Duality averaging of structure functions: An Experimental Perspective

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To study duality experimentally we wish to examine how the *resonances follow* the *scaling curve*

However, we must first define what we mean by these terms.

→ What do we mean by *resonances follow*?

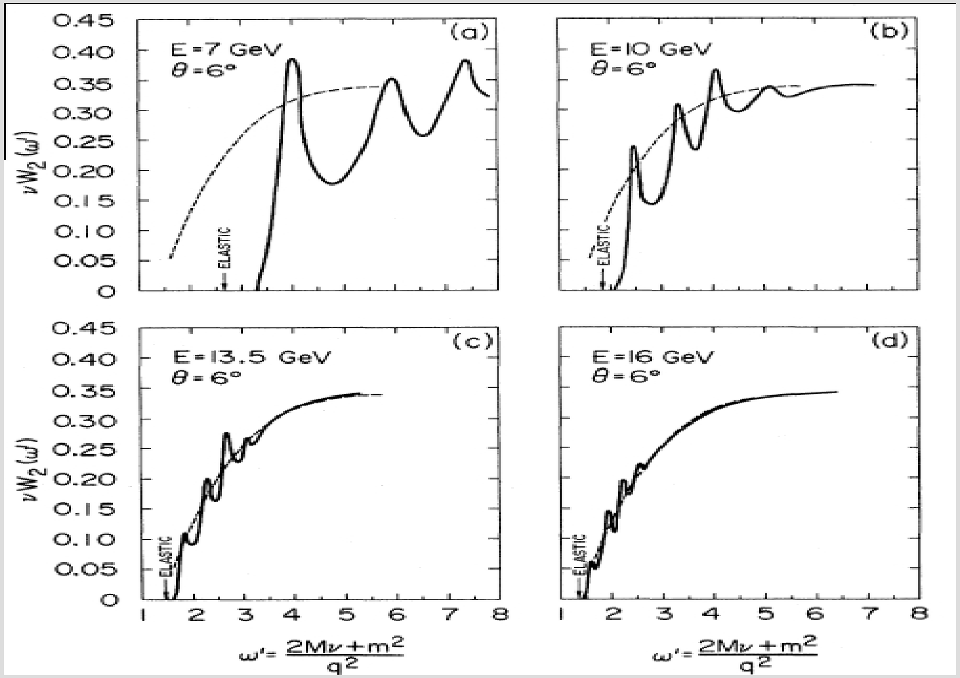
→ What *scaling curve*?

pure pQCD curve?

or

defined by data which must include

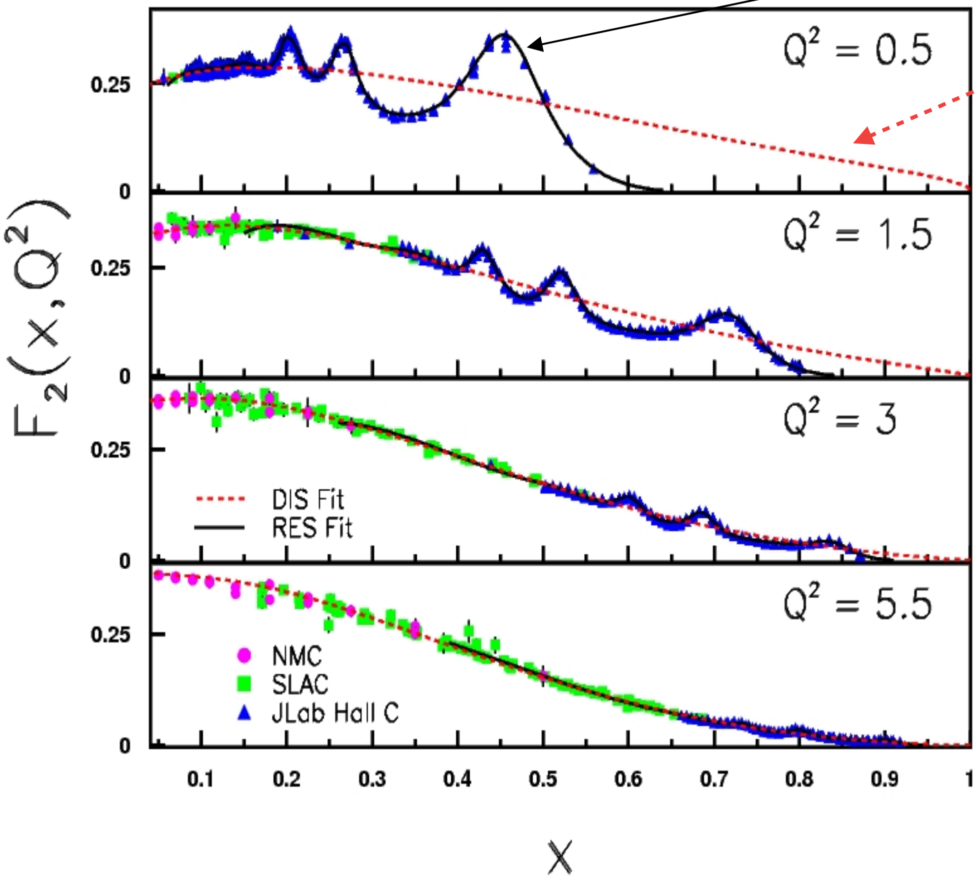
target mass (TM) and Higher-Twist (H-T)



Previously observed that resonance region proton F_2 *averages* to a scaling curve

Examine highest precision Resonance Region data on proton from Jlab Hall C E94-110

DIS fit – 'F2ALLM' H.Abramowicz and A.Levy, hep-ph/9712415
Res fit - E.C. and P.E. Bosted, PRC 81,055213



Resonance C-B fit reproduces data to ~3%

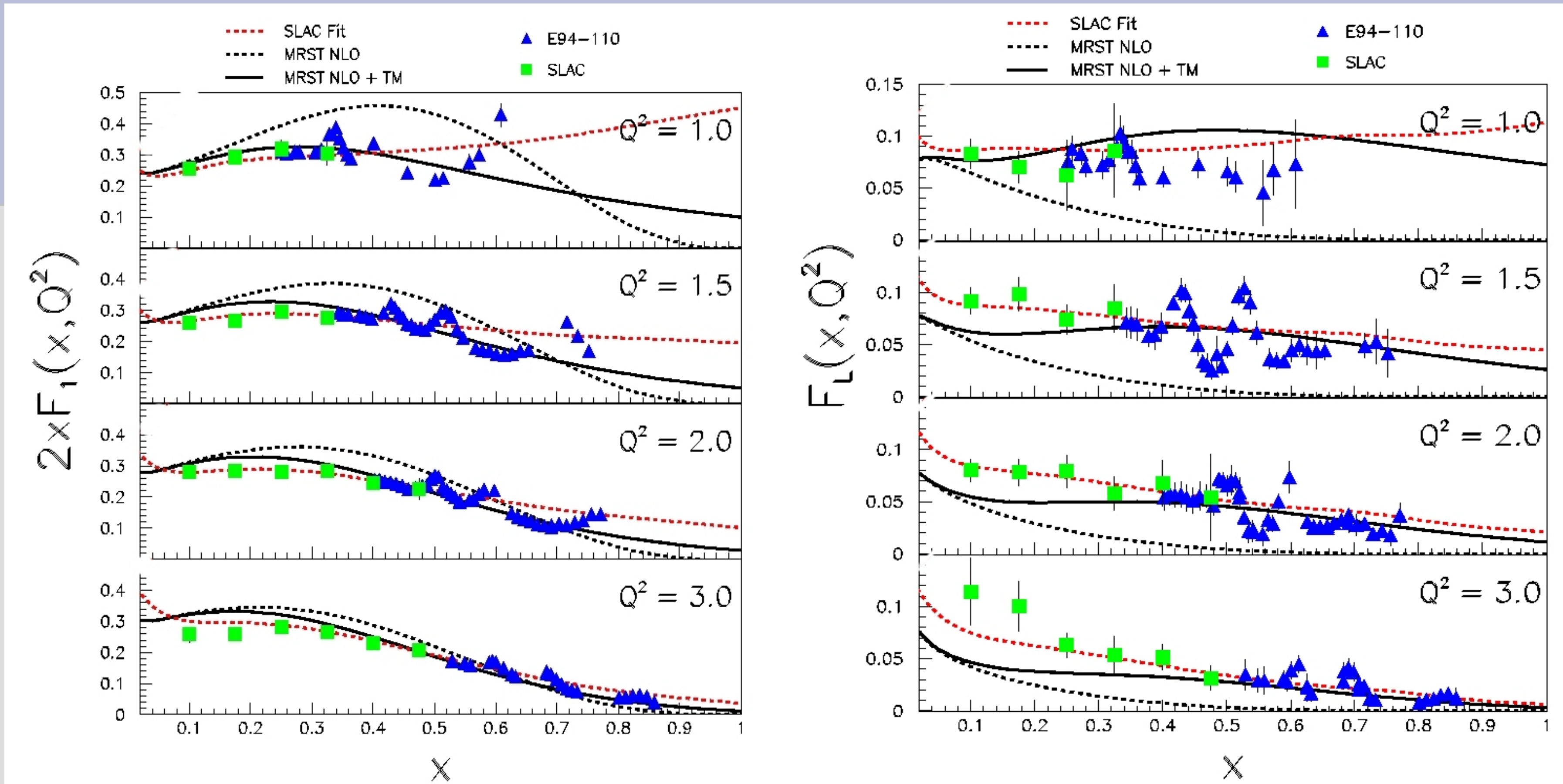
"DIS" fit is to larger W data all the way down to $Q^2 = 0$

→ Curve necessarily includes contributions beyond massless limit perturbative QCD:

I) Target Mass

II) Higher-Twist

Separated F_1 , F_L data from E94-110 provide more insight



→ Duality observed to hold at 10-20% level depending on the scaling curve chosen SLAC F2global (Whitlow) + R1990 (Tao) or MRST2004 PDF

→ Target Mass (TM) contributions can be significant at low Q^2, especially in F_L

