Hadron Masses and Factorization (in DIS)

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Hadronic vs. Partonic Degrees of Freedom

- $Q \approx 1$ GeV, large x.

- Approach kinematical issues in terms of what they reveal about underlying degrees of freedom.

Two Questions

- What are kinematical target mass approximations?

- When/how do they matter?

Target Mass Corrections

- Large number of TMC formalisms:

Brady, Accardi, Hobbs, Melnitchouk PHYSICAL REVIEW D 84, 074008 (2011)

  - OPE based
  - Feynman graph based
  - Higher twist
  - Standard factorization (Aivazis, Olness, Tung)

Standard setup

- Definition of a cross section

$$\mathrm{d}\sigma = \frac{|M^{e,P\to N}|^2}{2\lambda(s, m_e^2, M^2)^{1/2}} \frac{\mathrm{d}^3\mathbf{p}_1}{(2\pi)^3 2E_1} \frac{\mathrm{d}^3\mathbf{p}_2}{(2\pi)^3 2E_2} \cdots \times \frac{\mathrm{d}^3\mathbf{p}_N}{(2\pi)^3 2E_N} (2\pi)^4 \delta^{(4)} \left(P + l - \sum_{i=1}^{N} p_i \right)$$

$$E' \frac{\mathrm{d}\sigma}{\mathrm{d}^3\mathbf{l}'} = \frac{2\,\alpha_{\mathrm{em}}^2}{(s - M^2)\, Q^4}\; L_{\mu\nu} W^{\mu\nu}$$

Single photon exchange

$$W^{\mu\nu} = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) F_1 + \frac{(P^\mu - q^\mu P\cdot q/q^2)(P^\nu - q^\nu P\cdot q/q^2)}{P\cdot q} F_2$$

Massless Target Approximation (MTA)

- Exact:

$$P = \left(\sqrt{M^2 + P_z^2}, 0, 0, P_z\right) = \left(P^+, \frac{M^2}{2P^+}, \mathbf{0}_\mathrm{T}\right)$$

- The approximation:

$$P \to \tilde{P} = (P_z, 0, 0, P_z) = \left(P^+, 0, \mathbf{0}_\mathrm{T}\right)$$

$$2P \cdot q \to 2\tilde{P} \cdot q \qquad M^2/Q^2 \to 0$$

- Usually taken for granted at large Q and small x

MTA in Light-Cone Fractions

- Light-cone ratios:
  - No MTA: $$-\frac{q^+}{P^+} = x_{\rm N} \equiv \frac{2x_{\rm Bj}}{1+\sqrt{1+\frac{4x_{\rm Bj}^2 M^2}{Q^2}}}$$
  - MTA: $$-\frac{q^+}{P^+} = x_{\rm Bj} + O\left(\frac{x_{\rm Bj}^2 M^2}{Q^2}\right)$$

Structure Functions

$$W^{\mu\nu} = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) F_1\left(x_{\mathrm{N}}, Q\right) + \left(P^\mu - \frac{P \cdot q}{q^2} q^\mu\right)\left(P^\nu - \frac{P \cdot q}{q^2} q^\nu\right) \frac{F_2\left(x_{\mathrm{N}}, Q\right)}{P \cdot q}$$

Structure Functions

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$$F_1(x_{\mathrm{N}}, Q^2) \equiv \mathrm{P}_1(x_{\mathrm{N}}, Q^2, M^2)^{\mu\nu} W_{\mu\nu}$$

$$F_2(x_{\mathrm{N}}, Q^2) \equiv \mathrm{P}_2(x_{\mathrm{N}}, Q^2, M^2)^{\mu\nu} W_{\mu\nu}$$

$$\mathrm{P}_1(x_{\mathrm{N}}, Q^2, M^2)^{\mu\nu} \equiv -\frac{1}{2} g^{\mu\nu} + \frac{2Q^2 x_{\mathrm{N}}^2}{(M^2 x_{\mathrm{N}}^2 + Q^2)^2} P^\mu P^\nu$$

$$\mathrm{P}_2(x_{\mathrm{N}}, Q^2, M^2)^{\mu\nu} \equiv \frac{12 Q^4 x_{\mathrm{N}}^3\left(Q^2 - M^2 x_{\mathrm{N}}^2\right)}{\left(Q^2 + M^2 x_{\mathrm{N}}^2\right)^4}\left(P^\mu P^\nu - \frac{\left(M^2 x_{\mathrm{N}}^2 + Q^2\right)^2}{12 Q^2 x_{\mathrm{N}}^2} g^{\mu\nu}\right)$$

MTA

$$2P \cdot q \to 2\tilde{P} \cdot q \qquad M^2/Q^2 \to 0$$

$$F_1(x_{\mathrm{Bj}}, Q^2) \equiv \mathrm{P}_1(x_{\mathrm{Bj}}, Q^2, 0)^{\mu\nu} W_{\mu\nu}$$

$$F_2(x_{\mathrm{Bj}}, Q^2) \equiv \mathrm{P}_2(x_{\mathrm{Bj}}, Q^2, 0)^{\mu\nu} W_{\mu\nu}$$

$$W^{\mu\nu} \to \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) F_1(x_{\mathrm{Bj}}, Q^2) + \frac{(\tilde{P}^\mu - q^\mu \tilde{P} \cdot q/q^2)(\tilde{P}^\nu - q^\nu \tilde{P} \cdot q/q^2)}{\tilde{P} \cdot q} F_2(x_{\mathrm{Bj}}, Q^2)$$

