Duality in electron scattering: insights from theory

Wally Melnitchouk

Jefferson Lab

Outline

- What we (think we) understand from theory

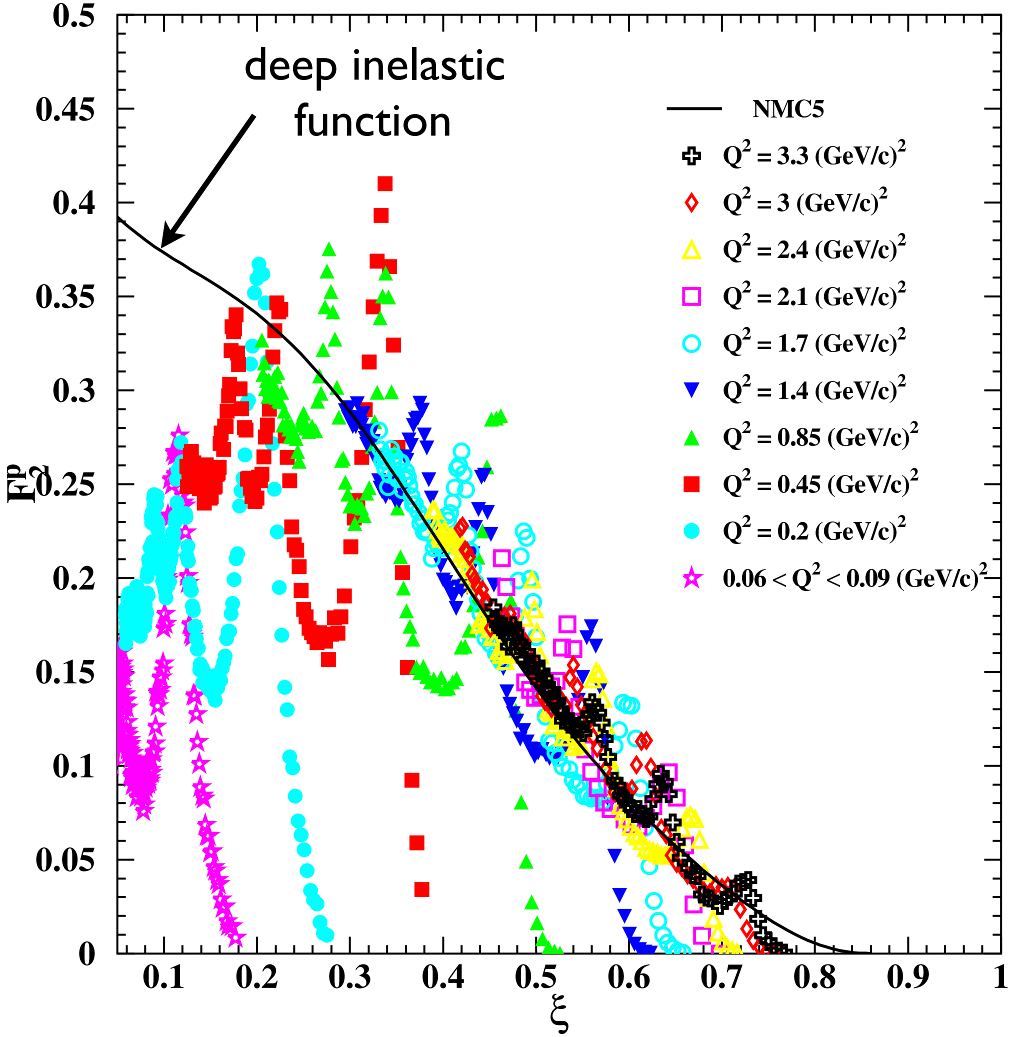
- Duality (global) in QCD

- Local duality

  $\longrightarrow$ insights from models

- What we don’t (yet) know … what do we do next?