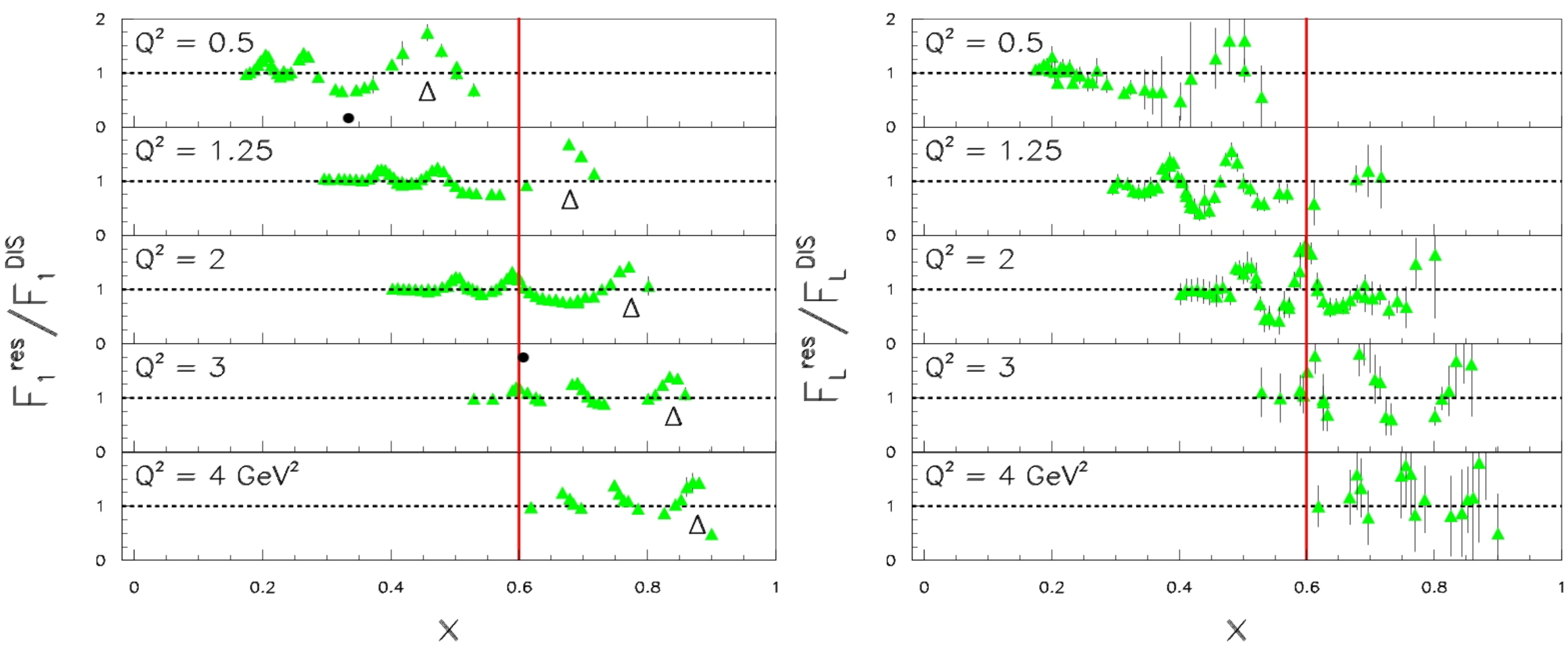
=> These are *necessary* for duality to hold at a reasonable level

Examine duality in F_1 and F_L relative to empirical DIS fit:

F_2ALLM fit to F_2 + $R = \sigma_L / \sigma_T$

H. Abramowicz and A.Levy, Hep-ph/9712415

K. Abe et.al Phys.Lett.B452:194-200,1999



Couple of important observations:

→ Many resonances pass though a given x for a large enough range of Q^2

→ DIS fit describes well the average Q^2 dependence of resonant Region structure

Several methods have been utilized for quantification, including:

(i) Compare Q^2 dependence of integral over local W^2 ranges to DIS scaling curve (or pQCD curve) see talk by I. Niculescu

(ii) Compare structure function moments at different orders (n=2,4,6,...) to the Q^2 dependence expected from pQCD+TM (scaling predictions).

(iii) Compare *truncated* moments defined over local W^2 ranges to pQCD evolution.

(iv) Calculate averaging over Q^2 for fixed x range

Truncated Moments

Originally developed to address lack of low *x* data

Forte and Magnea, PLB 448, 295 (1999); Forte, Magnea, Piccione, and Ridolfi, NPB 594, 46 (2001); Piccione PLB 518, 207 (2001); Kotlorz and Kotlorz, PLB 644, 284 (2007).

Idea: construct doubly truncated moments from

$$\overline{M}_n(\Delta x, Q^2) = \int_{\Delta x} dx\ x^{n-2}\ F_2(x, Q^2)$$

Truncated moments follow ***DGLAP-like evolution*** equations.

$$\frac{d\overline{M}_n(\Delta x, Q^2)}{d\log Q^2} = \frac{\alpha_s}{2\pi}\left(P'_{(n)} \otimes \overline{M}_n\right)(\Delta x, Q^2)$$

With modified splitting functions given by

$$P'_n(z, \alpha_S(Q^2)) = z^n P(z, \alpha_S(Q^2))$$

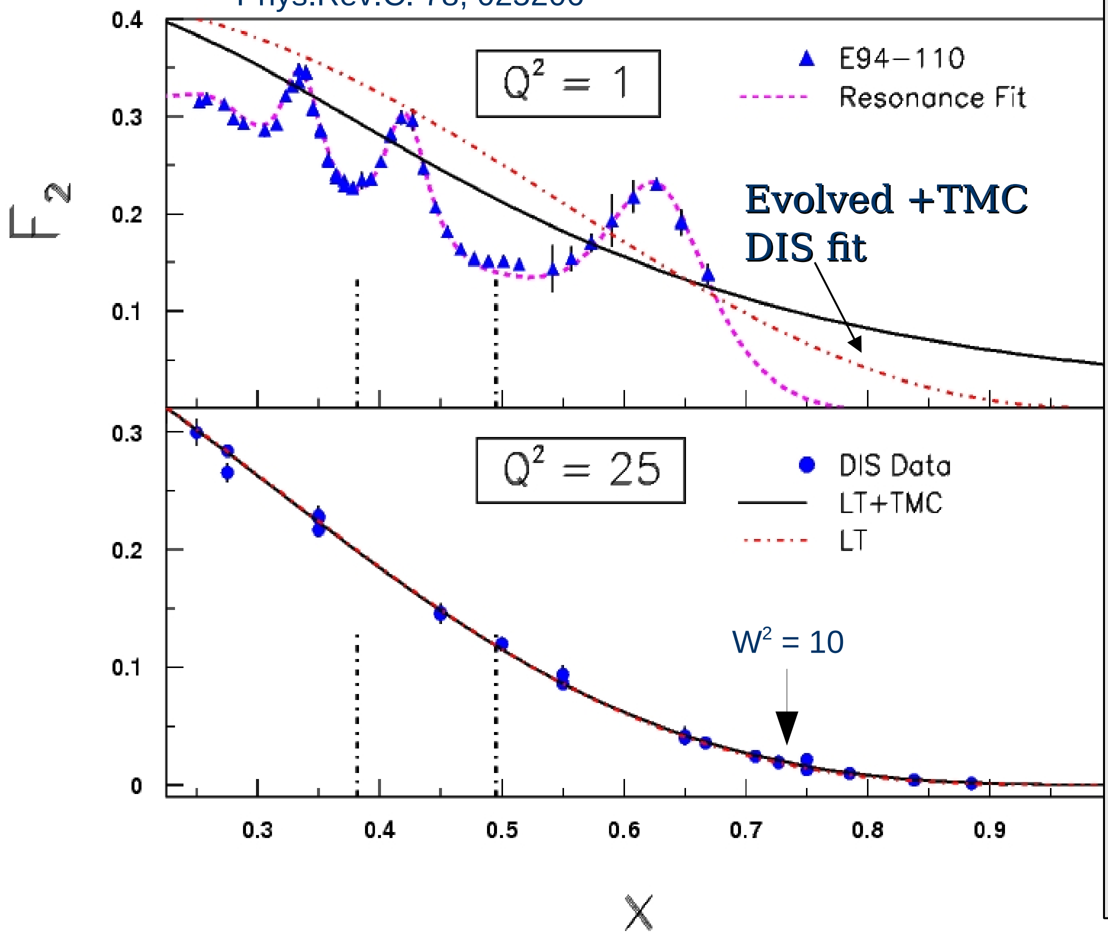
Allows study of ***regions in W*** within pQCD in well-defined, systematic way.

Truncated Moment Analysis: basic idea

Allows study of ***regions in W*** within pQCD framework

A. Psaker, W. Melnitchouk, MEC, C. Keppel

Phys.Rev.C. 78, 025206



→ Compare integral over select resonance regions to evolved scaling curve + TM

→ Scaling curve is empirical Fit to data at Q^2 = 25, where TM contribution has been separated from leading-twist via an unfolding Procedure.

→ Scaling curve is then evolved to lower Q^2 via non-singlet evolution before recalculating the TM contributions at the lower Q^2.

What DIS curve do we evolve?

- → Let data at Q^2 = 25 Gev^2 determine curve
- → Remove TM contribution via unfolding
- → Evolve with curve to lower Q^2 via non-singlet evolution
- → Include TM contribution at lower Q^2

Unfolding TM Contributions from data

In the Operator Product Expansion

$$F_2^{TM}(x,Q^2) = \frac{x^2}{r^3}\frac{F_2^{(0)}(\xi,Q^2)}{\xi^2} + 6\frac{M^2}{Q^2}\frac{x^3}{r^4}\int_\xi^1 dx' \frac{F_2^{(0)}(x',Q^2)}{x'^2} + 12\frac{M^4}{Q^4}\frac{x^4}{r^5}\int_\xi^1 dx' \int_{x'}^1 dx'' \frac{F_2^{(0)}(x'',Q^2)}{x''^2}$$

$$F_1^{TM}(x,Q^2) = \frac{x}{r}\frac{F_1^{(0)}(\xi,Q^2)}{\xi} + \frac{M^2}{Q^2}\frac{x^2}{r^2}\int_\xi^1 dx' \frac{F_2^{(0)}(x',Q^2)}{x'^2} + \frac{2M^4}{Q^4}\frac{x^3}{r^3}\int_\xi^1 dx' \int_{x'}^1 dx'' \frac{F_2^{(0)}(x'',Q^2)}{x''^2}$$

$$2xF_1^{TM} = \frac{F_2^{TM} - F_L^{TM}}{r^2}$$

$$2xF_1^{(0)} = F_2^{(0)} - F_L^{(0)}$$

$$r = 1 + \nu^2/Q^2 = \sqrt{1 + \frac{4M^2x^2}{Q^2}}$$

$$\xi = 2x/(1+r)$$

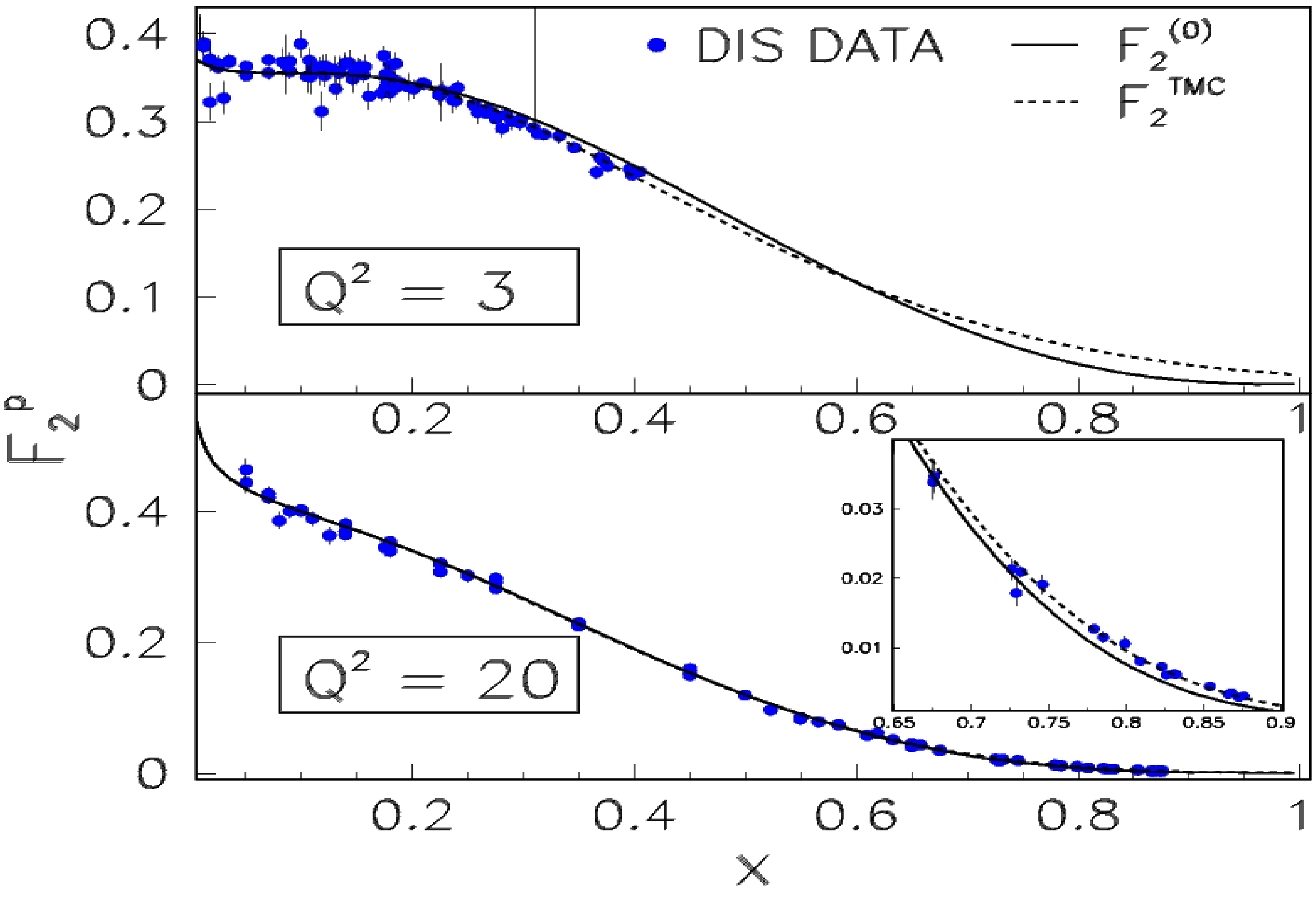
Parameterize $F_{2,L}^{M=0}$ (x,Q^2) and fit $F^{TM}_{2,L}(x,Q^2)$ to world data set => determine TMCs directly from data.

