Pion Valence Quark Distribution from Lattice Calculable Current-Current Correlations

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In Collaboration with R. Sufian, J. Karpie, K. Orginos, J. Qiu & D. Richards

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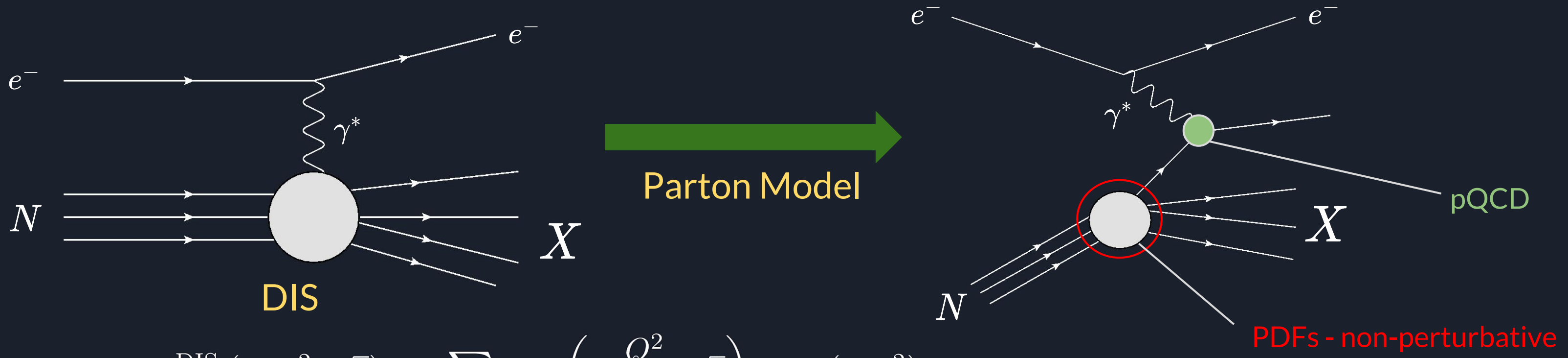
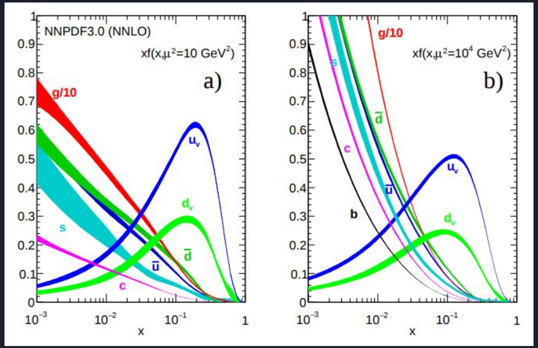
An Essential Window into Hadronic Structure



$$\sigma^{\text{DIS}}\left(x, Q^2, \sqrt{s}\right) = \sum_{a=q,\bar{q},g} C_a\left(x, \frac{Q^2}{\mu^2}, \sqrt{s}\right) \otimes f_a\left(x, \mu^2\right) + power\ corrections$$

➔ Essential for interpretation of high-energy scattering data & BSM searches at energy frontier



M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018)

- ➔ Models not a complete picture
- ➔ Global analysis techniques not uniquely defined
- ➔ Not fully known

Outline

- ➔ An essential window
- ➔ Lattice QCD and Light-Cone Physics?
- ➔ Lattice “Cross Sections”
- ➔ The pion - an economical theater
- ➔ Numerical Investigation
  - ◆ Ensemble specifics
  - ◆ Pion valence distribution
  - ◆ Results and comparison with literature
- ➔ Progress & Outlook

A Non-perturbative Study of QCD

➔ Low-energies & a strong coupling

$$\mathcal{L}_{\mathrm{QCD}} = \sum_f \overline{\psi}_f \left(i\not{D} - m_f \right) \psi_f - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu a}$$

➔ Real time path integral

$$\langle \hat{\mathcal{O}} \rangle = \mathcal{Z}^{-1} \int \mathcal{D}\psi \mathcal{D}\overline{\psi} \mathcal{D}G_\mu \hat{\mathcal{O}} \left[\psi, \overline{\psi}, G_\mu \right] e^{iS_{\mathrm{QCD}}[\psi, \overline{\psi}, G_\mu]}$$

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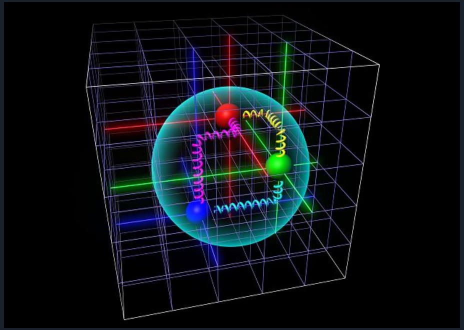
Uncountably infinite parameter space

➔ Lattice QCD → discretize and Wick rotate to imaginary time

$$\langle \hat{\mathcal{O}} \rangle_{\mathrm{E}} = \mathcal{Z}^{-1} \int \mathcal{D}\psi \mathcal{D}\overline{\psi} \mathcal{D}U \hat{\mathcal{O}} \left[\psi, \overline{\psi}, U \right] e^{-S^{\mathrm{E}}_{\mathrm{QCD}}\left[\psi,\overline{\psi},U\right]}$$

$$\mathcal{D}U = \prod_{n \in \Lambda} \prod_{\mu=1}^{4} \mathrm{dU}_\mu(\mathrm{n})$$

$$\mathcal{D}\psi\mathcal{D}\overline{\psi} = \prod_{n \in \Lambda} \prod_{f,\alpha,c} \mathrm{d}\psi_{\mathrm{f}}(\mathrm{n})^{\mathrm{c}}_\alpha \, \mathrm{d}\overline{\psi}_{\mathrm{f}}(\mathrm{n})^{\mathrm{c}}_\alpha$$



➔ Importance sampling

$$C_{2\mathrm{pt}}(\vec{p}, t) = \langle 0 | h(\vec{p}, t) h^\dagger(0) | 0 \rangle$$

$$C_{3\mathrm{pt}}(\vec{p}, \vec{q}; t, \tau) = \langle 0 | h(\vec{p}, t) \mathcal{O}(\vec{q}, \tau) h^\dagger(0) | 0 \rangle$$

$$\langle \mathcal{O} \rangle \sim \frac{1}{N} \sum_n \mathcal{O}_n$$

Tremendous success in studying QCD spectrum and accessing state-to-vacuum transitions

Towards the Extraction of PDFs from the Lattice

$$f_{a/h}(x) = \int \frac{\mathrm{d}\xi^-}{4\pi} e^{-ixP^+\xi^-} \langle P| \overline{\psi}_a(\xi^-, \mathbf{0_T}) W(\xi^-, 0) \gamma^+ \psi_a(0) |P\rangle$$

- ➔ All light-cone physics is lost in a Euclidean spacetime

$$g^{\mu\nu} = \mathrm{diag}(-1,-1,-1,-1) \quad \Longrightarrow \quad z^2 = 0 \ \text{?!}$$

- ➔ Moments calculations & the OPE
  - ◆ Power-divergent mixing/gauge noise for high moments
- ➔ Led to quasi/pseudo-PDF proposals

$$\mathcal{P}(x, z^2) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathrm{d}\omega e^{-ix\omega} \mathcal{M}(\omega, z^2)$$

K. Orginos, A. Radyushkin, J. Karpie, and S. Zafeiropoulos, Phys. Rev. D96, 094503 (2017), arXiv:1706.05373 [hep-ph]

$$\langle p| \overline{\psi}(z) \gamma^\alpha W(z) \psi(0) |p\rangle = 2p^\alpha \mathcal{M}_{\mathrm{p}}(\omega, z^2) + z^\alpha \mathcal{M}_{\mathrm{z}}(\omega, z^2)$$

$$\tilde{q}(x, P_3) = \int_{-\infty}^{\infty} \frac{\mathrm{d}z}{4\pi} e^{-izxP_3} \langle P| \overline{\psi}(z) \gamma^0 W(z) \psi(0) |P\rangle$$

X. Ji, Phys. Rev. Lett. 110, 262002 (2013), arXiv:1305.1539 [hep-ph]

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➢ Beyond quasi-/pseudo-PDF approaches

