Large transverse momentum in semi-inclusive deeply inelastic scattering

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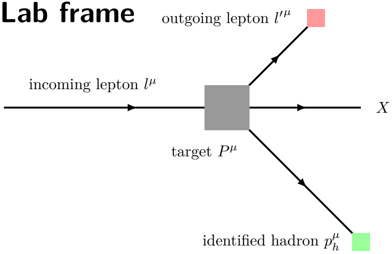
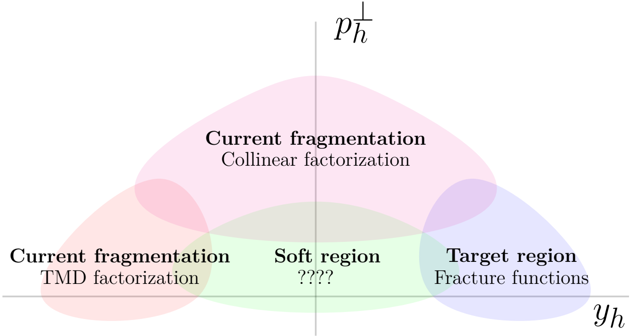
ODU/JLab

Hall C Winter Collaboration Meeting
JLab, 2019

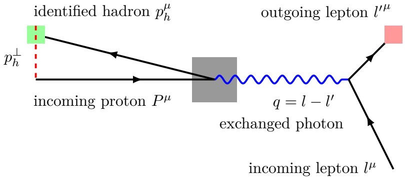
Theory overview

Lab frame





Breit frame



- **Key question** : How is $p_h^{\perp}$ generated at short distances?
- **Different regions** are sensitive to distinct physical mechanisms

Nucleon structure from SIDIS

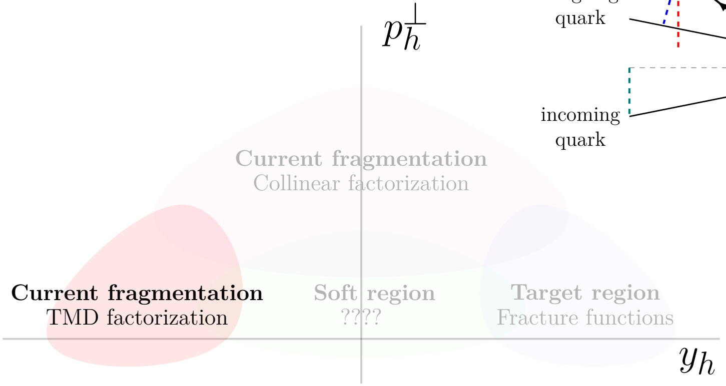
F_i	Standard label	β_i
F_1	$F_{UU,T}$	1
F_2	$F_{UU,L}$	ε
F_3	F_{LL}	$S_{\|\|}\lambda_e\sqrt{1-\varepsilon^2}$
F_4	$F_{UT}^{\sin(\phi_h+\phi_S)}$	$\|\vec{S}_\perp\|\varepsilon\sin(\phi_h+\phi_S)$
F_5	$F_{UT,T}^{\sin(\phi_h-\phi_S)}$	$\|\vec{S}_\perp\|\sin(\phi_h-\phi_S)$
F_6	$F_{UT,L}^{\sin(\phi_h-\phi_S)}$	$\|\vec{S}_\perp\|\varepsilon\sin(\phi_h-\phi_S)$
F_7	$F_{UU}^{\cos 2\phi_h}$	$\varepsilon\ \cos(2\phi_h)$
F_8	$F_{UT}^{\sin(3\phi_h-\psi_S)}$	$\|\vec{S}_\perp\|\varepsilon\sin(3\phi_h-\phi_S)$
F_9	$F_{LT}^{\cos(\phi_h-\phi_S)}$	$\|\vec{S}_\perp\|\lambda_e\sqrt{1-\varepsilon^2}\cos(\phi_h-\phi_S)$
F_{10}	$F_{UL}^{\sin\ 2\phi_h}$	$S_{\|\|}\varepsilon\sin(2\phi_h)$
F_{11}	$F_{LT}^{\cos\phi_S}$	$\|\vec{S}_\perp\|\lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_S$
F_{12}	$F_{LL}^{\cos\phi_h}$	$S_{\|\|}\lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\cos\phi_h$
F_{13}	$F_{LT}^{\cos(2\phi_h-\phi_S)}$	$\|\vec{S}_\perp\|\lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\cos(2\phi_h-\phi_S)$
F_{14}	$F_{UL}^{\sin\phi_h}$	$S_{\|\|}\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_h$
F_{15}	$F_{LU}^{\sin\phi_h}$	$\lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\sin\phi_h$
F_{16}	$F_{UU}^{\cos\phi_h}$	$\sqrt{2\varepsilon(1+\varepsilon)}\ \cos\phi_h$
F_{17}	$F_{UT}^{\sin\phi_S}$	$\|\vec{S}_\perp\|\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_S$
F_{18}	$F_{UT}^{\sin(2\phi_h-\phi_S)}$	$\|\vec{S}_\perp\|\sqrt{2\varepsilon(1+\varepsilon)}\sin(2\phi_h-\phi_S)$

