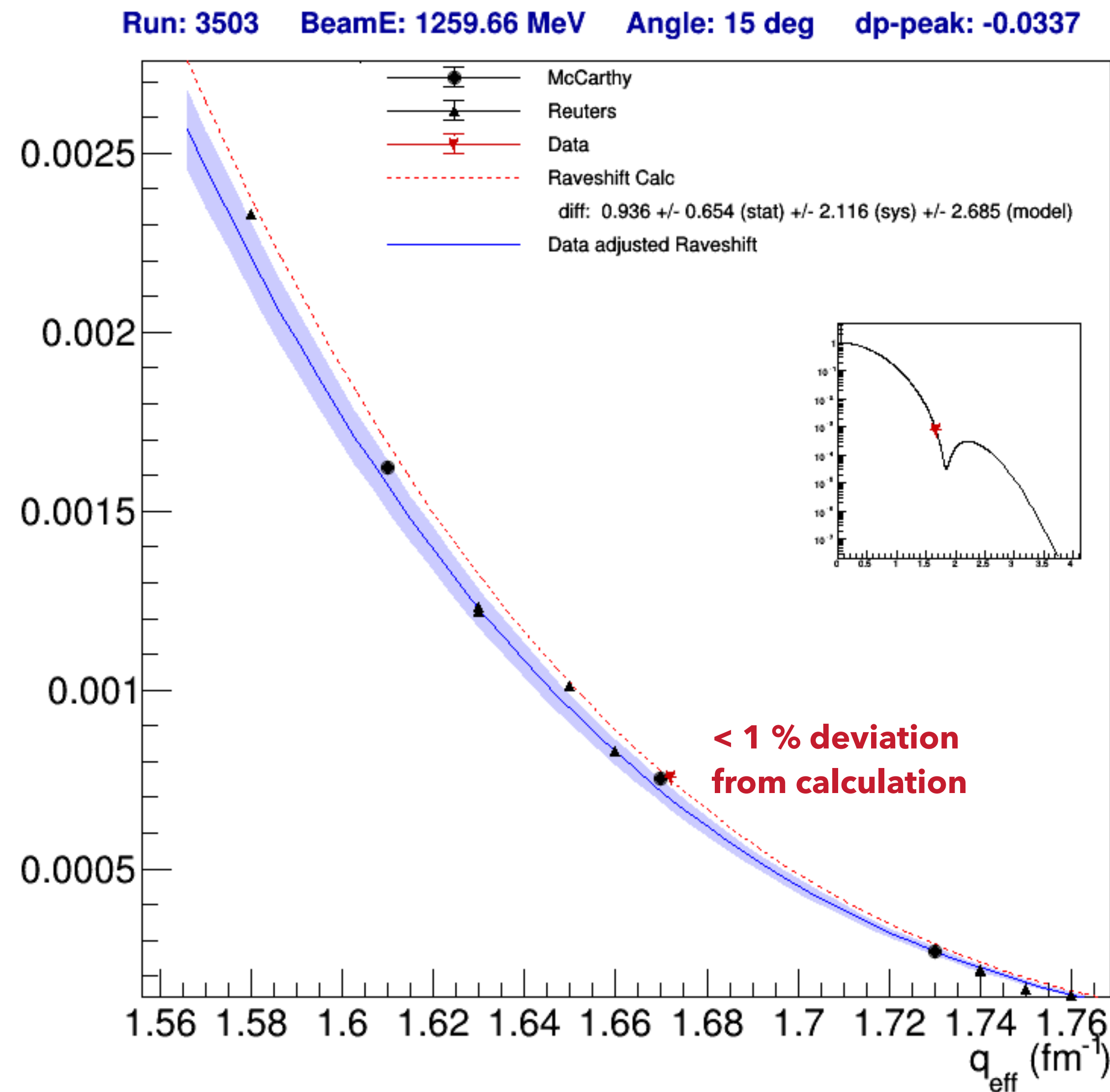
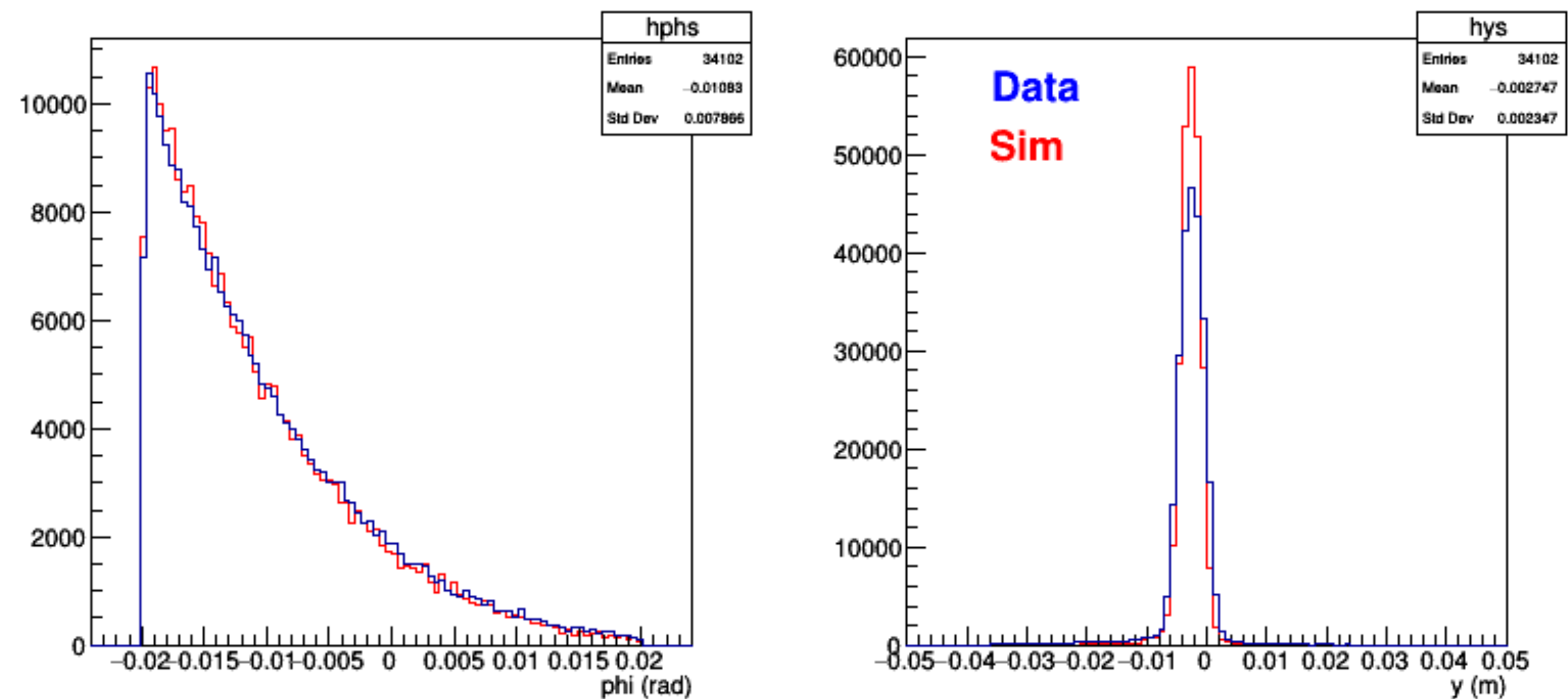


- ▶ Blue histograms are reconstructed data.
- ▶ Red histograms are monte-carlo:
  - ▶ Event sample generated from expected XS calculations (Fourier-Bessel fit to world data)
  - ▶ Radiative effects (internal, external, vertex) are handled, including exact bremsstrahlung distributions.
  - ▶ Resolution effects are applied by calculating the expected material effects of tracks passing through the VDC chamber materials.