- Hadronic Tensor (K. F. Liu et al., Phys. Rev. Lett. 72, Phys. Rev. D 59 & D 62)
- Auxiliary quark methods (U. Aglietti et al., Phys. Lett. B441; W. Detmold & C.J. D. Lin, Phys. Rev. D73; V. Braun & D. Mueller, Eur. Phys. J. C55)
- “OPE without OPE” (A. J. Chambers et al., Phys. Rev. Lett. 118)

Lattice "Cross Sections" (LCS)

Y. Q. Ma & J. W. Qiu, Phys. Rev. D 98, no. 7, 074021 (2018), arXiv:1404.6860 [hep-ph]
Y. Q. Ma & J. W. Qiu, Phys. Rev. Lett. 120, no. 2, 022003 (2018), arXiv:1709.03018 [hep-ph]

- Single-hadron matrix elements of renormalized non-local ops.

$$\sigma_{ij}^{\mu\nu}(\xi,p) = \langle h(p)|\,\mathcal{O}_{ij}^{\mu\nu}(\xi)\,|h(p)\rangle = \xi^4 \langle h(p)|\,\mathcal{J}_i^{\mu}(\xi/2)\,\mathcal{J}_j^{\nu}(-\xi/2)\,|h(p)\rangle$$

- Defining properties:
  - calculable in LQCD with Euclidean time
  - well-defined continuum limit; UV finite
  - share same collinear div. w/ PDFs

$$p \sim \sqrt{s}$$

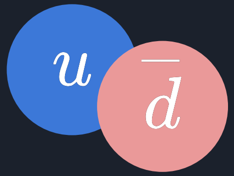
$$\xi^2 \sim \frac{1}{Q^2}$$

$$T_i\left(\omega,\xi^2\right) = \sum_{a=q,\bar{q},g} \int_{-1}^{1} \frac{\mathrm{d}x}{x}\, f_a\left(x,\mu^2\right) C_i^a\left(x\omega,\xi^2,\mu^2\right) + \mathcal{O}\left(\xi^2\Lambda_{\mathrm{QCD}}^2\right)$$

Pseudo-structure Functions — PDFs — Hard Coefficients

valid for any finite $\{\omega, p^2\xi^2\}$ provided $\left|\vec{\xi}\right| \ll \Lambda_{\mathrm{QCD}}^{-1}$

Pion Structure - A First Numerical LCS Study



- Chiefly from pionic Drell-Yan
  - E.g. J. S. Conway et al., Phys. Rev. D 39, 39 (1989)
  - J. Badier et all., Z. Phys. C18, 281 (1983)   B. Betev et al., Z. Phys. C28, 9 (1985)

- Importance of pion cannot be understated
  - DCSB and long-range $N - N$ interaction
  - nucleon quark sea flavor asymmetry

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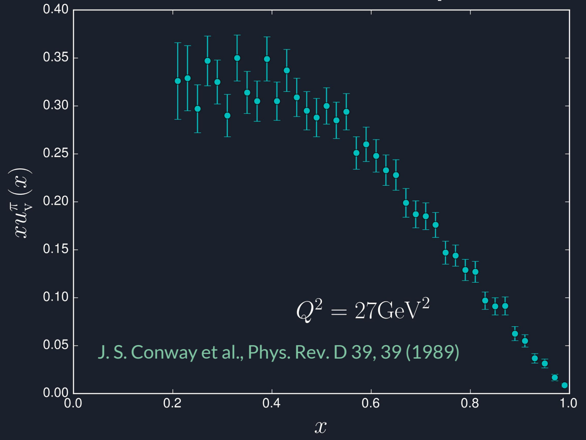
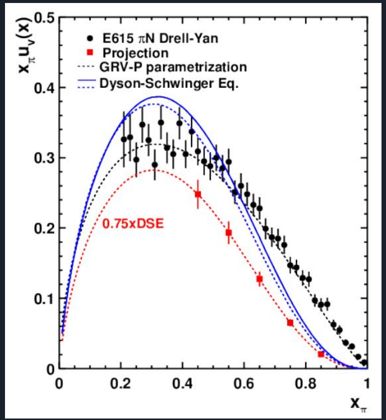
$\lim_{x\to 1} q_{\mathrm{v}}^{\pi}(x)$?

$(1-x)$ $(1-x)^2$



LO Analysis Conflicts with Expectations



PR12-15-006: TDIS @ JLab
(pion structure via Sullivan process)

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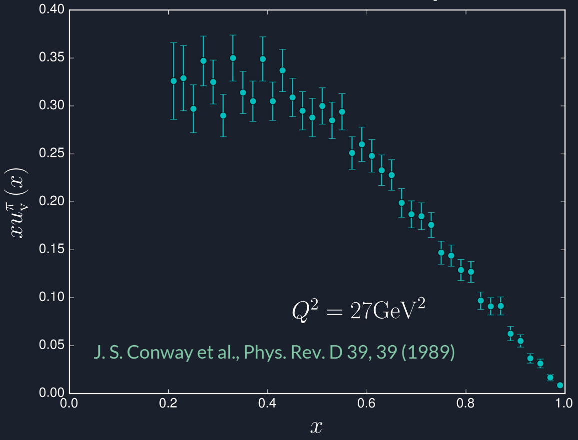
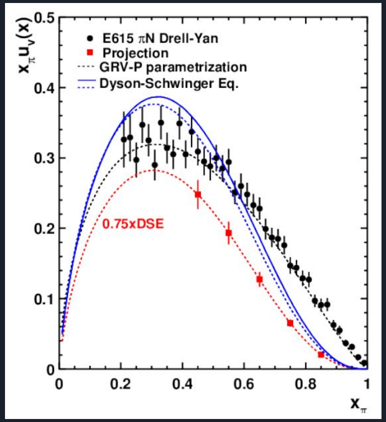


LO Analysis Conflicts with Expectations

Can a LCS calculation serve as a discriminator?



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Many Wick Contractions

➔ 4pt function with light-quark currents

$$\mathcal{J}_i = \overline{q}\Gamma_i q'$$

$$C_{4\text{pt}}\left(\xi, p, T, t\right) = \left\langle \Pi_p\left(\vec{z}, T\right) \mathcal{J}_{\Gamma'}\left(x_o + \xi, t\right) \mathcal{J}_{\Gamma}\left(x_0, t\right) \overline{\Pi}_p\left(\vec{y}, 0\right)\right\rangle$$

