Duality and Exotic Mesons

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Jefferson Lab
Joint Physics Analysis Center

Quark-Hadron Duality Workshop
James Madison University, September 2018

Jefferson Lab
Thomas Jefferson National Accelerator Facility

JPAC
Joint
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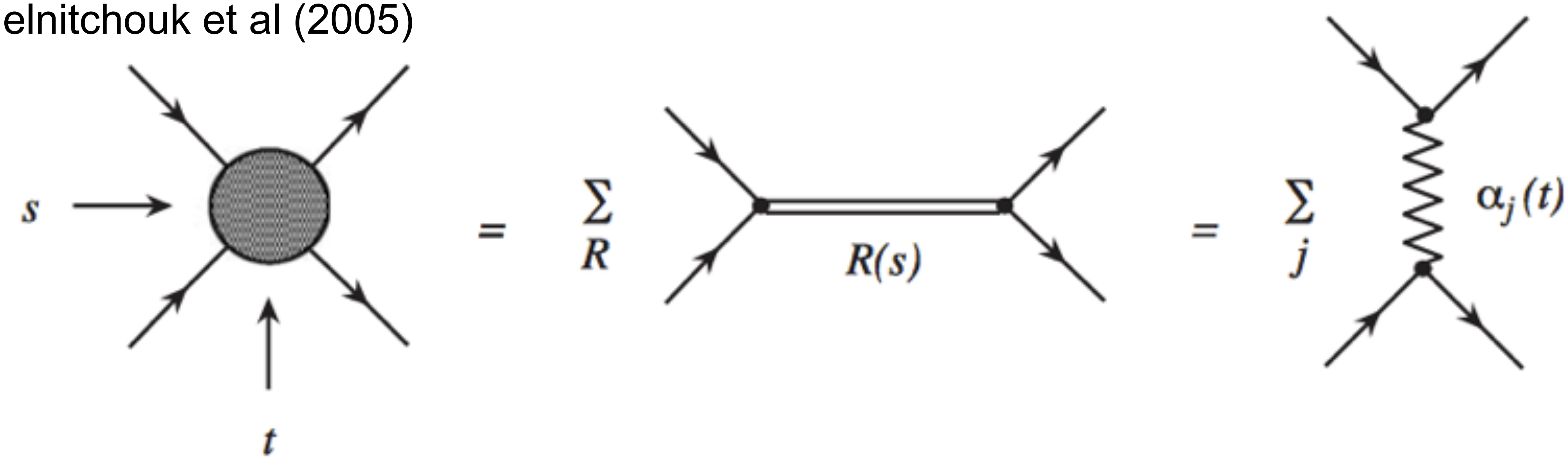
Center

Duality and the Veneziano dual model

taken from Melnitchouk et al (2005)



$$\frac{\Gamma[-\alpha(s)]\Gamma[-\alpha(t)]}{\Gamma[-\alpha(s)-\alpha(t)]} = \sum_{i=0}^{\infty} \frac{r_i(t)}{\alpha(s)-i} = \sum_{i=0}^{\infty} \frac{r_i(s)}{\alpha(t)-i}$$

properties of Veneziano dual model:

sum of narrow pole in s and t channel

‘Regge-like’ asymptotic

analytic formula

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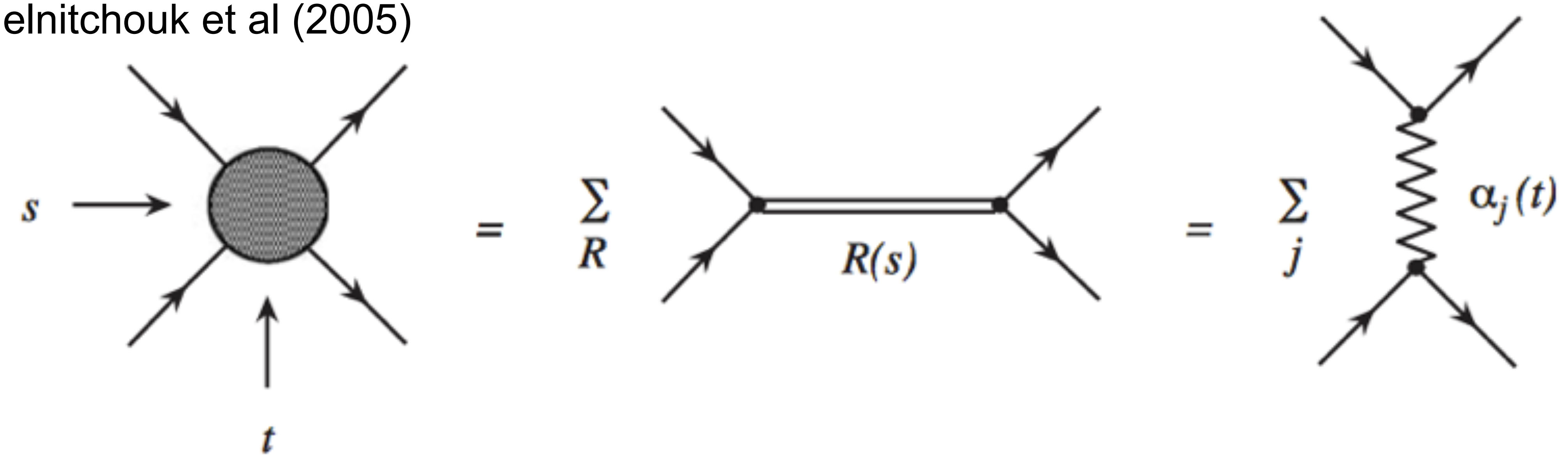

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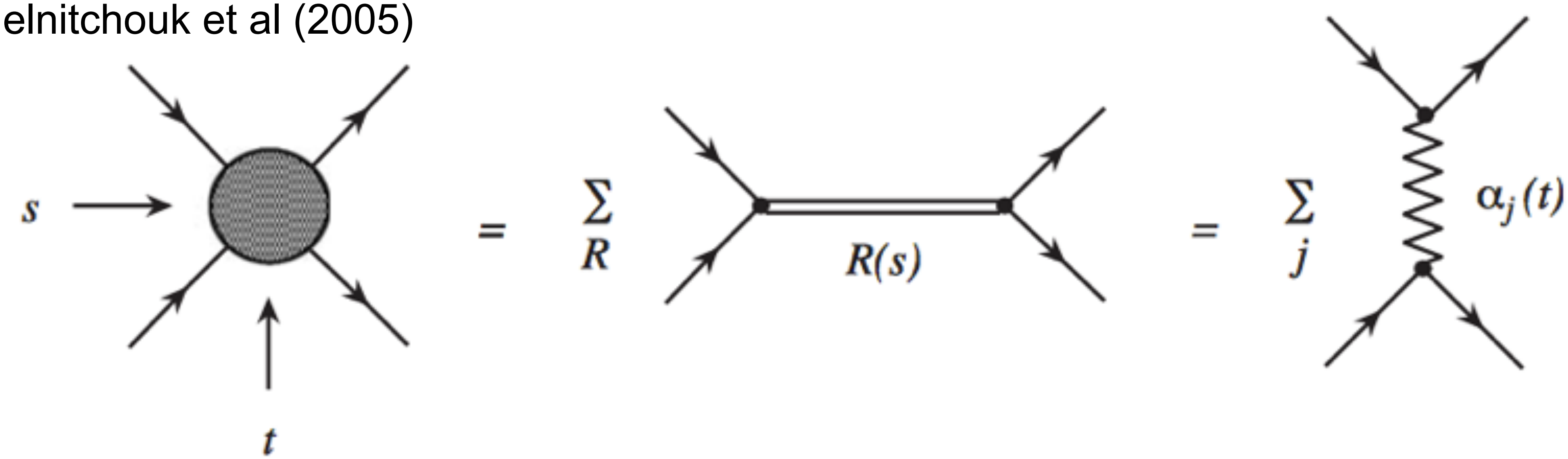

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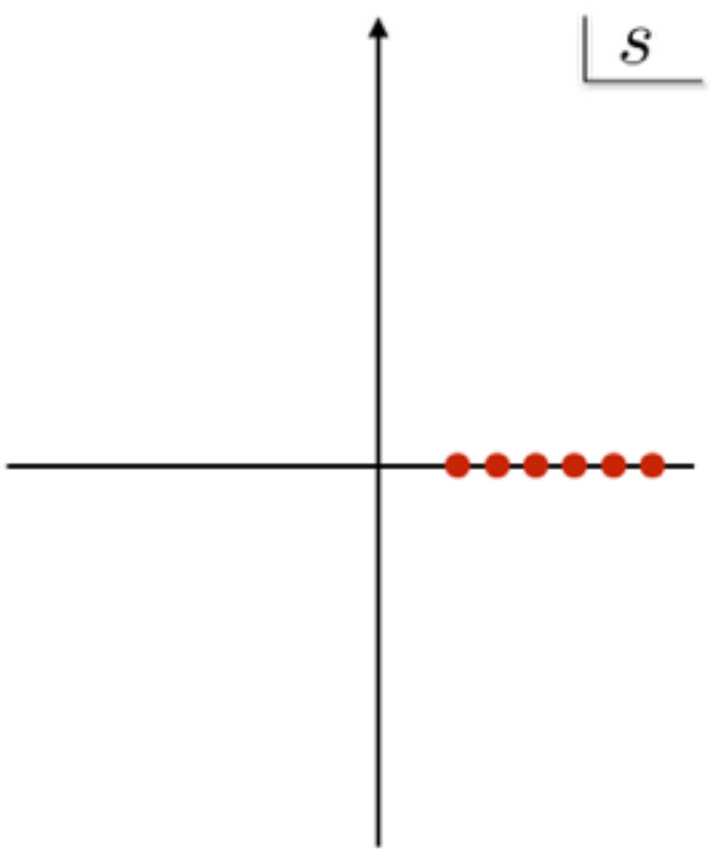

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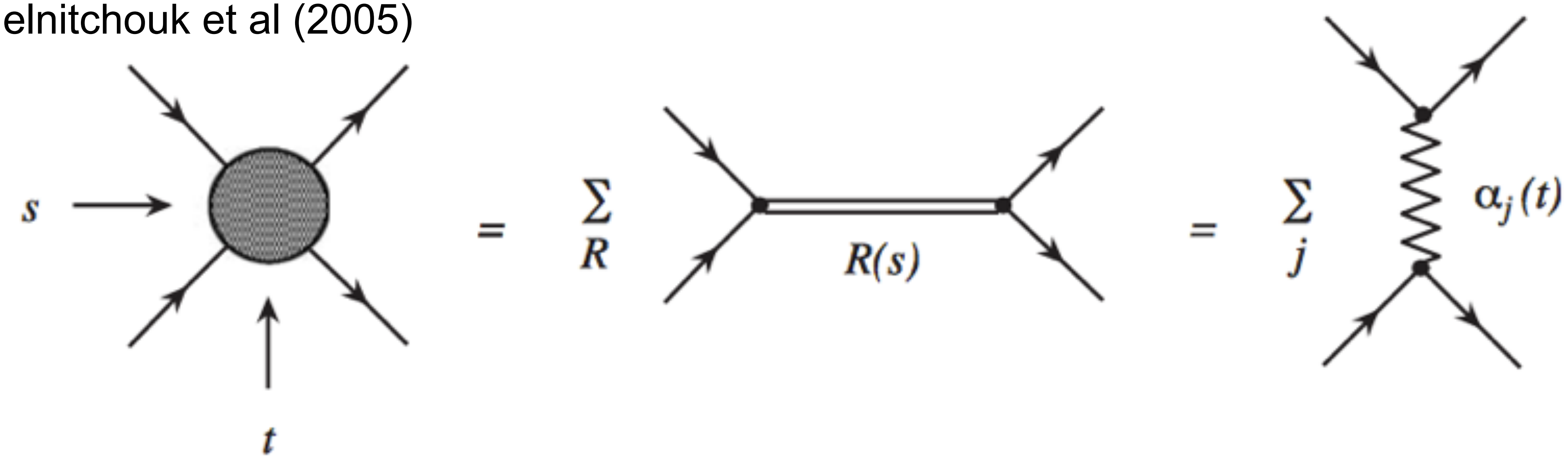

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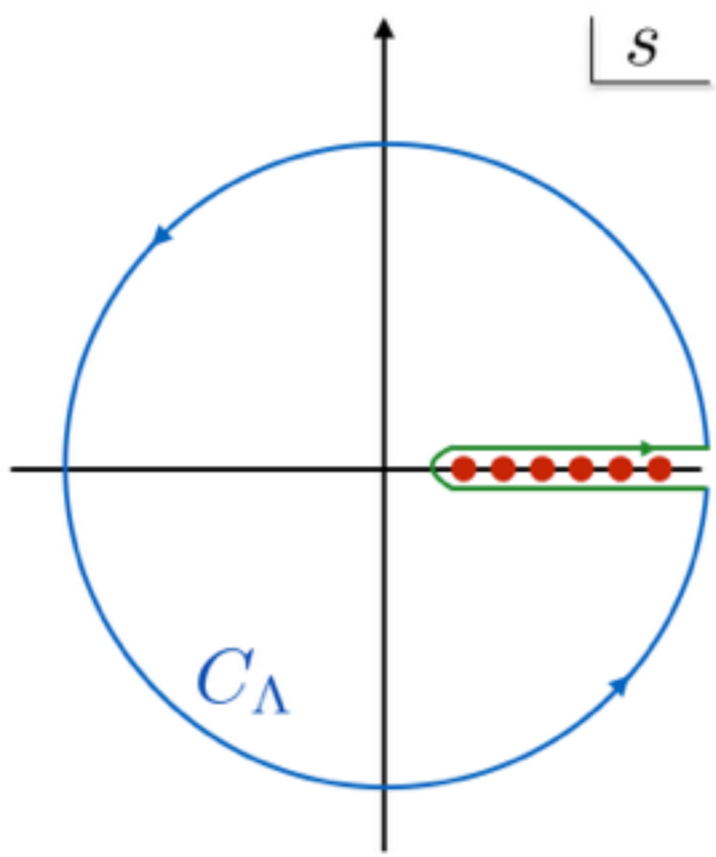

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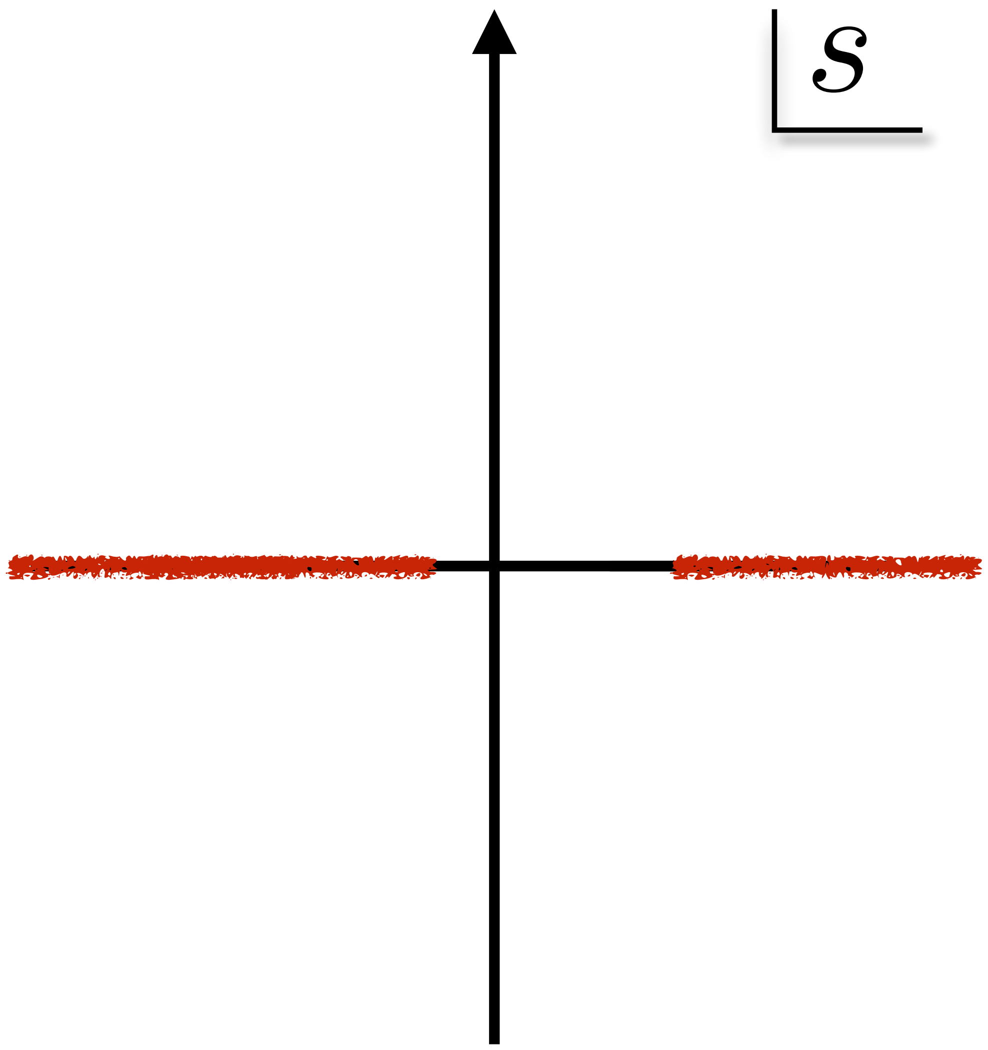
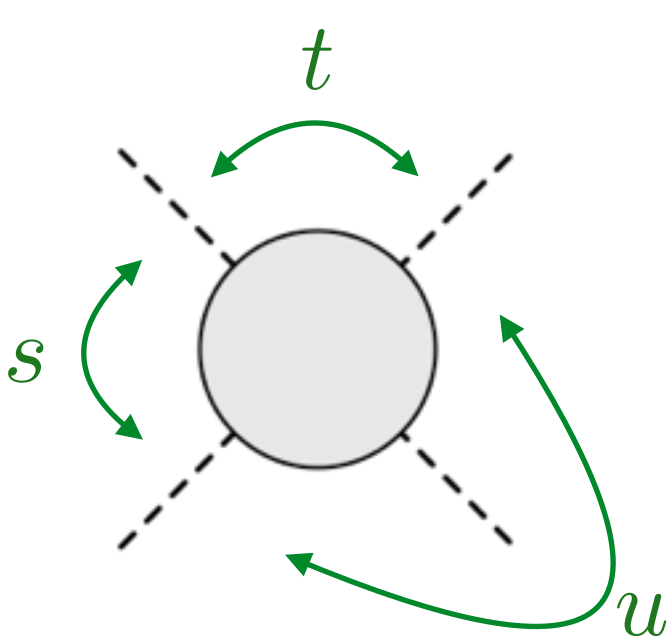
Finite Energy Sum Rules





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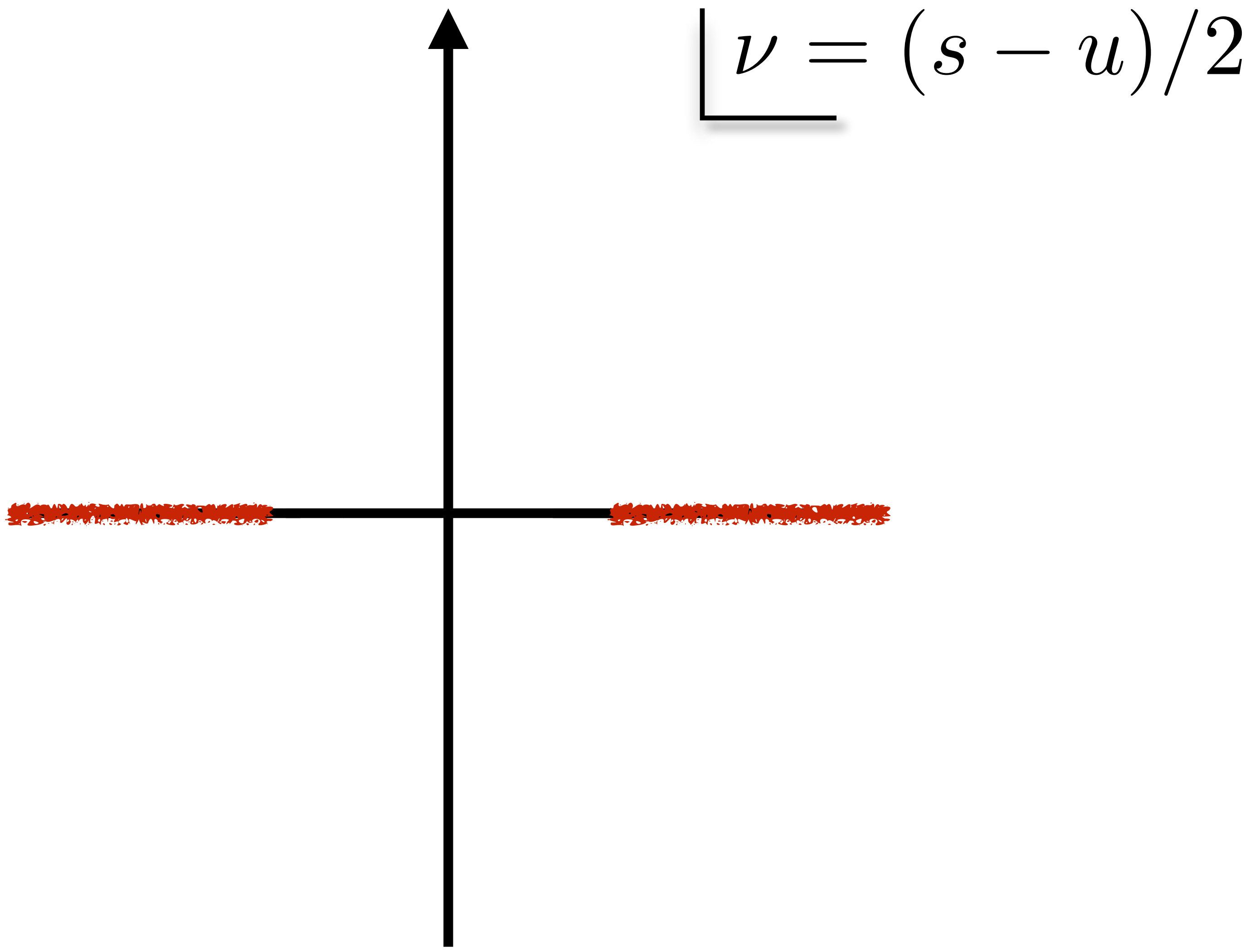
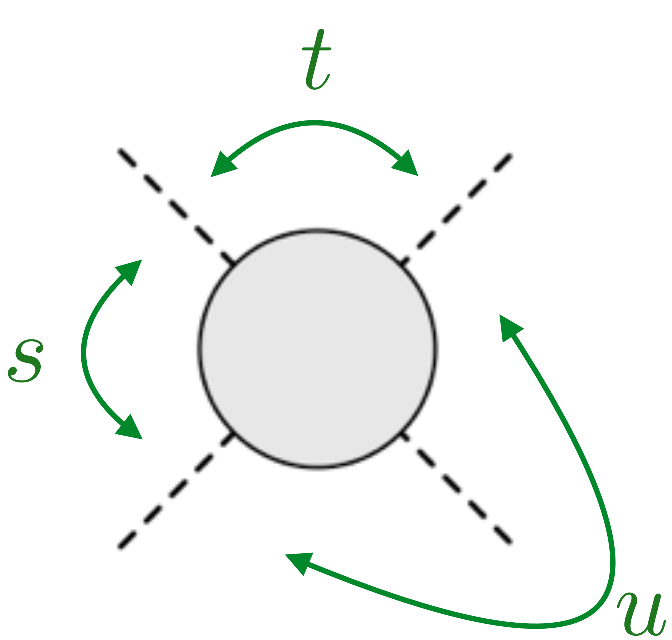
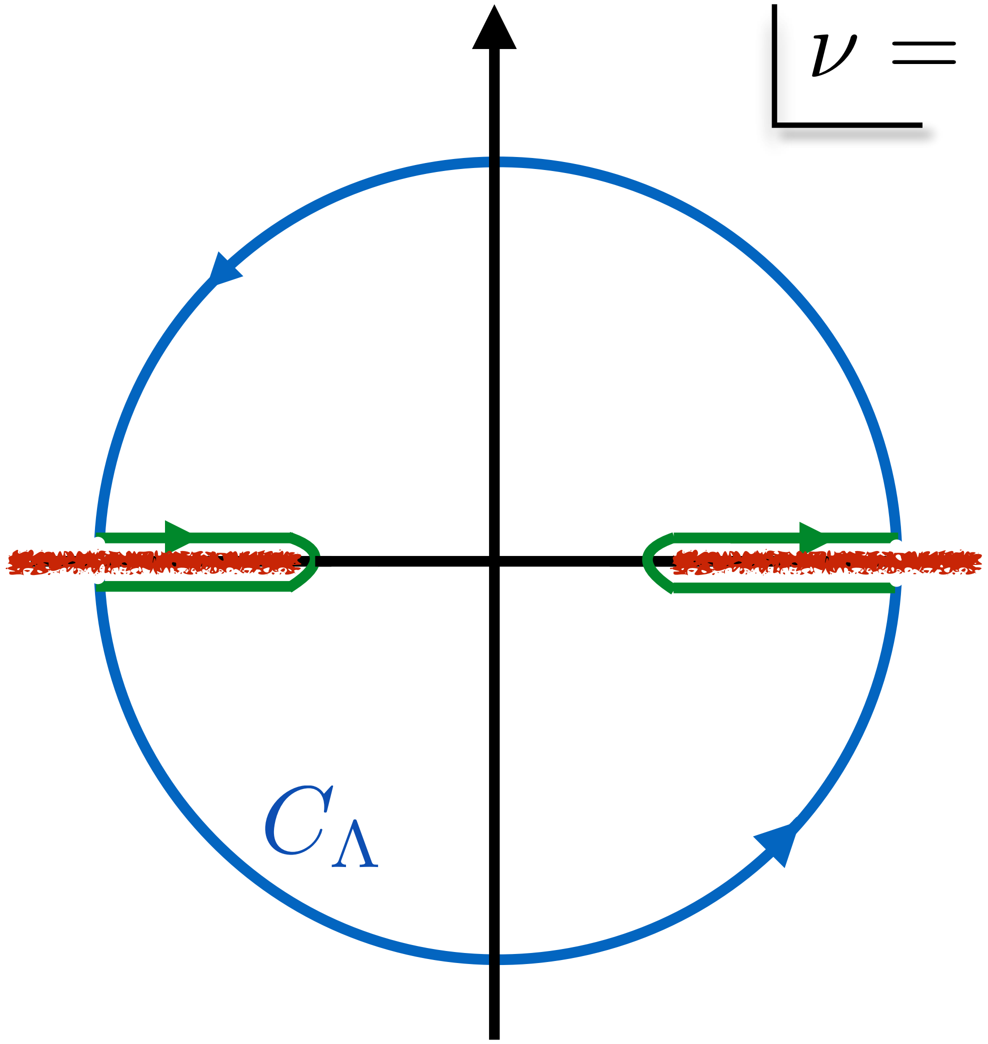
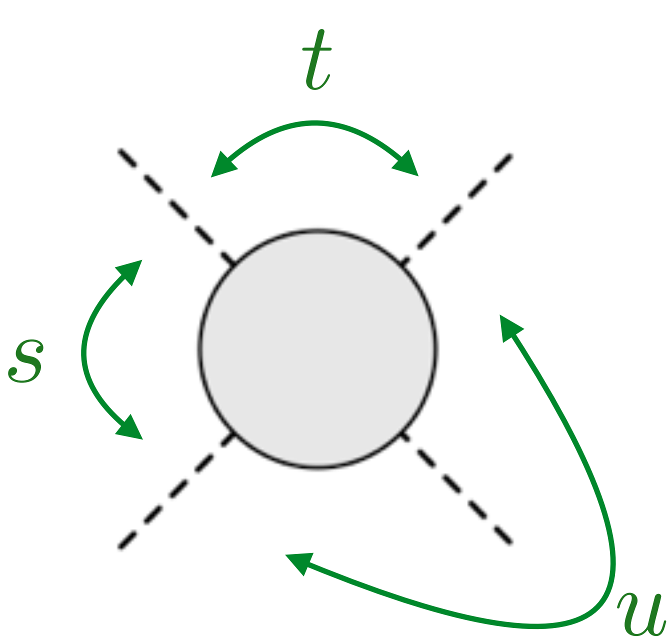