ONGOING STUDIES: ELASTIC CROSS-SECTIONS AND LOW MOMENTUM NORMALIZATION

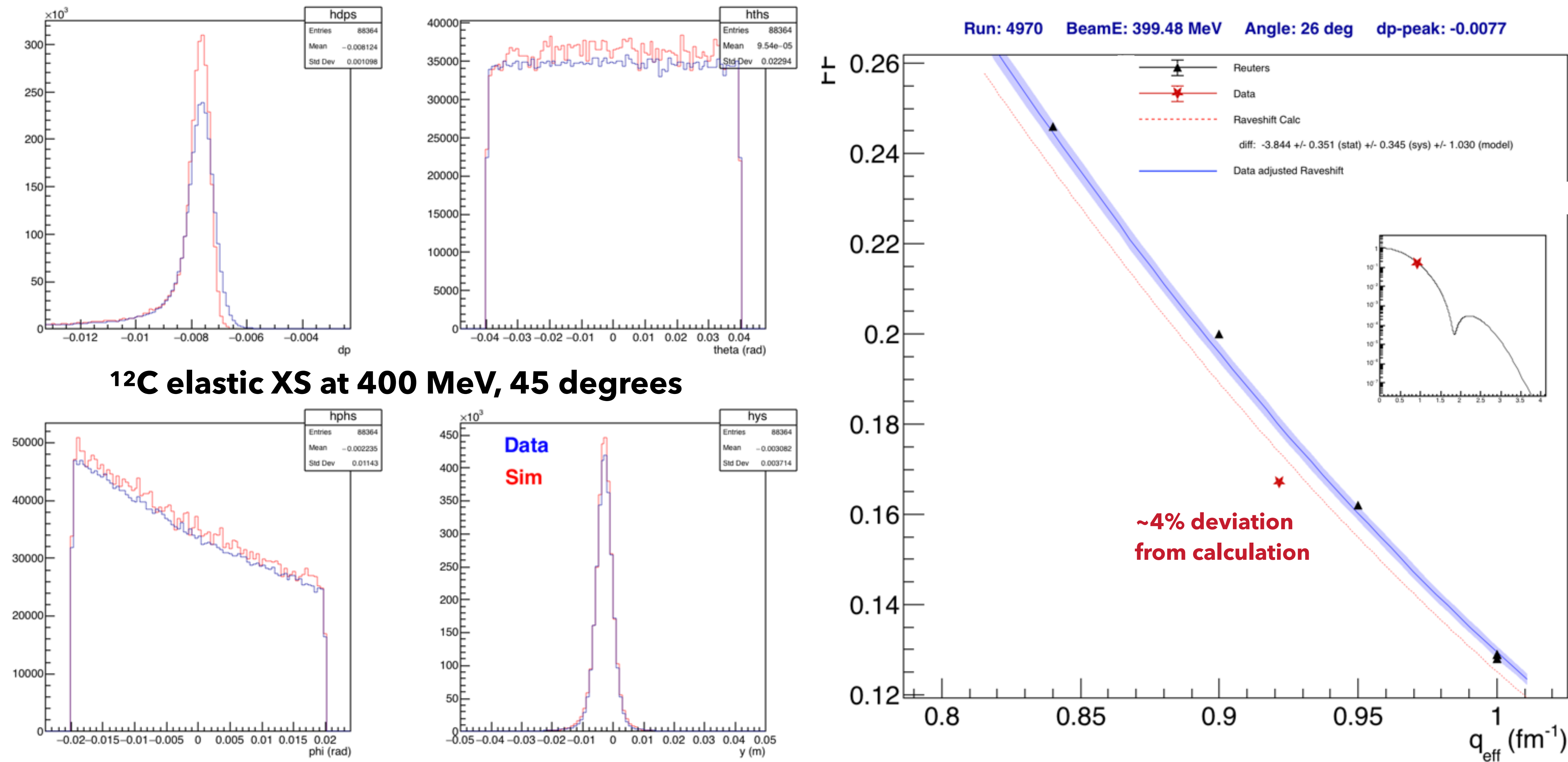


$^{12}\text{C}$  elastic XS at 1260 MeV, 15 degrees



< 1 % deviation  
from calculation

ONGOING STUDIES: ELASTIC CROSS-SECTIONS AND LOW MOMENTUM NORMALIZATION



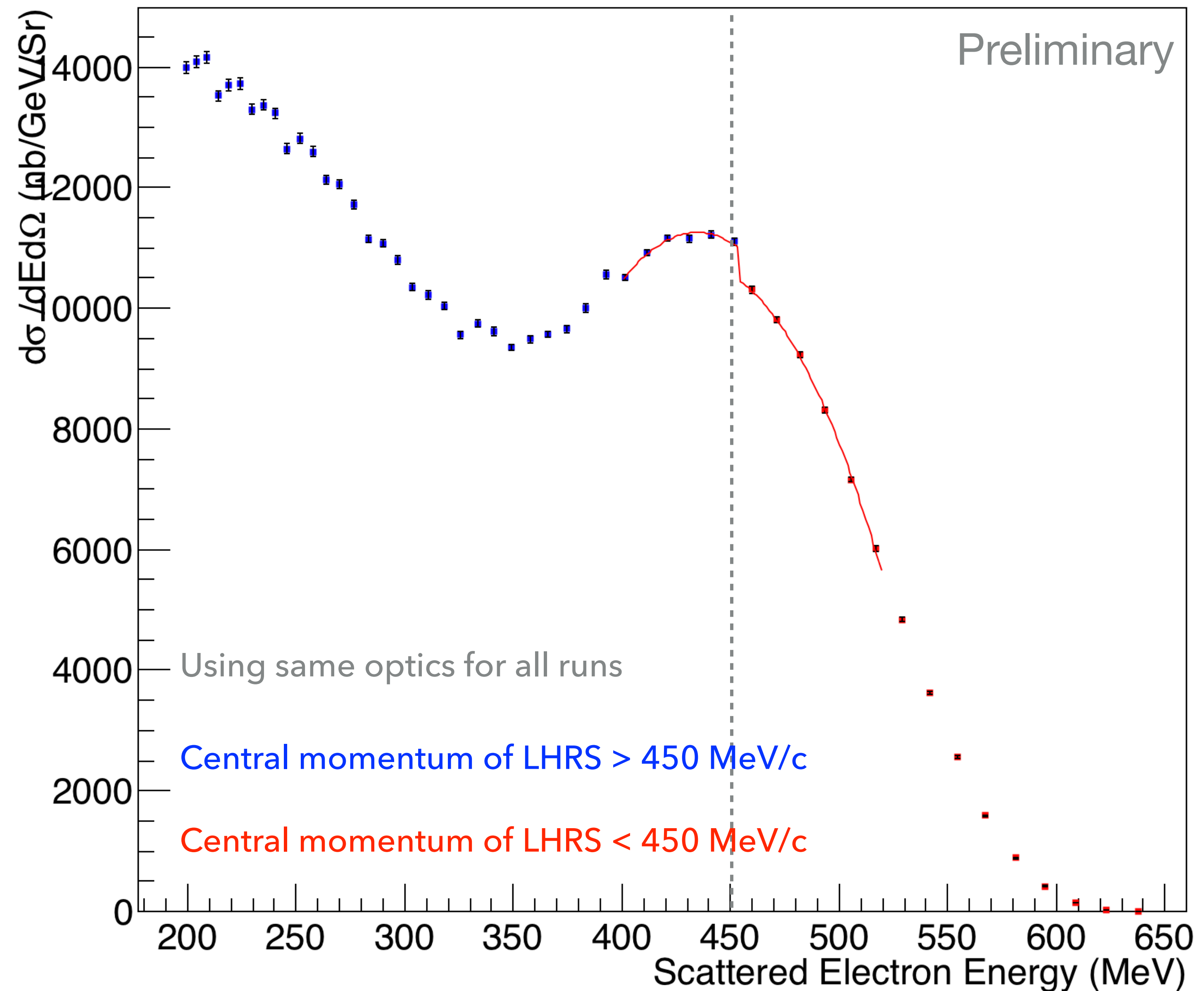
## RECENT UPDATES: LOW MOMENTUM MAGNET STUDY

- ▶ When the central momentum of an HRS is set below 450 MeV/c:
  - ▶ The NMR probe becomes quickly unreliable below 450 MeV/c to set the dipole field, so a gauss-meter is used instead.
  - ▶ In some way, the field-lock algorithm changes due to the change of probe.
    - ▶ This creates some differences in detector response when comparing above/below 450 MeV/c.