Factorization

- Power expansion

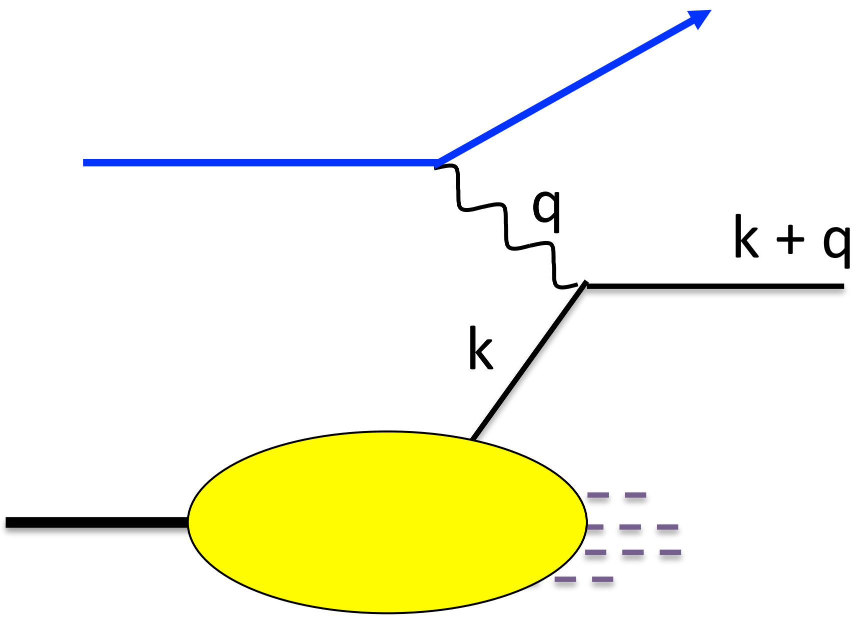
$$\frac{\mathrm{d}\sigma}{\mathrm{d}x_{\mathrm{Bj}}\ \mathrm{d}Q^2} = \int \mathrm{d}\xi\, \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\hat{x}_{\mathrm{Bj}}\ \mathrm{d}Q^2} f(\xi) + O\left(\frac{m^2}{Q^2}\right)$$

- m^2 = parton virtuality, transverse momentum, mass…

- What about hadron masses?

For now assume $M^2 \neq O(m^2)$

Factorization and partonic light-cone fractions



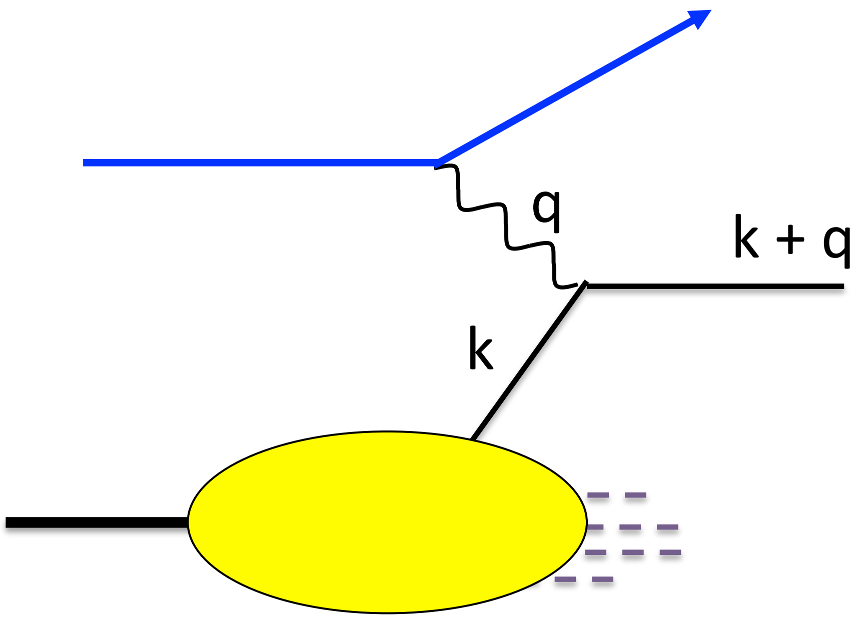
$$q = \left(-x_{\mathrm{N}} P^{+}, \frac{Q^2}{2 x_{\mathrm{N}} P^{+}}, \mathbf{0}_{\mathrm{T}} \right)$$