Duality in electron-proton scattering



Niculescu et al. (2000)

$$\xi = \frac{2x}{1+\sqrt{1+4M^2x^2/Q^2}}$$

- average over resonances
  (strongly Q^2 dependent)

  $\approx$ Q^2 independent
  scaling function

Duality and QCD

- Operator product expansion in QCD

$\longrightarrow$ expand *moments* of structure functions in powers of $1/Q^2$

$$M_n(Q^2) = \int_0^1 dx\ x^{n-2}\ F_2(x,Q^2)$$

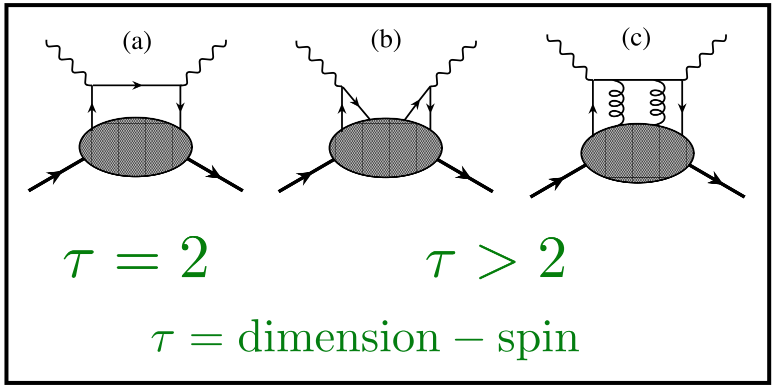
$$= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

matrix elements of operators with specific "twist" τ

$e.g.$ $\langle N|\,\overline{\psi}\,\gamma^+\psi\,|N\rangle$

$\langle N|\,\overline{\psi}\,\widetilde{G}^{+\nu}\gamma_\nu\,\psi\,|N\rangle$

etc.



Duality and QCD

- Operator product expansion in QCD

  $\longrightarrow$ expand *moments* of structure functions in powers of $1/Q^2$

$$M_n(Q^2) = \int_0^1 dx\ x^{n-2}\ F_2(x,Q^2)$$
$$= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