## LOW MOMENTUM MAGNET STUDY

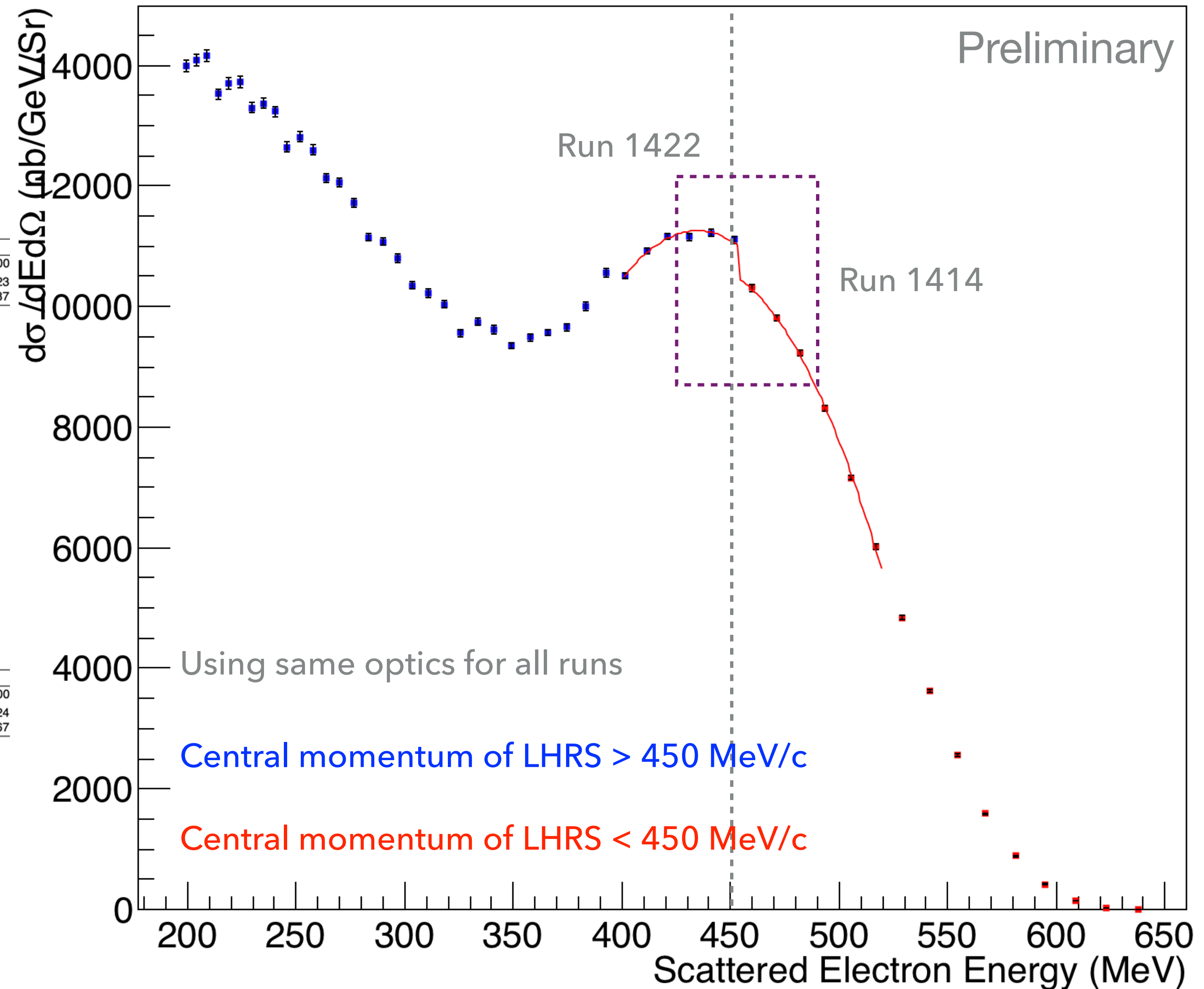
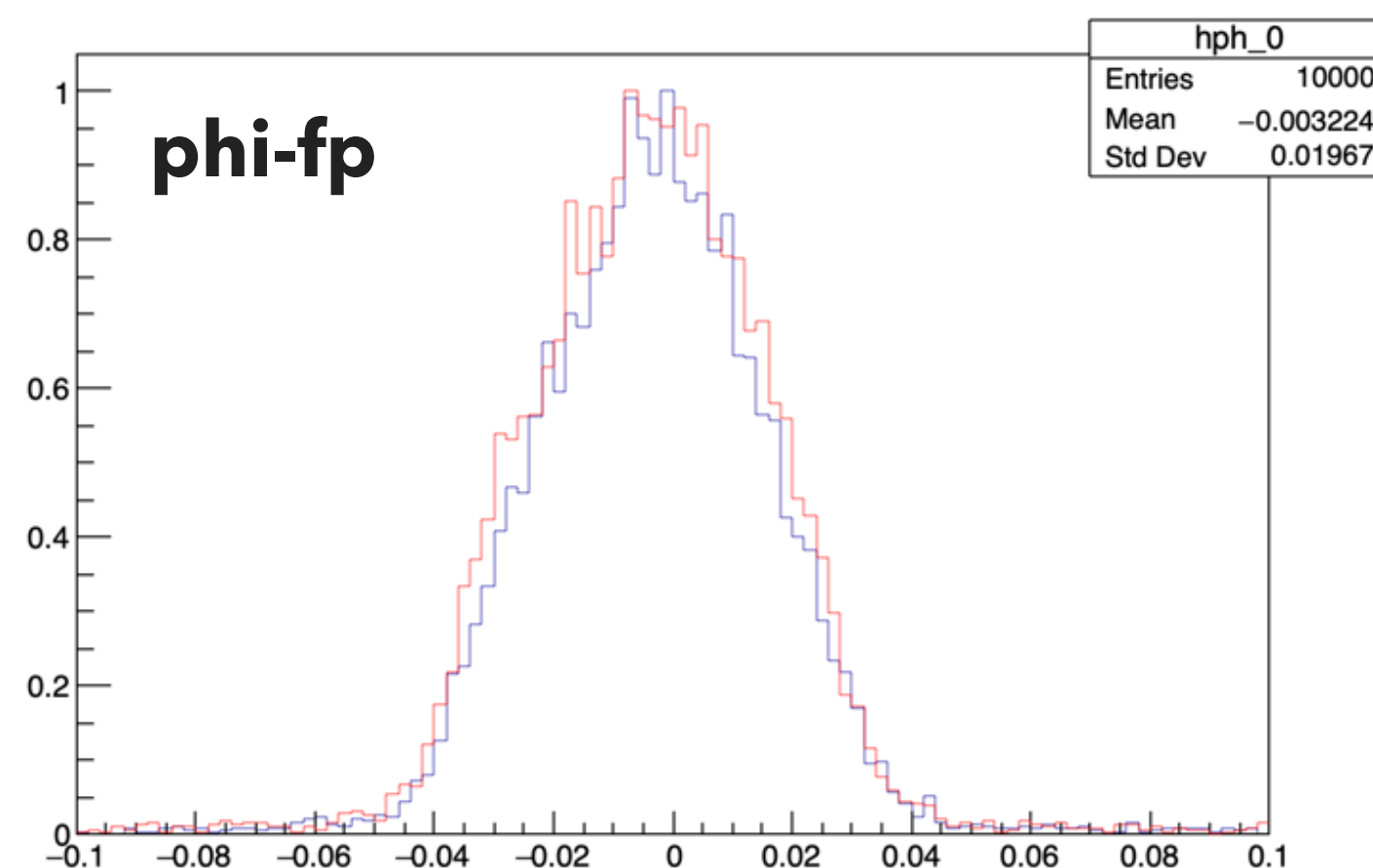
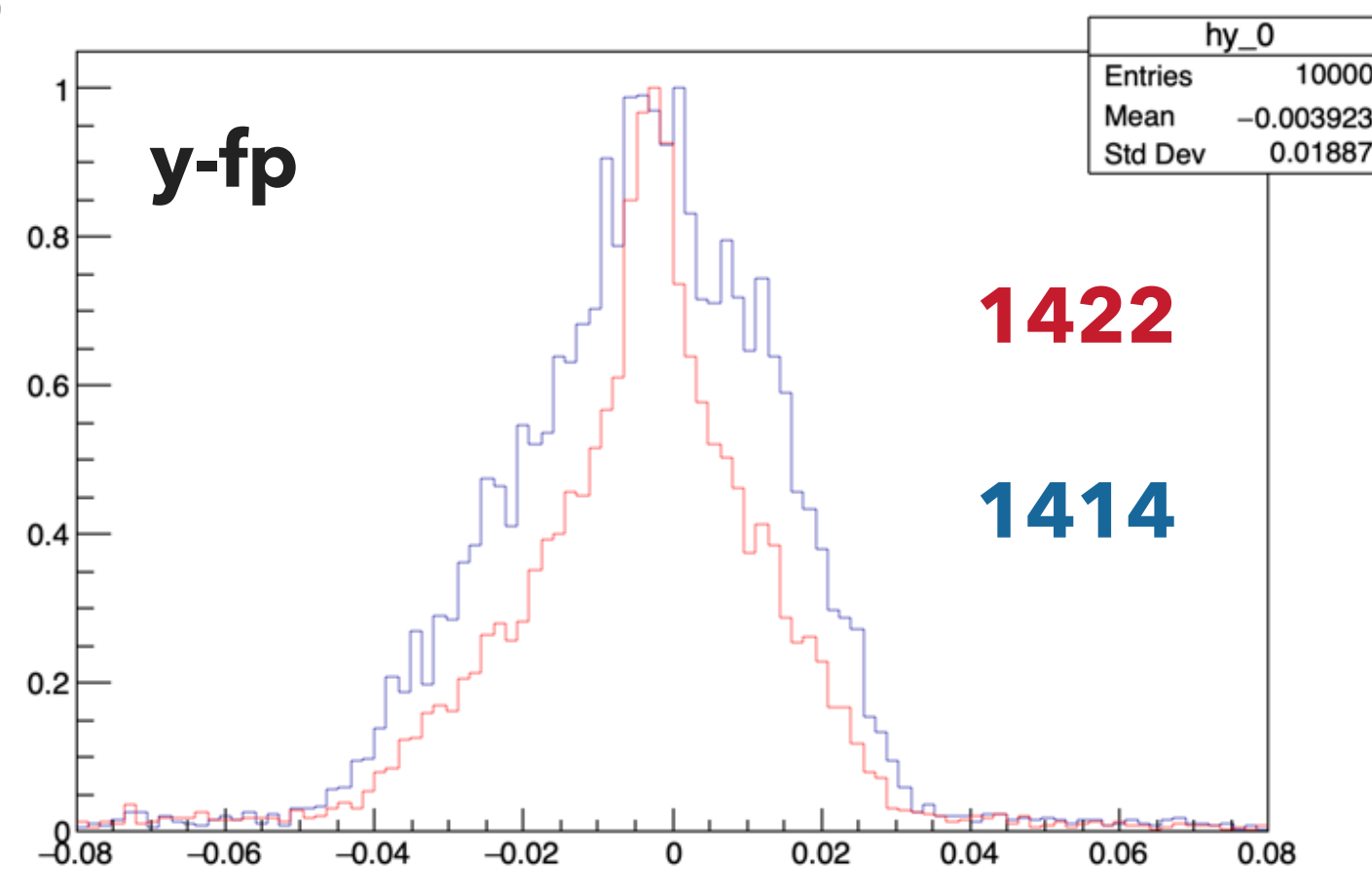
- Clear jump at switching point, what is happening?