- ***Not* a perturbative expansion**
- ***Assume that higher twist operators obey same formalism.***

*Proton charged lepton data on F_2 and F_L fit for **$0.3 < Q^2 < 250$** and **$x > 1x10^{-4}$***

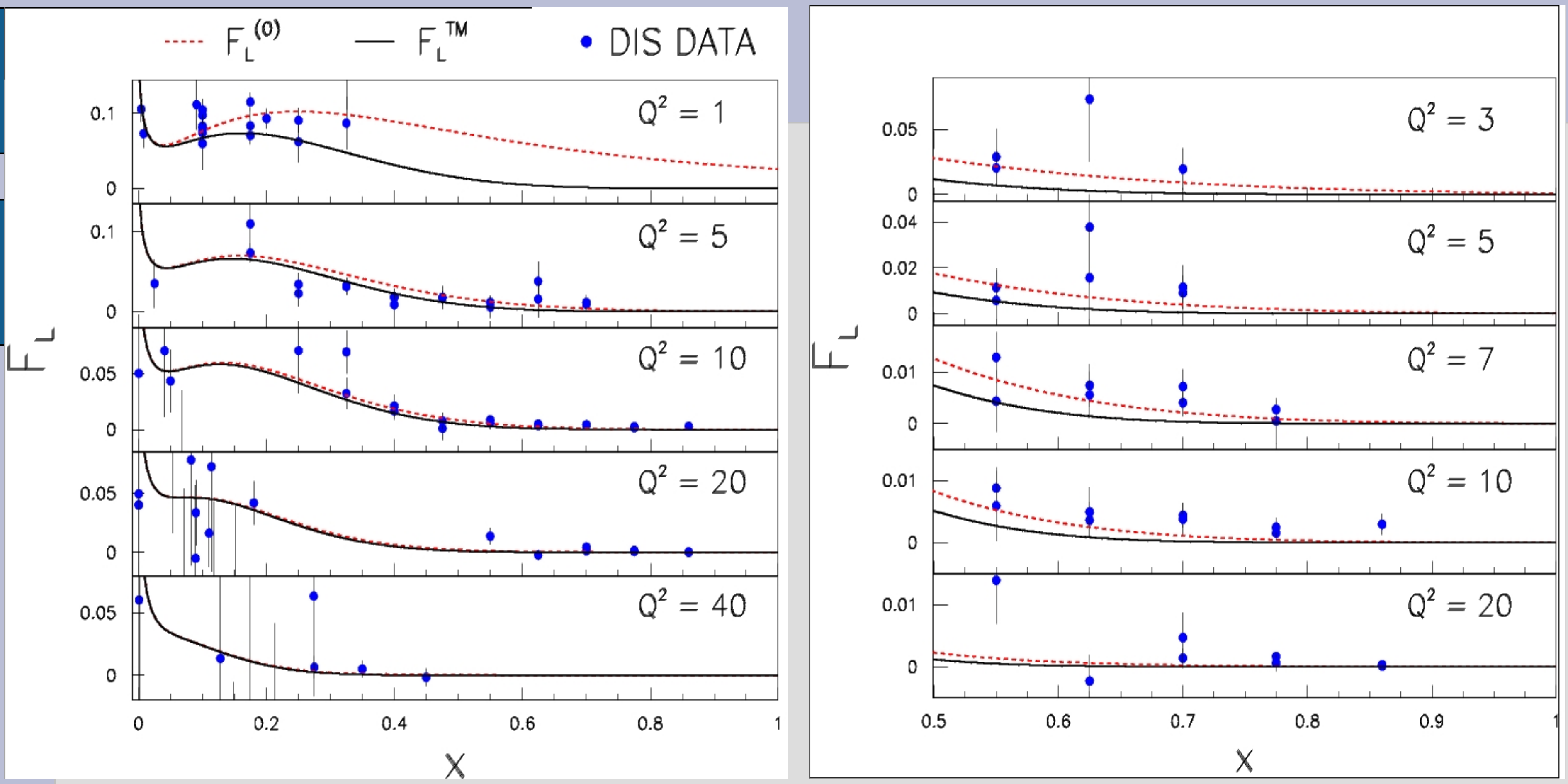
F_2^p results

(arXiv: 1201.0576 MEC, J. Blumlein, H. Bottcher)



F_L^p results from TMC fit

(arXiv: 1201.0576 MEC, J. Blumlein, H. Bottcher)

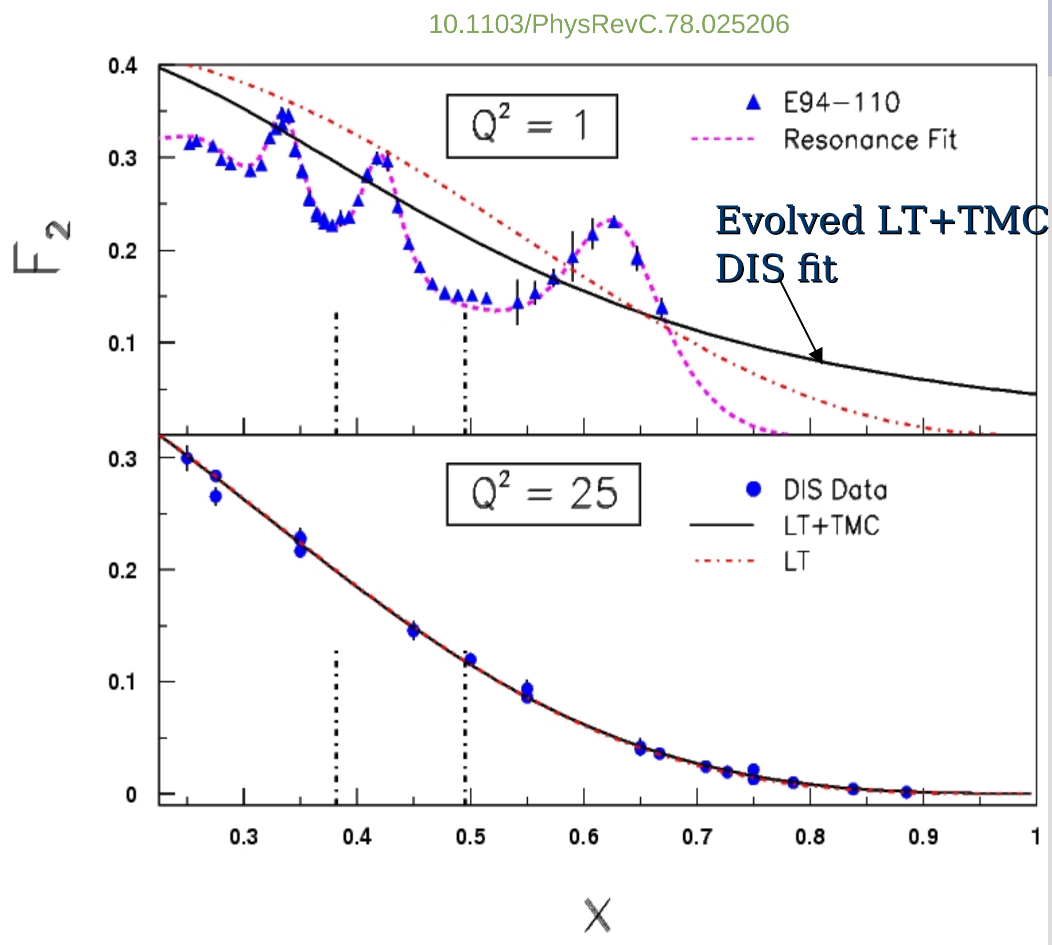


Can study → test pQCD evolution of extracted $F_{L,2}^{(o)}$

→ Further duality studies using as 'scaling' curve

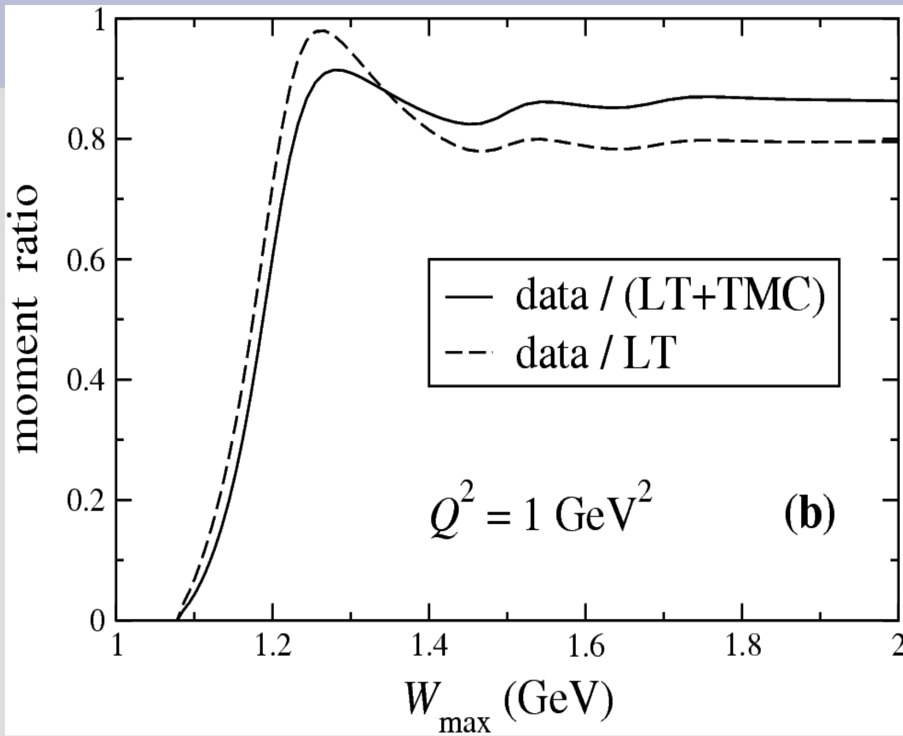
A. Psaker, W. Melnitchouk, MEC, C. Keppel