- If moment $\approx$ independent of Q^2

  $\longrightarrow$ "higher twist" terms $A_n^{(\tau>2)}$ small

- Duality $\longleftrightarrow$ suppression of higher twists

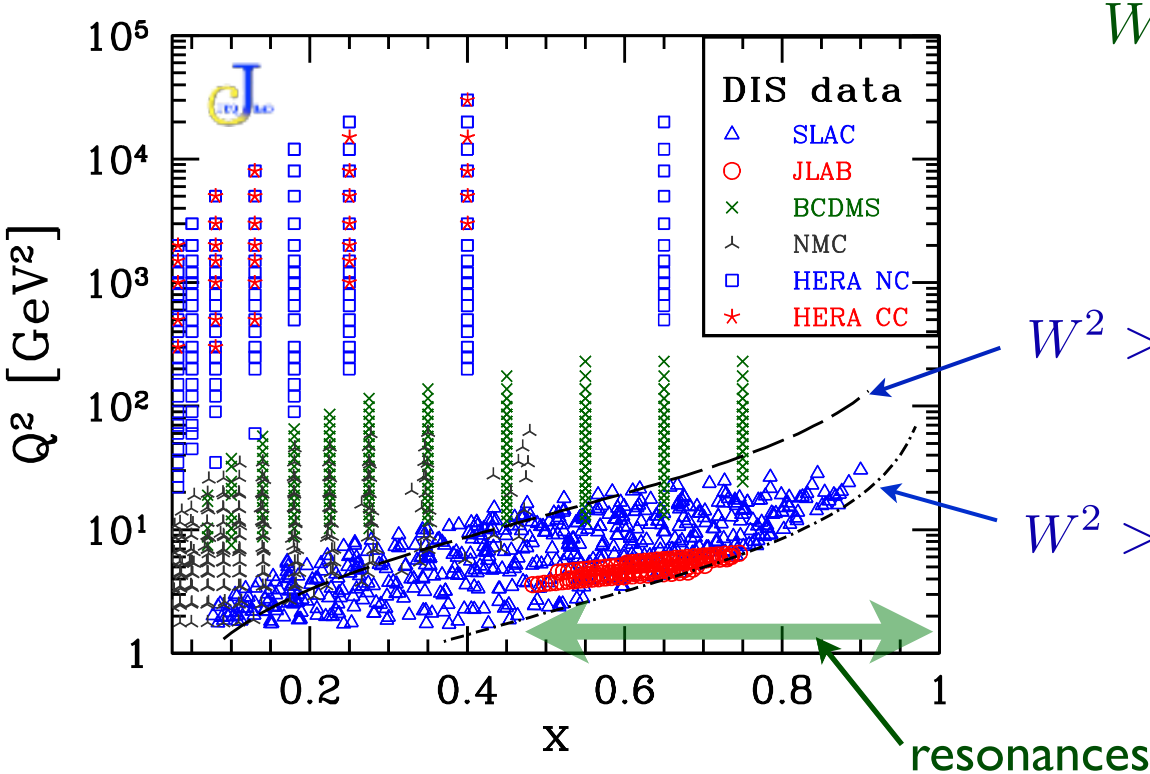
Duality and QCD

- <u>Note</u>: at finite Q^2, from kinematics *any* moment of *any* structure function (of *any* twist) must, by definition, include the resonance region

$$W^2 = M^2 + Q^2 \frac{(1-x)}{x} \quad \Longrightarrow \quad x_{\rm res} = \frac{Q^2}{W_{\rm res}^2 - M^2 + Q^2}$$

$$W_{\rm res} = 2 \text{ GeV} \quad \Longrightarrow \quad x_{\rm res} \approx 0.24$$

at $Q^2 = 1 \text{ GeV}^2$



Duality and QCD

- Note: at finite Q^2, from kinematics *any* moment of *any* structure function (of *any* twist) must, by definition, include the resonance region

- Resonance and DIS regions are intimately connected
$\longrightarrow$ resonances an *integral* part of scaling structure function

  e.g. in large-N_c limit, spectrum of zero-width resonances is "maximally dual" to quark-level (smooth) structure function

Resonances & twists

- Total "higher twist" is *small* at scales $Q^2 \sim \mathcal{O}(1\ \mathrm{GeV}^2)$

- On average, nonperturbative interactions between quarks and gluons not dominant (at these scales)

  $\longrightarrow$ nontrivial interference between resonances?

- Can we understand this dynamically, at quark level?

- Can we use resonance region data to learn about *leading twist* structure functions (and *vice versa*)?

  $\longrightarrow$ expanded data set has potentially significant implications for global quark distribution studies

Scaling functions from resonances

- Earliest attempts predate QCD

→ *e.g.* harmonic oscillator spectrum $M_n^2 = (n+1)\Lambda^2$
including states with spin = 1/2, ..., n+1/2
(n even: I = 1/2, n odd: I = 3/2)

Domokos et al. (1971)

→ at large Q^2 magnetic coupling dominates

$$G_n(Q^2) = \frac{\mu_n}{(1 + Q^2 r^2 / M_n^2)^2} \qquad r^2 \approx 1.41$$

→ in Bjorken limit, $\sum_n \longrightarrow \int dz\,, \quad z \equiv M_n^2/Q^2$

$$F_2 \sim (\omega' - 1)^{1/2} (\mu_{1/2}^2 + \mu_{3/2}^2) \int_0^\infty dz \frac{z^{3/2}(1 + r^2/z)^{-4}}{z + 1 - \omega' + \Gamma_0^2 z^2}$$

→ scaling function of $\omega' = \omega + M^2/Q^2 \qquad (\omega = 1/x)$

Scaling functions from resonances

- Earliest attempts predate QCD

→ *e.g.* harmonic oscillator spectrum $M_n^2 = (n+1)\Lambda^2$
including states with spin = 1/2, ..., n+1/2
(n even: I = 1/2, n odd: I = 3/2)