$E = 646.295000$  MeV, Angle =  $60.000300$  deg



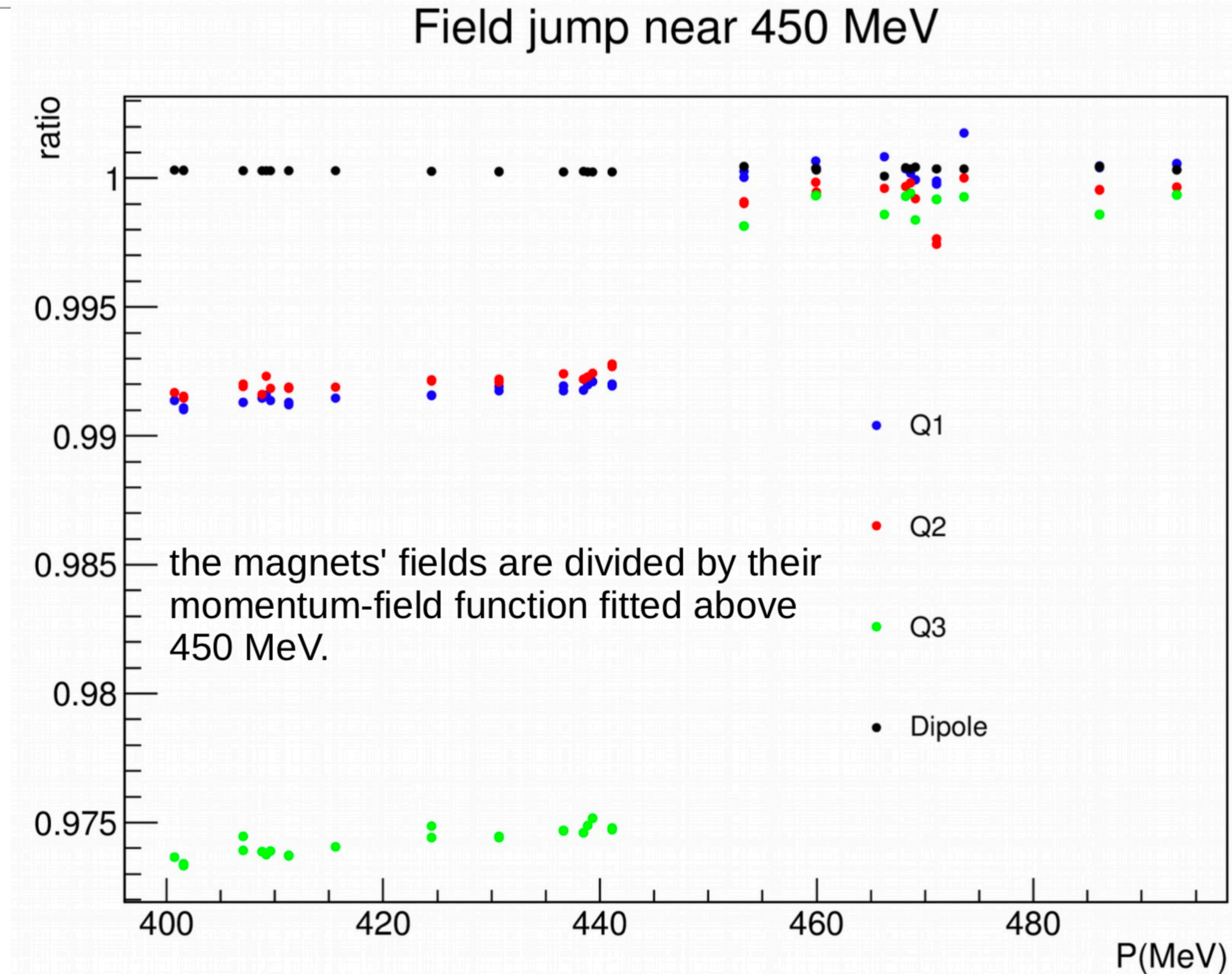
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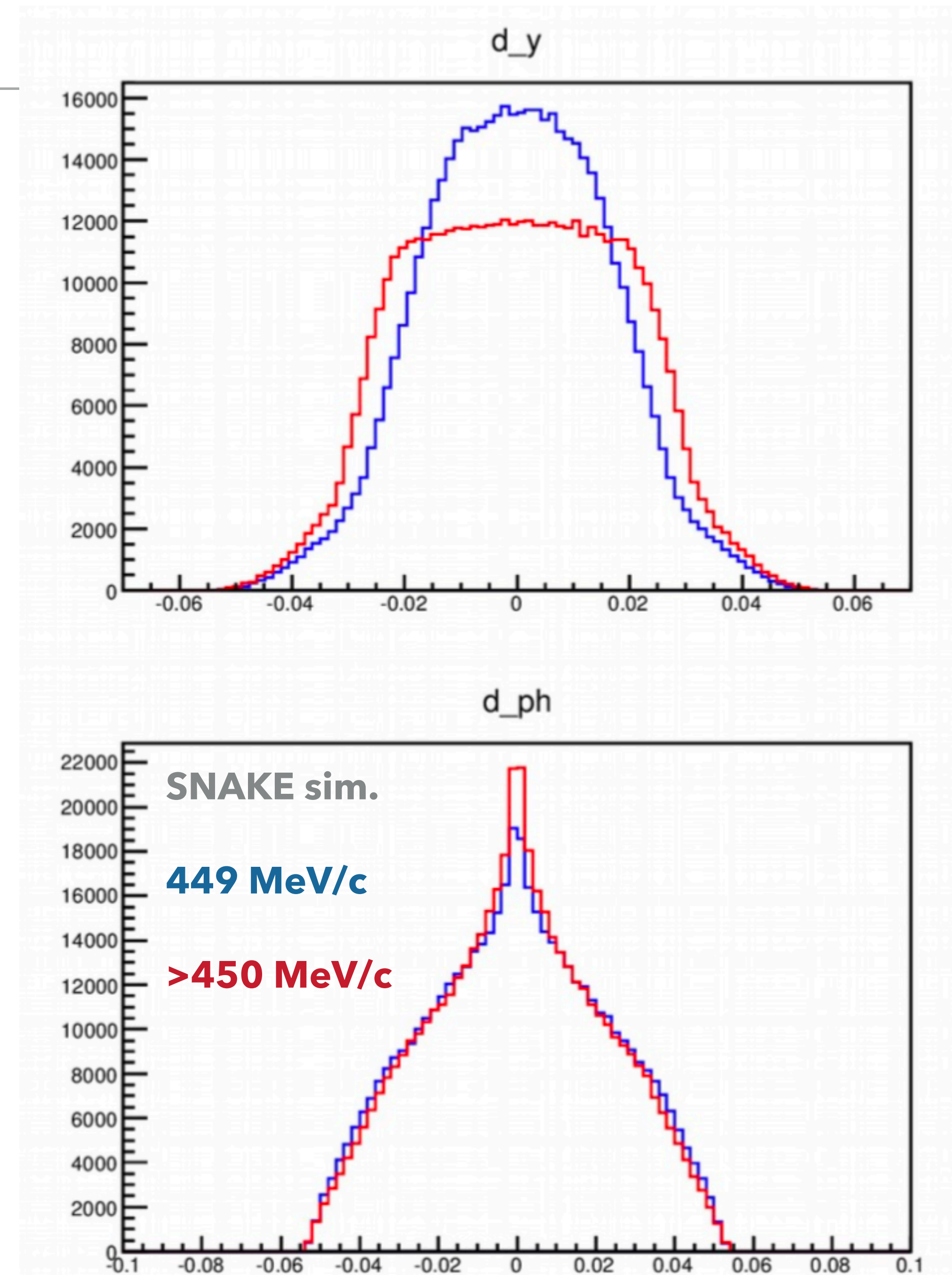
## LOW MOMENTUM MAGNET STUDY

- ▶ By fitting the gauss-meter field vs current, we can see a change in the set field below 450 MeV/c.
- ▶ The dipole remains mostly the same, but Q1 and Q2 change by  $\sim 0.8\%$  and Q3 changes by  $\sim 2.6\%$



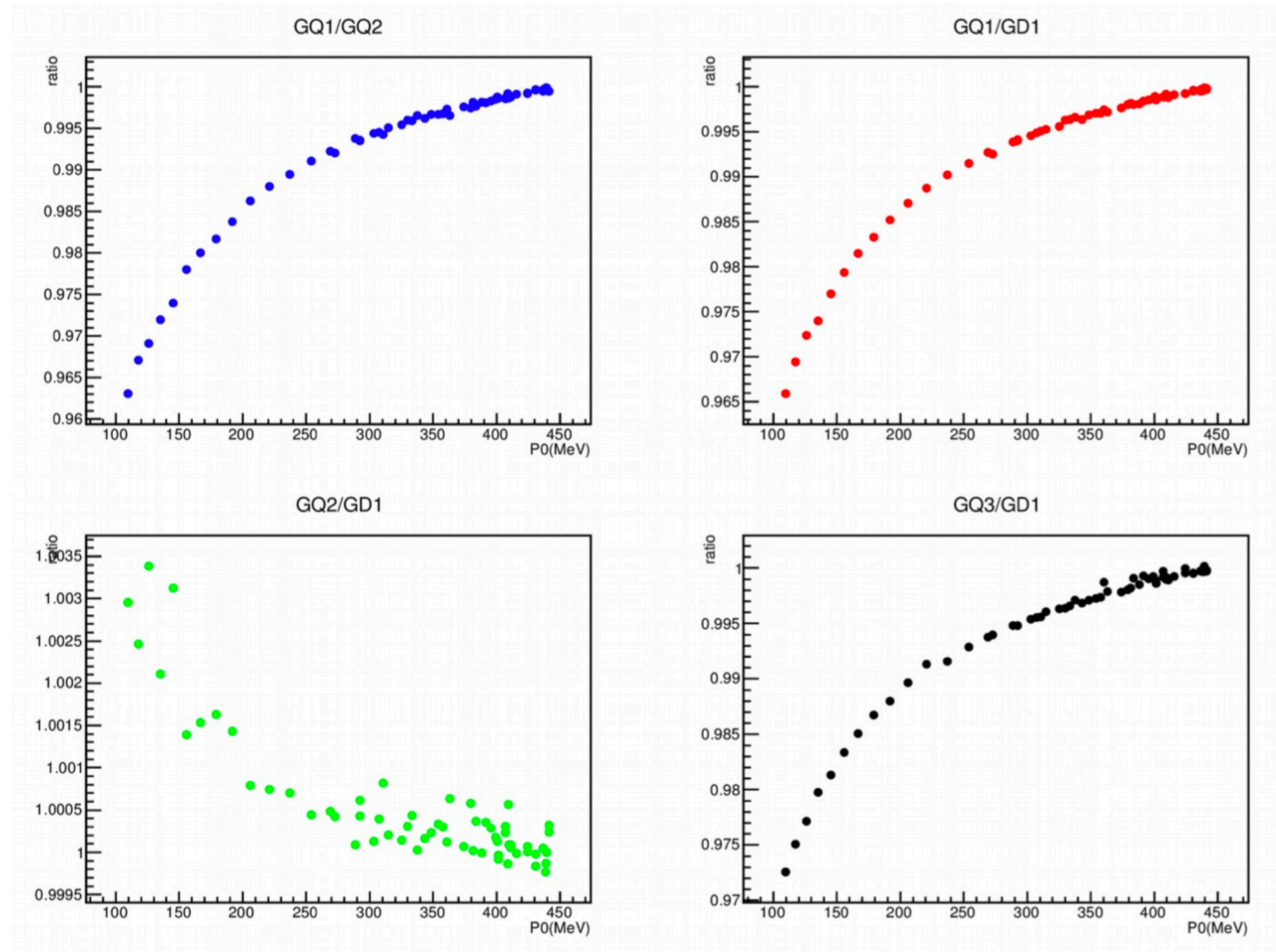
# LOW MOMENTUM MAGNET STUDY

- ▶ If we apply this field change into a SNAKE simulation, we can accurately reproduce the trend of the effect.