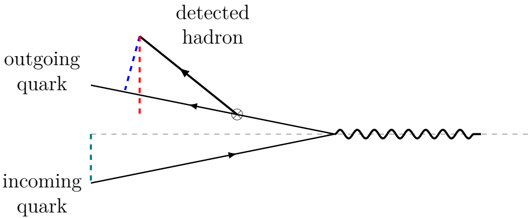
$$\frac{d\sigma}{dx\ dy\ d\Psi\ dz\ d\phi_h\ dP_{hT}^2} \sim \sum_{i=1}^{18} F_i(x,z,Q^2,P_{hT}^2)\beta_i$$

Name	Symbol	meaning
upol. PDF	f_1^q	U. pol. quarks in U. pol. nucleon
pol. PDF	g_1^q	L. pol. quarks in L. pol. nucleon
Transversity	h_1^q	T. pol. quarks in T. pol. nucleon
Sivers	$f_{1T}^{\perp(1)q}$	U. pol. quarks in T. pol. nucleon
Boer-Mulders	$h_1^{\perp(1)q}$	T. pol. quarks in U. pol. nucleon
Boer-Mulders	$h_1^{\perp(1)q}$	T. pol. quarks in U. pol. nucleon
⋮	⋮	⋮
FF	D_1^q	U. pol. quarks to U. pol. hadron
Collins	$H_1^{\perp(1)q}$	T. pol. quarks to U. pol. hadron
⋮	⋮	⋮

