Introducing $\pi_1(1400)$, $\pi_1(1600)$

- $\eta'(\pi)$ system with odd $L$ gives $J^P C = (1, 3, \ldots)^{--}$, exotic ("non-$q\bar{q}$") quantum numbers.
- $\pi_1(1400)$ claimed in the $\eta\pi$ final state (E852, Crystal Barrel)
- $\pi_1(1600)$ claimed in the $\rho\pi$ and $\eta'\pi$ (E852, VES), different from $\pi_1(1400)$.
- COMPASS experiment
  - confirms a peak in $\rho\pi$ and $\eta'\pi$ at $\sim 1.6$ GeV,
  - observes an additional structure at around $\sim 1.4$ GeV.
- Two different states are considered in the PDG...

COMPASS, PL,B740,303('15)
\[ \pi_1: \text{Introduction} \]

- \( J^{PC} = 1^{--} \) discards \( q\bar{q} \)
- \( l = 1 \) discards glueball.
- Molecular interpretation is very difficult.
- Tetraquark? Hybrid?
  - \( \pi_1(1600) \) is consistent with the expected lightest hybrid (1.7 – 1.9 GeV).
  - \( \pi_1(1400) \) could be interpreted as a tetraquark, but this brings more problems than solutions...

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The model

- COMPASS reaction $\pi p \to \eta^{(')} \pi p$ (190 GeV pion beam).
- Peripheral production dominated by Pomeron exchange ($P$).
- Effectively treated as a $P\pi \to \eta^{(')} \pi$ process ($t_1 \approx -0.1$ GeV$^2$).
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A. Rodas et al. (JPAC), 1810.04171

- Production amplitudes parameterized with $N/D$ method:

$$a^J_i(s) = q^{J-1}(s) p^J(s) \sum_k n^J_k(s) \left[ D^{-1}_J(s) \right]_{ki} ,$$

- $\eta^{(r)}\pi$ FSI embedded into $D^J(s)$ matrix:

$$D^J(s) = K^{-1}_J(s) - \frac{s}{\pi} \int ds' \frac{\rho N_J(s')}{s'(s'-s)} ,$$

- $K_J(s)$ matrix standard parameterization:

$$\left[ K^J(s) \right]_{ij} = \sum_R \frac{g^R_i g^R_j}{m^2_R - s} + c^J_{ij} + d^J_{ij} s \ (\ldots)$$
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The fit: results

Our reference model/fit
- Two $K$-matrix poles for $D$-wave (two $a_2$ states?)
- One single $K$-matrix pole for $P$-wave (one $\pi_1$ state?)

- $\chi^2_{d.o.f.} = 1.3$
- $D$-wave amplitudes show two peaks: two $a_2$ states?
- $P$-wave amplitudes show also two peaks, \(\sim 200\) MeV apart.
- They are commonly attributed to two different $\pi_1$ states.
- Important differences between $P$ and $D$ waves:
  - both $a_2$ show up in one channel ($\eta\pi$).
  - $a_2(1320)$ is very narrow.
- $P$ wave is different... How to know?
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Go to the complex plane!
### Spectroscopy

#### Pole Data

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### Conclusion

A single broad $\pi_1$ state ($\sim 1600$ MeV) is able to reproduce both set ($\eta \pi$ and $\eta' \pi$) of data.
**Spectroscopy**

A. Rodas *et al.* (JPAC), 1810.04171

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**P-wave poles**

**Q:** Is it possible to find two poles?

**A:** Yes, but those fits:

1. do not improve the fit quality,
2. the second poles is very unstable/not constrained,
3. requires a peculiar behaviour of the phase above 1.8 GeV.

**Conclusion**

A single broad $\pi_1$ state ($\sim 1600$ MeV) is able to reproduce both set ($\eta \pi$ and $\eta' \pi$) of data.
Correlations (inputs & outputs)

A. Rodas et al. (JPAC), 1810.04171
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The masses and widths of $\pi_1$ and both $a_2$ states are mostly not correlated to the production mechanism parameters.
Impact on GlueX and CLAS12

MA, V. Mathieu et al. (JPAC), *in preparation*

- Moments $H(LM)$ techniques [e.g. Chung, PR,D56(’97)] have been developed and used for spinless/unpolarized beam.

- For polarized beams, the spin-density matrix elements $\rho_{\lambda\lambda'}$ have been used, for instance, for vector meson production [e.g. Schilling, Seyboth, Wolf, NP,B15(’70)].
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For polarized beams, the spin-density matrix elements $\rho_{\lambda\lambda'}$ have been used, for instance, for vector meson production [e.g. Schilling, Seyboth, Wolf, NP,B15(’70)].