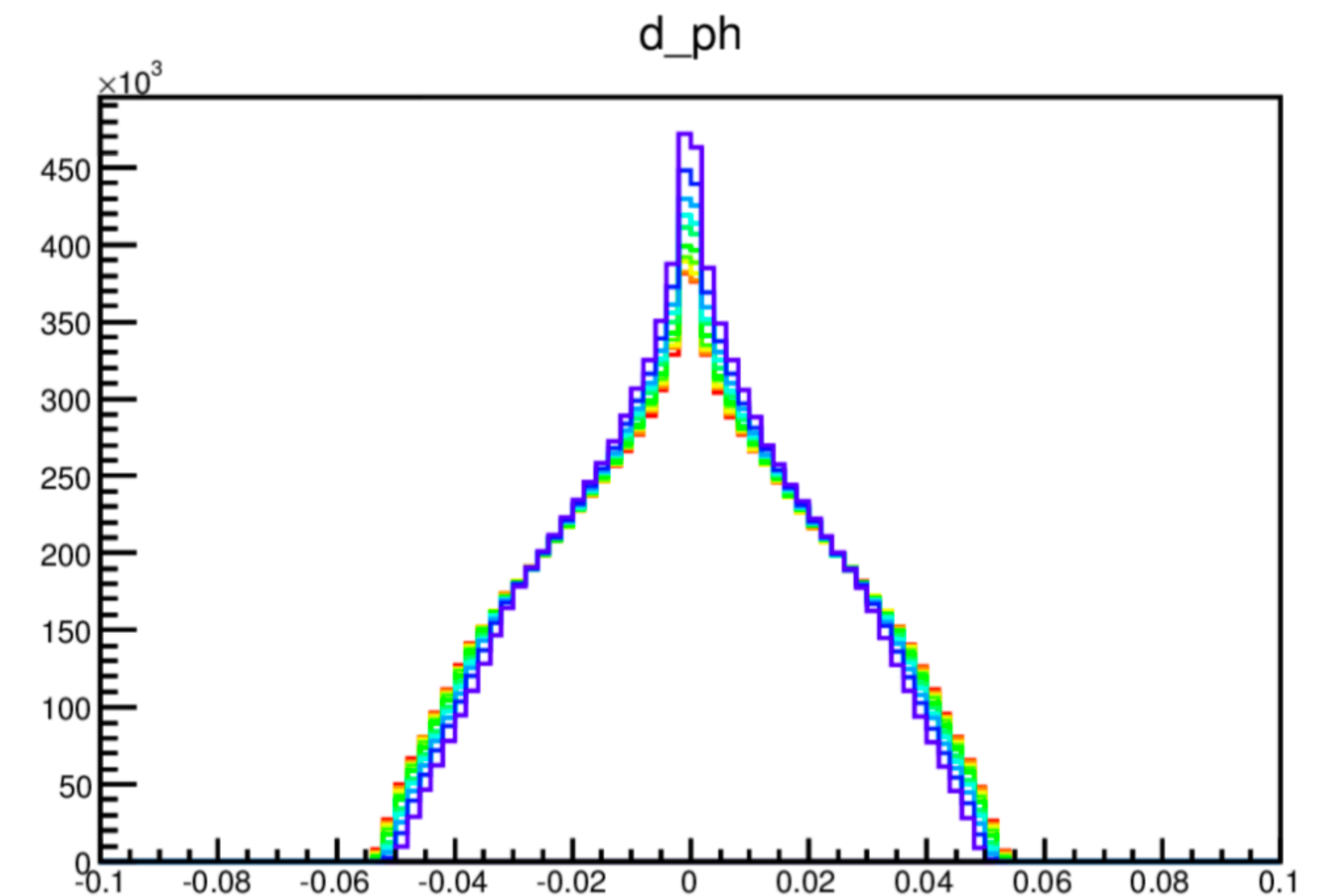
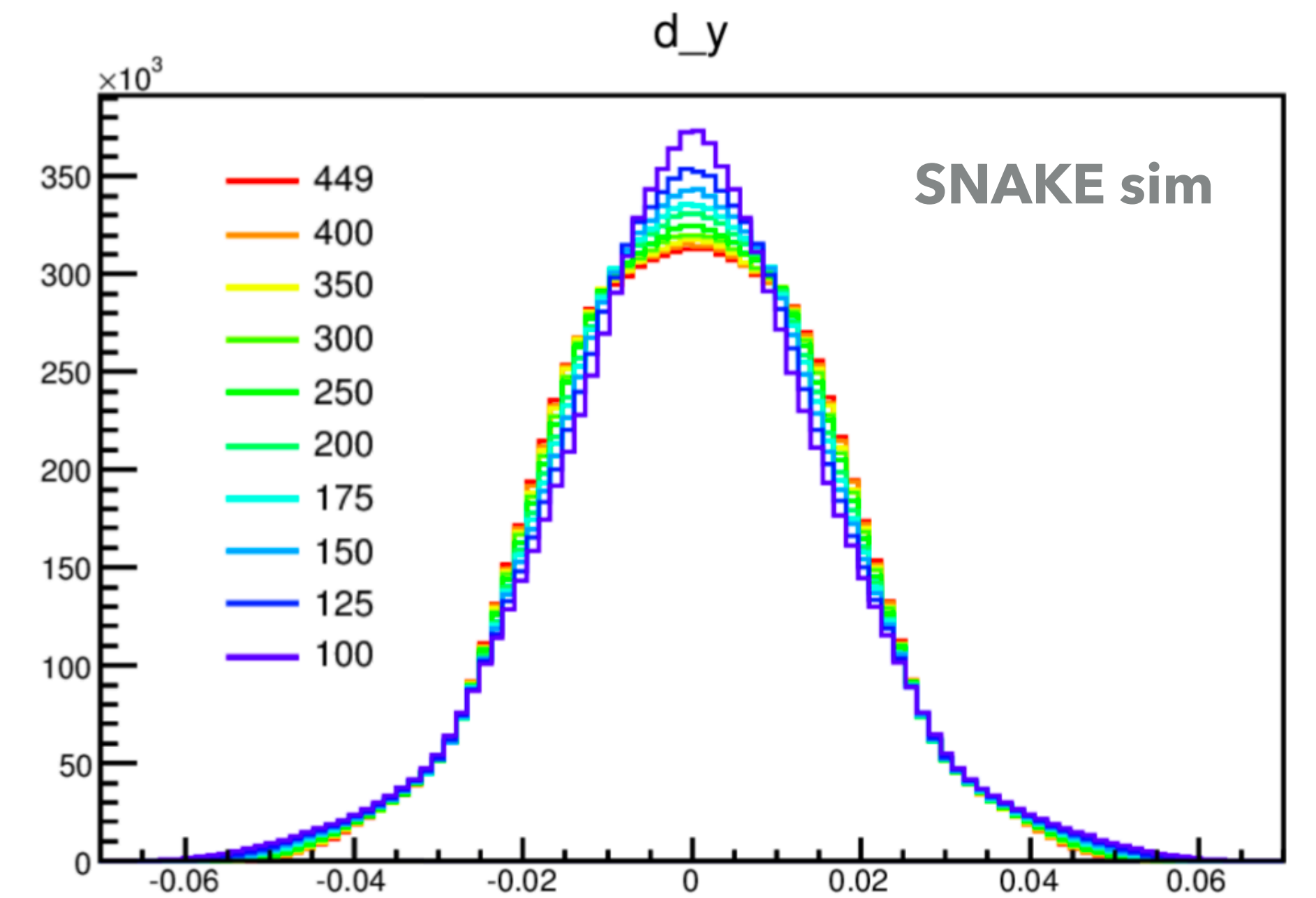
# LOW MOMENTUM MAGNET STUDY

- ▶ If we apply this field change into a SNAKE simulation, we can accurately reproduce the trend of the effect.
- ▶ The effect also appears to be momentum dependent.



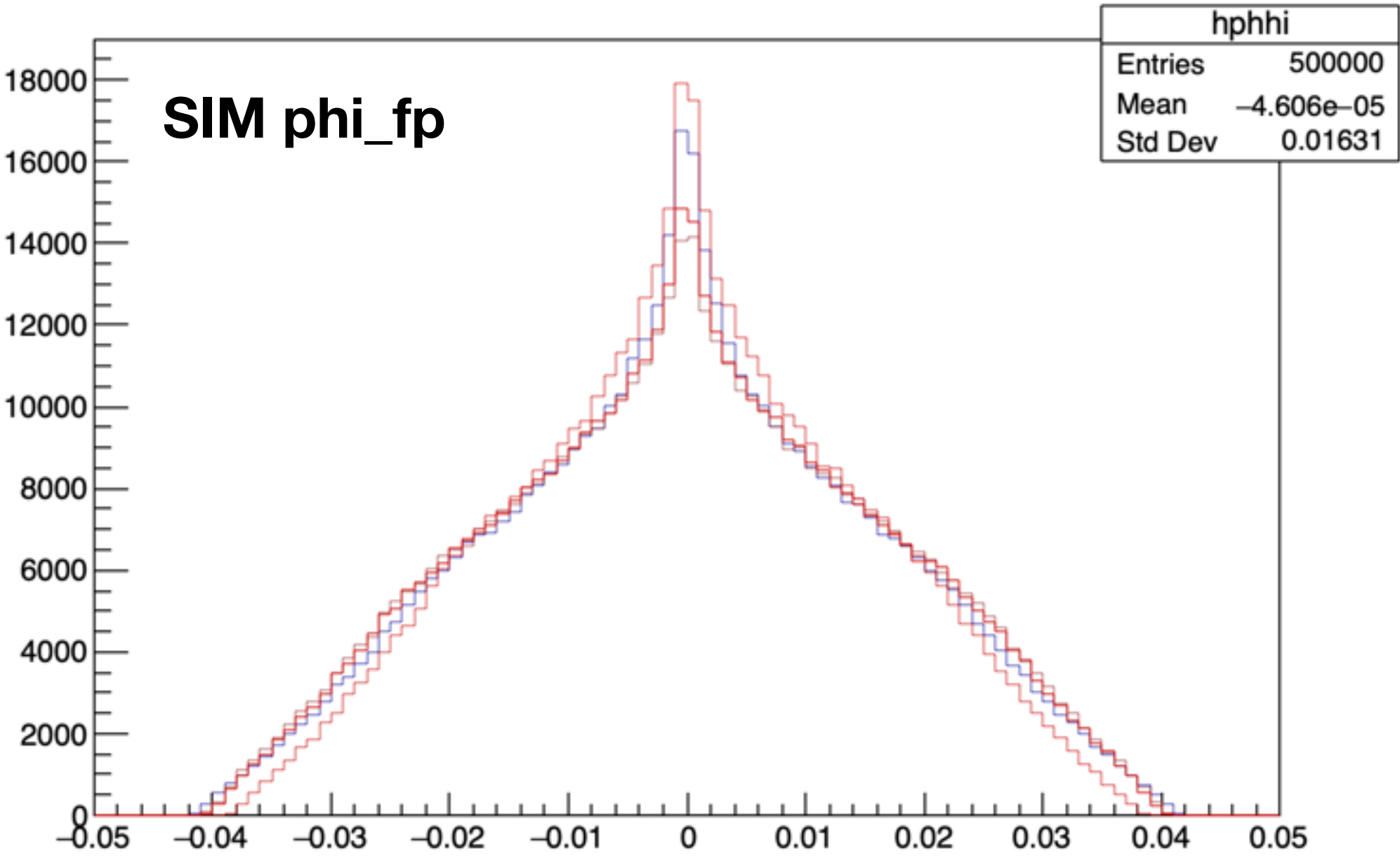
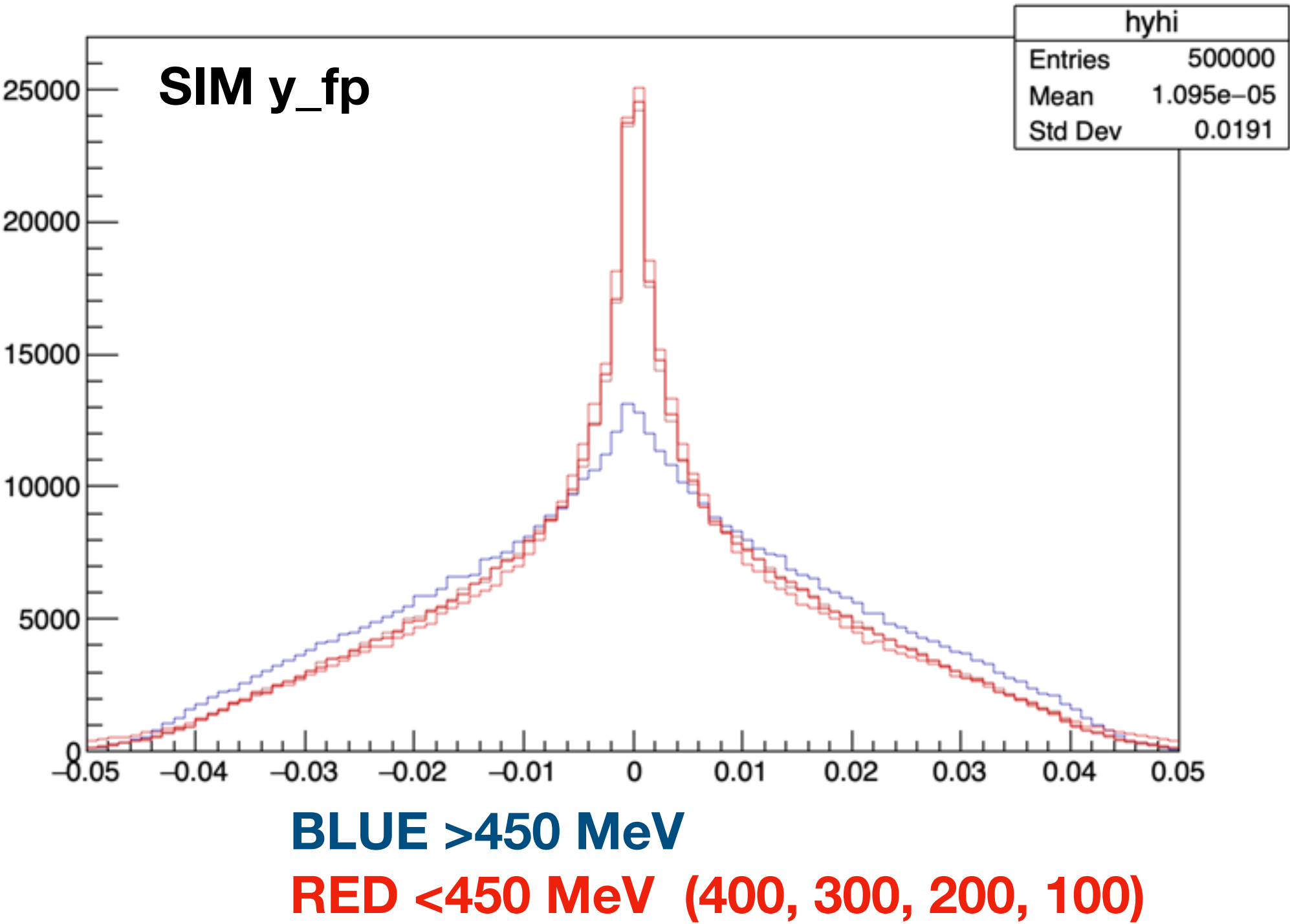
# LOW MOMENTUM MAGNET STUDY