Finite Energy Sum Rules

$$\nu = (s - u)/2$$





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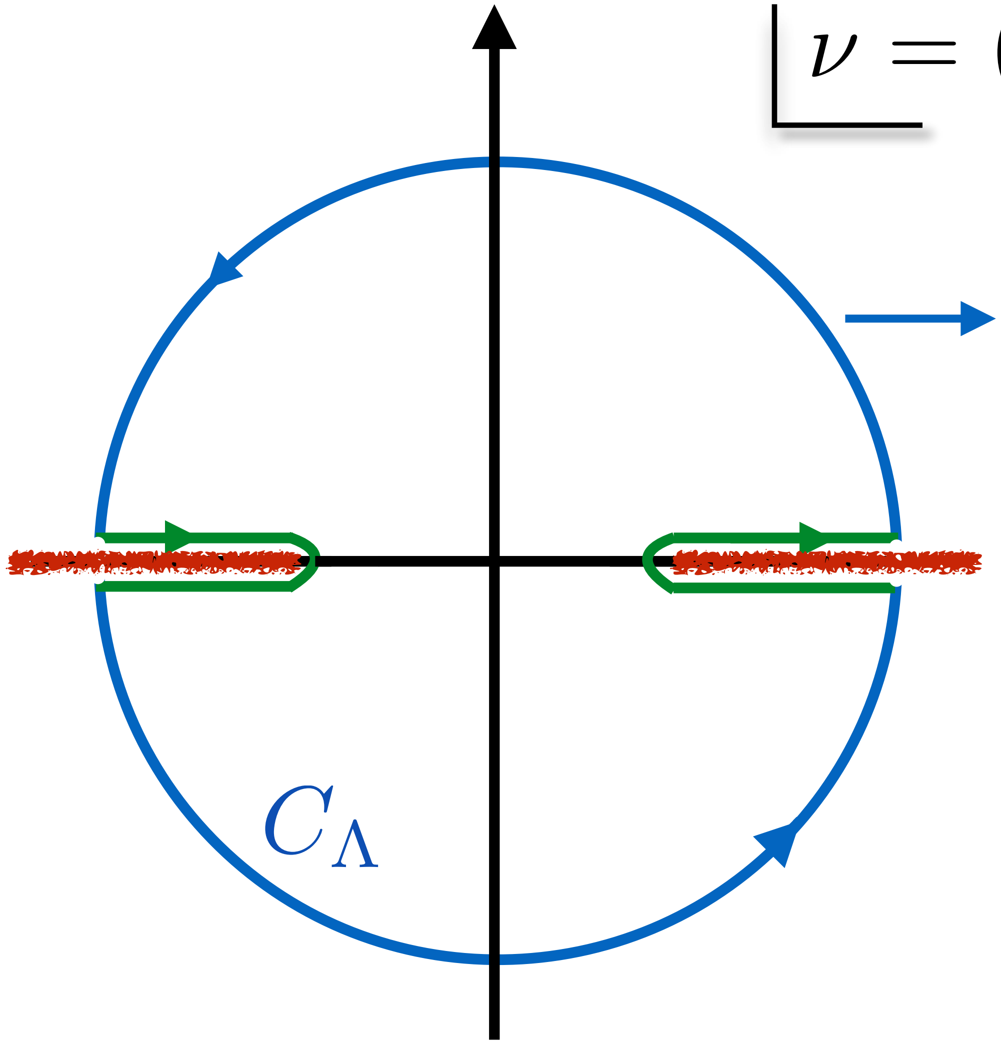
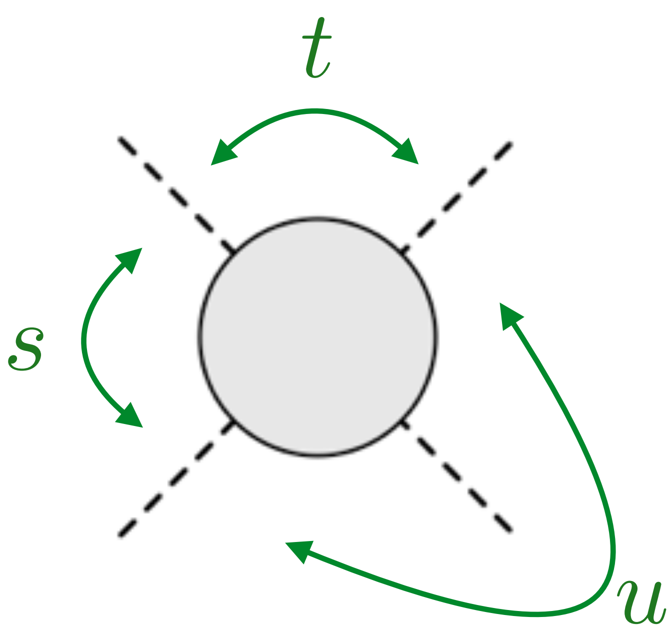


$$\text{Im}\ A(\nu,t) = \beta(t)\nu^{\alpha(t)}$$



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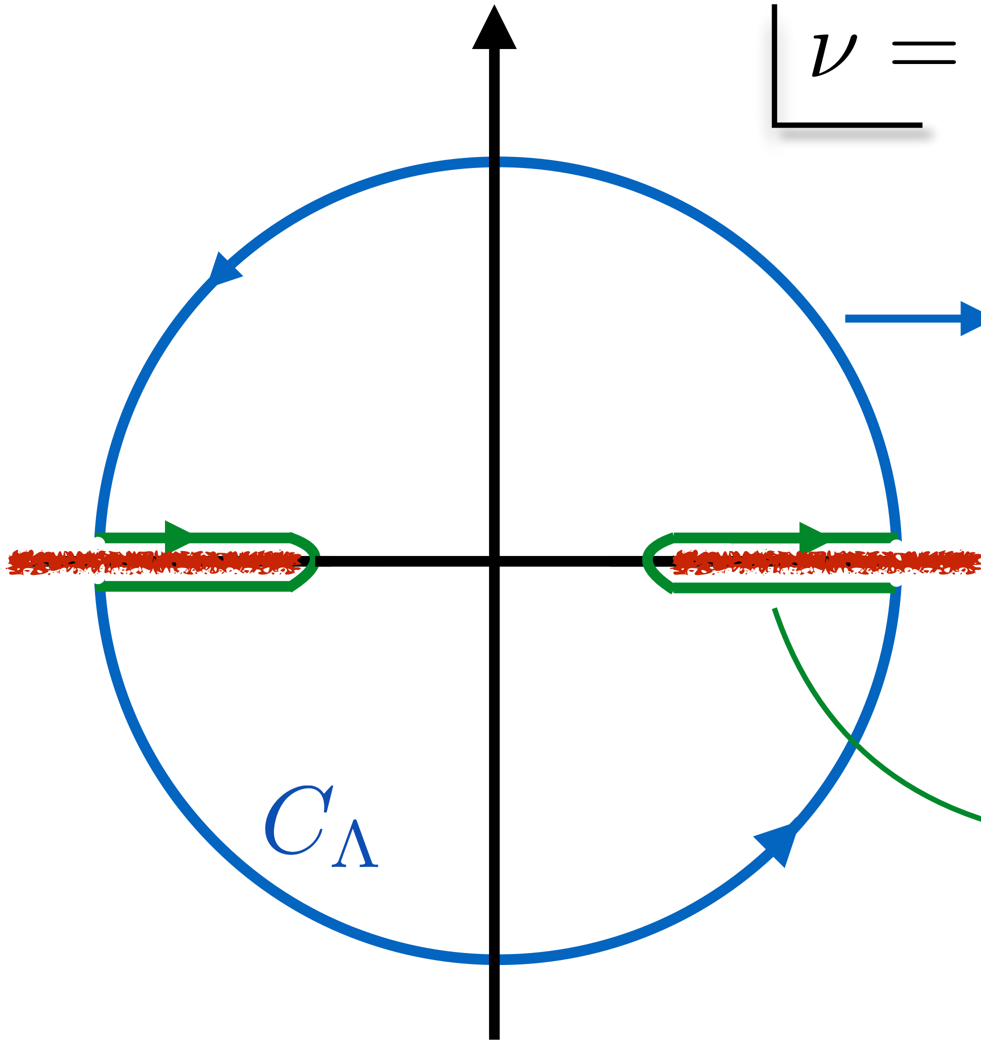

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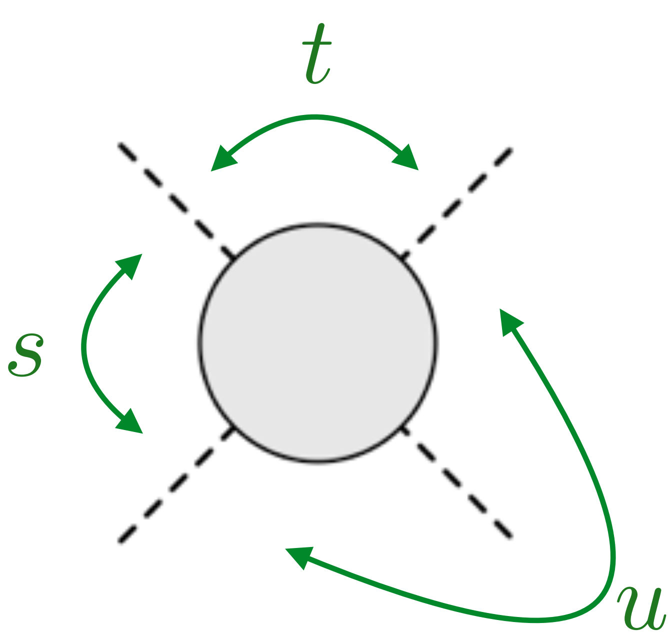

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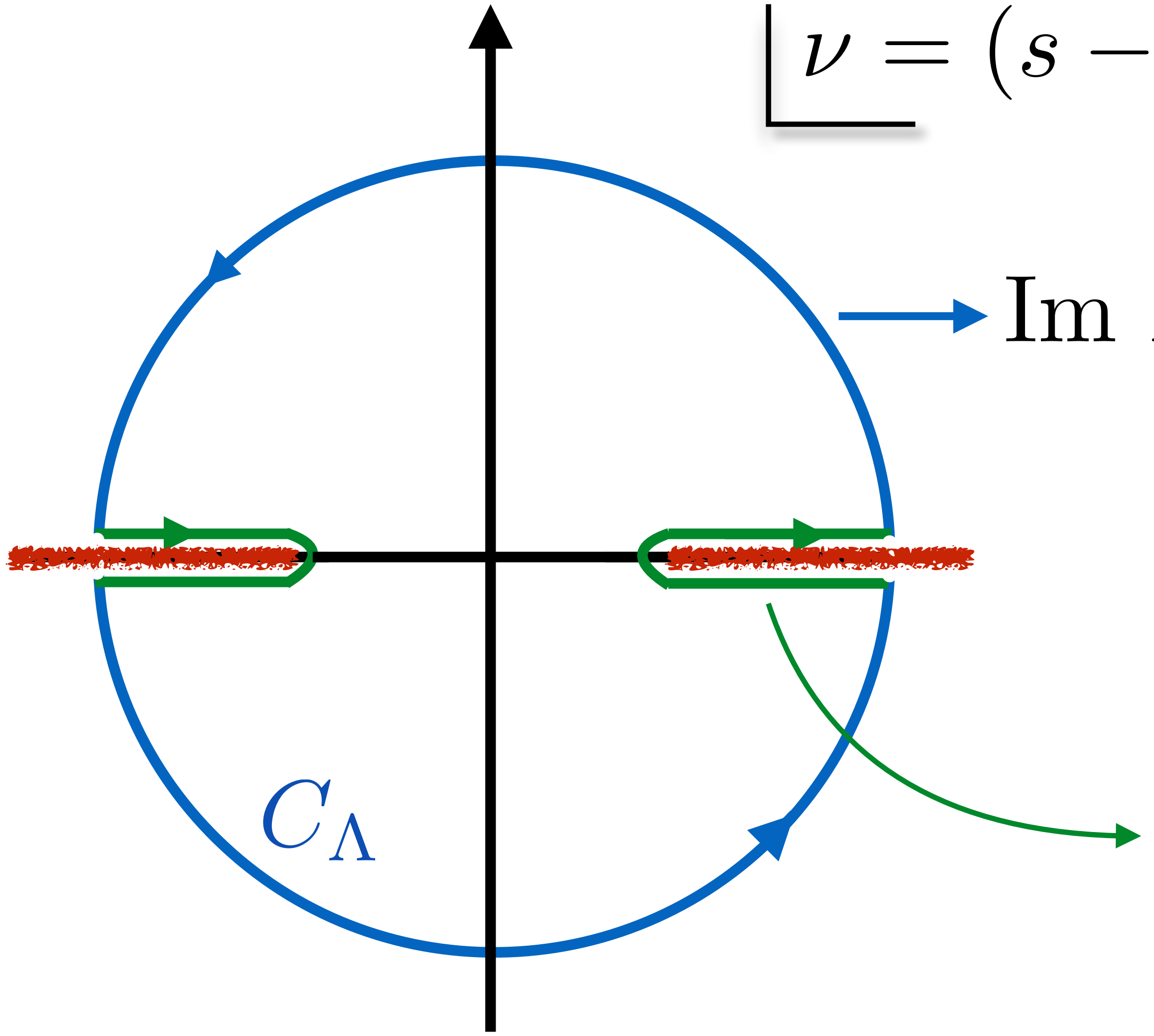

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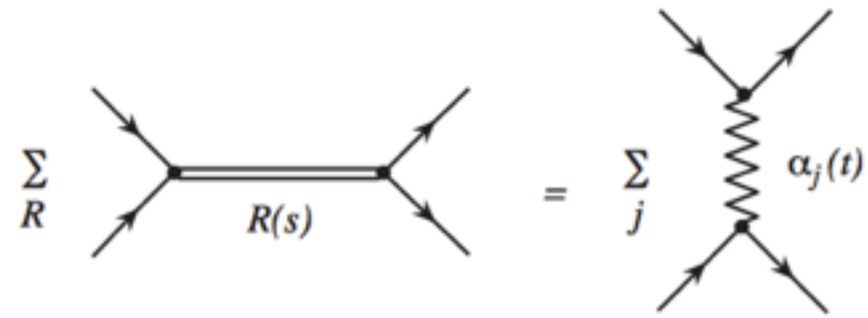

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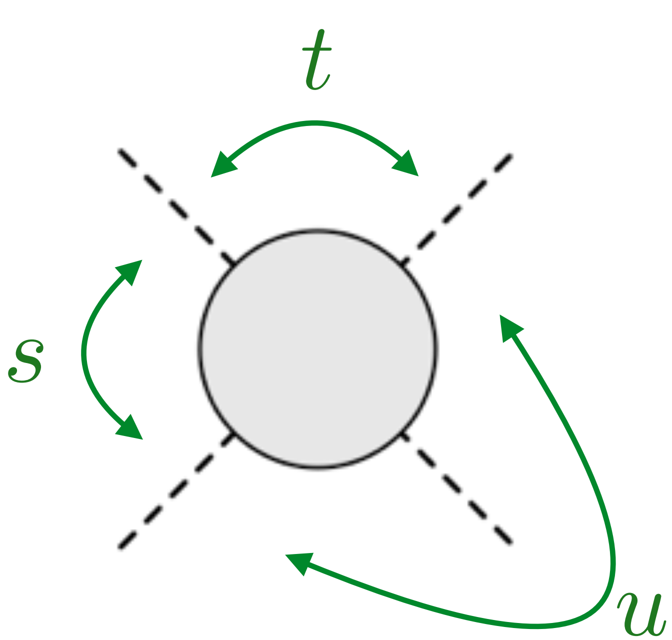

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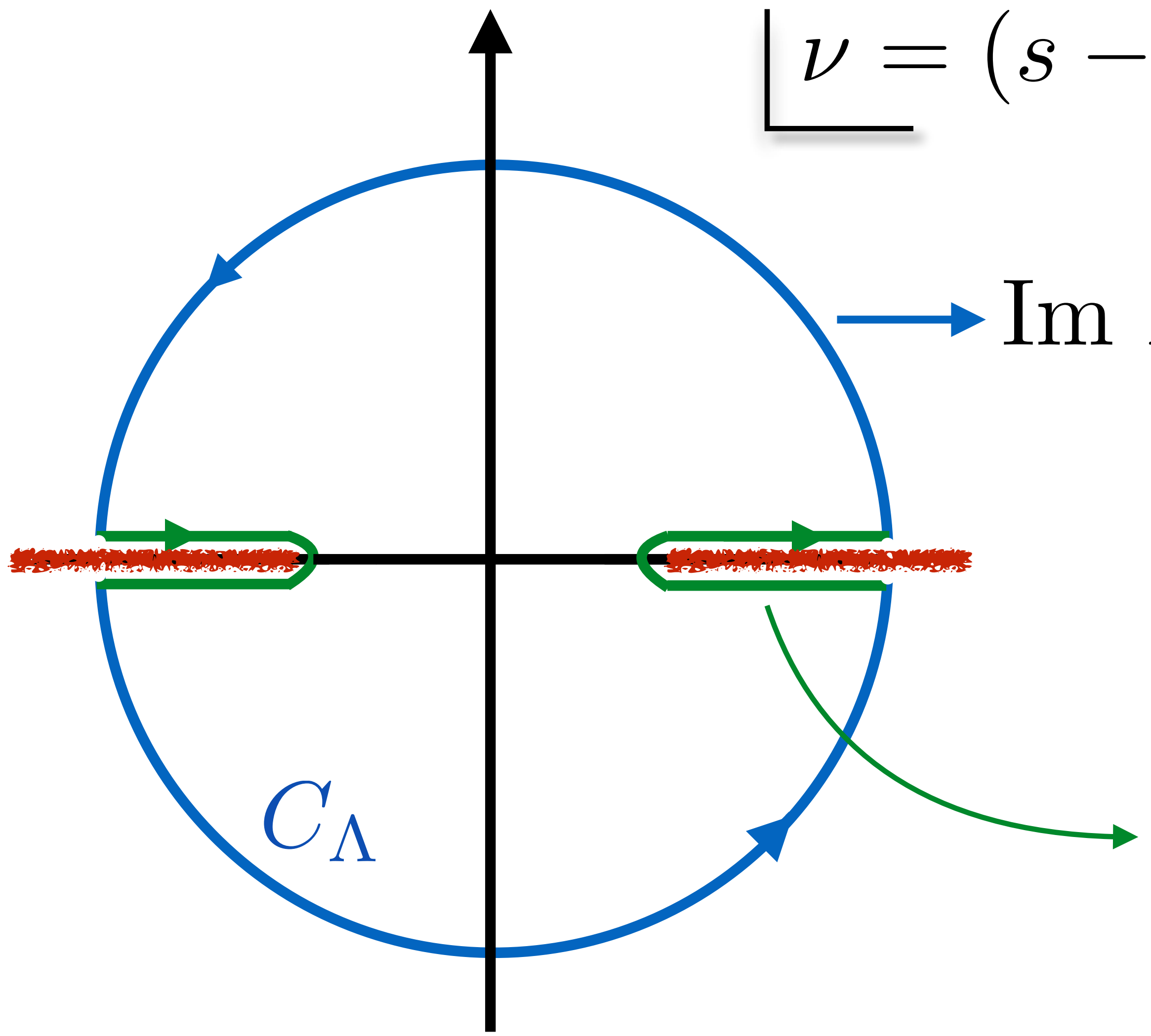

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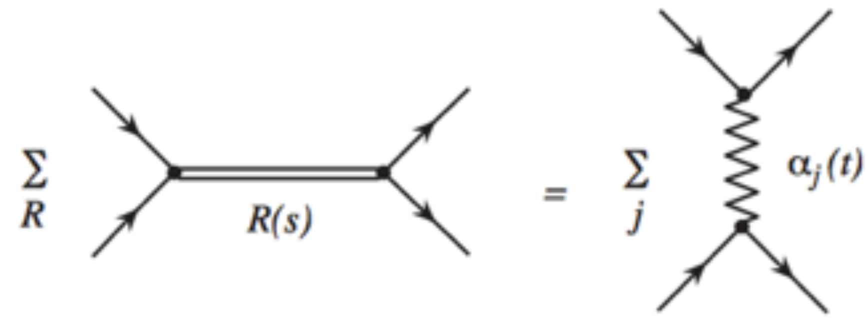