$$k^{+} = O\left(Q\right)$$

$$k^{2} = O\left(m^{2}\right)$$

$$(k + q)^{2} = O\left(m^{2}\right)$$

Factorization and partonic light-cone fractions



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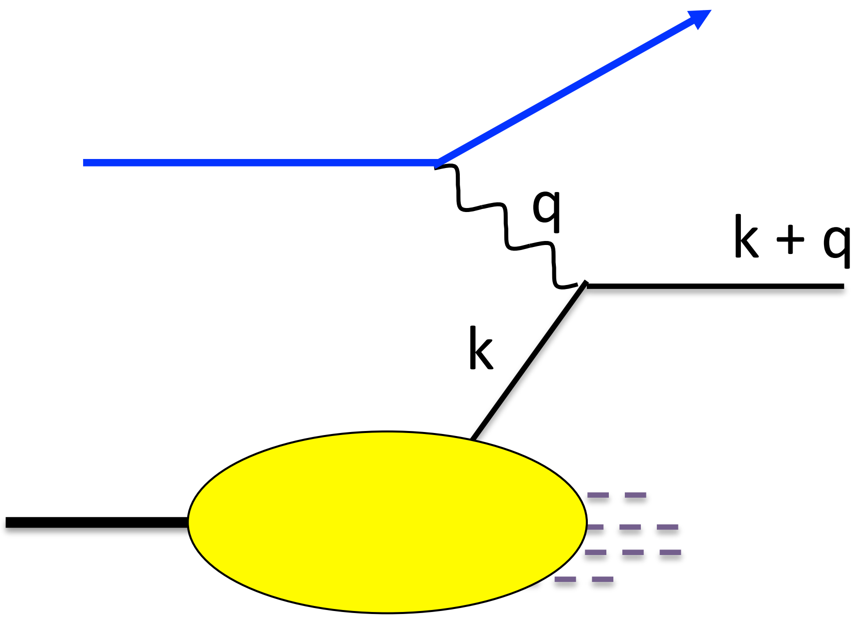
$$(k+q)^2 = O\left(m^2\right)$$

$$2k^+q^- + 2k^-q^+ - Q^2 + k^2 = O\left(m^2\right)$$

$$2k^+q^- = Q^2 + O\left(m^2\right)$$

Factorization and partonic light-cone fractions



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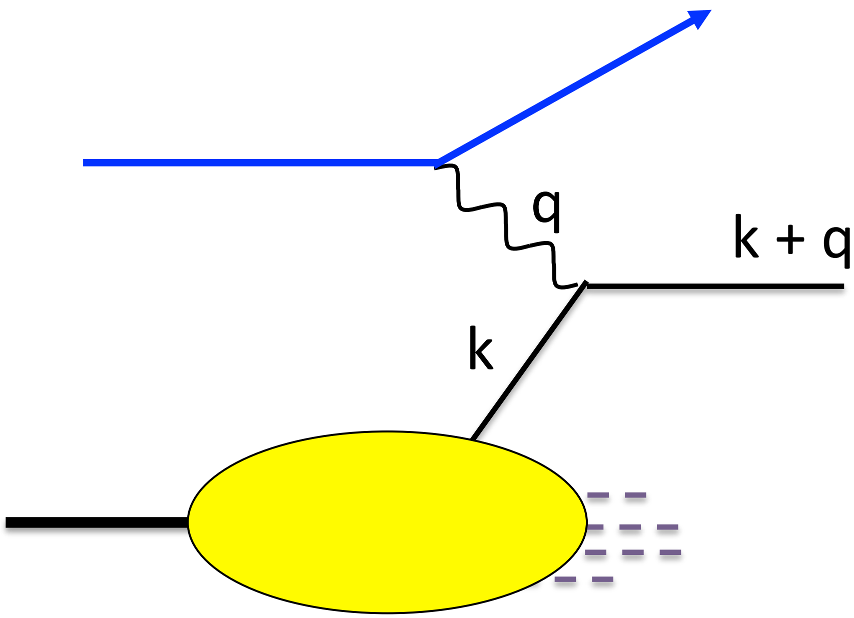
$$2k^+ q^- + 2k^- q^+ - Q^2 + k^2 = O\left(m^2\right)$$

$$2k^+ q^- = Q^2 + O\left(m^2\right)$$

$$\Rightarrow \quad \xi \equiv \frac{k^+}{P^+} = x_{\mathrm{N}} + O\left(\frac{m^2}{Q^2}\right)$$

Factorization and partonic light-cone fractions



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$$2k^+ q^- + 2k^- q^+ - Q^2 + k^2 = O\left(m^2\right)$$

$$2k^+ q^- = Q^2 + O\left(m^2\right)$$

$$\xi \equiv \frac{k^+}{P^+} = x_{\rm N} + O\left(\frac{m^2}{Q^2}\right)$$

$$= x_{\rm Bj} + O\left(\frac{x_{\rm Bj}^2 M^2}{Q^2}\right) + O\left(\frac{m^2}{Q^2}\right)$$

Factorization Power Series

- Drop $O(m^2/Q^2)$?: Necessary for factorization.

- Drop $O(x^2_{Bj} M^2/Q^2)$?: Not necessary for factorization.

MTA with factorization

- Make approximations with exact target momentum:

$$W^{\mu\nu} \to W^{\mu\nu}_{\text{fact}}$$

Introduce $O(m^2/Q^2)$ errors

- Then do MTA:

$$W^{\mu\nu}_{\text{fact}} \to W^{\mu\nu}_{\text{fact,TMC}}$$

Introduce $O(x_{Bj}{}^2M^2/Q^2)$ errors

Aivazis, Olness, Tung (AOT)

Phys. Rev. D 50, 3085 (1994)

- Normal factorization, just keeping exact mass.