The current region in SIDIS

small transverse momentum

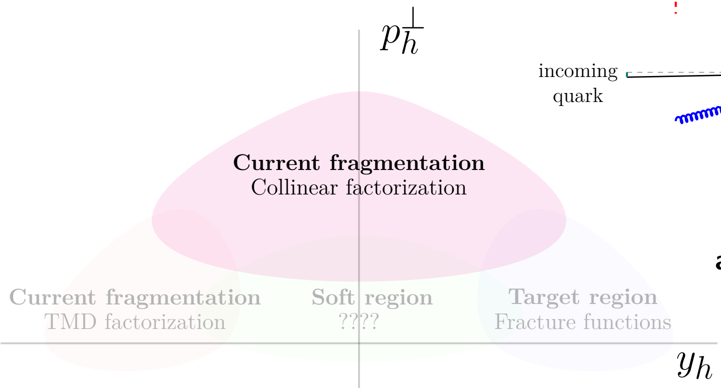


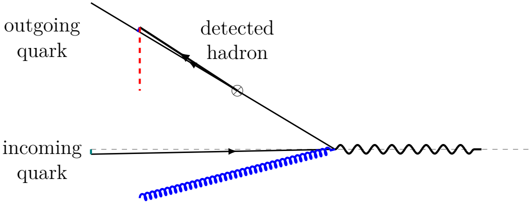


aka W

The current region in SIDIS

large transverse momentum

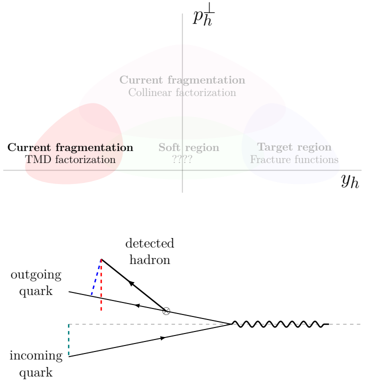




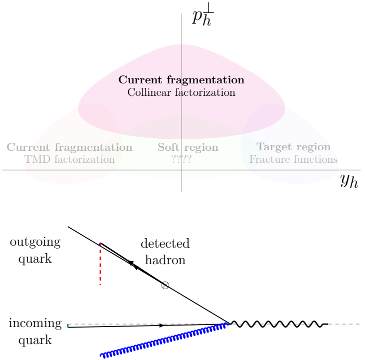
aka FO **(=fixed order)**

The current region in SIDIS

small transverse momentum W

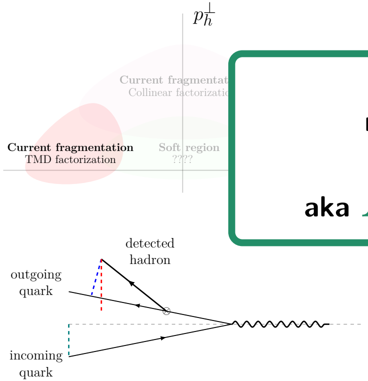


large transverse momentum FO

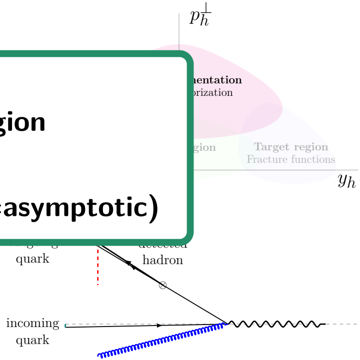


The current region in SIDIS

small transverse momentum W



large transverse momentum FO



matching region

aka ASY **(=asymptotic)**

The current region in SIDIS

- The formulation is based on a scale separation governed by the ratio

$$q_{\mathrm{T}}/Q$$

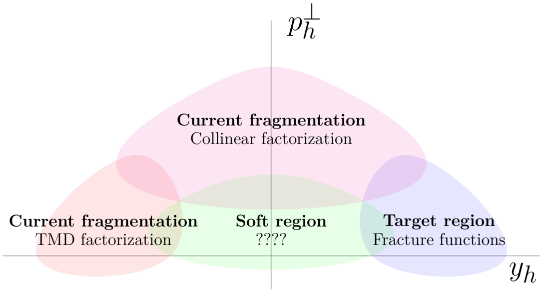
$$z = \frac{P \cdot p_h}{P \cdot q}, \qquad q_{\mathrm{T}} = p_h^{\perp}/z$$

- The cross section is built as

$$\frac{d\sigma}{dx dQ^2 dz dp_h^{\perp}} = \mathrm{W} + \mathrm{FO} - \mathrm{ASY} + \mathcal{O}(m^2/Q^2)$$

$$\sim \mathrm{W} \quad \text{for } q_{\mathrm{T}} \ll Q$$

$$\sim \mathrm{FO} \quad \text{for } q_{\mathrm{T}} \sim Q$$



Small transverse momentum Collins, Rogers PRD91 (2015)

$$\mathrm{W} = \sum_f H_f(Q,\mu) \int \frac{d^2\boldsymbol{b}_\mathrm{T}}{(2\pi)^2} e^{-i\boldsymbol{q}_\mathrm{T}\cdot\boldsymbol{b}_\mathrm{T}} F_{f/N}(x,\boldsymbol{b}_\mathrm{T},\mu,\zeta_F) D_{h/f}(z,\boldsymbol{b}_\mathrm{T},\mu,\zeta_D) + O(q_\mathrm{T}^2/Q^2)$$

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- **CSS evolution equation**

$$\frac{\partial \ln F_{f/N}(x,\boldsymbol{b}_\mathrm{T},\zeta_F,\mu)}{\partial \ln \sqrt{\zeta_F}} = \tilde{K}(\boldsymbol{b}_\mathrm{T},\mu)$$

+ Related to vacuum matrix elements of products of Wilson Lines
+ Independent of flavor, target and spin
+ Independent of x
+ Universal across TMDs and processes

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- **RG equations**

$$\frac{d\tilde{K}(\boldsymbol{b}_\mathrm{T},\mu)}{d\ln\mu} = -\gamma_K(\alpha_S(\mu))$$

$$\frac{d\ln F_{f/N}(\boldsymbol{b}_\mathrm{T},\mu)}{d\ln\mu} = \gamma_f(\alpha_S(\mu),1) - \frac{1}{2}\gamma_K(\alpha_S(\mu))\ln\frac{\zeta_F}{\mu^2}$$

$$\frac{d}{d\ln\mu}\ln H(Q,\mu) = -2\gamma_f(\alpha_S(\mu),1) + \gamma_K(\alpha_S(\mu))\ln\frac{Q^2}{\mu^2}$$

Small transverse momentum Collins, Rogers PRD91 (2015)