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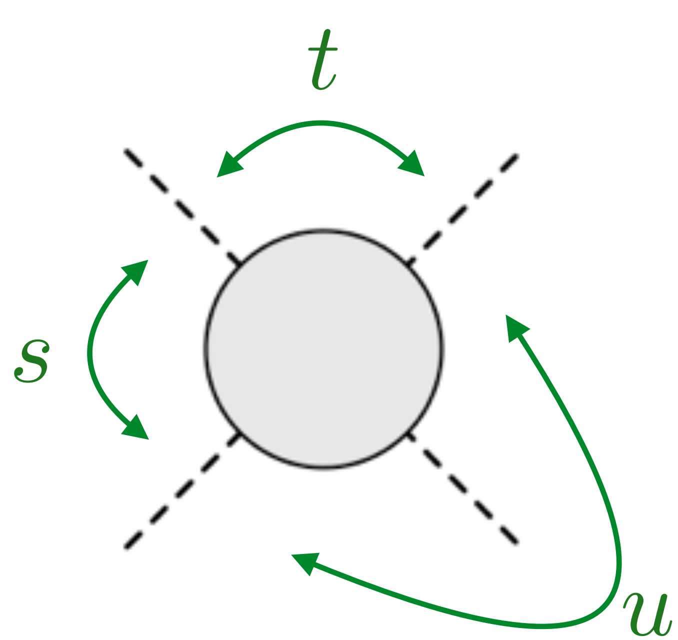

write a dispersion relation for $A(\nu, t_0)\nu^k$

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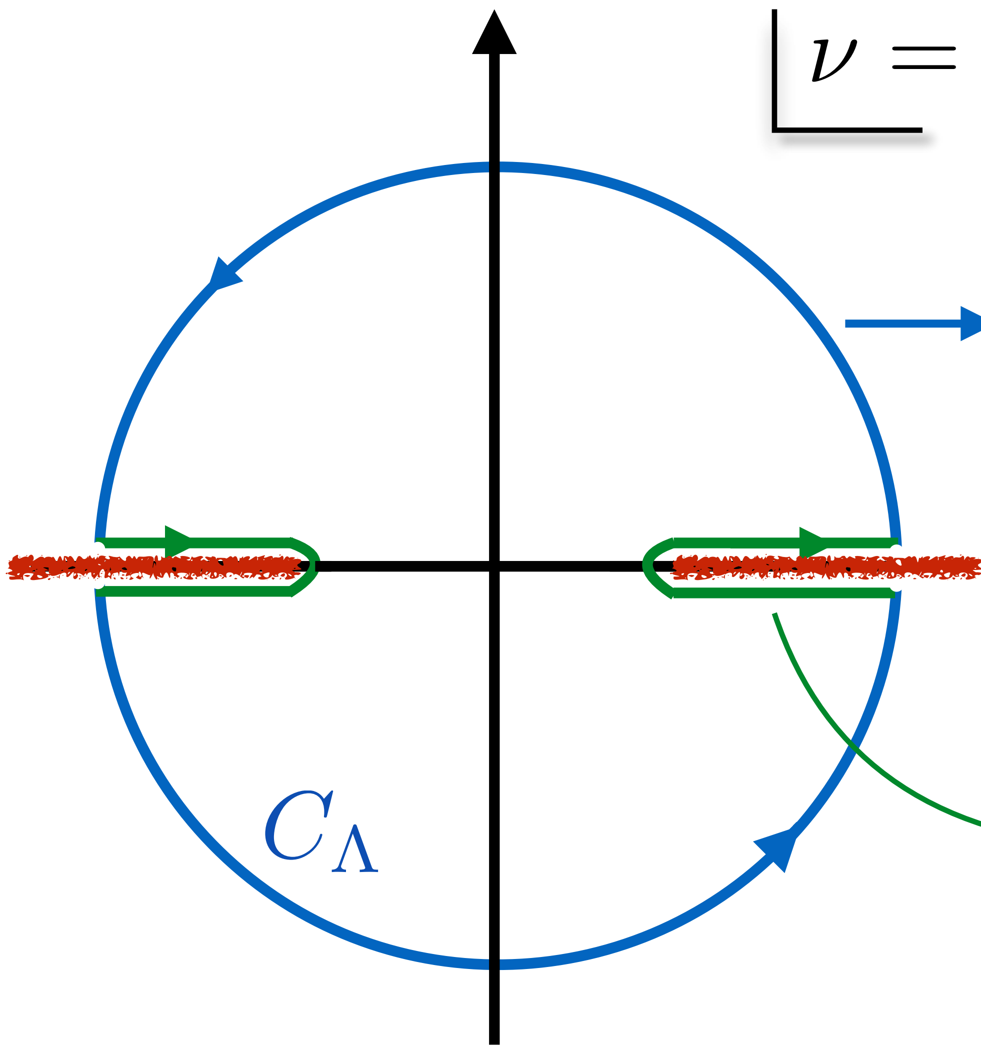

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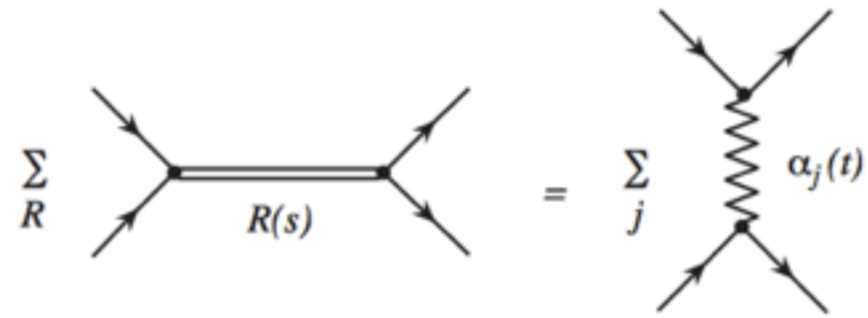

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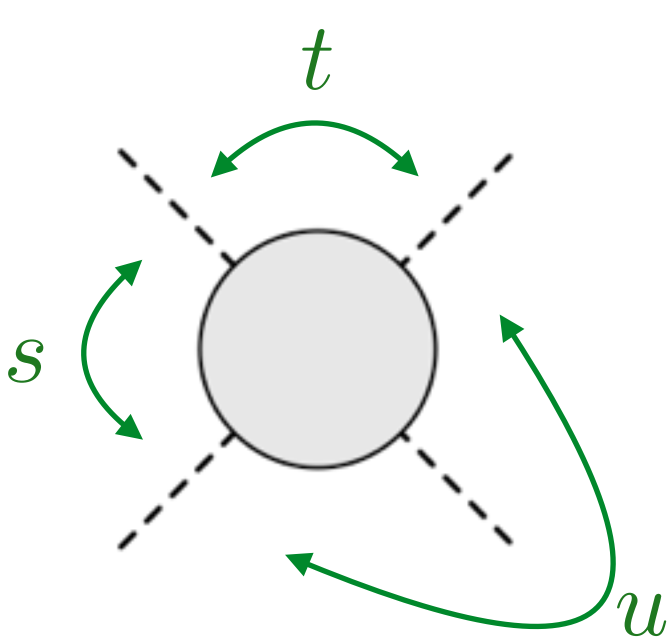

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$$\sigma(ab \to X) \propto \text{Im } A(\nu, t=0)$$

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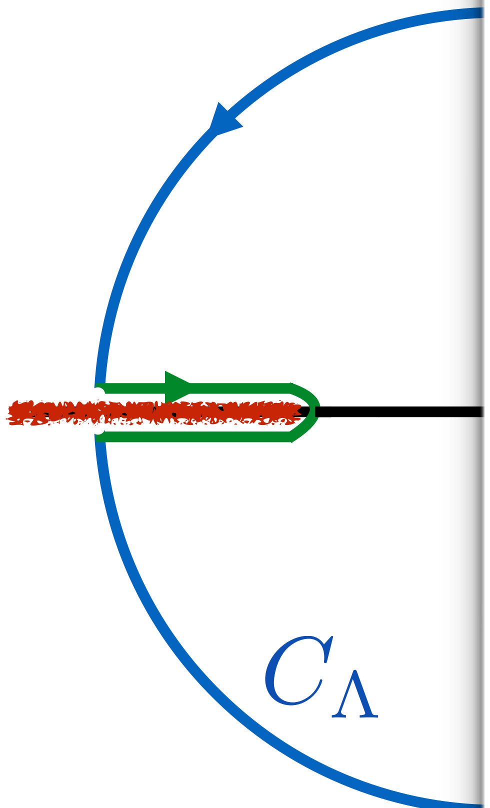

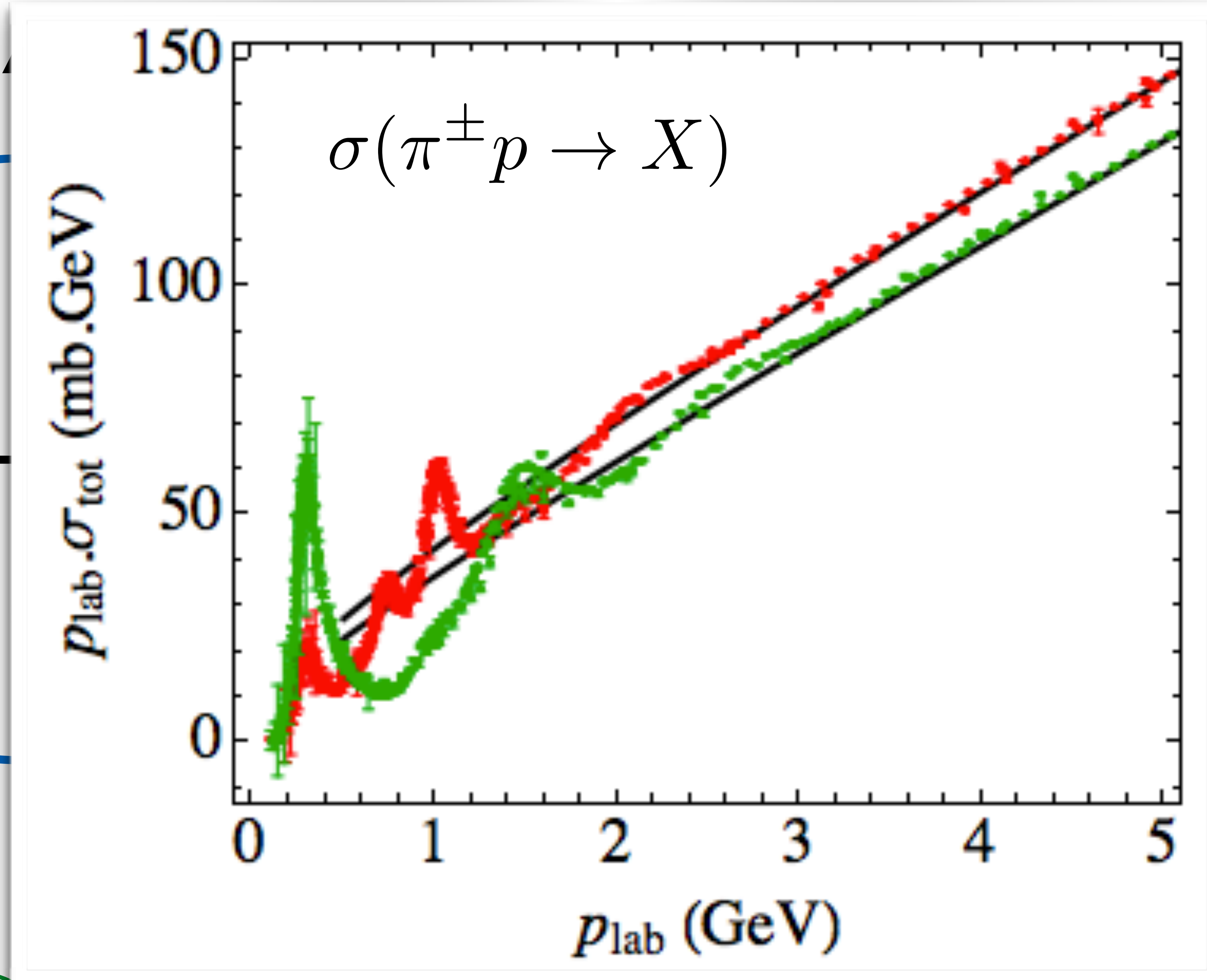

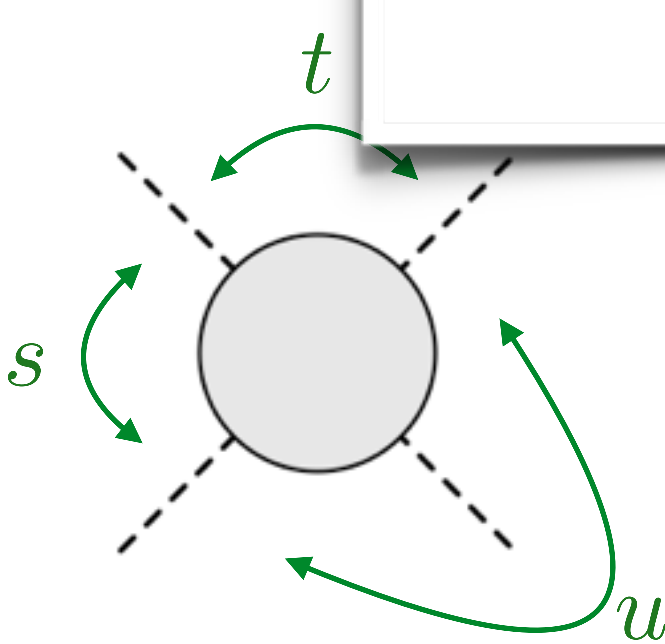

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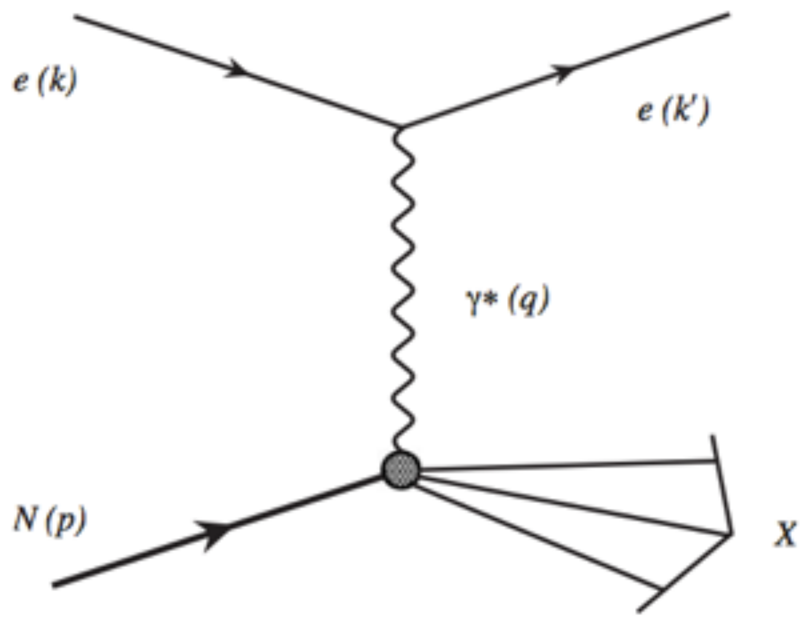
Application to election-nucleon scattering

$$e^- p \to e^- X$$



Data binned in Q^2

and ν **or** $x = Q^2/2M\nu$

$$\omega = 2M\nu/Q^2$$

Data related to forward elastic amplitude by optical theorem

Application to election-nucleon scattering

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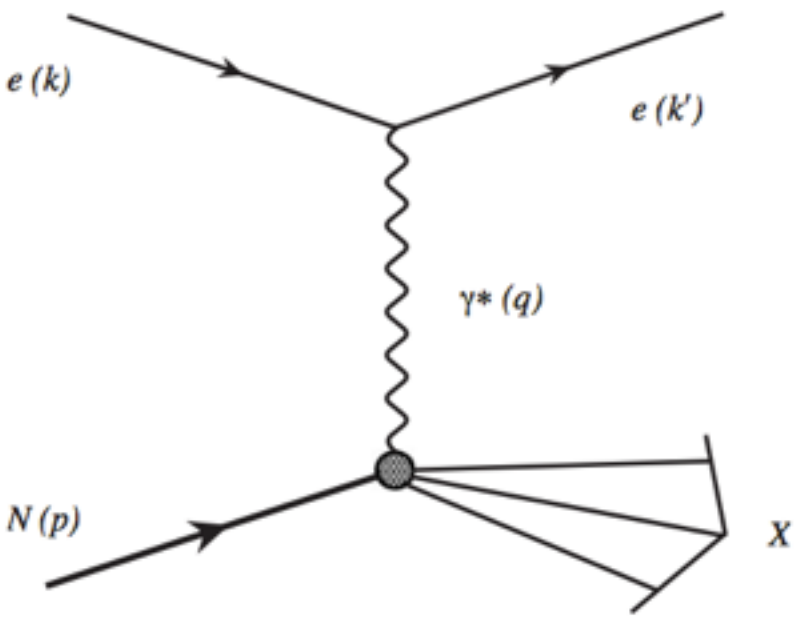
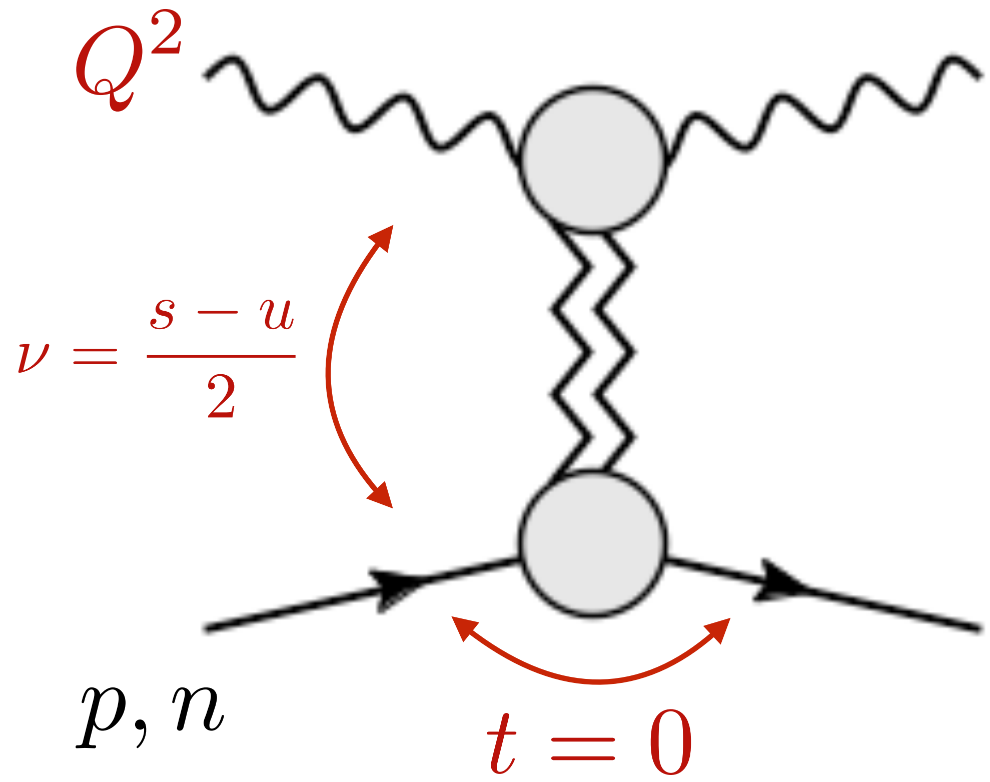


Elastic Amplitude



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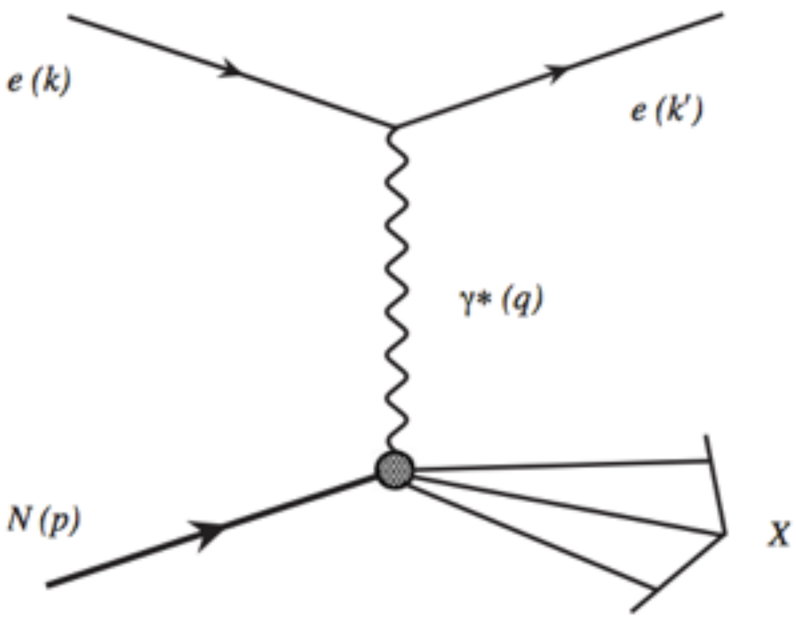

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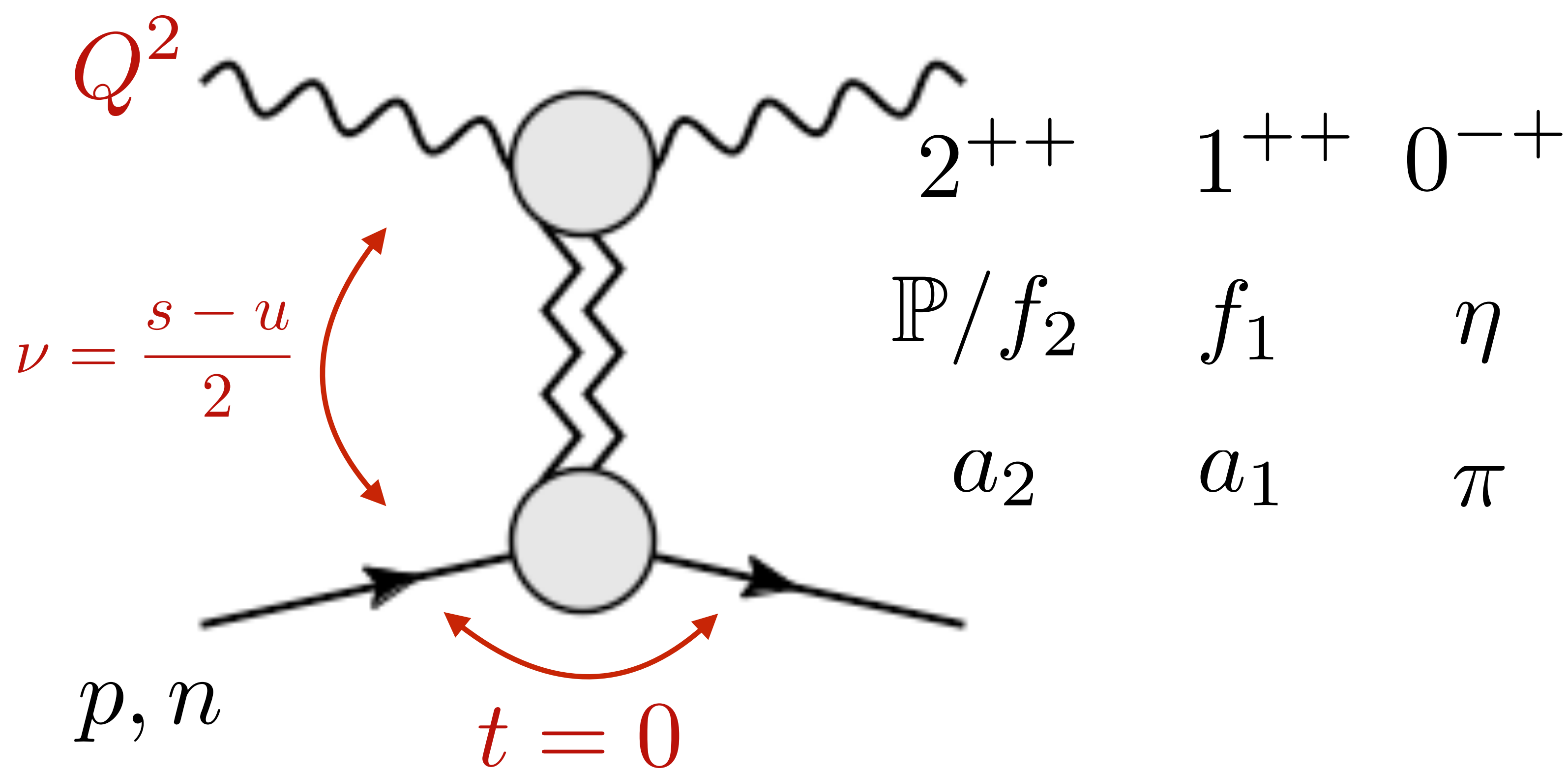

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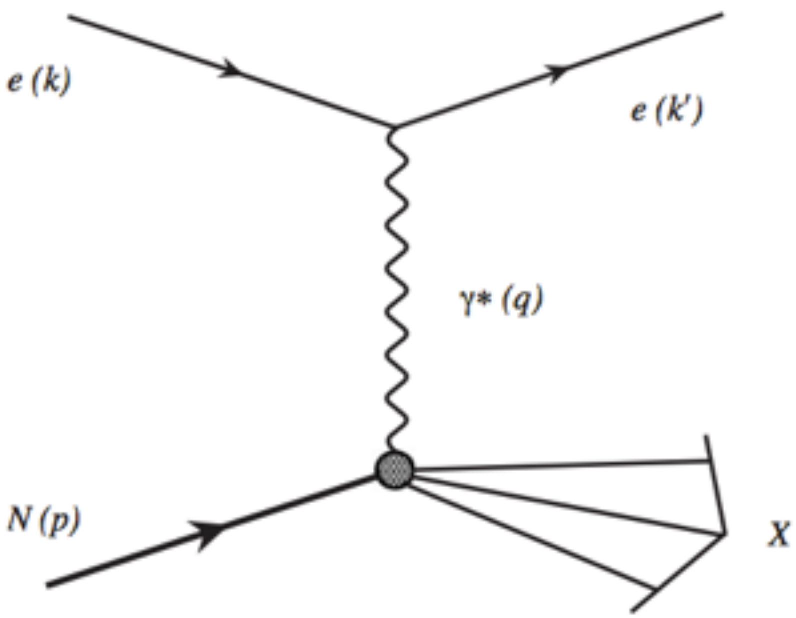