Phys.Rev.C. 78, 025206

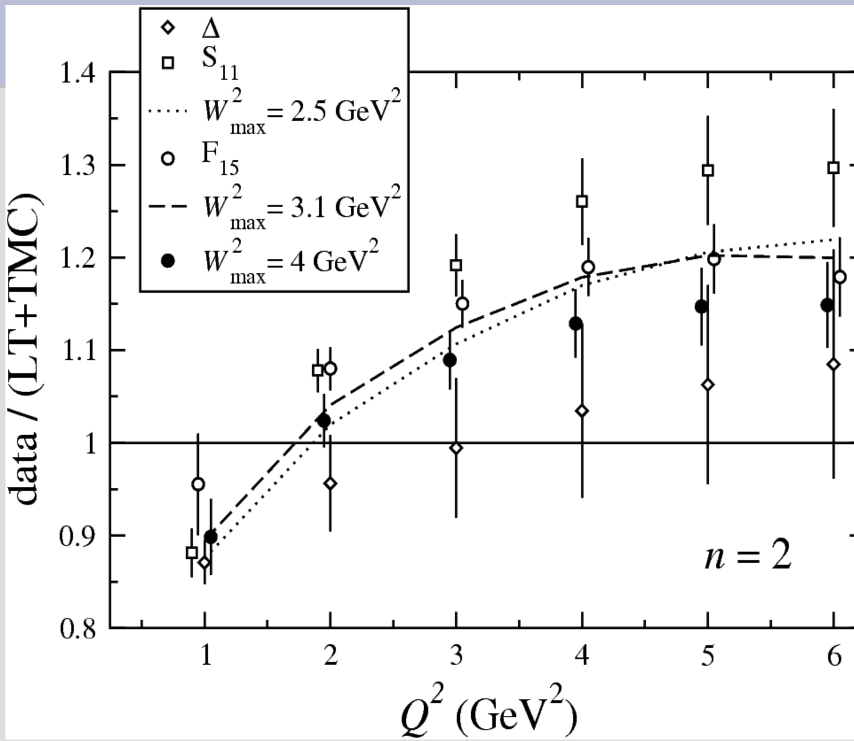


We now have a leading-twist curve that we can evolve to lower Q^2 and compare it's integral to resonance region integral over the same x region

Dependence of resonance data to LT+TMC moments ratios with W_{max} at $Q^2 = 1$



Q^2 dependence of truncated moment ratios



→ TM are important at low Q^2

→ Ratio is relatively flat for W_{Max} above Δ

→ ratios saturate at ~1.1-1.2 at large Q^2

→ Difference from unity has been interpreted as H-T contribution

Data seems to be telling us that the duality curve is no pure pQCD

Can we let the data tells us what the curve is?

How do we do the averaging?

Note the following:

1. Each resonance slides to higher x along the DIS fit

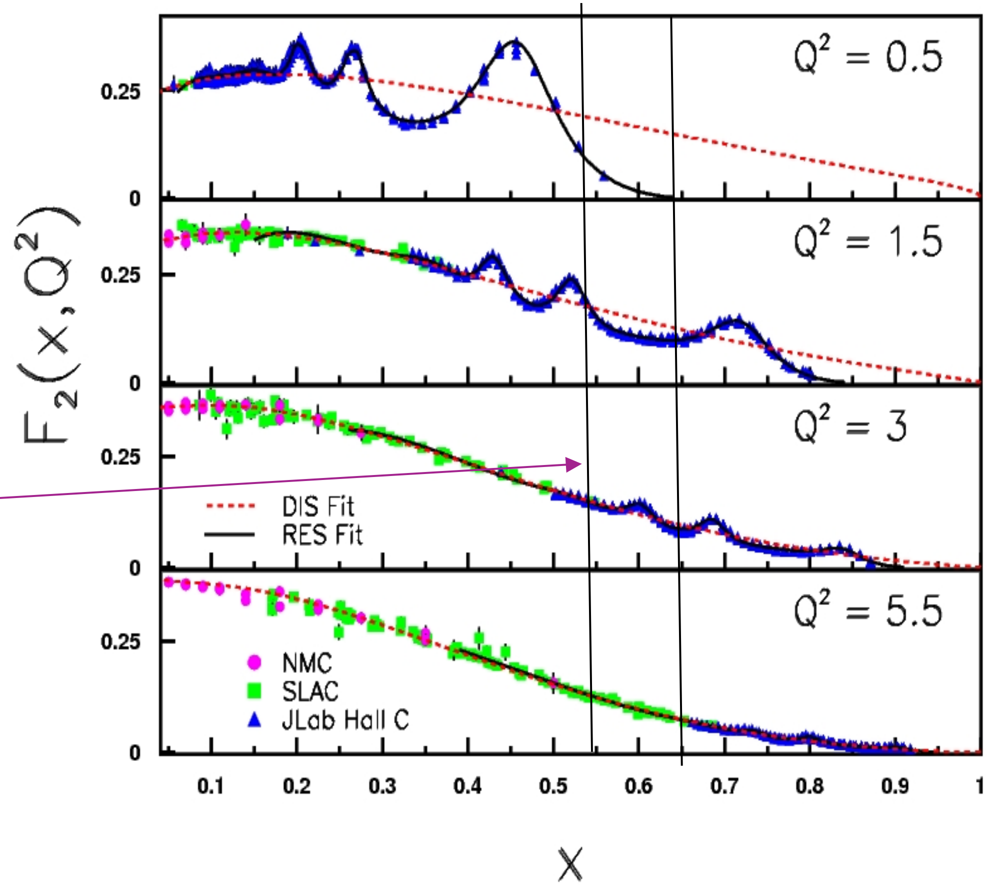
2. Averaging over a Q^2 range at fixed x effectively averages over a number of resonances including peaks and valleys.

=> Take out Q^2 dependence

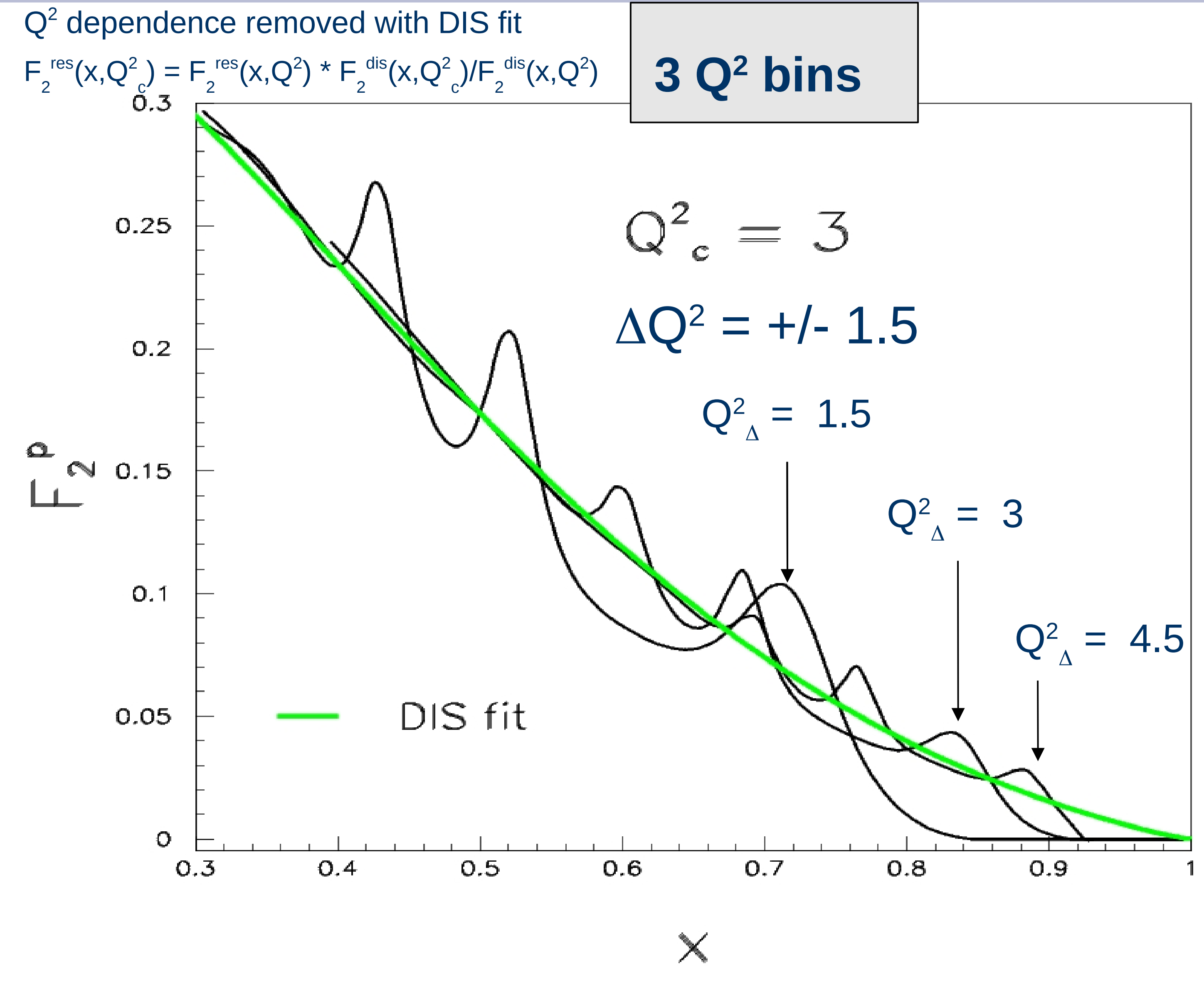
Using DIS curve then average over range in Q^2