Domokos et al. (1971)

→ in $\Gamma_n \to 0$ limit

$$F_2 \sim (\mu_{1/2}^2 + \mu_{3/2}^2) \frac{(\omega' - 1)^3}{(\omega' - 1 + r^2)^4}$$

cf. Drell-Yan-West relation

$$G(Q^2) \sim \left(\frac{1}{Q^2}\right)^m \quad \Longleftrightarrow \quad F_2(x) \sim (1-x)^{2m-1}$$

→ similar behavior found in many models

Einhorn (1976) ('t Hooft model)
Greenberg (1993) (NR scalar quarks in HO potential)
Pace, Salme, Lev (1995) (relativistic HO with spin)
Isgur et al. (2001) (transition to scaling)
....

Scaling functions from resonances

- Phenomenological analyses at finite Q^2

$\longrightarrow$ additional constraints from threshold behavior at $\mathbf{q} \to 0$ and asymptotic behavior at $Q^2 \to \infty$

$$\left(1 + \frac{\nu^2}{Q^2}\right) F_2^R = M\nu \ \left[|G_+^R|^2 + 2|G_0^R|^2 + |G_-^R|^2\right] \ \delta(W^2 - M_R^2)$$

Davidovsky, Struminsky (2003)

$\longrightarrow$ 21 isospin-1/2 & 3/2 resonances (with mass < 2 GeV)

$$\left|G_\pm^R(Q^2)\right|^2 = \left|G_\pm^R(0)\right|^2 \left(\frac{|\vec{q}|}{|\vec{q}|_0} \frac{\Lambda'^2}{Q^2 + \Lambda'^2}\right)^{\gamma_1} \left(\frac{\Lambda^2}{Q^2 + \Lambda^2}\right)^{m_\pm} \qquad m_{+,0,-} = 3, 4, 5$$

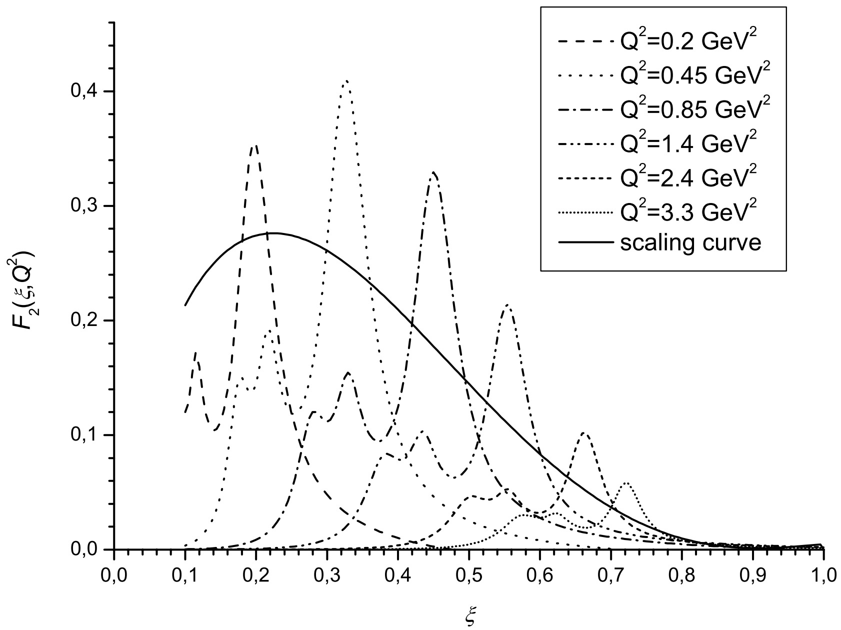
$$\left|G_0^R(Q^2)\right|^2 = C^2 \left(\frac{Q^2}{Q^2 + \Lambda''^2}\right)^{2a} \frac{q_0^2}{|\vec{q}|^2} \left(\frac{|\vec{q}|}{|\vec{q}|_0} \frac{\Lambda'^2}{Q^2 + \Lambda'^2}\right)^{\gamma_2} \left(\frac{\Lambda^2}{Q^2 + \Lambda^2}\right)^{m_0}$$