- ▶ If we apply this field change into a SNAKE simulation, we can accurately reproduce the trend of the effect.
- ▶ The effect also appears to be momentum dependent.



# LOW MOMENTUM MAGNET STUDY

- ▶ One can try to make an effective correction by rotating y and phi at the focal plane.



## LOW MOMENTUM MAGNET STUDY

- One can try to make an effective correction by rotating y and phi at the focal plane.

$$y'_{fp} = a_1 y_{fp} + a_2 \phi_{fp}$$

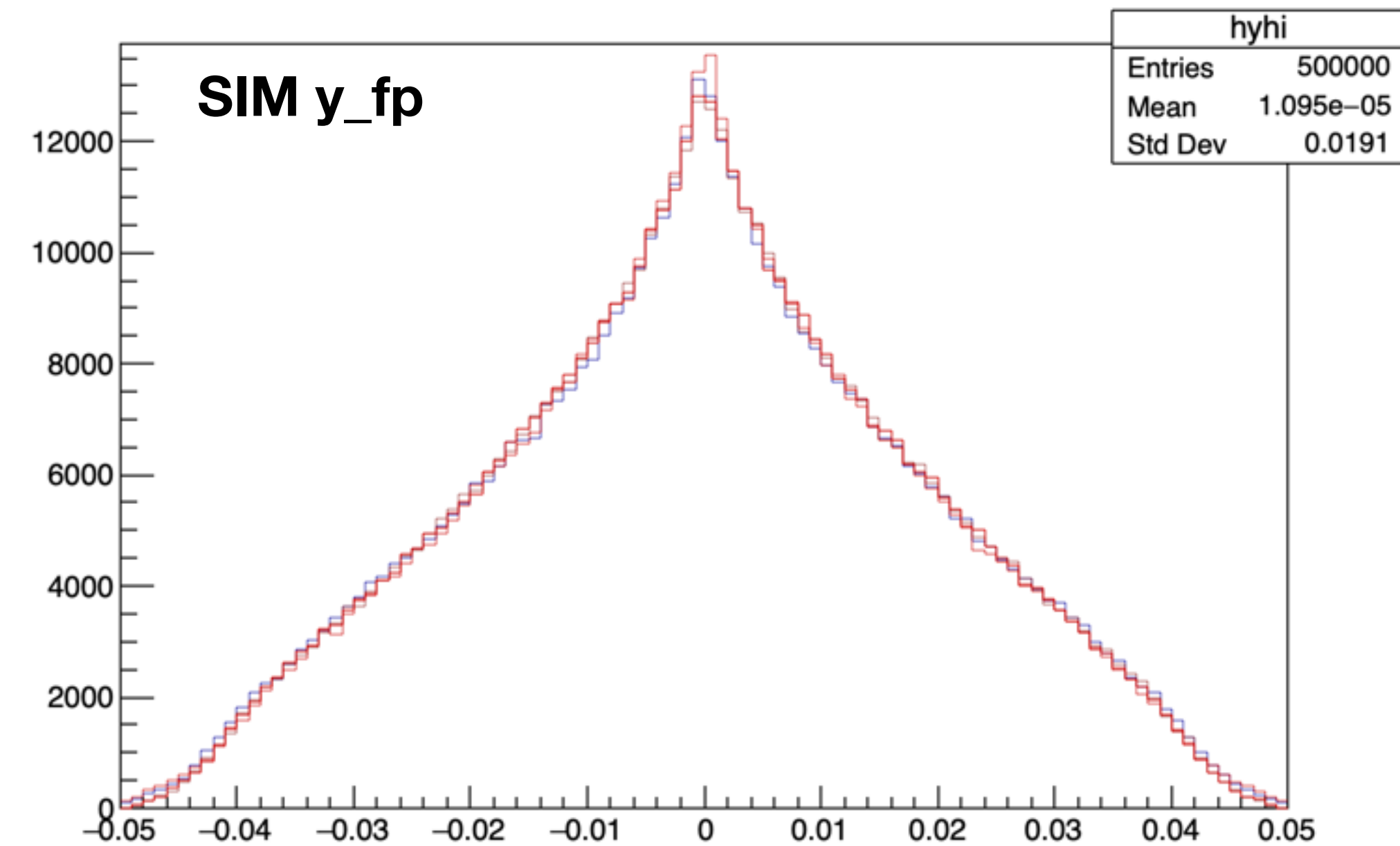
$$\phi'_{fp} = b_1 \phi_{fp} + b_2 y_{fp}$$

$$a_1 = 1 - \frac{1}{\sqrt{p}}$$

$$b_1 = 0.98 + \frac{2}{p} + \frac{1000}{p^2}$$

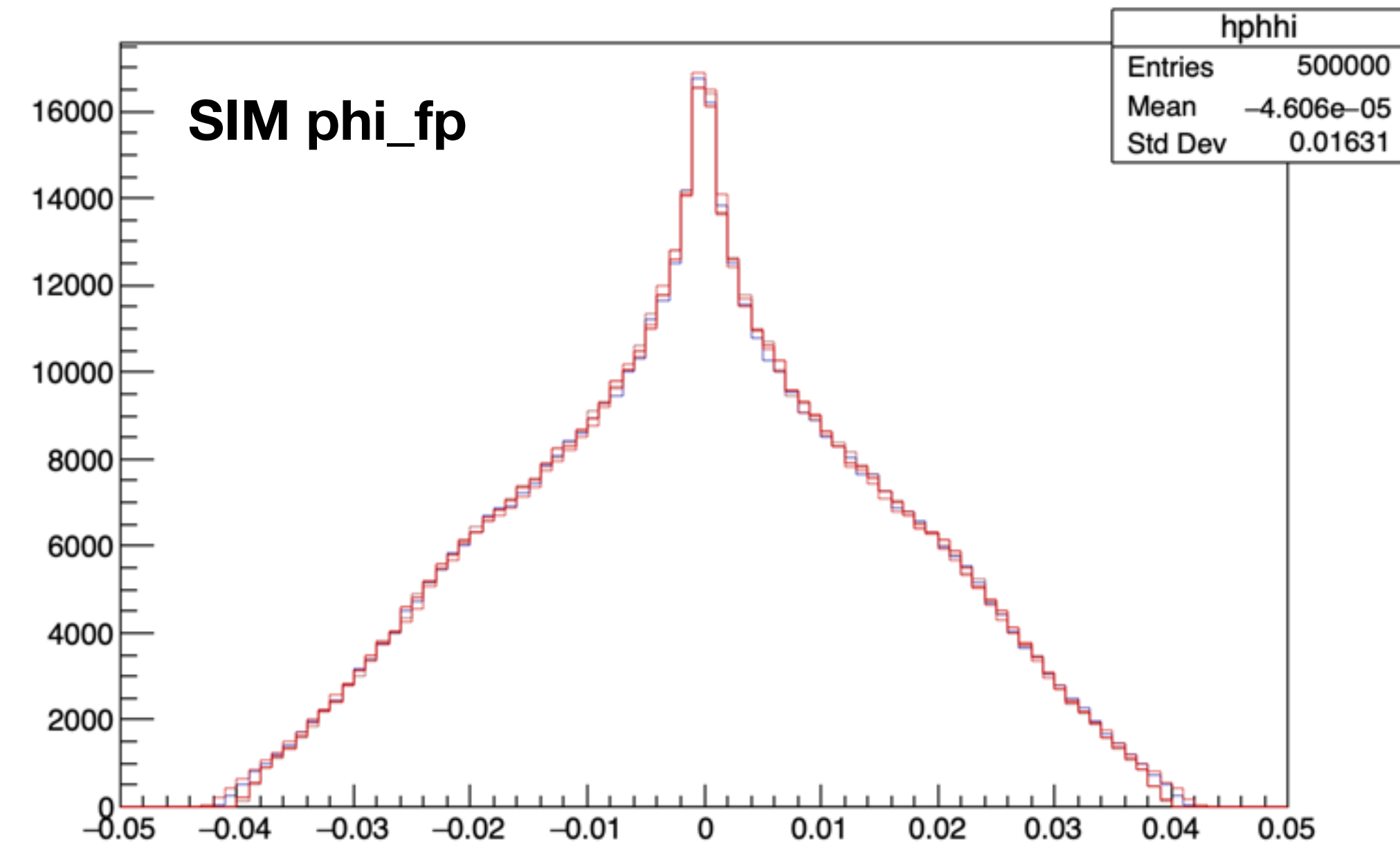
$$a_2 = 0.12 + \frac{1.6}{\sqrt{p}} + \frac{1.6}{p}$$