DIS fit – 'F2ALLM' H.Abramowicz and A.Levy, hep-ph/9712415

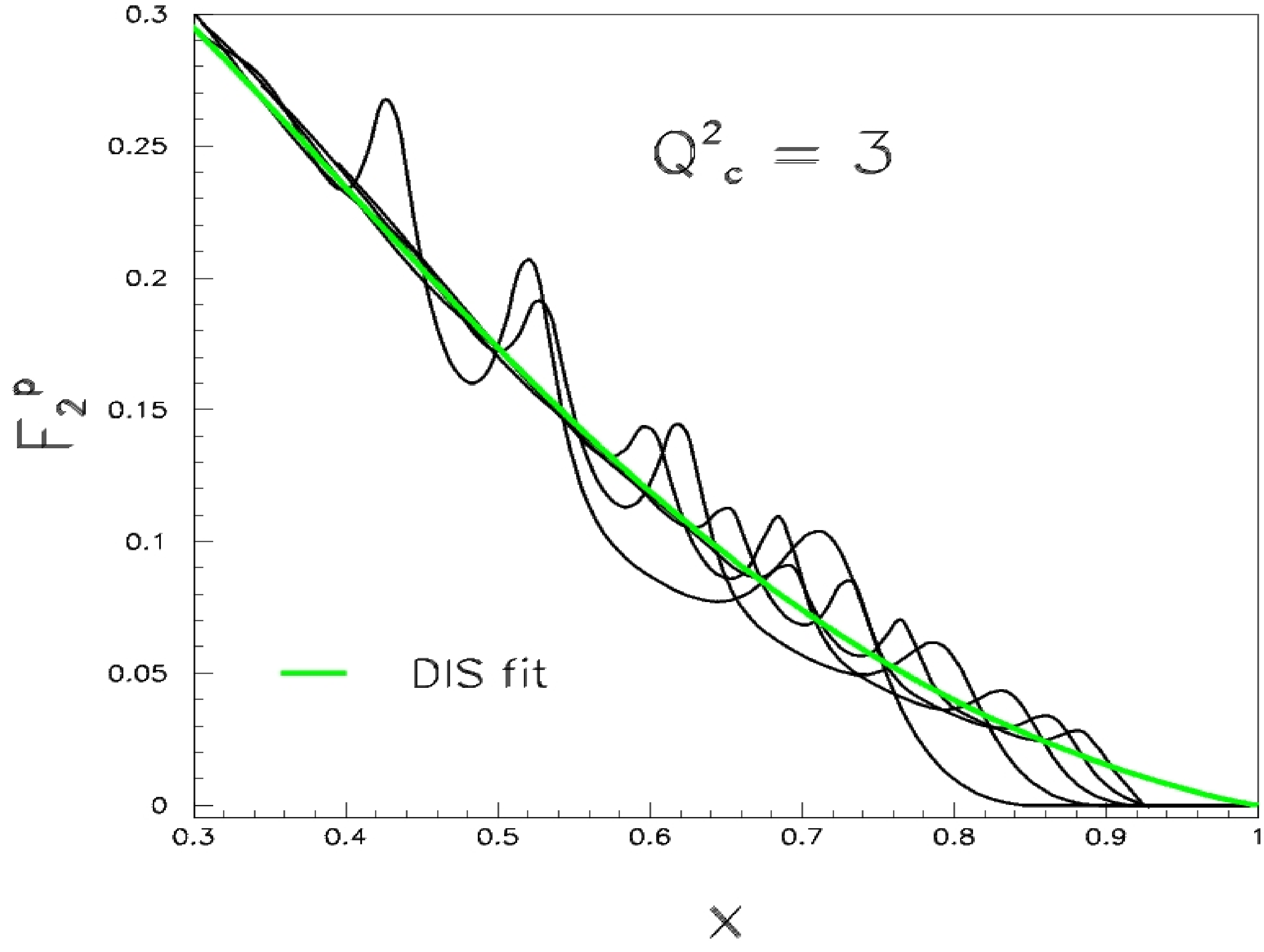
Res fit - E.C. and P.E. Bosted, PRC 81,055213