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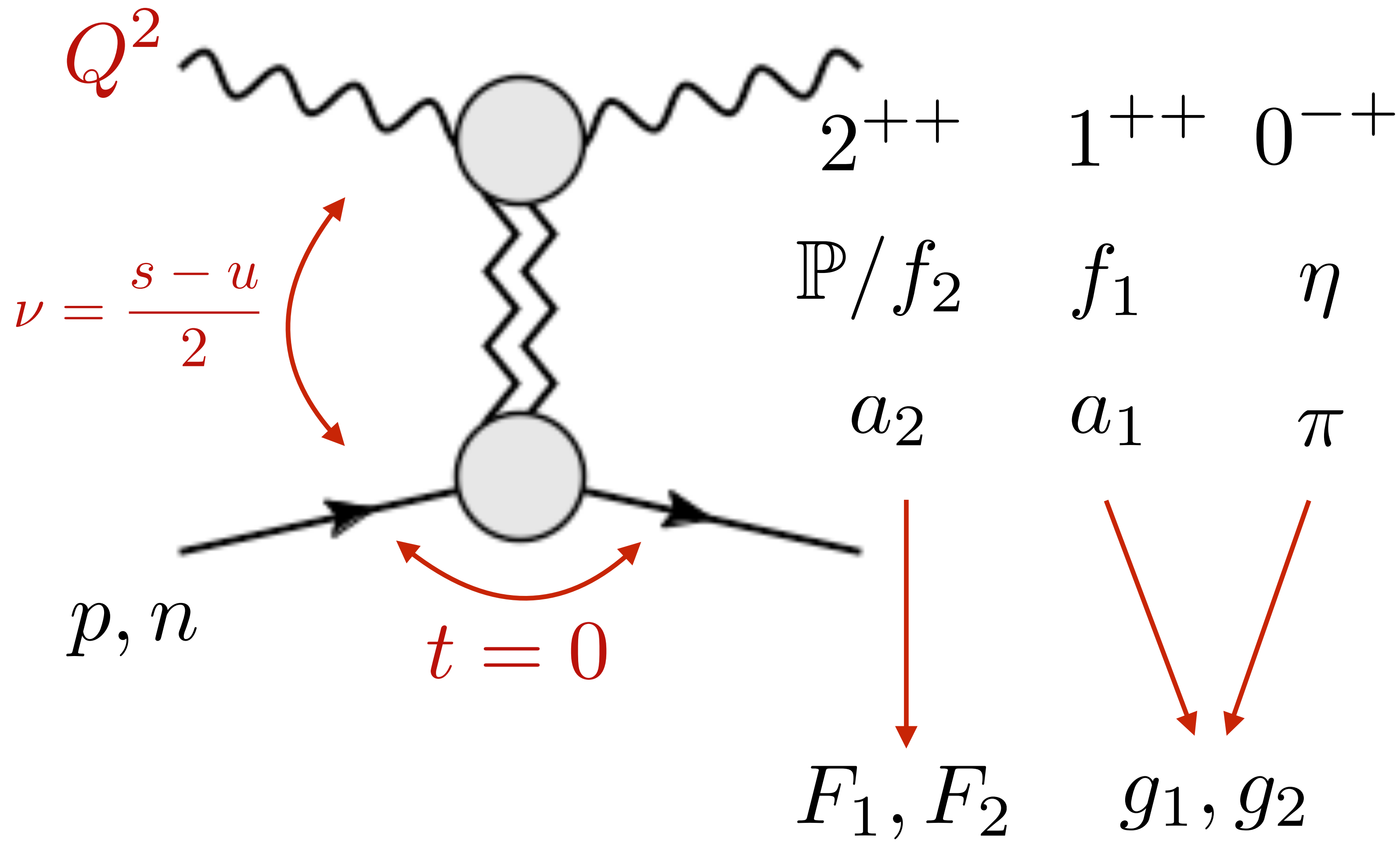

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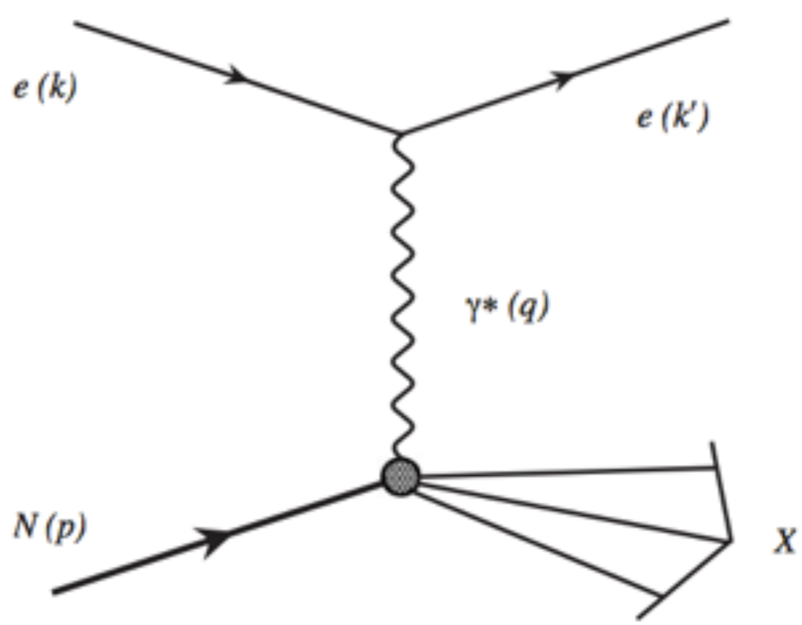

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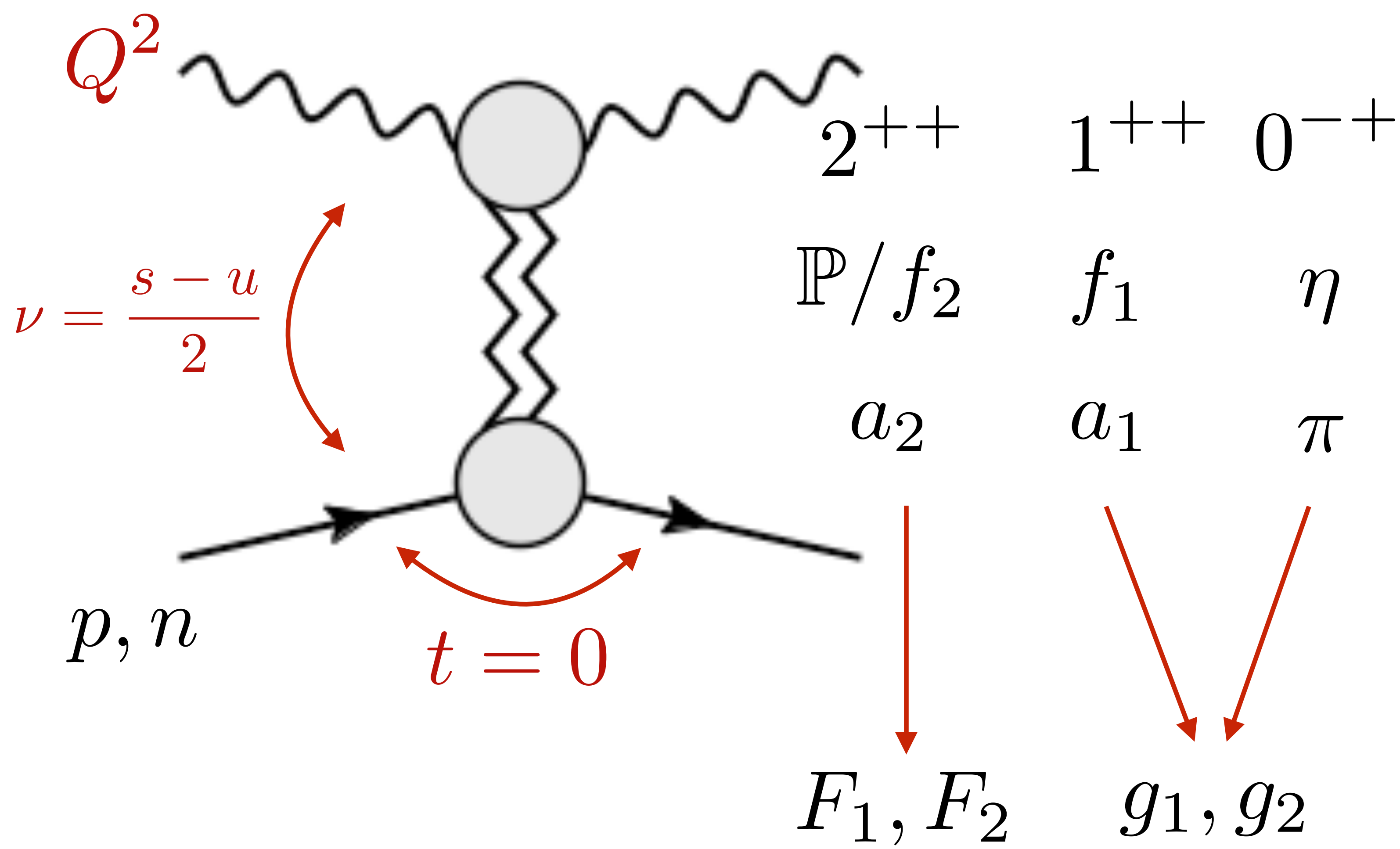

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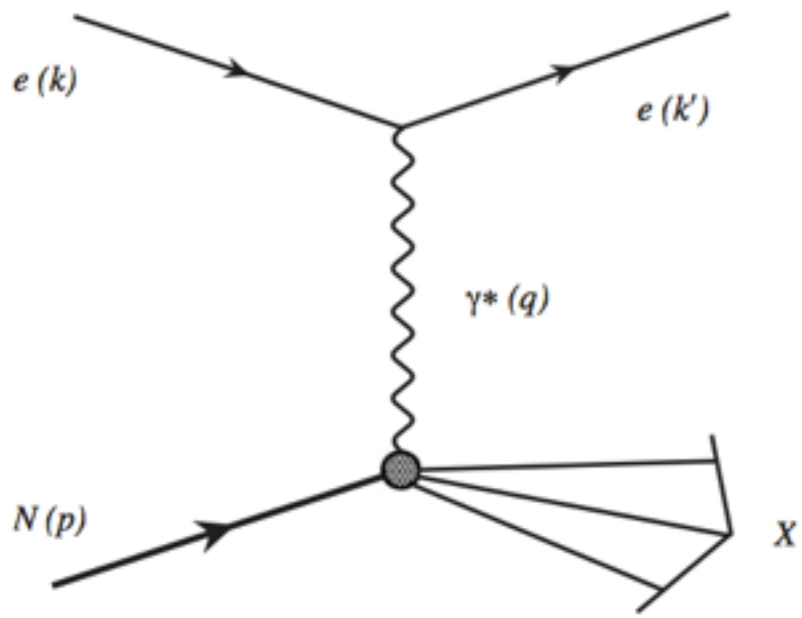
Data related to forward elastic amplitude by optical theorem

$I = 0$ **average proton/neutron target**

$I = 1$ **difference proton/neutron target**

Application to election-nucleon scattering

$$e^- p \to e^- X$$



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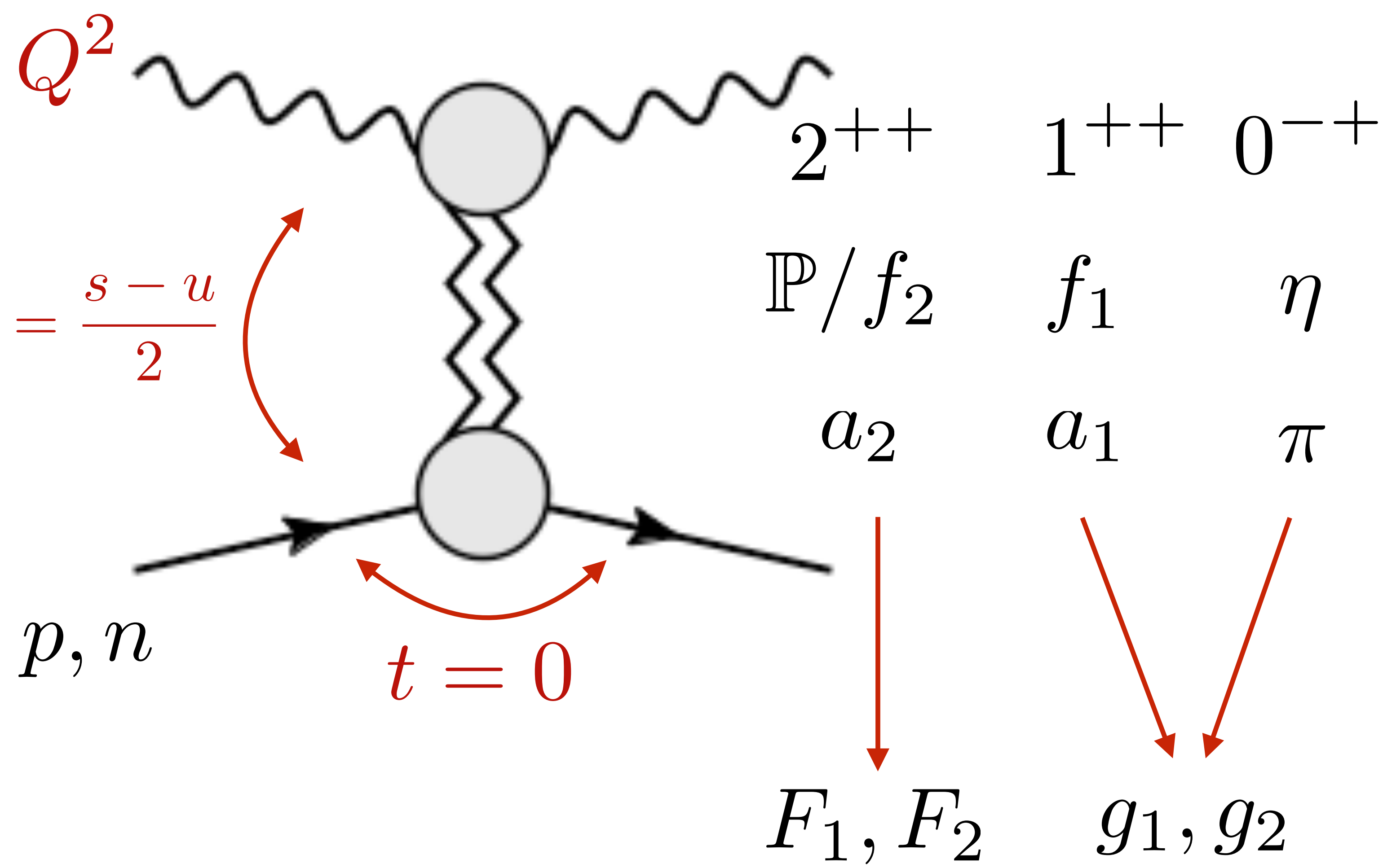

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Data related to forward elastic amplitude by optical theorem

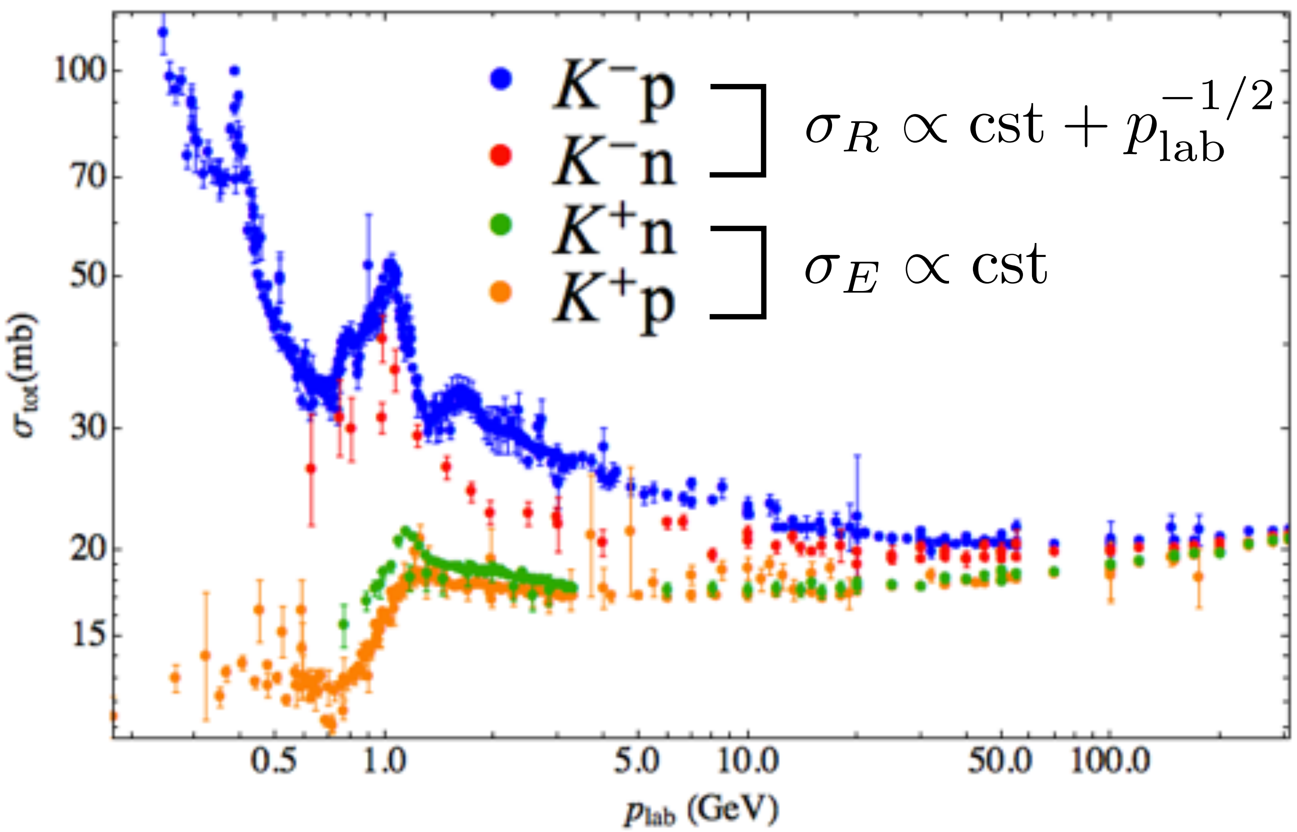
$$F^p_{1,2}(Q^2,\omega) + F^n_{1,2}(Q^2,\omega) \propto \omega^{\alpha_{\mathbb{P}}(Q^2)-1} + \omega^{\alpha_{\mathbb{R}}(Q^2)-1}$$

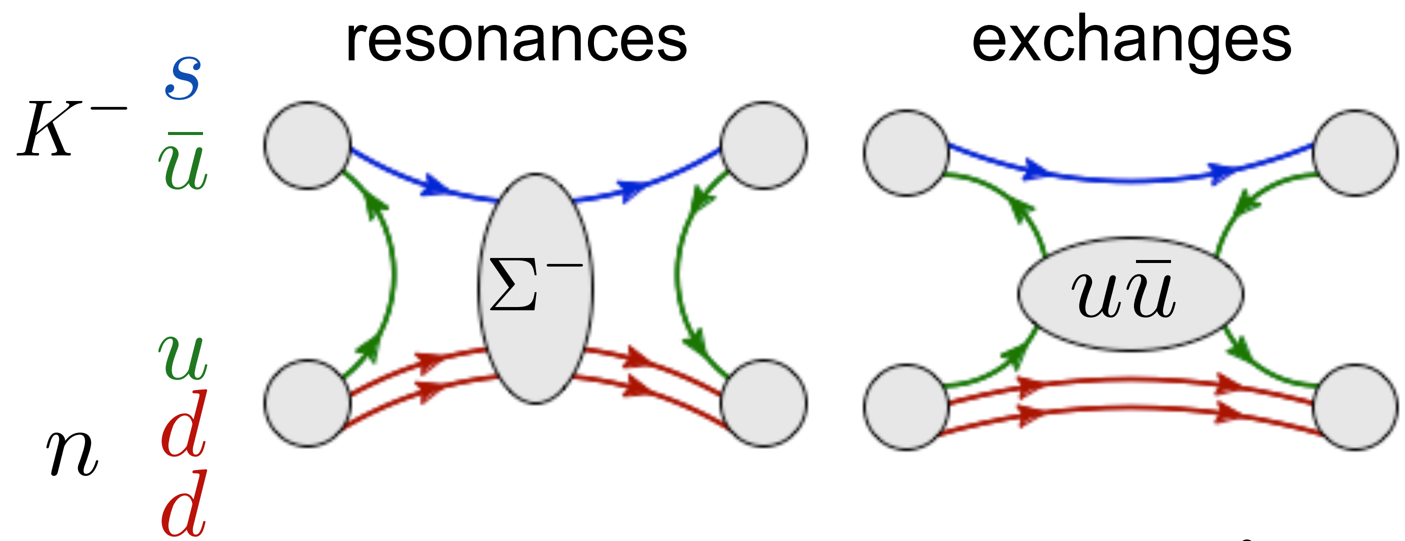
$$F^p_{1,2}(Q^2,\omega) - F^n_{1,2}(Q^2,\omega) \propto \omega^{\alpha_{\mathbb{R}}(Q^2)-1}$$

$$g^{p,n}_{1,2}(Q^2,\omega) \propto \omega^{\alpha_\pi(Q^2)-1}$$

Two-component Duality

Harari PRL20 (1968)





$u\bar{u} = \omega, \rho, f_2, a_2$

exotic channel (no resonance) → **flat cross section (Regge exchanges cancel out)**

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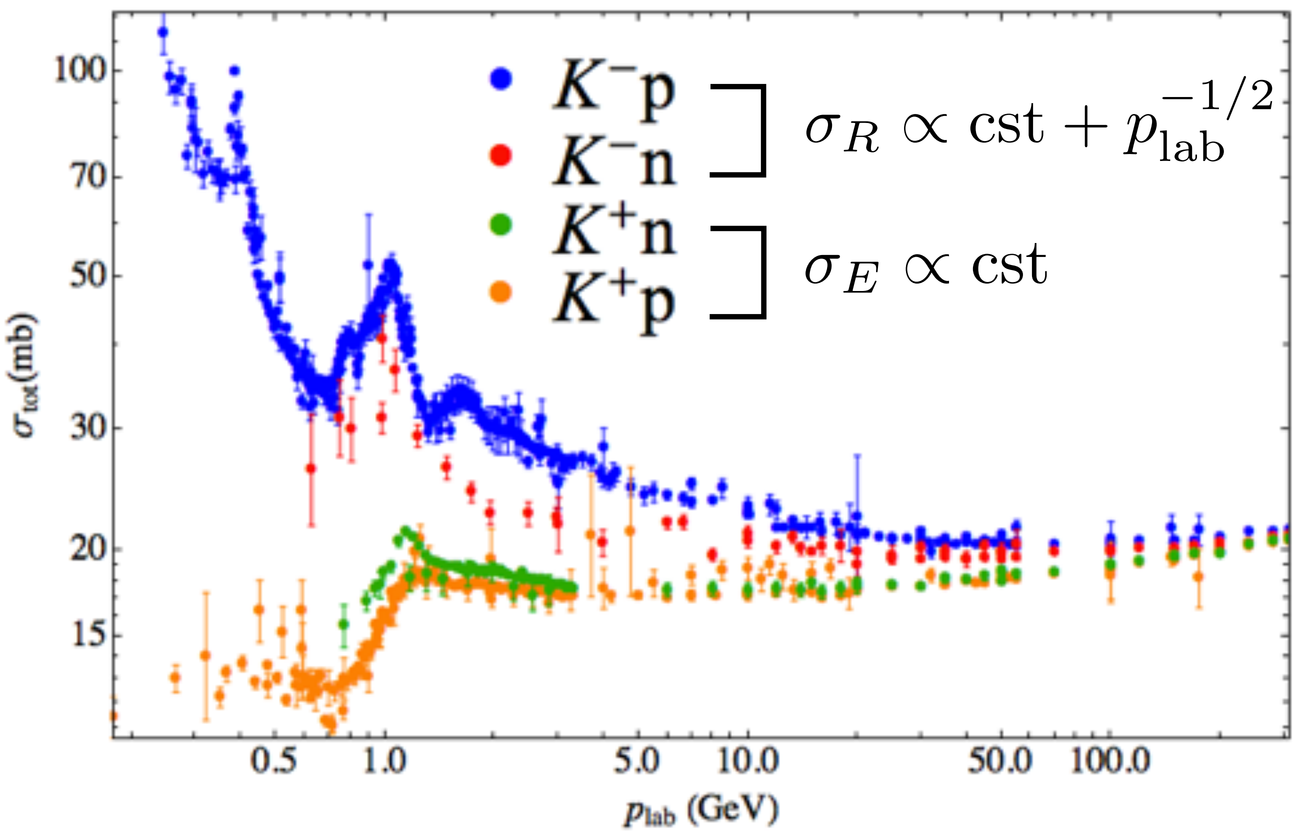