$$b_2 = -\frac{p}{8000}$$



**BLUE >450 MeV**

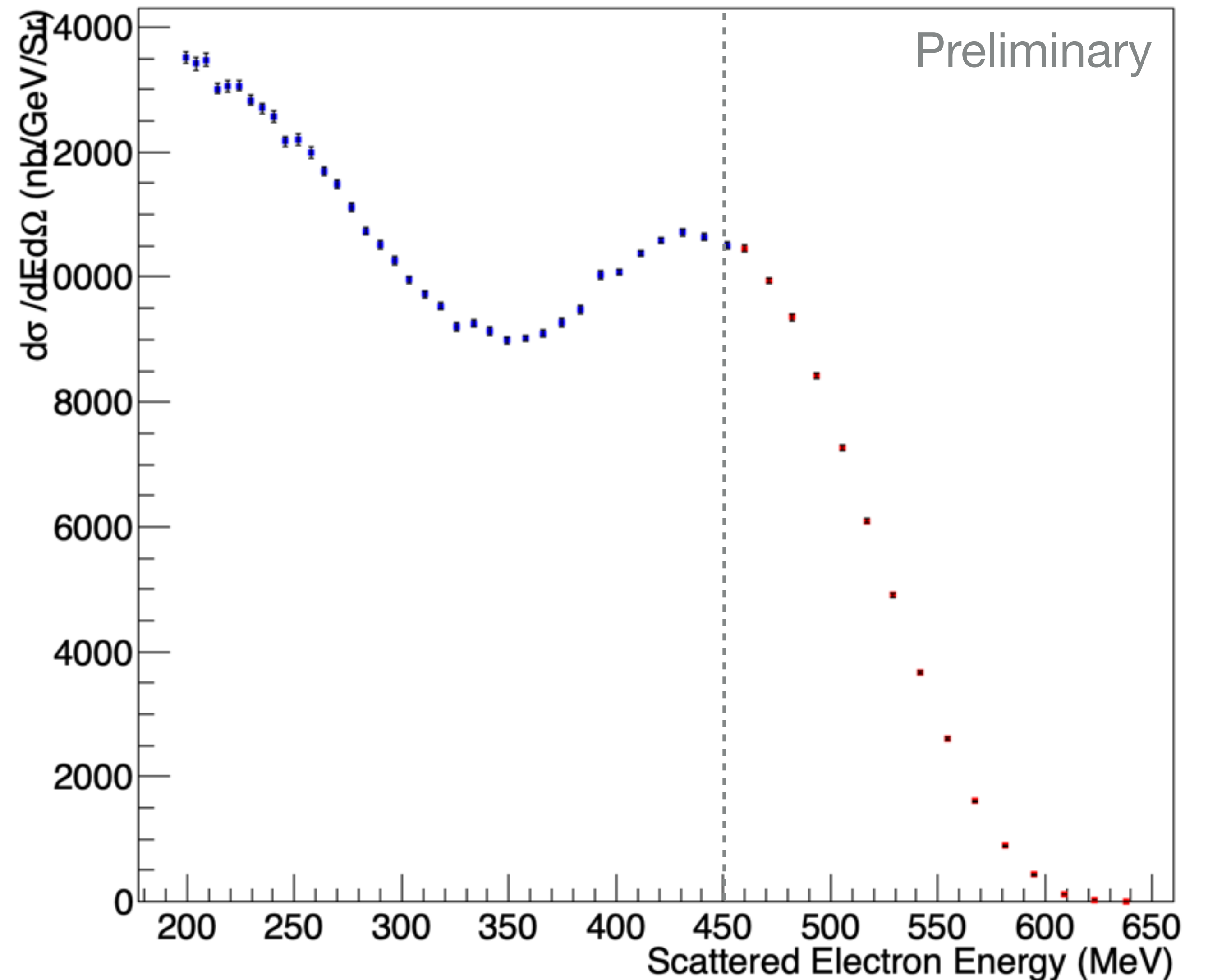
**RED <450 MeV (400, 300, 200, 100)**



## LOW MOMENTUM MAGNET STUDY

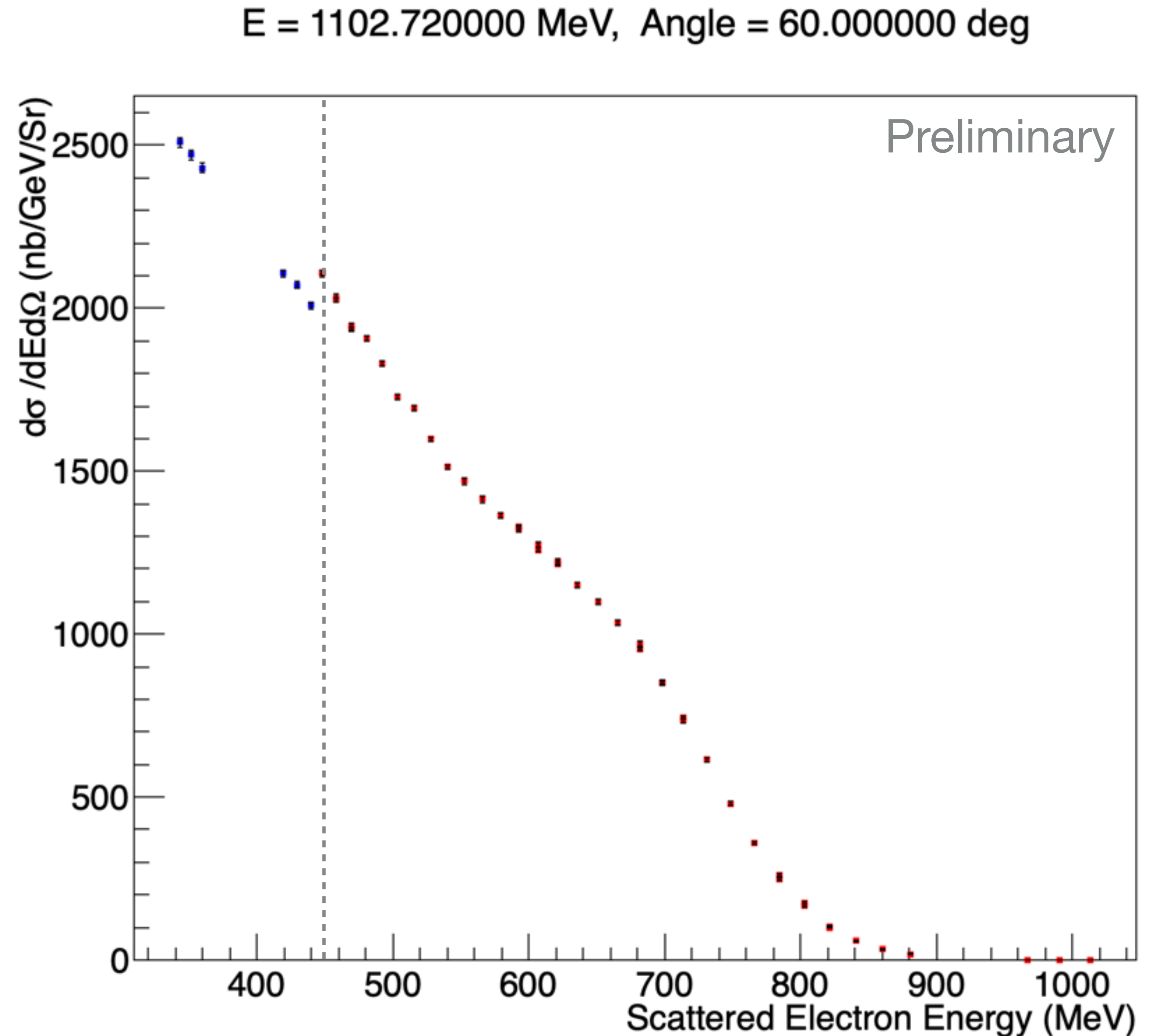
- ▶ Making correction seems to fix jump at some transitions.