Valid for $0 \leq q_{\rm T} \ll Q$

$$
\mathrm{W} = \sum_f H_f(Q,\mu) \int \frac{d^2\boldsymbol{b}_{\rm T}}{(2\pi)^2} e^{-i\boldsymbol{q}_{\rm T}\cdot\boldsymbol{b}_{\rm T}}
$$

$$
\times\, e^{-g_{f/N}(x,b_{\rm T},b_{\max})} \int_x^1 \frac{d\hat{x}}{\hat{x}} \boldsymbol{f}_{f/N}(\hat{x},\mu_{b_*}) \tilde{C}_{f/p}(x/\hat{x}, b_*, \mu_{b_*}^2, \alpha_S(\mu_{b_*}))
$$

$$
\times\, e^{-g_{h/f}(z,b_{\rm T},b_{\max})} \int_z^1 \frac{d\hat{z}}{\hat{z}^3} \boldsymbol{d}_{h/f}(\hat{z},\mu_{b_*}) \tilde{C}_{h/f}(z/\hat{z}, b_*, \mu_{b_*}^2, \alpha_S(\mu_{b_*}))
$$

$$
\times \left(\frac{Q^2}{Q_0^2}\right)^{-g_K(b_{\rm T},b_{\max})} \left(\frac{Q^2}{\mu_{b_*}^2}\right)^{\tilde{K}(b_*,\mu_{b_*})}
$$

$$
\times \exp\left[\int_{\mu_{b_*}}^{\mu_Q} \frac{d\mu'}{\mu'} \left[2\gamma(\alpha_S(\mu'),1) - \ln\frac{Q^2}{(\mu')^2}\gamma_K(\alpha_S(\mu'))\right]\right]
$$

Large transverse momentum

Valid for $q_{\mathrm{T}} \sim Q$

$$\mathrm{FO} = \sum_q e_q^2 \int_{\frac{q_{\mathrm{T}}^2}{Q^2}\frac{xz}{1-z}+x}^{1} \frac{d\xi}{\xi - x}\; H(\xi)\; \boldsymbol{f}_q(\xi,\mu)\; \boldsymbol{d}_q(\zeta(\xi),\mu) + O(\alpha_S^2) + O(m^2/q^2)$$

- **Attention**: $\left(\frac{q_{\mathrm{T}}^2}{Q^2}\frac{xz}{1-z} + x\right) < \xi < 1$
- **large q_{T} probes large ξ in PDFs**
- Can be useful in collinear global analysis

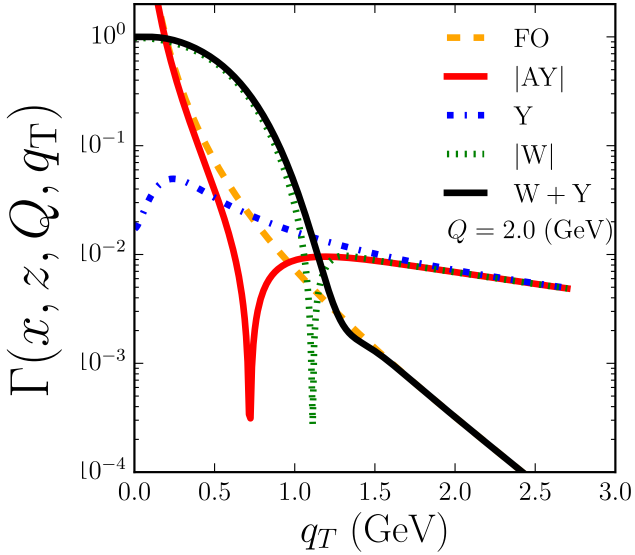
Matching region

Valid for $0 \ll q_{\mathrm{T}} \ll Q$

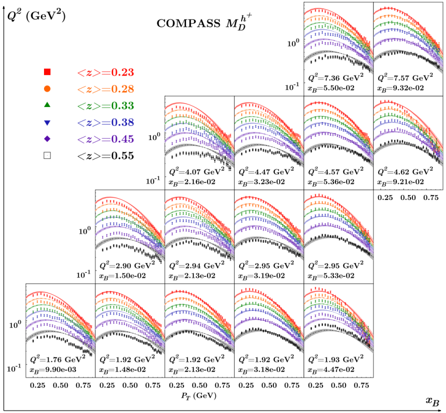
$$\mathrm{ASY} \sim \boldsymbol{d}(z;\mu) \int_x^1 \frac{d\xi}{\xi} \boldsymbol{f}(\xi;\mu) P(x/\xi) + \boldsymbol{f}(x;\mu) \int_z^1 \frac{d\zeta}{\zeta} \boldsymbol{d}(\zeta;\mu) P(z/\zeta)$$
$$+ 2C_F \boldsymbol{f}(x;\mu) \boldsymbol{d}(z;\mu) \left(\ln \left(\frac{Q^2}{q_{\mathrm{T}}} \right) - \frac{3}{2} \right)$$