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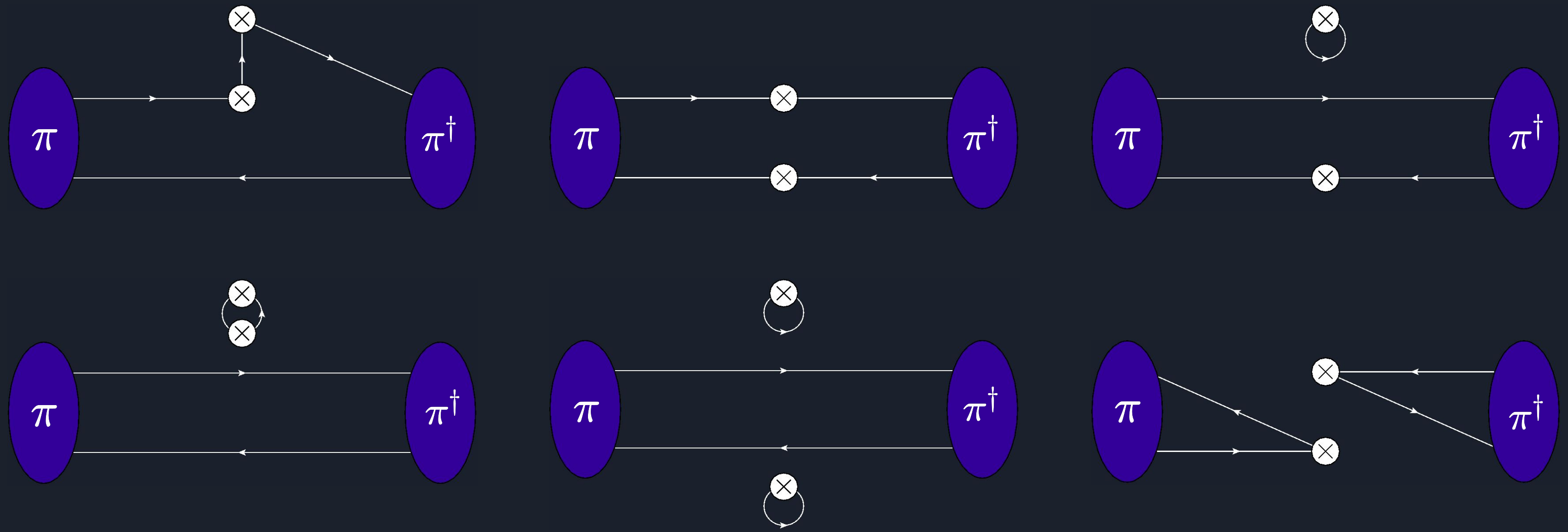
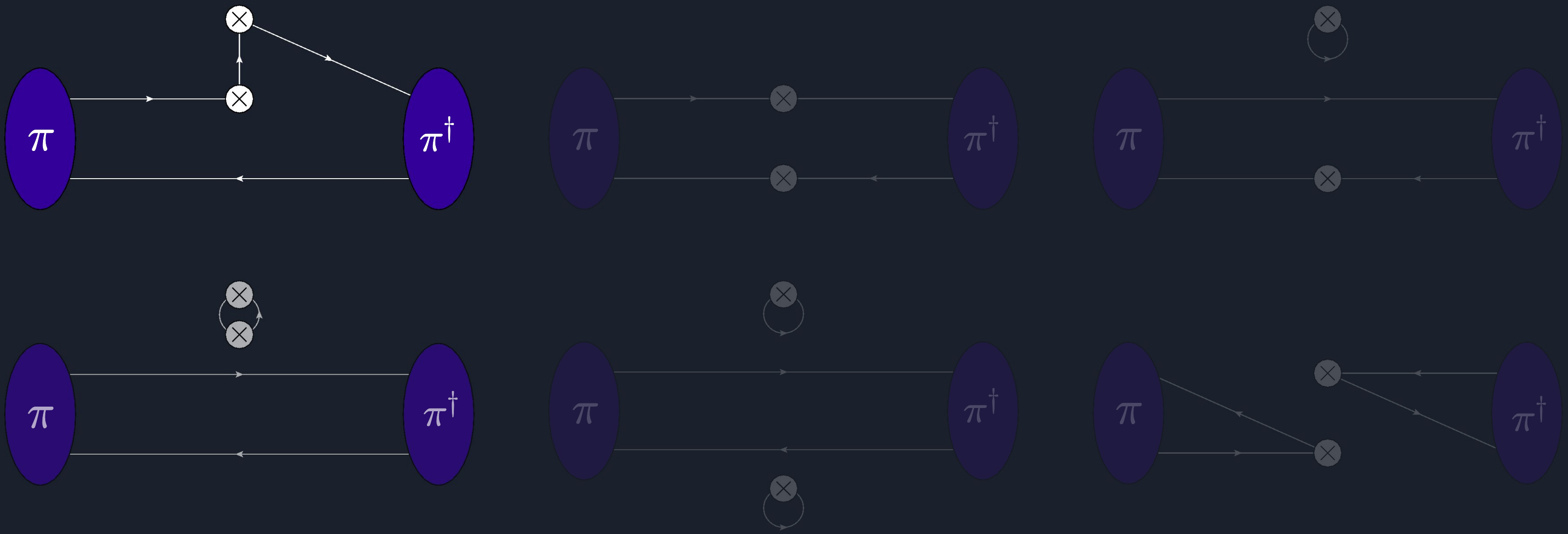


Many Wick Contractions

➔ Heavy-light currents ⋯⋯▷ $\mathcal{J}_i = \{\bar{q}\Gamma_i Q, \overline{Q}\Gamma_i q\}$

- sufficiently small ξ^2 and leading-twist contribution to PDF

W. Detmold & C.J. D. Lin, Phys.Rev. D73 (2006) 014501 hep-lat/0507007



Currents to Consider

- ➔ Vector/axial currents $\in$ pseudo-scalar hadrons

$$(\mathcal{PT})\,\mathcal{J}_{\mathrm{V}}^{\mu}(\xi)\,(\mathcal{PT})^{-1} = \mathcal{J}_{\mathrm{V}}^{\mu}(-\xi)$$
$$(\mathcal{PT})\,\mathcal{J}_{\mathrm{A}}^{\mu}(\xi)\,(\mathcal{PT})^{-1} = -\mathcal{J}_{\mathrm{A}}^{\mu}(-\xi)$$

$$\frac{1}{2}\left[\sigma_{\mathrm{VA}}^{\mu\nu}(\xi,p) + \sigma_{\mathrm{AV}}^{\mu\nu}(\xi,p)\right] = \epsilon^{\mu\nu\alpha\beta}\xi_\alpha p_\beta T_1\left(\omega,\xi^2\right) + \left(p^\mu\xi^\nu - \xi^\mu p^\nu\right)T_2\left(\omega,\xi^2\right)$$

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$$\frac{1}{2}\left[\sigma_{\mathrm{VA}}^{\mu\nu}(\xi,p)+\sigma_{\mathrm{AV}}^{\mu\nu}(\xi,p)\right]=\epsilon^{\mu\nu\alpha\beta}\xi_{\alpha}p_{\beta}T_{1}\left(\omega,\xi^{2}\right)+\left(p^{\mu}\xi^{\nu}-\xi^{\mu}p^{\nu}\right)T_{2}\left(\omega,\xi^{2}\right)$$

- Isolation of pseudo-structure functions

$$T_{1}\left(\omega,\xi^{2}\right)=\frac{\epsilon_{\mu\nu\alpha\beta}\xi^{\alpha}p^{\beta}}{2\left(\omega^{2}-p^{2}\xi^{2}\right)}\frac{1}{2}\left[\sigma_{VA}^{\mu\nu}(\xi,p)+\sigma_{AV}^{\mu\nu}(\xi,p)\right]$$

$$T_{2}\left(\omega,\xi^{2}\right)=\frac{\left(\xi_{\mu}p_{\nu}-p_{\mu}\xi_{\nu}\right)}{2\left(\omega^{2}-p^{2}\xi^{2}\right)}\frac{1}{2}\left[\sigma_{VA}^{\mu\nu}(\xi,p)+\sigma_{AV}^{\mu\nu}(\xi,p)\right]$$

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➔ Selecting: $p^{\mu} = \left(p^0, 0, 0, p^3\right)$ $\xi^{\mu} = \left(0, 0, 0, \xi^3\right)$ and $\{\mu = 1, \nu = 2\}$

$$T_1\left(\omega,\xi^2\right) = \frac{1}{p^0\xi^3}\frac{1}{2}\left[\sigma^{12}_{\mathrm{VA}}(\xi,p) + \sigma^{12}_{\mathrm{AV}}(\xi,p)\right]$$

Matching Kernels

➔ Factorized relation → asymptotic parton state → order-by-order in α_s

$$T_i^q\left(\omega,\xi^2\right)=\sum_a\int_{-1}^{1}\frac{\mathrm{d}x}{x}f_a^q\left(x,\mu^2\right)C_i^a\left(x\omega,\xi^2,\mu^2\right)$$

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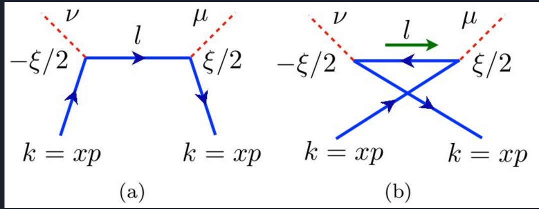

$$\begin{aligned}\mathcal{M}_{ij}^{(a)}&=\frac{\xi^4}{2}\sum\langle 0\mid\bar{u}_s(k)\,e^{ik\cdot\xi/2}\Gamma_i^\mu\psi(\xi/2)\,\overline{\psi}(-\xi/2)\,\Gamma_j^\nu e^{ik\cdot\xi/2}u_s(k)\mid 0\rangle\\&=\frac{\xi^4}{2}e^{ik\cdot\xi}\mathrm{Tr}\left[(\gamma\cdot k)\,\Gamma_i^\mu\int\frac{d^4l}{(2\pi)^4}\frac{i\gamma\cdot l}{l^2+i\epsilon}e^{-il\cdot\xi}\;\Gamma_j^\nu\right]\end{aligned}$$