  - MTA

$$W^{\mu\nu} = \int_{x_{\mathrm{Bj}}}^{1} \frac{\mathrm{d}\xi}{\xi} \hat{W}^{\mu\nu}(x_{\mathrm{Bj}}/\xi, q) f(\xi) + O\left(m^2/Q^2\right) + O\left(M^2/Q^2\right)$$

  - TMC

$$W^{\mu\nu} = \int_{x_{\mathrm{N}}}^{1} \frac{\mathrm{d}\xi}{\xi} \hat{W}^{\mu\nu}(x_{\mathrm{N}}/\xi, q) f(\xi) + O\left(m^2/Q^2\right)$$

- The only “pure” kinematical correction. Others involve assumptions about dynamics.

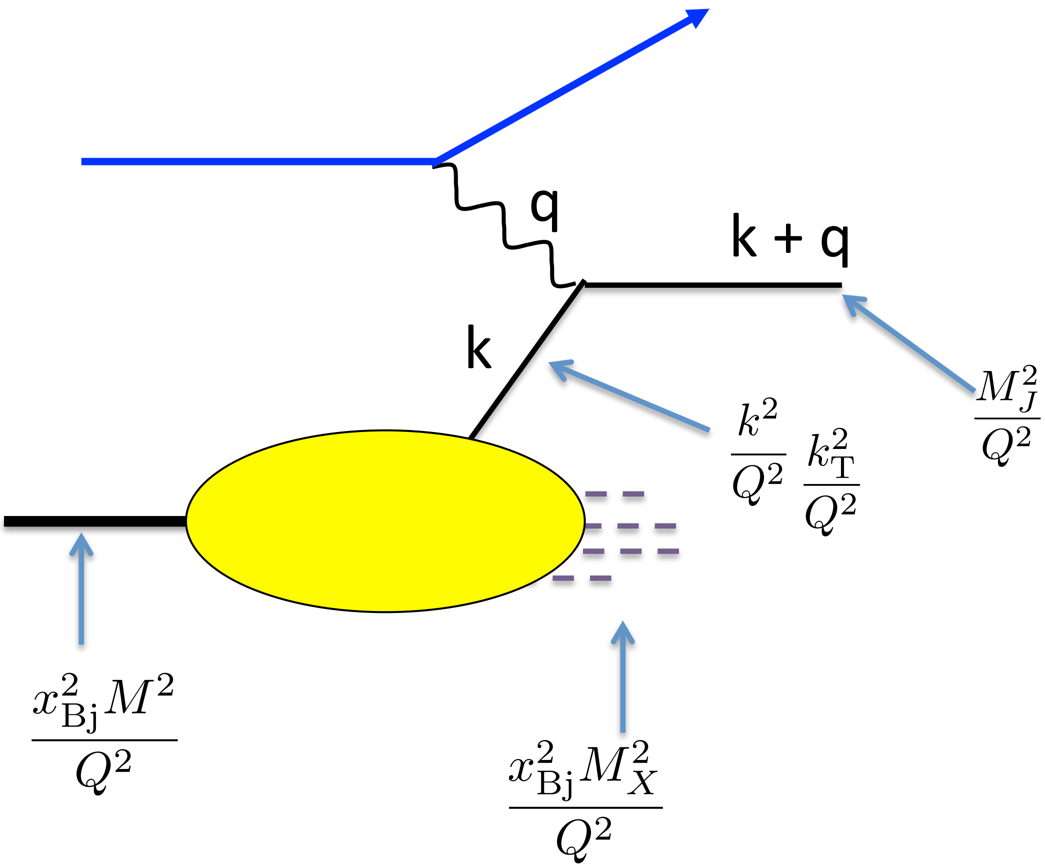
What if the target mass is important?

- How to test?
  - Scaling with Nachtmann rather than Bjorken variable?
  - Improved universality. Extend range of pQCD?

See N. Sato talk

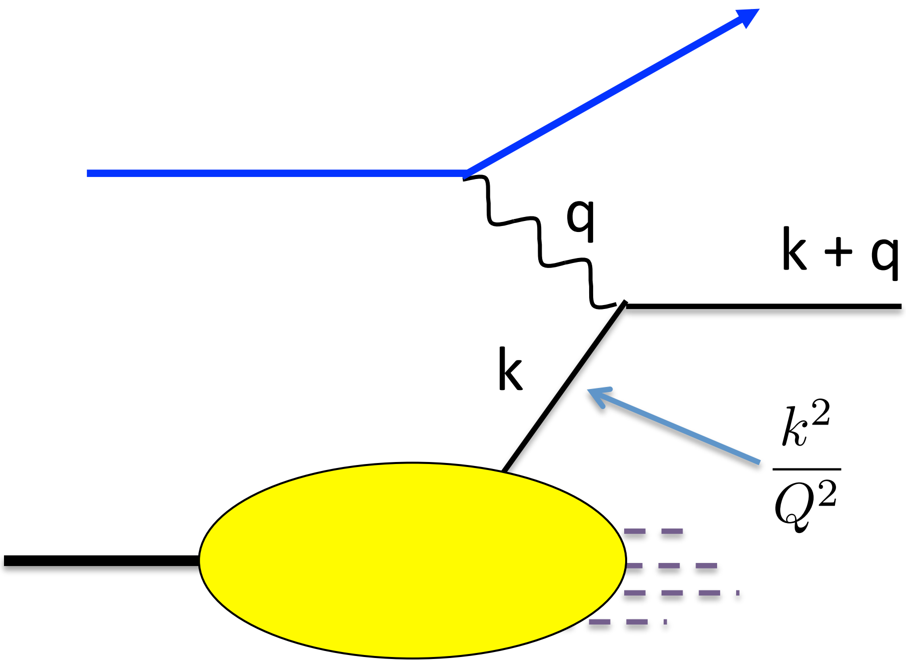
- Why does it give improvement? Something about nucleon structure?