$\longrightarrow$ in $x \to 1$ limit,

$$F_2(x) \sim (1 - x)^{m_+}$$

Scaling functions from resonances

- Phenomenological analyses at finite Q^2



21 isospin-1/2 & 3/2 resonances (mass < 2 GeV)

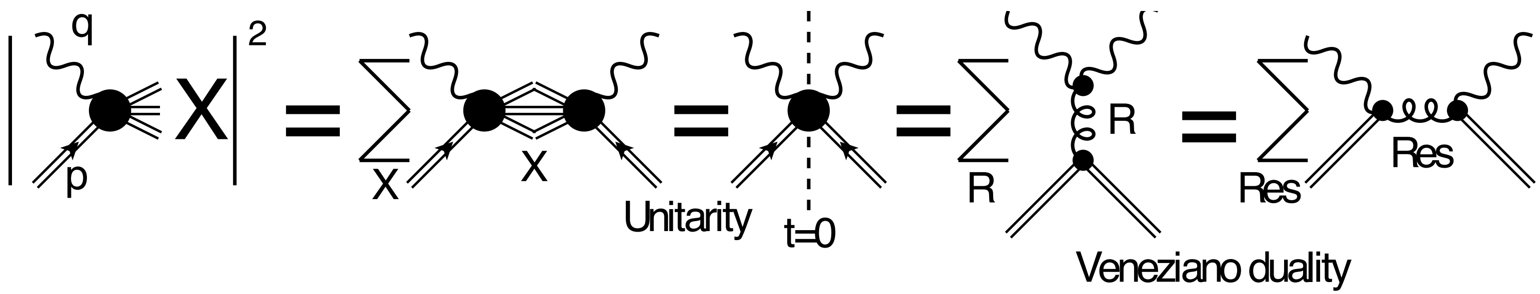
Davidovsky, Struminsky (2003)

$\longrightarrow$ valence-like structure of dual function suggests "two-component duality":

- valence (Reggeon exchange) dual to resonances $F_2^{(\rm val)} \sim x^{0.5}$
- sea (Pomeron exchange) dual to background $F_2^{(\rm sea)} \sim x^{-0.08}$

Scaling functions from resonances

- Explicit realization of Veneziano & Bloom-Gilman duality



$$V(s,t) = \frac{\Gamma(1-\alpha(s))\Gamma(1-\alpha(t))}{\Gamma(2-\alpha(s)-\alpha(t))}$$

$\rightarrow \; s^{\alpha(t)}$ high s, low $|t|$

$\longrightarrow$ Veneziano model not unitary, has no imaginary parts

$\longrightarrow$ generalization of narrow-resonance approximation, with nonlinear, complex Regge trajectories

$$D(s,t) = \int_0^1 dz \left(\frac{z}{g}\right)^{-\alpha_s(s(1-z))-1} \left(\frac{1-z}{g}\right)^{-\alpha_t(tz)-1}$$

"dual amplitude with Mandelstam analyticity" (DAMA) model

Jenkovszky et al.