⋮

$$\sigma_{\mathrm{VA}}^{\mu\nu(0)}+\sigma_{\mathrm{AV}}^{\mu\nu(0)}=\frac{1}{\pi^2}x\epsilon^{\mu\nu\alpha\beta}\xi_\alpha p_\beta\left(e^{ix\omega}+e^{-ix\omega}\right)$$

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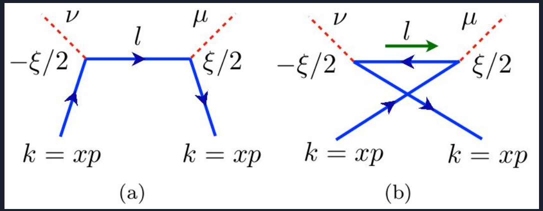
$$f_a^{q(0)}\left(x,\mu^2\right)=\delta(1-x)\delta^{qa}$$



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$$=\frac{\xi^4}{2}e^{ik\cdot\xi}\mathrm{Tr}\left[(\gamma\cdot k)\,\Gamma_i^{\mu}\int\frac{d^4l}{(2\pi)^4}\frac{i\gamma\cdot l}{l^2+i\epsilon}e^{-il\cdot\xi}\;\Gamma_j^{\nu}\right]$$

⋮

$$\sigma_{\mathrm{VA}}^{\mu\nu(0)}+\sigma_{\mathrm{AV}}^{\mu\nu(0)}=\frac{1}{\pi^2}x\epsilon^{\mu\nu\alpha\beta}\xi_\alpha p_\beta\left(e^{ix\omega}+e^{-ix\omega}\right)$$

$$T_i^{q(0)}\left(\omega,\xi^2\right)=K_i^{q(0)}\left(\omega,\xi^2\right)$$

$$T_1^{(0)}\left(\omega,\xi^2\right)\propto\left(e^{ix\omega}+e^{-ix\omega}\right)\qquad T_2^{(0)}\left(\omega,\xi^2\right)=0$$

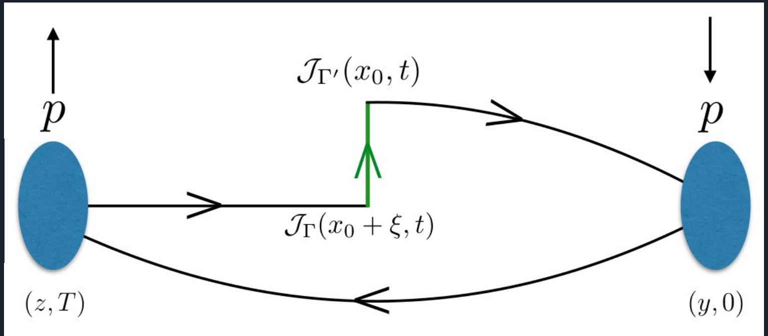
The Essence of the Calculation

➔ Randomly chosen source point (x_0, t)

$$G(\vec{y}, t'; x_0, t)^{ab}_{\alpha\beta} = \sum_{\gamma, c, \vec{x}} D^{-1}(\vec{y}, t'; \vec{x}, t)^{ac}_{\alpha\gamma} \delta^{cb} \delta_{\gamma\beta} \delta(\vec{x} - x_0)$$

➔ Momentum projections and src/snk seps.

- ...costly inversions



$$C_{4\text{pt}}(\xi, p, T, t) = \langle \Pi_p(\vec{z}, T) \mathcal{J}_{\Gamma'}(x_o + \xi, t) \mathcal{J}_{\Gamma}(x_0, t) \overline{\Pi}_p(\vec{y}, 0) \rangle$$

$$= \sum_{\vec{z}, \vec{z}} e^{-i(\vec{z} - \vec{z}) \cdot \vec{p}} \langle \bar{\tilde{d}} \gamma^5 \tilde{u}(\vec{z}, T) \bar{Q} \Gamma' u(x_0 + \xi, t) \bar{u} \Gamma Q(x_0, t) \bar{\tilde{u}} \gamma^5 \tilde{d}(\vec{y}, 0) \rangle$$

$$= \text{Tr}\left[I^p_q(x_0 + \xi, t; x_0, t) \Gamma' \gamma^5 G_Q(x_0 + \xi, t; x_0, t)^\dagger \gamma^5 \Gamma \right]$$

➢ Unrestricted energy flow

➢ Heavy auxiliary quark propagators

W. Detmold & C.J. D. Lin, Phys.Rev. D73 (2006) 014501 hep-lat/0507007

High-momenta essential

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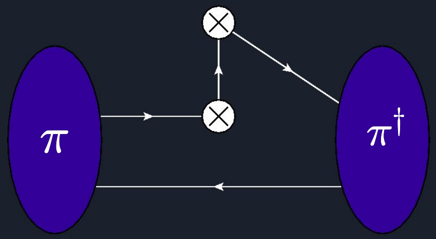

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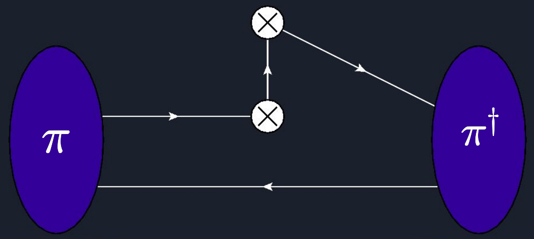


$$C_{4\mathrm{pt}}\left(\xi, p, T, t\right) = \langle \Pi_p\left(\vec{z}, T\right) \mathcal{J}_{\Gamma'}\left(x_o + \xi, t\right) \mathcal{J}_{\Gamma}\left(x_0, t\right) \overline{\Pi}_p\left(\vec{y}, 0\right) \rangle$$

$$= \sum_{\vec{z}, \vec{z}} e^{-i(\vec{z}-\vec{z})\cdot\vec{p}} \langle \bar{\tilde{d}} \gamma^5 \tilde{u}\left(\vec{z}, T\right) \bar{Q} \Gamma' u\left(x_0 + \xi, t\right) \bar{u} \Gamma Q\left(x_0, t\right) \bar{\tilde{u}} \gamma^5 \tilde{d}\left(\vec{y}, 0\right) \rangle$$

$$= \mathrm{Tr}\left[I^p_q\left(x_0 + \xi, t; x_0, t\right) \Gamma' \gamma^5 G_Q\left(x_0 + \xi, t; x_0, t\right)^\dagger \gamma^5 \Gamma \right]$$

➢ Unrestricted energy flow

➢ Heavy auxiliary quark propagators

W. Detmold & C.J. D. Lin, Phys.Rev. D73 (2006) 014501 hep-lat/0507007

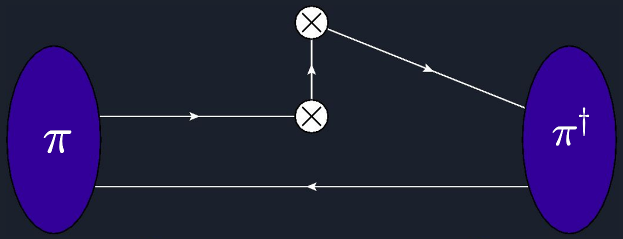
High-momenta essential

The Essence of the Calculation

- ➔ Randomly chosen source point (x_0, t)

$$G\left(\vec{y}, t'; x_0, t\right)^{ab}_{\alpha\beta} = \sum_{\gamma,c,\vec{x}} D^{-1}\left(\vec{y}, t'; \vec{x}, t\right)^{ac}_{\alpha\gamma} \delta^{cb}\delta_{\gamma\beta}\delta\left(\vec{x} - x_0\right)$$

- ➔ Momentum projections and src/snk seps.
  - ◆ ...costly inversions



$$C_{4\text{pt}}\left(\xi, p, T, t\right) = \langle \Pi_p\left(\vec{z}, T\right) \mathcal{J}_{\Gamma'}\left(x_o + \xi, t\right) \mathcal{J}_{\Gamma}\left(x_0, t\right) \overline{\Pi}_p\left(\vec{y}, 0\right)\rangle$$

$$= \sum_{\vec{z},\vec{z}} e^{-i(\vec{z}-\vec{z})\cdot\vec{p}} \langle \bar{\tilde{d}}\gamma^5\tilde{u}\left(\vec{z}, T\right) \bar{Q}\Gamma' u\left(x_0 + \xi, t\right) \bar{u}\Gamma Q\left(x_0, t\right) \bar{\tilde{u}}\gamma^5\tilde{d}\left(\vec{y}, 0\right)\rangle$$

$$= \text{Tr}\left[I^p_q\left(x_0 + \xi, t; x_0, t\right) \Gamma'\gamma^5 G_Q\left(x_0 + \xi, t; x_0, t\right)^\dagger \gamma^5\Gamma\right]$$