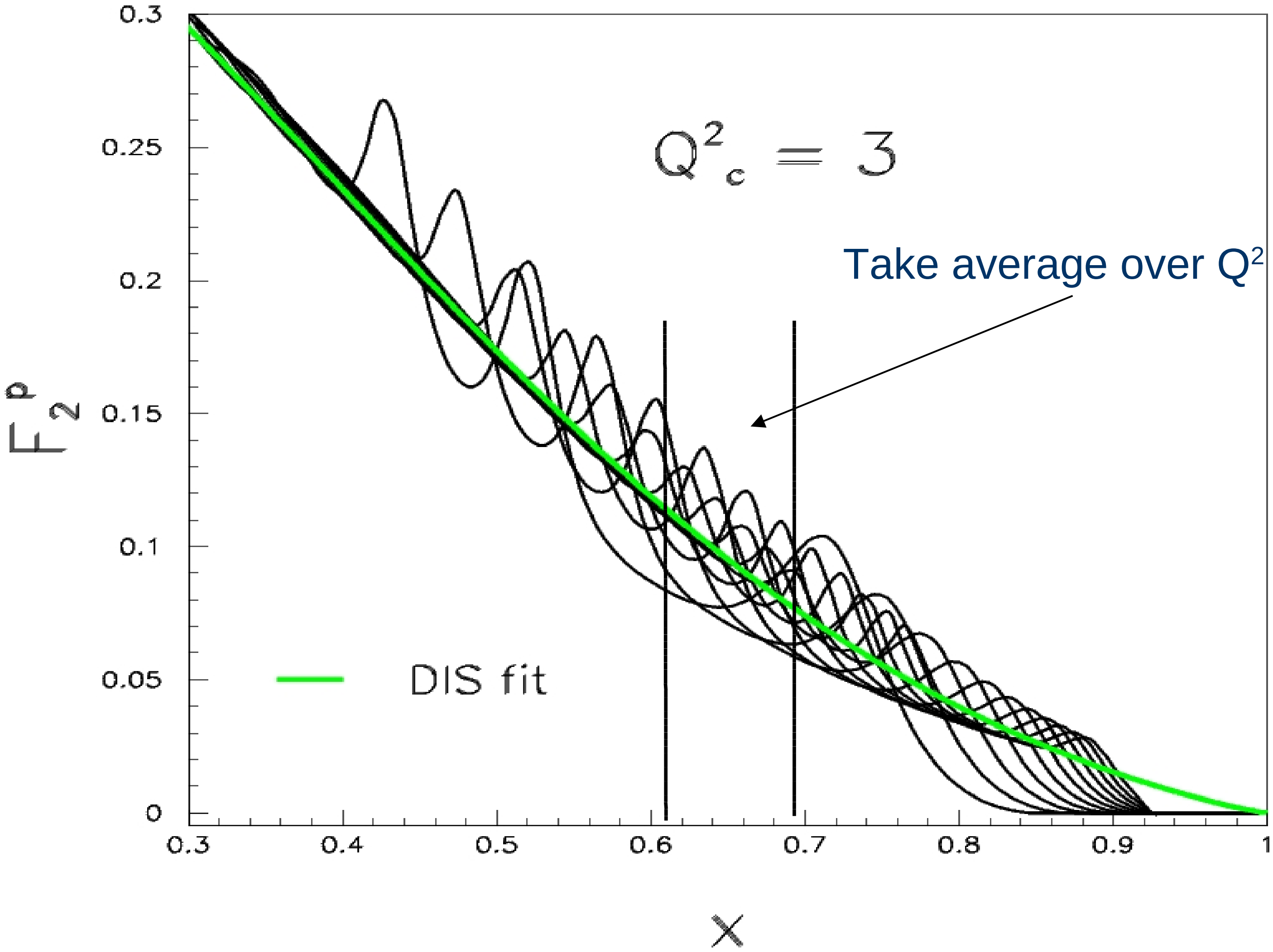
'DIS-like' duality averaging procedure



5 Q^2 bins

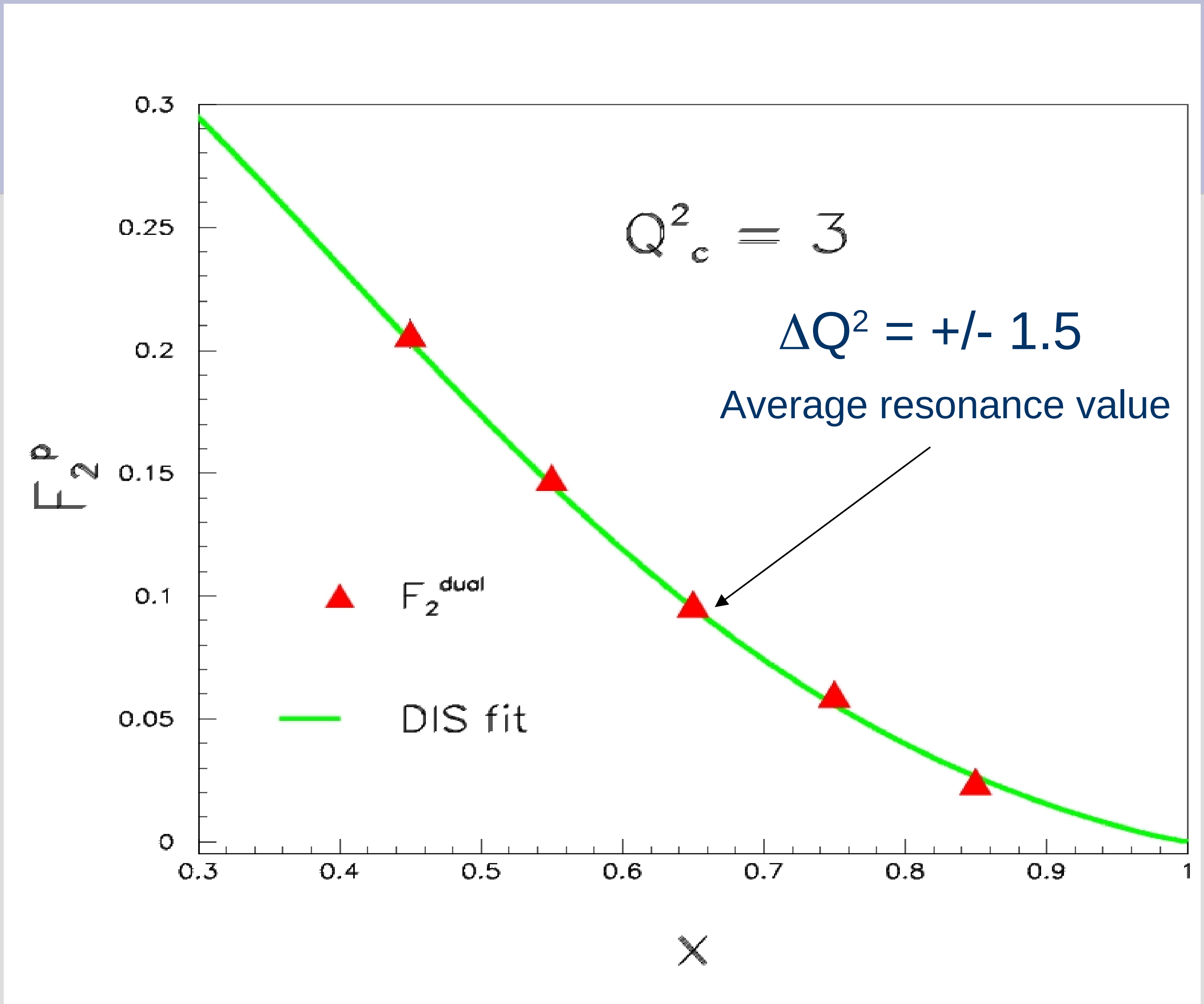


9 Q^2 bins

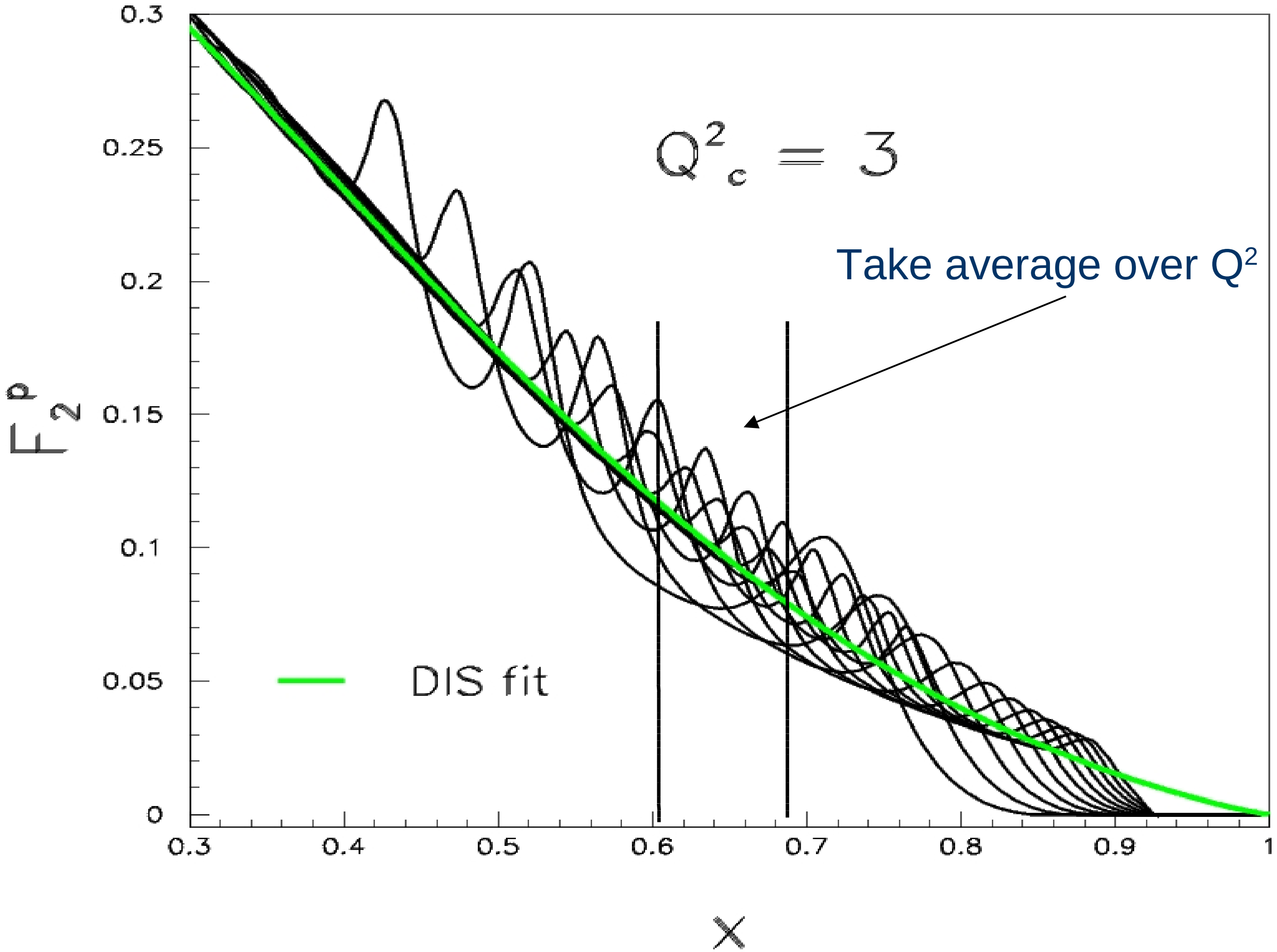


0.3
0.25
0.2
0.15
0.1
0.05
0
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
F_2^p
X
$Q^2_c = 3$
Take average over Q^2
DIS fit

Correct Model to data using average of data / (res model) in each x bin



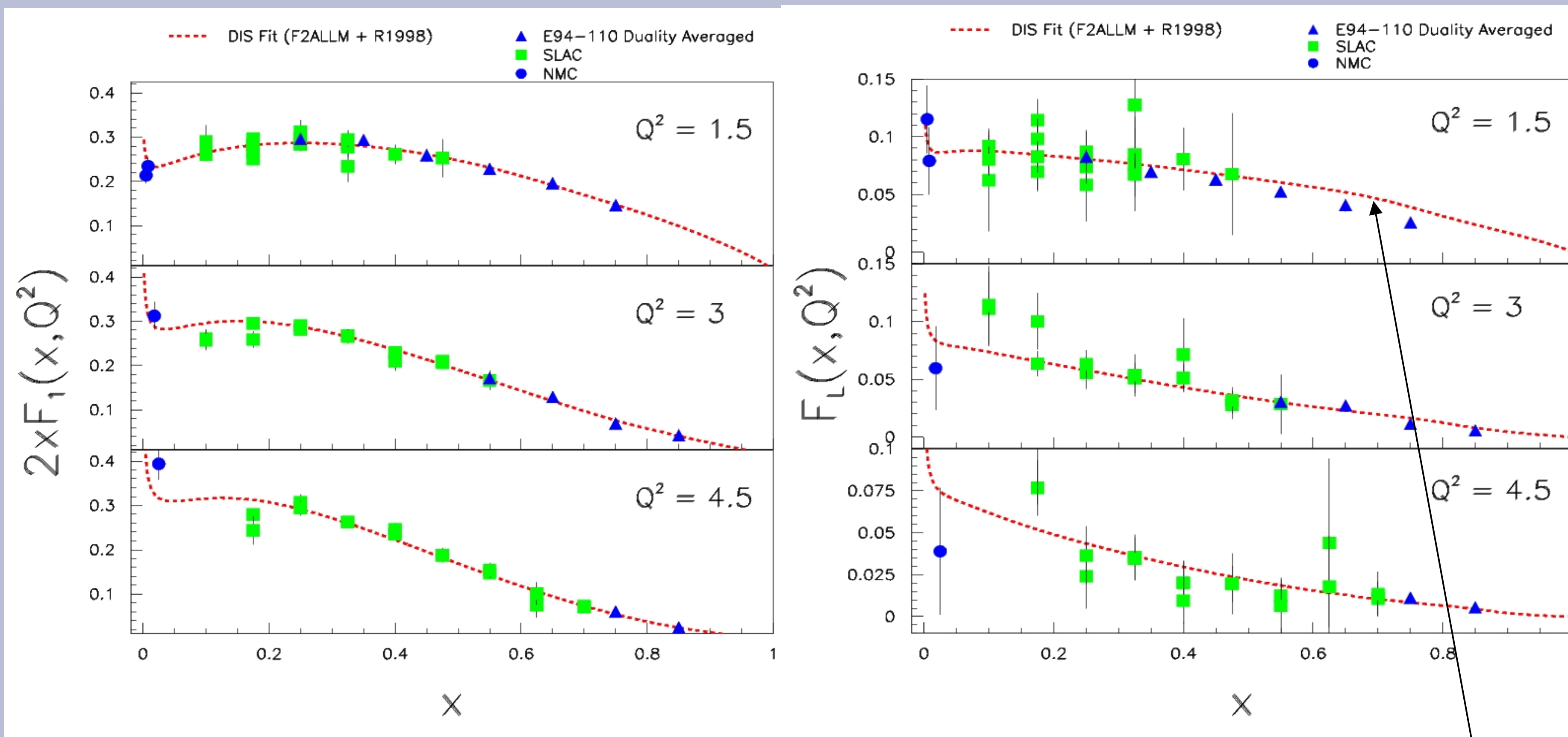
9 Q^2 bins



$Q^2_c = 3$
Take average over Q^2
DIS fit
F_2^p
x
0.3
0.25
0.2
0.15
0.1
0.05
0
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Duality averaging results for F_1 and F_L

ΔQ^2 = +/- 0.75



- **Good consistency with DIS and relatively smooth *x* dependence.**
- **Note different Q^2 dependence in averaged F_L from fit at lowest Q^2.**

Summary

→ Truncated moments indicate duality curve is not pure pQCD

=> Indicates that average H-T is not canceling in resonances,

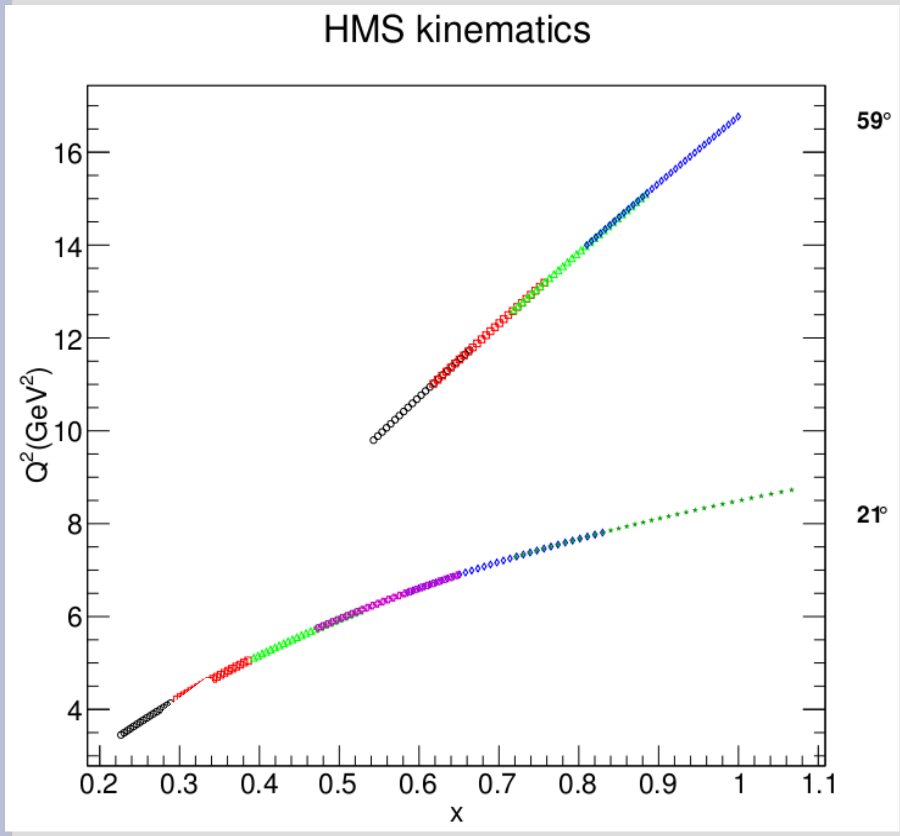
but the average H-T is the same as DIS.

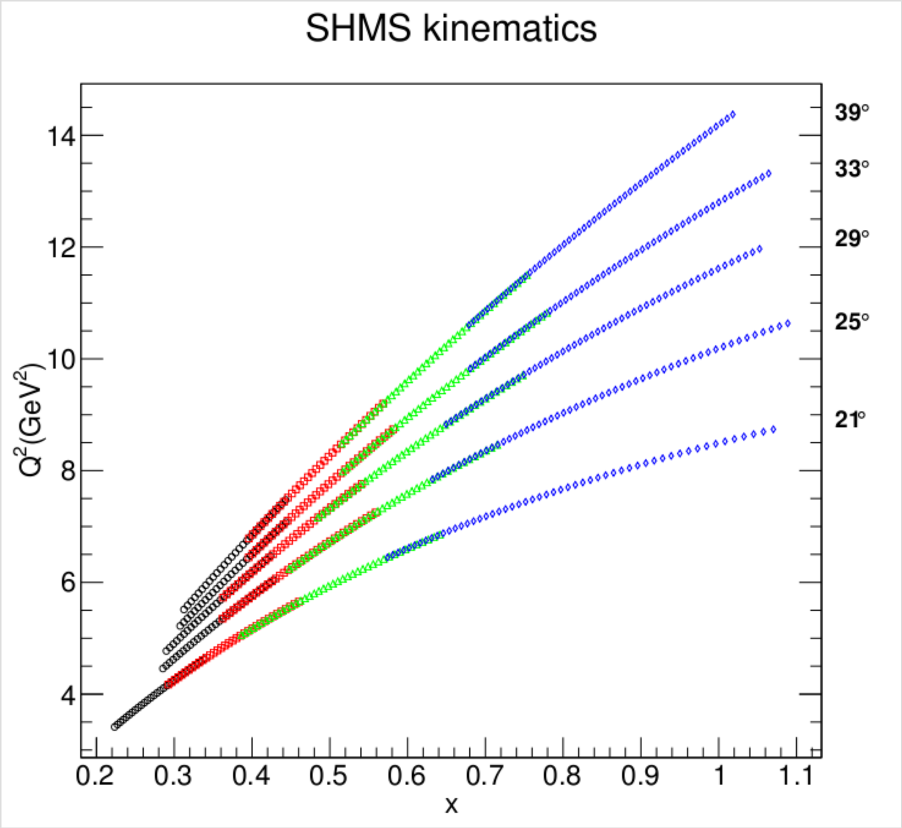
→ Duality averaging procedure developed to utilize Q^2 averages to Average over W^2 at fixed x.