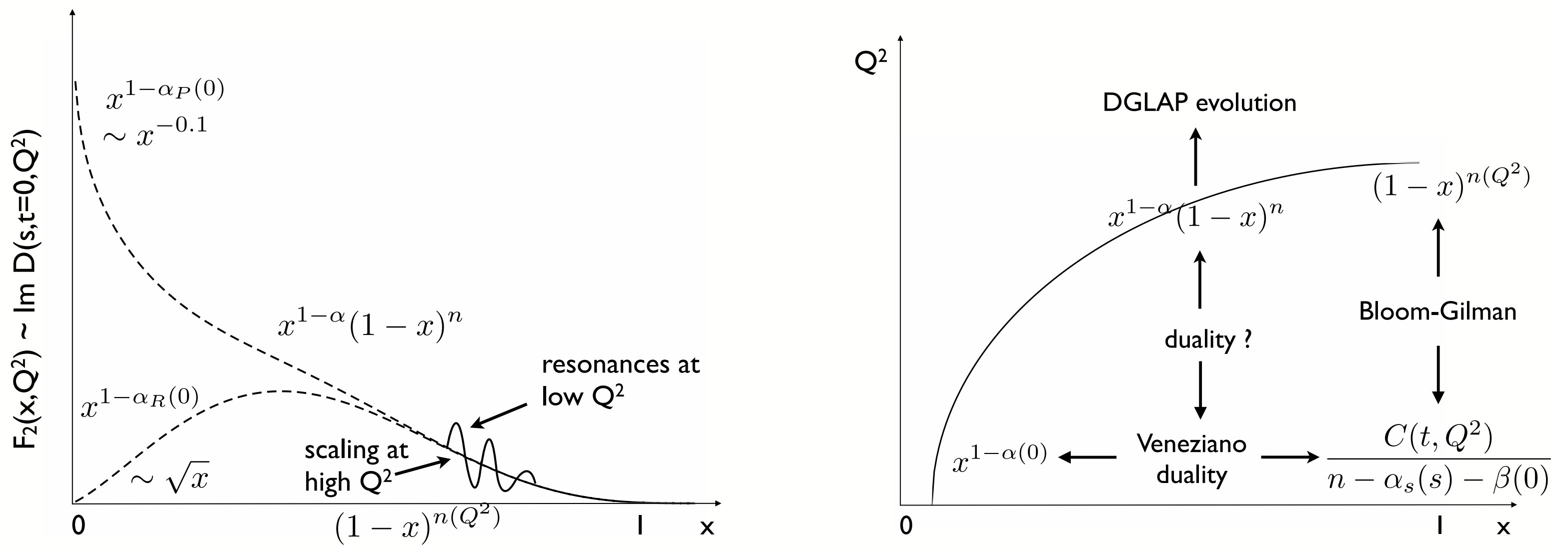
Scaling functions from resonances

- Explicit realization of Veneziano & Bloom-Gilman duality

  → for large x and Q^2, have power-law behavior

$$F_2 \sim (1-x)^{2\alpha_t(0)\ln 2g/\ln g}$$

  where parameter g can be Q^2 dependent



Jenkovszky, Magas, Londergan, Szczepaniak (2012)

More than one flavor?

- Consider simple quark model with spin-flavor symmetric wave function

low energy

$\longrightarrow$ *coherent* scattering from quarks $\quad d\sigma \sim \left(\sum_i e_i \right)^2$

high energy

$\longrightarrow$ *incoherent* scattering from quarks $\quad d\sigma \sim \sum_i e_i^2$

- For duality to work, these must be equal

$\longrightarrow$ how can *square of a sum* become *sum of squares*?

Dynamical cancellations

$\longrightarrow$ *e.g.* for toy model of two quarks bound in a harmonic oscillator potential, structure function given by

$$F(\nu, \mathbf{q}^2) \;\sim\; \sum_n \left|G_{0,n}(\mathbf{q}^2)\right|^2 \;\delta(E_n - E_0 - \nu)$$

$\longrightarrow$ charge operator $\Sigma_i \; e_i \exp(i\mathbf{q}\cdot\mathbf{r}_i)$ excites *even* partial waves with strength $\propto (e_1 + e_2)^2$
odd partial waves with strength $\propto (e_1 - e_2)^2$

$\longrightarrow$ resulting structure function

$$F(\nu, \mathbf{q}^2) \;\sim\; \sum_n \left\{(e_1+e_2)^2 \; G^2_{0,2n} + (e_1-e_2)^2 \; G^2_{0,2n+1}\right\}$$