Partonic interpretation of target mass effects



- Small scales
- Exact target mass useful if suppression by *partonic* scales is greater than target mass

Partonic interpretation of target mass effects



- Parton virtuality vs. hadron mass

$$\sim A \frac{x_{\rm Bj}^2 M^2}{Q^2} + B \frac{k^2}{Q^2}$$

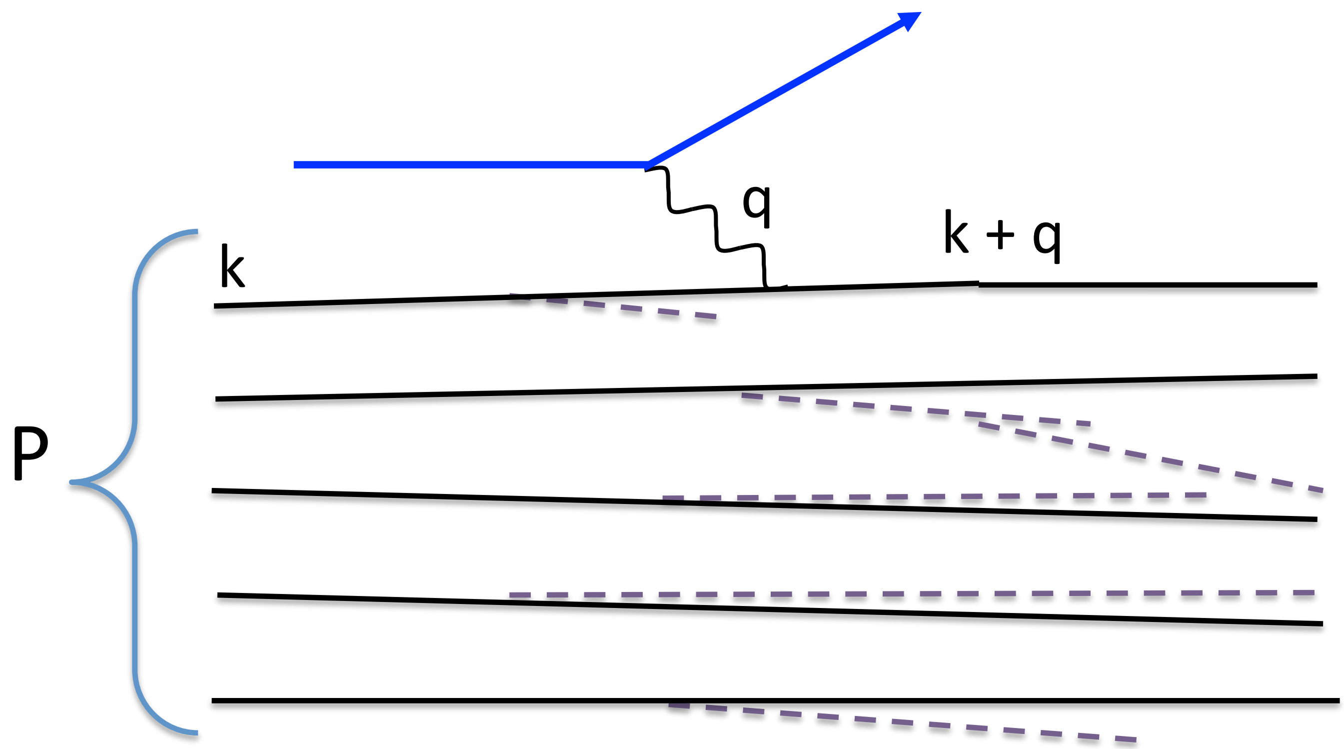
?? *??*

Partonic interpretation of target mass effects

- Two scales?

See E. Moffat talk

$$|k^2| \ll x_{\rm Bj}^2 M^2$$



Operator product expansion versus AOT

- AOT (direct factorization):
  - Direct power expansion in small partonic mass scales
  - Keep exact momentum expressions
- OPE:
  - Transform to Mellin moment space
  - Expand in 1/Q (both twist and target momentum)
  - Truncate twist
    - Identify leading M/Q part
    - Identify remaining series of M/Q
  - Invert leading M/Q ($F_1^{(0)}$)
  - Invert series of M/Q (F_1^{TMC})
  - Relate $F_1^{(0)}$ and F_1^{TMC}

Summary

- Normal factorization derivation naturally leads to x_N as scaling variable/independent variable.

- These are easy to retain (AOT).

- Sensitivity to a target mass might say something about nucleon structure.