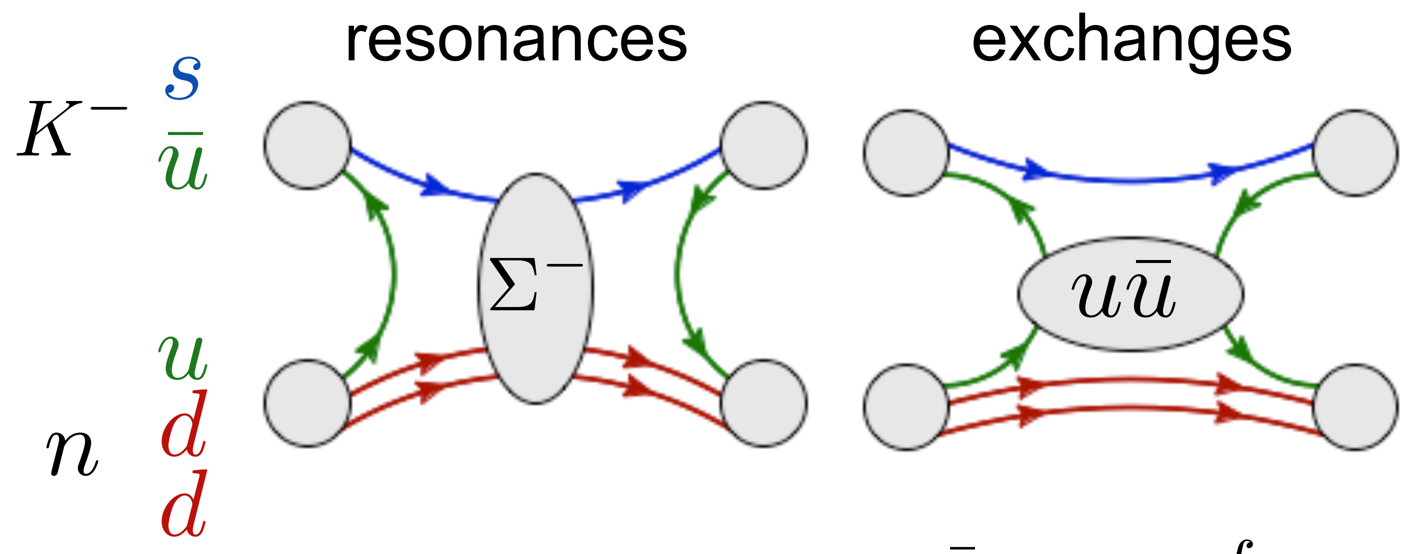

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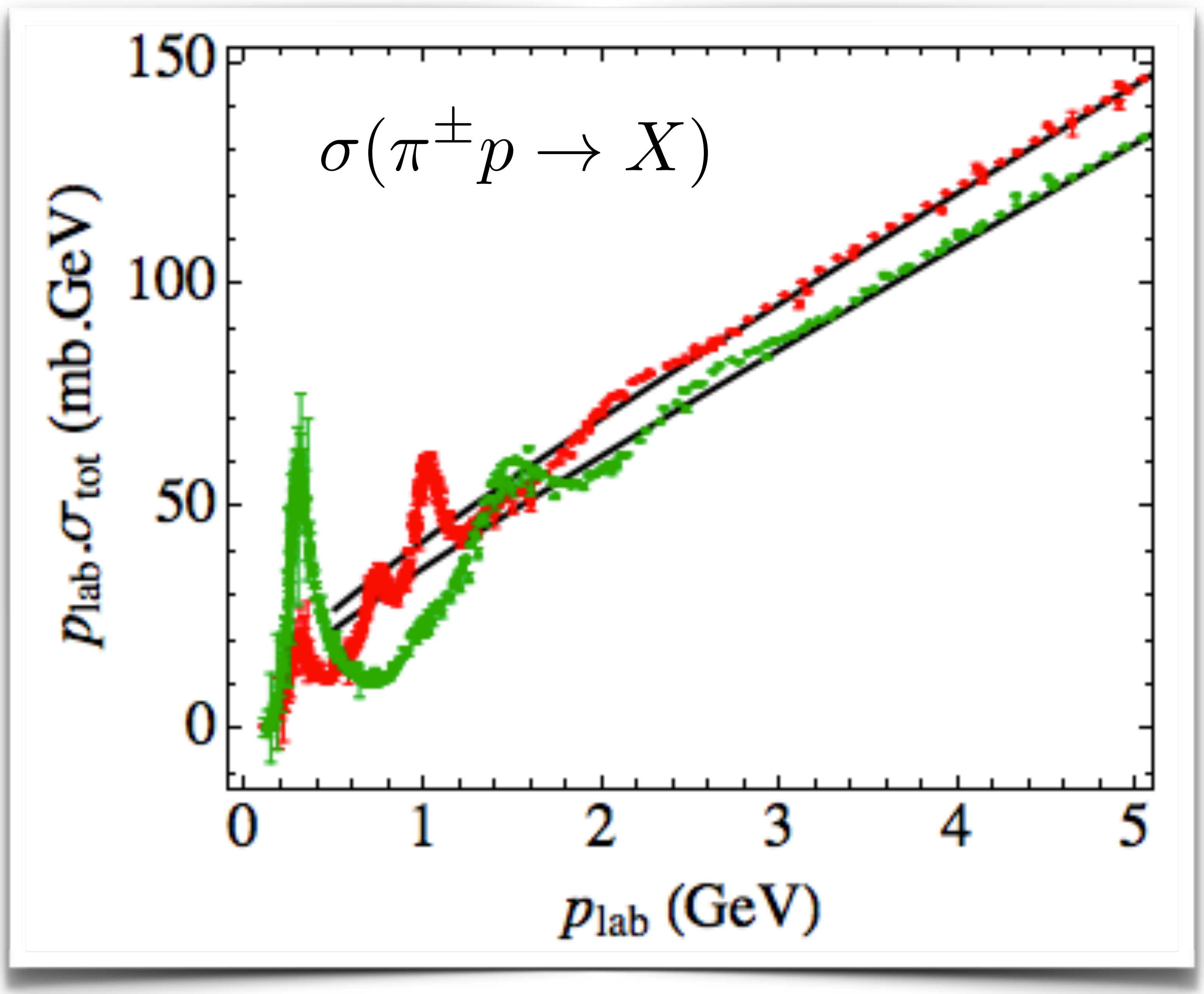
$$\int_{\nu_0}^{\Lambda} \mathrm{Im}\ A(\nu,t)d\nu = \beta_{\mathbb{P}}(t)\frac{\Lambda^{\alpha_{\mathbb{P}}(t)}}{\alpha_{\mathbb{P}}(t)+1} + \beta_{\mathbb{R}}(t)\frac{\Lambda^{\alpha_{\mathbb{R}}(t)}}{\alpha_{\mathbb{R}}(t)+1}$$

$$\mathrm{Im}\ A_{bkg} + \mathrm{Im}\ A_{res}$$

Application to pion-nucleon scattering

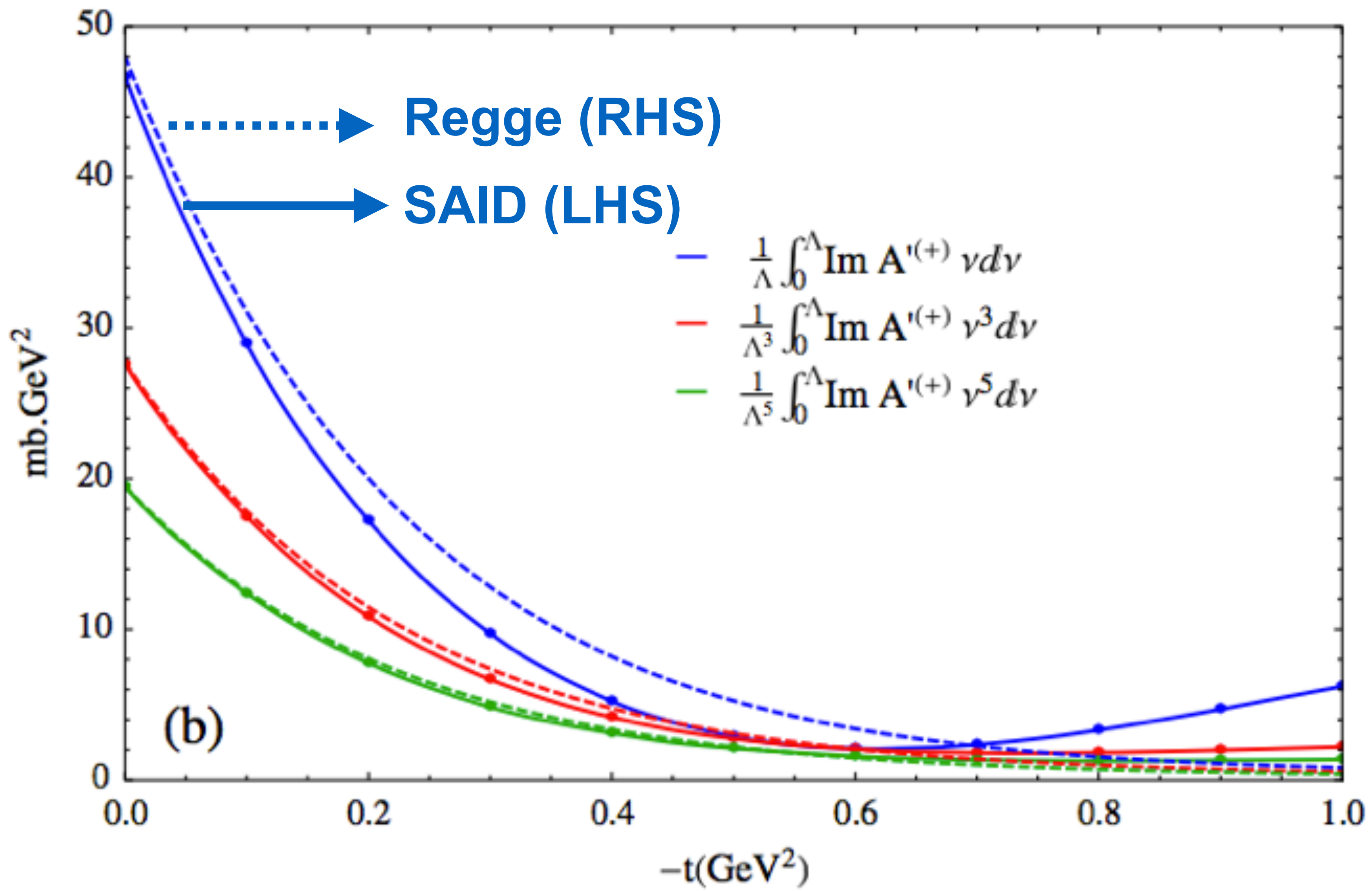
$$\int_{\nu_0}^{\Lambda} \text{Im } A(\nu, t_0) \frac{\nu^k}{\Lambda^k} d\nu = \beta(t_0) \frac{\Lambda^{\alpha(t_0)+1}}{\alpha(t_0) + k + 1}$$

$$\sigma(ab \to X) \propto \text{Im } A(\nu, t = 0)$$



Application to pion-nucleon scattering

$$\pi N \rightarrow \pi N$$

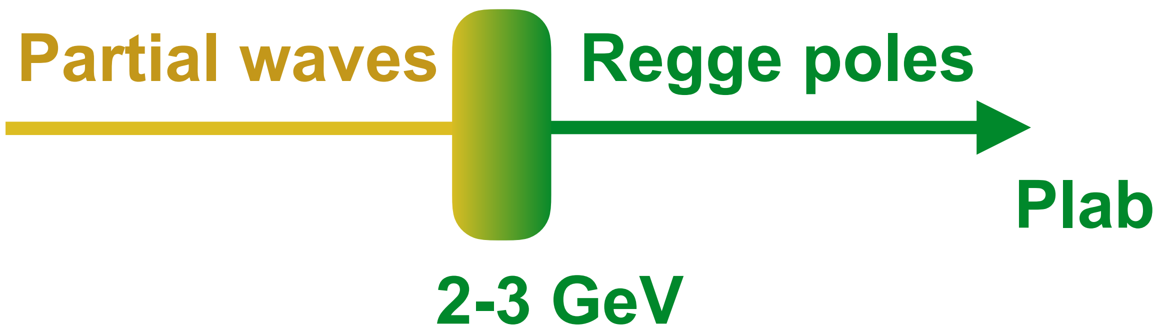


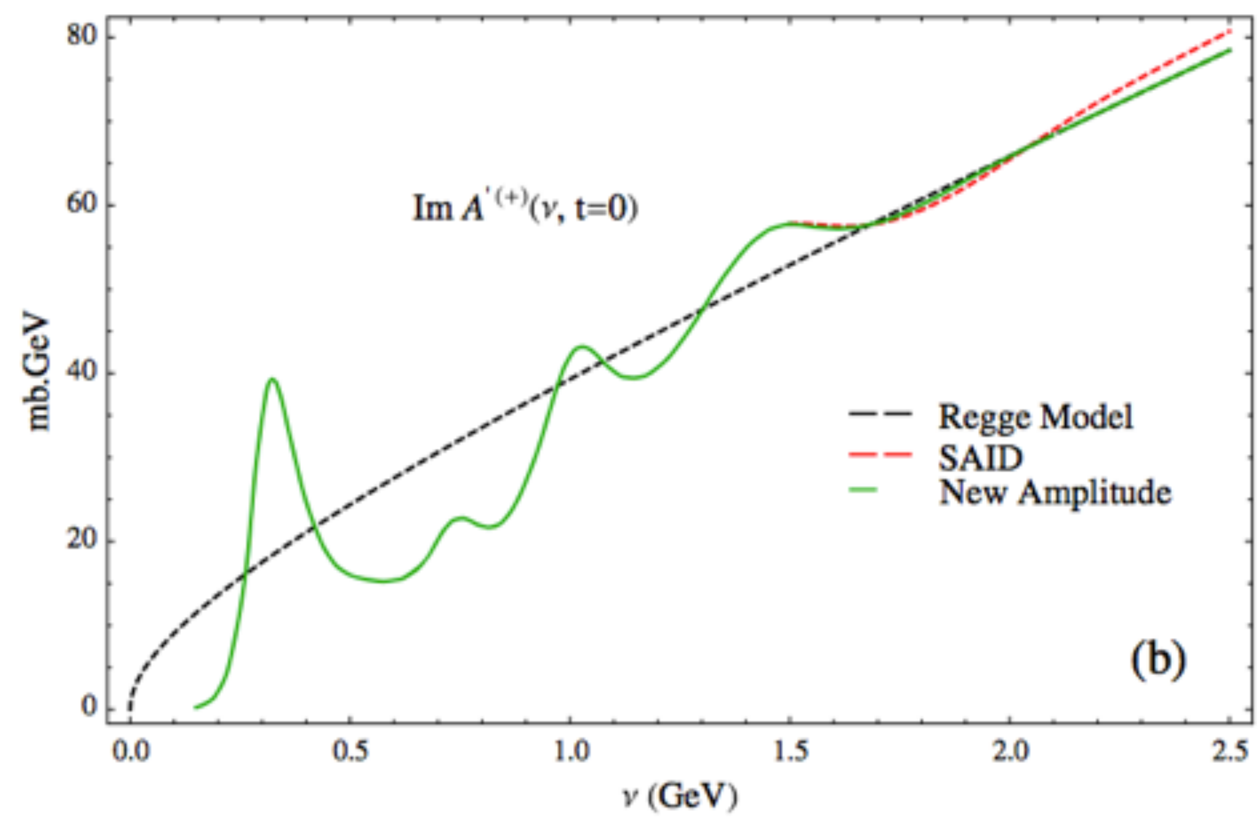
$$\frac{1}{\Lambda^k}\int_{\nu_0}^{\Lambda} \mathrm{Im}\ A(\nu,t)\nu^k d\nu = \frac{\beta(t)\Lambda^{\alpha(t)+1}}{\alpha(t)+k+1}$$

VM et al (JPAC) PRD92 (2015) ; arXiv:1506.01764

Application to pion-nucleon scattering

Match low energy (PW) and high energy (Regge) imaginary parts





Reconstruct the real part from the dispersion relation

$$A(\nu, t) = \frac{2}{\pi} \int_{\nu_0}^{\infty} \frac{\mathrm{Im}\, A(\nu', t)}{\nu'^2 - \nu^2} \nu' d\nu'$$

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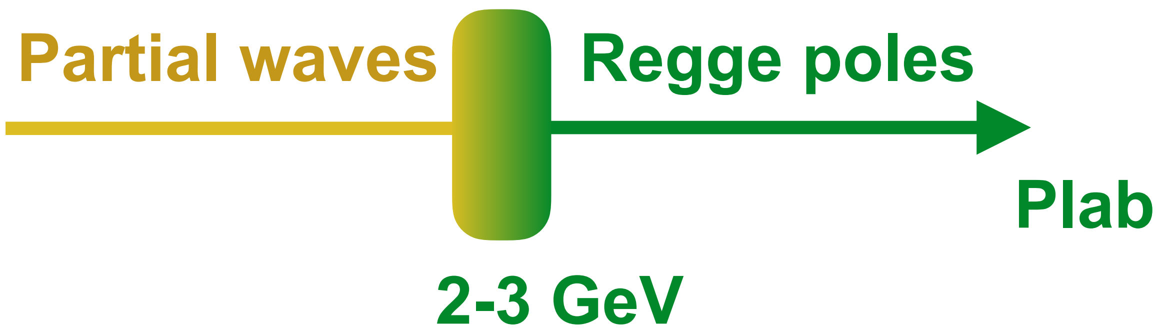

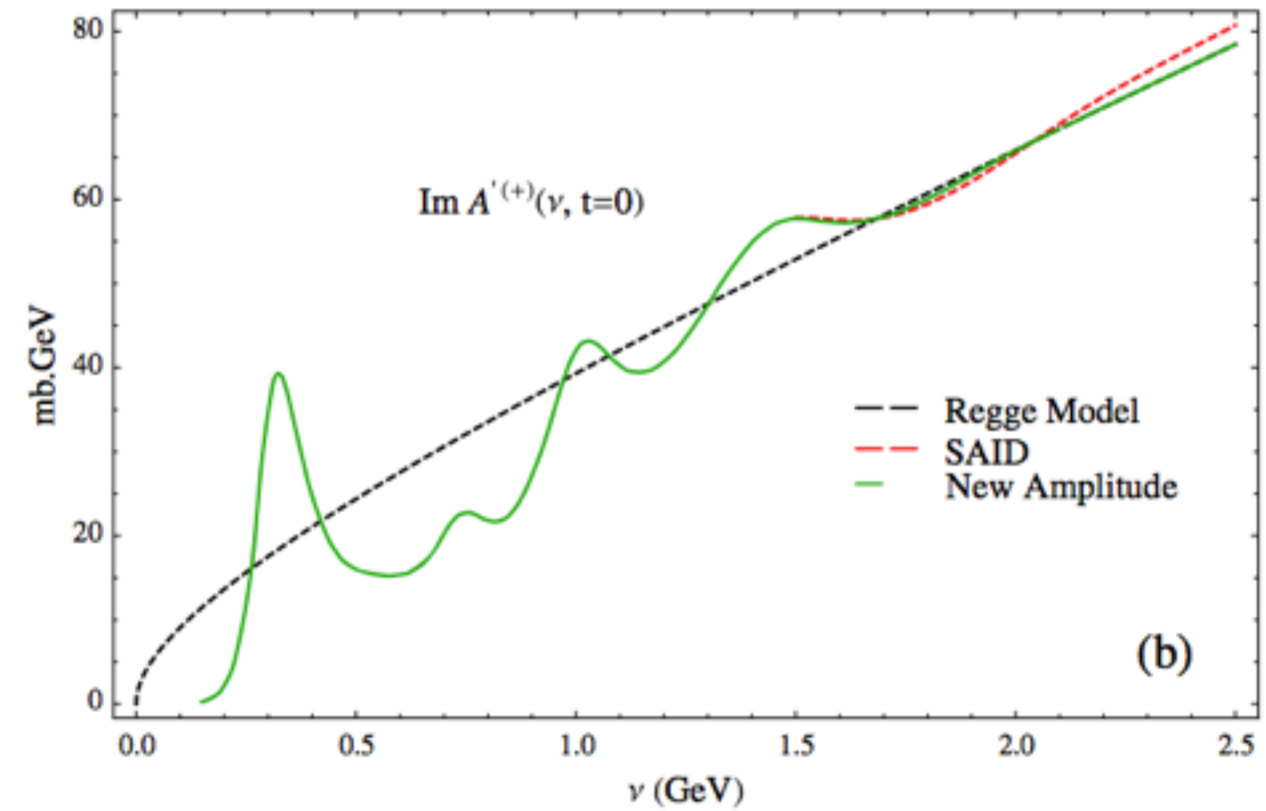

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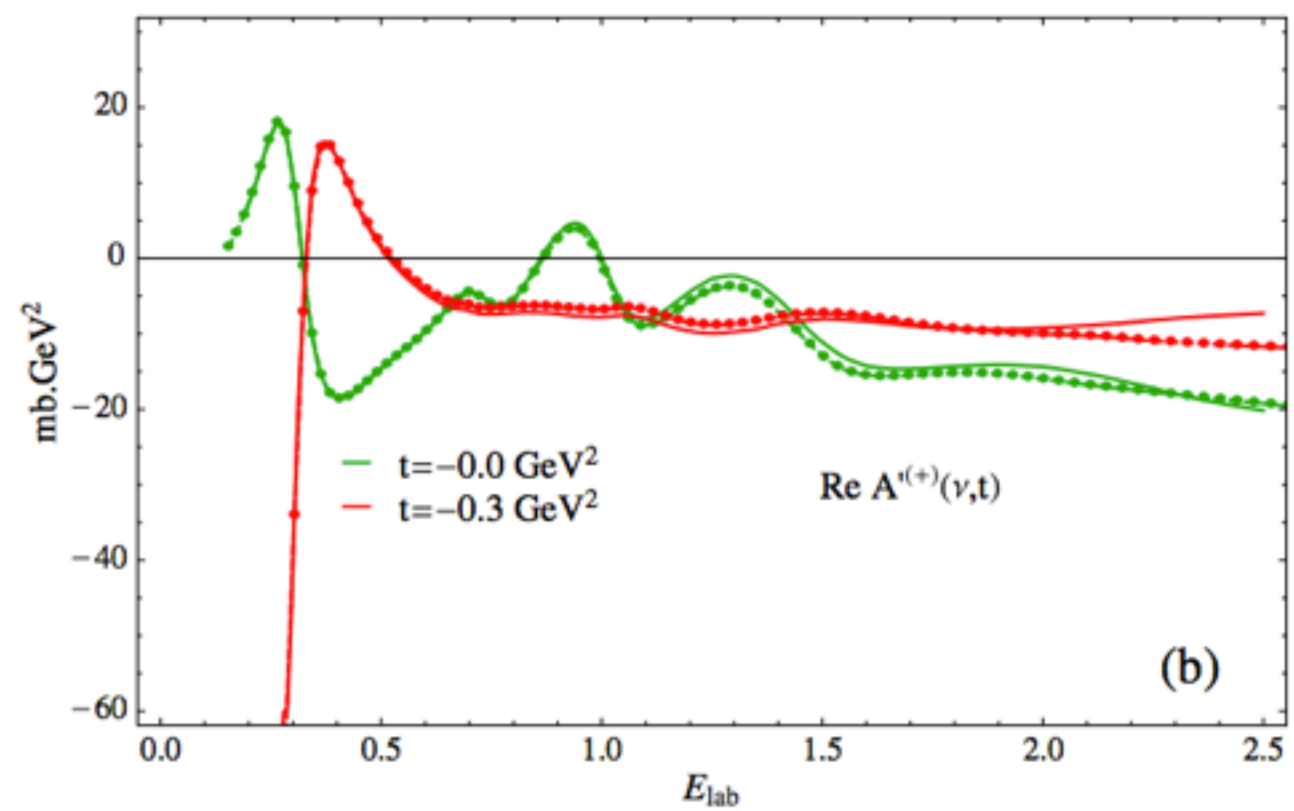

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High Energy Observables from Baryons

Effective residues extracted from baryons in $\gamma p \to \pi^0 p, \eta p$

$$\int_0^\Lambda \mathrm{Im}\ A_i(\nu,t)\nu^k d\nu = \beta_i(t)\frac{\Lambda^{\alpha(t)+k}}{\alpha(t)+k} \longrightarrow \beta_i(t) = \frac{\alpha(t)+k}{\Lambda^{\alpha(t)+k}}\int_0^\Lambda \mathrm{Im}\ A_i(\nu,t)\nu^k d\nu$$

Provide predictions for differential cross sections @JLab:

$$\frac{d\hat{\sigma}}{dt} = \frac{\nu^{2\alpha(t)-2}}{32\pi}\left[1+\tan^2\frac{\pi}{2}\alpha(t)\right]\left[\beta_1^2(t) - t\beta_4^2(t)\right]$$

VM et al. (JPAC) EPL122 (2018) ; arXiv:1708:07779
VM et al. (JPAC) PRD98 (2018) ; arXiv:1806.08414