+ Interpolates between W and FO
+ FO $-$ ASY $\equiv$ Y

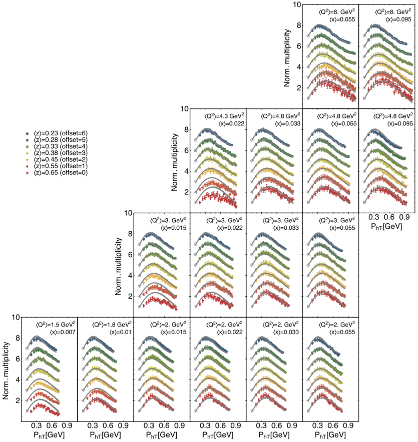
Toy example


FO
|AY|
Y
|W|
W + Y
$Q = 2.0$ (GeV)
$\Gamma(x, z, Q, q_T)$
q_T (GeV)
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0.0
0.5
1.0
1.5
2.0
2.5
3.0

Existing phenomenology



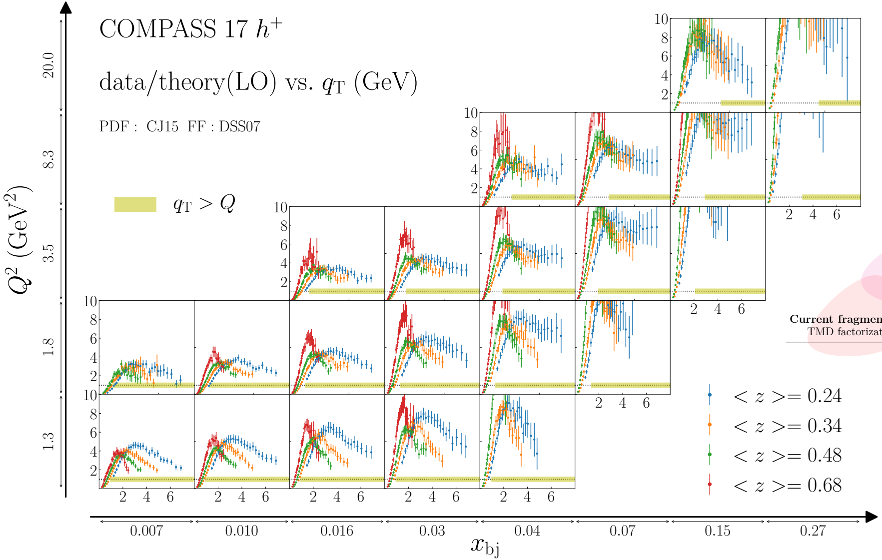
Anselmino et al

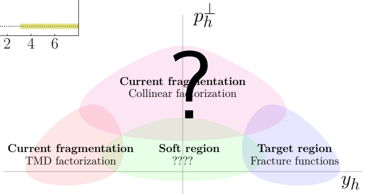


Bacchetta et al

- These analyses used only W (Gaussian, CSS) $\rightarrow$ no FO nor ASY
- Samples with $q_{\mathrm{T}}/Q \sim 1.63$ have been included
- **BUT TMDs are only valid for $q_{\mathrm{T}}/Q \ll 1$!**

FO @ LO predictions (DSS07) Gonzalez, Rogers, NS, Wang PRD98 (2018)