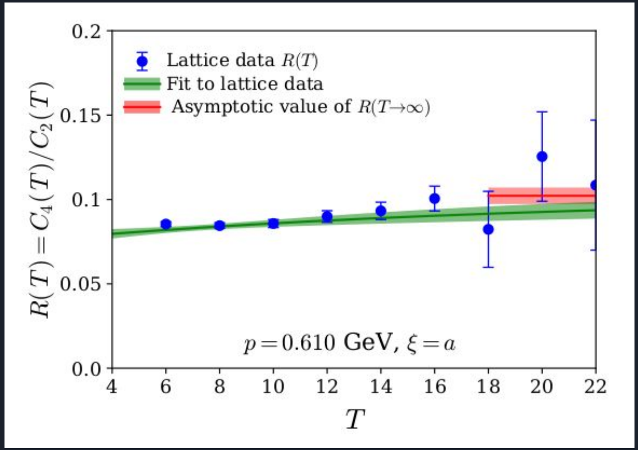
- ➢ Unrestricted energy flow
- ➢ Heavy auxiliary quark propagators

W. Detmold & C.J. D. Lin, Phys.Rev. D73 (2006) 014501 hep-lat/0507007

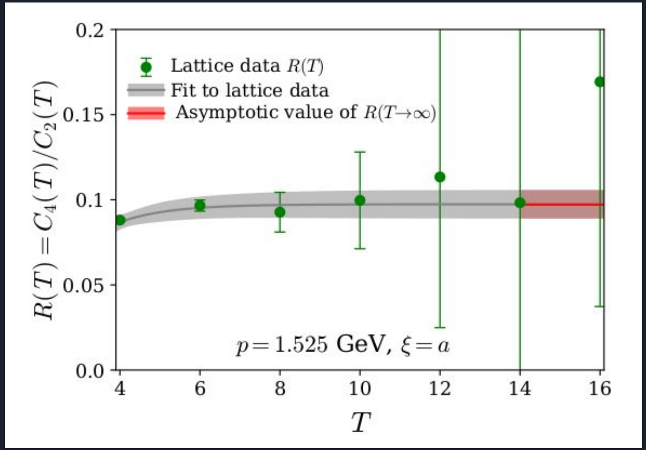
High-momenta essential

Ensemble & Typical Ratio Fits

Label	Lattice Spacing (a)	Pion Mass (m_π)	Lattice Dimensions
E1	0.127 fm	416 MeV	$32^3 \times 96$
E2	0.127 fm	416 MeV	$24^3 \times 64$
E3	0.09 fm	270 MeV	$32^3 \times 64$



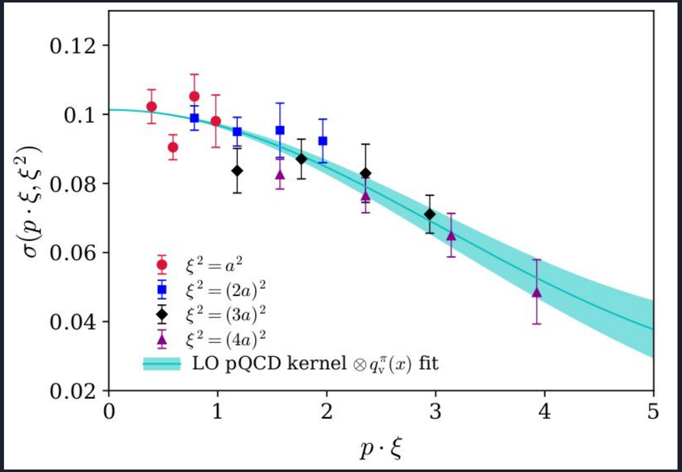
$$\frac{C_{4\mathrm{pt}}(T)}{C_{2\mathrm{pt}}(T)} = A + Be^{-\Delta E_{eff} T}$$



$$C_{2\mathrm{pt}}(p,T) = \langle \Pi_p(T)\,\overline{\Pi}_p(0)\rangle = \sum_n \frac{|Z_n|^2}{2E_n(p)} e^{-E_n(p)T}$$

$$C_{4\mathrm{pt}}(\xi,p,T,\tau) = \langle \Pi_p(\vec{z},T)\,\mathcal{J}_{\Gamma'}(\vec{x}_0+\vec{\xi},\tau)\mathcal{J}_{\Gamma}(\vec{x}_0,\tau)\overline{\Pi}_p(\vec{y},0)\rangle = \sum_{n',n} \frac{Z_{n'}(p')\,Z_n(p)^*}{4E_{n'}(p')\,E_n(p)} \langle n',p'|\,\mathcal{J}_2(\vec{x}_0+\vec{\xi})\mathcal{J}_1(\vec{x}_0)\,|n,p\rangle\, e^{-E_{n'}(p')T} e^{[E_{n'}(p')-E_n(p)]\tau}$$

The $T_1(\omega, \xi^2)$ Pseudo-structure Function



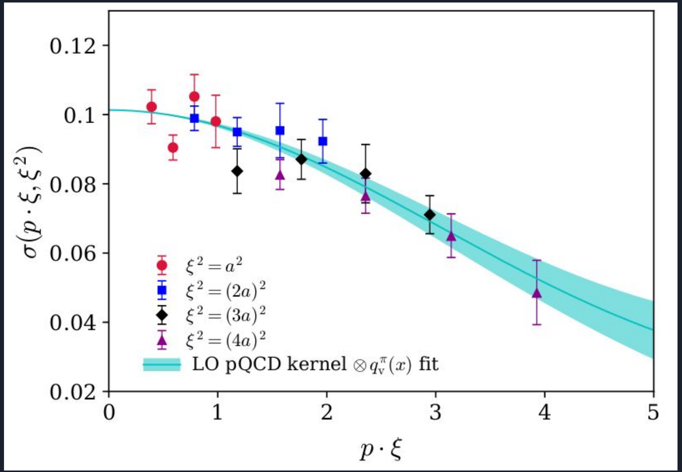
➔ How best to extract PDFs?

$$T_1^{(0)}(\omega, \xi^2) = \int_0^1 \mathrm{d}x \frac{1}{\pi^2} \cos(x\omega)\, q_V^\pi(x)$$

➢ An ill-posed inverse !

➢ Extra information needed

The $T_1\left(\omega,\xi^2\right)$ Pseudo-structure Function



- ➔ How best to extract PDFs?

$$T_1^{(0)}\left(\omega,\xi^2\right)=\int_0^1 \mathrm{d}x\,\frac{1}{\pi^2}\cos\left(x\omega\right)q_{\mathrm{V}}^{\pi}\left(x\right)$$

  - ➢ An ill-posed inverse !
  - ➢ Extra information needed

- ➔ Smooth, physically-motivated models

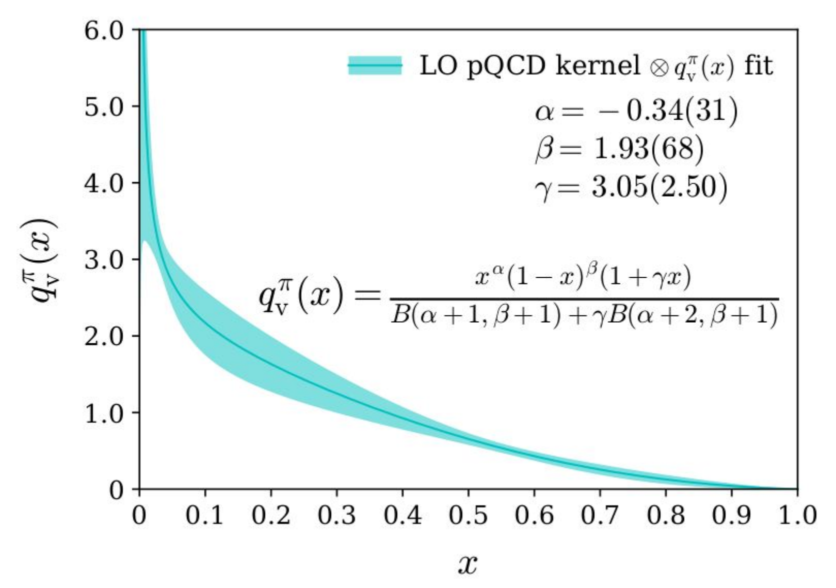
$$q_{\mathrm{v}}^{\pi}\left(x\right)=Nx^{\alpha}\left(1-x\right)^{\beta}\left(1+\rho\sqrt{x}+\gamma x\right)$$