High Energy Observables from Baryons

Effective residues extracted from baryons in $\gamma p \to \pi^0 p, \eta p$

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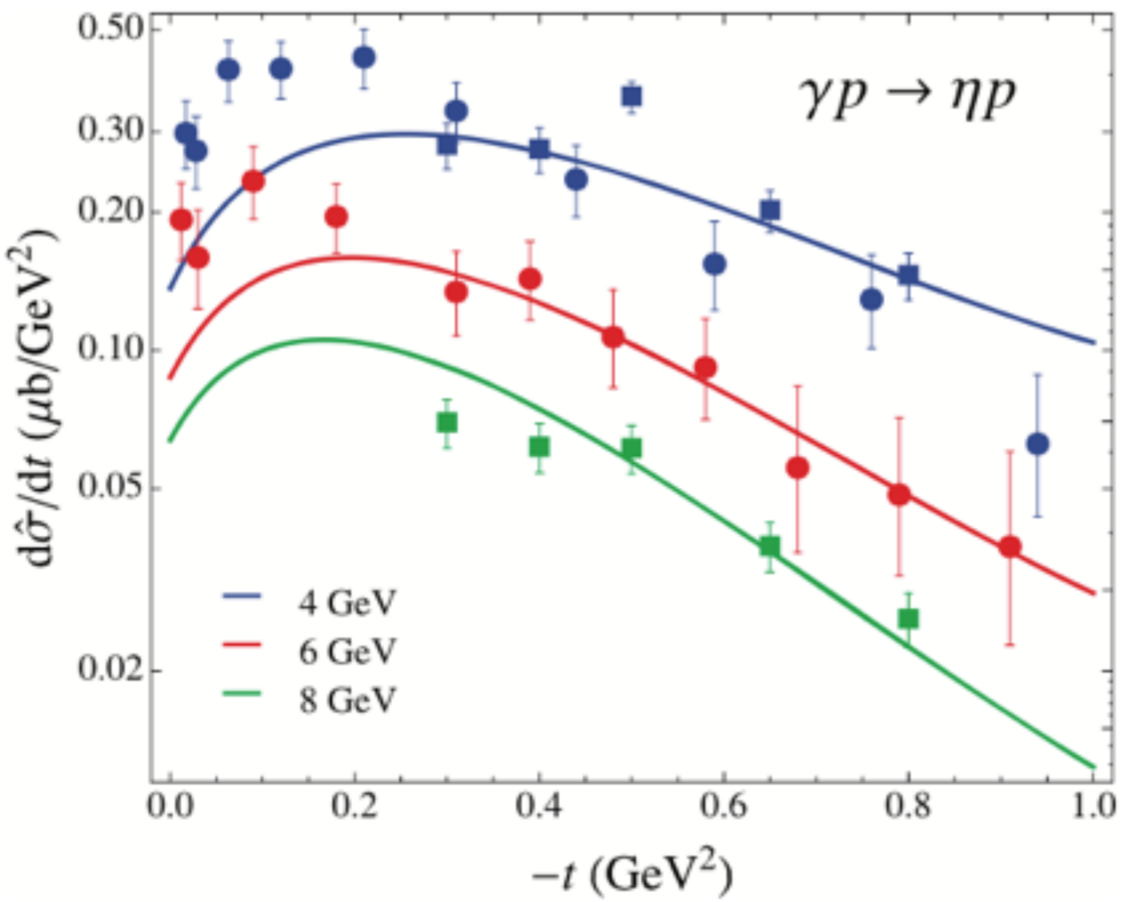

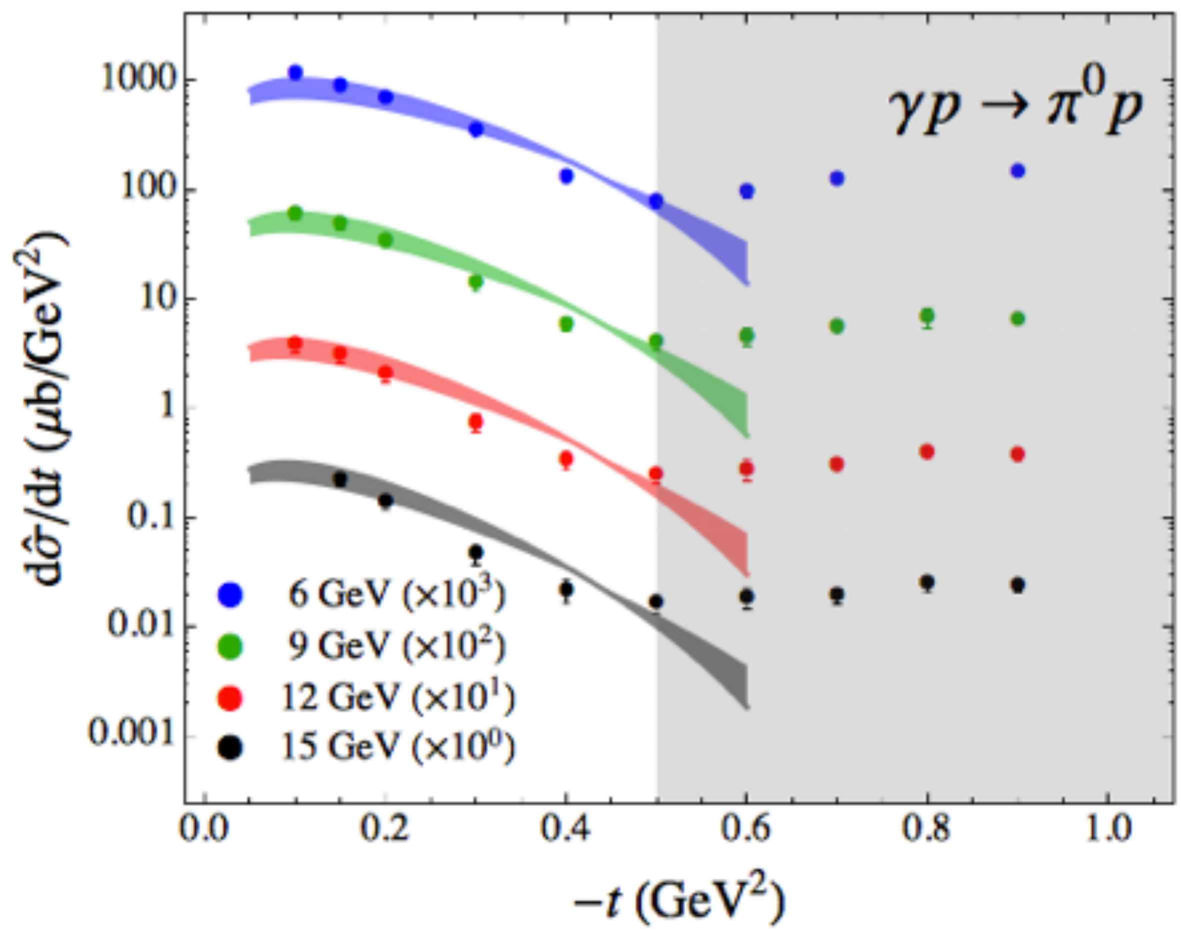

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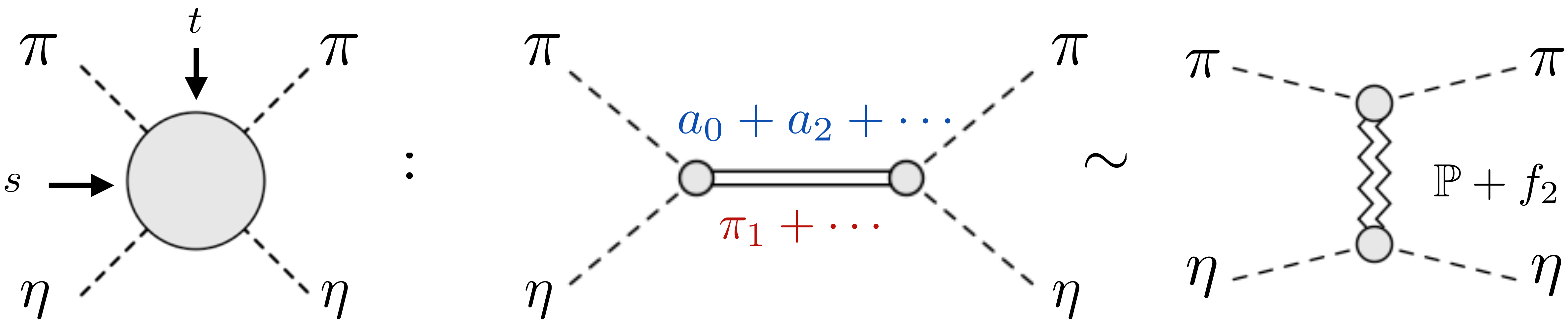
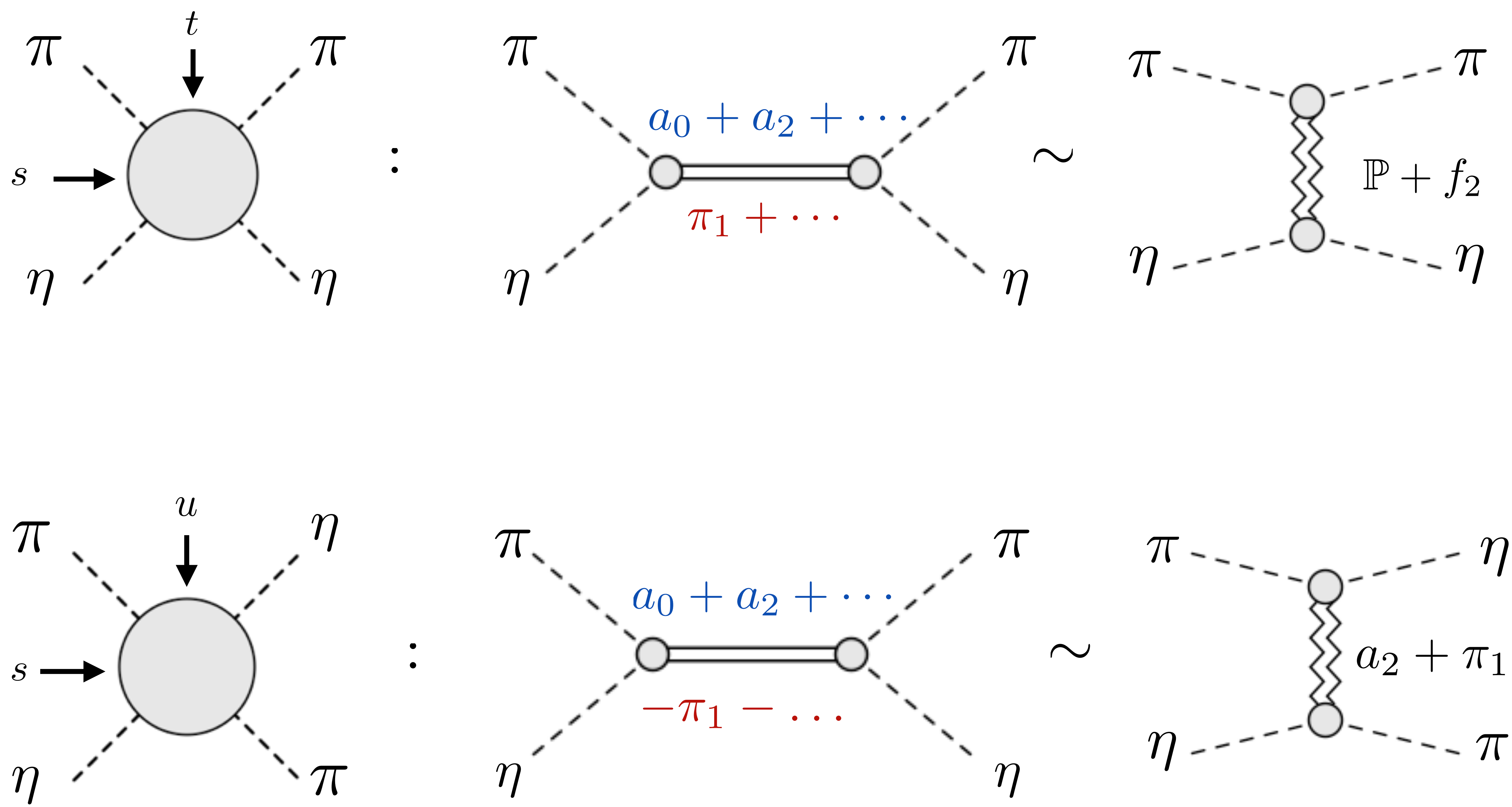
Duality for Exotic Meson



Duality for Exotic Meson

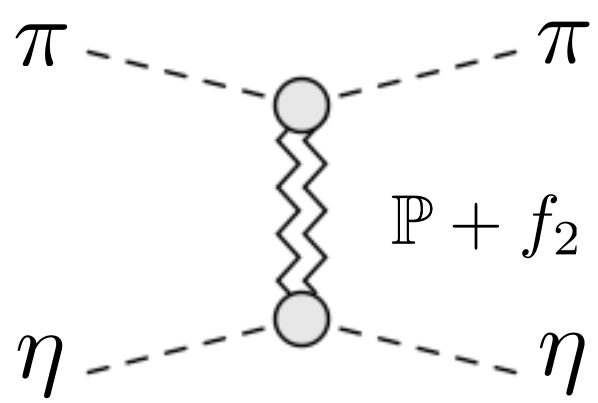


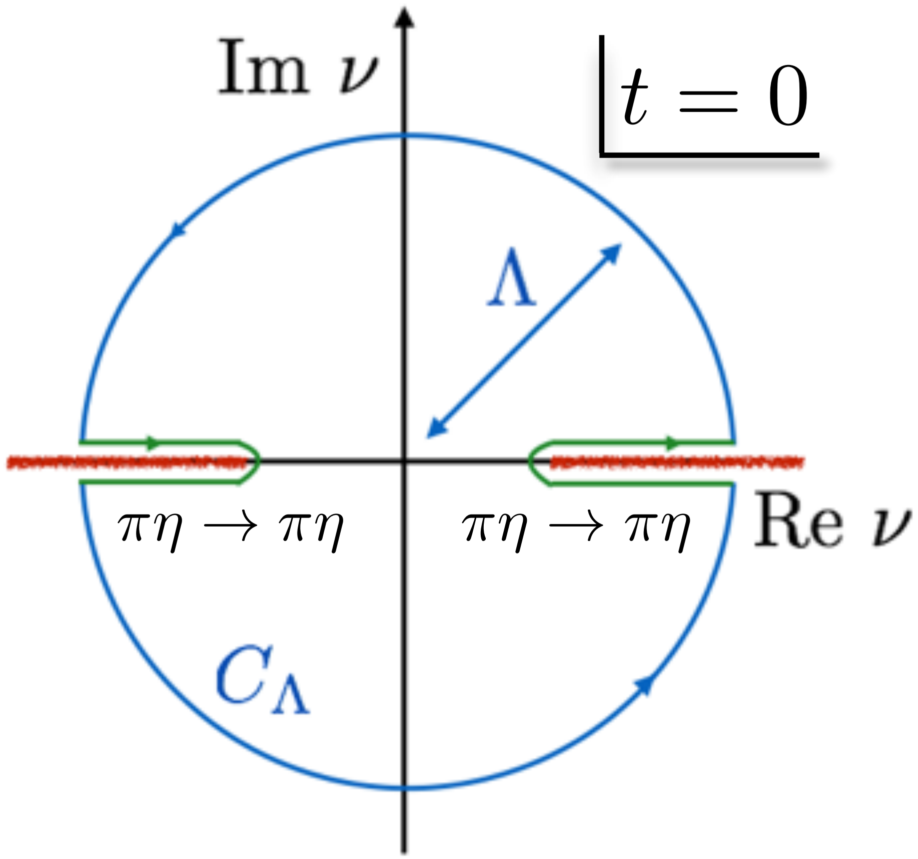
Forward and Backward Sum Rules

Forward:

$$\int_{\nu_0}^{\Lambda} \text{Im } A^{\pi\eta\to\pi\eta}(\nu, t=0)d\nu = \beta_{\mathbb{P}} \frac{\Lambda^{\alpha_{\mathbb{P}}+1}}{\alpha_{\mathbb{P}}+1} + \beta_f \frac{\Lambda^{\alpha_f+1}}{\alpha_f+1}$$

only even
signature exchange





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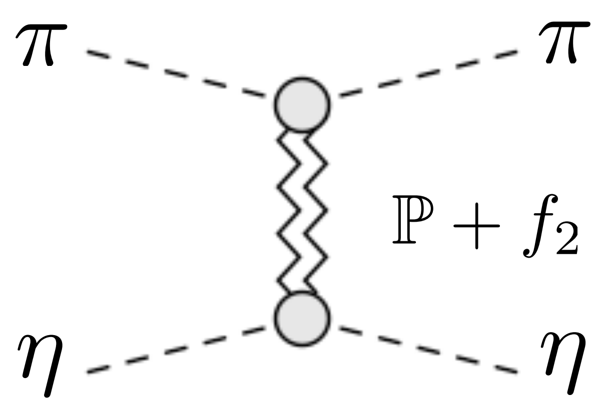
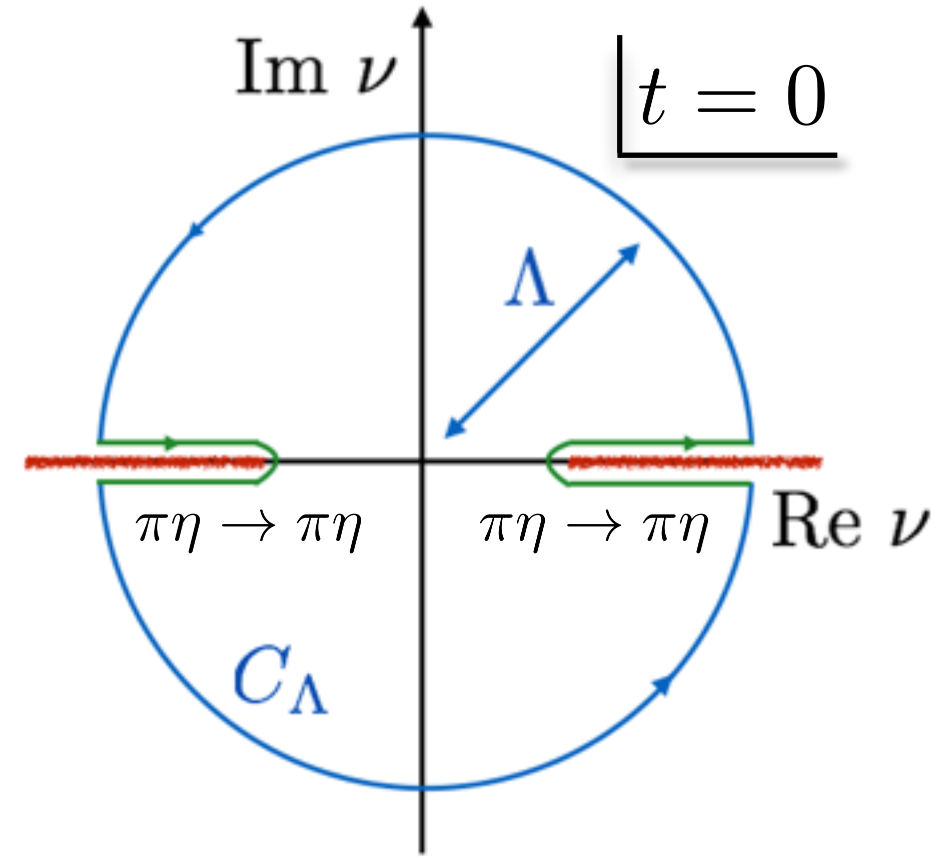


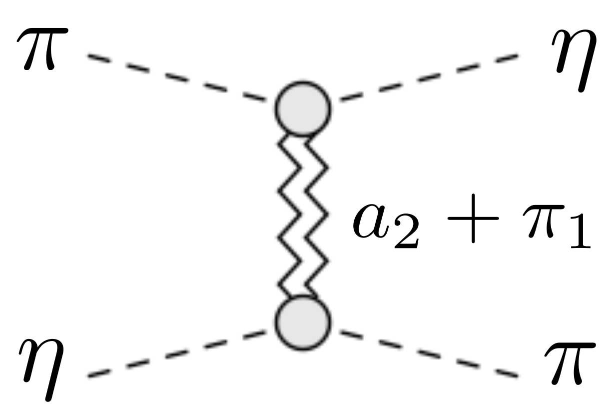
only even
signature exchange



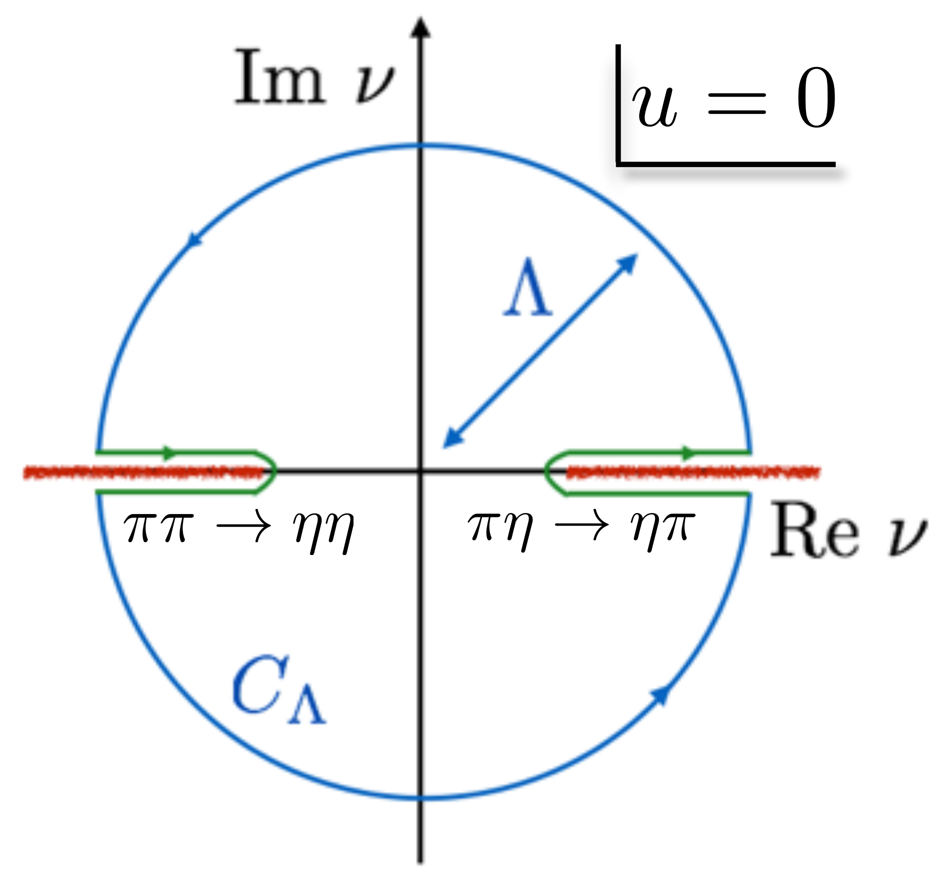
Backward:

$$\frac{1}{2}\int_{\nu_0}^{\Lambda} \text{Im } \left[A^{\pi\eta\to\pi\eta}(\nu, u=0) + A^{\pi\pi\to\eta\eta}(\nu, u=0)\right] d\nu = \beta_a \frac{\Lambda^{\alpha_a+1}}{\alpha_a+1}$$

$$\frac{1}{2}\int_{\nu_0}^{\Lambda} \text{Im } \left[A^{\pi\eta\to\pi\eta}(\nu, u=0) - A^{\pi\pi\to\eta\eta}(\nu, u=0)\right] \frac{\nu}{\Lambda} d\nu = \beta_\pi \frac{\Lambda^{\alpha_\pi+1}}{\alpha_\pi+1}$$



even and odd
signature exchange



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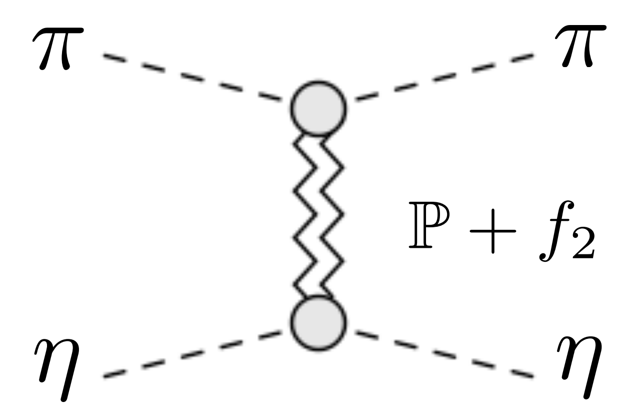

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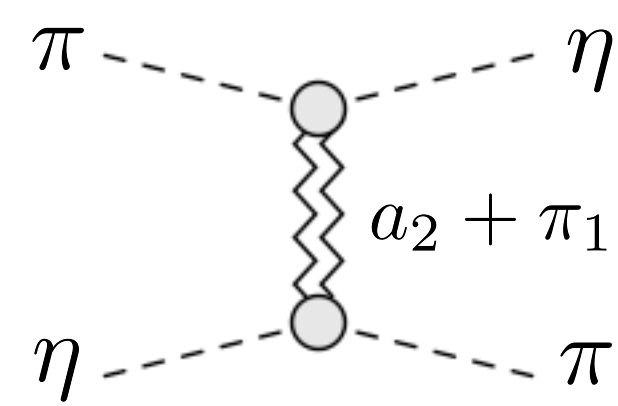