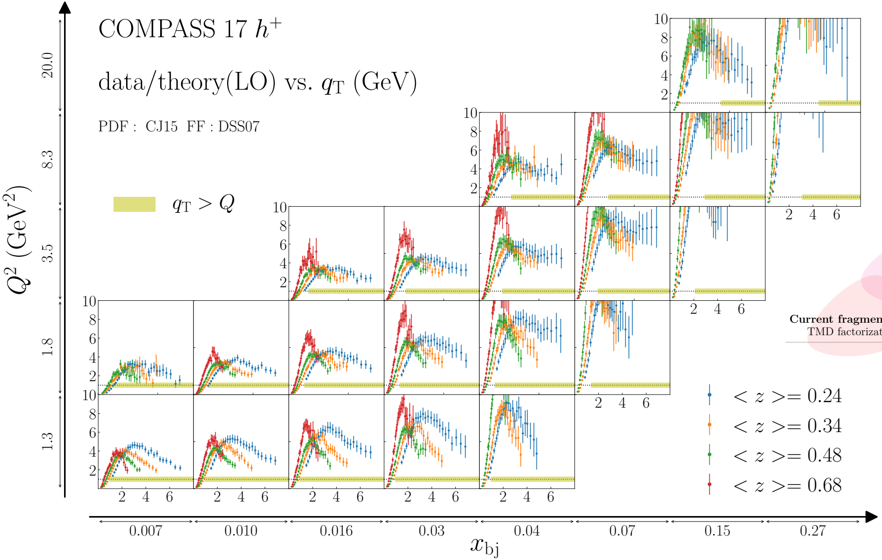


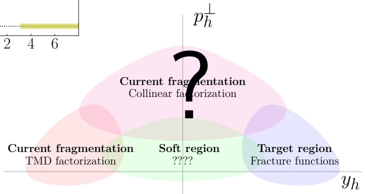
Trouble with large transverse momentum

$$\text{FO} = \sum_q e_q^2 \int_{\frac{q_\mathrm{T}^2}{Q^2}\frac{xz}{1-z}+x}^{1} \frac{d\xi}{\xi - x}\ H(\xi)\ \boldsymbol{f}_q(\xi,\mu)\ \boldsymbol{d}_q(\zeta(\xi),\mu) + O(\alpha_S^2) + O(m^2/q^2)$$

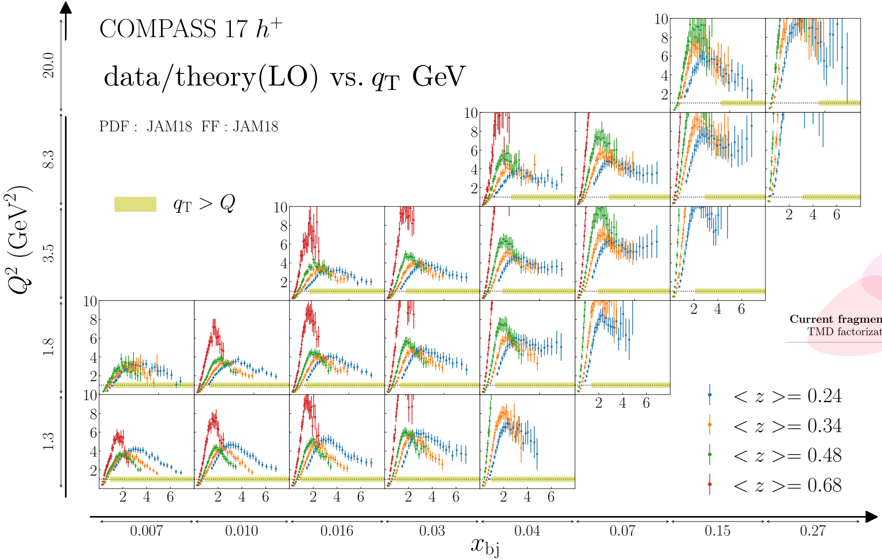
+ FFs needs to be updated?

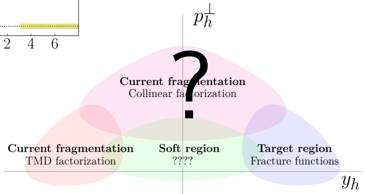
FO @ LO predictions (DSS07) Gonzalez, Rogers, NS, Wang PRD98 (2018)





FO @ LO predictions (JAM18) Gonzalez, Rogers, NS, Wang PRD98 (2018)



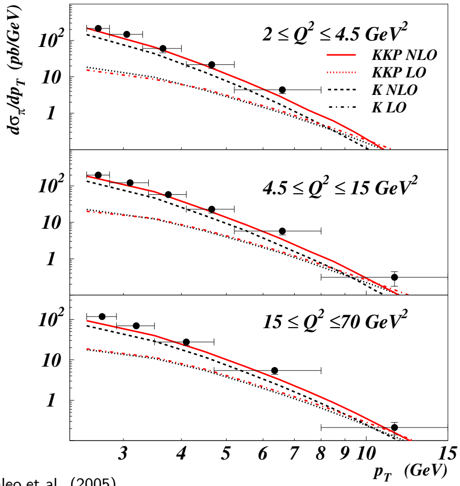


Trouble with large transverse momentum

$$\text{FO} = \sum_q e_q^2 \int_{\frac{q_T^2}{Q^2}\frac{xz}{1-z}+x}^{1} \frac{d\xi}{\xi - x}\, H(\xi)\, \boldsymbol{f}_q(\xi,\mu)\, \boldsymbol{d}_q(\zeta(\xi),\mu) + O(\alpha_S^2) + O(m^2/q^2)$$