- To develop the necessary formalism to extract moments from the photoproduction of two pseudoscalar mesons with a polarized beam.
- Demonstrate with a simple model the usage of this generalization of the formalism.

Start from:

$$
\frac{d\sigma}{d\Omega} = I(\Omega, \Phi) = \sum_{\lambda, \lambda'} \sum_{\lambda_1, \lambda_2} T_{\lambda;\lambda_1,\lambda_2}(\Omega) \rho_{\lambda, \lambda'}(\Phi) T^*_{\lambda';\lambda_1,\lambda_2}(\Omega)
$$

And introduce polarized moments $H^\alpha(LM)$
Polarized moments: summary

- Introduce polarized intensities:
  \[ I(\Omega, \Phi) = I^0(\Omega) + \bar{P}_\gamma(\Phi) \cdot \bar{I}(\Omega) . \]

- Decompose \( I^\alpha(\Omega) \) into moments \( H^\alpha(LM) \),
  \[ I^\alpha(\Omega) = \sum_{L,M} \frac{2L + 1}{4\pi} H^\alpha(LM) D^{L*}_{M0}(\Omega) , \]
  \[ H^\alpha(LM) = \int d\Omega \ I^\alpha(\Omega) \ D^L_{M0}(\Omega) . \]

MA, V. Mathieu et al. (JPAC), in preparation

- Expand \( T_{\lambda;\lambda_1,\lambda_2} \) in \( \eta\pi \) partial waves:
  \[ T_{\lambda;\lambda_1,\lambda_2}(\Omega) = \sum_{\ell,m} T^{\ell m}_{\lambda;\lambda_1,\lambda_2} Y^m_{\ell}(\Omega) , \]

- SDME for arbitrary \( \ell, \ell' \):
  \[ (\rho_\alpha)^{\ell \ell'}_{mm'} = \sum_{\lambda, \lambda', \lambda_1, \lambda_2} T^{\ell m}_{\lambda;\lambda_1,\lambda_2} \frac{\sigma_{\lambda \lambda'}}{2} T^{\ell' m'}_{\lambda';\lambda_1,\lambda_2} \]

- Express \( H^\alpha(LM) \) in terms of SDME:
  \[ H^\alpha(LM) = \sum_{\ell, \ell', m, m'} \left( \frac{2\ell' + 1}{2\ell + 1} \right) \langle \ell'0, L0 | \ell0 \rangle \times \langle \ell' m', LM | \ell m \rangle (\rho_\alpha)^{\ell \ell'}_{mm'} \]
Predictions with a simple model

Simplifying assumptions:
- $\ell_{\text{max}} = 2$ in $\eta\pi$ system ($L_{\text{max}} = 4$ in $H(LM)$),
- $|\lambda - m| \leq 1$,
- Only positive naturality waves are included,
- Three resonances are included: $a_0(980)$, $\pi_1(1600)$, $a_2(1320)$.

\[
[\ell]_m^{(+)} = N_R \left( \delta_R \sqrt{-t} \right)^{|m-1|} \Delta_R(m_{\eta\pi}) P_V(s, t),
\]

\[
\Delta_R(m_{\eta\pi}) = \frac{x_R m_R \Gamma_R}{m_R^2 - m_{\eta\pi}^2 - i \Gamma_R},
\]

\[
P_V(s, t) = \Gamma (1 - \alpha_V(t)) \left( 1 - e^{-i\pi \alpha(t)} \right) s^\alpha(t),
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MA, V. Mathieu et al. (JPAC), in preparation
Predictions with a simple model (II)

Beam asymmetry $\Sigma$:

$$\int d\Omega \ I(\Omega, \Phi) \equiv \sigma^0 (1 + P_\gamma \Sigma \cos(2\Phi))$$

Equivalently,

$$\Sigma = -\frac{H^1(00)}{H^0(00)} = -\frac{\int d(\Omega) \ I^1(\Omega)}{\int d(\Omega) I^0(\Omega)}$$

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M. Albaladejo (JLab): JPAC update November 15, 2018
CLAS Collaboration Meeting
Summary

- We have shown that a single $\pi_1$ state is able to describe the $\eta(\nu)\pi$ lineshape in $\pi p \rightarrow \eta(\nu)\pi p$ data by COMPASS. We find no evidence for two separate states [$\pi_1(1400)$ and $\pi_1(1600)$].
  
  A. Rodas et al. (JPAC), 1810.04171

- We are currently working in the formalism of polarized moments $H^\alpha(LM)$ for the production of $\eta\pi$ with polarized photon beam (GlueX, CLAS).
  
  MA, V. Mathieu et al. (JPAC), in preparation

- We are also working in many other projects: KT equations for $\pi\pi$ FSI and crossing in processes $X \rightarrow 3\pi$, 3 body unitarity, ...

- Suggestions of projects to work in are welcome!