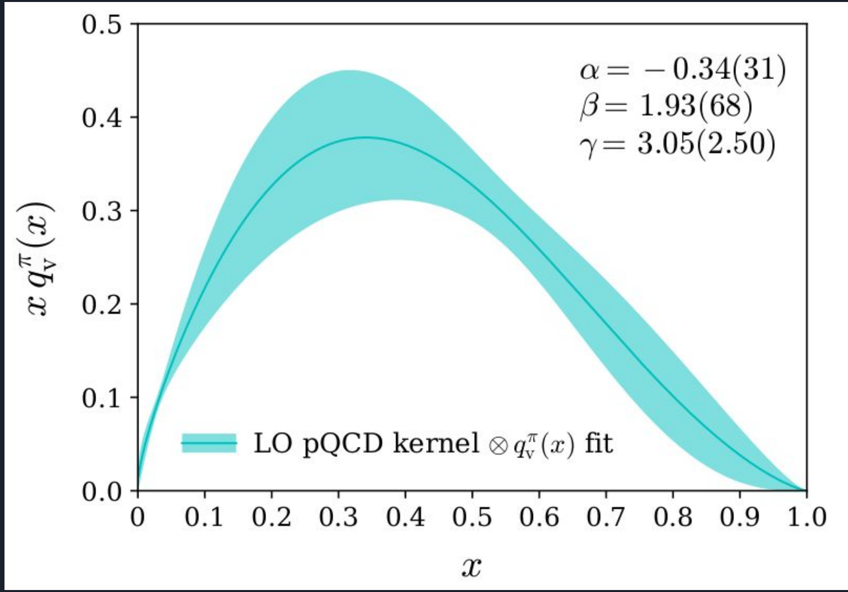
$$\int_0^1 \mathrm{d}x q_{\mathrm{v}}^{\pi}\left(x\right)=1$$

- ➔ ROOT for numerical integration

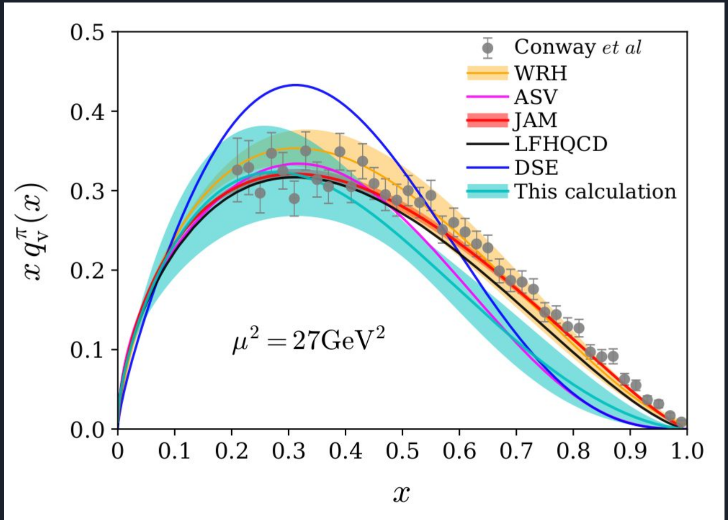
Rene Brun & Fons Rademakers, Nucl. Inst. & Meth. in Phys. Res. A 389 (1997) 81-86

Extracted Valence PDF





Comparison with the Literature



R. Sufian, J. Karpie, *CE* et al., (2019) arXiv:1901.03921 [hep-lat]
To appear in PRD

- ➔ μ_0^2 commensurate $1/\xi^2$'s $\sim 1\ \text{GeV}^2$
- ➔ consistent w/ NLL threshold soft-gluon resummation effects (ASV)
- ➔ favors $\lim_{x\to 1} (1-x)^2$
- ➔ no statistically meaningful change in distribution as μ_0^2 varied

➔ Appear to be capturing essential physics --- gluons esp. important in pion

P. C. Barry et al., Phys. Rev. Lett. 121, 152001 (2018), arXiv:1804.01965 [hep-ph]

In Progress

- ➔ Populating $T_1\left(\omega, \xi^2\right)$ with off-axis separations
  - $\vec{\xi} \perp \vec{p}$ maintaining $\omega \neq 0$
  - improving $C_{4\mathrm{pt}}$ statistics → Momentum projections as a time-slice average

Discretization effects...

G. S. Bali et al., Phys. Rev. D 98, no. 9, 094507 (2018), arXiv:1807.06671 [hep-lat]
G. S. Bali et al., Eur. Phys. J. C78 (2018) no. 3, 217, arXiv:1709.04325 [hep-lat]

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Momentum projections as a time-slice average

- ➔ Completing isolation of NLO kernels
  - logarithmic corrections (ξ and μ_0 related)
  - power corrections & higher-twist effects

In Progress

- ➔ Populating $T_1\,(\omega, \xi^2)$ with off-axis separations
  - ◆ $\vec{\xi} \perp \vec{p}$ maintaining $\omega \neq 0$
  - ◆ improving $C_{4\mathrm{pt}}$ statistics

Discretization effects...

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Momentum projections as a time-slice average

- ➔ Completing isolation of NLO kernels
  - ◆ logarithmic corrections (ξ and μ_0 related)
  - ◆ power corrections & higher-twist effects
- ➔ Finite volume effects?

R. Briceño et al., Phys. Rev. D 98, no. 1, 014511 (2018), arXiv:1805.01034 [hep-lat]

Label	Lattice Spacing (a)	Pion Mass (m_π)	Lattice Dimensions
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E2	0.127 fm	416 MeV	$24^3 \times 64$
E3	0.09 fm	270 MeV	$32^3 \times 64$

- ➔ Power of LCS formalism lies in a “global” analysis of calculated matrix elements
  - ◆ analyzing $\{\gamma_\mu - \gamma_\mu, \gamma_5 - \gamma_5\}$
  - ◆ Can LCS data be competitive enough to include in global analyses?

BACKUPS

More on quasi/pseudo-distributions

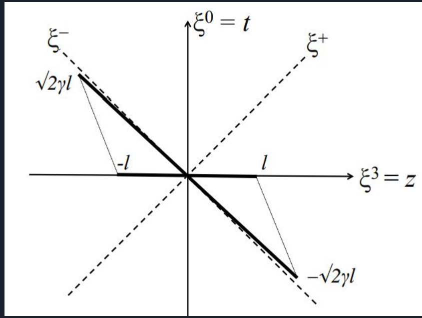
$$\tilde{q}(x, P_3) = \int_{-\infty}^{\infty} \frac{\mathrm{d}z}{4\pi} e^{-izxP_3} \langle P | \overline{\psi}(z) \gamma^0 W(z) \psi(0) | P \rangle$$

Fourier transform over length of Wilson line

$$\tilde{q}(x, \mu^2, P_3) = \int_{-1}^{1} \frac{\mathrm{d}y}{|y|} C\left(\frac{x}{y}, \frac{\mu}{P_3}\right) q(y, \mu^2) + \mathcal{O}\left(\Lambda_{\mathrm{QCD}}^2 / P_3^2, M_N^2 / P_3^2\right)$$

X. Ji, Phys. Rev. Lett. 110, 262002 (2013), arXiv:1305.1539 [hep-ph]

$$\mathcal{P}(x, z^2) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathrm{d}\omega e^{-ix\omega} \mathcal{M}(\omega, z^2)$$