$E = 646.295000 \text{ MeV}$ , Angle =  $60.000300 \text{ deg}$



## LOW MOMENTUM MAGNET STUDY

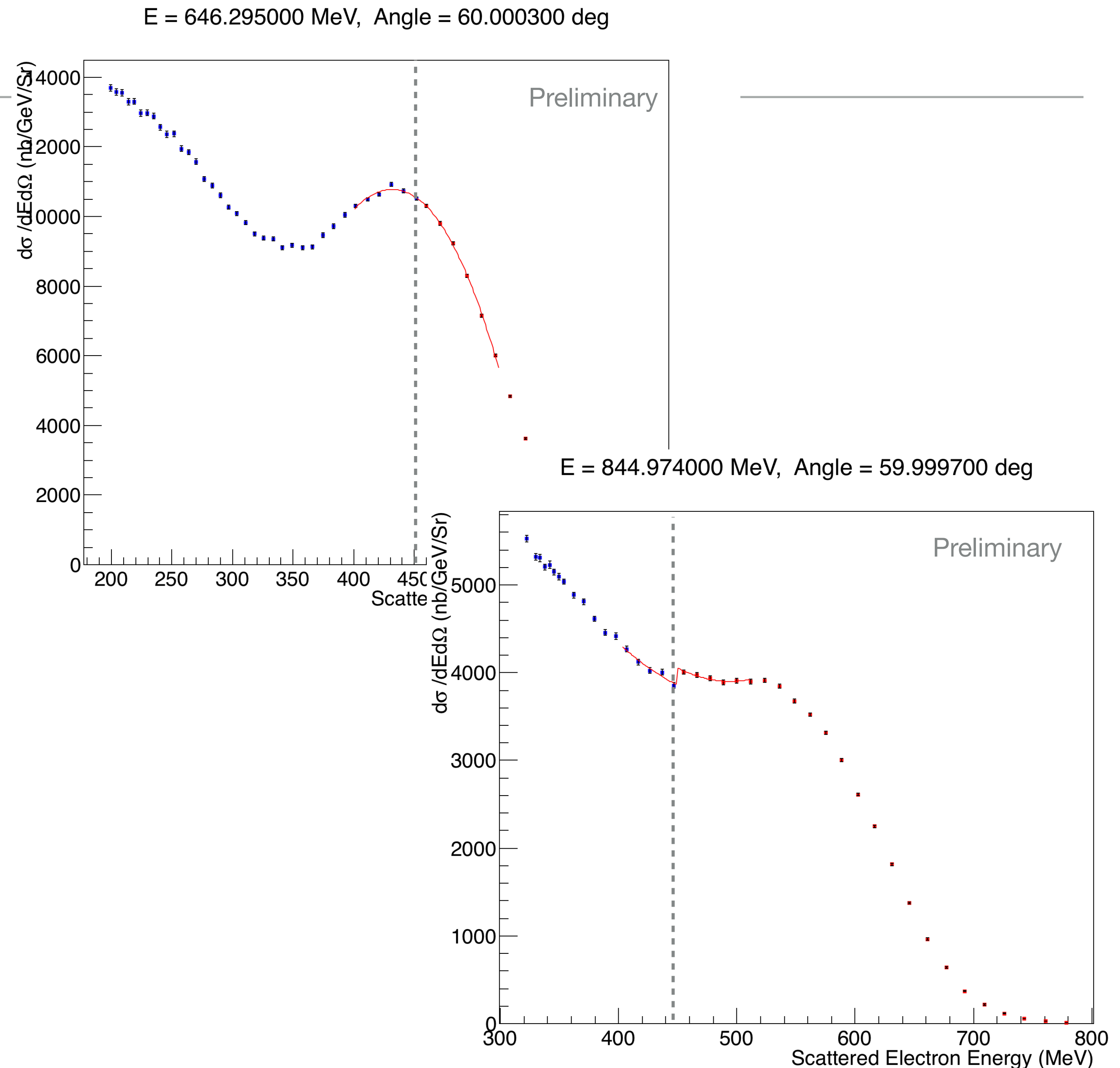
- ▶ Making correction seems to fix jump at some transitions.
- ▶ We still see some discontinuity with other spectra:



# THE COULOMB SUM RULE IN NUCLEI

## LOW MOMENTUM MAGNET STUDY

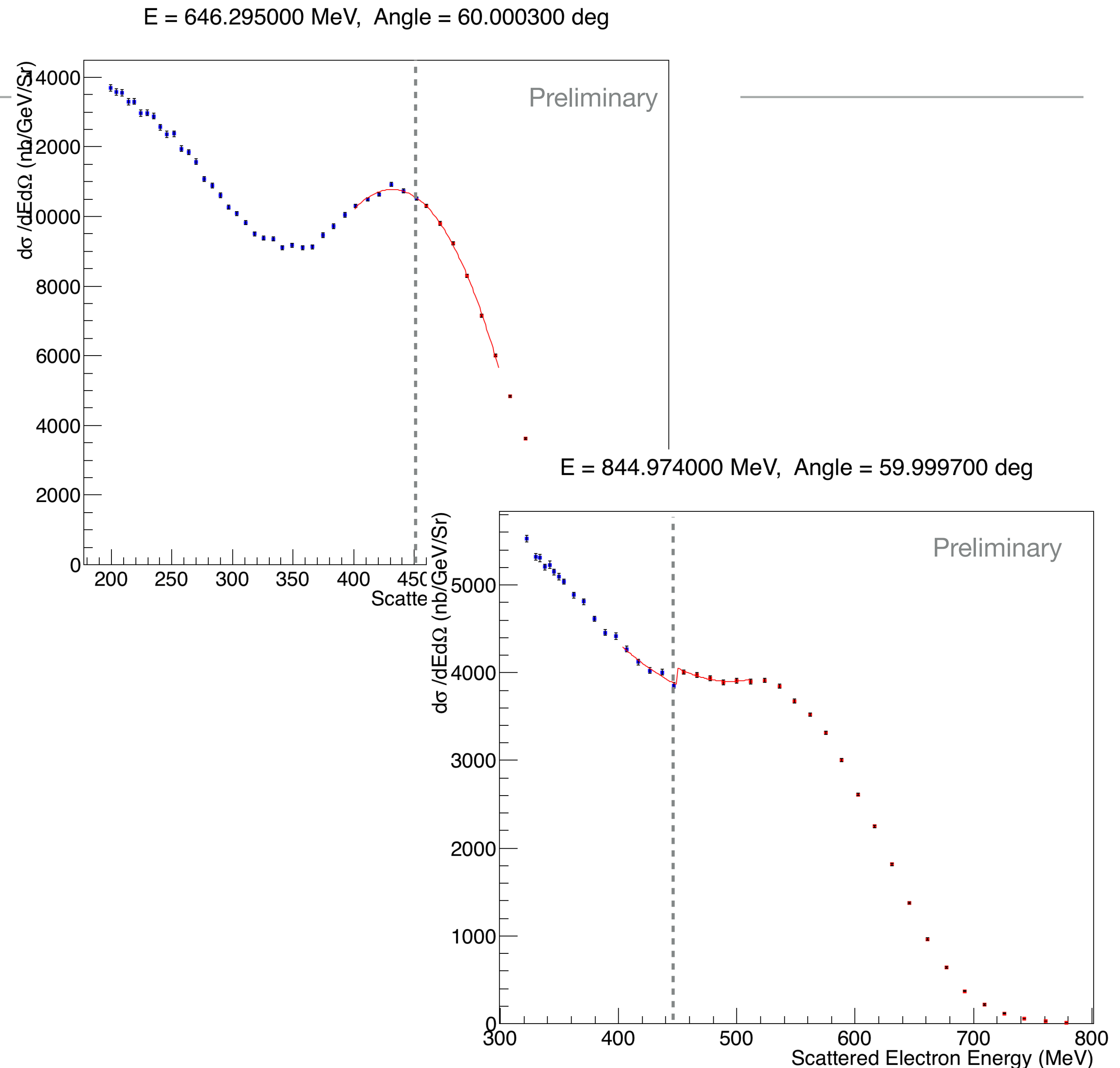
- ▶ Making correction seems to fix jump at some transitions.
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- ▶ We also have  $< 450$  MeV/c optics, which seem to effectively correct some spectra, but not others:



# THE COULOMB SUM RULE IN NUCLEI

## LOW MOMENTUM MAGNET STUDY

- ▶ Making correction seems to fix jump at some transitions.
- ▶ We still see some discontinuity with other spectra:
- ▶ We also have  $< 450$  MeV/c optics, which seem to effectively correct some spectra, but not others:
- ▶ **INVESTIGATION IN PROGRESS**



## SUMMARY / LOOKING AHEAD

### ▶ Recent efforts:

- ▶ Our understanding of the behavior of the response of the HRS at very low central momentum settings has improved.
- ▶ Need to understand all cross-section continuum spectra through the 450 MeV/c point.
- ▶ Need to understand discrepancies in  $< 450$  MeV/c elastic calculations.

### ▶ Looking ahead:

- ▶ We need to understand what absolute normalizations are needed; the goal is to have all cross-sections to  $\sim 1\%$  level.
- ▶ LHRS/ RHRS (redundant measurements) comparisons need to be revisited with latest updates to low momentum HRS response.
- ▶ The Iron and Carbon CSR is very close to completion (expected this year).
  - ▶ Following those, we need to focus on extended target extraction (Helium and Lead in Hydrogen).

## THANK YOU!!!

Kalyan Allada, Korand Aniol, Jon Arrington, **Hamza Atac**, Todd Averett, Herat Bandara, Werner Boeglin, Alexandre Camsonne, Mustafa Canan, **Jian-Ping Chen**, Wei Chen, Khem Chirapatpimol, **Seonho Choi**, Eugene Chudakov, Evaristo Cisbani, Francesco Cusanno, Rafelle De Leo, Chiranjib Dutta, Cesar Fernandez-Ramirez, David Flay, Salvatore Frullani, Haiyan Gao, Franco Garibaldi, Ronald Gilman, Oleksandr Glamazdin, Brian Hahn, Ole Hansen, Douglas Higinbotham, Tim Holmstrom, Bitao Hu, Jin Huang, Yan Huang, Florian Itard, Liyang Jiang, Xiaodong Jiang, **Kai Jin**, Hoyoung Kang, Joe Katich, Mina Katramatou, Aidan Kelleher, Elena Khrosinkova, Gerfried Kumbartzki, John LeRose, Xiaomei Li, Richard Lindgren, Nilanga Liyanage, Joaquin Lopez Herraiz, Lagamba Luigi, Alexandre Lukhanin, Michael Paolone, Maria Martinez Perez, Dustin McNulty, **Zein-Eddine Meziani**, Robert Michaels, Miha Mihovilovic, Joseph Morgenstern, Blaine Norum, **Yomin Oh**, Michael Olson, Makis Petratos, Milan Potokar, Xin Qian, **Yi Qiang, Arun Saha, Brad Sawatzky, Elaine Schulte**, Mitra Shabestari, Simon Sirca, Patricia Solvignon, Jeongseog Song, **Nikolaos Sparveris, Ramesh Subedi, Vincent Sulkosky**, Jose Udias, Javier Vignote, Eric Voutier, Youcai Wang, John Watson, Yunxiu Ye, Xinhui Yan, **Huan Yao**, Zhihong Ye, Xiaohui Zhan, Yi Zhang, Xiaochao Zheng, Lingyan Zhu

PEOPLE

and  
**Hall-A collaboration**

**PhD Students**

**Spokespersons**

**Run Coordinators**