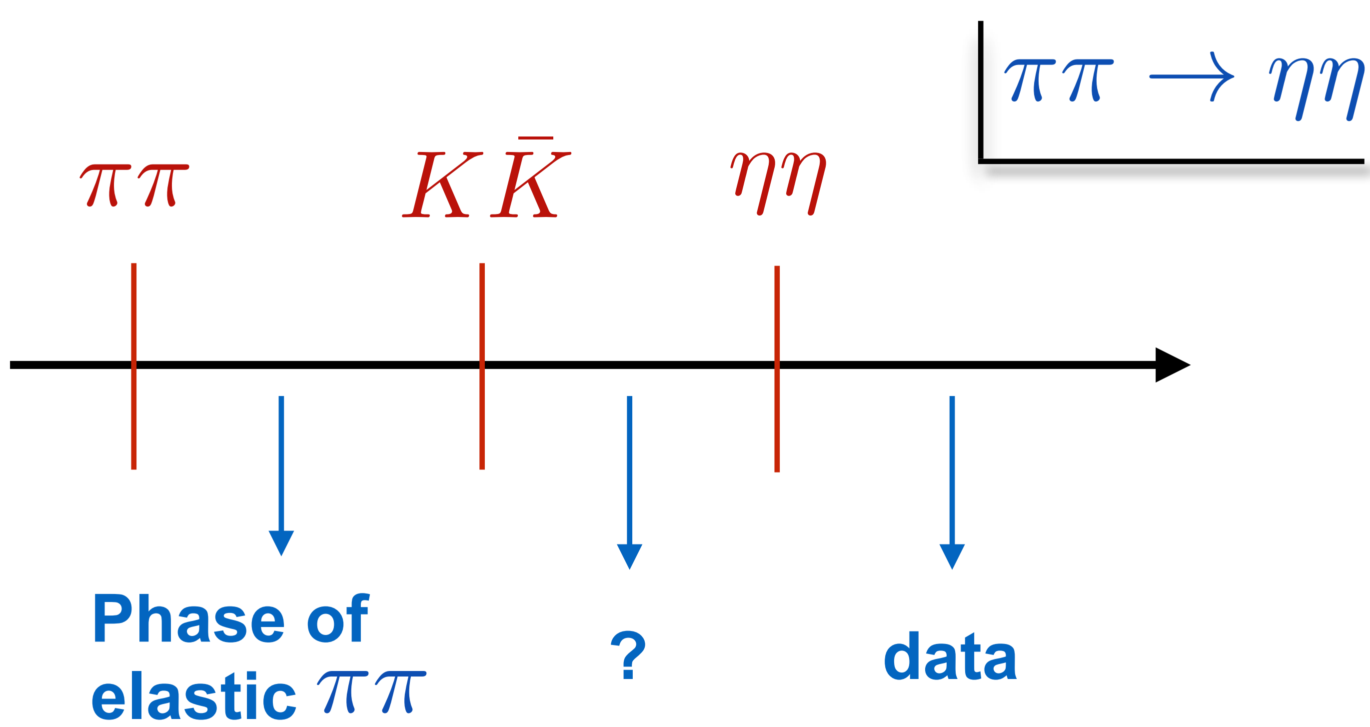

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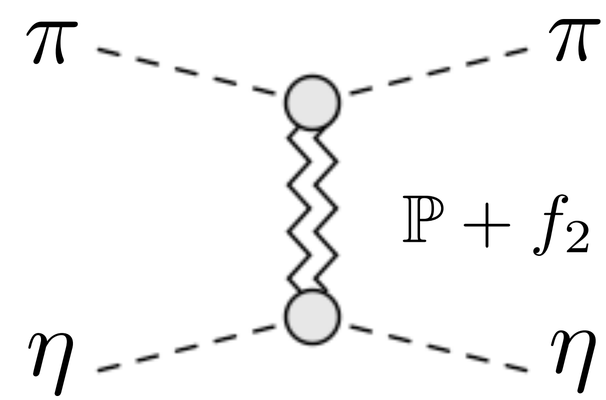

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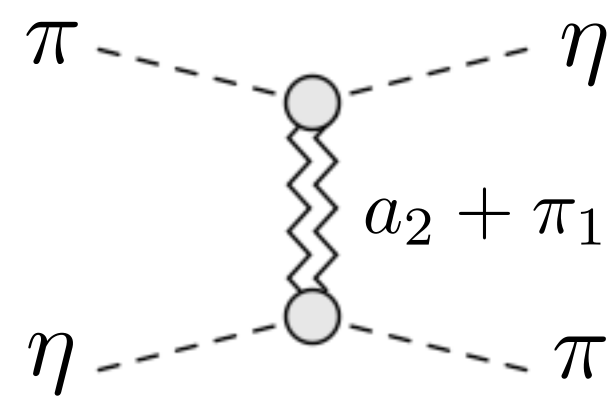
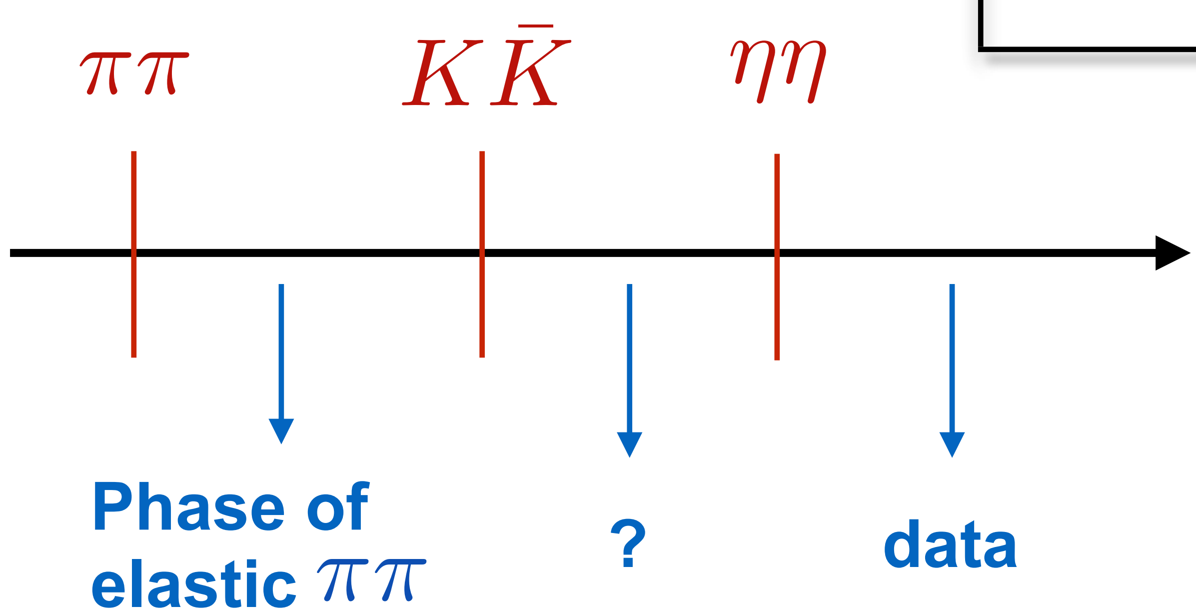


$$\text{Im } A^{\pi\eta\to\pi\eta}(\nu, u=0) \approx \text{Im } A^{\pi\pi\to\eta\eta}(\nu, u=0)$$

$\pi\pi \to \eta\eta$



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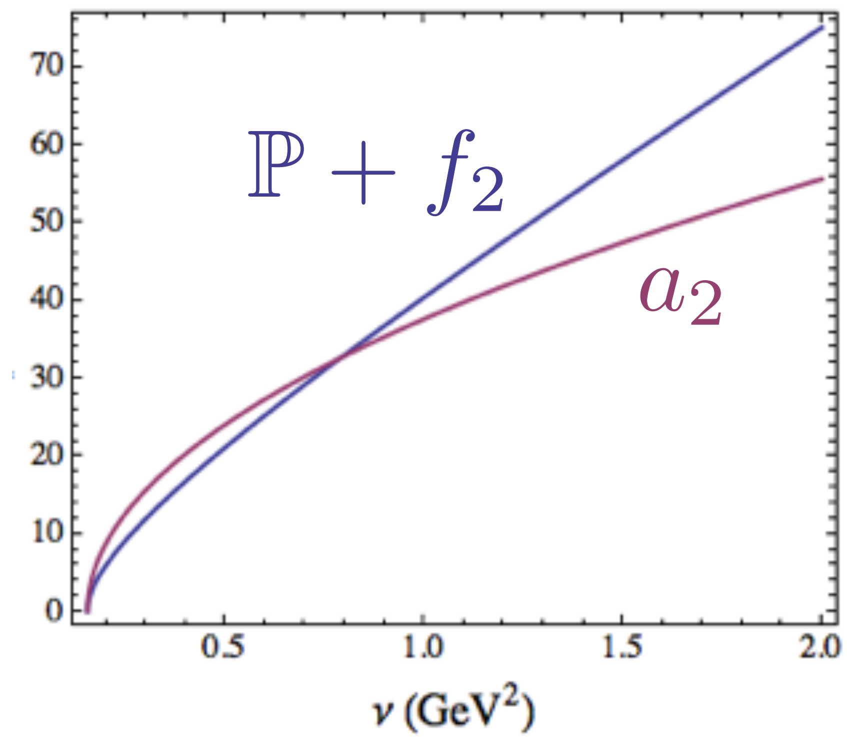

Combine and PW projection:

$$3\int_{\nu_0}^{\Lambda} \mathrm{Im}\ t_1(\nu)P_1(z)d\nu = \beta_{\mathbb{P}} \frac{\Lambda^{\alpha_{\mathbb{P}}+1}}{\alpha_{\mathbb{P}}+1} + \beta_f \frac{\Lambda^{\alpha_f+1}}{\alpha_f+1} - \beta_a \frac{\Lambda^{\alpha_a+1}}{\alpha_a+1} + \mathcal{O}(m_\eta^2 - m_\pi^2)$$

Forward and Backward Sum Rules

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$$\int_{\nu_0}^{\Lambda} \text{Im } A^{\pi\eta\to\pi\eta}(\nu, t=0)d\nu = \beta_{\mathbb{P}} \frac{\Lambda^{\alpha_{\mathbb{P}}+1}}{\alpha_{\mathbb{P}}+1} + \beta_f \frac{\Lambda^{\alpha_f+1}}{\alpha_f+1}$$

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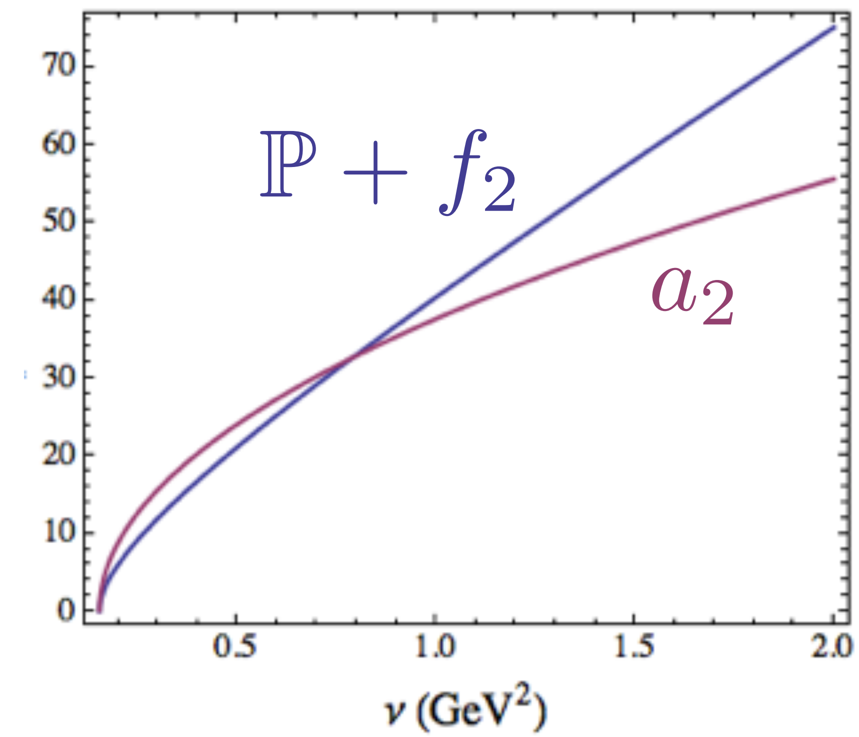
$$\int_{\nu_0}^{\Lambda} \text{Im } A^{\pi\eta\to\pi\eta}(\nu, u=0)d\nu = \beta_a \frac{\Lambda^{\alpha_a+1}}{\alpha_a+1}$$



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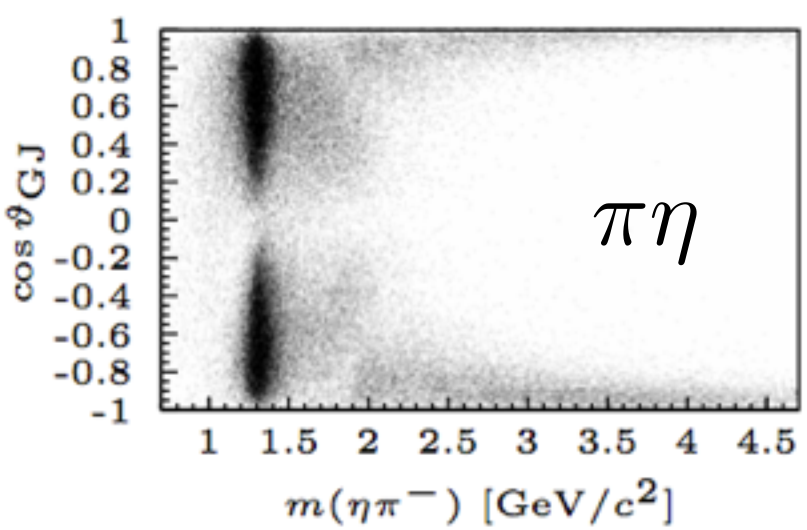
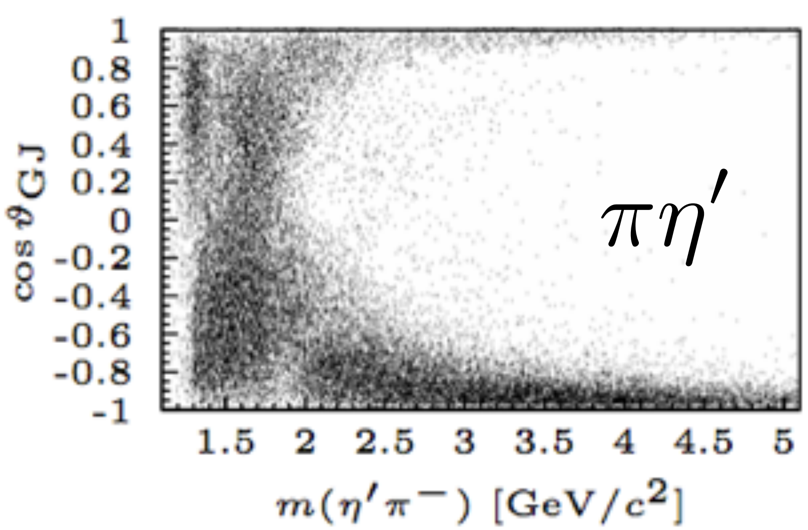
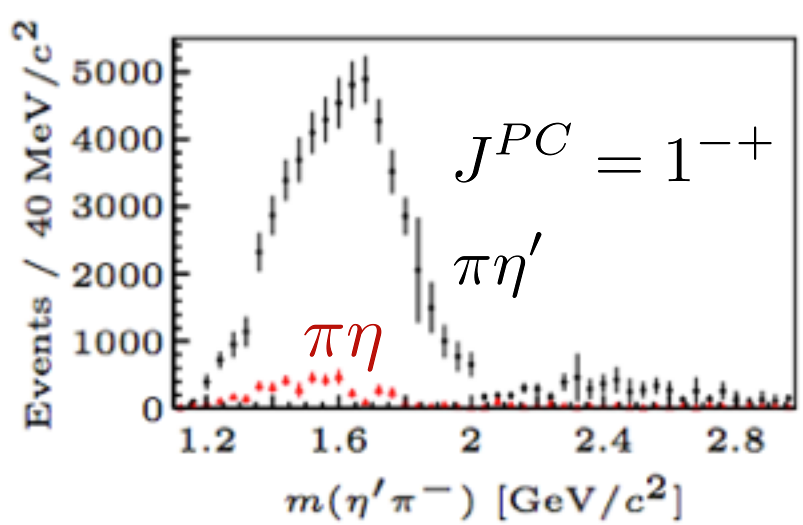




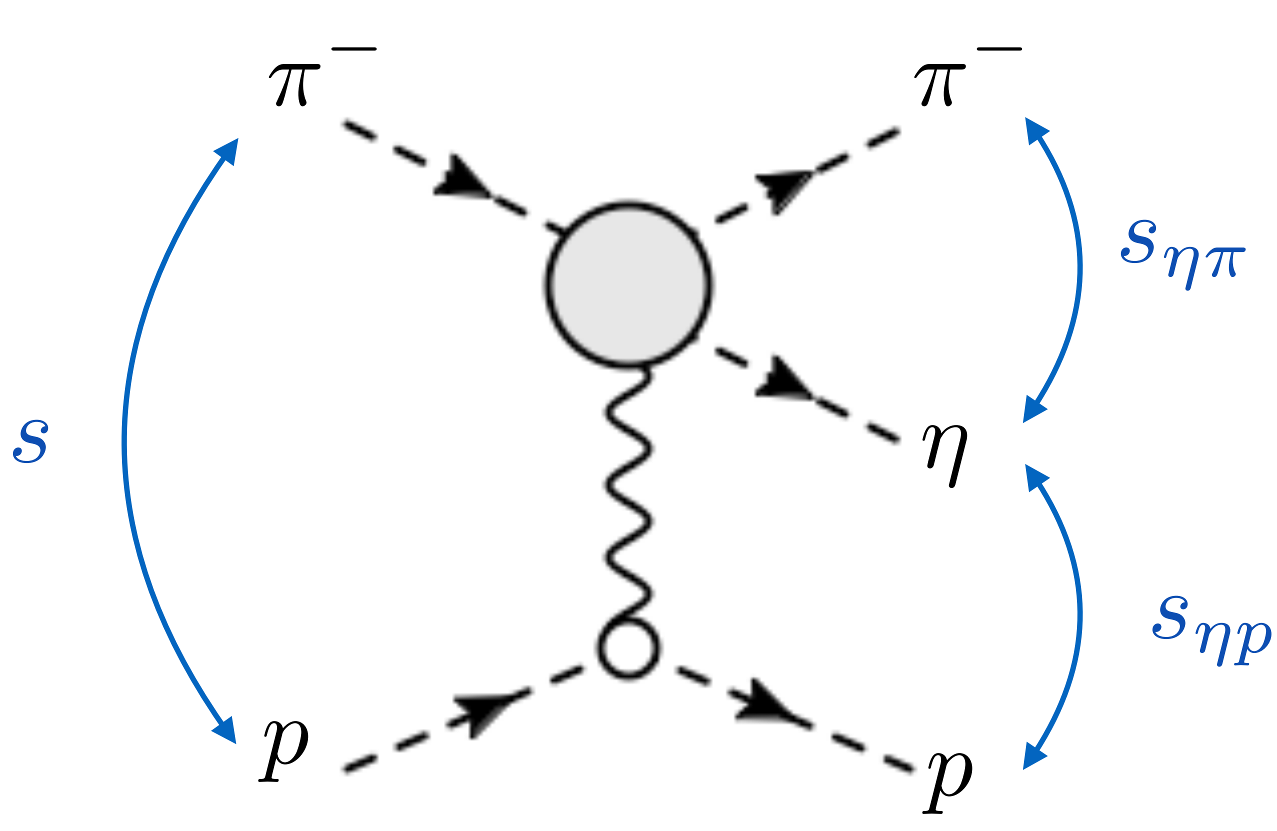


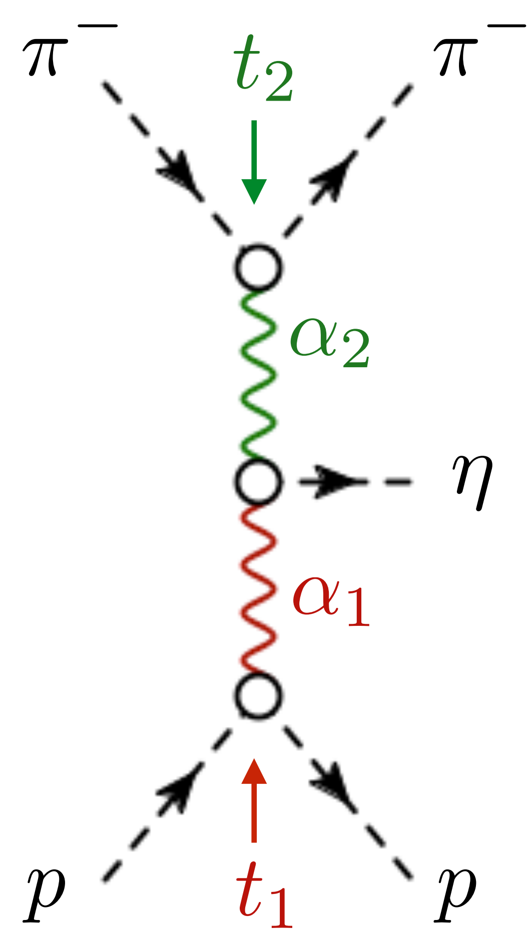
Dispersion Relation for 2-to-3

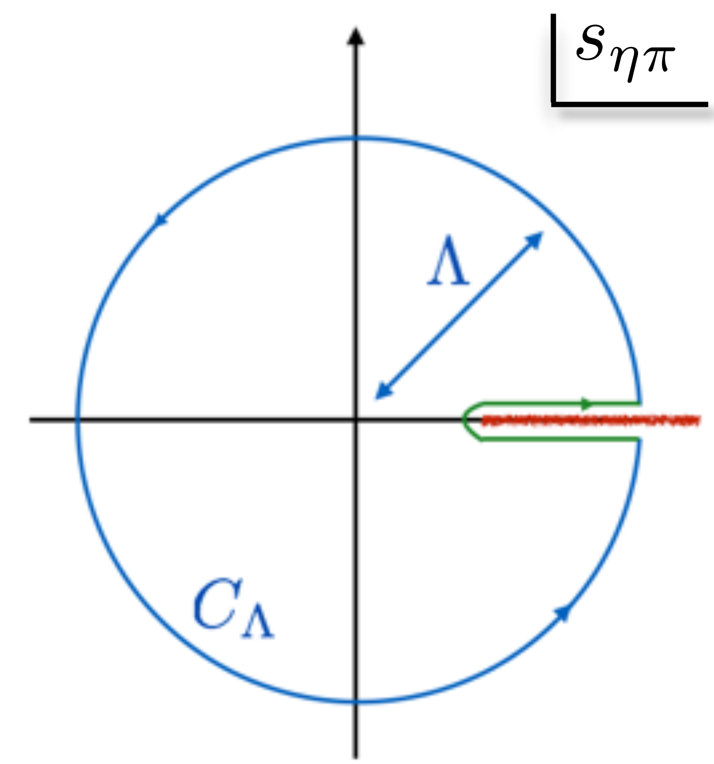
$$A = (-s)^{\alpha_1}\left\{ \sum_{i=0}^{\infty} \frac{\Gamma(1+i-\alpha_1)}{\Gamma(i+1)} \left(-\frac{s_{\eta p}}{s}\right)^{i} \left[\oint \frac{dJ_2}{2\pi i}\Gamma(\alpha_1 - J_2 - i)(-s_{\eta\pi})^{J_2-\alpha_1+i} R(J_2, \alpha_1 - i, t_1, t_2)\right] + \left(\frac{s_{\eta p}}{s}\right)^{\alpha_1-\alpha_2} \sum_{i=0}^{\infty} \frac{\Gamma(1+i-\alpha_1)}{\Gamma(i+1)} \Gamma(\alpha_2-\alpha_1-i)\beta(\alpha_2-i, t_1, t_2)\left(\frac{s_{\eta\pi}s_{\eta p}}{s}\right)^{i} \right\}$$

infinite number of subtractions

Reggeon-particle amplitude



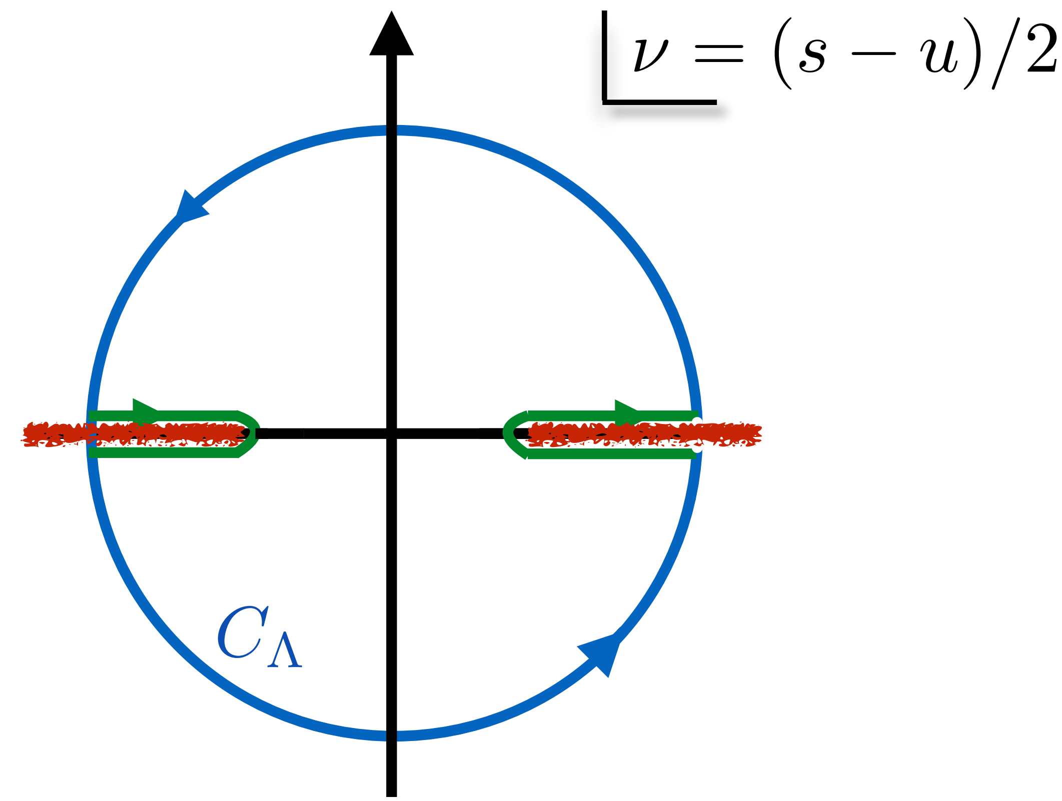




Consider $t_1,\ t_2,\ s_{\eta p}/s$ **fixed**

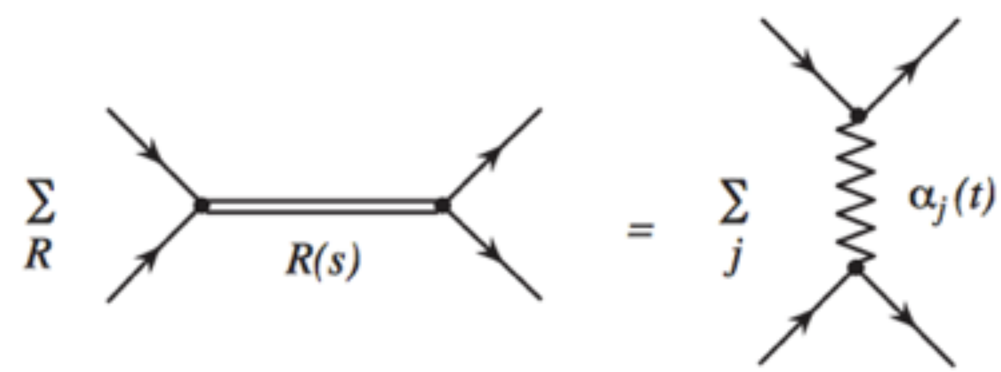
Summary

Duality can be expressed by sum rules (FESR) thanks to dispersion relations



Use FESR to contrain resonances by high energy data

Use FESR to make predictions for high energy experiments



Ongoing developments of FESR for 2-to-3 reactions (production of 2 mesons)