+ $O(\alpha_S^2)$ corrections might be important

order α_S^2 corrections to FO



Daleo,et al. (2005)
PRD.71.034013

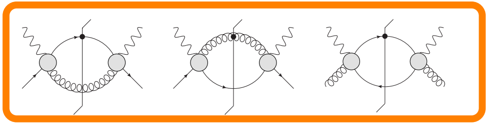
- There are strong indications that order α_S^2 corrections are very important
- An order of magnitude correction at small p_T.
- As a sanity check, we need to have an independent calculation

$O(\alpha_S^2)$ calculation (J. Gonzalez-Hernandes, T.C Rogers, NS, B. Wang - in preparation)

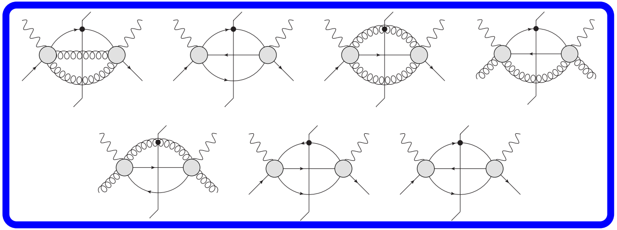
$$W^{\mu\nu}(P,q,P_H) = \int_{x-}^{1+} \frac{d\xi}{\xi} \int_{z-}^{1+} \frac{d\zeta}{\zeta^2} \hat{W}_{ij}^{\mu\nu}(q, x/\xi, z/\zeta) f_{i/P}(\xi) d_{H/j}(\zeta)$$

$$\{\mathrm{P}_g^{\mu\nu} \hat{W}_{\mu\nu}^{(N)}; \mathrm{P}_{PP}^{\mu\nu} \hat{W}_{\mu\nu}^{(N)}\} \equiv \frac{1}{(2\pi)^4} \int \{|M_g^{2\to N}|^2; |M_{pp}^{2\to N}|^2\} \, \mathrm{d}\Pi^{(N)} - \text{Subtractions}$$