K. Cichy & M. Constantinou, arXiv:1811.07248v1 [hep-lat]

Fourier transform Ioffe time - fixed length Wilson line

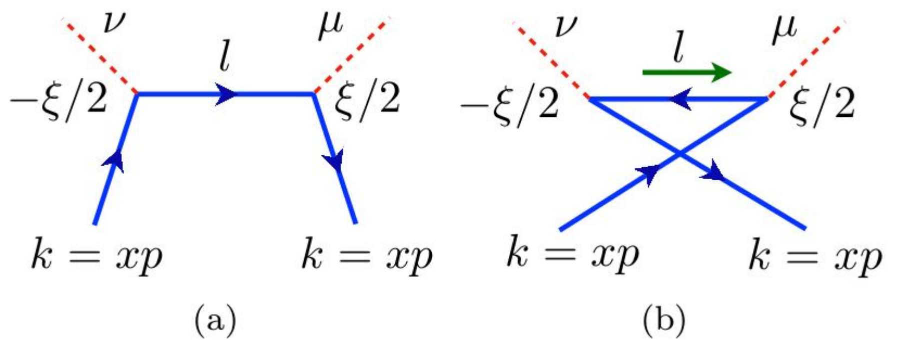
K. Orginos, A. Radyushkin, J. Karpie, and S. Zafeiropoulos, Phys. Rev. D96, 094503 (2017), arXiv:1706.05373 [hep-ph]

Leading Order Coefficient Functions - Coordinate Space

$$\mathcal{M}_{ij}^{(a)} = \frac{\xi^4}{2}\sum_s \langle 0 \mid \overline{u}_s(k)\, e^{ik\cdot\xi/2}\Gamma_i^\mu \psi(\xi/2)\,\overline{\psi}(-\xi/2)\,\Gamma_j^\nu e^{ik\cdot\xi/2} u_s(k) \mid 0\rangle$$

$$= \frac{\xi^4}{2} e^{ik\cdot\xi}\mathrm{Tr}\left[(\gamma\cdot k)\,\Gamma_i^\mu \int \frac{d^4 l}{(2\pi)^4}\frac{i\gamma\cdot l}{l^2+i\epsilon}e^{-il\cdot\xi}\;\Gamma_j^\nu\right]$$

$$\mathcal{M}_{ji}^{(b)} = \frac{\xi^4}{2} e^{-ik\cdot\xi}\mathrm{Tr}\left[(\gamma\cdot k)\,\Gamma_j^\nu \int \frac{d^4 l}{(2\pi)^4}\frac{-i\gamma\cdot l}{l^2+i\epsilon}e^{-il\cdot\xi}\;\Gamma_i^\mu\right]$$



$$\sigma_{\mathrm{VA}}^{\mu\nu(0)} + \sigma_{\mathrm{AV}}^{\mu\nu(0)} = \tfrac{1}{\pi^2} x \epsilon^{\mu\nu\alpha\beta}\xi_\alpha p_\beta \left(e^{ix\omega} + e^{-ix\omega}\right)$$

$$T_1^{(0)}\left(\omega, \xi^2\right) \propto \left(e^{ix\omega} + e^{-ix\omega}\right)$$

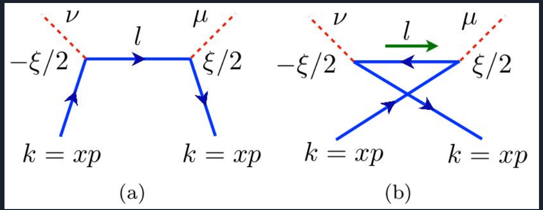
$$T_2^{(0)}\left(\omega, \xi^2\right) = 0$$

Matching Kernels & the Pion Valence Distribution

➔ Factorized relation → asymptotic parton state → order-by-order in α_s

$$T_i^q\left(\omega,\xi^2\right)=\sum_a\int_{-1}^{1}\frac{\mathrm{d}x}{x}f_a^q\left(x,\mu^2\right)C_i^a\left(x\omega,\xi^2,\mu^2\right)$$

$$f_a^{q(0)}\left(x,\mu^2\right)=\delta(1-x)\delta^{qa}$$



$$T_i^{q(0)}\left(\omega,\xi^2\right)=K_i^{q(0)}\left(\omega,\xi^2\right)$$

$$T_1^{(0)}\left(\omega,\xi^2\right)\propto\left(e^{ix\omega}+e^{-ix\omega}\right)\qquad T_2^{(0)}\left(\omega,\xi^2\right)=0$$

$$\mathcal{M}_{ij}^{(a)}=\frac{\xi^4}{2}\sum\langle 0\mid\overline{u}_s(k)\,e^{ik\cdot\xi/2}\Gamma_i^{\mu}\psi(\xi/2)\,\overline{\psi}(-\xi/2)\,\Gamma_j^{\nu}e^{ik\cdot\xi/2}u_s(k)\mid 0\rangle$$

$$=\frac{\xi^4}{2}e^{ik\cdot\xi}\mathrm{Tr}\left[(\gamma\cdot k)\,\Gamma_i^{\mu}\int\frac{d^4l}{(2\pi)^4}\frac{i\gamma\cdot l}{l^2+i\epsilon}e^{-il\cdot\xi}\;\Gamma_j^{\nu}\right]$$

⋮

$$\sigma_{\mathrm{VA}}^{\mu\nu(0)}+\sigma_{\mathrm{AV}}^{\mu\nu(0)}=\frac{1}{\pi^2}x\epsilon^{\mu\nu\alpha\beta}\xi_\alpha p_\beta\left(e^{ix\omega}+e^{-ix\omega}\right)$$

➔ To ensure access to valence distribution

- ◆ momentum space pseudo-structure fns.

$$\tilde{T}_1\left(\tilde{x},\xi^2\right)\equiv\int\frac{\mathrm{d}\omega}{2\pi}e^{-i\tilde{x}\omega}T_1\left(\omega,\xi^2\right)\propto\int\frac{\mathrm{d}\omega}{2\pi}e^{-i\tilde{x}\omega}\int_0^1\frac{\mathrm{d}x}{x}q(x)\underbrace{\left(e^{ix\omega}+e^{-ix\omega}\right)}_{\mathrm{L.O.}}$$

$$\propto\left[q(\tilde{x})+q(-\tilde{x})\right]=q_{\mathrm{v}}(\tilde{x})$$

Y. Q. Ma & J. W. Qiu, Phys. Rev. Lett. 120, no. 2, 022003 (2018), arXiv:1709.03018 [hep-ph]

Physics in a Discretized Euclidean Box

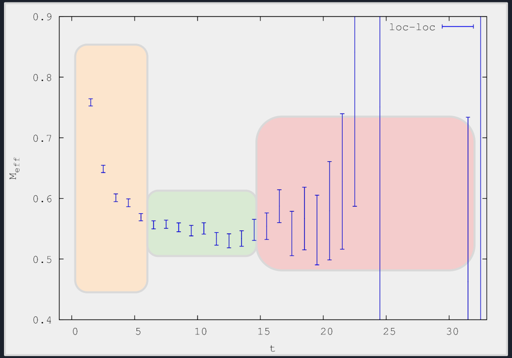
- ➔ Solution of QCD from first-principles
- ➔ "No free lunch!"
  - ◆ signal-to-noise problems

$$C(t) = \langle \mathcal{O}(t)\, \mathcal{O}^\dagger(0) \rangle \sim e^{-m_N t}$$
$$\sigma^2_{C(t)} = \langle C(t)\, C^\dagger(t) \rangle - \langle C(t) \rangle^2 \sim e^{-3 m_\pi t}$$
$$C(t) / \sigma^2_{C(t)} \sim e^{-\left(m_N - \frac{3}{2} m_\pi\right) t}$$

  - ◆ excited-state contamination
  - ◆ reduced symmetry groups

$$O(4) \longrightarrow H(4)$$

  - ◆ discretization/finite volume effects
  - ◆ quark mass dependence
- ➔ Systematics can be *systematically removed* with further computation...