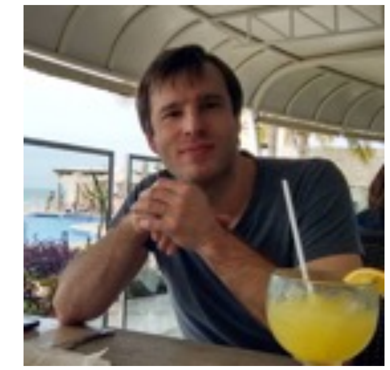
$$\int_{\nu_0}^{\Lambda} \mathrm{Im}\; A(\nu, t_0)\frac{\nu^k}{\Lambda^k}d\nu = \beta(t_0)\frac{\Lambda^{\alpha(t_0)+1}}{\alpha(t_0)+k+1}$$

Joint Physics Analysis Center

Emilie Passemar Indiana U. | **Andrew Jackura** Indiana U. | **Nathan Sherrill** Indiana U. | **Tim Londergan** Indiana U.

Astrid Hiller Blin Mainz U. | **Misha Mikhasenko** Bonn U. | **VM** JLab | **Jannes Nys** Ghent U. | **Adam Szczepaniak** Indiana U.

JPAC *Joint Physics Analysis Center*



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Interactive webpage: http://www.indiana.edu/~jpac/

Joint Physics Analysis Center

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JPAC acknowledges support from DOE and NSF

NEWS

Photoproduction:

1. High energy model for π photoproduction constrained by finite energy sum rules: $\gamma N \to \pi N$ page
2. High energy model for $\pi\Delta$ photoproduction beam asymmetry: (in construction)
3. High energy model for ρ^0, ω, ϕ spin density matrix elements: $\gamma p \to V p$ page
4. High energy model for η' photoproduction beam asymmetry: $\gamma p \to \eta^{(\prime)} p$ page
5. High energy model for η photoproduction: $\gamma p \to \eta p$ page
6. High energy model for π^0 photoproduction: $\gamma p \to \pi^0 p$ page
7. High energy model for J/ψ photoproduction: $\gamma p \to J/\psi p$ page

Hadroproduction:

1. Pion-nucleon scattering:
   - Amplitudes $\pi N \to \pi N$ amplitude page
   - Finite energy sum rules $\pi N \to \pi N$ FESR page
2. Kaon-nucleon scattering: $\bar{K}N \to \bar{K}N$ page

Light Meson Decay:

1. η meson into three pions: $\eta \to 3\pi$ page
2. vector meson into three pions: $\omega, \phi \to 3\pi$ page

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JPAC acknowledges support from DOE and NSF

Resources

- **Publication:** [Mat15a]
- **Fortran:** Fortran file, Input file, Output file
- **C/C++:** AmpTools class, C/C++ file, AmpTools class header
- **Mathematica:** notebook , converted in text
- **Data:** Anderson, All data
- **Contact person:** Vincent Mathieu
- **Last update:** November 2015

Description of the Fortran code: [show/hide]
Description of the C/C++ code: [show/hide]

Run the code

Choose the beam energy in the lab frame E_γ , the other variable (t or $\cos\theta$) and its minimal, maximal, and increment values.
If you choose t (cos) only the min, max and step values of t ($\cos\theta$) are read.

E_γ in GeV 9

t cos

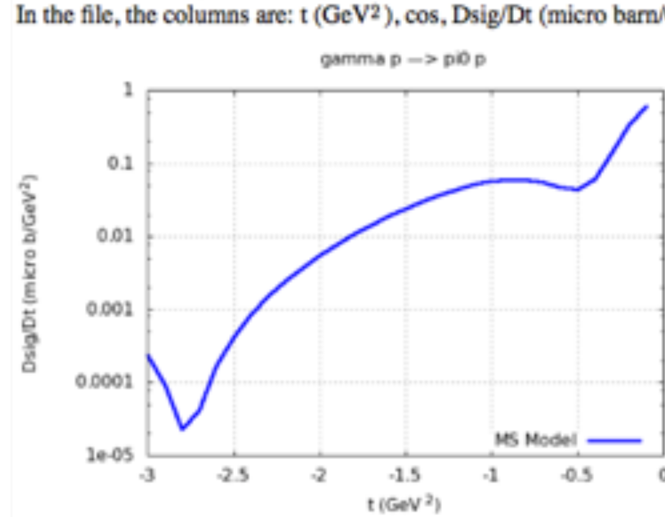
t in GeV² (min max step) -3 -0.1 0.1

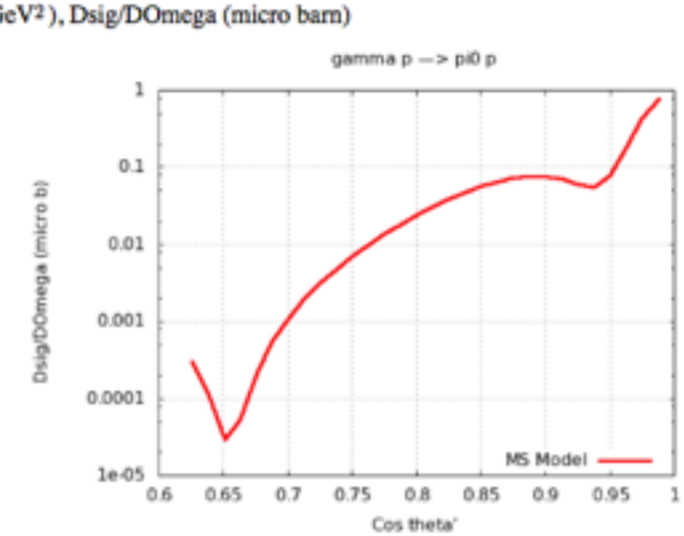
$\cos\theta$ (min max step) 0.5 0.95 0.01

Start reset

beam energy: 9 GeV
Observable: differential cross section
X variable: t with interval -3:0.1:-0.1

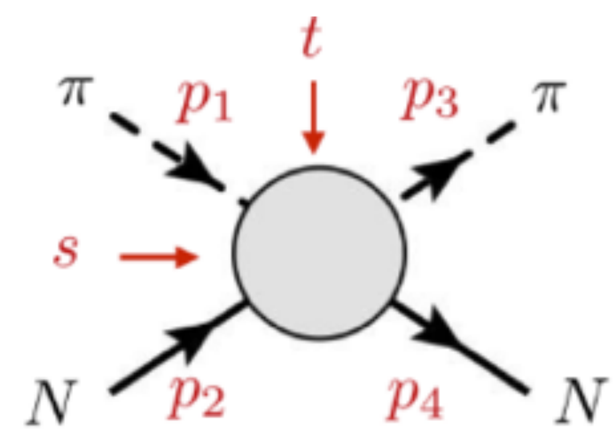
Download the output file, the plot with Ox=t , the plot with Ox=cos .
In the file, the columns are: t (GeV²), cos, Dsig/Dt (micro barn/GeV²), Dsig/DOmega (micro barn)





Interactive webpage: http://www.indiana.edu/~jpac/index.html

INDIANA UNIVERSITY



Upload your partial waves: Choose File no file selected

Or choose a model: SAID JuBo BnGo KH80 ANL-O

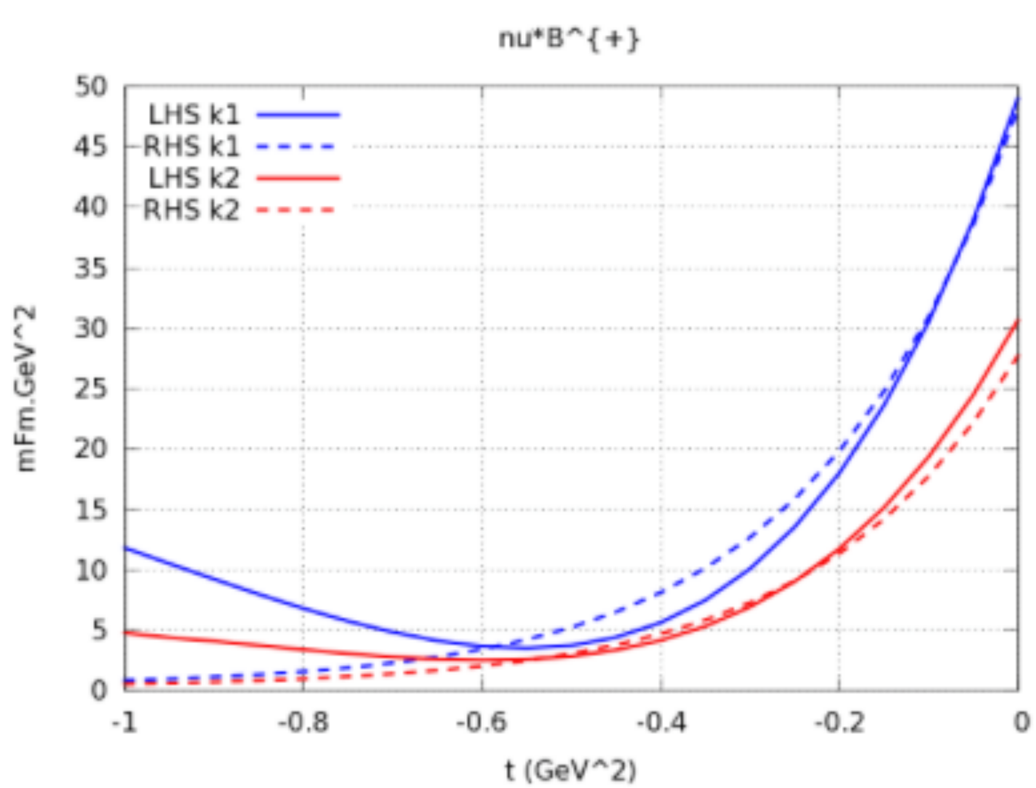
Range of t in GeV^2: min = -1.0 max = 0.0 step = 0.05

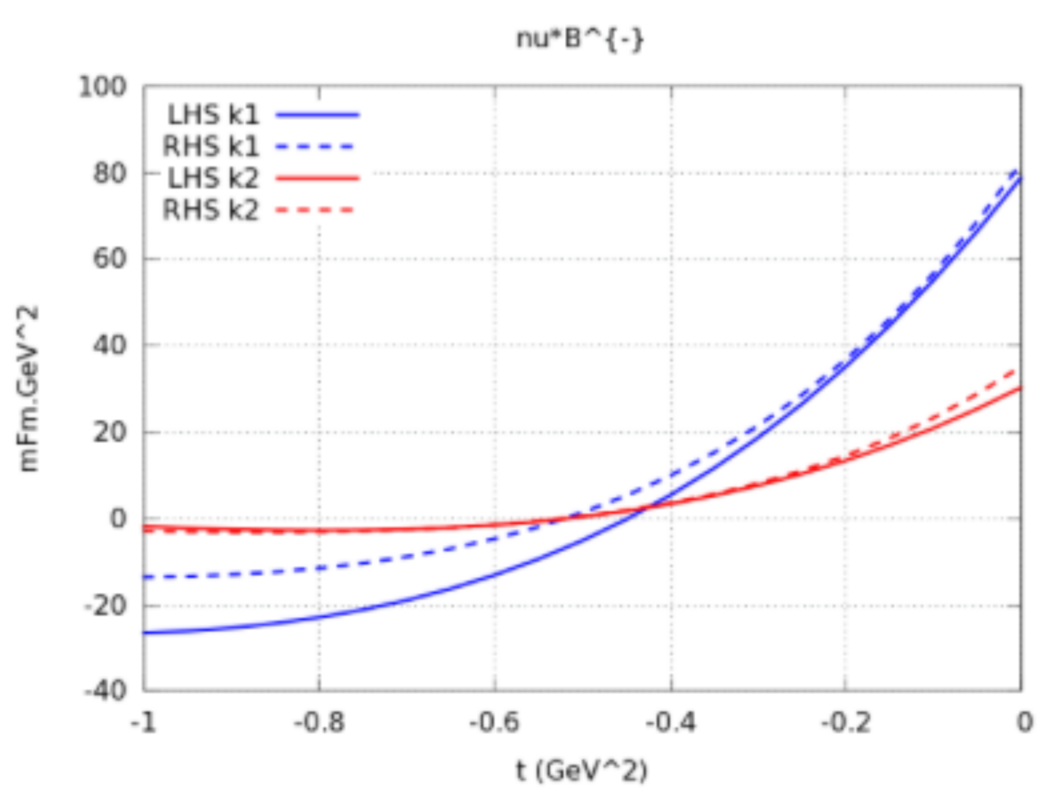
Cutoff: $E_{\text{lab}}^{\max}$ = 2.00 GeV

Moments: k1 = 0 k2 = 2

Regge parameters [show/hide]

Start



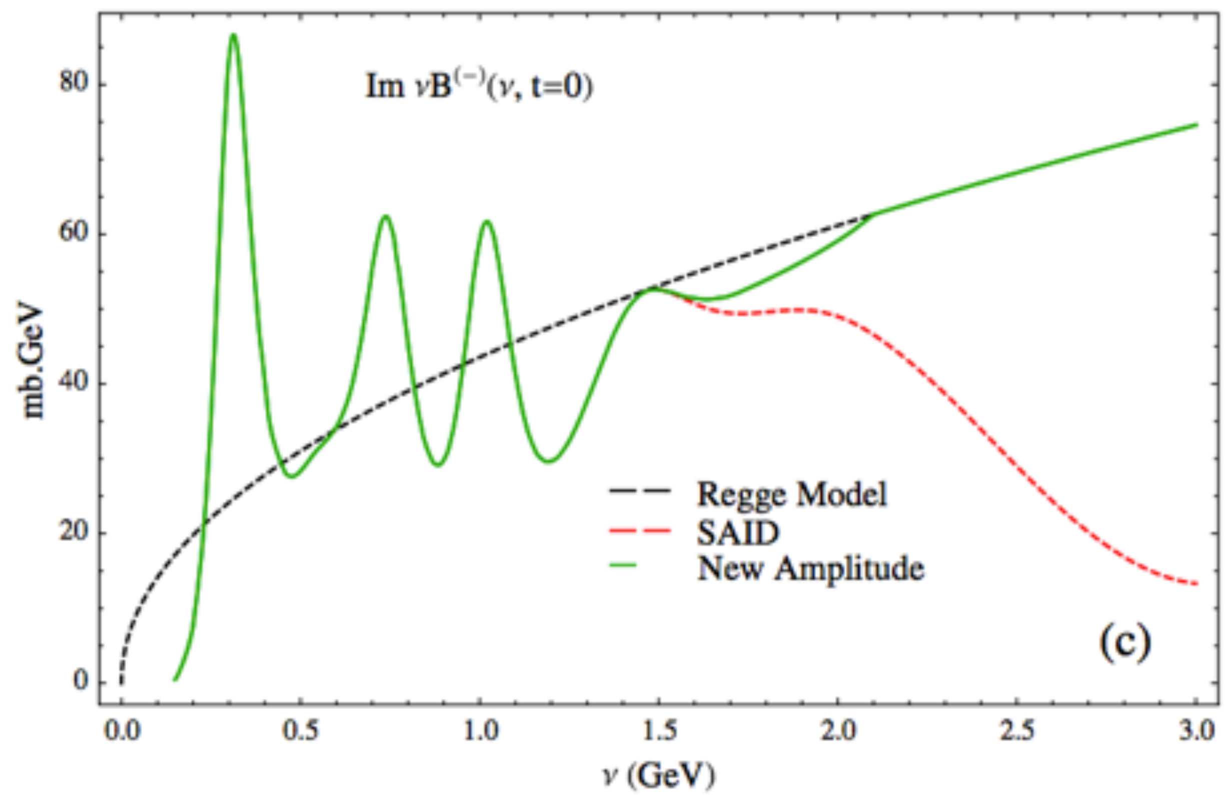


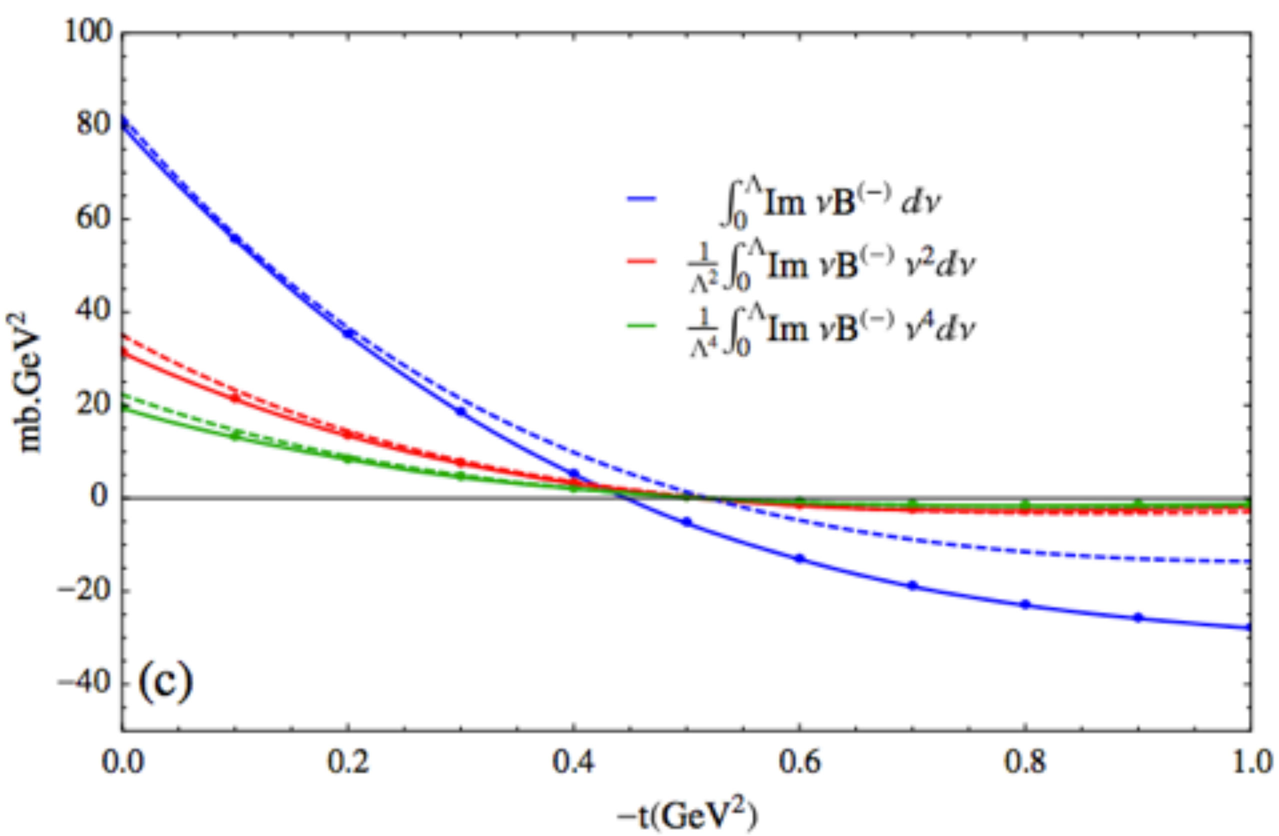
Backup Slides

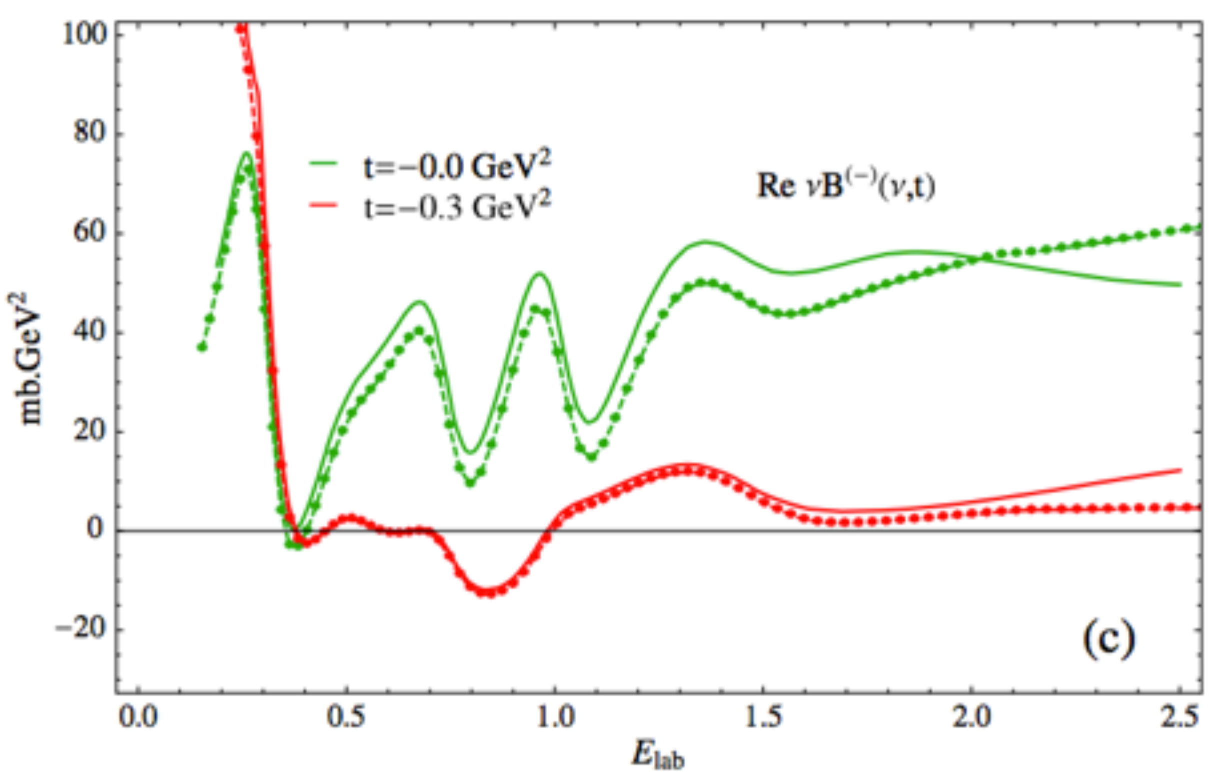
Application to pion-nucleon scattering

Similar results for the other amplitude

$$T = \bar{u}(p_4, \lambda_4)\left(A + \frac{1}{2}(\not{p}_1 + \not{p}_3)\, B\right) u(p_2, \lambda_2)$$







VM et al (JPAC) PRD92 (2015) ; arXiv:1506.01764

Duality for Exotic Meson