Born/Virtual

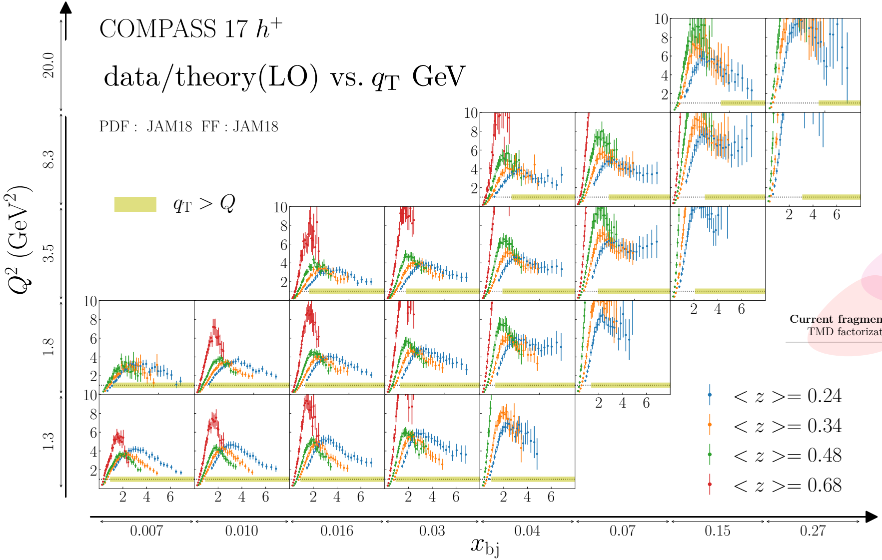


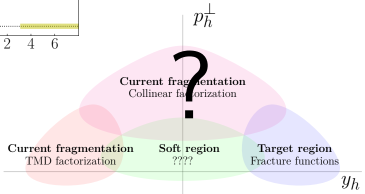
Real



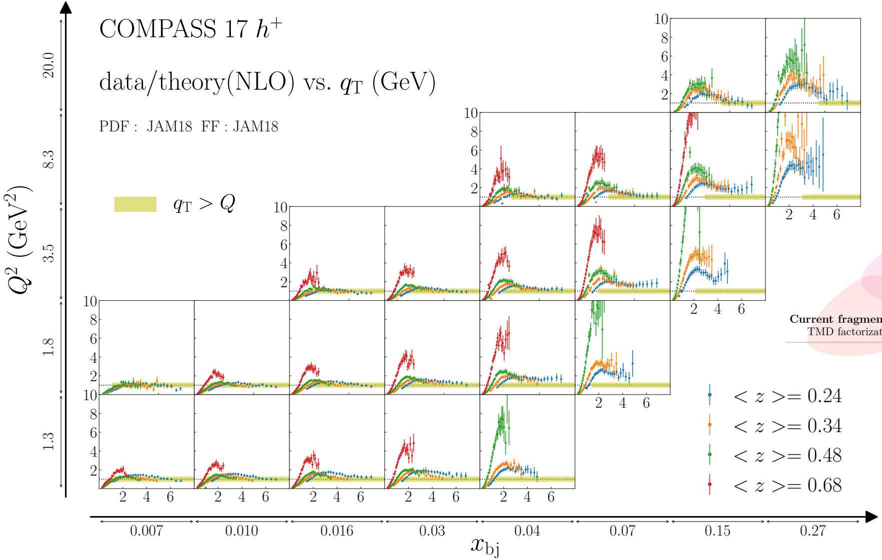
- ✓ Generate all $2 \to 2$ and $2 \to 3$ squared amplitudes
- ✓ Evaluate $2 \to 2$ virtual graphs (Passarino-Veltman)
- ✓ Integrate 3-body PS analytically
- ✓ Check cancellation of IR poles

FO @ LO predictions (JAM18)

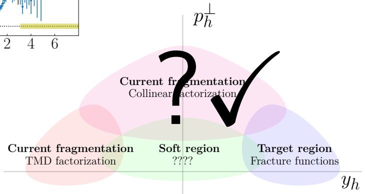




FO @ NLO (JAM18)



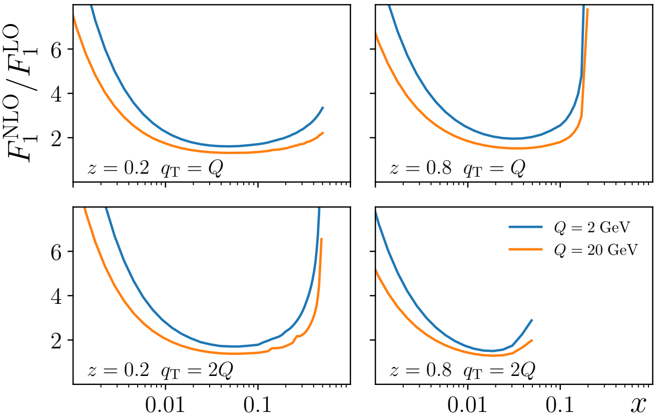
COMPASS 17 h^+
data/theory(NLO) vs. q_T (GeV)
PDF : JAM18 FF : JAM18
$q_T > Q$
Q^2 (GeV^2)
20.0
8.3
3.5
1.8
1.3
x_{bj}
0.007
0.010
0.016
0.03
0.04
0.07
0.15
0.27
$< z >= 0.24$
$< z >= 0.34$
$< z >= 0.48$
$< z >= 0.68$



$p_h^{\perp}$
Current fragmentation
Collinear factorization
Current fragmentation
TMD factorization
Soft region
????
Target region
Fracture functions
y_h

Understanding the large x

(J. Gonzalez-Hernandes, T.C Rogers, NS, B. Wang - in preparation)

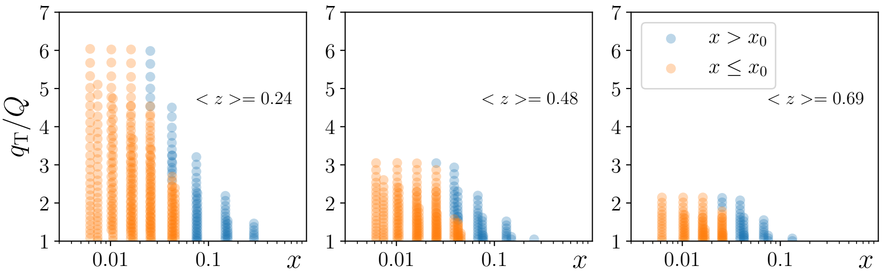


- Large corrections threshold corrections are observed
- The x at the minimum can be used as an indicator of where such corrections are expected to be large

Understanding the large x

(J. Gonzalez-Hernandes, T.C Rogers, NS, B. Wang - in preparation)

COMPASS kinematics



- The blue region might receive large threshold corrections
- This can potential explain why the $O(\alpha_S^2)$ fail to describe the data at large x

Summary and outlook

- SIDIS large q_{T} has a potential impact on collinear PDFs/FFs
- $O(\alpha_S^2)$ corrections are crucial to describe the data
- New global analysis of collinear PDFs/FFs that includes large q_{T} SIDIS is required (in progress @ JAM)
- The threshold region might require resummation techniques