Momentum Smearing

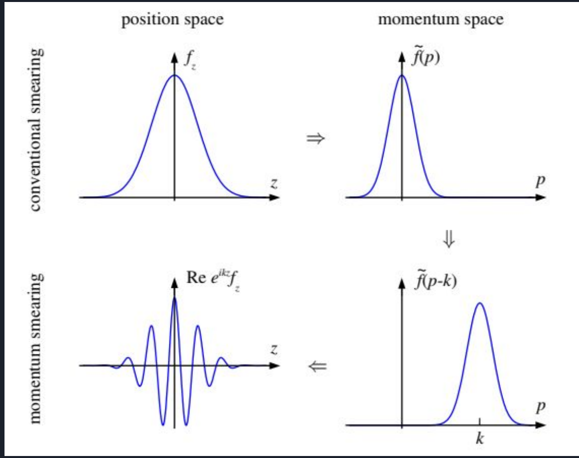
➔ Dilemma >>>>>>>> Large Ioffe time ⟶ large momentum

$$\tilde{q}(\vec{x},t) = \sum_{\vec{y}} S(\vec{x},\vec{y})\, q(\vec{y},t)$$

$$\longrightarrow \sum_{\vec{y}} \exp\left(-\frac{|\vec{x}-\vec{y}|^2}{2\sigma^2}\right) q(\vec{y},t)$$

$$\sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}}\tilde{q}(\vec{x},t) = f_0 \sum_{\vec{z}} \exp\left(-|\vec{z}|^2/2\sigma^2 + i\vec{p}\cdot\vec{z}\right) q(t)$$

$$\propto \exp\left(-\frac{\sigma^2|\vec{p}|^2}{2}\right) q(0)$$



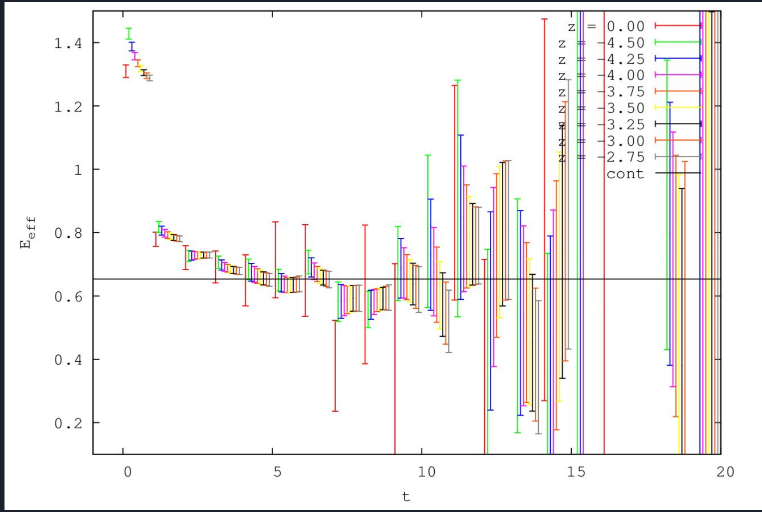
G. S. Bali et al. Phys. Rev. D93, 094515 (2016), arXiv:1602.05525 [hep-lat]

➔ Rotate gauge links $\{U_\mu[x]\}$

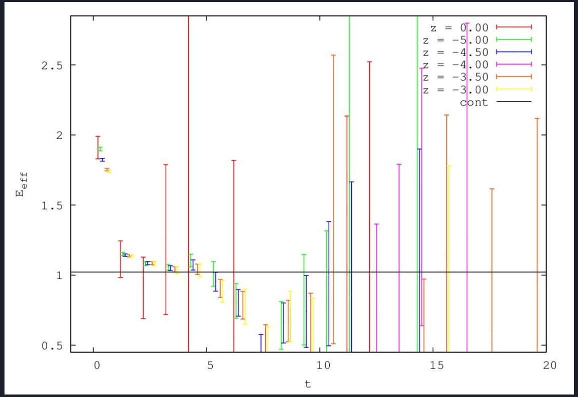
$$\tilde{U}_\mu[x] = e^{i\frac{2\pi}{L}\zeta d_\mu} U_\mu[x]$$

*** But what are these phases? ***

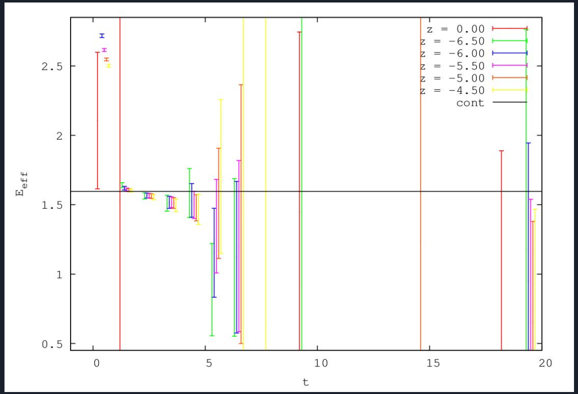
Scanning for Momentum Phases



$\vec{p} = (0, 0, 3)$

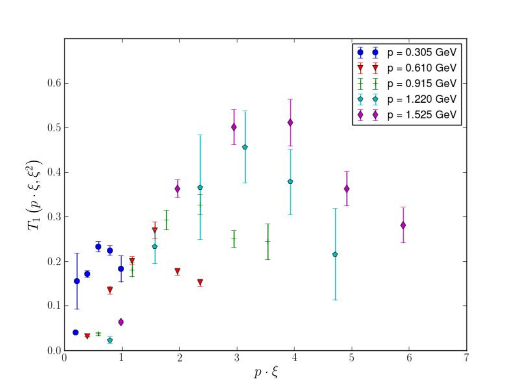


$\vec{p} = (0, 0, 5)$



$\vec{p} = (0, 0, 8)$

Preliminary E2 Pseudo-structure Functions



$$\sigma^{\mu\nu}_{\mathrm{VV}}(\xi, p) = p^\mu p^\nu T_1(\omega, \xi^2) + \frac{1}{2}(p^\mu \xi^\nu + \xi^\mu p^\nu) T_2(\omega, \xi^2) + g^{\mu\nu} T_3(\omega, \xi^2) + \xi^\mu \xi^\nu T_4(\omega, \xi^2)$$

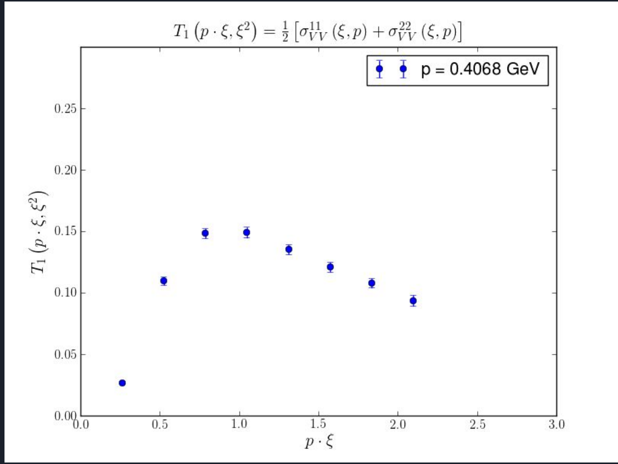
Vector-Vector of E1 ensemble

Many Combinations….

$$p^\mu = (p^0, 0, 0, p^z) \quad \xi^\mu = (0, 0, 0, \xi^z) \Rightarrow T_3(\omega, \xi^2) \propto \frac{1}{2}\left[\sigma^{11}_{\mathrm{VV}}(\xi, p) + \sigma^{22}_{\mathrm{VV}}(\xi, p)\right]$$

$$p^\mu = (p^0, 0, 0, p^z) \quad \xi^\mu = (0, \xi^x, 0, \xi^z) \Rightarrow T_3(\omega, \xi^2) \propto \sigma^{22}_{\mathrm{VV}}(\xi, p)$$

$$p^\mu = (p^0, 0, 0, p^z) \quad \xi^\mu = (0, 0, \xi^y, \xi^z) \Rightarrow T_3(\omega, \xi^2) \propto \sigma^{11}_{\mathrm{VV}}(\xi, p)$$



Euclidean vs. Minkowski Matrix Elements

R. A. Briceño et al., Phys. Rev. D 96, no. 1, 014502 (2017), arXiv:1703.06072

- Correlation fns. carry spacetime signature info
  - time-indep matrix elements do not
- quasi-PDF carries no info about spacetime signature
- spacetime signature of ops. only arise when time-evolved
  - $\hat{\mathcal{O}}(0, \{\vec{\xi}\})$ should carry no E vs. M distinction
- With no temporal extent ⟶ $\sigma_{ij}^{\mu\nu}(\xi, p)_{\rm E} = \sigma_{ij}^{\mu\nu}(\xi, p)_{\rm M}$
  - 1-loop perturbative calculation (scalar field theory)
  - all-orders derivation of LSZ reduction in Euclidean space