=> Let data determine fit

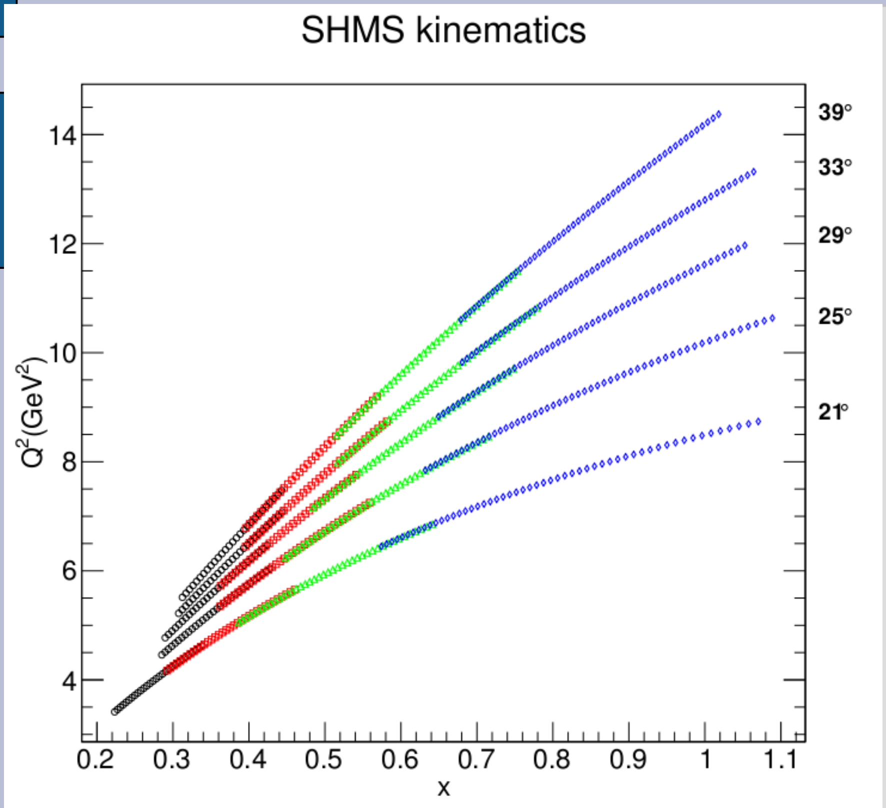
→ New 12 GeV Jlab data will allow for duality averages at larger Q^2

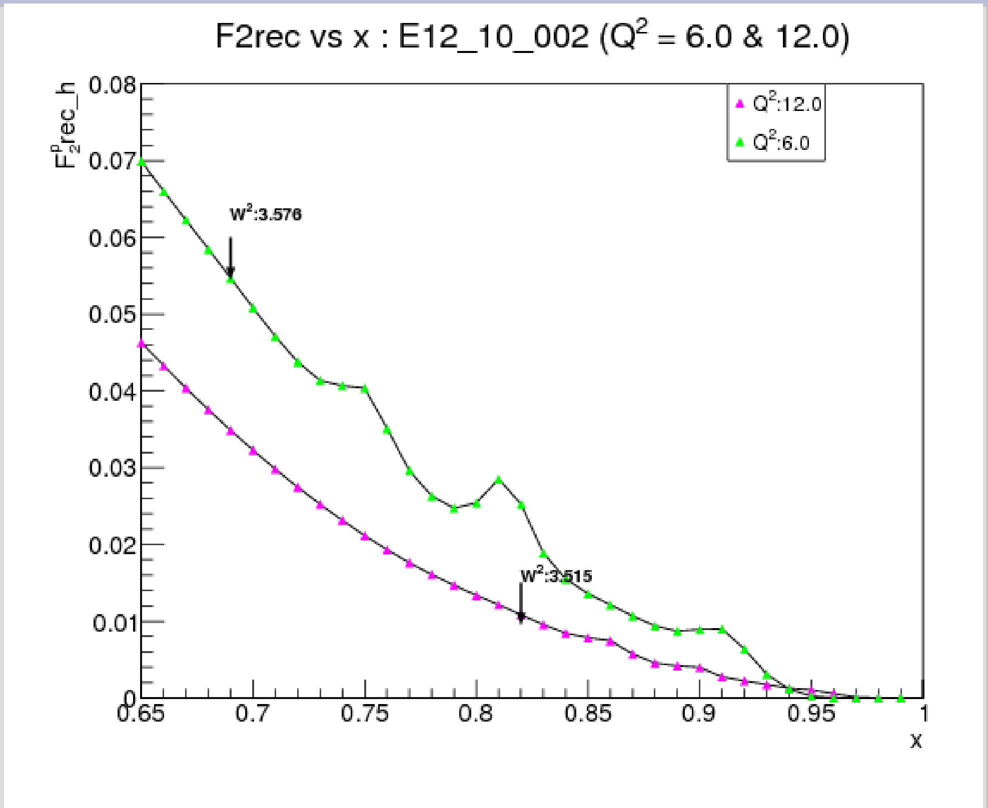
Future Studies From E12-10-002





Future Studies From E12-10-002

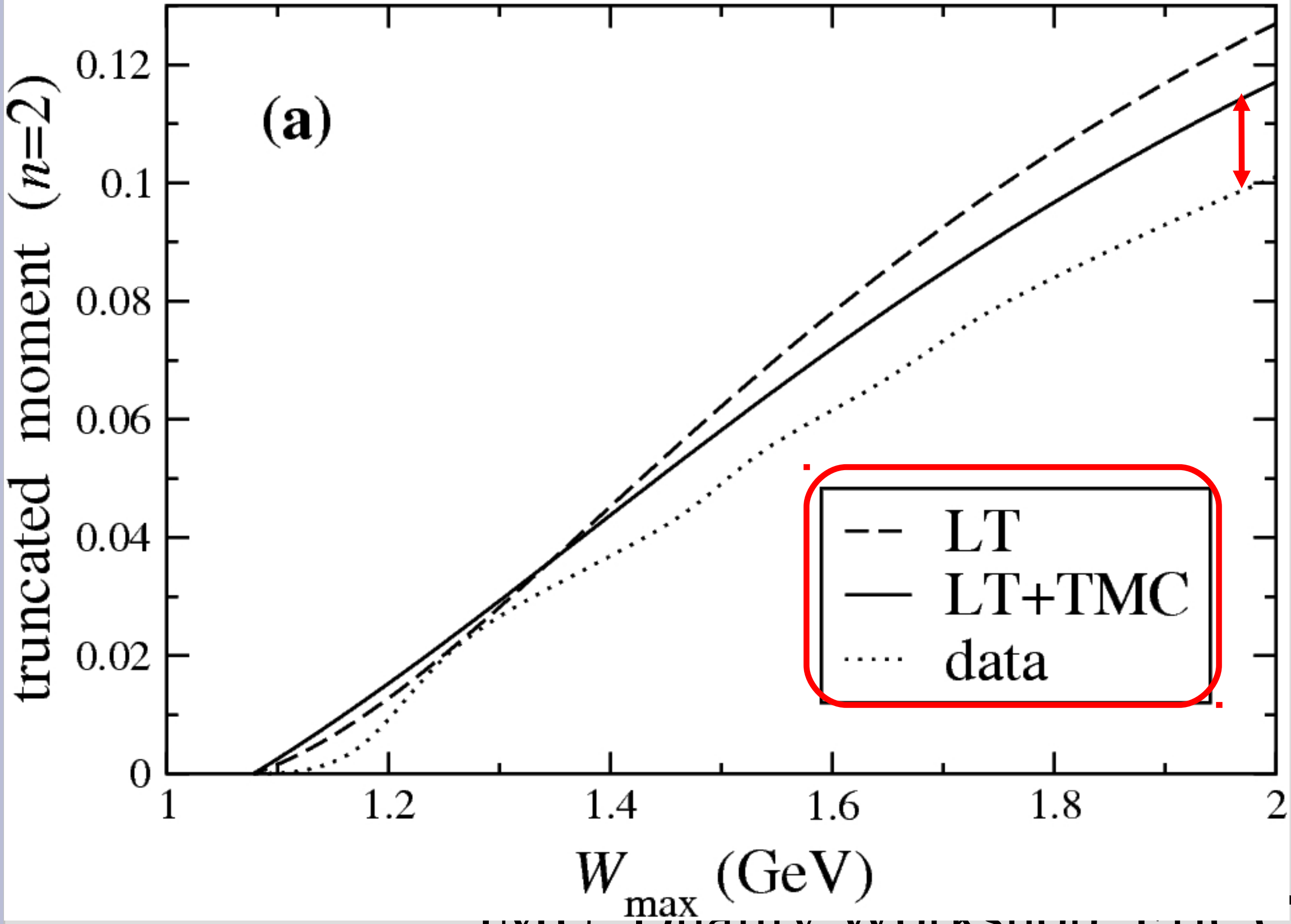




Backup

Truncated Moment Analysis (NLO) of Hall C F_2 Data

- *Assume* data at highest Q^2 (25 GeV^2) is entirely leading twist
- Evolve (target mass corrected fit) as **NS**, with uncertainty evaluated, from Q^2 = 25 GeV^2 down to lower Q^2

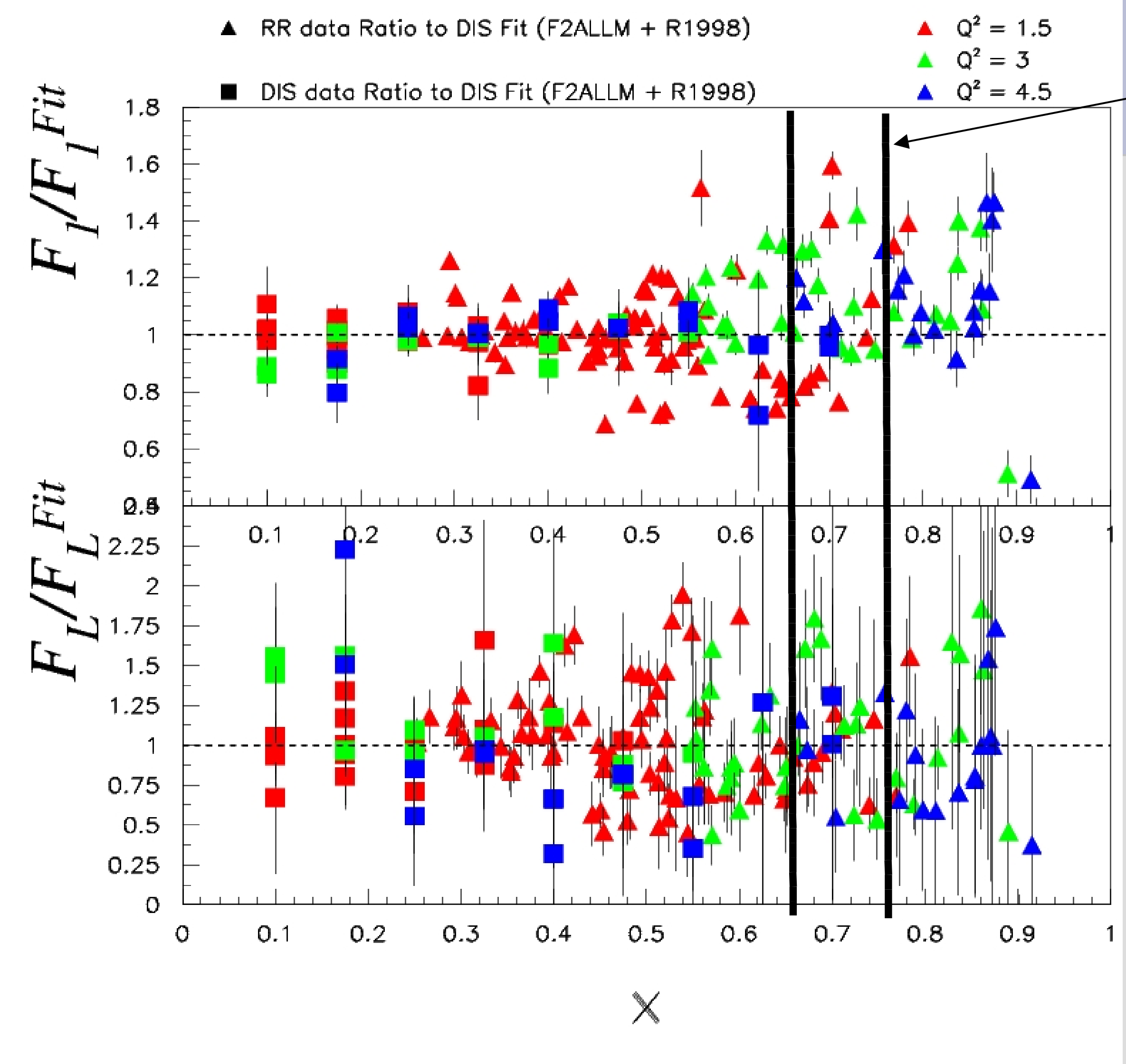


This difference quantifies the higher twist.