  – Compare Proton, Kaon, Pion, Nucleus targets

Backup

Operator product expansion versus AOT

- AOT (direct factorization):
  - Direct power expansion in small partonic mass scales
  - Keep exact momentum expressions
- OPE:
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  - Relate $F_1^{(0)}$ and F_1^{TMC}

$$\rho^2 \equiv 1 + \frac{4x_{\mathrm{Bj}}^2 M^2}{Q^2}$$

OPE-based

- Georgi-Politzer (1976)

$$F_1^{\mathrm{TMC}}(x_{\mathrm{Bj}}, Q^2) = \frac{1+\rho}{2\rho} F_1^{(0)}(x_{\mathrm{N}}, Q^2) + \frac{\rho^2 - 1}{4\rho^2} \int_{x_{\mathrm{N}}}^{1} \frac{\mathrm{d}u}{u^2} F_1^{(0)}(u, Q^2) + \frac{(\rho^2-1)^2}{8x_{\mathrm{Bj}}\rho^3} \int_{x_{\mathrm{N}}}^{1} \mathrm{d}u \int_{u}^{1} \frac{\mathrm{d}v}{v^2} F_1^{(0)}(v, Q^2)$$

- What is $F_{1,2}^{(0)}$?

As exact structure function

- Power series:

$$F_2 = \frac{2}{x_{\rm Bj}} \sum_{l=0}^{\infty} \frac{1}{x_{\rm Bj}^{2l}} \sum_{j=0}^{\infty} \left(\frac{M^2}{Q^2} \right)^j N_{j,l} C^{2l+2j} A_{2j+2l+2}$$

Drop Higher Twist

As exact structure function

- Power series:

$$F_2 = \frac{2}{x_{\rm Bj}} \sum_{l=0}^{\infty} \frac{1}{x_{\rm Bj}^{2l}} \sum_{j=0}^{\infty} \left(\frac{M^2}{Q^2}\right)^j N_{j,l} C^{2l+2j} A_{2j+2l+2}$$

- Integer Mellin moments:

$$\int_0^1 x_{\rm Bj}^{n-2} F_2 = \sum_{j=0}^{\infty} \left(\frac{M^2}{Q^2}\right)^j \bar{N}_{n,j} C^{n+2j} A_{2j+n}$$

Series in α_s
OPE provides info about *integer values*

As exact structure function

- Power series:

$$F_2 = \frac{2}{x_{\mathrm{Bj}}} \sum_{l=0}^{\infty} \frac{1}{x_{\mathrm{Bj}}^{2l}} \sum_{j=0}^{\infty} \left(\frac{M^2}{Q^2}\right)^j N_{j,l} C^{2l+2j} A_{2j+2l+2}$$

- Integer Mellin moments:

$$\int_0^1 x_{\mathrm{Bj}}^{n-2} F_2 = \sum_{j=0}^{\infty} \left(\frac{M^2}{Q^2}\right)^j \bar{N}_{n,j} C^{n+2j} A_{2j+n}$$