$\longrightarrow$ if states degenerate, *cross terms* $(\sim e_1 e_2)$ *cancel* when averaged over nearby *even and odd parity* states

Close, Isgur (2001)

Dynamical cancellations

→ duality is realized by summing over at least one complete set of *even* and *odd* parity resonances

→ in NR Quark Model, even & odd parity states generalize to **56** (L=0) and **70** (L=1) multiplets of spin-flavor SU(6)

representation	$^2\mathbf{8}[\mathbf{56}^+]$	$^4\mathbf{10}[\mathbf{56}^+]$	$^2\mathbf{8}[\mathbf{70}^-]$	$^4\mathbf{8}[\mathbf{70}^-]$	$^2\mathbf{10}[\mathbf{70}^-]$	Total
F_1^p	$9\rho^2$	$8\lambda^2$	$9\rho^2$	0	λ^2	$18\rho^2+9\lambda^2$
F_1^n	$(3\rho+\lambda)^2/4$	$8\lambda^2$	$(3\rho-\lambda)^2/4$	$4\lambda^2$	λ^2	$(9\rho^2+27\lambda^2)/2$

λ (ρ) = (anti) symmetric component of ground state wave function

Close, WM (2003, 2009)

Dynamical cancellations

$\longrightarrow$ in SU(6) limit $\lambda = \rho$, with relative strengths of $N \rightarrow N^*$ transitions

$SU(6):$	$[\mathbf{56}, \mathbf{0}^+]^2\mathbf{8}$	$[\mathbf{56}, \mathbf{0}^+]^4\mathbf{10}$	$[\mathbf{70}, \mathbf{1}^-]^2\mathbf{8}$	$[\mathbf{70}, \mathbf{1}^-]^4\mathbf{8}$	$[\mathbf{70}, \mathbf{1}^-]^2\mathbf{10}$	*total*
F_1^p	9	8	9	0	1	27
F_1^n	4	8	1	4	1	18

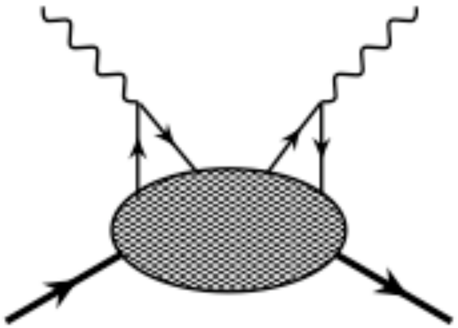
$\longrightarrow$ summing over all resonances in $\mathbf{56}^+$ and $\mathbf{70}^-$ multiplets

$$\frac{F_1^n}{F_1^p} = \frac{18}{27} = \frac{2}{3}$$

$\longrightarrow$ at the quark level, n/p ratio is

$$\frac{F_1^n}{F_1^p} = \frac{4d+u}{d+4u} = \frac{6}{9} = \frac{2}{3} \quad ! \quad \text{if } u = 2d$$

Accidental cancellations of charges?



<u>cat's ears diagram</u> (4-fermion higher twist $\sim 1/Q^2$)

$$\propto \sum_{i \neq j} e_i \, e_j \sim \Big(\sum_i e_i\Big)^2 - \sum_i e_i^2$$

coherent *incoherent*

<u>*proton*</u> $\quad \mathrm{HT} \sim 1 - \left(2 \times \frac{4}{9} + \frac{1}{9}\right) = 0 \,!$

<u>*neutron*</u> $\quad \mathrm{HT} \sim 0 - \left(\frac{4}{9} + 2 \times \frac{1}{9}\right) \neq 0$