smallest x (low x = high W), largest integration range

highest x, smallest integration range

A closer look at Res/DIS ratios ...



Averaging RR measurements for $0.65 < x < 0.75$ gives nearly same F_1 and F_L as DIS!

...and Q^2 dependence at fixed x is the same.

How can we use this observation to determine **duality averaged data**?

F_L^p Data Sets

Data Set	Q^2_{Min} (GeV^2)	x_{min}	Q^2_{Max} (GeV^2)	x_{max}	# Data Points
BCDMS [1]	15	0.07	50	0.65	10
EMC [2]	15	0.041	90	0.369	28
NMC [3]	1.31	0.0045	20.6	0.11	10
SLAC (Whitlow [18])	0.63	0.1	20	0.86	90
SLAC (E140x [19])	0.5	0.1	3.6	0.50	4
H1 [?]	25	0.00062	90	0.0036	5
E99-118 [20]	0.273	0.077	1.67	0.320	7

Fit Form

$$F^{(0)}_{2,L}(x) = Ax^B(1-x)^C(1+D\sqrt{x}+Ex),$$

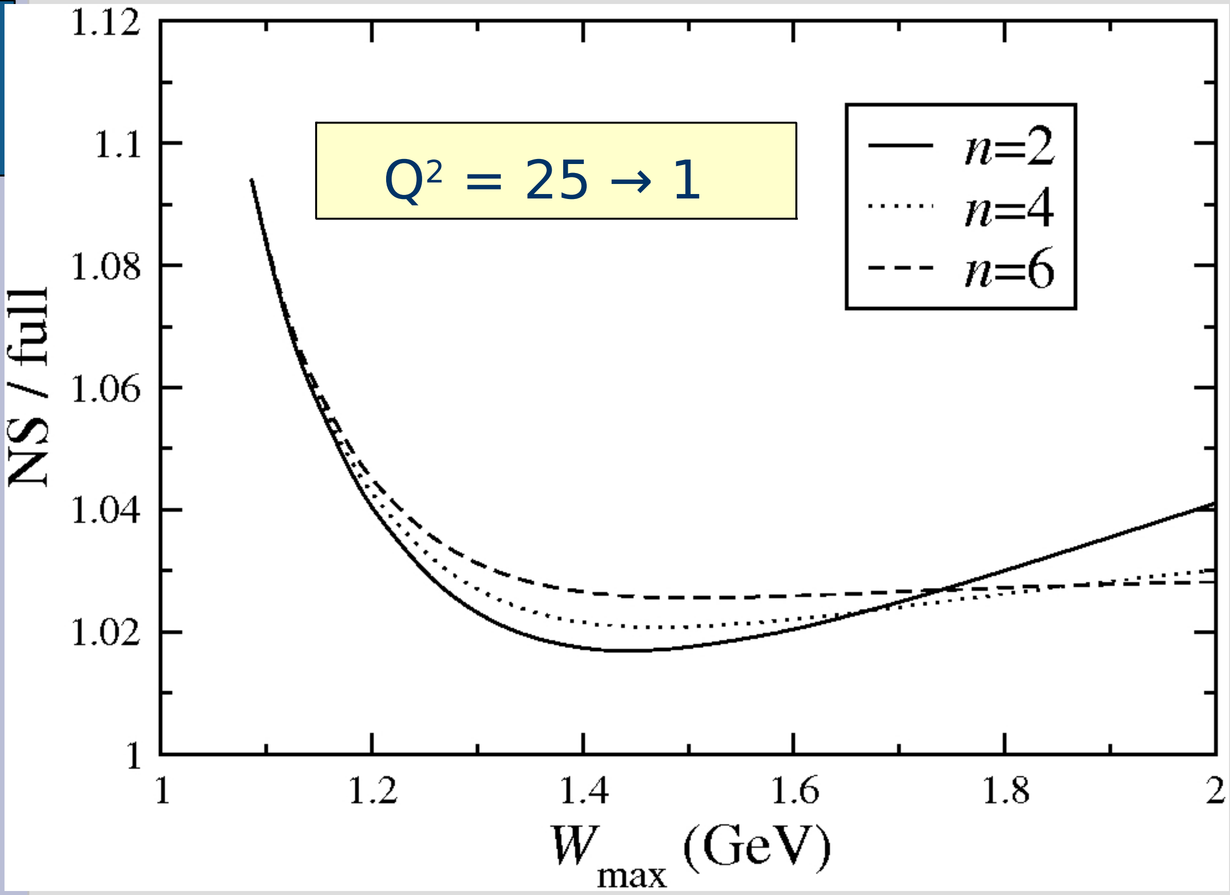
F_2 parameter Q^2 dependence

$$A(Q^2) = A_1 + A_2 e^{-Q^2/A_3} + A_4 \log(0.3^2 + Q^2)$$

*Same form for **A**, **B**, **C**, **D**, and **E***

First check Non-Singlet vs full evolution.

Evolve F_2 from MRST PDFs from Q^2 = 25 to 1 GeV^2 using both **N-S** and full (**N-S** + **Singlet**).



Largest difference for n=2 ***moments***

~4% effect

Higher order (higher n) moments dominated by larger *x* (smaller W) regime

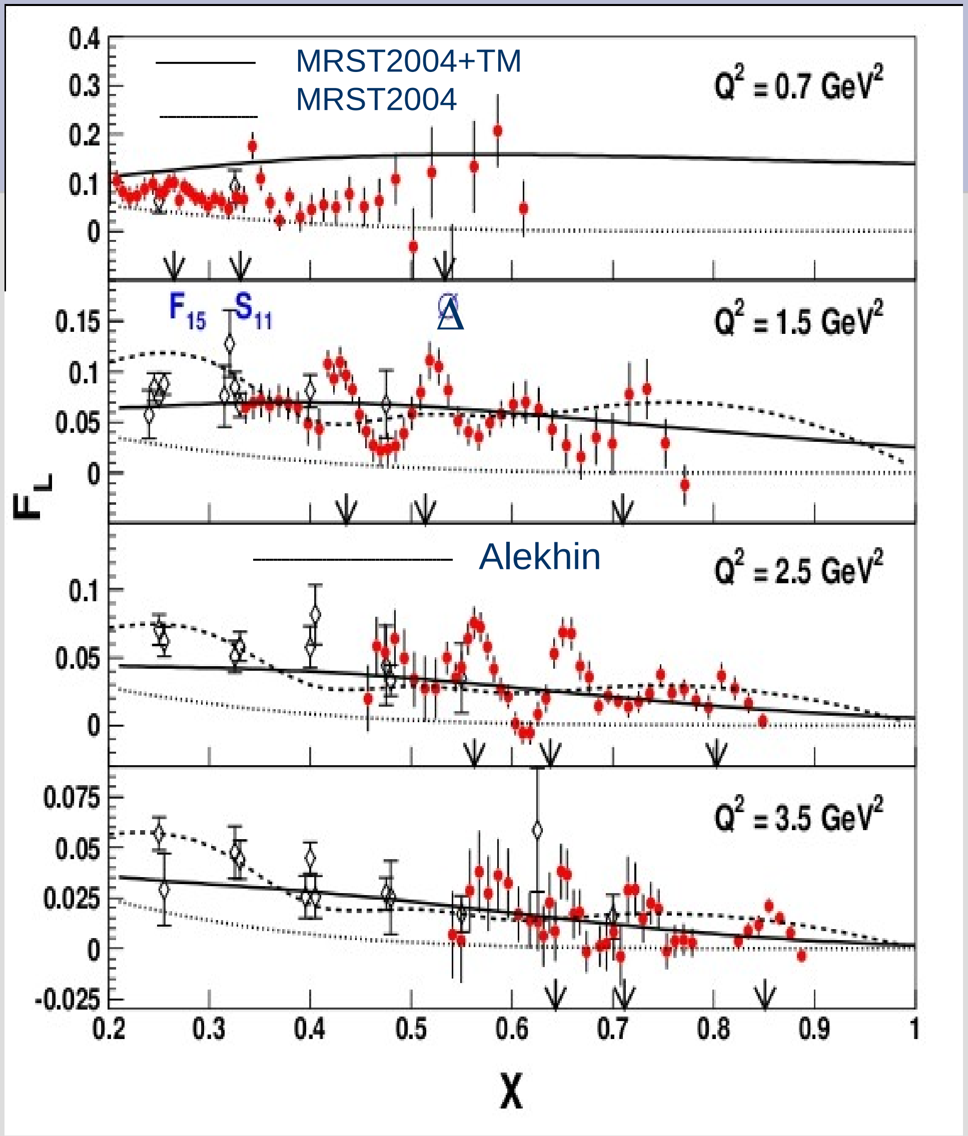
Recall - high W corresponds to low x - glue increasingly more important.
Becomes dominant uncertainty.

E94-110: proton F_L in resonance region

→ First observation of quark-hadron duality in F_L.

→ TM corrections are critical
Component of scaling function.

- Duality is considerably broken for $Q^2 < 4$ without this contribution



To compared Data to QCD Moments using PDFs, must correct for known TM effects

In massless limit only operators with spin = n contributes to n^{th} Cornwall-Norton (CN) moments,

$$M_2^{(n)}(Q^2) = \int dx\, x^{n-2} F_2^{(0)}(x)$$

$F_2^{(0)}$ Massless limit SF

This is not true for finite M^2/Q^2. However,

$$\mu_2^n(Q^2) = \int_0^1 dx \frac{\xi^{n+1}}{x^3}\left[\frac{3+3(n+1)r+n(n+2)r^2}{(n+2)(n+3)}\right] F_2^{TMC}(x, Q^2)$$

projects out pure spin-n contribution – *Nachtmann (1973)*

Here F_2^{TMC} is the *experimental* structure functions.

For consistency, it should be true that $\mu_2^n(Q^2) = \int dx\, x^{n-2} F_2^{(0)}(x)$