- Leading twist, zero mass

$$\int_0^1 \mathrm{d}y\, y^{n-2} F_2^{(0)}(y, Q^2) \equiv C^n A_n$$

Extend to non-integer values

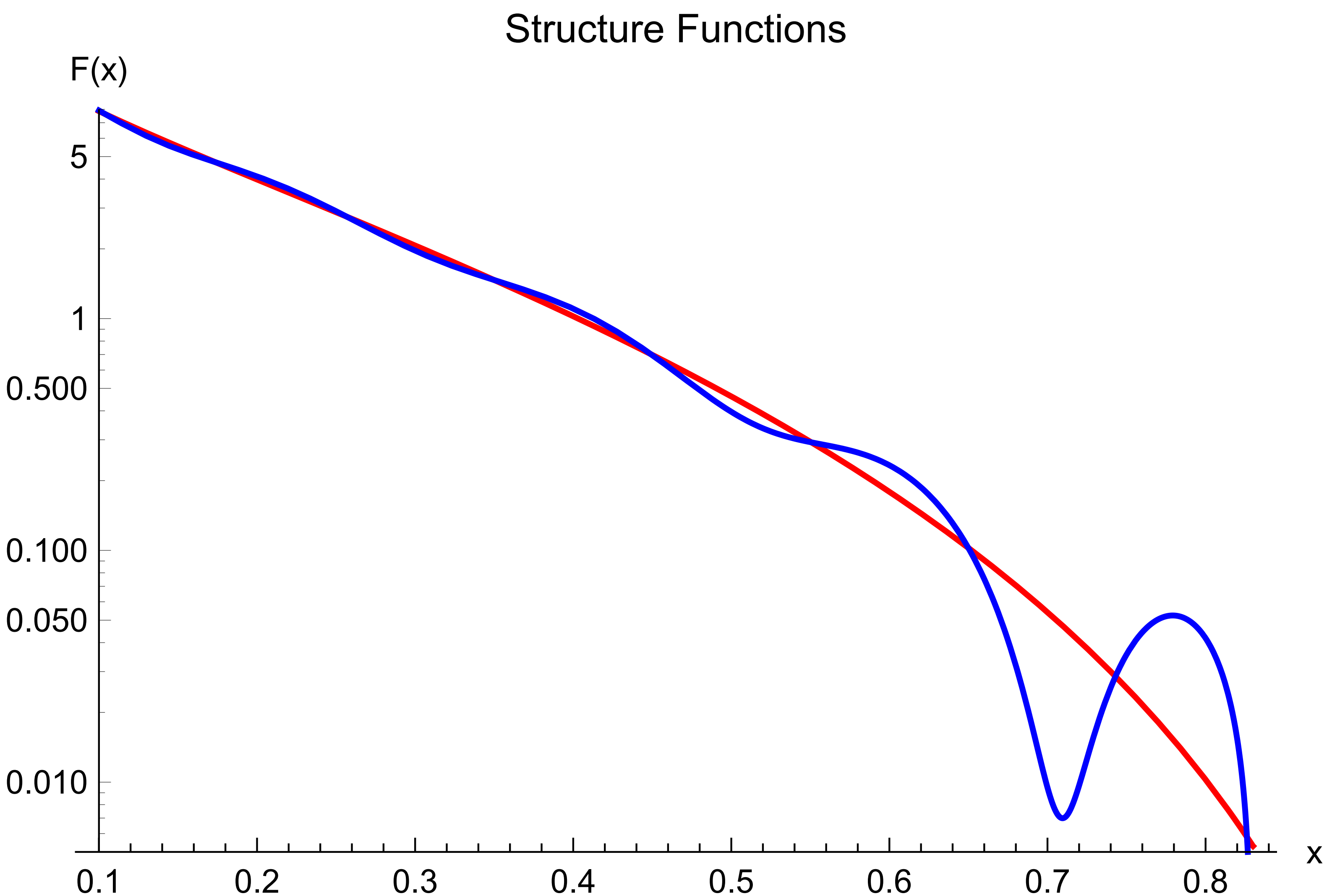
As exact structure function

- Invert:

$$F_2 = \frac{1}{2\pi i} \int_{-i\infty}^{i\infty} \mathrm{d}n \, x_{\mathrm{Bj}}^{1-n} \left(\frac{M^2}{Q^2} \right)^j \bar{N}_{n,j} C^{n+2j} A_{2j+n}$$

$$F_1^{\mathrm{TMC}}(x_{\mathrm{Bj}}, Q^2) = \frac{1+\rho}{2\rho} F_1^{(0)}(x_{\mathrm{N}}, Q^2) + \frac{\rho^2 - 1}{4\rho^2} \int_{x_{\mathrm{N}}}^{1} \frac{\mathrm{d}u}{u^2} F_1^{(0)}(u, Q^2) + \frac{(\rho^2-1)^2}{8 x_{\mathrm{Bj}} \rho^3} \int_{x_{\mathrm{N}}}^{1} \mathrm{d}u \int_u^1 \frac{\mathrm{d}v}{v^2} F_1^{(0)}(v, Q^2)$$

- Functions with equal moments up to N = 13



As fit to phenomenological structure function

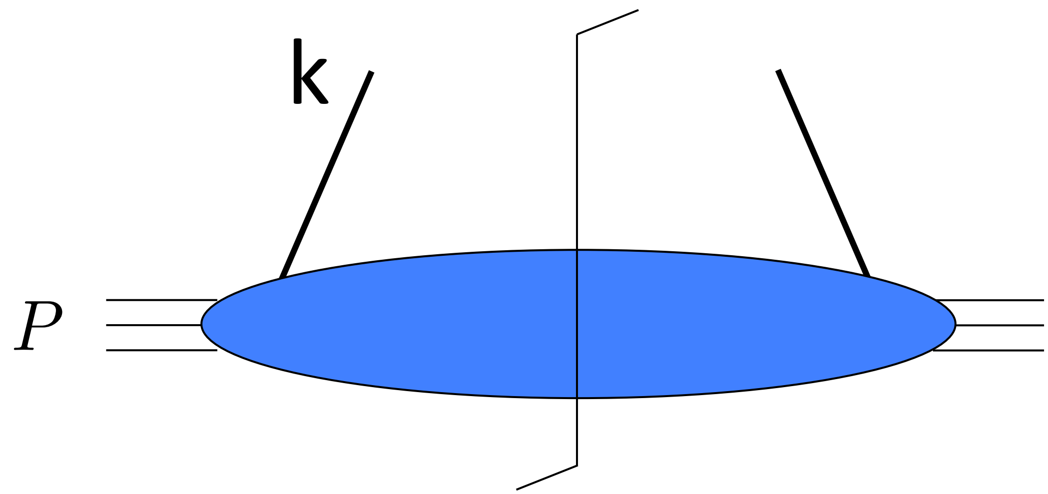
$$F_2 = \sum_j C_{j/i}^{\alpha_s^n}(x/\xi, Q) \otimes f_{i/P}(\xi; Q)$$

- Finite order hard part
- Parametrization of pdf
- Fit needed all the way to x = 1
- Theoretical leading twist ≠ pheno fit near x = 1

As a parton density

- Ellis-Furmanski-Petronzio (Intro - 1982)