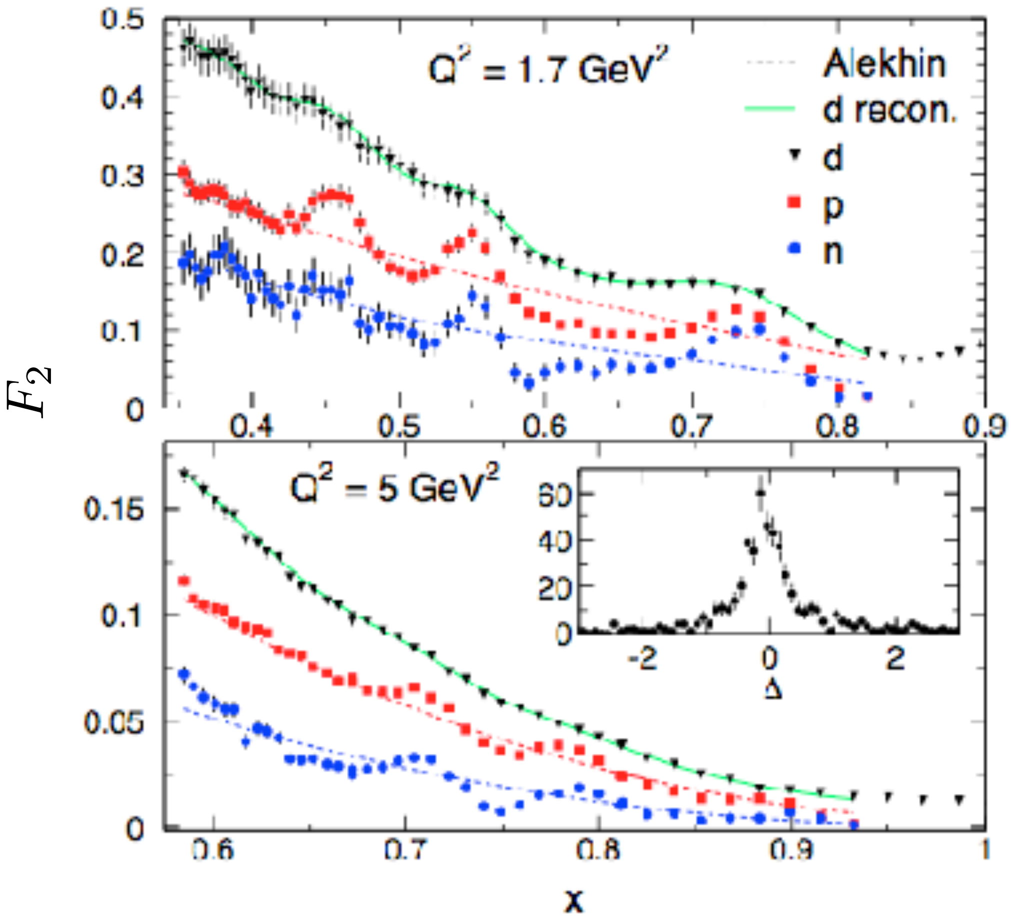
$\longrightarrow$ duality in proton a *coincidence*!

$\longrightarrow$ should <u>*not*</u> hold for neutron !!

Brodsky (2000)

Duality in electron-neutron scattering

- No free neutron targets, but iterative method allows neutron resonance structure to be extracted from deuteron & proton data



Malace, Kahn, WM, Keppel (2010)

→ *locally*, violations of duality in resonance regions < 15−20% (largest in Δ region)

→ evidence that duality is *<u>not</u> accidental*, but a general feature of resonance−scaling transition!

→ ***C. Keppel & I. Niculescu talks***

Outlook and open questions

- Confirmation of duality (experimentally & theoretically) suggests origin in dynamical cancelations between resonances

  → explore more realistic descriptions based on phenomenological $\gamma^* N N^*$ form factors

- Era of "quantitative duality" — need to define the extent to which duality "works"

- Is duality between (high energy) continuum and resonances, or between total (resonance + background)?

  → "resonance region" vs. "resonances"

  → incorporate nonresonant background in same framework + quantum mechanics

- Where does duality *not* work (and why)?


Thank you!