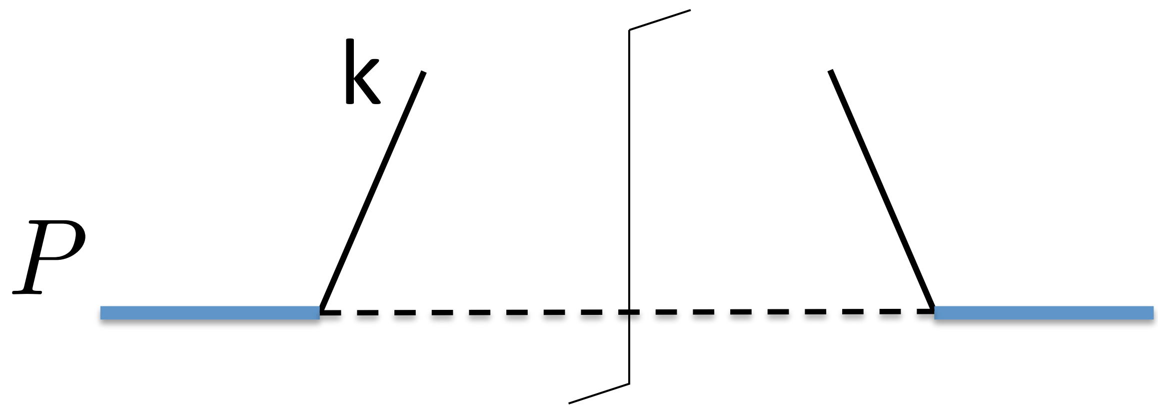
$$\int dk^{-} d^{2}\mathbf{k}_t$$



- Impose $k^2 = 0$ but allow $k_T > 0$

As a parton density

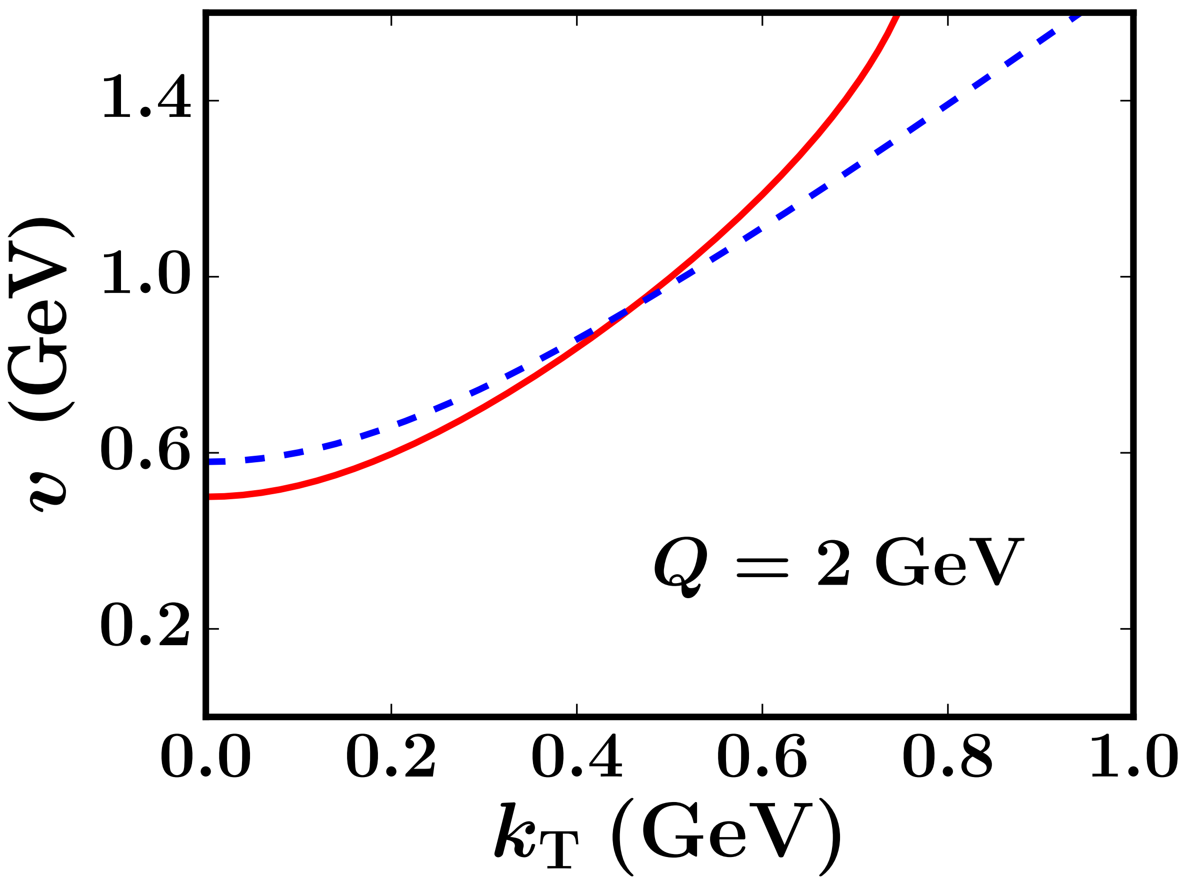
- In low order Yukawa theory



$$\int \frac{dw^-}{(2\pi)}\, e^{-i\xi P^+ w^-}\ \langle P|\,\bar{\psi}_0(0,w^-,\mathbf{0}_t)\frac{\gamma^+}{2}\psi_0(0,0,\mathbf{0}_t)\,|P\rangle$$

As a parton density

- In low order Yukawa theory



Moffat et al., (2017)

- $v = \sqrt{-k^2}$
- Blue = in pdf
- Red = in unapproximated graph